THEME 2

# Hierarchical Structure Formation.

# Press-Schechter formalism

Connect nonlinear collapse to initial, linear density field.

Is it possible to make predictions on the outcome of nonlinear collapse on the basis of mere initial density field?

$\Rightarrow$ Central Assumption:

Even if field nonlinear, amplitude of large wavelength modes in final field close to predictions linear theory.

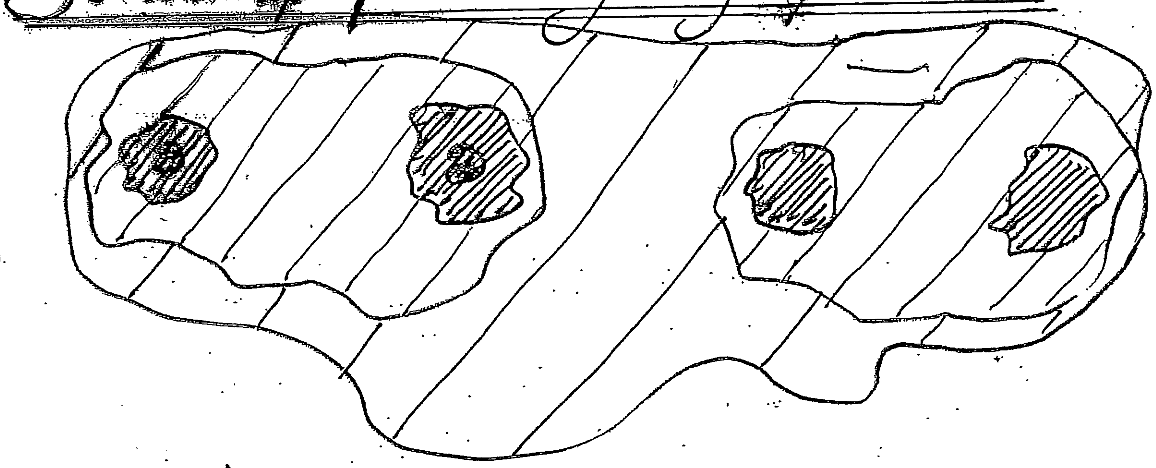
When overdensity spherical, its nonlinear collapse time can be precisely predicted in terms of initial linear overdensity

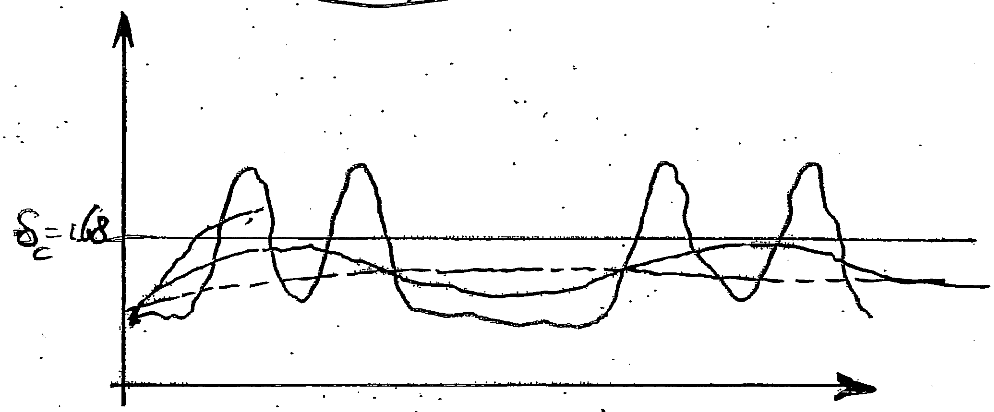
e.g. $\Omega = 1$ : $\quad a_{coll} = \dfrac{1.69}{\Delta_{0,lin}}$

$\Delta_{0,lin}$: tophat averaged overdensity within radius $R$.

# Hierarchies and the Press-Schechter approach.

## Bottom-up picture galaxy formation:






collapse
$(\cdot\ \delta_{lin}(R) > \delta_c = 1.68)$

$R = 0.5\,h^{-1}\,Mpc \rightarrow a_{coll}(0.5\,h^{-1}\,Mpc)$
$R = 2\,h^{-1}\,Mpc \rightarrow a_{coll}(2\,h^{-1}\,Mpc)$
$R = 5\,h^{-1}\,Mpc \quad a_{coll}(5\,h^{-1}\,Mpc)$

$a_{coll}(0.5\,h^{-1}\,Mpc) < a_{coll}(2\,h^{-1}\,Mpc) < a_{coll}(5\,h^{-1}\,Mpc)$

* Look at successive volume fractions, for a sequence of "smoothing" radii: the extra volume collapsed as $R/M \uparrow$ has collapsed in these higher mass objects.

⇒ • Identification location and properties collapsed objects of typical scale by tracing in initial density field those peaks that at some epoch $a$ will have collapsed:

• e.g. simple spherical collapse:

$$a_{coll} = \frac{1.69}{\Delta_{0,lin}}$$

$$\Rightarrow \text{if } \Delta_{lin}(a) \geq 1.69 \quad \text{Collapse}:$$

• Probability that fluctuation on scale $R_f$ has some linear density deviation

$$\delta = \frac{\delta\rho}{\rho}:$$

$$P(\delta) = \frac{1}{\sqrt{2\pi}\,\sigma(M)} \exp\left\{-\frac{\delta^2}{2\sigma^2(M)}\right\}$$

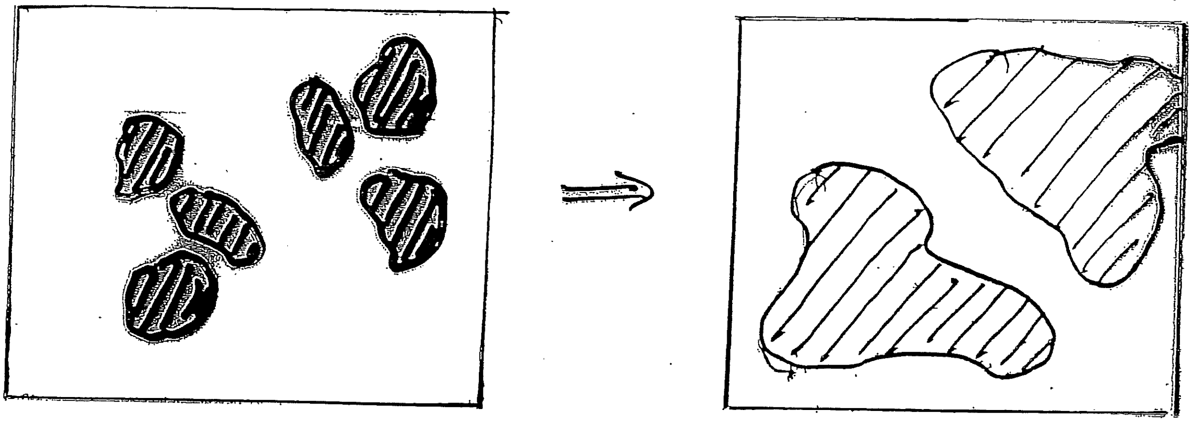
• Note: scale $R_f \xleftrightarrow{\text{corresp.}}$ mass scale $M$

$$M = \frac{4\pi}{3}\bar{\rho}\, R_f^3$$

$$\Downarrow$$

$$\sigma^2(M) = \left\langle \left(\frac{\delta\rho}{\rho}\right)^2 \right\rangle = \langle \delta^2 \rangle$$

mass scale $M_1$ mass scale $M_2$



$\delta(M_1) > 1.6g$ $\delta(M_2) > 1.6g$

- fraction of volume on mass scale $M$, exceeding critical density $\delta_c$:

$$F(M) = \frac{1}{\sqrt{2\pi}\,\sigma(M)} \int_{\delta_c}^{\infty} \exp\left\{ -\frac{\delta^2}{2\sigma^2(M)} \right\} d\delta$$

$$\boxed{F(M) = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\delta_c}{\sqrt{2}\,\sigma}\right)\right]}$$

infamous factor 2!!

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt.$$

- Mass Distribution:

# objects mass range $[M, M+dM]$:

$$n(M)\,dM = \underbrace{\left|\frac{dF}{dM}\right| dM}_{\text{change volume fraction}} \cdot \frac{\rho_0}{M}$$

get total mass

divide mass individual clump

$$\boxed{n(M) = \frac{\rho_0}{M^2}\frac{dF}{d\ln M} = \frac{\rho_0}{M^2}\left|\frac{d\ln\sigma}{d\ln M}\right|\sqrt{\frac{2}{\pi}}\,\nu e^{-\frac{\nu^2}{2}}}$$

- Multiplicity function:

$$\frac{M^2 f(M)}{\rho_0}.$$

- $\frac{\delta\rho}{\rho} = \nu\sigma(M)$

Describes at any time $t$ mass spectrum depending on Primordial Mass Spectrum: $\sigma(M)$!!

Power-law spectrum: $P(k) \propto k^n$

$$\sigma^2(M) = \left\langle\left(\frac{\delta\rho}{\rho}\right)^2\right\rangle = \langle\delta^2\rangle = A M^{-(3+n)/3}$$

$$\Rightarrow \frac{\delta_c}{\sqrt{2}\,\sigma(M)} = \frac{\delta_c}{\sqrt{2}A^{1/2}} M^{(3+n)/6} = \left(\frac{M}{M_*}\right)^{(3+n)/6}.$$

Reference Mass:

$$\boxed{M^* = \left(\frac{2A}{\delta_c^2}\right)^{3/3+n}.}$$

- <u>Reference mass $M_*$</u> :

$$M_* = \left(\frac{2A}{\delta_c^2}\right)^{3/(3+n)}.$$

$$\delta(M) \propto t^{2/3} \Rightarrow \sigma^2(M) = \delta^2(M) \propto t^{4/3}.$$

$\uparrow$ $\Omega=1$, growing mode

$$\Longrightarrow \quad M^* \propto A^{3/3+n} \propto t^{4/3+n}$$

$$\Longrightarrow \quad \boxed{M^* = M_0^* \left(\frac{t}{t_0}\right)^{4/(3+n)}}$$

Evolution mass spectrum through mediating role characteristic mass $M^*$

$$\Rightarrow \quad \boxed{n(M) = \frac{1}{2\sqrt{\pi}}\left(1+\frac{n}{3}\right)\frac{\bar{\rho}}{M^2}\left(\frac{M}{M_*}\right)^{(3+n)/6} \exp\left\{-\left(\frac{M}{M_*}\right)^{3+n/3}\right\}}$$

non-power law spectrum : $n(k) \propto \frac{d\ln P}{d\ln k}$ (at e.g. char. galaxy scale):

$$\boxed{N(M) = \frac{\bar{\rho}}{\sqrt{\pi}}\frac{\gamma}{M^2}\left(\frac{M}{M^*}\right)^{\gamma/2} \exp\left\{-\left(\frac{M}{M^*}\right)^{\gamma}\right\}}$$

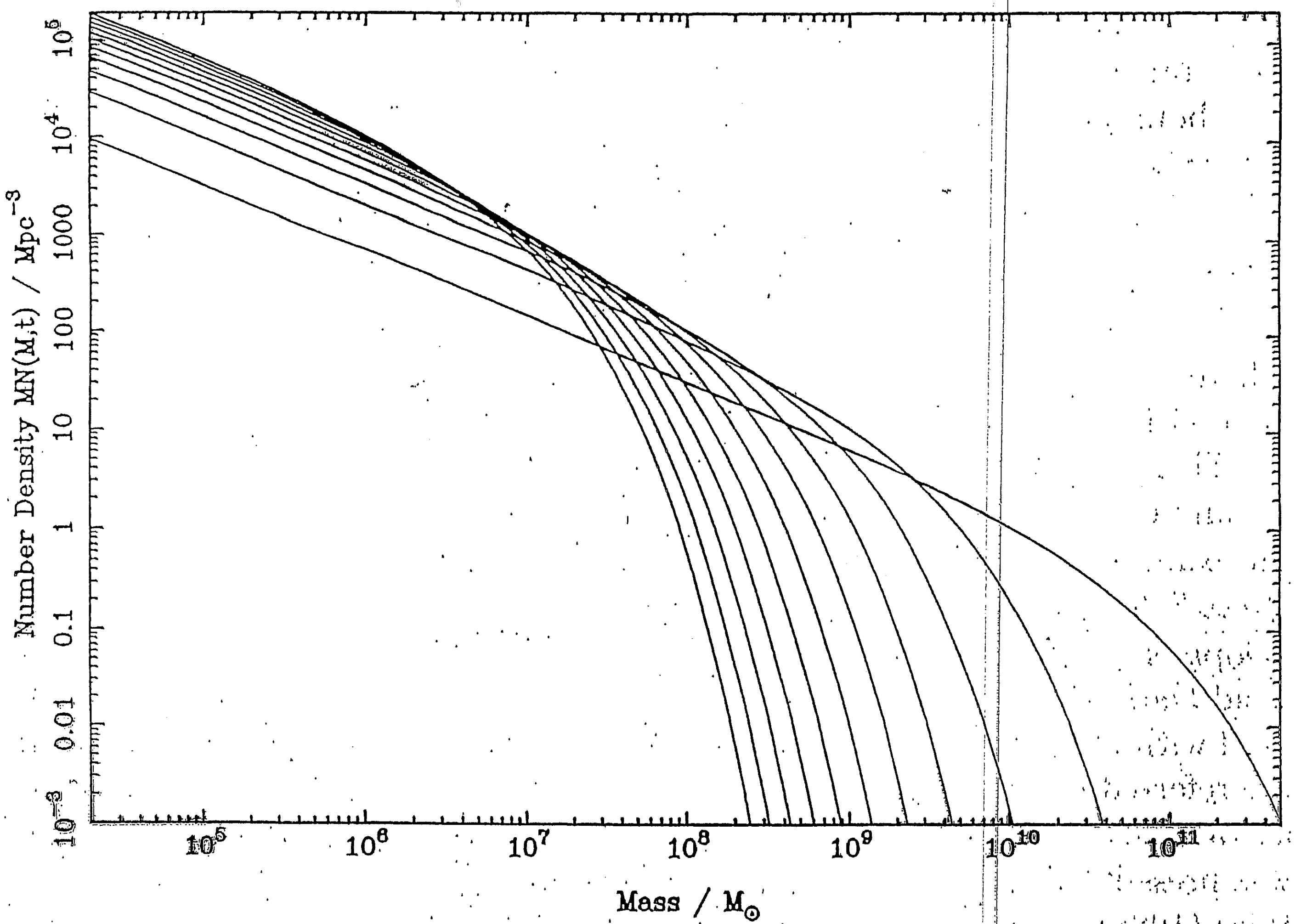



Fig. 16.4. Illustrating the variation of the form of the Press-Schechter mass function as a function of cosmic time (Courtesy of Dr. Andrew Blain).

# Dissipation

## Dissipationless collapse :

pure gravitational collapse
$\Downarrow$
(violent relaxation) —— Limited mass.

- $R_{vir} = \frac{1}{2} R_{max}$
- recall:

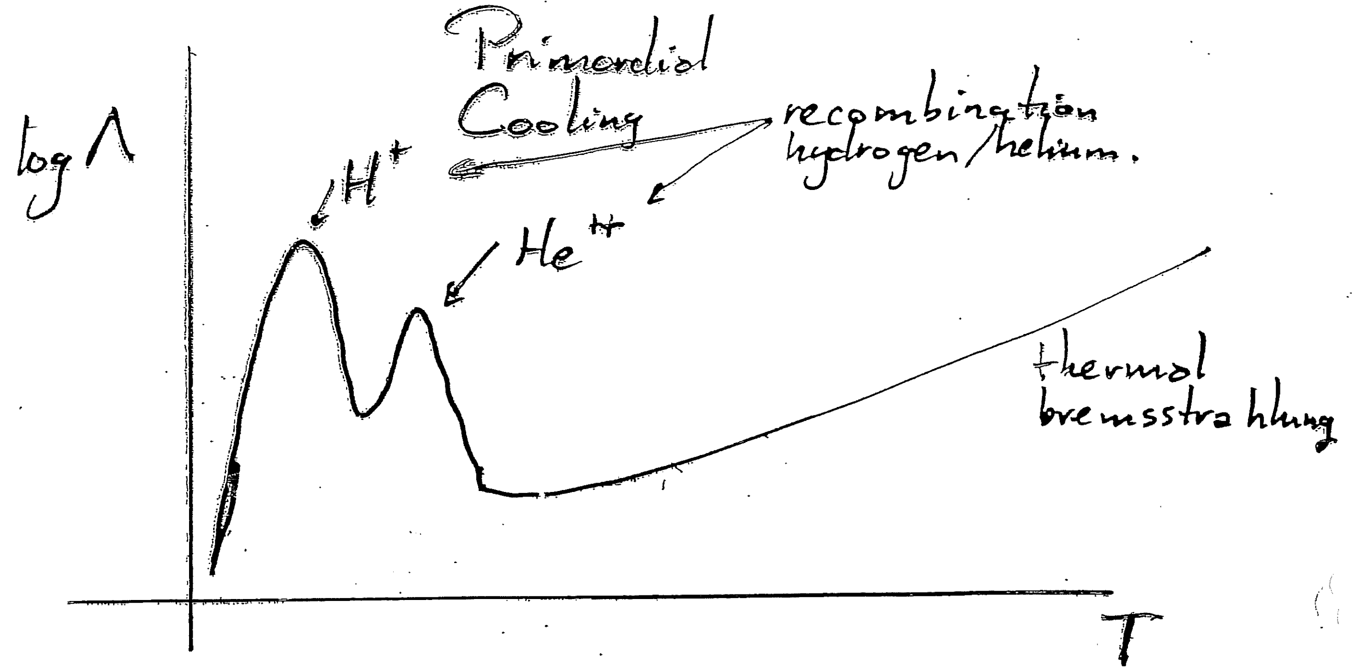
galaxy mass: $M \approx 10^{12} M_\odot$
$\Downarrow$
$R_f \approx 1 h^{-1} Mpc$
$\Downarrow$
$R_{max} >> R_f \Rightarrow \underline{\underline{R_{vir} \hat{=} h^{-1} Mpc}}$

$\boxed{\text{BUT}}$ : galaxies much smaller !!!

## How to collapse a galaxy further ?

$\boxed{\text{Dissipation}}$

get rid of your internal energy
$\Downarrow$
removal pressure
$\Downarrow$
further collapse.




## How effective is cooling:

Cooling Rate:

$$\tau_{cool} = \frac{E}{|dE/dt|} = \frac{3 N k_B T}{N^2 \Lambda(T)}$$

$\Updownarrow$

compare with dynamic time scale:

$$\tau_{dyn} \simeq \frac{1}{\sqrt{G\rho}} \propto \frac{1}{\sqrt{N}}.$$

- when $\tau_{cool} \ll \tau_{dyn}$ : effective collapse: Galaxy formation

  $\tau_{cool} \approx \tau_{Hubble}$ : only gravitational collapse.

  $\tau_{grav} \approx \tau_{Hubble}$ : no collapse.

## Effective dissipation mechanism

**RADIATION**

- Crucial for formation of stars in Molecular Clouds.

  ↓

- collapse continues through effective radiation losses

  ↓

- collapse of cloud so dense, it becomes optically thick.

  ↓

- heating dust grains $\Rightarrow$ 60-100 K.

  ↓ radiating at far-infrared (cloud transparent)

(But first stars? No dust (heavy elements) $\Rightarrow H_2$ molecule cooling ? )

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- **How about galactic scales :**

$$\boxed{\frac{dE}{dt} = -\underbrace{N^2}_{\substack{\text{density}\\ \text{nuclei}}} \underbrace{\Lambda(T)}_{\text{cooling}}}$$

Cooling

Primordial Plasma:

- $T > 10^6 K$ - Thermal Bremsstrahlung $\Lambda \propto N^2 T^{1/2}$
- $T < 10^6 K$ - free-bound tr. } - hydrogen
  bound-bound tr. } - ionized

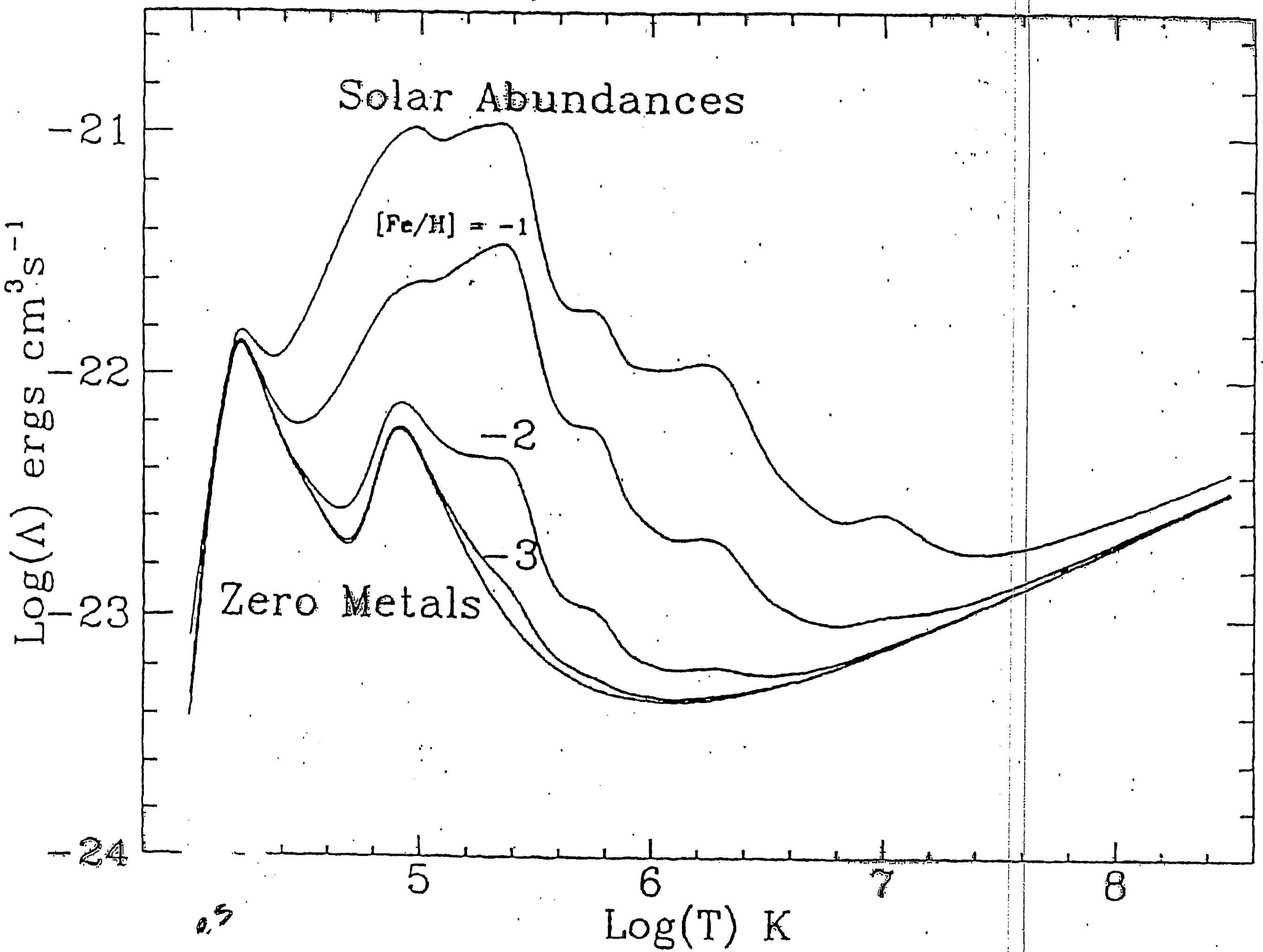


**Fig. 16.2.** The cooling rate per unit volume $\Lambda(T)$ of an astrophysical plasma of number density 1 nucleus $cm^{-3}$ by radiation for different cosmic abundances of the heavy elements ranging from zero metals to the present abundance of the heavy elements as a function of temperature $T$ (Silk and Wyse 1993, after Sutherland and Dopita 1993). In the zero metal case, the two maxima of the cooling curve are associated with the recombination of hydrogen ions and doubly ionised helium (see also Sect. 19.5 and Fig. 19.3).

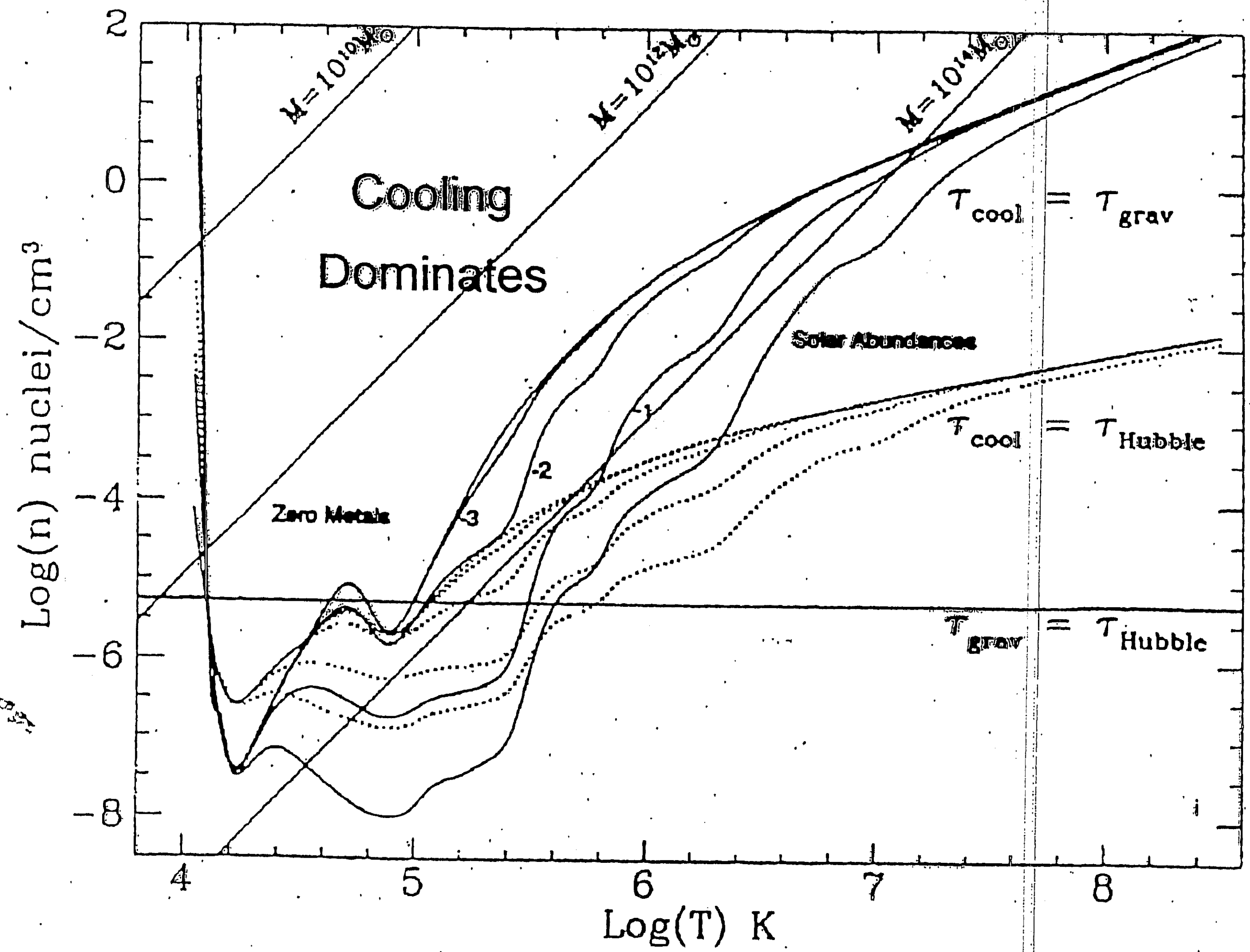


Fig. 16.3. A number density–temperature diagram showing the locus defined by the condition that the collapse time of a region $\tau_{dyn}$ should be equal to the cooling time of the plasma by radiation $\tau_{cool}$ for different abundances of the heavy elements (after Silk and Wyse 1993). Also shown are lines of constant mass, a cooling time of $10^{10}$ years (dotted lines), and the density at which the perturbations are of such low density that they do not collapse in the age of the Universe.