## Cosmology, lect. 7

#### **Thermal History**

## FRW

## Thermodynamics

## FRW Dynamics

$$\ddot{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3p}{c^2}\right)a + \frac{\Lambda}{3}a$$

$$\dot{a}^{2} = \frac{8\pi G}{3}\rho a^{2} - \frac{kc^{2}}{R_{0}^{2}} + \frac{\Lambda}{3}a^{2}$$

To find solutions a(t) for the expansion history of the Universe, for a particular FRW Universe ,

one needs to know how the density  $\rho(t)$  and pressure p(t)evolve as function of a(t)

FRW equations are implicitly equivalent to a third Einstein equation, the energy equation,

$$\dot{\rho} + 3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{a}}{a} = 0$$

#### FRW Dynamics: Adiabatic Cosmic Expansion

Important observation: the energy equation,

$$\dot{\rho} + 3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{a}}{a} = 0$$

is equivalent to stating that the change in internal energy

$$U = \rho c^2 V$$

of a specific co-expanding volume V(t) of the Universe, is due to work by pressure:

$$dU = -p \, dV$$

Friedmann-Robertson-Walker-Lemaitre expansion of the Universe is

Adiabatic Expansion

#### FRW Dynamics: Thermal Evolution

Adiabatic Expansion of the Universe:

• Implication for Thermal History

• Temperature Evolution of cosmic components

For a medium with adiabatic index **y**:

 $TV^{\gamma-1} = cst$ 

Radiation (Photons) $\gamma = \frac{4}{3}$  $T = \frac{T_0}{a}$ Monatomic Gas<br/>(hydrogen) $\gamma = \frac{5}{3}$  $T = \frac{T_0}{a^2}$ 

# Radiation & Matter

## **Cosmic Radiation**

The Universe is filled with thermal radiation, the photons that were created in The Big Bang and that we now observe as the Cosmic Microwave Background (CMB).

The CMB photons represent the most abundant species in the Universe, by far !

The CMB radiation field is PERFECTLY thermalized, with their energy distribution representing the most perfect blackbody spectrum we know in nature. The energy density  $u_{\mathbb{R}}(T)$  is therefore given by the Planck spectral distribution,

$$u_{\nu}(T) = \frac{8\pi h\nu^{3}}{c^{3}} \frac{1}{e^{h\nu/kT} - 1}$$

At present, the temperature T of the cosmic radiation field is known to impressive precision,

$$T_0 = 2.725 \pm 0.001 K$$

## CMB Radiation Field Blackbody Radiation



Waves / centimeter

Intensity, 10<sup>-4</sup> ergs / cm<sup>2</sup> sr sec cm<sup>-1</sup>

## **Cosmic Radiation**

With the energy density  $u_v(T)$  of CMB photons with energy hv given, we know the number density  $n_v(T)$  of such photons:

$$n_{\nu}(T) = \frac{u_{\nu}(T)}{h\nu} = \frac{8\pi\nu^2}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

The total number density  $n_{\gamma}(T)$  of photons in the Universe can be assessed by integrating the number density  $n_{\gamma}(T)$  of photons with frequency v over all frequencies,

$$n_{\gamma}(T) = \int_{0}^{\infty} n_{\nu}(T) d\nu =$$

$$= \int_{0}^{\infty} \frac{8\pi v^{2}}{c^{3}} \frac{1}{e^{h\nu/kT} - 1} d\nu = 60.4 \left(\frac{kT}{hc}\right)^{3}$$

$$T = 2.725 K$$

$$I$$

$$I$$

$$R_{\gamma}(T) = 412 cm^{-3}$$

## **Baryon-Photon Ratio**

Having determined the number density of photons, we may compare this with the number density of baryons,  $n_b(T)$ . That is, we wish to know the PHOTON-BARYON ratio,

The baryon number density is inferred from the baryon mass density. here, for simplicity, we have assumed that baryons (protons and neutrons) have the same mass, the proton mass  $m_p \sim 1.672 \times 10^{-24}$  g. At present we therefore find

$$n_{b} = 1.12 \times 10^{-5} \ \Omega_{b} h^{2} \ g \ cm^{-3}$$
$$\eta_{0} = \frac{n_{\gamma}}{n_{B}} \approx 3.65 \times 10^{7} \frac{1}{\Omega_{b} h^{2}} \ g \ cm^{-3}$$

We know that  $\Omega_{b} \sim 0.044$  and  $h \sim 0.72$ :  $\eta_{0} = \frac{n_{\gamma}}{n_{b}} \approx 1.60 \times 10^{9}$ 

## **Baryon-Photon Ratio**

From simple thermodynamic arguments, we find that the number of photons is vastly larger than that of baryons in the Universe.

$$\eta_0 = \frac{n_{\gamma}}{n_b} \approx 1.60 \times 10^9$$

In this, the Universe is a unique physical system, with tremendous repercussions for the thermal history of the Universe. We may in fact easily find that the cosmic photon-baryon ratio remains constant during the expansion of the Universe,

$$n_b(t) = \frac{n_{b,0}}{a^3}$$
  

$$\eta = \frac{n_{\gamma}(t)}{n_b(t)} = \frac{n_{\gamma,0}}{n_{b,0}} = \eta_0$$
  

$$n_{\gamma}(t) \propto T(t)^3 \propto \frac{1}{a^3} \implies n_{\gamma}(t) = \frac{n_{\gamma,0}}{a^3}$$

## Entropy of the Universe

The photon-baryon ratio in the Universe remains constant during the expansion of the Universe, and has the large value of

$$\eta = \frac{n_{\gamma}(t)}{n_b(t)} = \frac{n_{\gamma,0}}{n_{b,0}} = \eta_0 = 1.60 \times 10^9$$

This quantity is one of the key parameters of the Big Bang. The baryon-photon ratio quantifies the ENTROPY of the Universe, and it remains to be explained why the Universe has produced such a system of extremely large entropy !!!!!

The key to this lies in the very earliest instants of our Universe !

#### **Cosmic Light (CMB): most abundant species**

# By far, the most abundant particle species in the Universe $n_v/n_B \sim 1.9$ billion

# Hot Big Bang:

## Thermal History

## **Adiabatic Expansion**

- The Universe of Einstein, Friedmann & Lemaitre expands *adiabatically*
- Energy of the expansion of the Universe corresponds to the decrease in the energy of its constituents
- The Universe COOLS as a result of its expansion !

 $T(t) \propto 1/a(t)$ 

## Equilibrium Processes

Throughout most of the universe's history (i.e. in the early universe), various species of particles keep in (local) thermal equilibrium via interaction processes:

$$\psi_i + \psi_j \quad \overleftrightarrow \quad \chi_i + \chi_j$$

Equilibrium as long as the interaction rate  $\Gamma_{int}$  in

the cosmos' thermal bath, leading to N<sub>int</sub> interactions in time t,

$$\Gamma_{\rm int} \implies N_{\rm int} = \int \Gamma_{\rm int}(t) dt$$

is much larger than the expansion rate of the Universe,

the Hubble parameter H(t):

$$\Gamma_{\rm int} \gg H(t)$$

#### Reconstruction Thermal History Timeline Strategy:

To work out the thermal history of the Universe, one has to evaluate at each cosmic time which physical processes are still in equilibrium.

Once this no longer is the case, a physically significant transition has taken place.

Dependent on whether one wants a crude impression or an accurately and detailed worked out description, one may follow two approaches:

#### □ Crudely:

Assess transitions of particles out of equilibrium, when they decouple from

thermal bath. Usually, on crude argument:

$$\Gamma_{\rm int} \gg H(t)$$
  $\Gamma_{\rm int} < H(t)$ 

#### □ Strictly:

evolve particle distributions by integrating the Boltzmann equation



#### **Thermal History:** Interactions

Particle interactions are mediated by gauge bosons:



# Hot Big Bang Eras

## **Cosmic Epochs**

#### Planck Epoch

Phase Transition Era

<u>Hadron Era</u>

<u>Lepton Era</u>

Radiation Era



GUT transition electroweak transition quark-hadron transition

muon annihilation neutrino decoupling electron-positron annihilation primordial nucleosynthesis

radiation-matter equivalence recombination & decoupling

Structure & Galaxy formation Dark Ages Reionization Matter-Dark Energy transition  $t < 10^{-43} sec$ 

 $10^{-43} \sec < t < 10^{5} \sec$ 

 $t \sim 10^{-5} sec$ 

 $10^{-5} \sec < t < 1 \min$ 

1 min < t < 379,000 yrs

t>379,000 yrs

#### History of the Universe in Four Episodes: I

On the basis of the

1) complexity of the involved physics

2) our knowledge of the physical processes

we may broadly distinguish four cosmic episodes:



#### History of the Universe in Four Episodes: II



#### History of the Universe in Four Episodes: III

 $10^{-3} \le t \le 10^{13} \text{ sec}$ 

Standard

<u>fundamental</u> <u>microphysics</u>: known very well

**Hot Big Bang** 

**Fireball** 

- primordial nucleosynthesis
- blackbody radiation: CMB

#### History of the Universe in Four Episodes: IV



# Origins:

#### the Planck Epoch

## Thermal History: Episode by Episode



• In principle, temperature T should rise to infinity as we probe earlier and earlier into the universe's history:

$$T \to \infty, \quad R \downarrow 0$$

• However, at that time the energy of the particles starts to reach values where quantum gravity effects become dominant. In other words, the de Broglie wavelength of the particles become comparable to their own Schwarzschild radius.

## Thermal History: Planck Epoch

Once the de Broglie wavelength is smaller than the corresponding Schwarzschild radius, the particle has essentially become a "quantum black hole":



These two mass scales define the epoch of quantum cosmology, in which the purely deterministic metric description of gravity by the theory of relativity needs to be augmented by a theory incorporating quantum effects: quantum gravity.

# Thermal History: Planck Epoch

On the basis of the expressions of the de Broglie wavelength and the Schwarzschild radius we may infer the typical mass scale, length scale and timescale for this epoch of quantum cosmology:

$$m_{Pl} \equiv \sqrt{\frac{\hbar c}{G}} \sim 10^{19} \,\text{GeV} \qquad \text{Planck Mass}$$

$$l_{Pl} \equiv \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35} \,\text{m} \qquad \text{Planck Length}$$

$$t_{Pl} \equiv \sqrt{\frac{\hbar G}{c^5}} \sim 10^{-43} \,\text{sec} \qquad \text{Planck Time}$$

Because our physics cannot yet handle quantum black holes, i.e. because we do not have any viable theory of quantum gravity we cannot answer sensibly questions on what happened before the Planck time. In other words, we are not able to probe the ultimate cosmic singularity ... some ideas of how things may have been do exist

## Statistical Equilibrium

## Maxwell-Boltzmann

Non-relativistic medium

For statistical equilibrium, the Maxwell-Boltzmann distribution specifies for a temperature T, the density of particles:

- number density n<sub>i</sub>
- particles with mass m<sub>i</sub>
- statistical weight g<sub>i</sub>
- chemical potential  $\mu_i$

$$n_{i} = g_{i} \frac{\left(2\pi m_{i} kT\right)^{3/2}}{\left(2\pi\hbar\right)^{3}} \exp\left\{\frac{\mu_{i} - m_{i} c^{2}}{kT}\right\}$$

# Echo of the Big Bang:

Recombination, Decoupling, Last Scattering

#### **CMB Discovery:** Penzias & Wilson



#### Cosmic Light (CMB): the facts



#### **Recombination & Decoupling**



## the Cosmic TV Show



#### Note:

far from being an exotic faraway phenomenon, realize that the CMB nowadays is counting for approximately 1% of the noise on your (camping) tv set ...

!!!! Live broadcast from the Big Bang !!!!

Courtesy: W. Hu

## Recombination & Decoupling

- T ~ 3000 K
- $z_{dec}=1089$  ( $\Delta z_{dec}=195$ );  $t_{dec}=379.000$  yrs
- Before the "Recombination Epoch Radiation and Matter are tightly coupled through Thomson scattering.
- The events surrounding "recombination" exist of THREE major (coupled, yet different) processes:

Recombination
Decoupling
Last scattering

protons & electrons combine to H atoms photons & baryonic matter no longer interact

meaning, photons have a last kick and go ...

# Recombination & DecouplingT ~ 3000 K $z_{dec}=1089$ ( $\Delta z_{dec}=195$ ); $t_{dec}=379.000$ yrs• Before this time, radiation and matter are tightly coupled throughThomson scattering: $e^- + \gamma \rightarrow e^- + \gamma$

Because of the continuing scattering of photons, the universe is a "fog".

A radical change of this situation occurs once the temperature starts to drop below  $T \sim 3000$  K. and electrons. Thermodynamically it becomes favorable to form neutral (hydrogen) atoms H (because the photons can no longer destory the atoms):  $p + e^{-} \leftrightarrow H$ 

This transition is usually marked by the word "recombination", somewhat of a misnomer, as of course hydrogen atoms combine just for the first time in cosmic history. It marks a radical transition point in the universe's history.

## **Recombination - Saha**

**Recombination Process:** 

$$p + e^- \rightleftharpoons H + \gamma$$

Statistical Equilibrium sets the density of electrons, protons and hydrogen atoms involved in the recombination process:



## **Recombination - Saha**

**Recombination Process:** 

$$p + e^- \rightleftharpoons H + \gamma$$

Taking along that for the chemical potentials

$$\mu_p + \mu_e = \mu_H$$

we find for the relation between the number densities

$$\frac{n_{H}}{n_{e}n_{p}} = \frac{g_{H}}{g_{e}g_{p}} \left(\frac{m_{H}}{m_{e}m_{p}}\right)^{3/2} \left(\frac{kT}{2\pi\hbar^{2}}\right)^{-3/2} \exp\left\{\frac{[m_{p}+m_{e}-m_{H}]c^{2}}{kT}\right\}$$

## **Recombination - Saha**

**Recombination Process:** 

$$p + e^- \rightleftharpoons H + \gamma$$

 $m_H / m_p \approx 1$ 

- mass electron small:
- binding energy hydrogen atom:
- weights g<sub>i</sub>:

$$g_e = 2, \quad g_p = 2, \quad g_H =$$

 $(m_e + m_p - m_H)c^2 = \chi = 13.6 \ eV$ 

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results in the Saha Equation,

$$\frac{n_H}{n_e n_p} = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left\{\frac{\chi}{kT}\right\}$$

which specifies the shifting ionization state as a function of shifting temperature T

#### **Recombination - Ionization**

**Recombination Process:** 

$$p + e^- \rightleftharpoons H + \gamma$$

Photon number density in blackbody bath temperature T:

Ionization fraction x:

$$n_{\gamma} = \frac{2.464}{\pi^2} \left( \frac{\kappa T}{\hbar c} \right) = 0.243 \left( \frac{\kappa T}{\hbar c} \right)$$
$$X = \frac{n_p}{n_p + n_H} \qquad n_e = n_p = Xn$$
$$n = n_p + n_H \simeq n_b$$

 $2 \Lambda 0 \Lambda (kT)^3 \qquad (kT)^3$ 

**Baryon-photon ratio η:** 

$$\eta = \frac{n_b}{n_{\gamma}} = \frac{n_p}{Xn_{\gamma}}$$

Proton number density at T:

 $n_p = 0.243 \, X \, \eta \left(\frac{kT}{\hbar c}\right)^2$ 



#### **Recombination - Ionization**

**Recombination Process:** 

$$p + e^- \rightleftharpoons H + \gamma$$

**Relation between temperature T and ionization fraction X:** 

$$\frac{1-X}{X^2} = 3.84\eta \left(\frac{kT}{m_e c^2}\right)^{3/2} \exp\left\{\frac{\chi}{kT}\right\}$$

Moment of recombination:

$$X = \frac{1}{2}$$
  $kT_{rec} \approx 0.323 \ eV = 3740 \ K$ 

#### Standard theory of H recombination (Peebles 1968, Zel'dovich et al 1968)



## Decoupling

**Decoupling:** 

$$\gamma + e^- \rightleftharpoons \gamma + e^-$$

Thomson scattering: Elastic scattering of photons off electrons

$$\sigma_e = 6.65 \times 10^{-29} m^2$$

Mean free path:

$$\lambda = \frac{1}{n_e \sigma_e}$$

**Interaction rate:** 

$$\Gamma = \frac{c}{\lambda} = n_e \sigma_e c$$

#### **Decoupling - Primordial Plasma**

**Decoupling:** 

$$\gamma + e^- \rightleftharpoons \gamma + e^-$$

Thomson scattering:

#### Fully ionized plasma:

$$n_{e} = n_{p} = n_{b}$$

$$n_{e} = n_{b} = \frac{n_{b,0}}{a^{3}}$$
Interaction rate:  $\Gamma = \frac{n_{b,0} \sigma_{e} c}{a^{3}} s^{-1} = \frac{4.4 \times 10^{-21}}{a^{3}} s^{-1}$ 
 $a = 10^{-5}$ : ~3 per week

#### **Decoupling - Primordial Plasma**

**Decoupling:** 

$$\gamma + e^- \rightleftharpoons \gamma + e^-$$

Thomson scattering:

#### Fully ionized plasma:

Interaction rate - Hubble expansion rate:  $\Gamma > H$  $\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} \implies H = \frac{H_0 \Omega_{r,0}^{-1/2}}{a^2} = 2.1 \times 10^{-20} \ s^{-1}$ radiation-dominated phase  $\Gamma = \frac{n_{b,0} \ \sigma_e c}{a^3} \ s^{-1} = \frac{4.4 \times 10^{-21}}{a^3} \ s^{-1}$ 

If fully ionized: decoupling at  $a \approx 0.023 \implies z \approx 42$ , T = 120 K

#### **Decoupling - Recombination**

**Decoupling:** 

$$\gamma + e^- \rightleftharpoons \gamma + e^-$$

Thomson scattering:

While plasma undergoes recombination, number density electrons n<sub>e</sub> decreases, substantially altering interaction rate:

$$\Gamma(z) = n_e(z)\sigma_e c = X(z)(1+z)^3 n_{b,0} \sigma_e c$$
  
= 4.4×10<sup>-21</sup> s<sup>-1</sup> X(z)(1+z)<sup>3</sup>

#### **Decoupling - Recombination**

**Decoupling:** 

$$\gamma + e^- \rightleftharpoons \gamma + e^-$$

Thomson scattering:

#### Interaction rate vs. Hubble expansion rate:

$$\frac{H^2}{{H_0}^2} = \frac{\Omega_{m,0}}{a^3} = \Omega_{m,0} (1+z)^3 \implies H(z) = 2.1 \times 10^{-18} (1+z)^{3/2} s^{-1}$$
  
Decoupling when  $\Gamma < H$   
$$1 + z_{dec} = \frac{43.0}{X(z_{dec})^{2/3}}$$

Saha equation value X(z):

 $z_{dec} \approx 1\overline{130}$ 

#### Last Scattering

Thomson scattering:

$$\gamma + e^- \rightleftharpoons \gamma + e^-$$

**Probability scattering in time interval dt:** 

$$dP = \Gamma(t) \ dt$$

Expected number of scatterings since time t when CMB photon seen at t0,

$$\tau(t) = \int_{t}^{t_0} \Gamma(t) \, dt$$

This is Optical Depth !

#### Last Scattering

Thomson scattering:

$$\gamma + e^- \rightleftharpoons \gamma + e^-$$

Last Scattering Epoch:

time t for which

$$\tau(t) = \int_{t}^{t_0} \Gamma(t) \, dt = 1$$

Last scattering epoch:

$$\tau(a) = \int_{a}^{1} \Gamma(a) \frac{da}{\dot{a}} = \int_{a}^{1} \frac{\Gamma(a)}{H(a)} \frac{da}{a} \implies \tau(z) = \int_{0}^{z} \frac{\Gamma(z)}{H(z)} \frac{dz}{1+z} = 0.0035 \int_{0}^{z} X(z) (1+z)^{1/2} dz$$

$$z_{ls} \approx z_{dec} \approx 1100$$



We can only see the surface of the cloud where light

## Recombination & Decoupling

 In summary, the recombination transition and the related decoupling of matter and radiation defines one of the most crucial events in cosmology. In a rather sudden transition, the universe changes from



After z<sub>dec</sub>, z<z<sub>dec</sub>

- universe practically neutral
- photons propagate freely
- pressure only by

baryons:

$$p = n k T$$

• (photon pressure negligible)

Origin CMB Photons

Important Issues: • When were the CMB photons produced ? • How did they become a blackbody/thermal radiation field

 At which time were they scattered for the last time (in other words, what are we looking at ?)

Origin CMB Photons

#### $T < 10^9 K$

 $t \sim 1 \min, z \sim 10^9$ 

Origin CMB photons:

 $e^+ + e^- \leftrightarrow 2\gamma$ 

most were produced when

electrons & positrons annihilated each other

• (a few perhaps even at reheating phase inflation)

## CMB thermalization

- At the onset certainly not thermally distributed energies
- Photons keep on being scattered back and forth until z ~ 1089, the epoch of recombination.
- Thermal equilibrium (blackbody spectrum) of photons reached within
   2 months after their creation

Blackbody Spectrum produced through three scattering processes

- Compton scattering
- Free-free scattering
- Double Compton scattering

## **CMB** Thermalization

Thermalization through three scattering processes

- Compton scattering
- Free-free scattering
- Double Compton scattering

+ dominant energy redistribution + creates new photons to adjust spectrum to Planck

While Compton scattering manages to redistribute the energy of the photons, it cannot adjust the number of photons. Free-free scattering and Double Compton scattering manage to do so ...

2 But ...

?

only before  $z < 10^5$ , after that the interaction times too long ....

## **CMB** Thermalization

Following this thermalization, a perfect blackbody photon spectrum has emerged:

$$I_{\nu}(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

This is the ULTIMATE proof of the HOT BIG BANG

Note: after z ~ 10<sup>5</sup> till recombination, the interaction between electrons and photons exclusively by Thomson Scattering

## **CMB** hermalization



Chluba & Silk 2015

## **First Three Minutes:**

#### **Big Bang Nucleosynthesis**

#### Photon Energy Early Universe

Radiation-dominated phase:

$$T(t) \approx 10^{10} K \left(\frac{t}{1 \text{ sec}}\right)^{-1/2}$$

$$kT \approx 1 \ MeV\left(\frac{t}{1 \ \text{sec}}\right)^{-1/2}$$

## **Big Bang Nucleosynthesis**

p/n ~1/7: 1 min na BB



Mass Fraction Light Elements

24%4He nucleitracesD, 3He, 7Li nuclei75%H nuclei (protons)



Between 1-200 seconds after Big Bang, temperature dropped to 10<sup>9</sup> K:

Fusion protons & neutrons into light atomic nuclei

#### Neutron-Proton – before Neutrino Decoupling

**Before Neutrino Decoupling:** 

equilibrium between protons & neutrons through 2 weak interactions:P

$$n + v_e \rightleftharpoons p + e^-$$

$$n + e^+ \rightleftharpoons p + \overline{v}_e$$

we find for the relation between the number densities

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} \exp\left\{\frac{[m_n - m_p]c^2}{kT}\right\}$$

## Neutron-Proton - Saha



## **Deuterium-Neutron - Saha**



## **Big Bang Reaction Network**



## **Nucleus Binding Energies**







## Helium Abundance – Neutrino's

