Echo of the Big Bang

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Cosmic Microwave Background

Echo of the Big Bang
Nobelprijs Natuurkunde 2006

COBE (1992):
- John Mather
  DIRBE: temperature
  blackbody
- George Smoot
  DMR: fluctuations
  embryonic
  structure

DIRBE: temperature

Hubble Ultra Deep Field (HUDF)

Modern universe

13.7

HDF

1.0

0.7 - 0.4

Age of the universe (billions of years)
Cosmic Microwave Background: Some Facts

Radiation Field of the Universe:
- Discovered in 1965 (serendipitously) by Penzias & Wilson, Nobelprize 1978.
- Thermal radiation pervading throughout the whole Universe.
- As yet it has a temperature of $T = 2.725 \text{ K}$.

1) By far CMB photons represent the most abundant species in the Universe:
   - $n_\gamma \sim n_B = 415 \text{ cm}^{-3}$
   - For comparison: $n_{\gamma / n_B} = 1.9 \times 10^9$ (second: cosmic neutrino's)
   - Stellar photons: negligible (integrated over all stars at all times)

Cosmic Microwave Background: Some Facts
Cosmic Microwave Background: Some Facts

7) CMB highly (impressively) isotropic:
   - in each direction on the sky the radiation has almost exactly
     the same temperature/intensity
   - temperature anisotropies VERY SMALL, in the order of
     \[ \frac{\Delta T}{T} < 10^{-5} \]
   - for comparison:
     Planet Earth’s highest mountain would be in the order 10-25 m

8) The electromagnetic spectrum of the CMB PERFECTION
   Thermal Blackbody (most accurately measured BB spectrum ever):
   \[ L_\nu(T) = \frac{2\pi^2\nu^3}{c^2\exp\left(\frac{\nu}{kT}\right) - 1} \]
Cosmic Microwave Background: Some Facts

6) CMB photons Last Scattered
379,000 yrs. after Big Bang
at a redshift $z=1089$ (i.e., expansion factor $a(t)=1/1089$)

7) Following the -- Decoupling of Radiation and (Baryonic) Matter
- Recombination Hydrogen Atoms
  (as protons and electrons combine)

8) At recombination $T \approx 3000 \text{ K}$, the (CMB) sky would look red
Since then, gradual cooling of radiation through expansion Universe:
- cosmic redshift photons

9) The CMB photons created at much earlier epoch !!!
  Last surge: positron-electron annihilation,
  1 min. after Big Bang, redshift $z \sim 10^9$

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Cosmic Microwave Background

COBE (1992):
- Accurate measurement
- Planck spectrum CMB
- First detection angular temperature perturbations
  ($\theta \sim 7^\circ$): Sachs-Wolfe effect
**Cosmic Microwave Background**

**COBE (1992):**
Three instruments:
- FIRAS: Far-Infrared Absolute Spectrophotometer
- DIRBE: Diffuse Infrared Background Experiment
- DMR: Differential Microwave Radiometer

John Mather
COBE-DIRBE:
- Temperature, blackbody
  - Most accurately measured Blackbody Spectrum Ever !!!!!

**Spectrum Blackbody Radiation**
Primordial Anisotropies

CMB sky

COBE–DMR Map of CMB Anisotropy

George Smoot
DMR

Fluctuations, embryonic structure
North Galactic Hemisphere South Galactic Hemisphere

-100 μK +100 μK

Primordial Anisotropies

CMB sky

the early 2-year COBE map
Noise, noise, ..., noise ...
Key to the Universe
CMB Radiation, the cosmic radiation field of the Universe,
Cosmic Treasure Trove:

1) Direct probe of Thermal State of the early primordial Universe,
the Universe before Decoupling,
   Ultimate Proof Reality of Hot Big Bang
2) Direct reflection of Primordial Structure of the Universe,
   the Embryonic State
   of all Structure in today's Universe
3) Through its simplicity (linearity) the ultimate diagnostic tool for
   measuring the Universe:
   Cosmic Parameters
4) Link on early decisive hypothetical/theoretical
   Inflationary Epoch

The Simple Universe:
Cosmic Geometry & Expansion
Hubble Expansion

\[ v = H \cdot r \]

Edwin Hubble
(1889-1953)

... its geometry rules the world, the world rules its geometry...

... Spacetime becomes a dynamic continuum, integral part of the structure of the cosmos ... curved spacetime becomes force of gravity

\[ R^\alpha{}_{\beta} - \frac{1}{2} g^\alpha{}_{\beta} R = - \frac{8\pi G}{c^4} T^\alpha{}_{\beta} \]
Geometry of the Universe

K=\pm 1
\Omega_0 > 1

K=0
\Omega_0 = 1

\Omega_0 < 1
K=-1

Friedmann-Robertson-Walker-Lemaitre Universe

$$\ddot{R} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) R + \frac{\Lambda}{3} R$$

$$\dot{R}^2 = \frac{8\pi G}{3} \rho R^2 + \frac{\Lambda}{3} R^2 - kc^2$$
Geometry & Dynamics:
Friedmann-Robertson-Walker-Lemaitre Universe

Three factors (interrelated) determine Evolution & Fate Universe:

1) The energy content of the Universe: cosmic (energy) density: $\rho(t)$
2) The curvature $k$
3) The cosmological constant $\Lambda$
**Dynamics:**

Friedmann-Robertson-Walker-Lemaitre Universe

Cosmological (energy) densities are typically in the order of that of the critical energy density of Universe. Currently,

\[ \rho_{\text{crit}} = \frac{3H_0^2}{8\pi G} = 1.8791h^2 \times 10^{-29} \text{ g cm}^{-3} \]

**Geometry & Dynamics:**

Friedmann-Robertson-Walker-Lemaitre Universe

Evolution & Fate of the universe determined by one major factor:

\[ \Omega \equiv \frac{\rho}{\rho_{\text{crit}}} = \frac{8\pi G \rho}{3H^2} \]

Curvature determined by the Energy Density:

\[ k = \frac{H^2 R^2}{c^2} (\Omega - 1) \]
Geometry & Dynamics:
Friedmann-Robertson-Walker-Lemaitre Universe

Evolution & Fate of the universe determined by one major factor:

\[
\Omega \equiv \frac{\rho}{\rho_{\text{crit}}} = \frac{8\pi G \rho}{3H^2}
\]

Total energy density the sum of various contributions,

\[
\Omega_{\text{tot}} = \Omega_{\text{matter}} + \Omega_{\text{rad}} + \Omega_{\Lambda}
\]

Cosmology:
“quest for two issues”:
Content & Parameters
In addition to the constituents mentioned in the diagram, there are contributions by e.g. gravitational waves, magnetic fields, etc. However, given the poor constraints on their contribution henceforth we will not take them into consideration.
Cosmic Energy Inventory

<table>
<thead>
<tr>
<th>Energy Class</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 dark energy</td>
<td>0.72 \pm 0.03</td>
</tr>
<tr>
<td>1.1 dark energy</td>
<td>0.72 \pm 0.03</td>
</tr>
<tr>
<td>1.2 dark matter</td>
<td>0.23 \pm 0.03</td>
</tr>
<tr>
<td>1.3 primordial gravitational waves</td>
<td>\lesssim 10^{-10}</td>
</tr>
<tr>
<td>2 primordial thermal remnants</td>
<td>0.996 \pm 0.0005</td>
</tr>
<tr>
<td>2.1 electromagnetic radiation</td>
<td>\rho_\gamma \approx 10^{-5}</td>
</tr>
<tr>
<td>2.2 neutrinos</td>
<td>10^{-5}</td>
</tr>
<tr>
<td>2.3 perturbative nuclear binding energy</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>3 baryon rest mass</td>
<td>0.045 \pm 0.003</td>
</tr>
<tr>
<td>3.1 warm intergalactic plasma</td>
<td>0.024 \pm 0.003</td>
</tr>
<tr>
<td>3.3 cold dark matter</td>
<td>0.016 \pm 0.003</td>
</tr>
<tr>
<td>3.5 white dwarfs</td>
<td>0.00006 \pm 0.00002</td>
</tr>
<tr>
<td>3.7 black holes</td>
<td>0.00006 \pm 0.00002</td>
</tr>
<tr>
<td>3.9 molecular gas</td>
<td>0.00016 \pm 0.00002</td>
</tr>
<tr>
<td>3.11 planets</td>
<td>10^{-5}</td>
</tr>
<tr>
<td>3.12 condensed matter</td>
<td>10^{-5}</td>
</tr>
<tr>
<td>3.13 sequestered in massive black holes</td>
<td>10^{-5} (1 + c_s)</td>
</tr>
<tr>
<td>4 primordial gravitational binding energy</td>
<td>\approx 10^{-6}</td>
</tr>
<tr>
<td>4.1 virialized halos of galaxies</td>
<td>10^{-6}</td>
</tr>
<tr>
<td>4.2 clusters</td>
<td>10^{-6}</td>
</tr>
<tr>
<td>4.3 large-scale structure</td>
<td>10^{-6}</td>
</tr>
</tbody>
</table>

Cosmic Constituents

The energy content of the Universe may be broadly divided into various classes, dependent on how their energy density evolves in time. The three most important ones are:

- **Matter:** \( \rho_m \propto a(t)^{-3} \)
- **Radiation:** \( \rho_{\text{rad}} \propto a(t)^{-4} \)
  \[ \rho_v = a(t)^{-3(1+w)} \iff p = w \rho_v \]
- **Dark Energy:** \( \rho_{\Lambda} = \text{cst.} \)

- \( w = -1 \)
Concordance

“Vanilla”

Cosmology

(WMAP3

parameters)

EADN summerschool, Leiden, July 1995 ...

of course, since some years ...

ACDM Universe deemed “concordant” ...

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>$70.9^{+2.4}_{-3.3}$ km s$^{-1}$ Mpc$^{-1}$</td>
<td>Hubble parameter</td>
</tr>
<tr>
<td>$\Omega_0$</td>
<td>$0.044^{+0.0042}_{-0.00055}$</td>
<td>Baryon density</td>
</tr>
<tr>
<td>$\Omega_m$</td>
<td>$0.266^{+0.025}_{-0.040}$</td>
<td>Total matter density (baryons + dark matter)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$0.079^{+0.029}_{-0.032}$</td>
<td>Optical depth to reionization</td>
</tr>
<tr>
<td>$A_s$</td>
<td>$0.813^{+0.042}_{-0.052}$</td>
<td>Scalar fluctuation amplitude</td>
</tr>
<tr>
<td>$n_s$</td>
<td>$0.948^{+0.015}_{-0.018}$</td>
<td>Scalar spectral index</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Derived parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_c$</td>
</tr>
<tr>
<td>$\Omega_{\Lambda}$</td>
</tr>
<tr>
<td>$z_{ion}$</td>
</tr>
<tr>
<td>$\sigma_8$</td>
</tr>
<tr>
<td>$t_0$</td>
</tr>
</tbody>
</table>
From the Friedmann equations, it is straightforward to appreciate that cosmic expansion is an adiabatic process:

\[ \dot{\rho} + 3 \left( \rho + \frac{p}{c^2} \right) \frac{\dot{a}}{a} = 0 \]

\[ dU = -p dV \leftrightarrow \begin{cases} U = \rho c^2 V & \text{internal energy} \\ V \propto a^3 & \text{cosmic volume} \end{cases} \]

In other words, there is no "external power" responsible for "pumping" the tube...
Adiabatic Expansion

\[ p \propto \rho^\gamma \quad \Rightarrow \quad TV^{\gamma-1} = \text{cst.} \]

\[ \begin{align*}
\gamma &= \frac{5}{3} \quad \Rightarrow \quad T_b \propto V^{-2/3} \propto a^{-2} \\
\gamma &= \frac{4}{3} \quad \Rightarrow \quad T_{\text{rad}} \propto V^{-1/3} \propto a^{-1}
\end{align*} \]

 Cosmic expansion is Adiabatic. Temperature History

Hot Big Bang

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- **Big Bang**: 0
- **Inflation**: \(10^{-32}\) Sec.
- **Quark Soup**: 1 Second
- **Big Freeze Out**: 300,000 Years
- **Parting Company**: 1

Age of the Universe
### Episodes of Thermal History

<table>
<thead>
<tr>
<th>Epoch</th>
<th>Time Range</th>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Planck Epoch</strong></td>
<td>$t &lt; 10^{-43}$ sec</td>
<td>GUT transition</td>
</tr>
<tr>
<td><strong>Phase Transition Era</strong></td>
<td>$10^{-43} &lt; t &lt; 10^5$ sec</td>
<td>electroweak transition, quark-hadron transition</td>
</tr>
<tr>
<td><strong>Hadron Era</strong></td>
<td>$t \sim 10^{-5}$ sec</td>
<td>muon annihilation</td>
</tr>
<tr>
<td><strong>Lepton Era</strong></td>
<td>$10^{-5} &lt; t &lt; 1$ min</td>
<td>neutrino decoupling, electron-positron annihilation, primordial nucleosynthesis</td>
</tr>
<tr>
<td><strong>Radiation Era</strong></td>
<td>$1$ min $&lt; t &lt; 379,000$ yrs</td>
<td>radiation-matter equivalence, recombination &amp; decoupling</td>
</tr>
<tr>
<td><strong>Post-Recombination Era</strong></td>
<td>$t &gt; 379,000$ yrs</td>
<td>Structure &amp; Galaxy formation, Dark Ages, Reionization, Matter-Dark Energy transition</td>
</tr>
</tbody>
</table>

### Hot Big Bang: What it explains

- **Olber’s Paradox:**
  - The night sky is dark.
  - Finite age Universe ($13.7$ Gyr)

- **Hubble Expansion**
  - Uniform expansion, with expansion velocity = distance: $v = Hr$

- **Explanation of Helium Abundance:**
  - Light chemical elements formed ($H, He, Li, ...$) after $\sim 3$ minutes...

- **The Cosmic Microwave Background Radiation:**
  - The $2.725 K$ radiation blanket, remnant left over hot ionized plasma (neutral universe, $379,000$ years after Big Bang)

- **Distant, deep Universe indeed looks different...**
The Hot Big Bang: Inflationary Universe

FRW Big Bang: What it cannot explain

- Flatness Problem: the Universe is remarkably flat, and was even (much) flatter in the past
- Horizon Problem: the Universe is nearly perfectly isotropic and homogeneous, much more so in the past
- Monopole Problem: There are hardly any magnetic monopoles in our Universe
- Fluctuations, seeds of structure: Structure in the Universe: origin
FRW Big Bang extended: Inflationary Universe

- Essential Ingredient/Extension
  Standard Cosmology
  Inflationary Universe

  Phase transition
  Early Universe
  - GUT transition:
    \[ t \approx 10^{-36} \text{ sec} \]
    -(false) vacuum potential
    induces exponential (de Sitter) expansion
  - Universe blows up by factor \[ N > 10^{60} \]

FRW Big Bang extended:
Inflationary Universe

Explains:
- Horizon Problem
- Flatness Problem
- Monopole Problem

And:
- Origin of Structure
Cosmic Radiation

Most ubiquitous, most pervasive, constituent of the Universe: Radiation.

- photons $\gamma$
- neutrinos $\nu$

Two major components of relativistic (massless) species:
**Cosmic Radiation**

1) Number Density CMB photons:

\[
\begin{align*}
n_\gamma(T) &= \frac{8\pi}{3} \int_0^\infty \frac{\nu^2 d\nu}{e^{\frac{k\nu}{h\pi}} - 1} = 60.4 \left(\frac{kT}{h\pi}\right)^3 \\
n_\gamma &= 60.4 \left(\frac{kT}{h\pi}\right)^3 \approx 410 (1+z)^3 \text{ cm}^{-3}
\end{align*}
\]

Present Number Density: \( n_\gamma \sim 410 \text{ cm}^{-3} \)

**Cosmic Redshift**

\[
1 + z = \frac{1}{a} \iff \begin{cases} 
\lambda_{em} = \lambda_0 \\
\lambda_{obs} = \frac{a(t_{obs})}{a(t_{em})} \lambda_0
\end{cases}
\]

\[
z \equiv \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}
\]
2) Number photons/neutrinos conserved
   - photon number density \( n_{\text{rad}} \propto a(t)^{-3} \)

3) As a result of the cosmic expansion,
   - wavelength of a photon redshifts:
     \( \lambda \propto a(t)^{-1} \)
   - photon energy \( \varepsilon_{\text{rad}} \propto a(t)^{-1} \)

4) Energy density radiation evolves:
   \[ \rho_{\text{rad}} \propto a(t)^{-4} \propto (1+z)^{4} \]

5) Energy density at present negligible:
   \[ \Omega_{\text{rad}} \approx 10^{-5} \]

6) Dynamically increasingly important in early Universe,
   dominant over matter before equivalence epoch

\[ 1 + z_{\text{eq}} = 4.0 \times 10^{4} \Omega_{m} h^{2} \]
7) In terms of Number Density, Cosmic Photons have ALWAYS been dominant, Most abundant species in the Universe. By FAR !!!!!!!!!

5) Ratio Baryons to Photons
Entropy Universe

\[
\frac{n_B}{n_\gamma} \approx 10^9
\]

Fundamental Property !!!!!!
Universe very Peculiar Physical System

\[ n_B(x) = n_{B,0}(1 + z)^3 \]
\[ n_\gamma(x) = n_{\gamma,0}(1 + z)^3 \]
\[ \eta(x) = \frac{n_B(x)}{n_\gamma(x)} = \frac{n_{B,0}}{n_{\gamma,0}} = \eta_0 \]
Before this redshift, electrons and photons are in thermal equilibrium. After the temperature drops below $T \sim 10^9$ K, the electrons and positrons annihilate, leaving a sea of photons.

As they absorb the total entropy $s$ of the $e^+, e^-, \gamma$ plasma, the photons acquire a temperature $T_\gamma >$ neutrino temperature $T_\nu$.

At this redshift the majority of photons of the Cosmic Microwave Background are generated.

At the onset certainly not thermally distributed energies

Photons keep on being scattered back and forth until $z \sim 10^9$, the epoch of recombination.

Thermal equilibrium (blackbody spectrum) of photons reached within 2 months after their creation

Blackbody Spectrum produced through three scattering processes

- Compton scattering
- Free-free scattering
- Double Compton scattering
CMB Thermalization

- Thermalization through three scattering processes
  - Compton scattering + dominant energy redistribution
  - Free-free scattering + creates new photons to
  - Double Compton scattering adjust spectrum to Planck

- While Compton scattering manages to redistribute the energy of the photons, it cannot adjust the number of photons. Free-free scattering and Double Compton scattering manage to do so ...

- But ...
  - only before \( z < 10^5 \), after that the interaction times too long ...

- Following this thermalization, a perfect blackbody photon spectrum has emerged:

\[
I_\nu(T) = \frac{2\hbar\nu^3}{c^2} \frac{1}{\exp \left( \frac{\hbar\nu}{kT} \right) - 1}
\]

- This is the ULTIMATE proof of the HOT BIG BANG

- Note: after \( z \approx 10^5 \) till recombination, the interaction between electrons and photons exclusively by Thomson Scattering.
Spectrum
Blackbody Radiation

Cosmic Microwave Background Spectrum from COBE

Theory and observation agree.

CMB
Recombination &
Decoupling
Before the "Recombination Epoch"
- Radiation and Matter are tightly coupled through Thomson scattering.

The events surrounding "recombination" exist of THREE major (coupled, yet different) processes:

- **Recombination**: protons & electrons combine to \( H \) atoms
- **Decoupling**: photons & baryonic matter no longer interact
- **Last scattering**: meaning, photons have a last kick and go ...

\[ T \approx 3000 \text{ K} \]
\[ z_{\text{dec}} = 1089 \quad (\Delta z_{\text{dec}} = 195); \quad t_{\text{dec}} = 379,000 \text{ yrs} \]

Before this time, radiation and matter are tightly coupled through Thomson scattering:

Because of the continuing scattering of photons, the universe is a "fog".

A radical change of this situation occurs once the temperature starts to drop below \( T \approx 3000 \text{ K} \) and electrons. Thermodynamically, it becomes favorable to form neutral (hydrogen) atoms \( H \) (because the photons can no longer destroy the atoms):

\[ e^- + \gamma \rightleftharpoons e^- + \gamma \]
\[ p + e^- \rightleftharpoons H \]

This transition is usually marked by the word "recombination", somewhat of a misnomer, as of course hydrogen atoms combine just for the first time in cosmic history. It marks a radical transition point in the universe’s history.
Recombination history

As temperature changes:
- Shifting ionization can be followed through Saha equation (note: on "wrong" premise of equilibrium)
- Recombination should happen at $T \sim 4000$ K
- But: far too many CMB photons, it is not equilibrium process!!!

Recombination & Decoupling

- Note that the decoupling transition occurs rather sudden at $T \sim 3000$ K, with a "cosmic photosphere" depth of only $\Delta x_{dec} \sim 195$ (at $z \sim 10^9$).
- The cosmological situation is highly exceptional. Under more common circumstances the (re)combination transition would already have taken place at a temperature of $T \sim 10^4$ K.
- Due to the enormous amount of photons in the universe, signified by the abnormally high cosmic entropy,
  \[
  \frac{n_\gamma}{n_B} \approx 10^9
  \]
  even long after the temperature dropped below $T \sim 10^4$ K there are still sufficient photons to keep the hydrogen ionized (i.e. there are still plenty of photons in the Wien part of the spectrum).
- Recombination therefore proceeds via a 2-step transition, not directly to the groundstate of hydrogen. The process is therefore dictated by the rate at which $Ly\alpha$ photons redshift out of the $Ly\alpha$ rest wavelength. For $n_\gamma/n_B \sim 10^9$ this occurs at $T \sim 3000$ K.
Standard theory of H recombination (Peebles 1968, Zel'dovich et al 1968)

Recombination Process not entirely trivial:

- ground state could be reached via Lya transition (2P - 1S)

DOES NOT WORK !!!!! large abundance Lya Ionization

- Recombination in parts: forbidden transition = 2-photon emission: 2S - 1S

"Takes ~ 8.23 s⁻¹ much slower than 'direct', and thus recombination occurs late...

at T~3000 K

H⁺ + e⁻

radiative recombination + photoionization

3s  3p  3d

2s  2p

Lyman-α resonance escape

1s

Big Bang

We can only see the surface of the cloud where light
Cosmic Photons

Note:
far from being an exotic faraway phenomenon, realize that the CMB nowadays is counting for approximately 1% of the noise on your tv set ...

Courtesy: W. Hu
In summary, the recombination transition and the related decoupling of matter and radiation defines one of the most crucial events in cosmology. In a rather sudden transition, the universe changes from:

**Before** $z > z_{\text{dec}}$
- universe fully ionized
- photons incessantly scattered
- pressure dominated by radiation:
  $$p = \frac{1}{3} \mu T^4$$

**After** $z < z_{\text{dec}}$
- universe practically neutral
- photons propagate freely
- pressure only by baryons:
  $$p = n k T$$
- (photon pressure negligible)

**Ripples in the Universe**
Structure in the Universe

Map of the Universe at Recombination Epoch:
- 379,000 years after Big Bang
- Subhorizon perturbations: primordial sound waves
- $\Delta T/T < 10^{-5}$

Cosmic Microwave Background

WMAP
Primordial Gaussian Perturbations

Gravitational Instability

\[ g(r, t) = \frac{1}{a} \nabla \phi = \frac{3 \Omega H^2}{8\pi} \int dx' \delta(x', t) \frac{(x' - x)}{|x' - x|^3} \]
Millennium Simulation

Millennium Simulation: LCDM

(courtesy: Virgo/V. Springel)

Millennium Simulation

Millennium Simulation: LCDM

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Rippling Origins

Predictions:
- Gaussian fluctuations
- Adiabatic fluctuations (radiation & matter equally perturbed)
- Near scale-free potential perturbations:
  \[ P(k) \sim k^n \]
  \[ n \approx 0.96 \]

Inflationary Origins

Inflationary Origin of Cosmic Structure:
- Primordial Quantum Noise (due to uncertainty principle)
- Becomes seeds for structure today
- At inflation the fluct's expanded to superhorizon size

Predictions:
- Gaussian fluctuations
- Adiabatic fluctuations (radiation & matter equally perturbed)
- Near scale-free potential perturbations:
**Primordial Perturbation Growth**

Once fluctuations "entered" the cosmic horizon, they can start growing...

Growth dependent on a series of modulating processes, e.g.,

- Baryon perturbations: held up by pressure (Jeans)

- Dark Matter:
  - starts growth at horizon entrance
  - damps below free-streaming
  - CDM vs. HDM

- Cosmic Expansion regime:
  - radiation dominance: no growth DM pert.
  - matter dominance

**Cold Dark Matter Cosmologies**

\[ P_{\text{CDM}}(k) \propto \frac{k^\alpha}{[1 + (16.1q)^2 (5.46q)^2 + (6.71q)^2]^{3/2}} \times \frac{\ln 1 + 2.34q}{(2.34q)^2} \]

- Standard Scenario: Perturbations in medium of Cold Dark Matter particles
- axions, neutrinos, etc...
- Results in a hierarchical scenario of structure formation
Cold Dark Matter Cosmologies

Standard Scenario:
- Perturbations in medium of Cold Dark Matter particles
  - axions, neutralinos, ...
- Results in a hierarchical scenario of structure formation

\[
P_{\text{CDM}}(k) \propto \frac{k^3}{[1 + 3.89 q + (16.1 q)^2 + (3.46 q^2 + (6.71 q^3))^{1/2}] \left(\frac{\ln 1 + 2.34 q}{2.34 q^2}\right)}
\]

\[
q = \frac{k}{\Gamma}
\]

\[
\Gamma = \Omega_{\text{m}}, h \exp \left(-\Omega_{\text{b}} - \frac{\Omega_{\text{m}}}{\Omega_{\text{m,0}}}\right)
\]

Rippling The Photons
CMB Perturbations

- As a result of perturbations in the gravitational potential, photons experience a frequency shift.
- While travelling through a perturbation:
  - Gravitational Redshift
  - (Relativistic) Time Dilation
- Combined effect: Sachs-Wolfe Effect

Sachs-Wolfe Effect

\[
\frac{\Delta T}{T} \sim \frac{1}{3} \frac{\Delta \Phi}{c^2}
\]
Cosmic Microwave Background

Map of the Universe at Recombination Epoch:

- 379,000 years after Big Bang
- Superhorizon perturbations in gravitational potential (Sachs-Wolfe)
- $\Delta T/T < 10^{-5}$
Horizon Problem Illustrated

Cosmic Microwave Background

COBE measured fluctuations: $> 7''$
Size Horizon at Recombination spans angle $\sim 1^o$

How can it be that regions totally out of thermal contact, would have the same temperature?
COBE measured fluctuations: $> 7^\circ$
Size Horizon at Recombination spans angle $\sim 1^\circ$
COBE proved existence superhorizon fluctuations: Inflation prediction !!!!!
Cosmic Microwave Background

Map of the Universe at Recombination Epoch:
- 379,000 years after Big Bang
- Subhorizon perturbations: primordial sound waves
- $\Delta T/T < 10^{-5}$

Temperature Anisotropies

Temperature Perturbations in terms of Spherical Harmonics:

$$T(\theta, \phi) = \sum_{l,m} a_{lm} Y^m_l(\theta, \phi)$$

$$\phi \sim \frac{\pi}{l} \sim \frac{180^\circ}{l}$$
Temperature Anisotropies

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CMB Power Spectrum

Angular power spectrum

\[ C_l \propto \langle a_{lm} a^*_{lm} \rangle \]

\[ C_l \propto l(l+1) \]

\[ P(k) \propto k \]
Music of the Spheres

Cosmic Microwave Background

- small ripples in primordial matter & photon plasma
- gravity compresses primordial photon gas, photon pressure resists:
- compressions and rarefactions in photon gas:
  sound waves
- sound waves not heard, but seen:
  - compressions: (photon) temperature higher
  - rarefactions: lower
Seeing Sound

- Colliding electrons, protons and photons forms a plasma
- Acts like a gas
- Compressional disturbance propagates in the plasma through collisions

Unlike sound in the air:
- air molecules travel $= 10^5$ cm before colliding
- in primordial plasma, photons travel $10^4$ pc

- we do not hear it but see it in the CMB
- compression heats the gas resulting in a hot spot in the CMB

Piper at the Gates of Dawn

- Like blowing into a flute or an open pipe
- Spectrum of sound contains a
  Fundamental frequency & Harmonic overtones

<table>
<thead>
<tr>
<th>Inflation</th>
<th>1st peak</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2nd peak</td>
</tr>
<tr>
<td></td>
<td>recombination</td>
</tr>
<tr>
<td></td>
<td>3rd peak</td>
</tr>
<tr>
<td></td>
<td>4th peak</td>
</tr>
</tbody>
</table>
- Inflation is the source of sound waves at the beginning of time.

- Sound waves are frozen at recombination, yielding a harmonic spectrum of frequencies that reach maximum displacement.

\[
P(k) \propto \langle a_m a_n^* \rangle \propto l(l+1)
\]

\[
P(k) \propto k
\]
Harmonic Signature

- Identify structure and composition of the Universe
  - through detailed examination of the pattern of overtones on the fundamental frequency
  - much like using them for a music instrument

- Observed frequency spectrum consistent with inflationary origin:
  - spectrum of cosmic sound has harmonics at integer ratios of fundamental

- Without inflation, fluctuations should have been generated at intermediate times
  - This would have destroyed the harmonic structure of the peaks (like drilling holes in an organ pipe)

Music of the Spheres

Total Angular CMB Spectrum
modulated by combination of several effects,
the Primary Perturbations

1) Sachs-Wolfe potential fluctuations,
2) Acoustic perturbations
   as the corresponding potential fluct.
   enter horizon and start to collapse
3) Integrated Sachs-Wolfe
   potential perturbations:
   - Early ISW: matter/radiation at recomb.
   - Late ISW: expansion influence curvature & cosmological constant
4) Doppler perturbations
   velocity fluct. accompanying potential pert.
5) Silk Damping
   radiation damping of fluctuations
**Modulating Influences**

- **Silk Damping:**
  - photons diffuse out of matter perturbations
  - fluctuations with size $<\text{photon free-streaming length}$ get suppressed
  - harmonic structure beyond third peak seriously damped

- **Integrated Sachs-Wolfe effect:**
  - damping/boosting temperature fluctuations due to decay/growth potential perturbations:
    - *Early ISW:* while still radiation-dominated, potential DM fluct's grow less, suppression of temp. fluct.
    - *Late ISW:* as Dark Energy takes over universe, potential wells decay (due to accelerated expansion)
Universe Measured

Cosmic Parameters

The WMAP CMB temperature power spectrum
### Friedmann-Robertson-Walker-Lemaitre Universe

**Old Universe – New Numbers**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_m$</td>
<td>1.02 ± 0.20</td>
</tr>
<tr>
<td>$\Omega_b$</td>
<td>0.05 ± 0.01</td>
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<tr>
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</tr>
<tr>
<td>$\Omega_L$</td>
<td>0.71 ± 0.10</td>
</tr>
<tr>
<td>$H_0$</td>
<td>70 ± 10 km s$^{-1}$ Mpc$^{-1}$</td>
</tr>
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<td>$\Omega_{b}h^2$</td>
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</tr>
<tr>
<td>$\Omega_{r}h^2$</td>
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</tr>
<tr>
<td>$\Omega_{r}h^2$</td>
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</tbody>
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**Curvature Measured**

### WMAP: 1st year data

- $\Omega_m$: 0.99 (95% CL)
- $\Omega_b$: 0.05 ± 0.01
- $\Omega_f$: 0.02 ± 0.01
- $\Omega_k$: 0.05 ± 0.01
- $\Omega_L$: 0.71 (95% CL)
- $H_0$: 70 ± 10 km s$^{-1}$ Mpc$^{-1}$
- $\Omega_{b}h^2$: 0.02 ± 0.01
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- $\Omega_{L}h^2$: 0.70 ± 0.10
- $\theta$: 0.01 ± 0.001
**Harmonic Signature**

- Spectrum cosmic sound:
  - integer ratios fundamental
  - competition between gravity vs. pressure: dependent on phase
  - fundamental + odd mode: gravity along sonic motion
  - even multiples: gravity fights sonic motion

![Harmonic Signature Graph](image)

---

**Geometry of the Universe: Music of the Spheres**

**Measuring the Geometry of the Universe:**
- Object with known physical size, at large cosmological distance
- Measure angular extent on sky
- Comparison yields light path

*Physical Object*:
- Sound waves in primordial matter-radiation plasma: wavelength $\lambda_s$
- Observable at surface of epoch recombination, at which photons were last scattered

![Geometry of space](image)

*WMAP*
Measuring the Geometry of the Universe:

- Object with known physical size, at large cosmological distance
- Measure angular extent on sky
- Comparison yields light path

Geometry of space

Friedmann-Robertson-Walker-Lemaitre
Universe

Flat (\Omega = 1)
Open (\Omega < 1)
Closed (\Omega > 1)
Music of the Spheres
The Cosmic Tonal Ladder

The Cosmic Microwave Background Temperature Anisotropies: the Embryonic Universe

Curvature: CMB

Curvature influences:
- angular scale of given physical fluct.
- evolution potential wells
Cosmic Constituents: Matter

- Baryonic Matter
- Nonbaryonic Dark Matter
Baryonic Matter

- Baryonic “drag” suppresses fluctuation
- low second peak: baryon density comparable to photon density

Baryonic Matter

Fukugita & Peebles 2004

Note:
• STARS are but a fraction of the total amount of baryonic matter
• There is still a large amount of undetected baryonic matter: hiding as warm Intergalactic Gas (WHIM)?
Baryonic Matter: primordial nucleosynthesis

From measured light element abundances:

\[ \eta \equiv \frac{n_B}{n_\gamma} \]

0.005 \lesssim \Omega_b h^2 \lesssim 0.026

Baryonic Matter: CMB

Due to baryon drag in the primordial baryon-photon gas, 2nd peak in CMB spectrum is suppressed:

\[ \Omega_b h^2 \approx 0.0224 \pm 0.0009 \]
\[ \Omega_b \approx 0.044 \pm 0.004 \]
Dark Matter

It is the nonbaryonic Matter that is responsible for the existence of Structure in the Universe !!!

If it had not been there: no substantial structure

Clusters of Galaxies

Coma Cluster

Courtesy: O. Lopez-Cruz
Clusters: X-ray intracluster gas

Hydrostatic Equilibrium:

\[
\frac{GM(r)}{r^2} = -\frac{k_B T}{\mu m_H} \left[ \frac{d \log \rho}{dr} + \frac{d \log T}{dr} \right]
\]

Emission:
Bremsstrahlung: \( L \sim \rho^2 \)

Clusters: Gravitational Lensing

Strong Lensing Arcs:
Abell 2218
\( z=0.175 \)

Galaxy Cluster Abell 2218
HST • WFPC2
NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08
## Nonbaryonic Dark Matter

### Two major classes of Dark Matter:

**MACHOs**
- massive compact halo objects
  - brown dwarfs
  - stellar remnants (black holes, neutron stars)
  - primordial black halos

**WIMPs**
- weakly interacting massive particles:
  - hot dark matter e.g. massive neutrinos
  - cold dark matter axions, neutralinos
  - warm dark matter

---

## Dark Matter

- **Dark Matter:**
  - Retains the gravitational potential,
  - while baryonic matter oscillates as stable sound wave
  - otherwise, decay gravitational potential

---

### Recombination

- dark matter
- no dark matter
Dark Matter

- Responsible for keeping up gravitational potential perturbations
- Third peak: dark matter density

Matter: CMB

(Dark) Matter determines the depth of the potential wells, influencing the amplitude of the acoustic fluctuations
** Cosmic Acceleration **

Our Universe

Present: ACCELERATES

Past: DECELERATED

Cosmic tug of war
The force of dark energy surpasses that of dark matter as time progresses.
Dark Energy: CMB

- Sunyaev-Zel'dovich Effect
- Gravitational Lensing CMB
- Reionization: polarization
- Integrated Sachs-Wolfe Effect
- Rees-Sciama Effect
- Vishniac Effect
- ...
Polarization CMB

Richest Source of Information on Primordial Universe:

Two modes:
- \( \mathbf{E} \)-mode: Doppler motions, recombination, \( \text{Reionization} \)
- \( \mathbf{B} \)-mode: Gravitational Lensing, Primordial (Inflationary) Gravitational Waves

\[
\left( \frac{(l+1)C}{2\pi} \right)^{1/2} \quad [\mu K]
\]
Cosmic Microwave Background

Lifted Cosmology into
the realm
Of Precision Cosmology