FRW Universe & The Hot Big Bang:
From the Friedmann equations, it is straightforward to appreciate that cosmic expansion is an adiabatic process:

$$\dot{\rho} + 3 \left( \rho + \frac{p}{c^2} \right) \frac{\dot{a}}{a} = 0$$

In other words, there is no "external power" responsible for "pumping" the tube...
Translating the adiabatic expansion into the temperature evolution of baryonic gas and radiation (photon gas), we find that they cool down as the Universe expands:

\[ p \propto \rho^\gamma \quad \implies \quad TV^{\gamma-1} = \text{cst.} \]

\[
\begin{align*}
\gamma = \frac{5}{3} & \quad \implies \quad T_b \propto V^{-2/3} \propto a^{-2} \\
\gamma = \frac{4}{3} & \quad \implies \quad T_{\text{rad}} \propto V^{-1/3} \propto a^{-1}
\end{align*}
\]
Adiabatic Expansion

Thus, as we go back in time and the volume of the Universe shrinks accordingly, the temperature of the Universe goes up. This temperature behaviour is the essence behind what we commonly denote as

Hot Big Bang

From this evolution of temperature we can thus reconstruct the detailed

Cosmic Thermal History
The Universe: the Hot Big Bang

Timeline: the Cosmic Thermal History
Equilibrium Processes

Throughout most of the universe’s history (i.e. in the early universe), various species of particles keep in (local) thermal equilibrium via interaction processes:

$$\psi_i + \psi_j \leftrightarrow \chi_k + \chi_k$$

Equilibrium as long as the interaction rate $\Gamma_{\text{int}}$ in the cosmos’ thermal bath, leading to $N_{\text{int}}$ interactions in time $t$,

$$\Gamma_{\text{int}} \implies N_{\text{int}}(t) = \int \Gamma_{\text{int}}(t) \, dt$$

is much larger than the expansion rate of the Universe, the Hubble parameter $H(t)$:

$$\Gamma_{\text{int}} \gg H(t)$$
Brief History of Time
Reconstructing Thermal History Timeline

**Strategy:**

To work out the thermal history of the Universe, one has to evaluate at each cosmic time which physical processes are still in equilibrium. Once this no longer is the case, a physically significant transition has taken place. Dependent on whether one wants a crude impression or an accurately and detailed worked out description, one may follow two approaches:

- **Crudely:**
  
  Assess transitions of particles out of equilibrium, when they decouple from thermal bath. Usually, on crude argument:

  \[ \Gamma_{int} \gg H \quad \text{to} \quad \Gamma_{int} < H \]

- **Strictly:**
  
  evolve particle distributions by integrating the Boltzmann equation
Particle interactions are mediated by gauge bosons: photons for the electromagnetic force, the $W$ bosons for weak interactions, and gluons for the strong force (and gravitons for the gravitational force). The strength of the interaction is set by the coupling constant, leading to the following dependence of the interaction rate $\Gamma$, on temperature $T$:

(i) mediated by massless gauge boson (photon):

$$\frac{\Gamma_{int}}{H} \sim \alpha^2 \frac{m_{Pl}}{T}$$

$\alpha$: coupling strength

$m_{Pl}$: Planck mass

(ii) mediated by massive gauge boson ($W^+/-, Z^0$)

$$\frac{\Gamma_{int}}{H} \sim G_x^2 \frac{m_{Pl}}{T^3}$$

$G_x$: coupling strength

$m_{Pl}$: Planck mass
On the basis of the 1) complexity of the involved physics and 2) our knowledge of the physical processes we may broadly distinguish four cosmic episodes:

(I) Planck Era

$t < 10^{-43}$ sec

fundamental physics:
- totally unknown

Origin universe

???
History of the Universe in Four Episodes: II.

(II)

$10^{-43} < t < 10^{-3}$ sec

**VERY early universe**

- **fundamental physics:**
  - poorly known
  - speculative

- **$\Omega_{\text{tot}}$:** curvature/flatness
- **$\Omega_b (n_b/n_\gamma)$**
- `exotic` dark matter
- primordial fluctuations

**Products**
History of the Universe in Four Episodes: III.

(III)

$10^{-3} < t < 10^{13} \text{ sec}$

Standard Hot Big Bang Fireball

- primordial nucleo-synthesis
- blackbody radiation: CMB

fundamental microphysics: known very well

Products
History of the Universe in Four Episodes: IV.

(IV)

$t > 10^{13}$ sec

Post (Re)Combination

universe

- structure formation:
  - stars, galaxies, clusters...

complex macrophysics:
- Fundamentals known
- complex interplay

Products
Episodes of Thermal History

**Planck Epoch**
$t < 10^{-43}$ sec

**Phase Transition Era**
$10^{-43}$ sec < $t$ < $10^5$ sec

 **Hadron Era**
$t \sim 10^{-5}$ sec

**Lepton Era**
$10^{-5}$ sec < $t$ < 1 min

**Radiation Era**
$1$ min < $t$ < 379,000 yrs

**Post-Recombination Era**
$t > 379,000$ yrs

- GUT transition
- Electroweak transition
- Quark-hadron transition
- Muon annihilation
- Neutrino decoupling
- Electron-positron annihilation
- Primordial nucleosynthesis
- Radiation-matter equivalence
- Recombination & decoupling
- Structure & Galaxy formation
- Dark Ages
- Reionization
- Matter-Dark Energy transition
In principle, temperature $T$ should rise to infinity as we probe earlier and earlier into the universe’s history:

$$T \rightarrow \infty, \quad R \downarrow 0$$

However, at that time the energy of the particles starts to reach values where quantum gravity effects become dominant. In other words, the de Broglie wavelength of the particles become comparable to their own Schwarzschild radius.
Once the de Broglie wavelength is smaller than the corresponding Schwarzschild radius, the particle has essentially become a “quantum black hole”:

\[
\begin{align*}
\text{de Broglie wavelength: } & \quad \lambda_B = \frac{2\hbar\pi}{mc} \\
\leq \\
\text{Schwarzschild radius: } & \quad \lambda_S = \frac{2Gm}{c^2}
\end{align*}
\]

These two mass scales define the epoch of quantum cosmology, in which the purely deterministic metric description of gravity by the theory of relativity needs to be augmented by a theory incorporating quantum effects: quantum gravity.
**Thermal History: Planck Epoch**

On the basis of the expressions of the de Broglie wavelength and the Schwarzschild radius we may infer the typical mass scale, length scale and timescale for this epoch of quantum cosmology:

\[ m_{Pl} \equiv \sqrt{\frac{\hbar c}{G}} \sim 10^{19} \text{ GeV} \]

\[ l_{Pl} \equiv \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35} \text{ m} \]

\[ t_{Pl} \equiv \sqrt{\frac{\hbar G}{c^5}} \sim 10^{-43} \text{ sec} \]

Because our physics cannot yet handle quantum black holes, i.e. because we do not have any viable theory of quantum gravity we cannot answer sensibly questions on what happened before the Planck time. In other words, we are not able to probe the ultimate cosmic singularity ... some ideas of how things may have been do exist ...
In the Planck epoch, before the universe is 1 hundred-million-trillion-trillionth (10\(^{-44}\)) sec old, the density reaches values higher than \(\rho \sim 10^{94} \text{ g/cm}^3\) and temperatures in excess of \(T \sim 10^{32} \text{ K}\).

Quantum fluctuations of spacetime, on the scale of the Planck scale and Planck time are now of cosmic magnitude. Space and time are inextricably and discontinuously. As was pictured by J. Wheeler, spacetime under these conditions looks like a chaotic foam.

Spacetime is a foam of quantized black holes, and space and time no longer exist in the sense that we would understand. There is no “now” and “then”, no “here” and “there”, for everywhere is torn into discontinuities.

Then, due to the cosmic expansion, temperatures drop below \(T \sim 10^{32} \text{ K}\), and the unified “superforce” splits into a force of Gravity and a GUT force.
Thermal History: Episode by Episode

Phase Transition Era

$10^{-43} \text{ sec} < t < 10^{-5} \text{ sec}$

- The universe is filled by a plasma of relativistic particles: quarks, leptons, gauge bosons, Higgs bosons, ...

- During this epoch, as the universe expands and cools down, it undergoes various phase transitions, as a result of Spontaneous Symmetry Breaking
We may identify three major phase transitions during this era:

- GUT transition \( z \sim 10^{27-29} \)
- Electroweak transition \( z \sim 10^{15} \)
- Quark-Hadron transition \( z \sim 10^{11-12} \) (\( t \sim 10^{-5} \) s)

Phase Transition Era

\( 10^{-43} \text{ sec} < t < 10^{-5} \text{ sec} \)
• **Baryon non-conserving processes**

It is possible that the origin of the present-day excess of matter over antimatter finds its origin in the GUT phase transition.

• **Inflationary Epoch**

It is conceivable that the GUT transition may be identified with the phase transition that gave rise to a rapid exponential de Sitter expansion, in which the universe expanded by ~ 60 orders of magnitude (and in which its horizon shrank accordingly). Primordial density perturbations, the seeds of cosmic structure, may have been generated during this episode.
Thermal History: Episode by Episode

Lepton Era

\[ 10^{-5} \text{ sec} < t < 1 \text{ min}; \]
\[ 130 > T > 0.5 \text{ MeV} \quad 10^{12} \text{ K} > T > 5 \times 10^9 \text{ K} \]

Four major events occur during the lepton era:

- **Annihilation muons**
  \[ T \sim 10^{12} \text{ K} \]

- **Neutrino Decoupling**
  \[ T \sim 10^{10.5} \text{ K}; \quad z \sim 10^{10} \]

- **Electron-Positron Annihilation**
  \[ T < 10^9 \text{ K}; \quad z \sim 10^9, \quad t \sim 1 \text{ min} \]

- **Primordial Nucleosynthesis**
  \[ T \sim 10^9 \text{ K}; \quad t \sim 200 \text{ sec (3 min)} \]
Neutrino Decoupling

\[ T \sim 10^{10.5} \text{ K} \]
\[ t \sim 10^{-5} \text{ sec, } z \sim 10^{10} \]

- Weak interactions, e.g.

\[ e^+ + e^- \leftrightarrow \nu + \bar{\nu} \]

get so slow that neutrinos decouple from the \( e^+, e^-, \gamma \) plasma. Subsequently, they proceed as a relativistic gas with its own temperature \( T_{\nu} \).

- Because they decouple before the electron-positron annihilation, they keep a temperature \( T_{\nu} \) which is lower than the photon temperature \( T_\gamma \) (which gets boost from released annihilation energy):

\[ T_{\nu} = \left( \frac{4}{11} \right)^{1/3} T_\gamma \sim 1.95 \text{ K} \]

- The redshift of neutrino decoupling, \( z \sim 10^{10} \), defines a surface of last neutrino scattering, resulting in a “Cosmic Neutrino Background” with present-day temperature \( T \sim 1.95 \text{ K} \). A pity it is technically not feasible to see it!
Before this redshift, electrons and photons are in thermal equilibrium. After the temperature drops below $T \sim 10^9 \text{ K}$, the electrons and positrons annihilate, leaving a sea of photons.

$$e^+ + e^- \rightarrow 2\gamma$$

temperature drops below $T \sim 10^9 \text{ K}$, the electrons and positrons annihilate, leaving a sea of photons.

As they absorb the total entropy $s$ of the $e^+, e^-, \gamma$ plasma, the photons acquire a temperature $T_\gamma > \text{ neutrino temperature } T_\nu$. 
At this redshift the majority of photons of the Cosmic Microwave Background are generated.

- These photons keep on being scattered back and forth until $z \sim 10^{89}$, the epoch of recombination.
- Within 2 months after the fact, thermal equilibrium of photons is restored by a few scattering processes:
  - free-free scattering
  - Compton scattering
  - double Compton scattering

The net result is the perfect blackbody CMB spectrum we observe nowadays.
Primordial Nucleosynthesis

$T \sim 10^9 \text{K} \sim 0.1 \text{MeV}$

$t \sim 200 \text{sec} \sim 3 \text{min}$

• At the end of these “first three minutes” we find an event that provides us with the first direct probe of the Hot Big Bang, the nucleosynthesis of the light chemical elements, such as deuterium, helium and lithium.

• The prelude to this event occurs shortly before the annihilation of positrons and electrons. The weak interactions coupling neutrons and protons

\[
p + e^- \leftrightarrow n + \nu
\]

\[
p + \bar{\nu} \leftrightarrow n + e^+
\]

can no longer be sustained when the temperature drops below $T \sim 10^9 \text{K}$, resulting in a

Freeze-out of Neutron-Proton ratio:

\[
\frac{N_n}{N_p} \approx \frac{1}{6}
\]
Primordial Nucleosynthesis

- Note that from the ratio $N_n/N_p \sim 1/6$ we can already infer that if all neutrons would get incorporated into $^4\text{He}$ nuclei, around 25% of the baryon mass would involve Helium! Not far from the actual number...

- After freeze-out of protons and neutrons, a number of light element nucleons forms through a number of nuclear reactions involving the absorption of neutrons and protons:

  - Deuterium
    
    $$n + p \leftrightarrow D + \gamma$$

  - $^3\text{He}$
    
    $$D + p \rightarrow ^3\text{He}$$

  - $^4\text{He}$
    
    $$^3\text{He} + n \rightarrow ^4\text{He}$$

  - and traces of $^7\text{Li}$ and $^9\text{Be}$
Heavier nuclei will not form anymore, even though thermodynamically preferred at lower temperatures: when $^4\text{He}$ had formed, the temperature and density have simply below too low for any significant synthesis.

The precise abundances of the light elements depends sensitively on various cosmological parameters.

Particularly noteworthy is the dependence on the ratio of baryons to photons (proportional to the entropy of the universe), setting the # neutrons and protons available for fusion:

$$\eta \equiv \frac{n_B}{n_\gamma}$$

By comparing the predicted abundances as function of $\eta$, one can infer the density of baryons in the universe, $\Omega_B$ (see figure).
On the basis of the measured light element abundances, we find a rather stringent limit on the baryon density in the universe:

\[0.005 \lesssim \Omega_b h^2 \lesssim 0.026\]

This estimate of the baryon density from primordial nucleosynthesis is in perfect agreement with the completely independent estimate of the baryon density from the second peak in the angular power spectrum of the WMAP temperature perturbations:

\[
\begin{align*}
\Omega_b h^2 &\approx 0.0224 \pm 0.0009 \\
\Omega_b &\approx 0.044 \pm 0.004
\end{align*}
\]

This should be considered as a truly astonishing vindication of the Hot Big Bang.

Not that these nuclear reactions also occur in the Sun, but at a considerably lower temperature: \(T \sim 1.6 \times 10^7\) K. The fact that they occur in the early universe only at temperatures in excess of \(10^9\) K is due to the considerably lower density in the early universe:

\[
\begin{align*}
\rho_{\odot,\text{centre}} &\approx 10^2 \text{ g cm}^{-3} \\
\rho_{\text{univ},t=3\text{min}} &\approx 10^{-8} \text{ g cm}^{-3}
\end{align*}
\]
The radiation era begins at the moment of annihilation of electron-positron pairs.

After this event, the contents of the universe is a plasma of photons and neutrinos, and matter (after nucleosynthesis mainly protons, electrons and helium nuclei, and of course the unknown “dark matter”).

During this era, also called “Plasma Epoch”, the photons and baryonic matter are glued together. The protons and electrons are strongly coupled by Coulomb interactions, and they have the same temperature. The electrons are coupled to the radiation by means of Compton scattering. Hence, baryons and radiation are in thermal equilibrium.
Thermal History: Episode by Episode

Radiation Era

$t > 1 \text{ min}; \quad T < 5 \times 10^9 \text{ K}$

- Two cosmic key events mark the plasma era:
  - Radiation-Matter transition (equivalence matter-radiation)
    $z_{eq} \sim 2 \times 10^4$
  - Recombination & Decoupling
    $z \sim 1089; \quad t \sim 279,000 \text{ yrs}$
 Radiation-Matter Equality

\[ z_{eq} \sim 2 \times 10^4 \]

- The time of matter-radiation equality represents a crucial dynamical transition of the universe.

- Before \( z_{eq} \) the dynamics of the universe is dominated by \textit{Radiation}. After equivalence \textit{Matter} takes over as the dominant component of the universe.

\[
\begin{align*}
  z > z_{eq} & : \quad \rho_{\text{rad}} > \rho_m \\
  z < z_{eq} & : \quad \rho_{\text{rad}} < \rho_m
\end{align*}
\]

- Because the energy density of radiation diminishes with the fourth power of the expansion of the universe, while the density of matter does so with the third power, the ratio between radiation and matter density is an increasing function of \( a(t) \):

\[
\rho_{\text{rad}} \propto a(t)^{-4} \quad \leftrightarrow \quad \rho_m \propto a(t)^{-3}
\]

\[
\frac{\rho_m}{\rho_{\text{rad}}} = 4.3 \times 10^4 \, \Omega_m h^2 \frac{1}{1 + z}
\]
Radiation-Matter Equality

- The redshift $z_{eq}$ at which the radiation and matter density are equal to each other can then be inferred:

\[ 1 + z_{eq} = 4.0 \times 10^4 \Omega_m h^2 \]

- Because of the different equation of state for matter and radiation (and hence their different density evolution), the universe changes its expansion behaviour:

  - radiation-dominated $z > z_{eq}$: $a(t) \propto t^{1/2}$
  - matter-dominated $z < z_{eq}$: $a(t) \propto t^{2/3}$

- This has dramatic consequences for various (cosmic structure formation) processes, and we can find back the imprint of this cosmic transition in various phenomena.

- Note that the universe underwent a similar transition at a more recent date. This transition, the "Matter-Dark Energy Equality" marks the epoch at which dark energy took over from matter as dynamically dominant component of the universe.
Recombination Epoch

$T \sim 3000 \text{ K}$

$z_{\text{dec}} = 1089 \quad (\Delta z_{\text{dec}} = 195); \quad t_{\text{dec}} = 379,000 \text{ yrs}$

• Before this time, radiation and matter are tightly coupled through bremsstrahlung:

$e^- + \gamma \leftrightarrow e^- + \gamma$

Because of the continuing scattering of photons, the universe is a “fog”.

• A radical change of this situation occurs once the temperature starts to drop below $T \sim 3000 \text{ K}$. and electrons. Thermodynamically it becomes favorable to form neutral (hydrogen) atoms $H$ (because the photons can no longer destroy the atoms):

$p + e^- \leftrightarrow H$

• This transition is usually marked by the word “recombination”, somewhat of a misnomer, as of course hydrogen atoms combine just for the first time in cosmic history. It marks a radical transition point in the universe’s history.
Recombination Epoch

- This happened 279,000 years after the Big Bang, according to the impressively accurate determination by the WMAP satellite (2003).
- Major consequence of recombination:

Decoupling of Radiation & Matter

- With the electrons and protons absorbed into hydrogen atoms, the Photons decouple from the plasma, their mean free path becoming of the order of the Hubble radius. The cosmic “fog” lifts:

universe transparent

- The photons assume their long travel along the depths of the cosmos. Until some of them, Gigaparsecs further on and Gigayears later, are detected by telescopes on and around a small planet in some faraway corner of the cosmos...
Recombination & Decoupling

In summary, the recombination transition and the related decoupling of matter and radiation defines one of the most crucial events in cosmology. In a rather sudden transition, the universe changes from

**Before** $z > z_{\text{dec}}$
- universe fully ionized
- photons incessantly scattered
- pressure dominated by radiation:

\[
 p = \frac{1}{3} a T^4
\]

**After** $z < z_{\text{dec}}$
- universe practically neutral
- photons propagate freely
- pressure only by baryons:

\[
 p = n k T
\]
- (photon pressure negligible)
Recombination & Decoupling

• Note that the decoupling transition occurs rather sudden at $T \sim 3000 \text{ K}$, with a “cosmic photosphere” depth of only $\Delta z_{\text{dec}} \sim 195$ (at $z \sim 1089$).

• The cosmological situation is highly exceptional. Under more common circumstances the (re)combination transition would already have taken place at a temperature of $T \sim 10^{4} \text{ K}$.

• Due to the enormous amount of photons in the universe, signified by the abnormally high cosmic entropy,

$$\frac{n_{\gamma}}{n_{B}} \approx 10^{9}$$

even long after the temperature dropped below $T \sim 10^{4} \text{ K}$ there are still sufficient photons to keep the hydrogen ionized (i.e. there are still plenty of photons in the Wien part of the spectrum).

• Recombination therefore proceeds via a 2-step transition, not directly to the groundstate of hydrogen. The process is therefore dictated by the rate at which $\text{Ly}_{\alpha}$ photons redshift out of the $\text{Ly}_{\alpha}$ rest wavelenght. For $n_{\gamma}/n_{B} \sim 10^{9}$ this occurs at

$$T \sim 3000 \text{ K}$$
The photons that are currently reaching us, emanate from the Surface of Last Scattering located at a redshift of $z \sim 1089$.

The WMAP measurement of the redshift of last scattering confirms the theoretical predictions (Jones & Wyse 1985) of a sharply defined last scattering surface.

The last scattering surface is in fact somewhat fuzzy, the photons arrive from a "cosmic photosphere" with a narrow redshift width of $\Delta z \sim 195$. 

We can only see the surface of the cloud where light was last scattered.
Recombination Epoch

- The photons emanating from the last scattering surface, freely propagating through our universe, define a near isotropic sea of radiation.

- Shortly after they were created at the time of electron-positron annihilation, $z \sim 10^9$, the photon bath was thoroughly thermalized. It thus defines a most perfect blackbody radiation field:

$$I_\nu(T) = \frac{2\hbar \nu^3}{c^2} \frac{1}{\exp\left(\frac{\hbar \nu}{kT}\right) - 1}$$

- Due to the cosmic expansion, the radiation field has in the meantime cooled down to a temperature of $T = 2.725 \text{ K} \ (\pm 0.002 \text{ K}, \text{ WMAP})$.

- This cosmic radiation we observe as the **Cosmic Microwave Background**
Recombination Epoch

Cosmic Microwave Background

- first discovered serendipitously by Penzias & Wilson in 1965, and reported in their publication “… an excess measurement …”, without doubt should be regarded as one of the principal scientific discoveries of the 20th century.

- Its almost perfect blackbody spectrum is the ultimate proof of a hot and dense early phase: the Hot Big Bang

... the Nobel prize for the discovery of the CMB followed in 1978 ...
Recombination Epoch

Cosmic Microwave Background

- The amazingly precise blackbody nature of the CMB was demonstrated by the COBE satellite (1992).
- The spectral energy distribution in the figure is so accurately fit by a Planckian spectrum that the error bars are smaller than the thickness of the solid (blue) curve (see figure) !!!
- Note that the corresponding CMB photon number density is:

\[ n_\gamma \sim 410 \text{ cm}^{-3} \]
The CMB is a fabulously rich treasure trove of information on the primordial universe.

In the accompanying figure you see three milestones of CMB research:

2) The COBE satellite (1992), first discovery of primordial perturbations.
3) WMAP (2003), detailed temperature perturbations “fix” the universe’s parameters.
After recombination/decoupling, while the universe expands it gradually cools down (baryonic matter faster than radiation once they are entirely decoupled). We can identify various major processes and transitions during these long-lasting eons ...

◊ Structure & Galaxy Formation $z \sim 10^{89.0}$
◊ Dark Ages $z \sim 10^{89.10/20}$
◊ Reionization $z \sim 20-6 ?$
◊ Matter-Dark Energy transition $z \sim 0.3$
Post-Recombination Era: The last five billion years

While the universe moved itself into a period of accelerated exponential expansion as it came to be dominated by “Dark Energy”, stars and galaxies proceeded with their lives. Stars died, new and enriched ones arose out of the ashes. Alongside the newborn stars, planets emerged:

One modest and average yellowish star, one of the two hundred billion denizens of a rather common Sb spiral galaxy called “Milky Way”, harboured a planetary system of around 9 planets - a few of them rocky, heavy clumps with loads of heavy elements...

One of them bluish, a true pearl in the heavens...
This planet, Earth it is called, became home to remarkable creatures ... some of which evolved sophisticated brains. The most complex structures in the known universe ...

Some of them started using them to ponder about the world in which they live ... Pythagoras, Archimedes, Albert Einstein were their names ... they took care of an astonishing feat: they found the universe to be understandable, how truly perplexing!

A universe thinking about itself ... and thinking it understands ...

Post-Recombination Era:

The last five billion years