Cosmology,

lect. 6a

Dark Energy

General Relativity:

Einstein Field Equations

Einstein Field Equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

Metric tensor: $g_{\mu\nu}$

Energy-Momentum tensor: $T_{\mu\nu}$

$$T_{\mu\nu} = \left(\rho + \frac{p}{c^2} \right) U^{\mu} U^{\nu} - p g^{\mu\nu}$$

Einstein Field Equation

Einstein Tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

$$G_{\mu\nu;\nu} = T_{\mu\nu;\nu} = 0$$

Einstein Tensor only rank 2 tensor for which this holds:

$$G_{\mu\nu} \propto T_{\mu\nu}$$

Einstein Field Equation

also: $$g_{\mu\nu;\nu} = 0$$

Freedom to add a multiple of metric tensor to Einstein tensor:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

Λ : Cosmological Constant

Einstein Field Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

Dark Energy

$$G^{\mu\nu} = -\frac{8\pi G}{c^4}\left(T^{\mu\nu} + T^{\mu\nu}{}_{vac}\right)$$

$$T^{\mu\nu}{}_{vac} \equiv \frac{\Lambda c^4}{8\pi G} g^{\mu\nu}$$

Einstein Field Equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Einstein Field Equation

curvature side

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu}$$

energy-momentum side

Equation of State

$$T^{\mu\nu}{}_{vac} \equiv \frac{\Lambda c^4}{8\pi G} g^{\mu\nu}$$

restfraame

$$T^{\mu\nu}{}_{vac} \equiv \frac{\Lambda c^4}{8\pi G} \eta^{\mu\nu}$$

$$\eta^{00} = 1, \quad \eta^{ii} = -1$$

$$T_{\mu\nu} = \left(\rho + \frac{p}{c^2} \right) U^{\mu} U^{\nu} - p g^{\mu\nu}$$

restframe:

$$\left. \begin{aligned} T^{00}{}_{vac} &= \rho_{vac} c^2 \\ T^{ii}{}_{vac} &= p \end{aligned} \right\} \Rightarrow \quad \begin{aligned} \rho_{vac} c^2 &= \frac{\Lambda c^4}{8\pi G} \\ p &= -\frac{\Lambda c^4}{8\pi G} \end{aligned}$$

Equation of State

$$\rho_{vac} c^2 = \frac{\Lambda c^4}{8\pi G}$$

$$p = -\frac{\Lambda c^4}{8\pi G}$$

$$p_{vac} = -\rho_{vac} c^2$$

Dynamics

Relativistic Poisson Equation:

$$\nabla^2 \phi = 4\pi G \left(\rho + \frac{3p}{c^2} \right)$$

$$\rho_{vac} + \frac{3p_{vac}}{c^2} = -2\rho_{vac} < 0; \qquad \rho_{vac} = \frac{\Lambda}{8\pi G}$$

⇓

$$\nabla^2 \phi < 0$$

Repulsion !!!

the Source:

Dark Energy

Dark Energy & Cosmic Acceleration

Nature Dark Energy:

(Parameterized) Equation of State

$$p(\rho) = w\rho c^2$$

Cosmic Acceleration:

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a$$

Gravitational Repulsion:

$$p = w\rho c^2 \quad \Leftrightarrow \quad w < -\frac{1}{3} \quad \Rightarrow \quad \ddot{a} > 0$$

Dark Energy: Identity & Nature

Huge and ever growing list of suggestions on

identity & nature of Dark Energy:

- Cosmological Constant
- Cosmic Backreaction (inhomogeneities)
- Modified Gravity
- Quintessence, in a variety of flavours
- Phantom Energy
- Chameleon Energy
- Chaplygin gas
- Agegraphic DE
- ….

Dark Energy = Vacuum Energy

Ya. Zel'dovich - 1960s
S. Weinberg - 1989

Cosmological Constant to be identified with zero-point vacuum energy ?

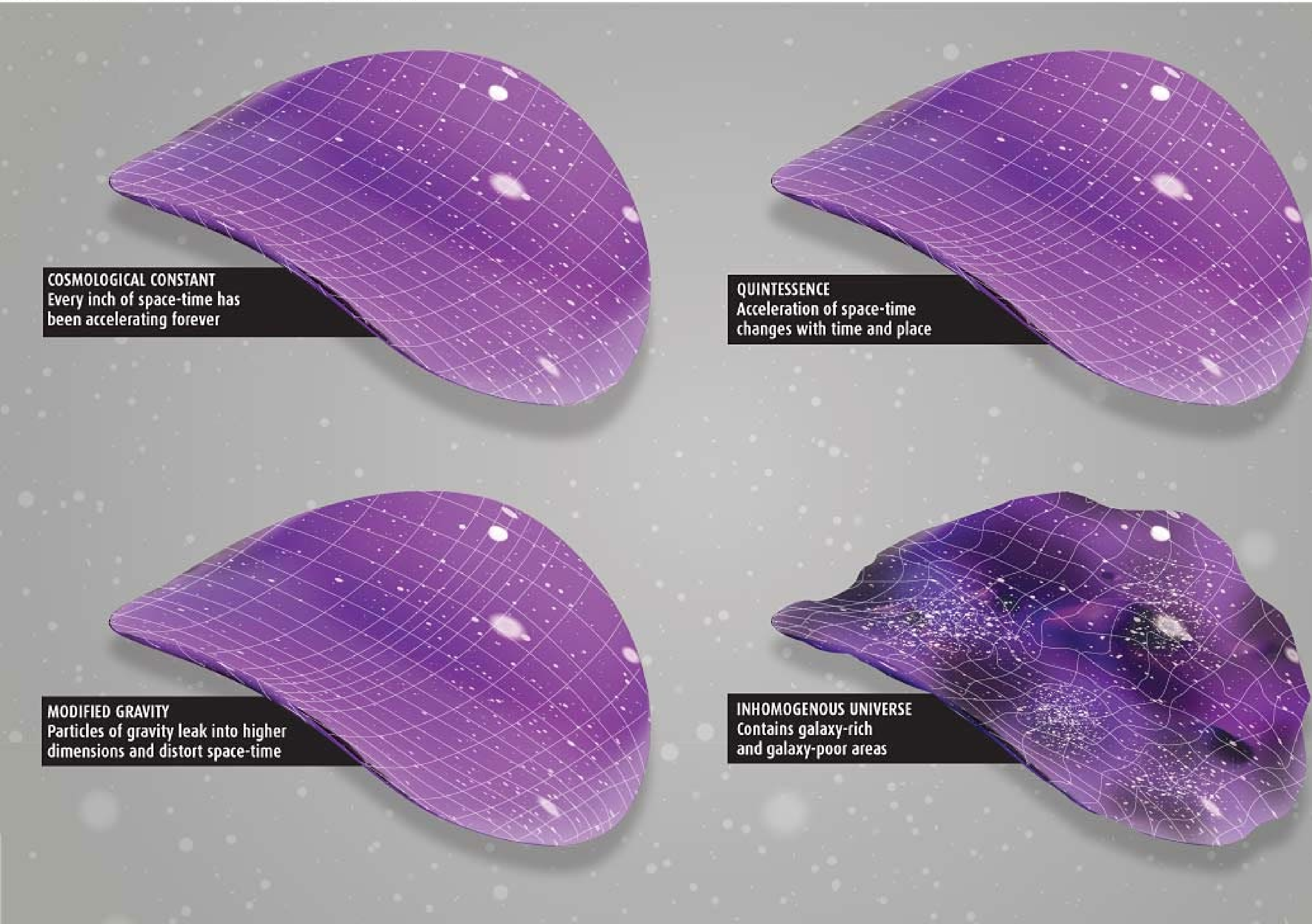
minor problem:

1st order estimate off by 120 orders magnitude:

$\sim 10^{120}$

FOUR WAYS TO EXPAND THE UNIVERSE

How space-time looks depends on the nature of dark energy



Dark Energy & Cosmic Acceleration

DE equation of State

$$p(\rho) = w\rho c^2$$

$$\rho_w(a) = \rho_w(a_0)\, a^{-3(1+w)}$$

Cosmological Constant:

$\Lambda :$ $w = -1$ $\rho_w = cst.$

-1/3 > w > -1:

$\rho_w \propto a^{-3(1+w)}$ $1 + w > 0$ **decreases with time**

Phantom Energy:

$\rho_w \propto a^{-3(1+w)}$ $1 + w < 0$ **increases with time**

Phantom Energy:

De Big Rip ?

Dynamic Dark Energy

DE equation of State

$$p(\rho) = w\rho c^2$$

Dynamically evolving dark energy, parameterization:

$$w(a) = w_0 + (1-a)w_a \approx w_\phi(a)$$

$$\rho_w(a) = \rho_w(a_0)\ \exp\left\{-3\int_1^a \frac{1+w_\phi(a')}{a'}\,da'\right\}$$

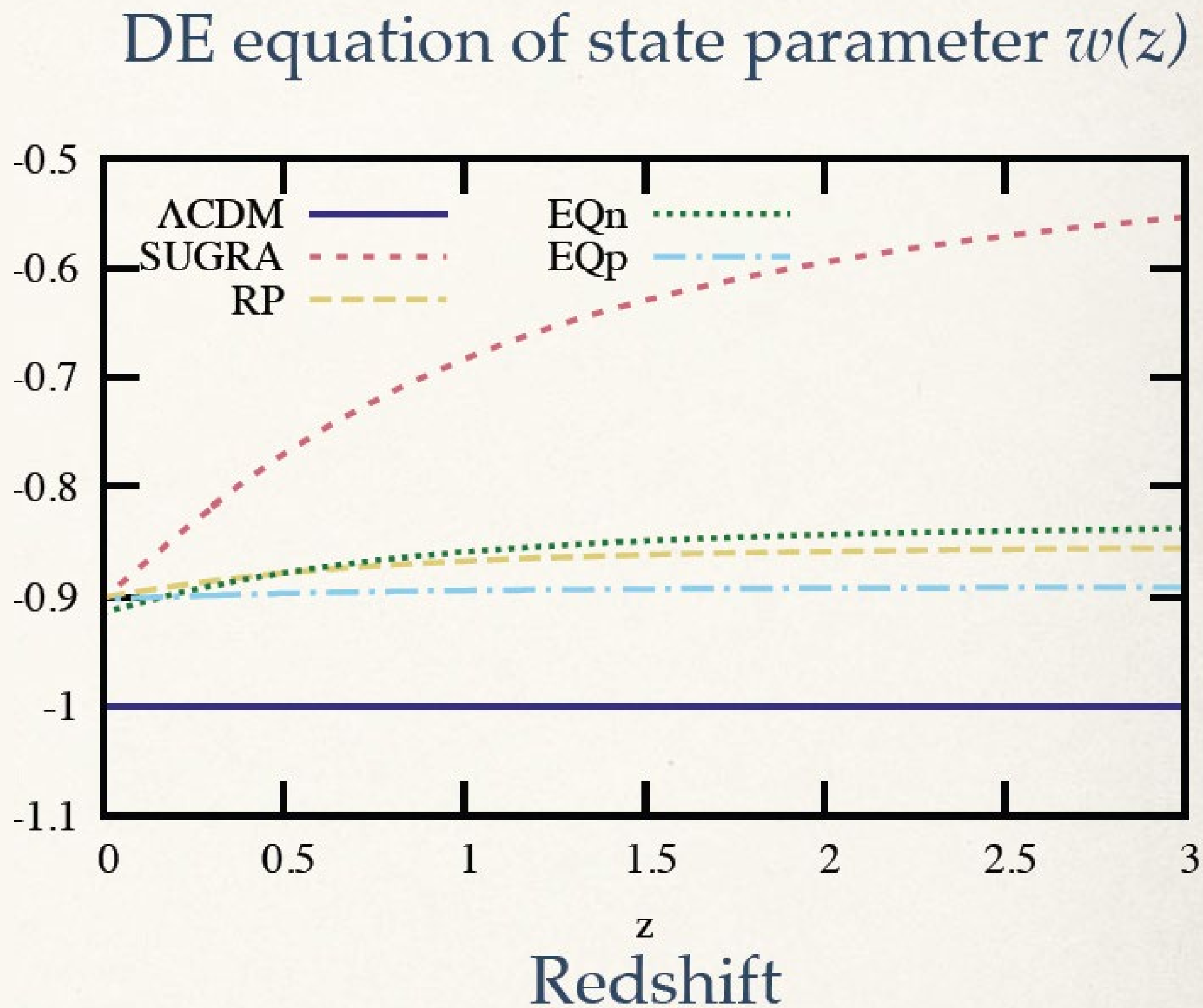
DE Equation of State

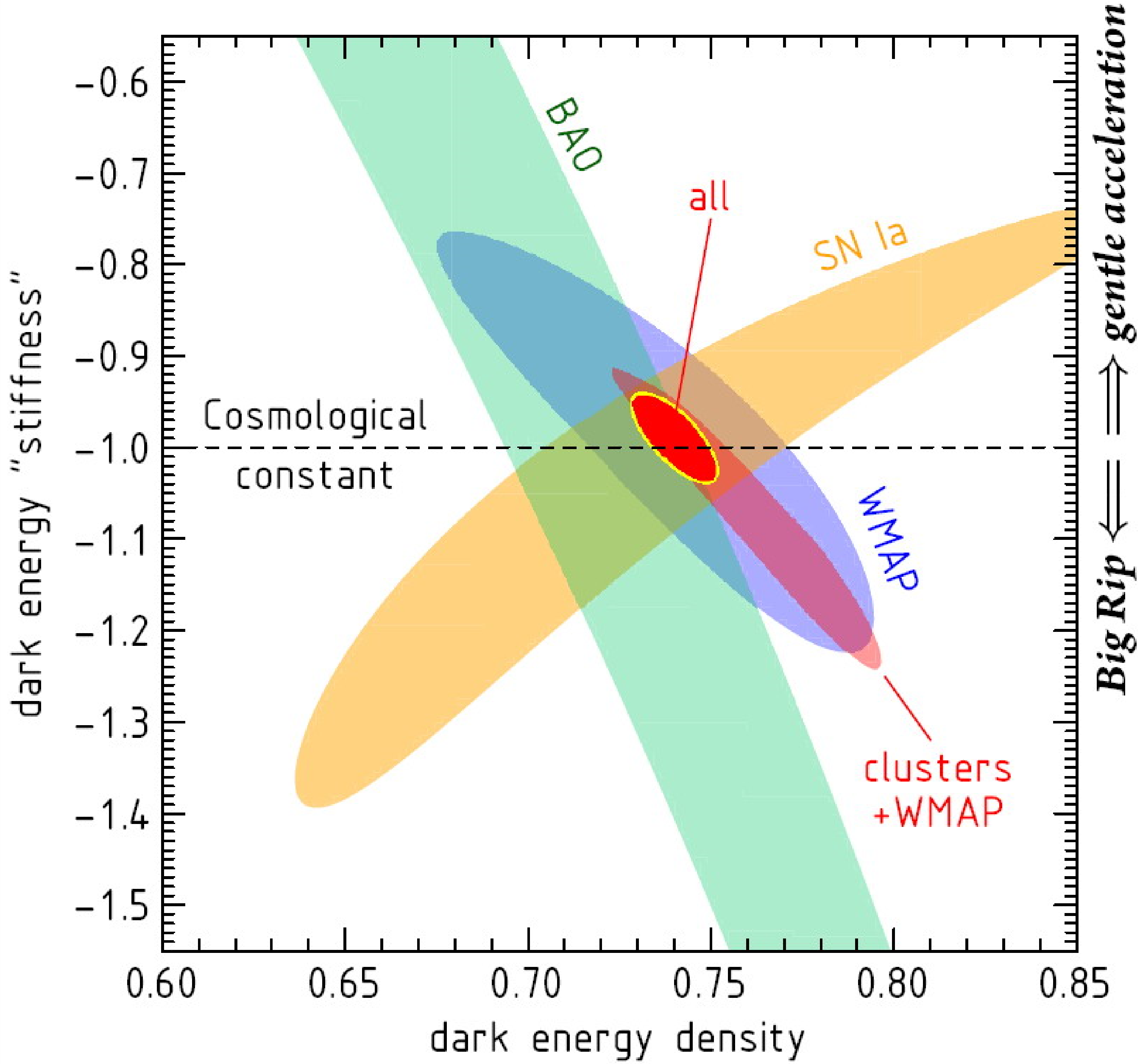
$$w(a) \equiv w_0 + w_a(1-a) \approx w_\phi(a)$$

$$H(a) = H_0\sqrt{\Omega_m a^{-3} + \Omega_\Lambda}$$

$$\Omega_\Lambda \to \Omega_w a^{-3(1+w)}$$

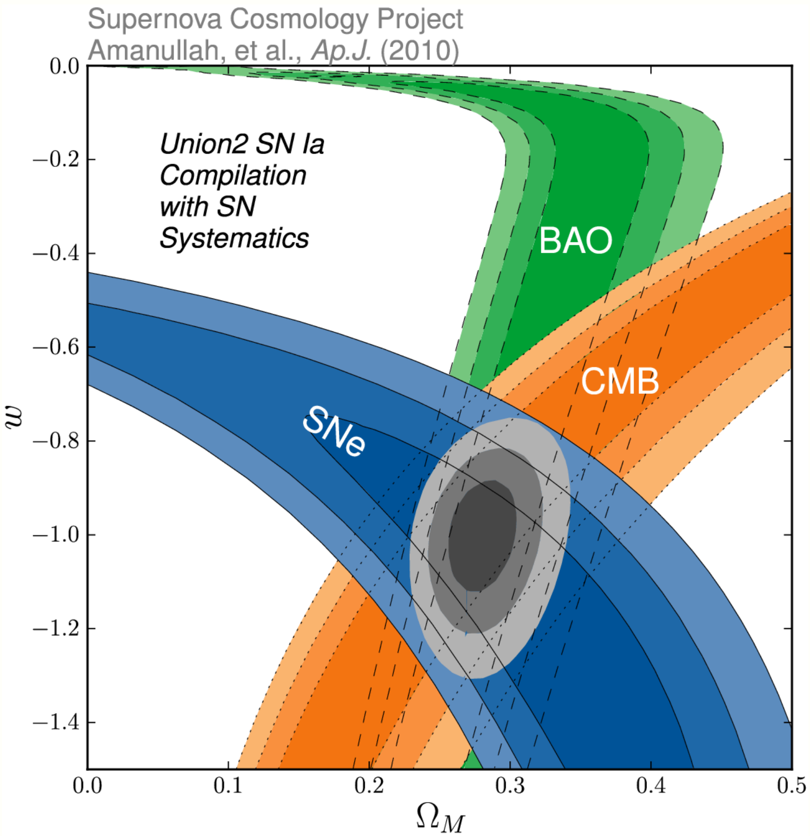
$$\Omega_\Lambda \to \Omega_\phi \exp\left(-3\int_1^a \frac{1+w_\phi(a')}{a'}da'\right)$$

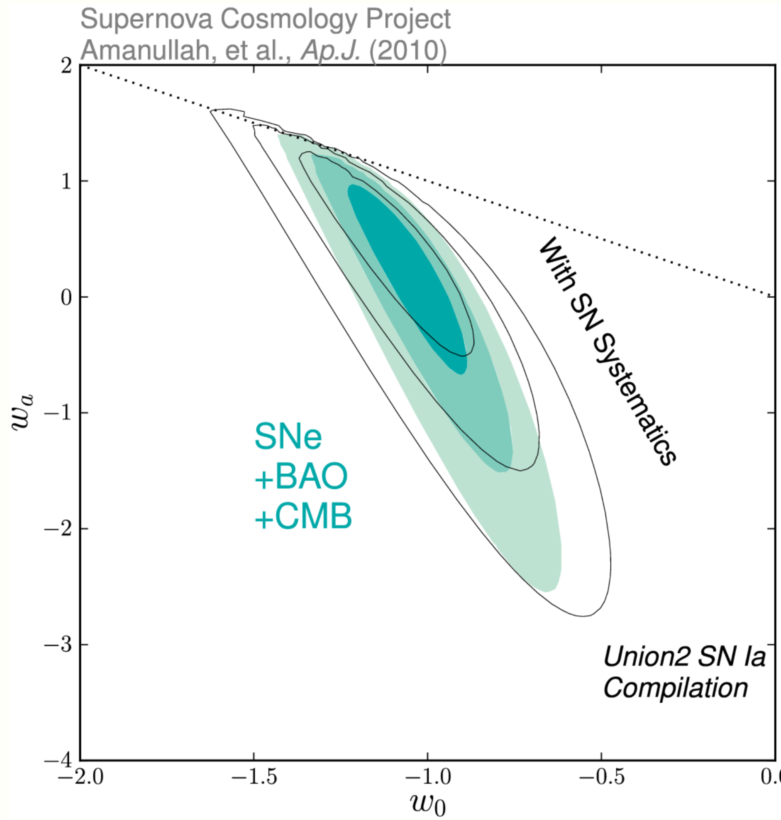




dark energy "stiffness"
dark energy density
BAO
all
SN Ia
WMAP
clusters +WMAP
Cosmological constant
Big Rip ⟸ ⟹ gentle acceleration
-0.6
-0.7
-0.8
-0.9
-1.0
-1.1
-1.2
-1.3
-1.4
-1.5
0.60
0.65
0.70
0.75
0.80
0.85

Dark Energy Eqn.State





SCP Union2 constraints (2010)

on values of matter density Ω_m
dark energy eqn. state w

on dynamical evolution dark energy:

eqn. state parameters w_0
w_a

Take-Home Facts

1. **Strong evidence Accelerated Expansion**
   - since supernova discovery, 100s SNIa observed over broader range redshifts
   - based solely upon supernova Hubble diagram, independent of General Relativity, very strong evidence expansion Universe accelerated recently

2. **Dark energy as cause cosmic acceleration**
   - within general relativity, accelerated expansion cannot be explained by any known form of matter or energy
   - it can be accommodated by a nearly smooth form of energy with large negative pressure, Dark Energy, that accounts for about 73% of the universe.

3. **Independent evidence dark energy**
   - Cosmic Microwave Background and Large Scale Structure data provide independent evidence, within context of CDM model of structure formation, that the universe is filled with a smooth medium accounting for 73% of the total energy content of the universe.
   - that came to dominate the dynamics of the universe once all observed structure had formed

Take-Home Facts

4. Vacuum energy as dark energy

- simplest explanation for dark energy is the energy associated with the vacuum
- mathematically equivalent to a cosmological constant
- However, most straightforward calculations of vacuum energy density from zero-point energies of all quantum fields lead to estimates which are a bit too large, in the order of $\sim 10^{120}$

5. Dark theories of Dark Energy

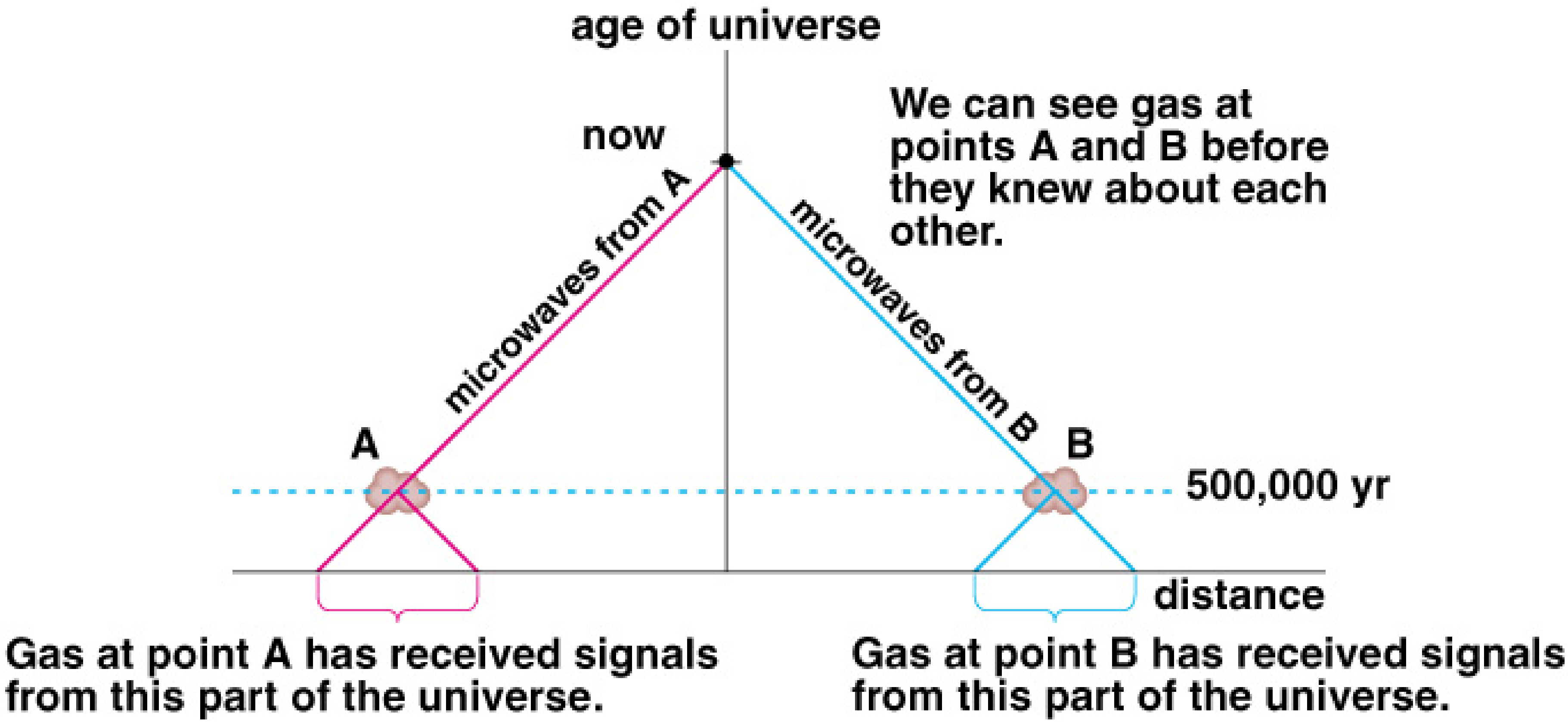
- There is no compelling theory of dark energy
- Beyond vacuum energy, man intriguing ideas: light scalar fields, additional spatial dimensions, etc.
- Many models involve time-varying dark energy

6. New Gravitational Theories ?

- alternatively, cosmic acceleration could be a manifestaation of gravitational physics beyond General Relativity
- however, as yet there is no self-consistent model for new gravitational physics that is consistent with large body of data that constrains theories of graavity.

Cosmic Future

Cosmic Horizons



Particle Horizon of the Universe:
distance that light travelled since the Big Bang

Cosmic Particle Horizon

Light travel in an expanding Universe:

- **Robertson-Walker metric:** $$ds^2 = c^2 dt^2 - a(t)^2 dr^2$$
- **Light:** $$ds^2 = 0$$

$$d_{Hor} = \int_0^t \frac{c\,dt'}{a(t')}$$

Horizon distance in comoving space

$\Leftrightarrow$

$$R_{Hor} = a(t) \int_0^t \frac{c\,dt'}{a(t')}$$

Horizon distance in physical space

Particle Horizon of the Universe:
distance that light travelled since the Big Bang

Cosmic Event Horizon

Light travel in an expanding Universe:

- **Robertson-Walker metric:** $ds^2 = c^2 dt^2 - a(t)^2 dr^2$
- **Light:** $ds^2 = 0$

$$d_{event} = \int_t^{\infty} \frac{c\,dt'}{a(t')}$$

Event Horizon distance in comoving space

⇔

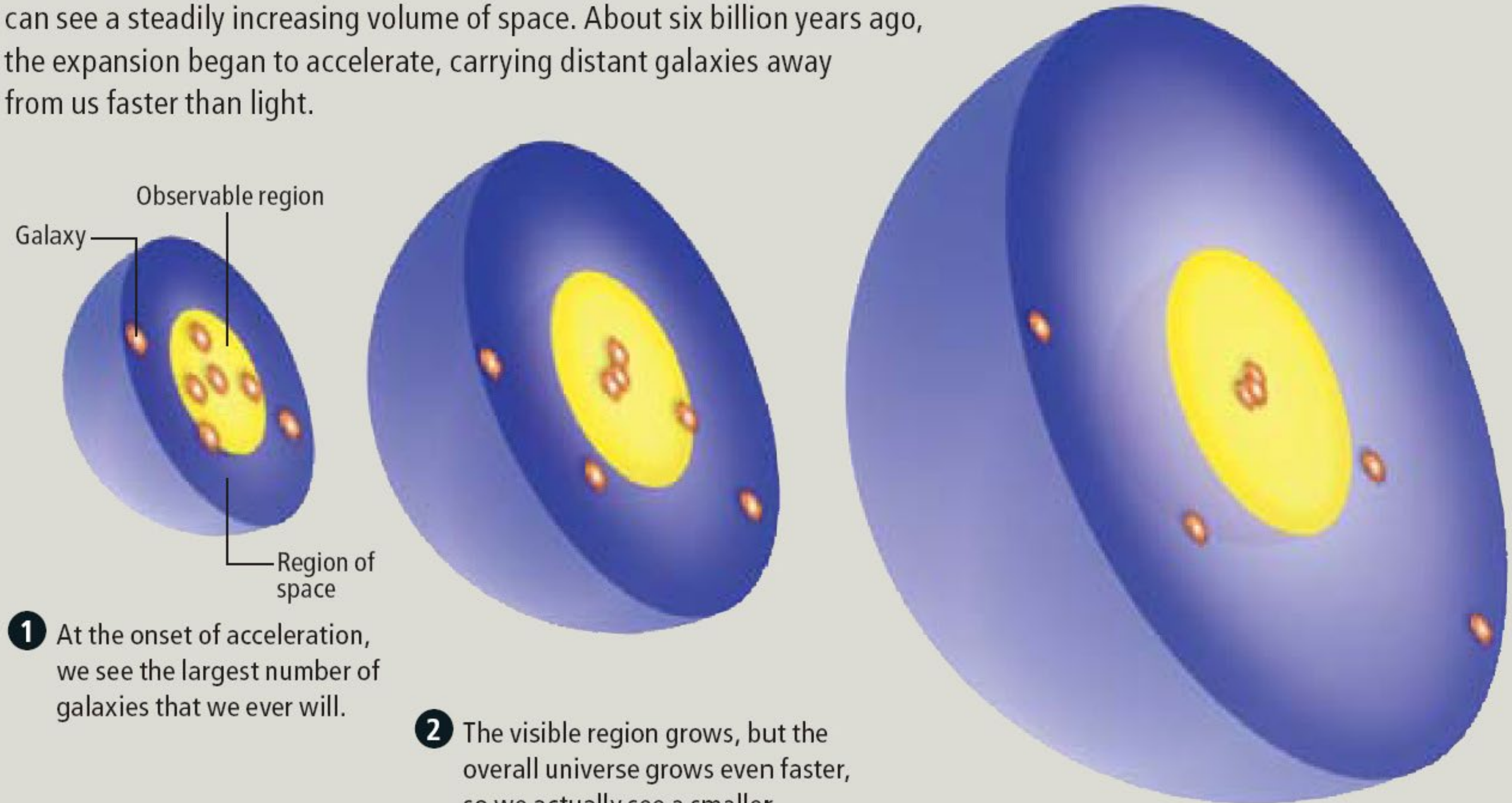
$$R_{event} = a(t) \int_t^{\infty} \frac{c\,dt'}{a(t')}$$

Event Horizon distance in physical space

Event Horizon of the Universe:
the distance over which one may still communicate …

EXPANDING UNIVERSE, SHRINKING VIEW

The universe may be infinite, but consider what happens to the patch of space around us (*purple sphere*), of which we see only a part (*yellow inner sphere*). As space expands, galaxies (*orange spots*) spread out. As light has time to propagate, we observers on Earth (or our predecessors or descendants) can see a steadily increasing volume of space. About six billion years ago, the expansion began to accelerate, carrying distant galaxies away from us faster than light.



1 At the onset of acceleration, we see the largest number of galaxies that we ever will.

2 The visible region grows, but the overall universe grows even faster, so we actually see a smaller fraction of what is out there.

3 Distant galaxies (those not bound to us by gravity) move out of our range of view. Meanwhile, gravity pulls nearby galaxies together.

NOTE:
Because space is expanding uniformly, alien beings in other galaxies see this same pattern.

Cosmic Fate

100 Gigayears:
the end of Cosmology

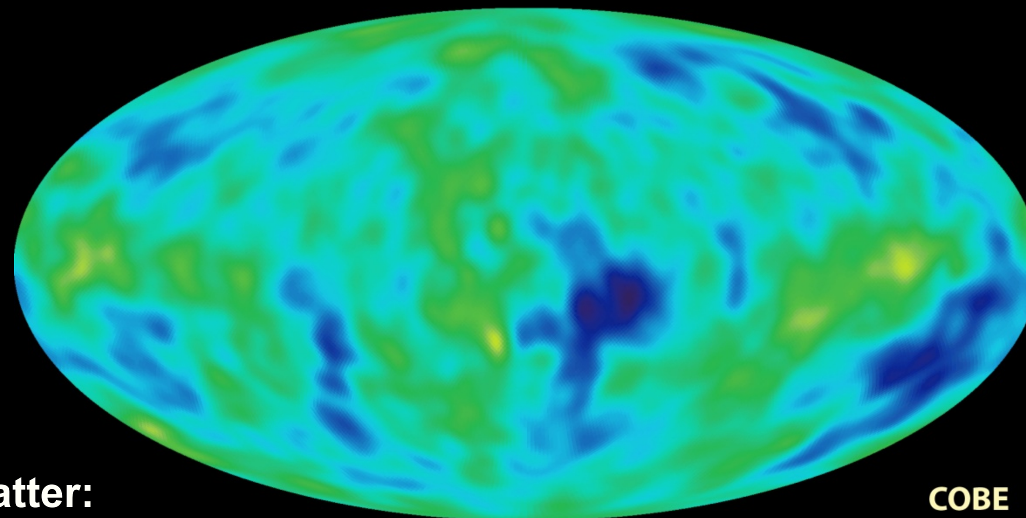
1990s:

the Brewing Crisis

Standard Cosmology ~ 1990

- FRW Universe

- augmented by Inflation
  - solved 4 fine-runing problems
  - accelerated expansion by
    factor 10^{60}
    ~ 10^{-36}-10^{-34} sec after Big Bang
  - firm prediction:
    Universe flat: $k=0$, $\Omega_{tot}=1$

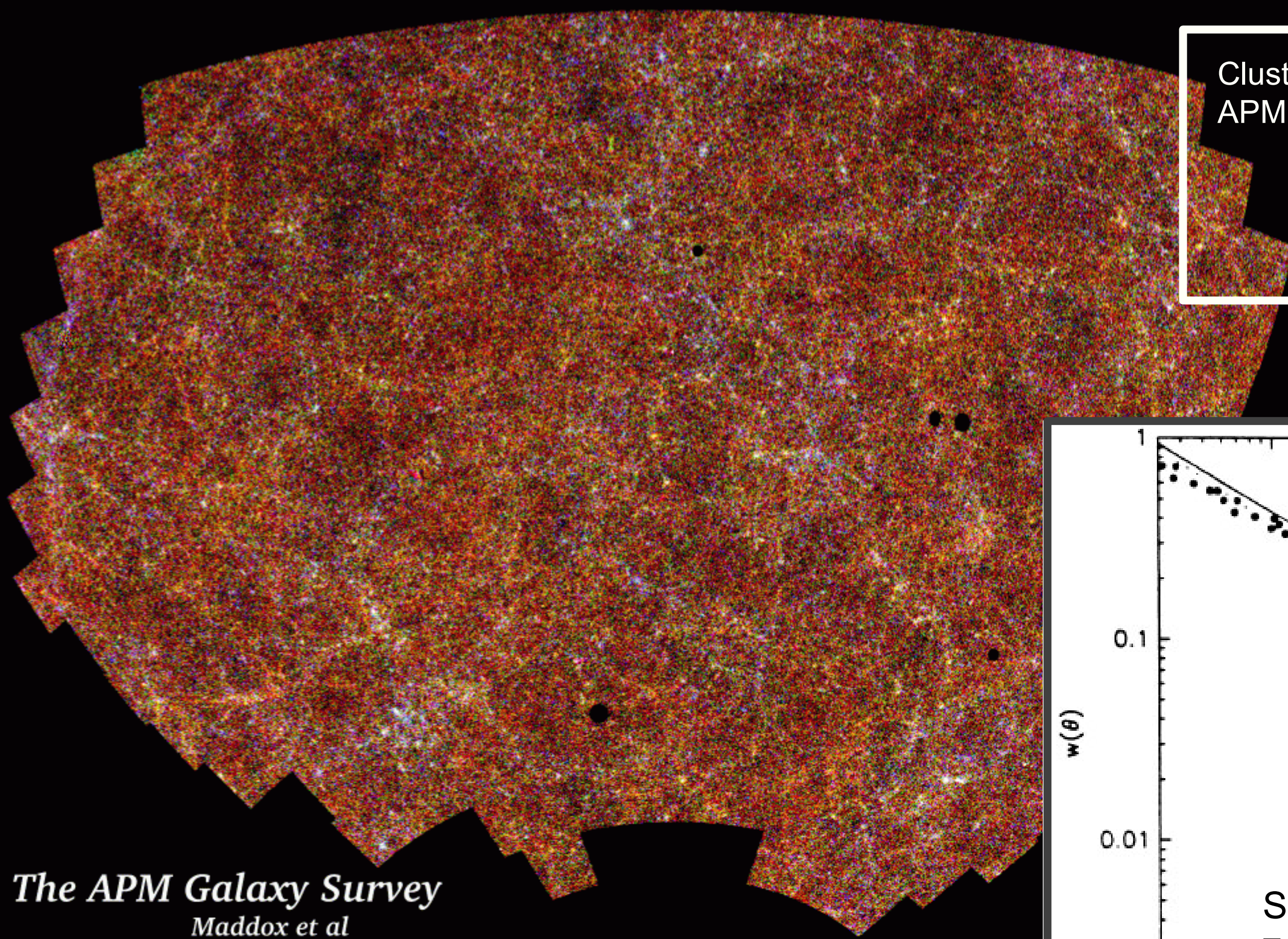


COBE

- Universe dominated by Dark Matter:
  - necessary to explain structure growth from primordial fluctuations, which COBE in 1992 had detected at 10^{-5} level
  - would have to make up 96% of matter density Universe
  - SCDM: "standard Cold Dark Matter", $\Omega_m=1.0$

- Succesfully explained large range of astronomical observations
  (or was made to explain these: "bias")

Galaxy Clustering

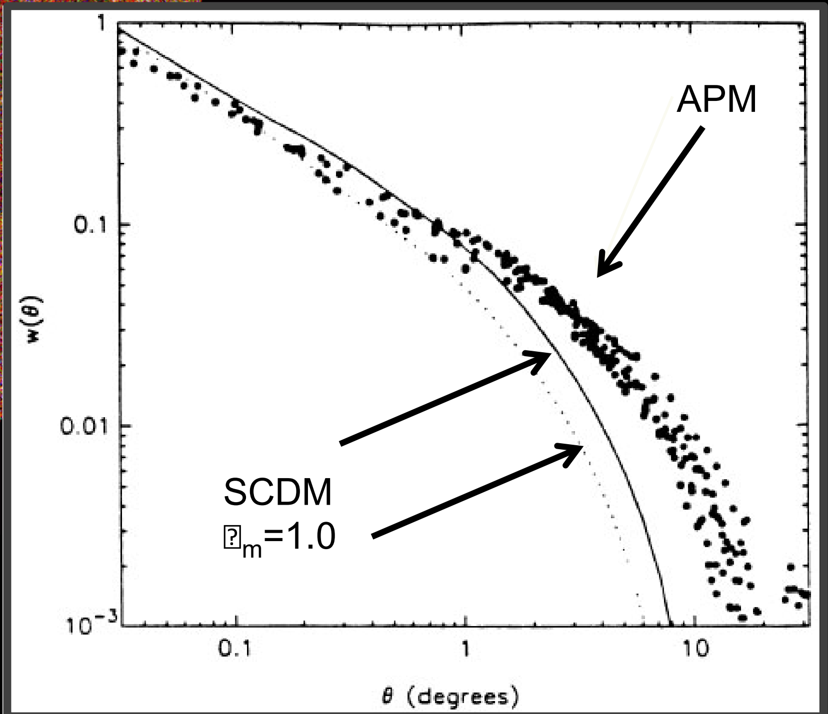


Clustering of galaxies in the plate-scanned APM sky galaxy survey (2 million gals):

$$dP(\theta) = \bar{n}^2\{1 + w(\theta)\}d\Omega_1 d\Omega_2$$

angular 2pt correlation function

Efstathiou, Sutherland & Maddox, 1990
Nature, 348, 705



"the Cosmological Constant and Cold Dark Matter"

"It is argued here that the success of the cosmological cold dark matter (CDM) model can be retained and the new observations of very large scale cosmological structures can be accommodated in a spatially flat cosmology in which as much as 80 percent of the critical density is provided by a positive cosmological constant. In such a universe, expansion was dominated by CDM until a recent epoch, but is now governed by the cosmological constant."

Cluster Baryon fraction

X-ray intracluster gas:

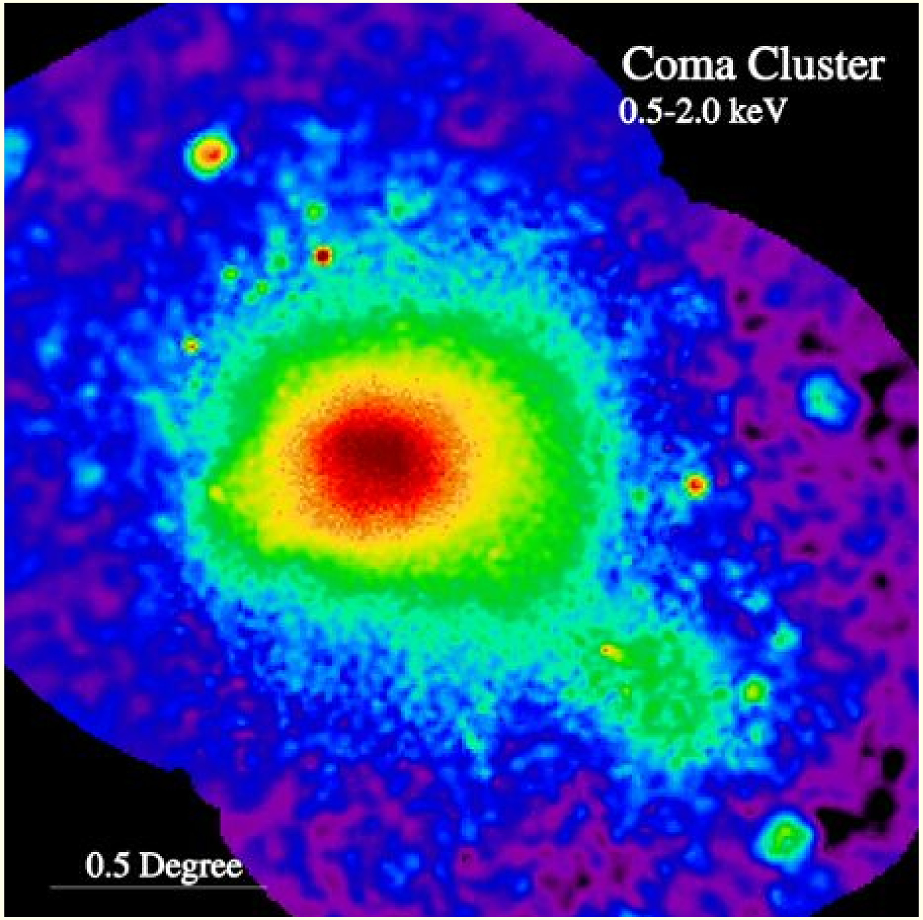
- Mass determination via hydrostatic equilibrium
- fraction mass in baryons (White et al.)

  $f_{baryon} \sim$ 1/6-1/7

- But,

  - if representative for Universe, AND
  - Ω_m=1.0
  - conflict with baryon density suggested by Big Bang nucleosynthesis: Ω_b=0.04

- Many other indications find Ω_m=0.3



ROSAT X-ray image Coma Cluster

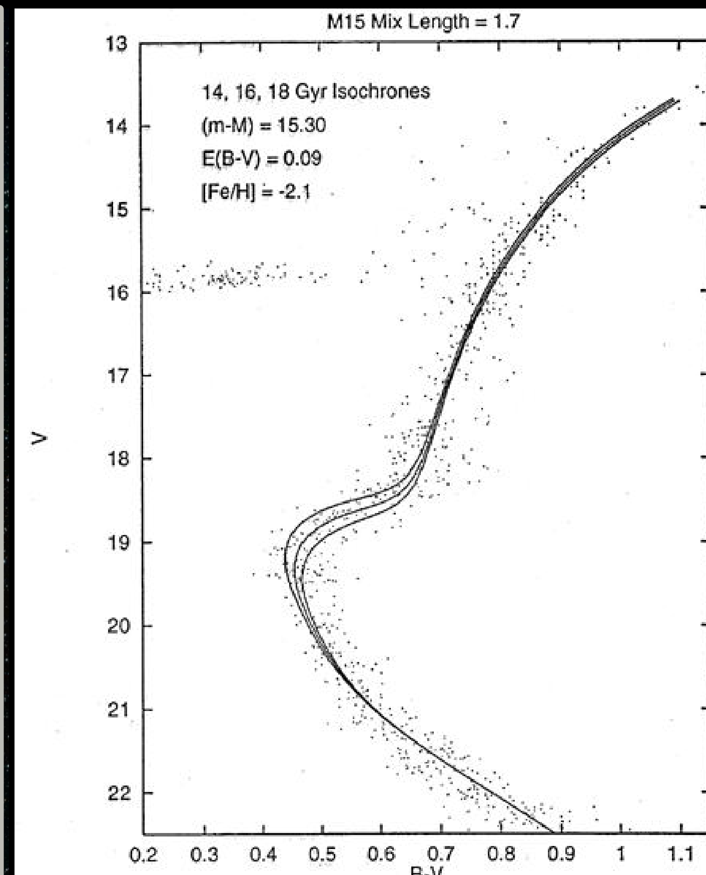
Cosmic Age Crisis

estimated age of the oldest stars in Universe far in excess of estimated age of matter-dominated FRW Universe:

Globular cluster stars: 13-15 Gyr
Universe: 10-12 Gyr

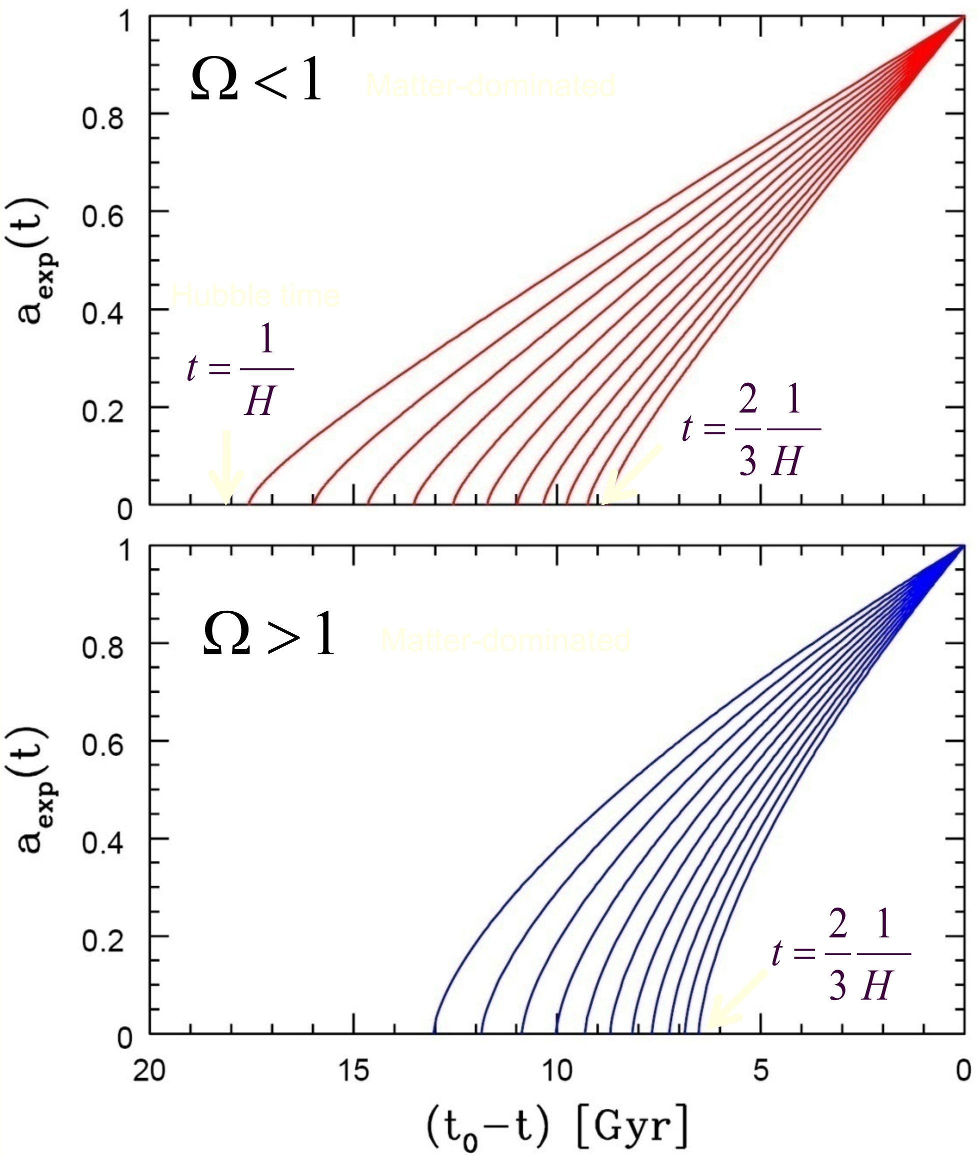
Globular Clusters

- Roughly spherical assemblies of 100,000-200,000 stars
- Radius ~ 20-50 pc: extremely high star density
- Globulars are very old, amongst oldest objects in local Universe
- Stars formed around same time: old, red, population
- Colour-magnitude diagram characteristic: accurate age determination on the basis of stellar evolution theories.



Typical 1980-1990s isochrone fit

Age of the Universe



Age of a FRW universe at
Expansion factor a(t)

$$H\,t = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{rad}}{a^2} + \frac{\Omega_m}{a} + \Omega_\Lambda a^2 + \left(1 - \Omega\right)}}$$

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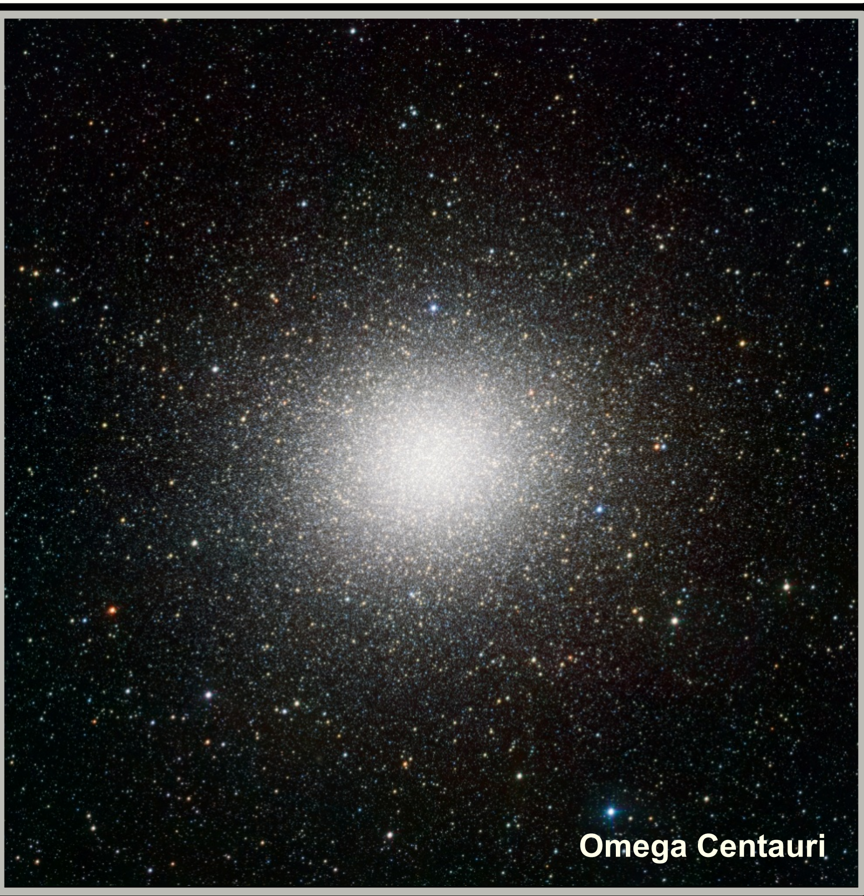
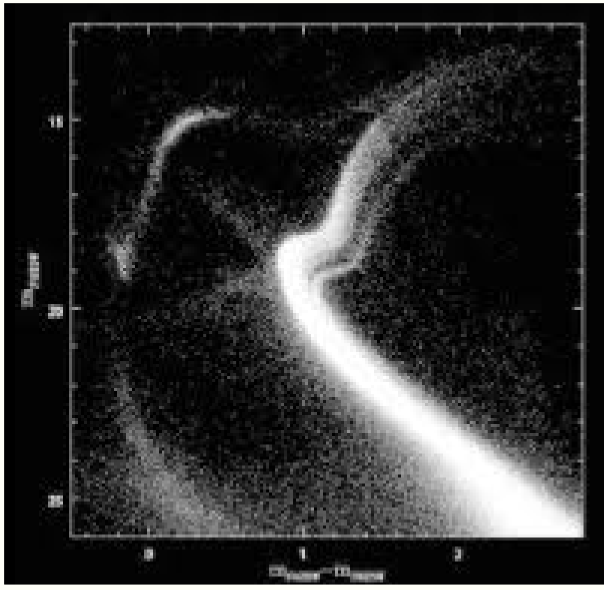


Omega Centauri

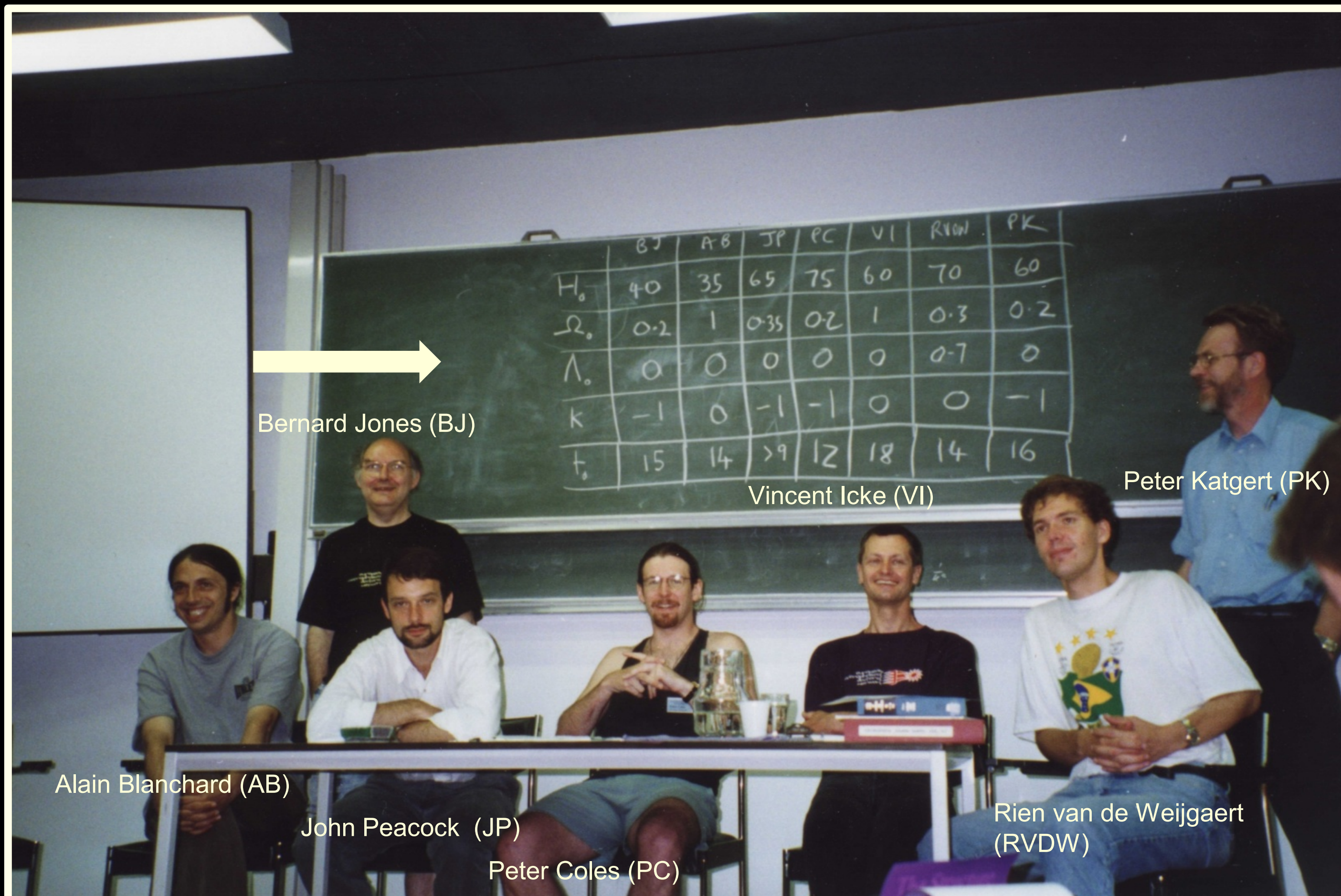


Typical 21st century colour-magnitude diagram:

multiple populations in Omega Centauri

1995: Cosmic Confusion

EADN Summerschool, July 1995, Leiden



“Rien, be real … “

John Peacock

Dark Energy:

Probes

Probes DE: additional

- **Clusters of Galaxies**

  number counts N(z),

  # formed clusters of galaxies as function of z sensitive to w & w'
- **Baryonic Oscillations (BAO)**

  cosmic yardstick, curvature: residual imprint in galaxy distribution

  acoustic oscillations primordial baryon-photon plasma
- **Integrated Sachs Wolfe (ISW)**

  imprint foreground large scale structure on CMB,

  via evolving potential perturbations

- **Clustering**

  clustering correlation function/power spectrum,

  directly probing cosmological scenario, BAO wiggles
- **Growth of clustering:**

  evolving growth rate f(Omega,z), probed via influence of

  redshift distortions on correlation functions
- **Voids:**

  evolving void shapes,

  probing tidal force field generated by large scale mass distribution
- **Morphology and Topology**

  sensitivity of topology, measured by homology (Betti numbers)

Dark Energy Probes: Comparison

Method	Strengths	Weaknesses	Systematics
Weak Lensing	Structure Growth + Geometric Statistical Power	CDM assumption	Image quality Photo-z
Supernovae SNIa	Purely Geometric Mature	Standard Candle assumption	Evolution Dust
BAO (Baryonic Acoustic Oscillation)	Largely Geometric Low systematics	Large samples required	Bias Nonlinearity
Cluster Population N(z)	Structure Growth + Geometric Xray+SZ+optical	CDM assumption	Determining mass Selection function

Type Ia Supernovae

Supernova Explosion & Host Galaxy

M51 supernovae

Type Ia Supernova Explosion

Type Ia Supernova

- Amongst the most energetic explosions in our Universe:

  $E \sim 10^{54}$ ergs
- During explosion the star is as bright as entire galaxy ! (ie. 10^{11} stars)

- Violent explosion Carbon-Oxygen white dwarfs:
- Embedded in binary, mass accretion from companion star
- When nearing Chandrasekhar Limit ($1.38\ M_{\odot}$), electron degeneracy pressure can no longer sustain star.
- while contracting under its weight, carbon fusion sets in, powering a
- catastrophic deflagration or detonation wave,
- leading to a violent explosion, ripping apart entire star

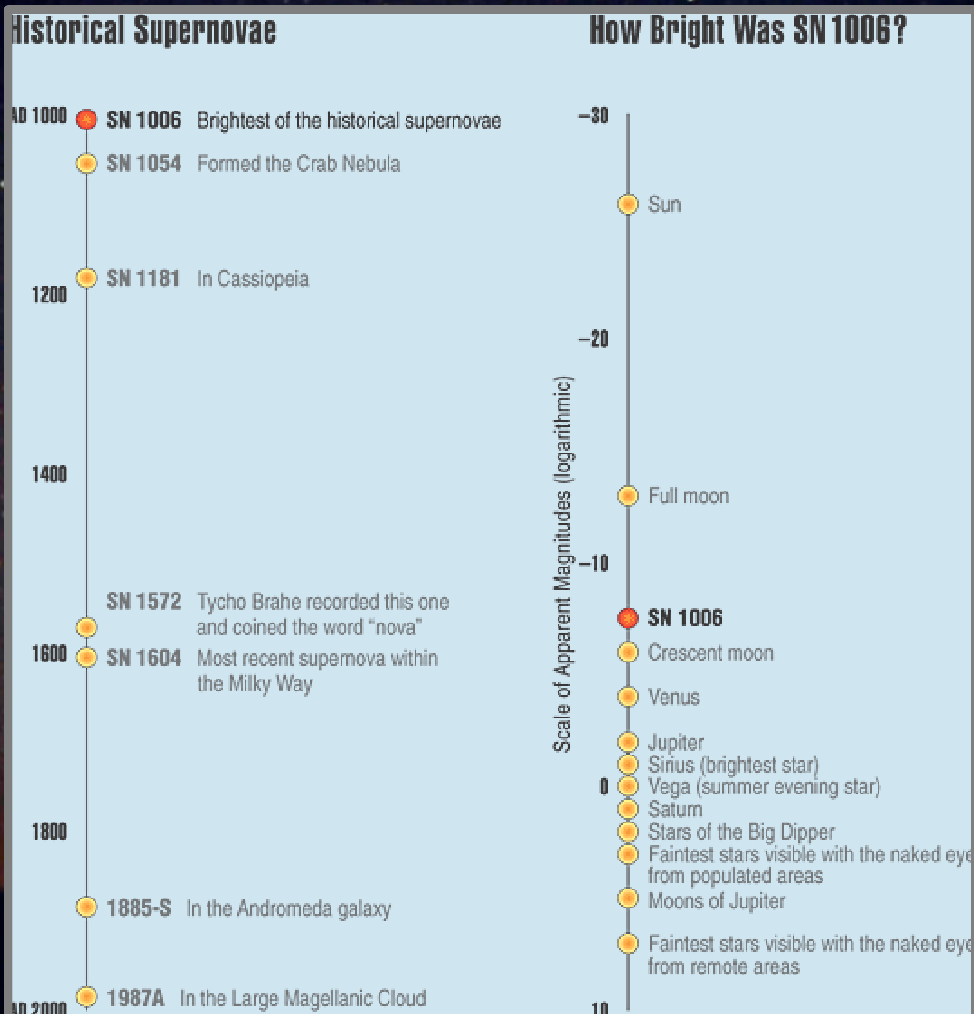
- Because exploding stars have nearly uniform progenitor ($\sim 1.38\ M_{\odot}$ white dwarf), their luminosity is almost the same: $M \sim -19.3$

  Standard Candle

SN1006

Supernova SN1006:
brightest stellar event recorded in history

SN1006

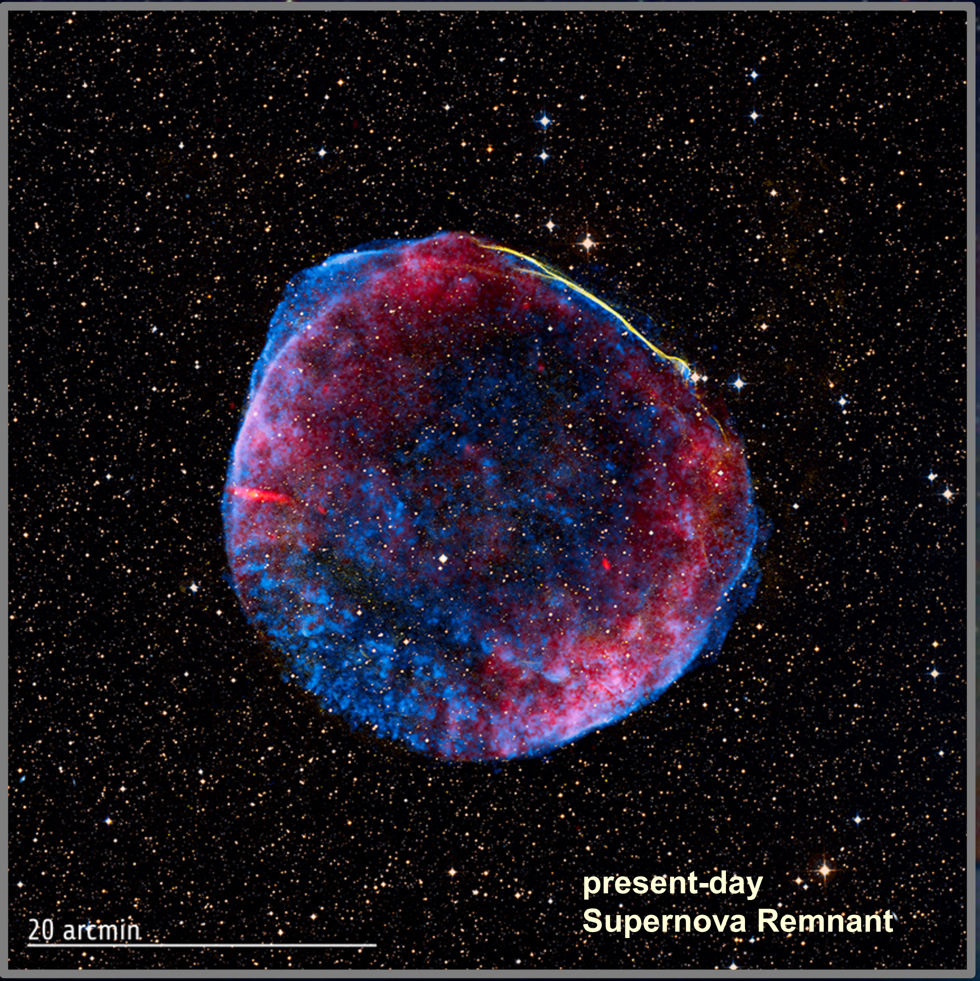


Supernova SN1006:

- **brightness:** m = -7.5
- **distance:** d=2.2 kpc
- **recorded:** China, Egypt, Iraq, Japan, Switzerland, North America

Supernova SN1006:
brightest stellar event recorded in history

SN1006



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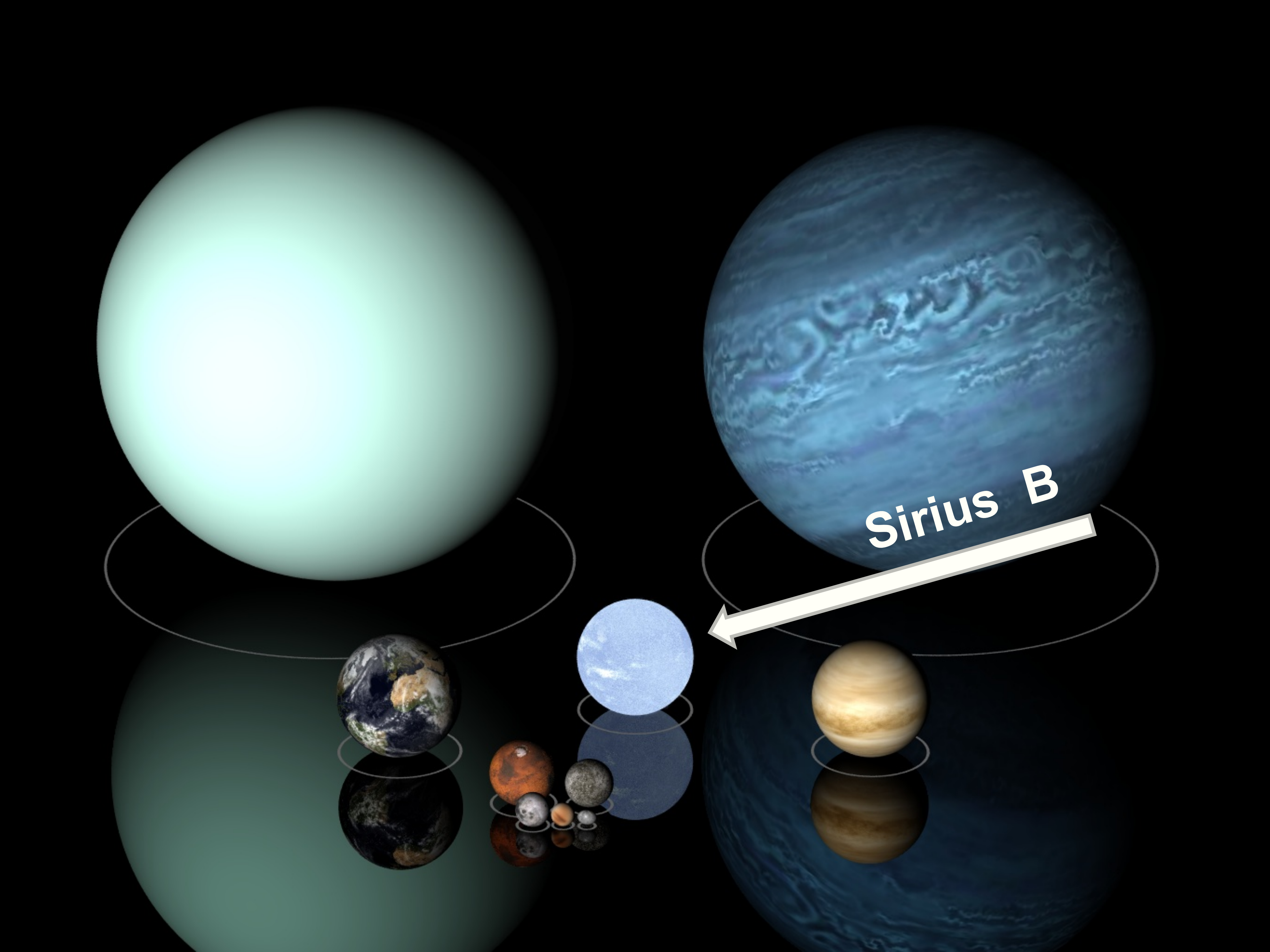
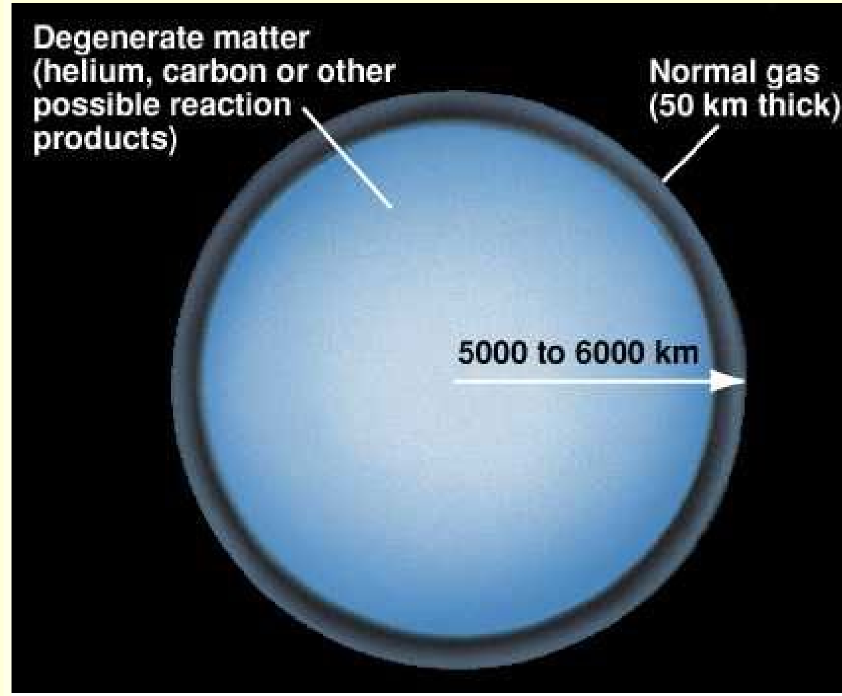
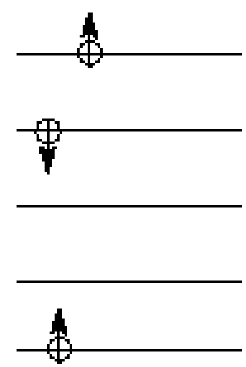
Supernova SN1006:
brightest stellar event recorded in history

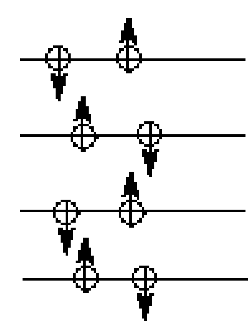
White Dwarfs


Sirius B





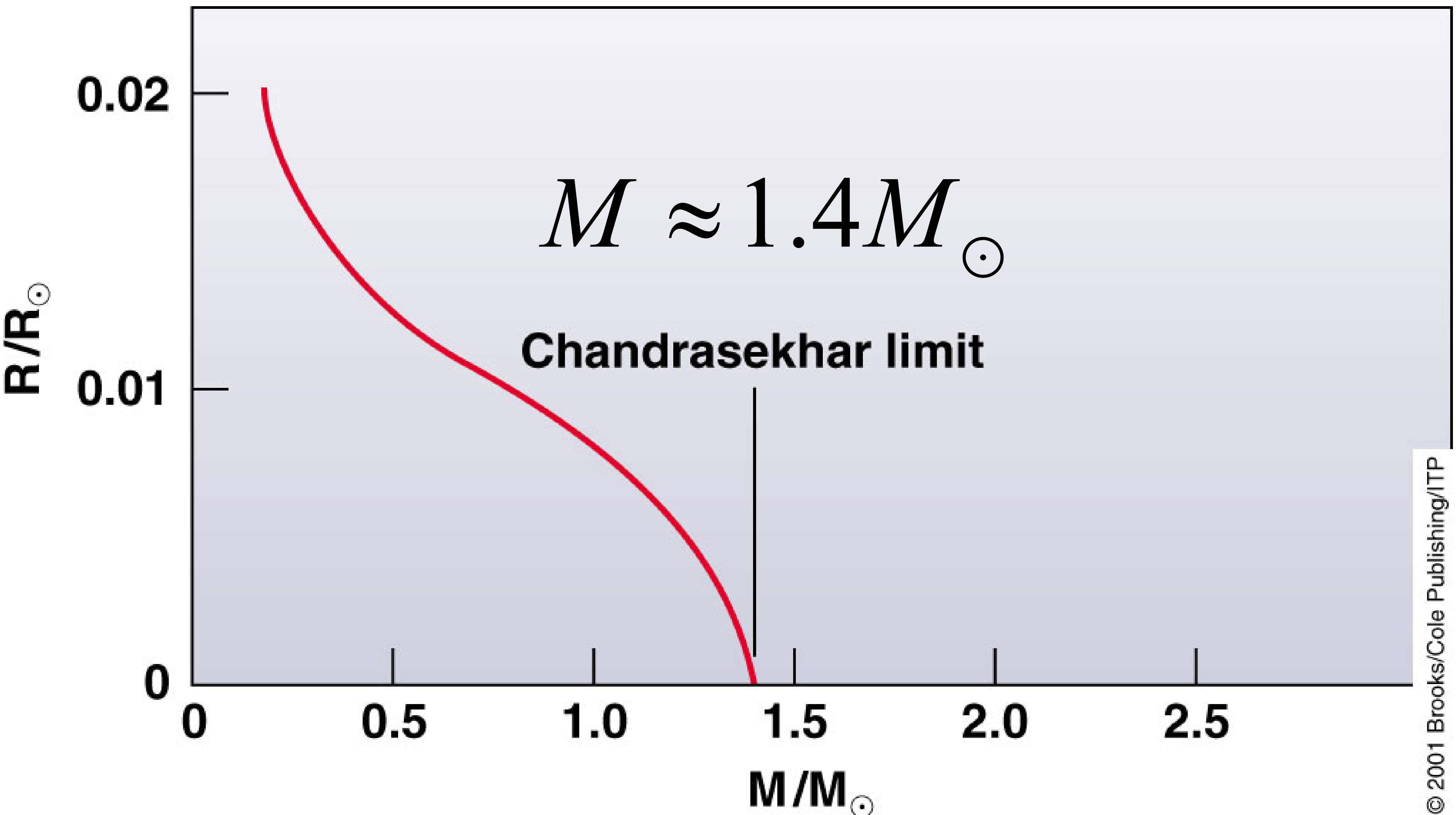
Regular gas: many unfilled energy levels. Particles free to move about and change energy levels.



Degenerate gas: all lower energy levels filled with two particles each (opposite spins). Particles **locked** in place.

Chandrasekhar Mass Limit

What is the maximum mass that can be supported by the dense compact material of a white dwarf star?



Supernova Lightcurves

SN 2007uy

6 January 2008

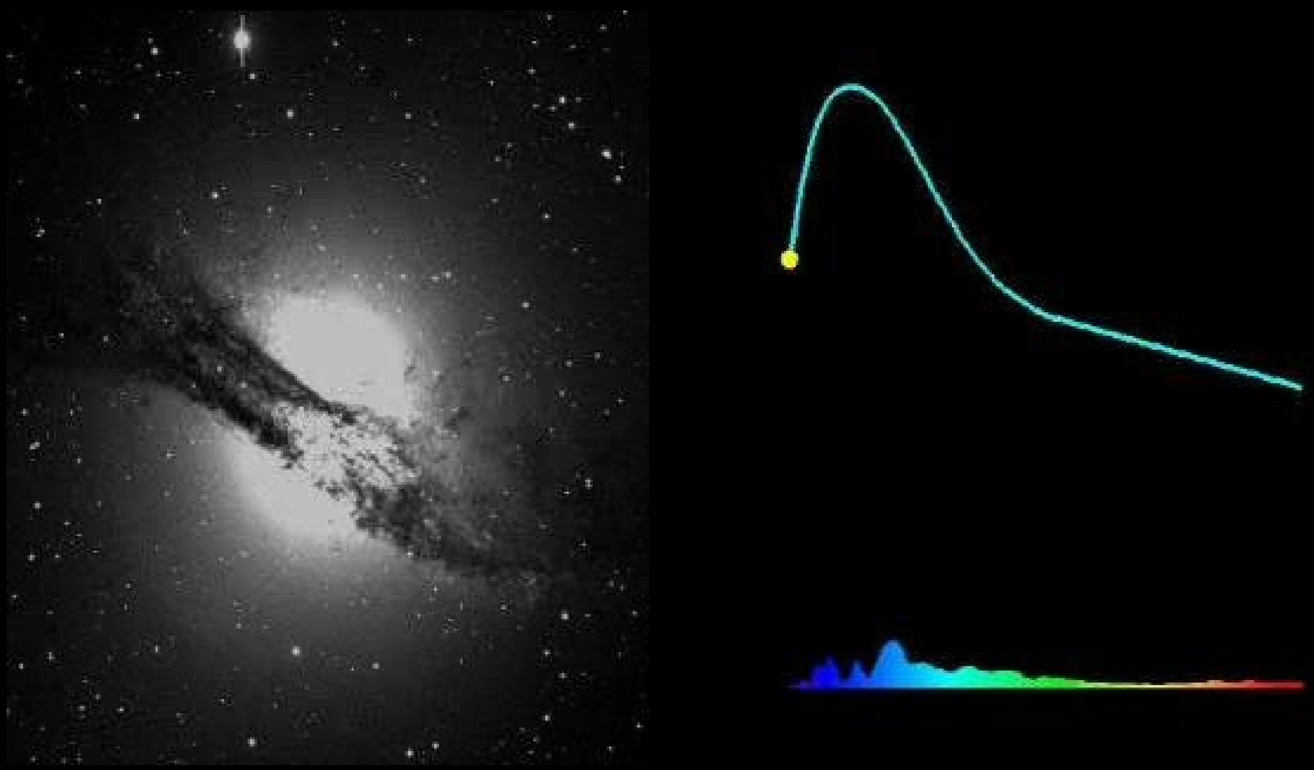
12 January 2008

10 February 2008

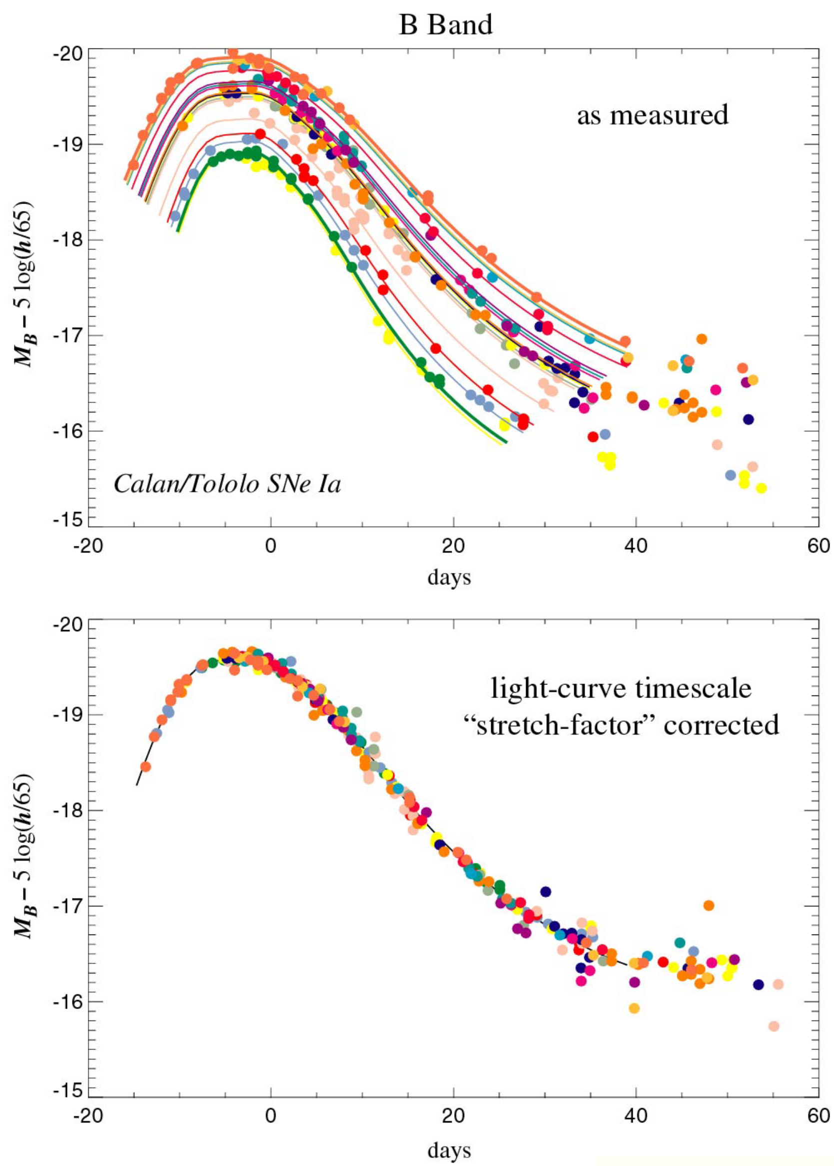
Supernova SN 2007uy in NGC2770

while fading, another supernova, SN2008D, went off in same galaxy

Supernova Lightcurve & Spectrum

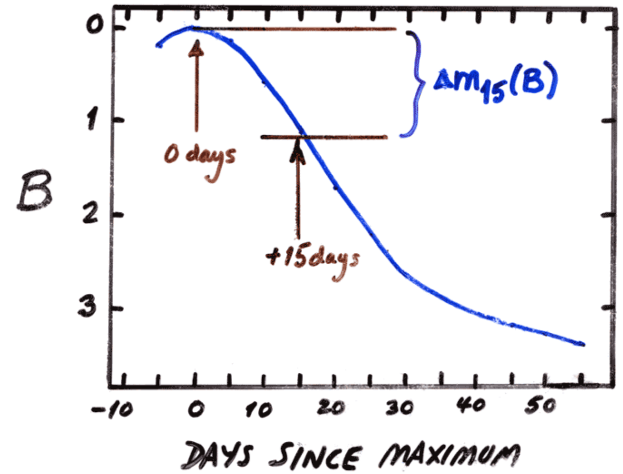


the Phillips Relation



Relationship between

- **peak luminosity of a Type Ia supernova**
- **speed of luminosity evolution after maximum light.**



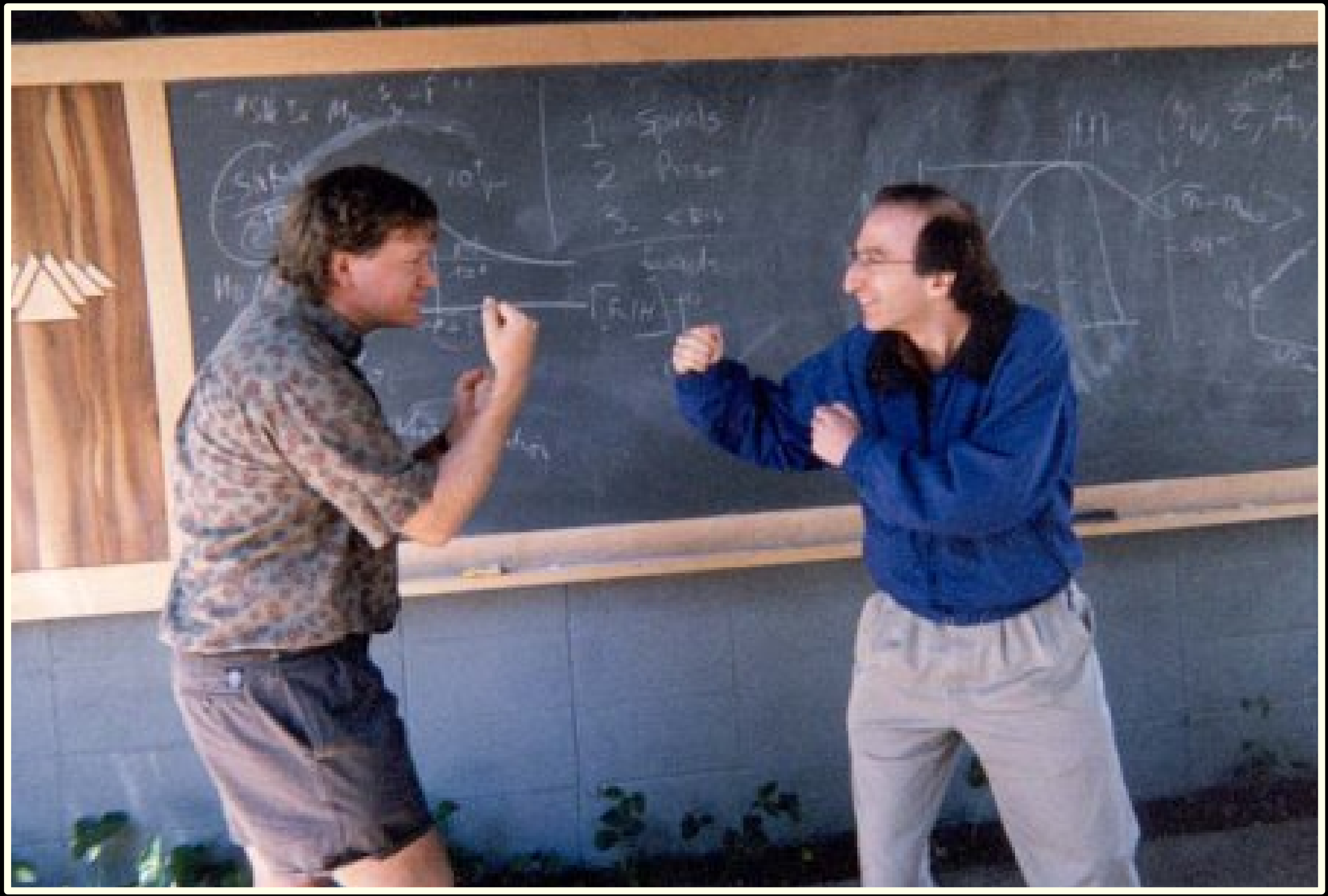
Mark Phillips (1993):

- **on the basis of Calan/Tololo Supernova Survey**

- **the faster a supernova fades after peak,**
- **the fainter its intrinsic peak luminosity**

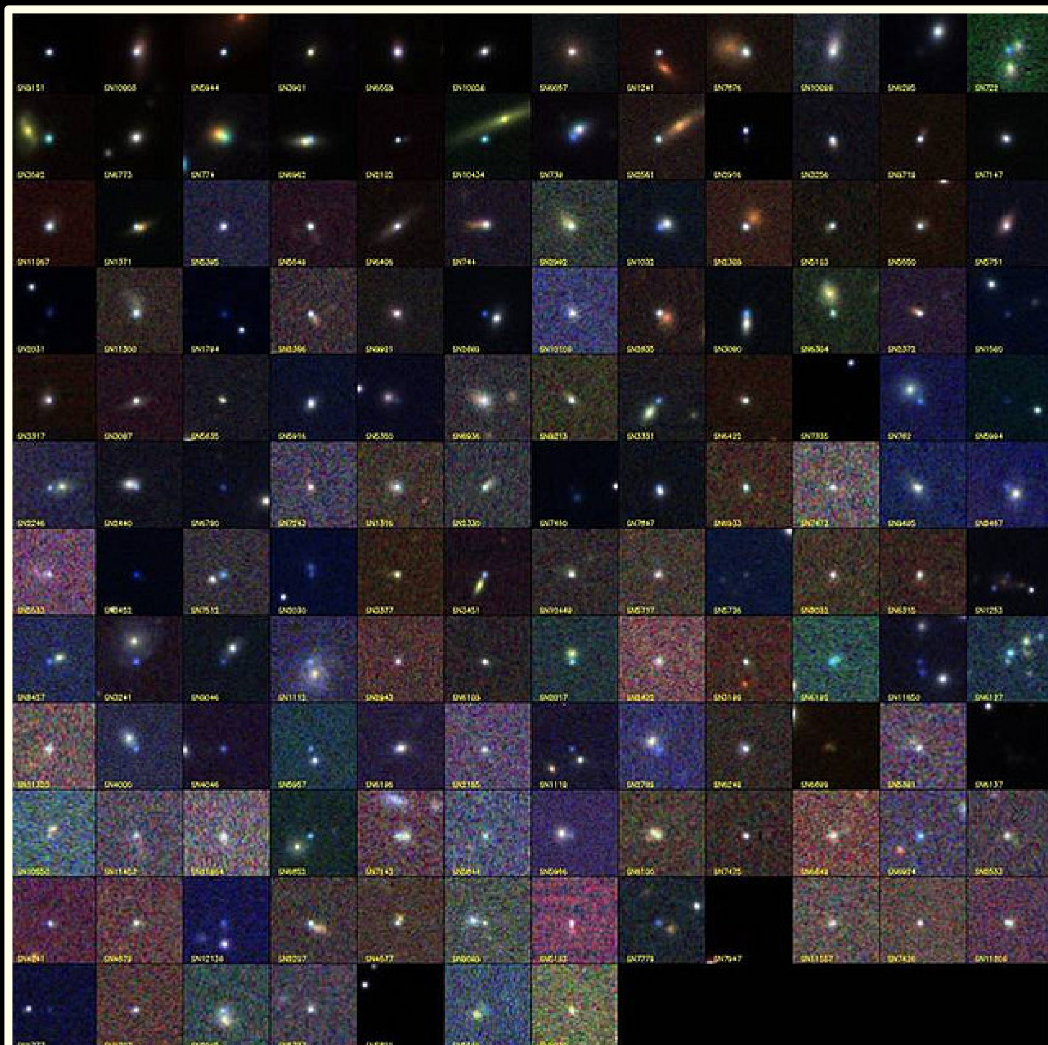
- **reduces scatter in Hubble diagram to <0.2 mag**
- **heuristic relationship, as yet not theoretically “understood”**

Supernova Cosmology Project



High-z Supernova Search Team

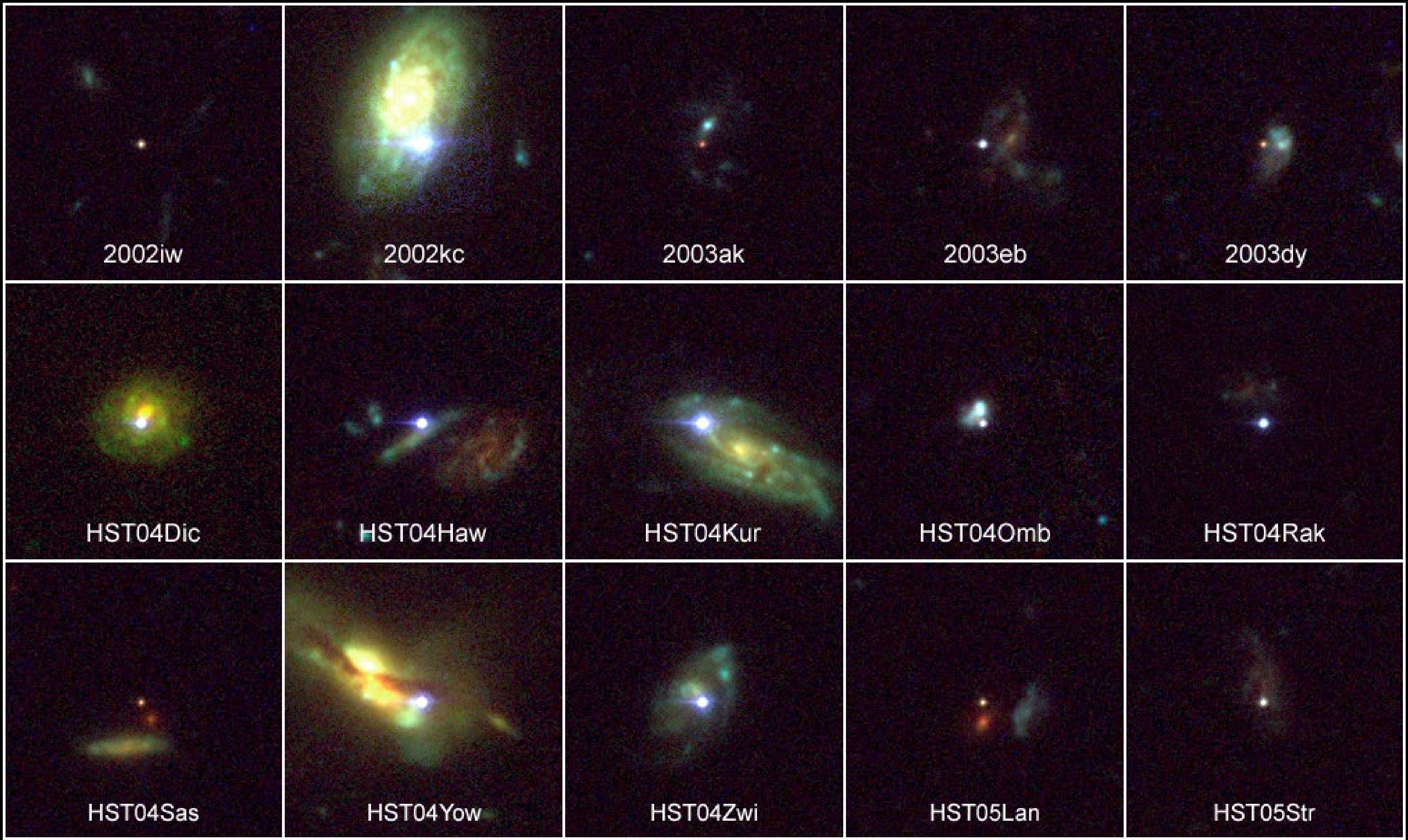
Supernova Cosmology Project



diligently monitoring millions of galaxies, in search for that one explosion …

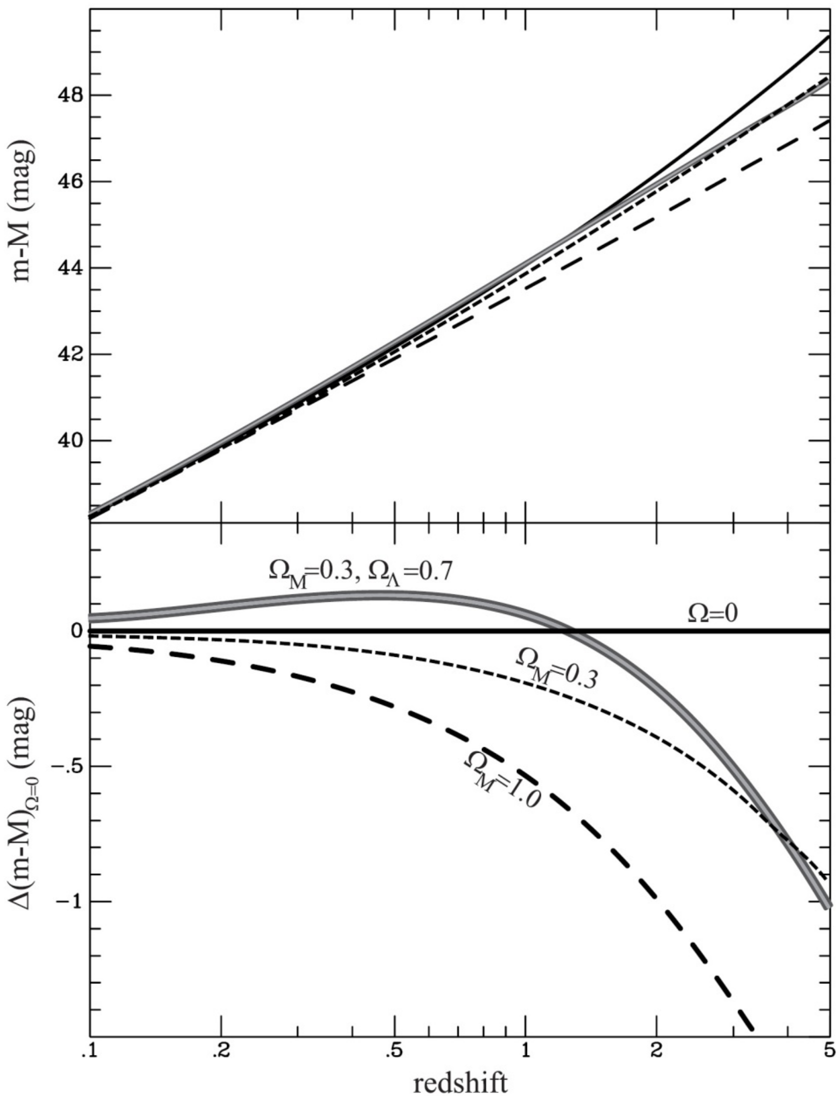
High-z Supernova Search Team

High-z SNIa: sample



Cosmic Acceleration
&
Cosmic Density

Cosmic Acceleration



Hubble Diagram high-z SNIa

- distance vs. redshift z
  m-M vs. redshift z

- determine:
  - absolute brightness of supernova Ia
  - from dimming rate (Phillips relation)

- measure:
  - apparent brightness of explosion

- translates into:
  - luminosity distance of supernova
  - dependent on acceleration parm. q

Luminosity Distance

For all general FRW Universe, the second-order luminosity distance-redshift relation, only depends on the *deceleration parameter* q_0:

$$D_L(z) = (1+z)D(z) = (1+z)R_c S_k\left(\frac{r}{R_c}\right)$$

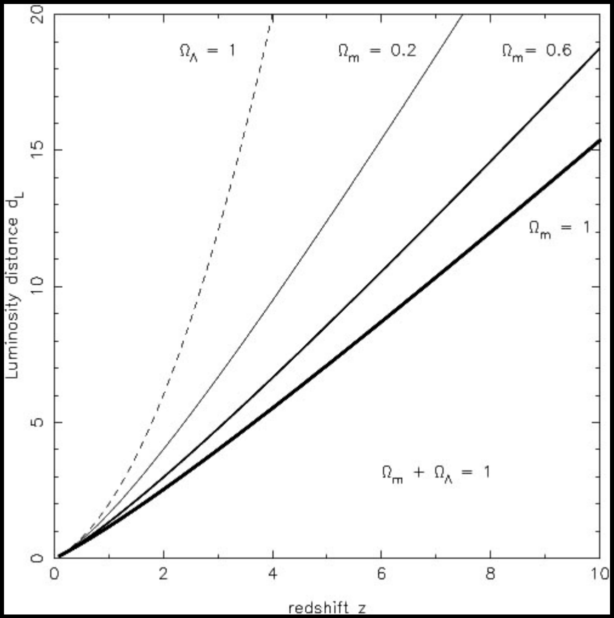
$$\simeq \frac{c}{H_0}(1+z)\left(z - \frac{1}{2}(1+q_0)z^2\right)$$

q_0 can be related to Ω_0 once the *equation of state* is known.

Luminosity Distance

matter-dominated FRW Universe

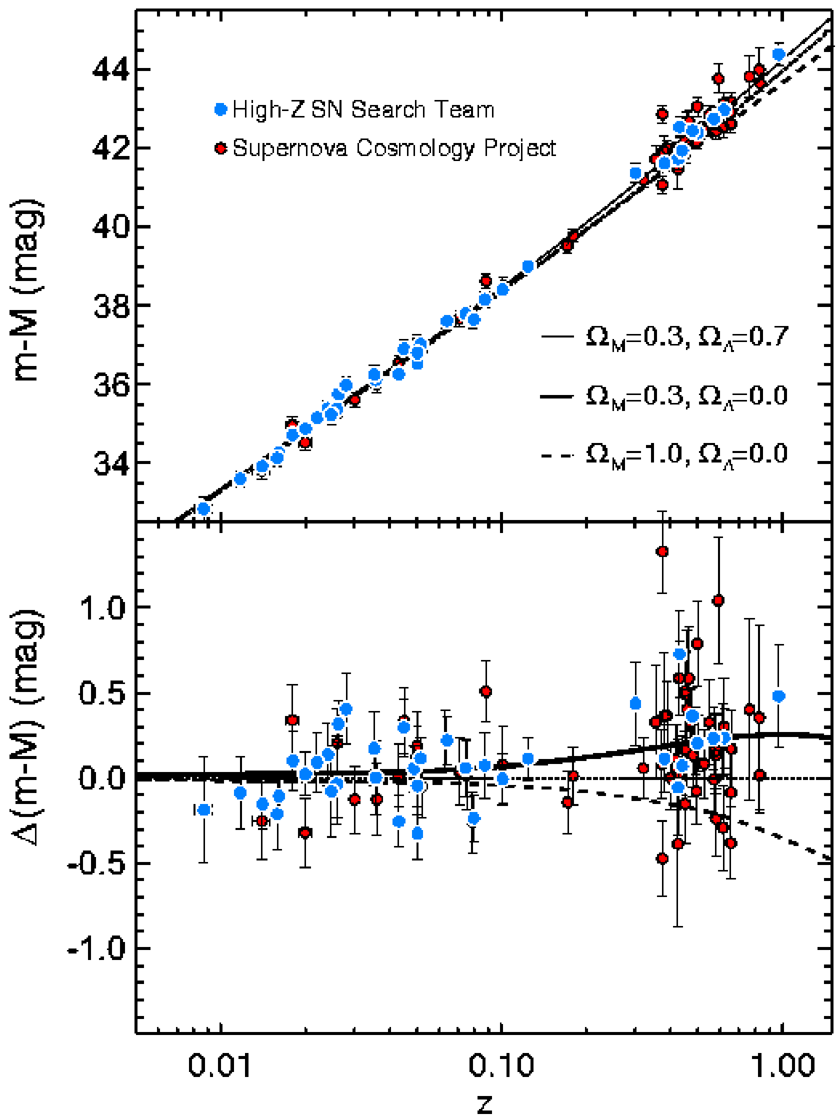
$$D_L = D(1+z) = (1+z) R_c S_k\left(\frac{r}{R_c}\right)$$



In a matter-dominated Universe, the luminosity distance as function of redshift is given by:

$$D_L(z) = (1+z) R_c S_k\left(\frac{r}{R_c}\right) = \frac{2c}{\Omega_0{}^2 H_0}\left\{\Omega_0 z + \left(\Omega_0 - 2\right)\left(\sqrt{1+\Omega_0 z} - 1\right)\right\}$$

Cosmic Acceleration



Hubble Diagram high-z SNIa

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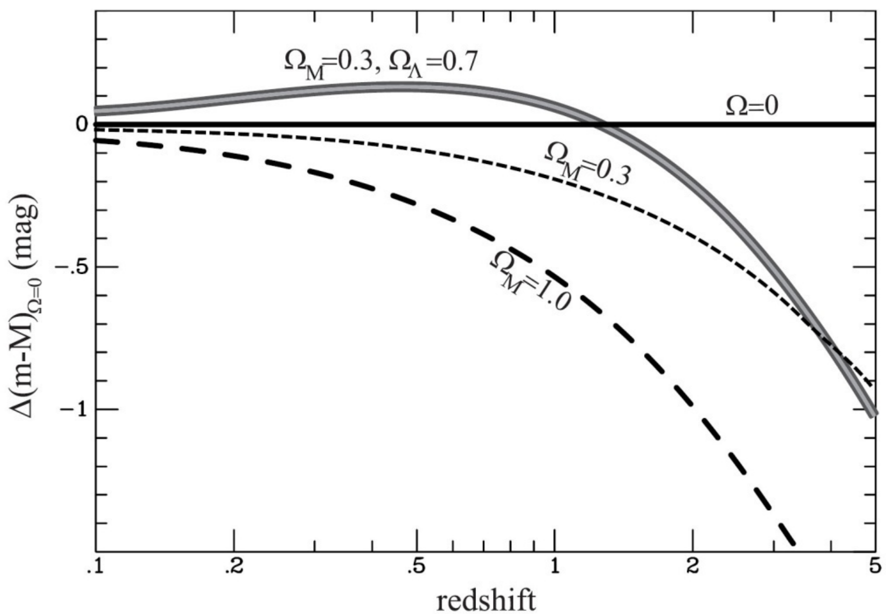
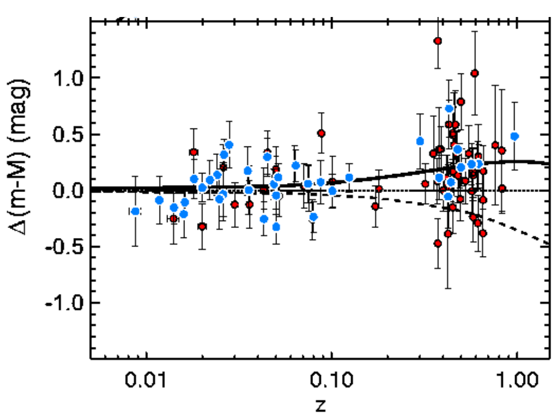
Cosmic Acceleration





Relative Hubble Diagram

Δ(m-M) vs. Redshift z

with Hubble diagram for empty Universe

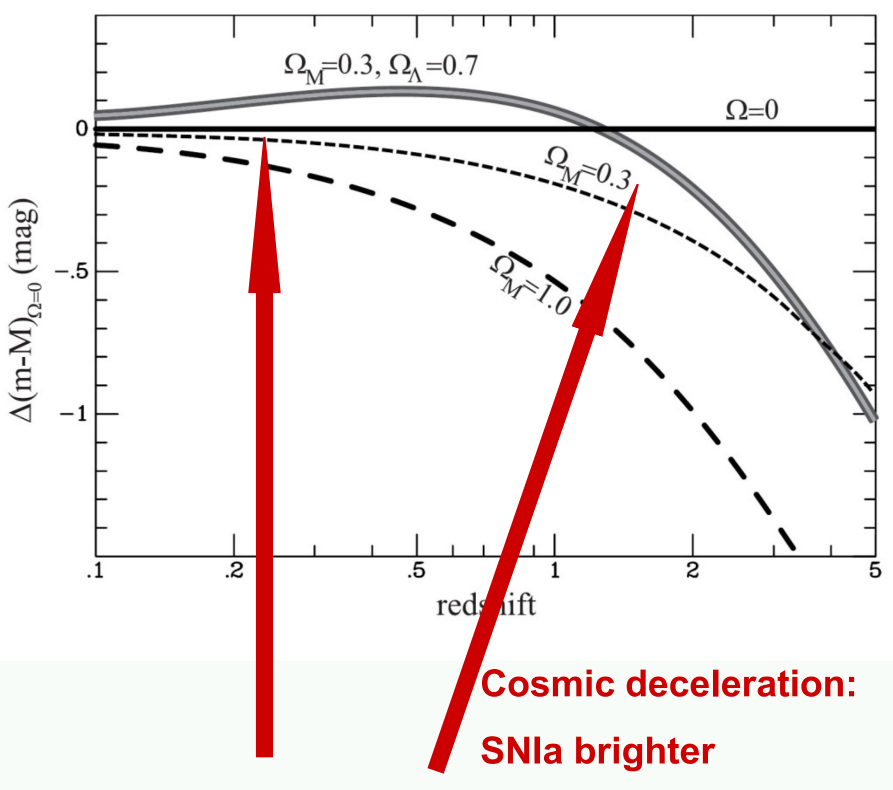
Ω_m=0.0, Ω_Λ=0.0

as reference.

Acceleration of the Universe:

$$q_0 \approx \frac{\Omega_m}{2} - \Omega_v \approx -0.55$$

Cosmic Deceleration



Cosmic deceleration:

SNIa brighter

Cosmic acceleration:

SNIa fainter

Before current Dark Energy epoch

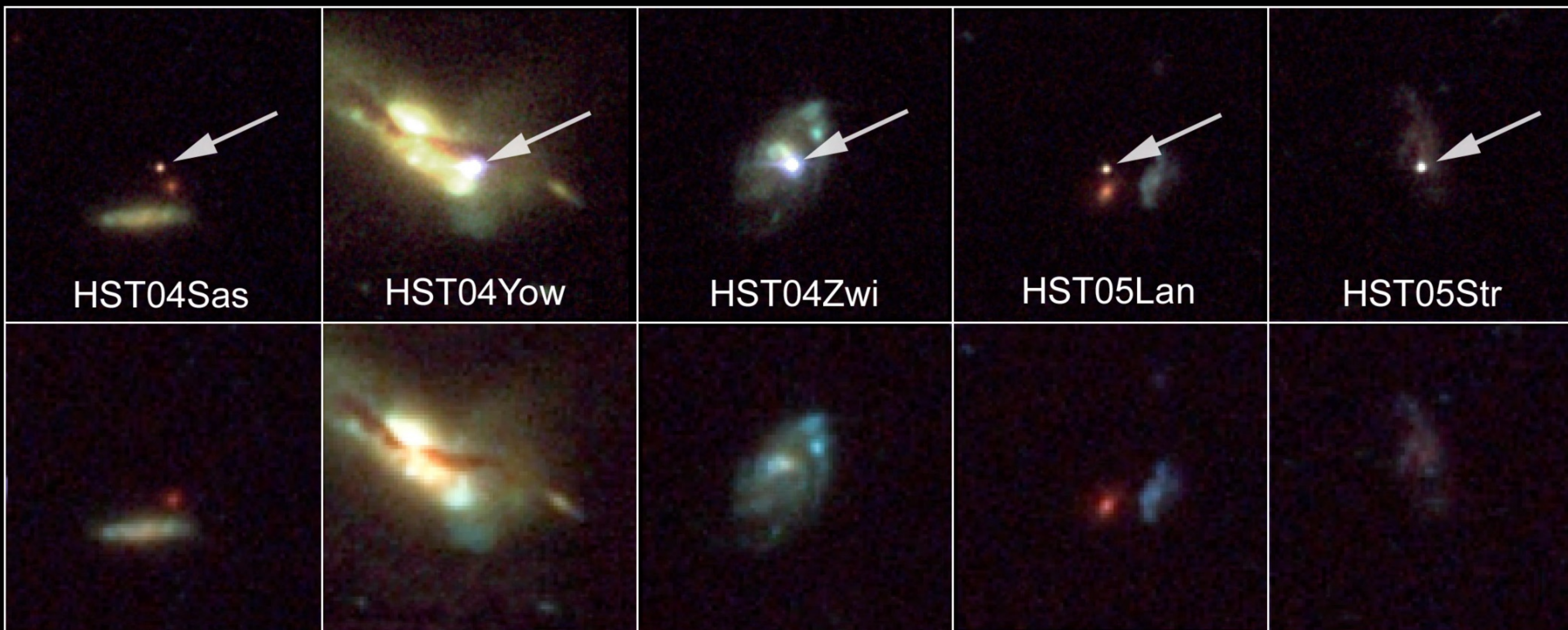
- **Universe dominated by matter:**

  Decelerating Expansion

- **Observable in SNIa at very high z:**

  z > 0.73

Beyond Acceleration: SNe Ia at z > 0.7

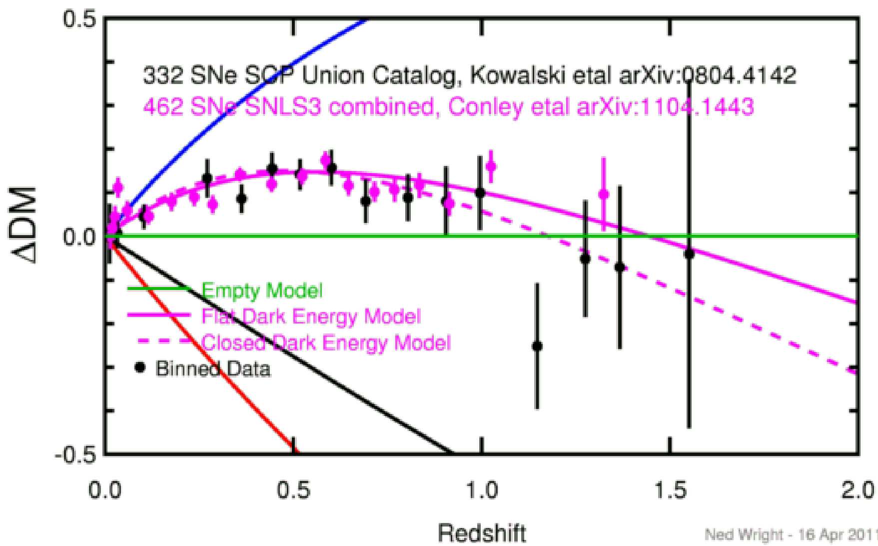


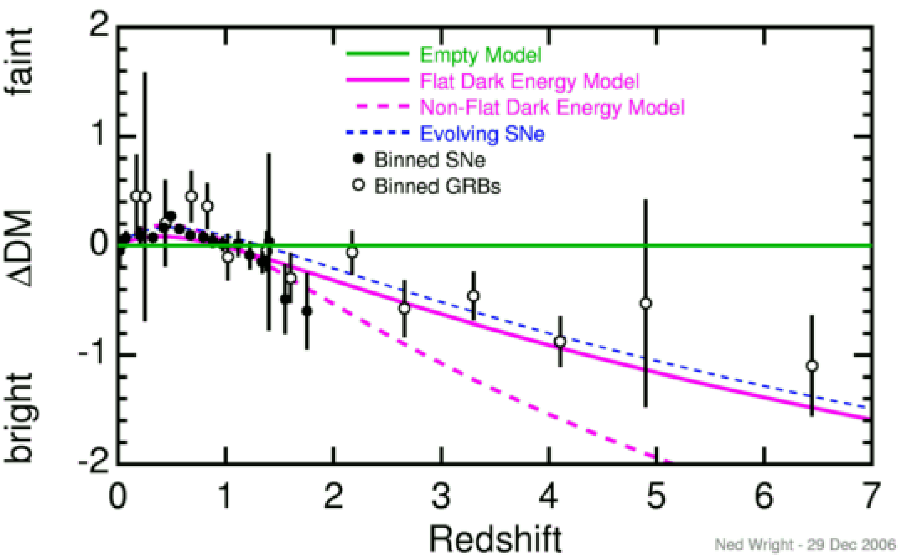
Five high-z SNIa, images HST-ACS camera

SNIa and host galaxies **lower panel: before**

top panel: after explosion)

Cosmic Deceleration





Before current Dark Energy epoch

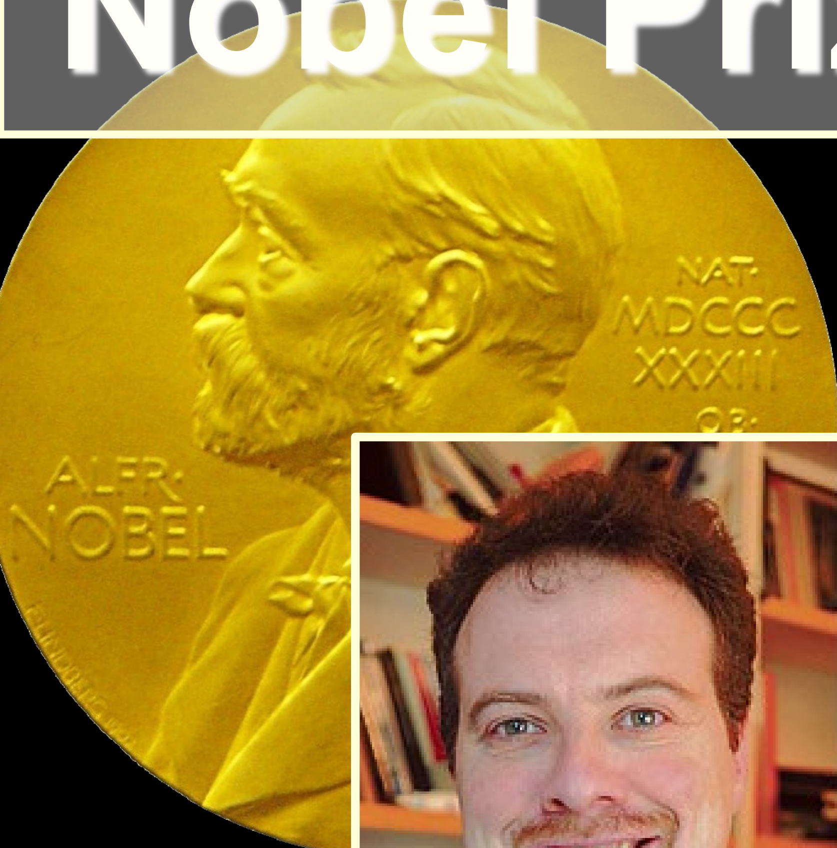
- **Universe dominated by matter:**

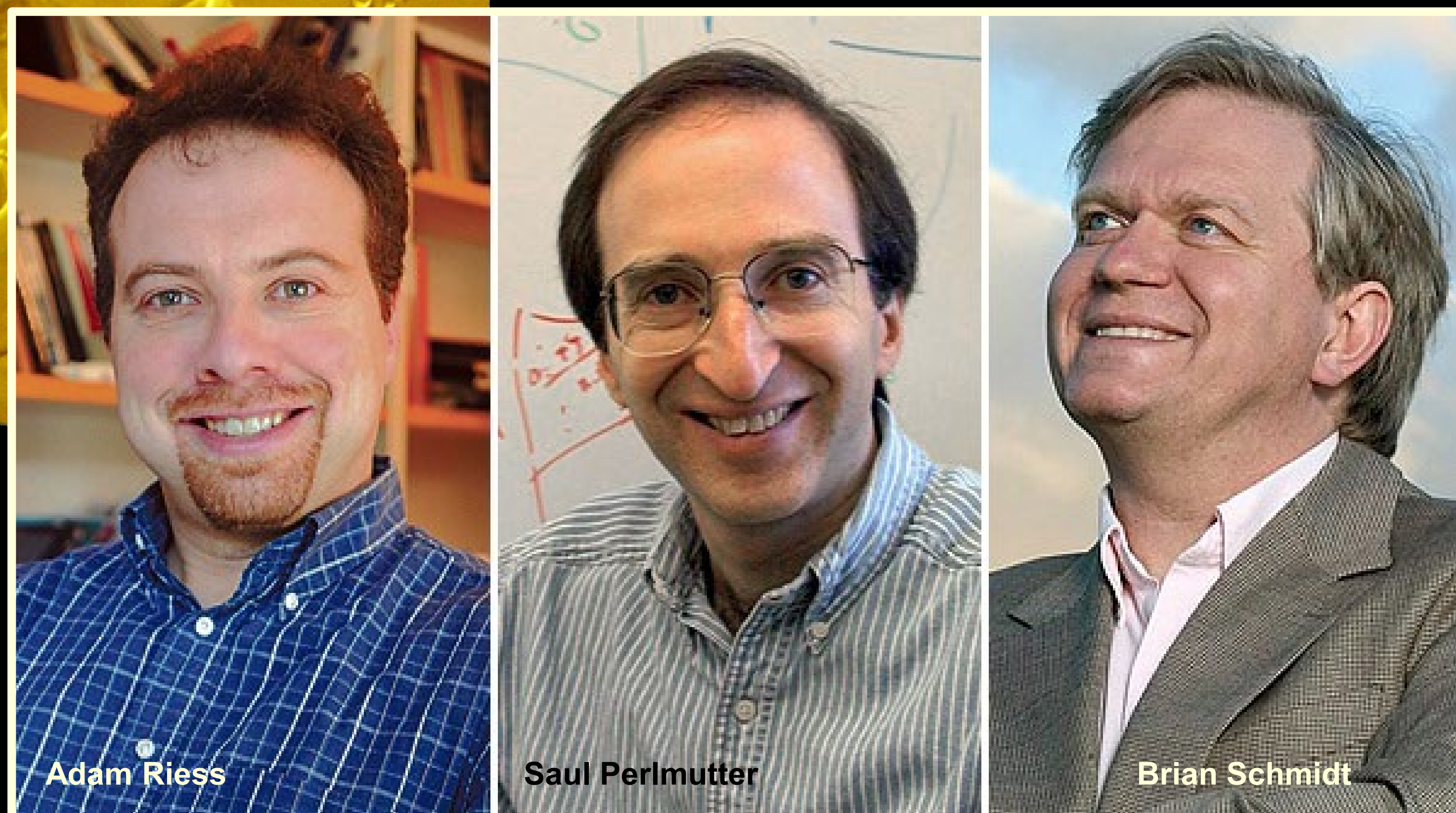
  Decelerating Expansion
  (“Einstein-de Sitter phase”)

- **Observable in SNIa at very high z:**

  z > 0.73

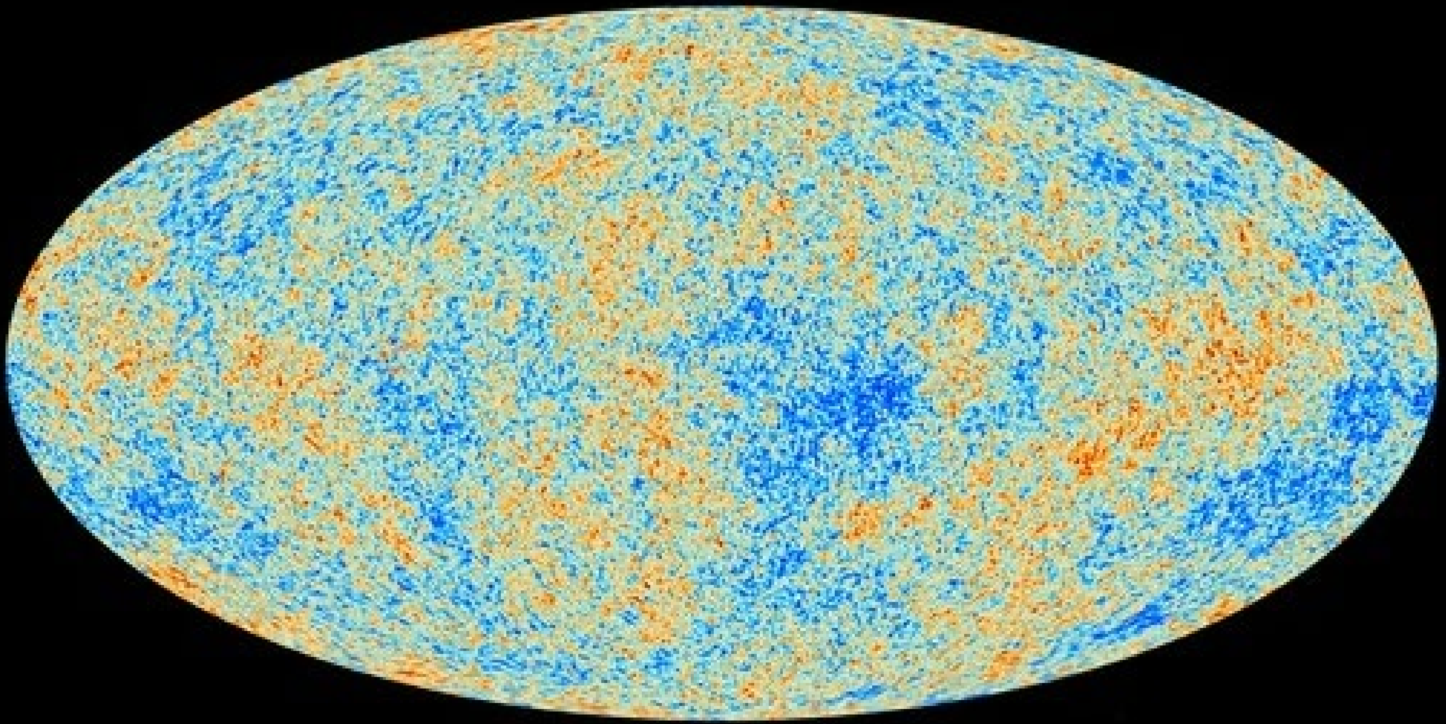
Nobel Prize Laureates





Cosmic Curvature Measured

Cosmic Microwave Background



Map of the Universe at Recombination Epoch (Planck, 2013):

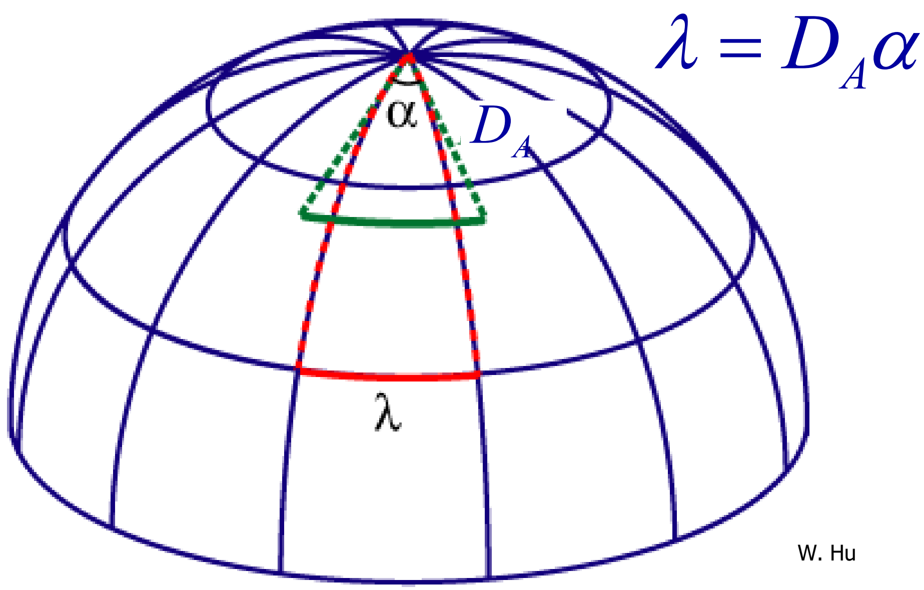
- 379,000 years after Big Bang
- Subhorizon perturbations: primordial sound waves
- ∆T/T < 10-5

Measuring Curvature

Measuring the Geometry of the Universe:

- **Object with known physical size, at large cosmological distance**
- **Measure angular extent on sky**
- **Comparison yields light path, and from this the curvature of space**

Geometry of Space



FRW Universe:
lightpaths described by Robertson-Walker metric

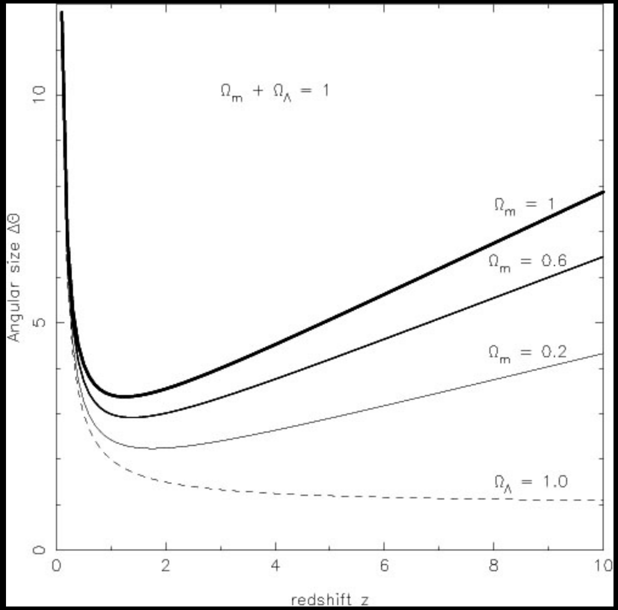
$$ds^2 = c^2 dt^2 - a(t)^2 \left\{ dr^2 + R_c^{\,2} S_k^{\,2}\left(\frac{r}{R_c}\right)\left[d\theta^2 + \sin^2\theta \, d\phi^2\right] \right\}$$

Here: angular diameter distance D_A: $\lambda = D_A \alpha$

Angular Size - Redshift
FRW Universe

$$\theta(z) = \frac{\ell}{D_A}$$

The angular size ?(z) of an object of physical size ? at a redshift z displays an interesting behaviour. In most FRW universes is has a minimum at a medium range redshift – z=1.25 in an Ω_m=1 EdS universe – and increases again at higher redshifts.



In a matter-dominated Universe, the angular diameter distance as function of redshift is given by:

$$D_A(z) = \frac{1}{1+z} R_c S_k\left(\frac{r}{R_c}\right) = \frac{2c}{H_0}\frac{1}{\Omega_0{}^2(1+z)^2}\left\{\Omega_0 z + (\Omega_0 - 2)\left(\sqrt{1+\Omega_0 z} - 1\right)\right\}$$

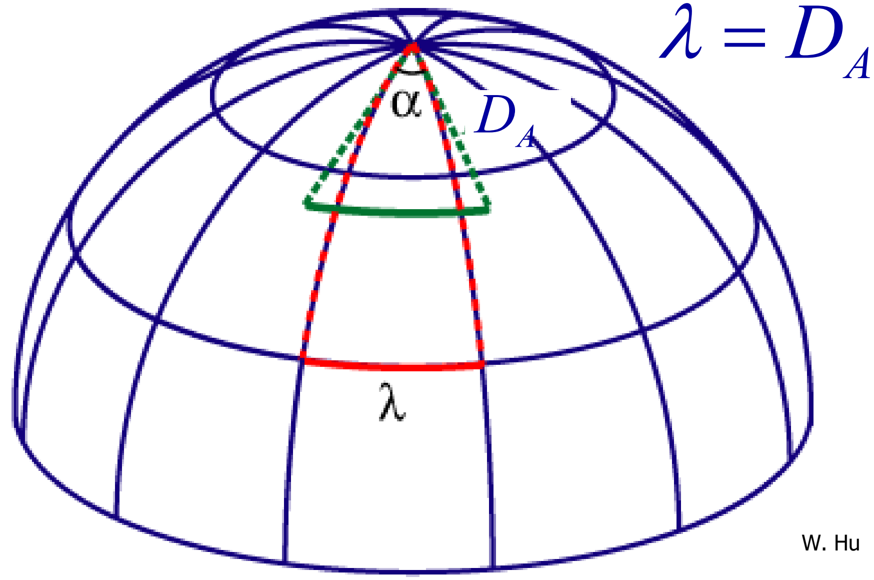
Measuring Curvature

- **Object with known physical size, at large cosmological distance:**
- **Sound Waves in the Early Universe !!!!**

⬇

Temperature Fluctuations
CMB

$$\lambda = D_A \alpha$$



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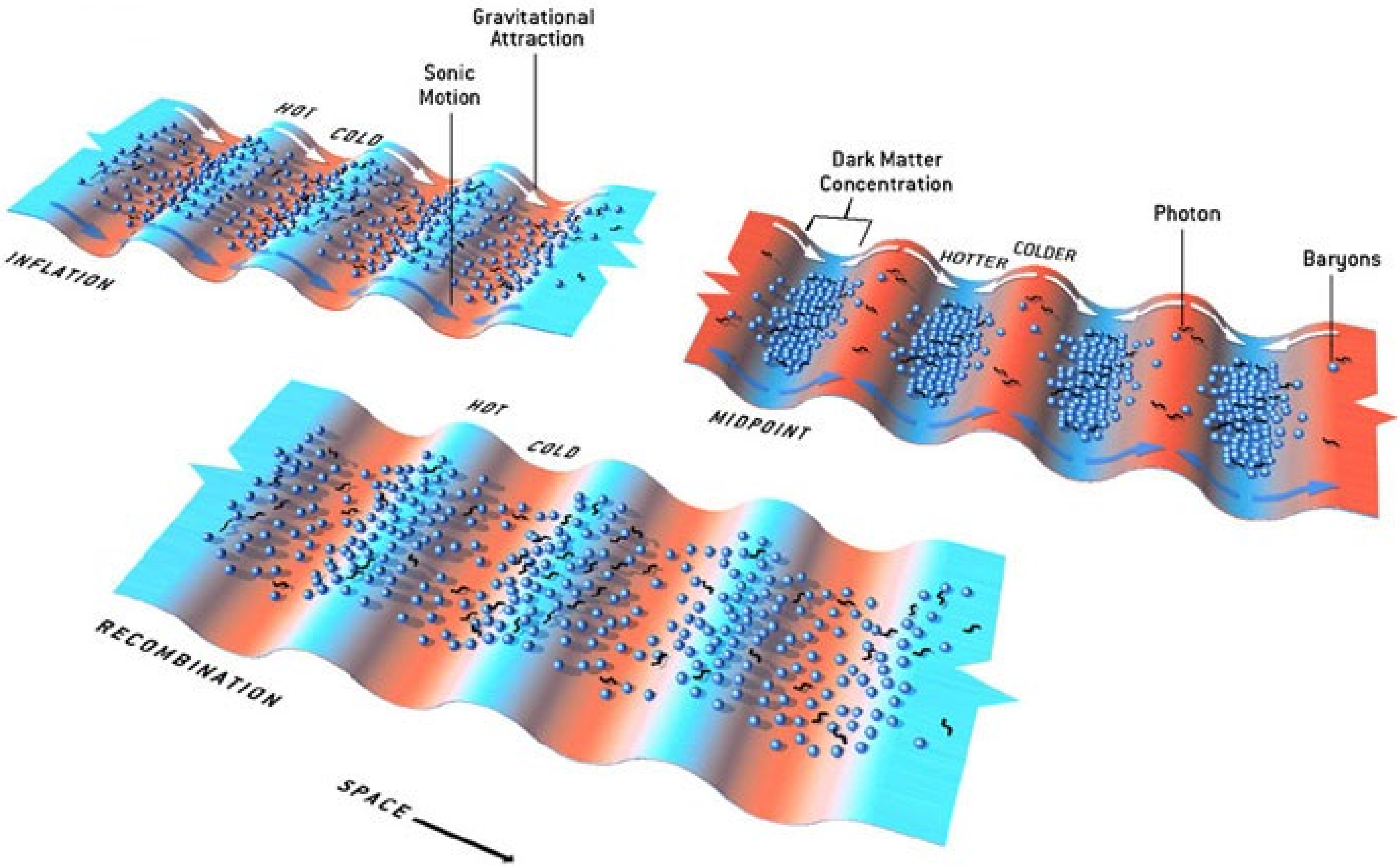
Here: angular diameter distance D_A: $\lambda = D_A \alpha$

Fluctuations-Origin



Music of the Spheres

- small ripples in
  primordial matter & photon distribution
- gravity:
  - compression primordial photon gas
  - photon pressure resists

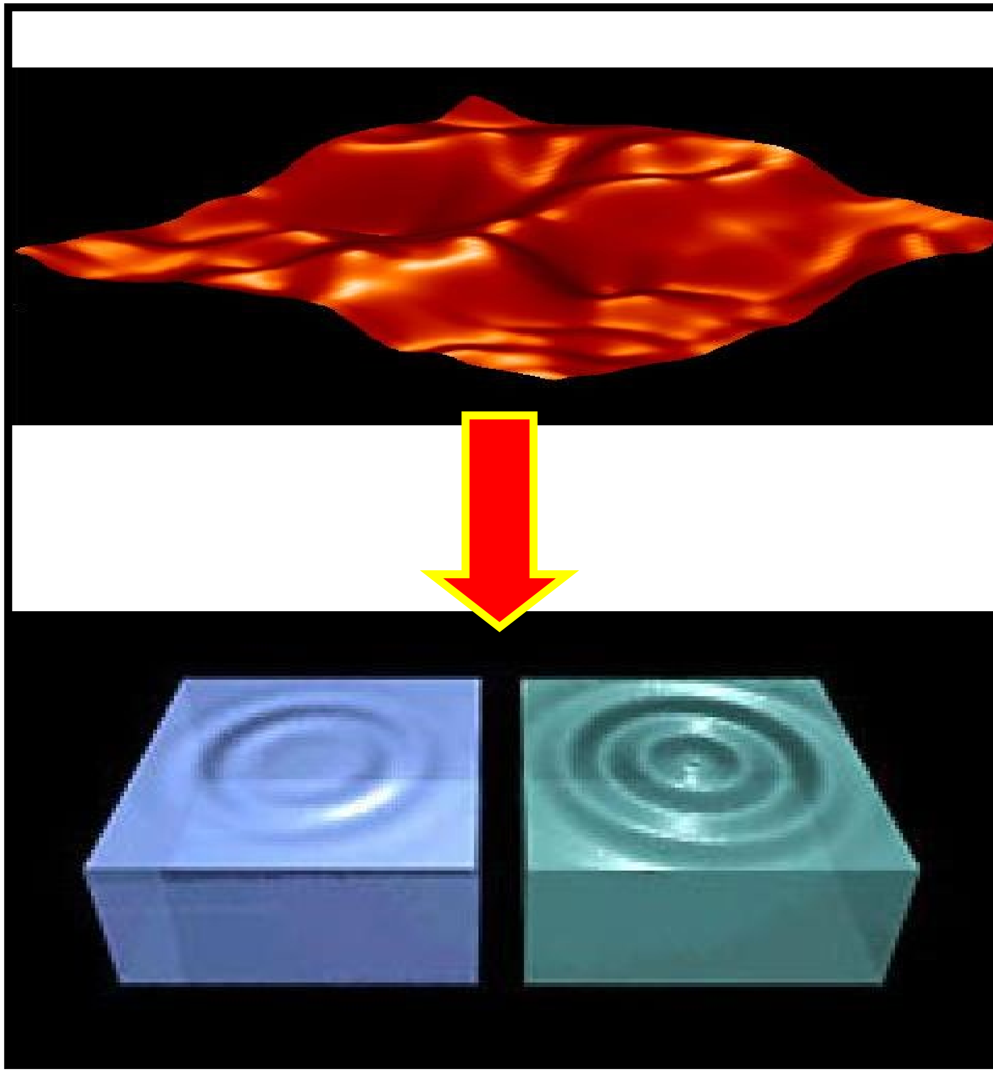
- compressions and rarefactions
  in photon gas: sound waves

- sound waves not heard, but seen:
  - compressions: (photon) T higher
  - rarefactions: lower

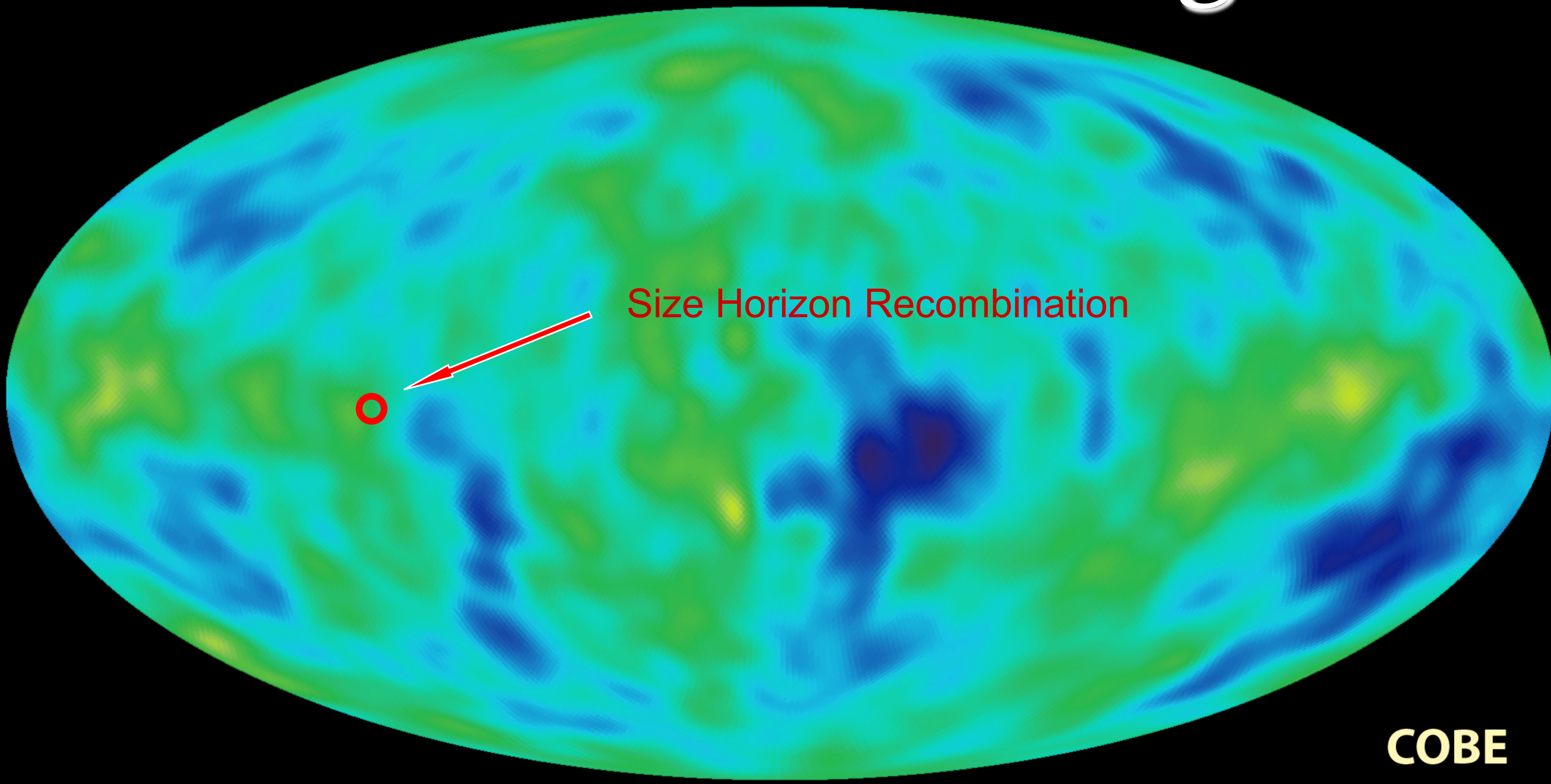
- fundamental mode sound spectrum
  - size of “instrument”:
  - (sound) horizon size last scattering

- Observed, angular size: $\theta \sim 1^{o}$
  - exact scale maximum compression, the
    “cosmic fundamental mode of music”

W. Hu



Cosmic Microwave Background

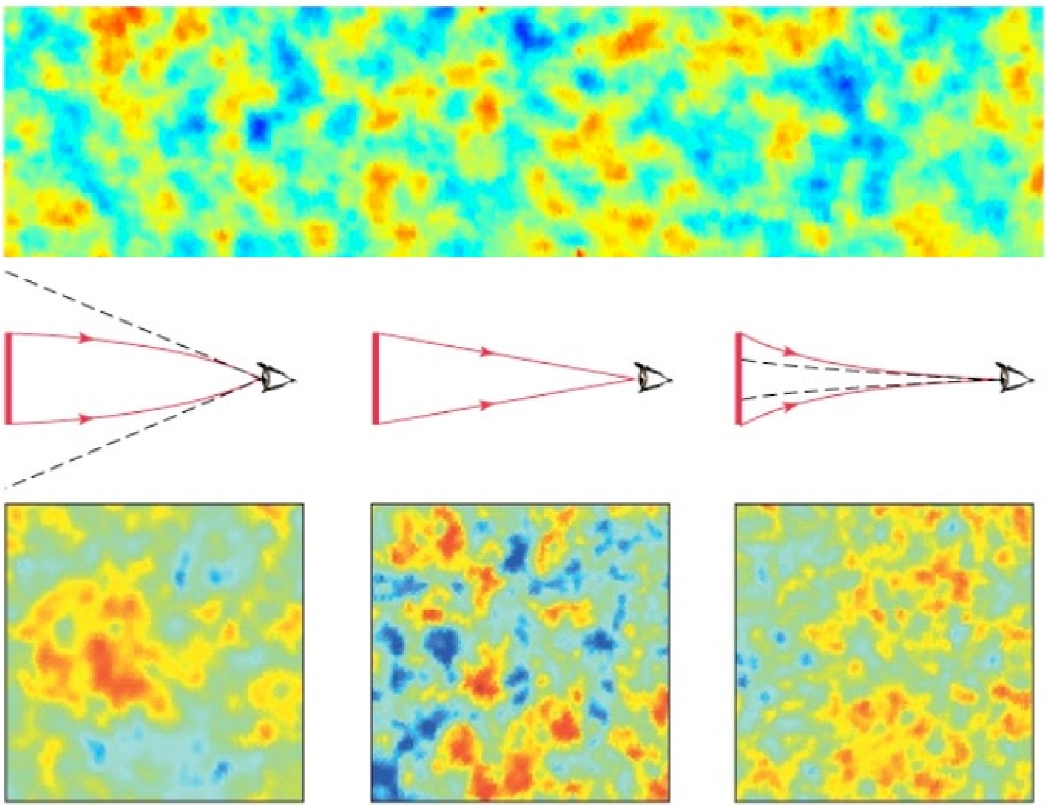


COBE measured fluctuations: > 7°
Size Horizon at Recombination spans angle ~ 1°

COBE proved that superhorizon fluctuations do exist: prediction Inflation !!!!!

Flat universe from CMB

- **First peak: flat universe**



Closed:
hot spots
appear larger

Flat:
appear as big
as they are

Open:
spots appear
smaller

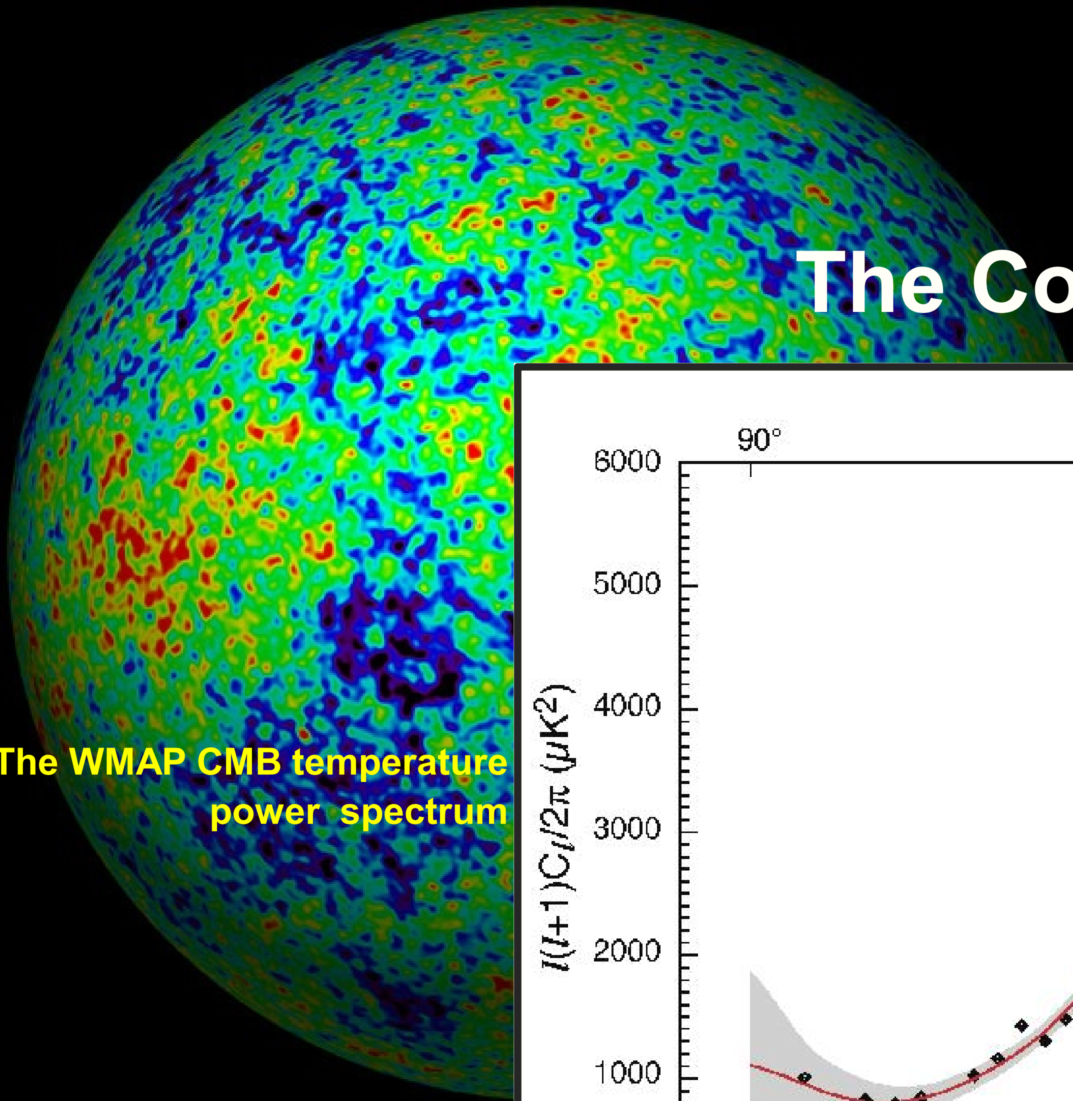
We know the redshift and the time it took for the light to reach us:

from this we know the
- length of the legs of the triangle
- the angle at which we are measuring the sound horizon.

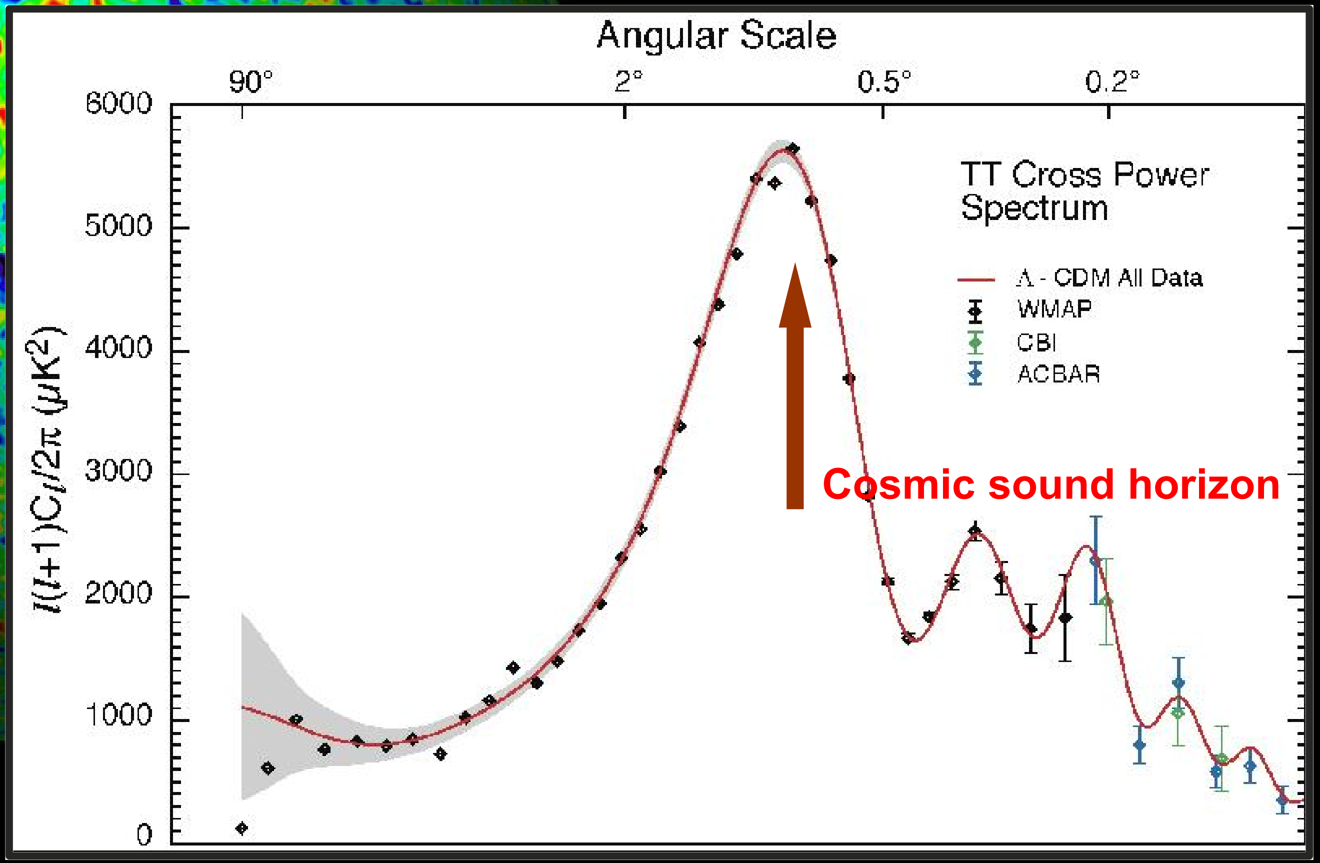
$$v \approx \frac{c}{\sqrt{3}}$$

$$\ell \approx 200/\sqrt{1-\Omega_k}$$

The Cosmic Tonal Ladder

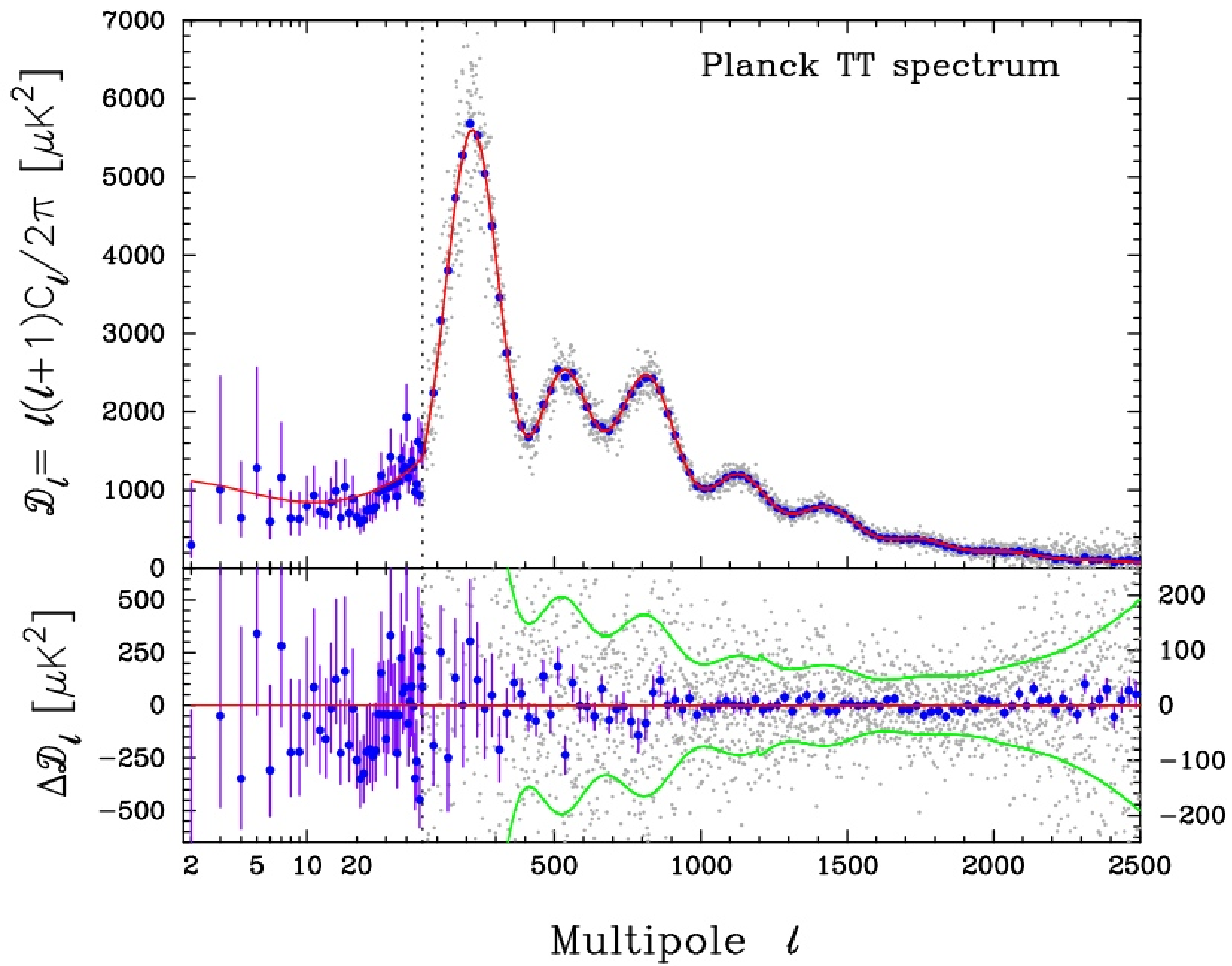


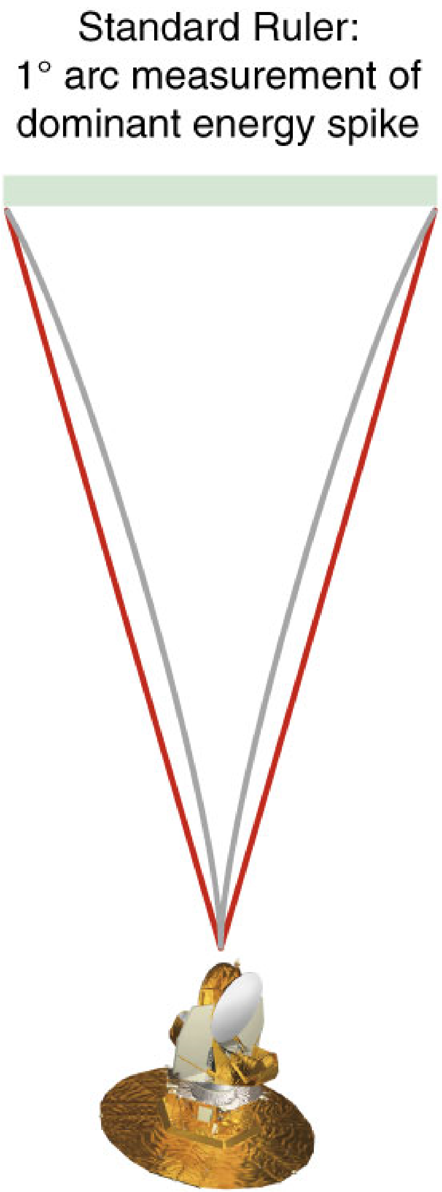
The WMAP CMB temperature power spectrum

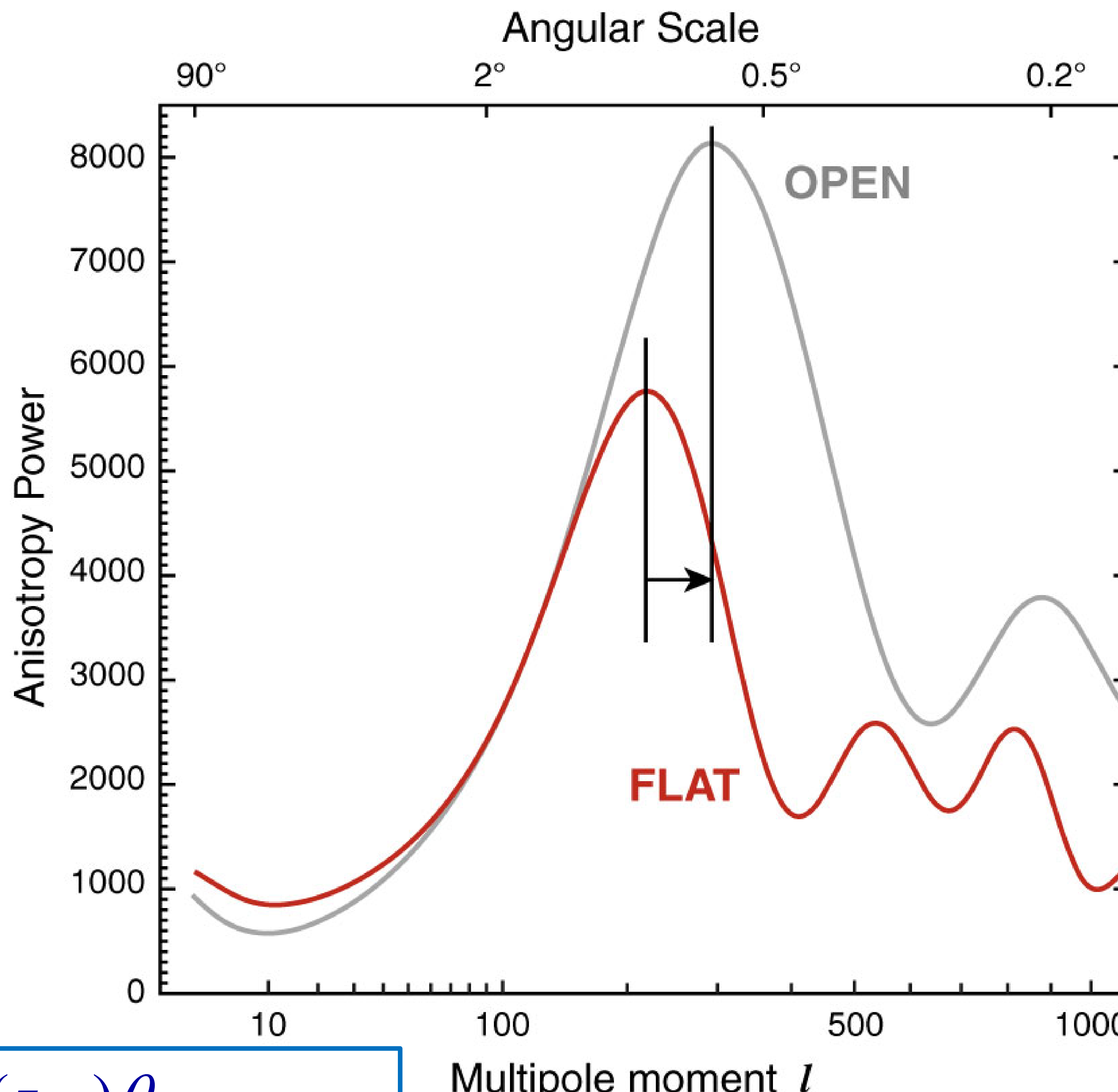


The Cosmic Microwave Background Temperature Anisotropies: Universe is almost perfectly FLAT !!!!

Planck CMB Temperature Fluctuations







$$\lambda_{sound\ horizon} = D_A(z_{rec})\,\theta_{CMB,\,1st\ peak}$$

FRW Universe: Curvature

There is a 1-1 relation between the total energy content of the Universe and its curvature. From FRW equations:

$$k = \frac{H^2 R^2}{c^2}(\Omega - 1)$$

$$\Omega = \Omega_{rad} + \Omega_m + \Omega_\Lambda$$

$\Omega < 1$	$k = -1$	*Hyperbolic*	*Open Universe*
$\Omega = 1$	$k = 0$	*Flat*	*Critical Universe*
$\Omega > 1$	$k = +1$	*Spherical*	*Close Universe*

Cosmic Curvature & Cosmic Density

$$q \approx \frac{\Omega_m}{2} - \Omega_\Lambda$$

$$k = \frac{H^2 R^2}{c^2}(\Omega_m + \Omega_\Lambda - 1)$$

SCP Union2 constraints (2010)

on values of matter density Ω_m
dark energy density Ω_Λ