

A cosmic background radiation map showing a complex network of blue and purple lines and dots, representing the structure of the universe. The lines radiate from a central bright white point, creating a starburst effect. The background is dark blue with numerous small white dots scattered throughout.

Cosmology,

lect. 6

Universe Measured

Galaxies & the Cosmos:

Distances & Motions

Distance Measurement

- Given the vast distances in the Universe, it is impossible to measure distances directly.
- Hence, we need to develop indirect methods that allow us to infer reliable estimates of the distances of objects.

• One of the most practical means is based on the comparison between

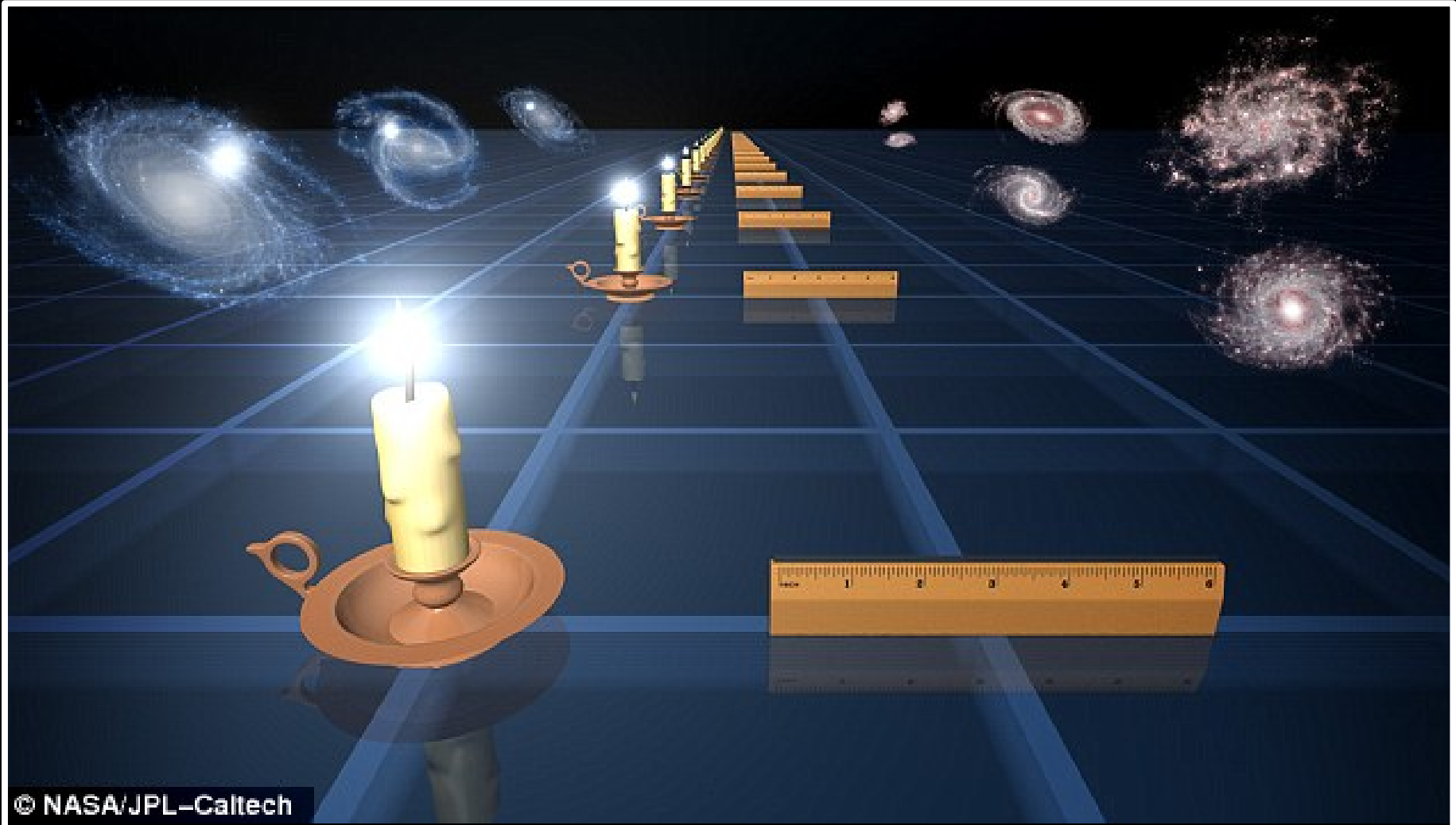
- observed brightness of an object
(*apparent brightness*)
- intrinsic brightness of an object
(*absolute brightness*)

Compare this with distance of streetlights :



Standard Candles

- To determine distances in the Universe, astronomers identify objects of which they know the intrinsic brightness: *standard candles*.



Standard Candles

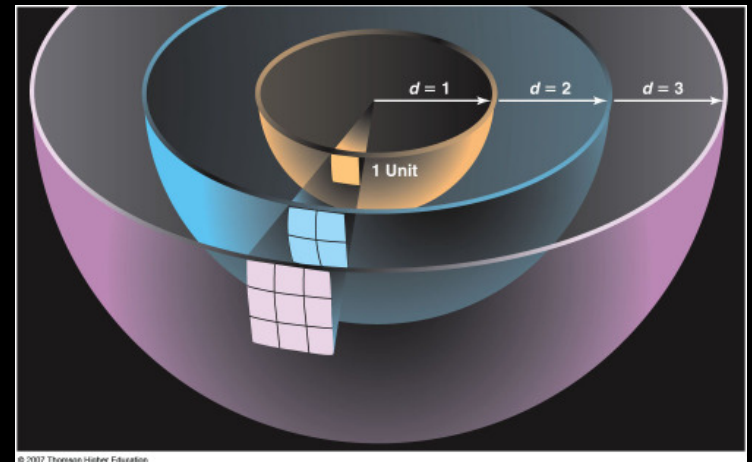
- To determine distances in the Universe, astronomers identify objects of which they know the intrinsic brightness:

Standard Candles

- Knowing the intrinsic luminosity/brightness L_{abs} of a star/object, and measuring its apparent brightness, or flux S (light through per unit area),

the distance D_L may simply be inferred from

$$S = \frac{L_{abs}}{4\pi D_L^2}$$

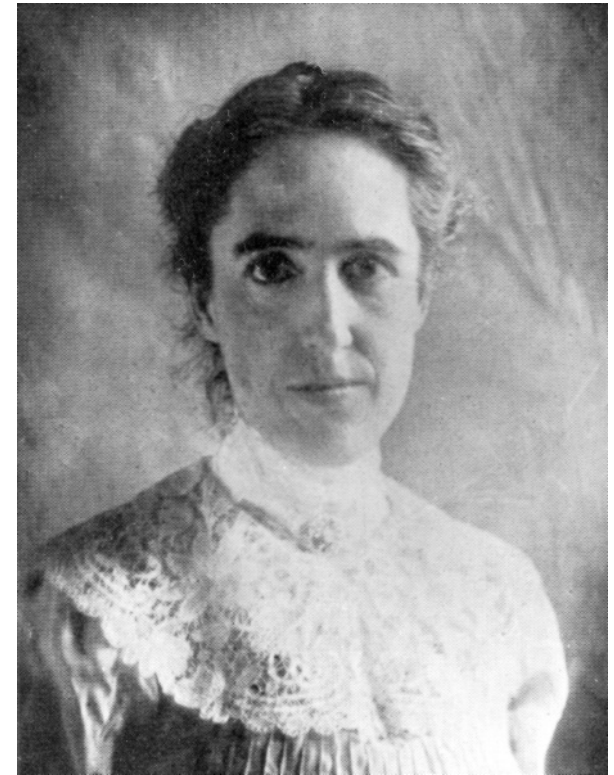


Cepheids: Period-Luminosity

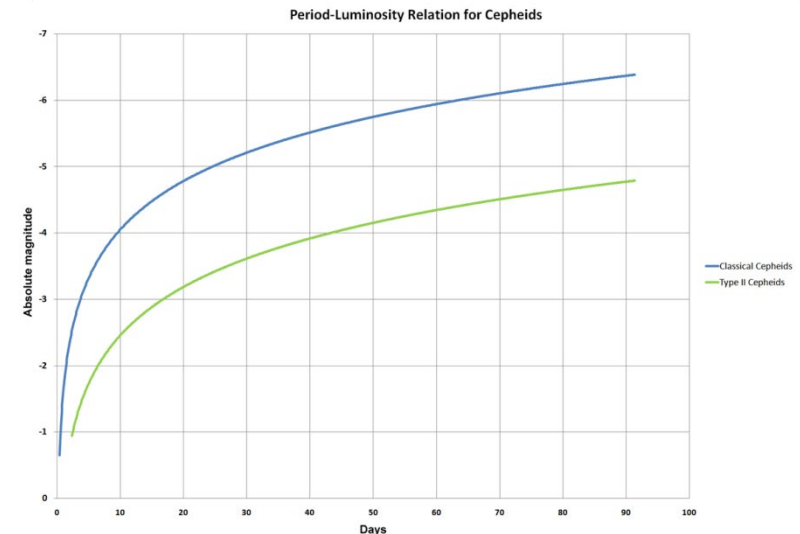
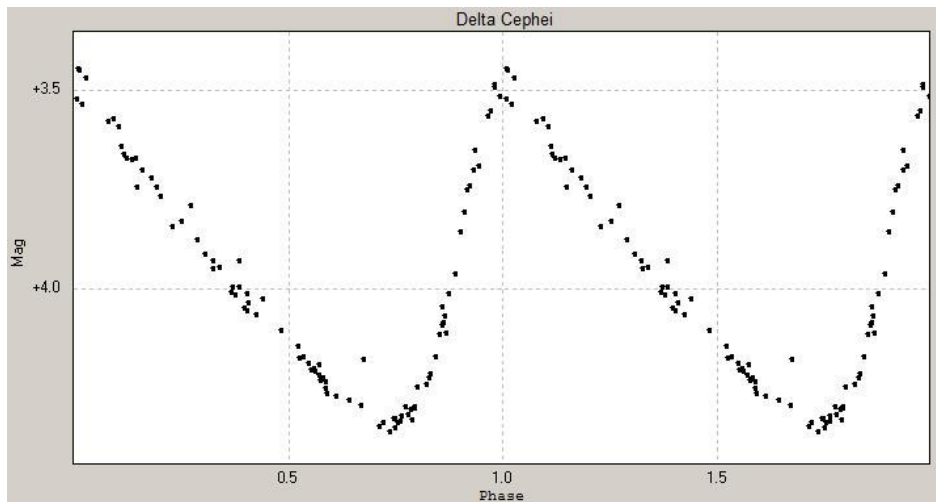
- To be able to determine cosmological distances, the reference Standard Candles
- need to be very bright objects/stars, whose intrinsic luminosity has been determined to high precision.
- It was **Henrietta Swan Leavitt (1868-1921)** who discovered that a particular type of variable stars, the Cepheid stars,
 - whose brightness varies as a result of their weeks long rhythmic pulsations –have a characteristic relation between
 - the period of their variation/pulsation
 - their intrinsic brightness
 - the so-called *Period-Luminosity relation*
- As individual Cepheid stars are very bright
 - up to 100,000 times the Sun's luminosity,
 - with masses in the order of 4-20 M_{\odot} -
 - they can be identified in other galaxies
- and the distance to those galaxies determined.



Henrietta Leavitt (1868-1921)



- Henrietta Swan-Leavitt started working in 1893 at Harvard College Observatory as one of the women human computers hired by Edward Pickering to measure and catalog brightness of stars on photographic plates.
- In this time, she made the fundamental discovery of the period-luminosity relation of Cepheid stars.
- During her lifetime she hardly got recognition for this discovery, which is one of astronomy's most significant ones as it allowed the measurement of extragalactic distances.
- Edwin Hubble used this relation to establish the distances to nearby galaxies and discover the expansion of the Universe.





M31-V1
"Most important single object in the
history of cosmology"



Andromeda-V1:

the object that changed the Universe

- October 6, 1923:
Edwin Hubble 45 minute exposure of the Andromeda galaxy M₃₁ with the 100 inch Hooker telescope at Mount Wilson
- Identifies 3 stars as N, thinking they are Novae
- Comparison with earlier plates of same region, he realizes one is a variable: VAR !
- And that it is a Cepheid variable, enabling the determination of the distance to M₃₁
- finding it is ~ 1 million lightyears
- Virtually overnight, our perception of the Universe, and cosmic distances, changed radically !



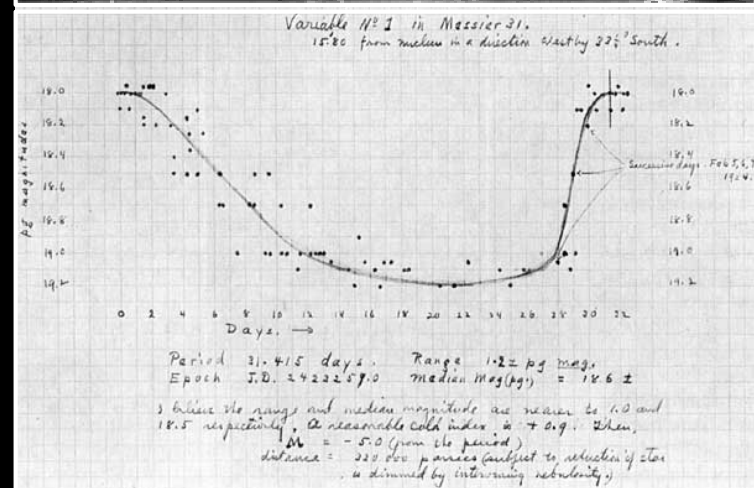
Edwin Hubble 1923
Carnegie Observatories
100-inch Telescope.



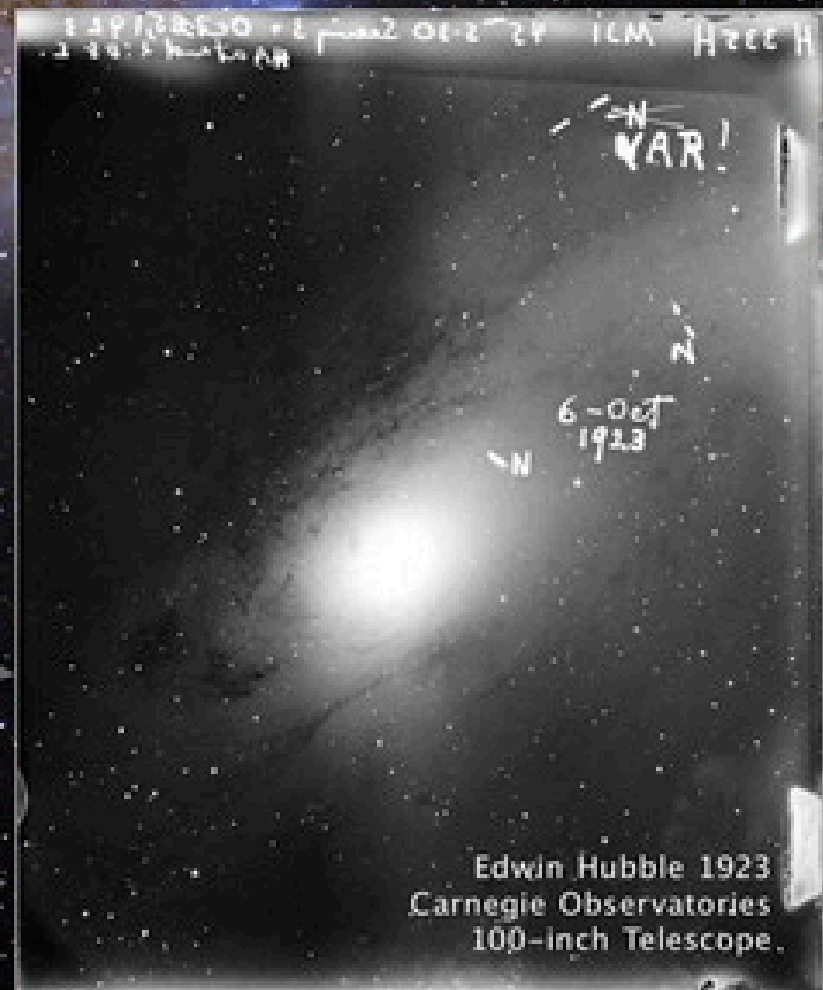
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- Virtually overnight, our perception of the Universe, - of cosmic scales and distances - changed in a radical and revolutionary way !

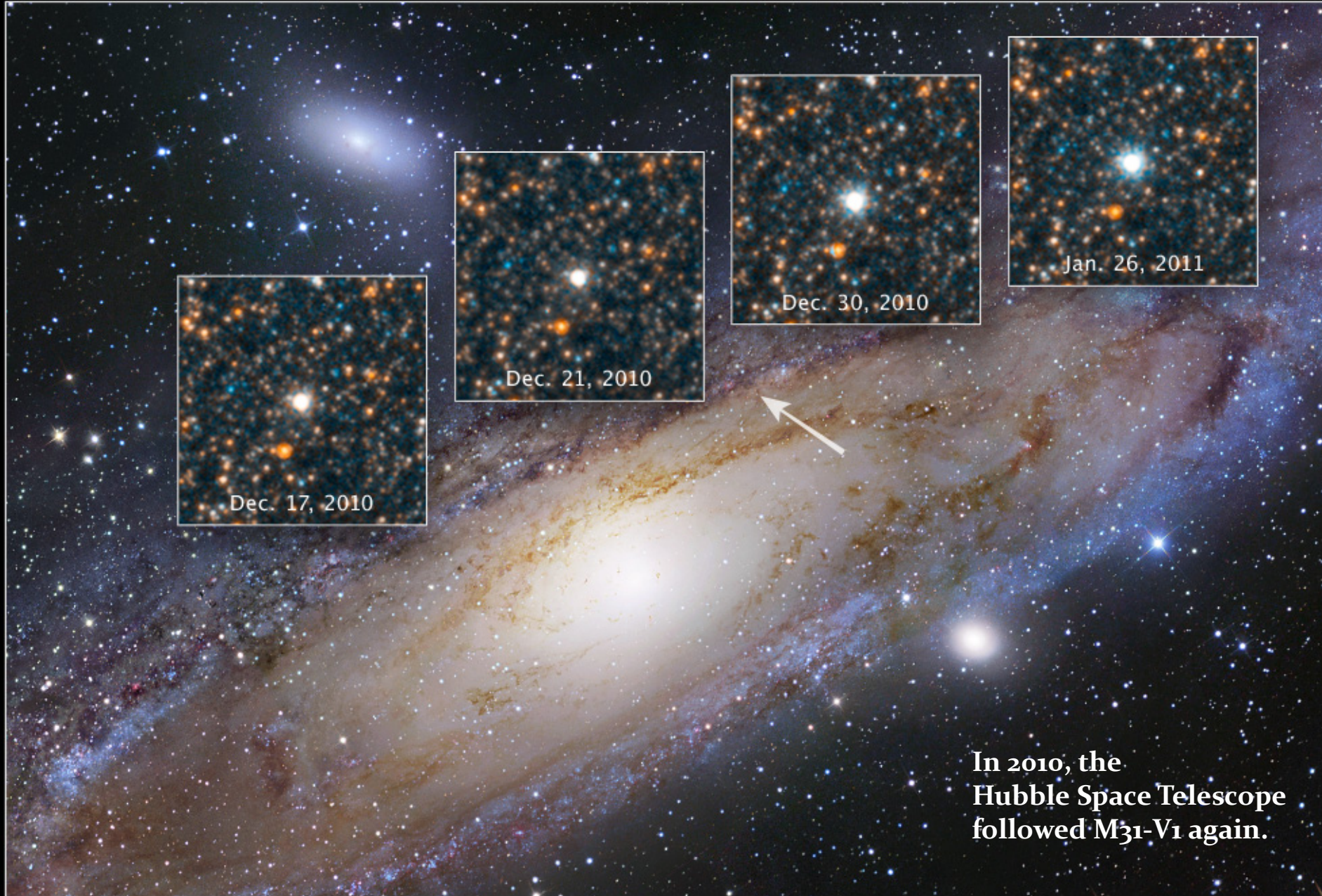


Andromeda-V1



Cepheid Variable Star V1 in M31

Hubble Space Telescope ■ WFC3/UVIS



In 2010, the Hubble Space Telescope followed M31-V1 again.

Galaxy Velocities: Redshift

- Velocity measurement:

redshift/blueshift of radiation emitted by a source (galaxy, star)

- Comparable to Doppler shift:

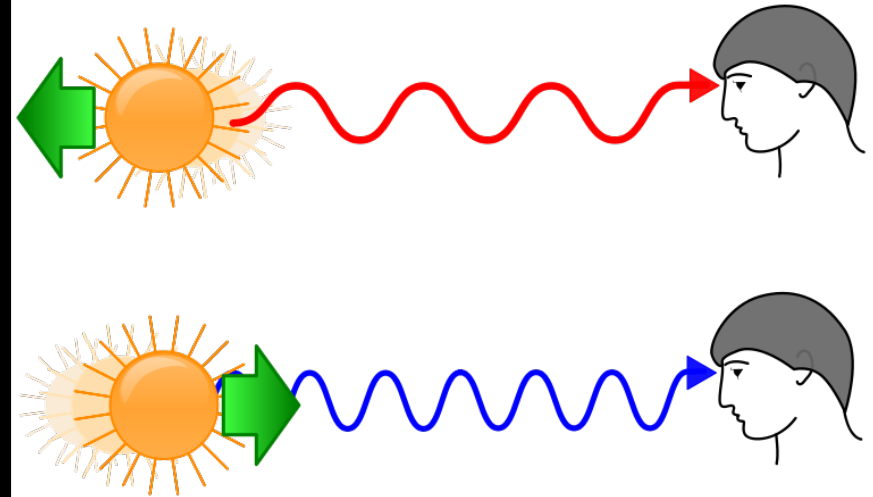
the wavelength of radiation emitted by a source changes as it has a velocity towards or away from us:

towards us:

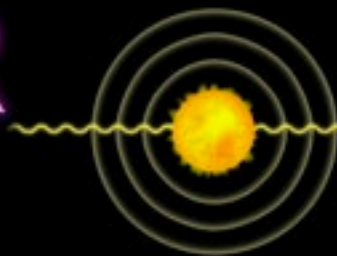
- towards shorter wavelength/higher frequency
- towards blue

away from us:

- towards larger wavelength/lower frequency
- towards red



DOPPLER EFFECT



When a star is stationary relative to an observer, the light produced looks the same no matter what direction it is seen from. Our sun is a good example of a star that is not moving much nearer or farther from the Earth.

If stars move either towards or away from our vantage point, however, the motion shifts the way their light looks to us.

RED SHIFT

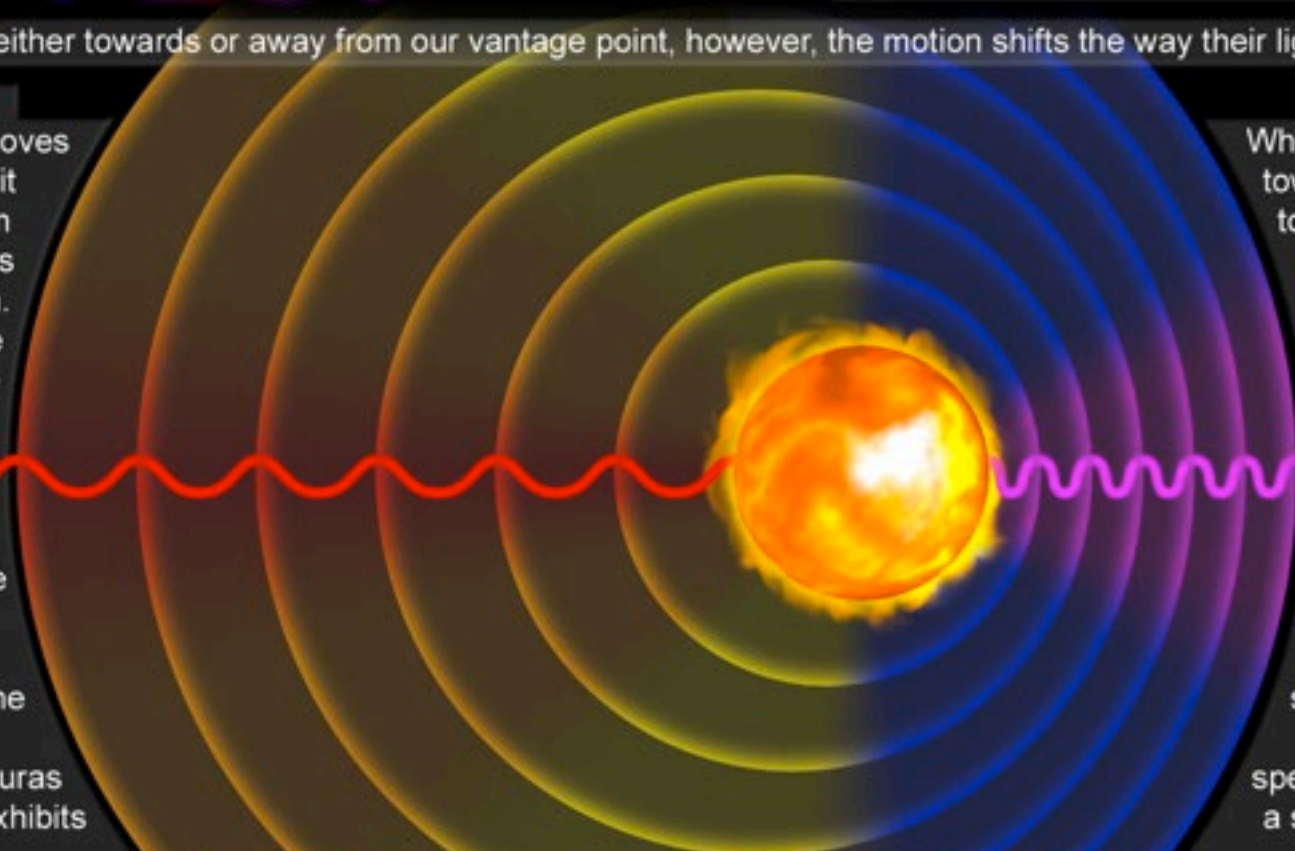
When a star moves away from us, it runs away from the light it emits in our direction. This makes the light waves we see expand.

Because the wavelengths are longer than usual, the light shifts toward the red side of the spectrum. Arcturus is a star that exhibits red shift.

BLUE SHIFT

When a star moves toward us, it starts to catch up to the light it emits in our direction. This makes the light waves we see contract.

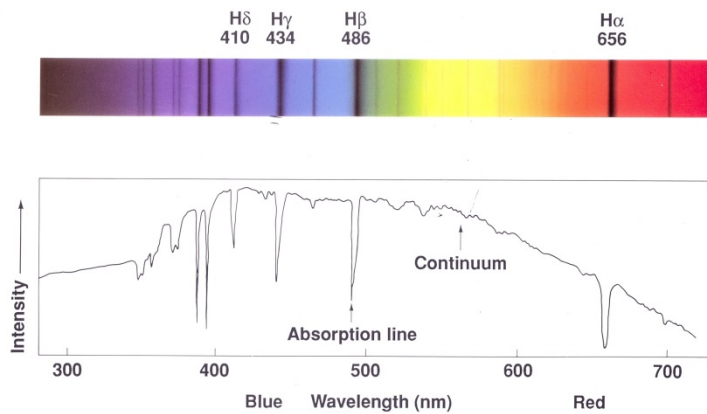
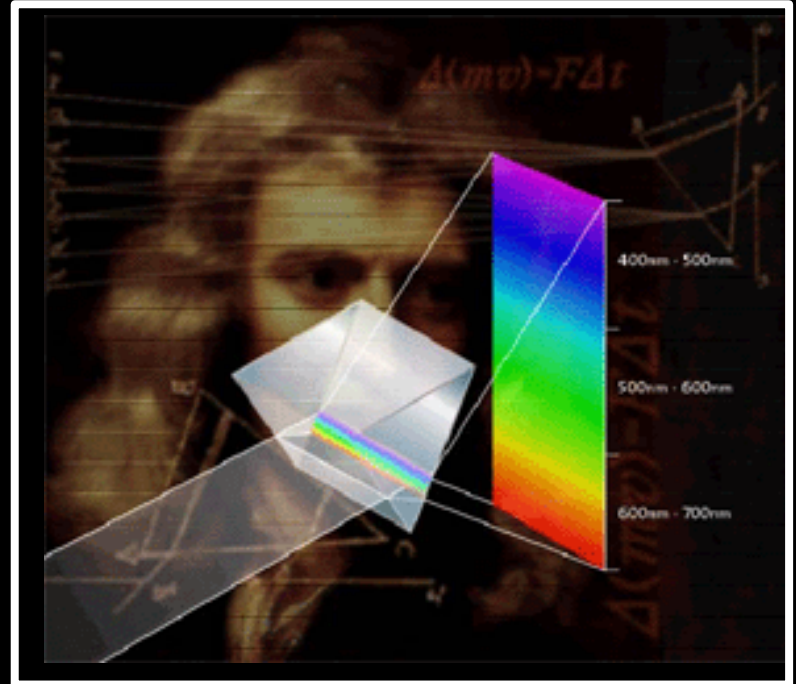
Because the wavelengths are shorter than usual, the light shifts toward the blue side of the spectrum. Sirius is a star that exhibits blue shift.



Most shifts can not be seen with the naked eye, but astronomers can measure them to learn whether other stars are advancing or receding.

Stellar Spectra & Spectral Lines

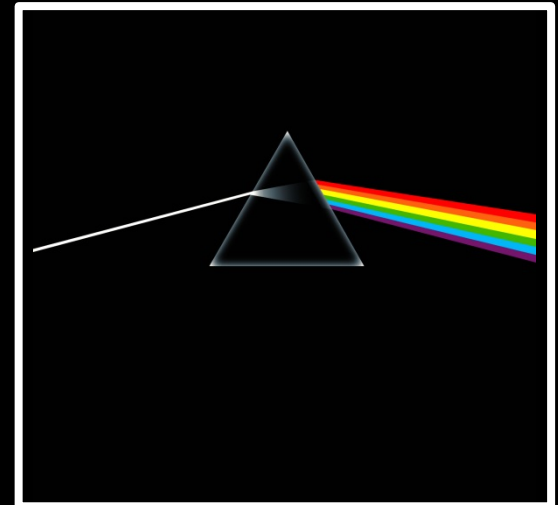
- Look at the spectrum of the light emitted by a galaxy:
- Spectrum: energy distribution of light
red: lower energy
blue: higher energy
Example: use prism to dissect light
- In the spectrum of stars, you see a large number of lines:
 - light/photons of specific energy/frequencies absorbed by atoms & molecules in the atmospheres of stars
 - the frequencies of these spectral lines are fixed, by the *quantum laws* governing the structure and dynamics of atoms



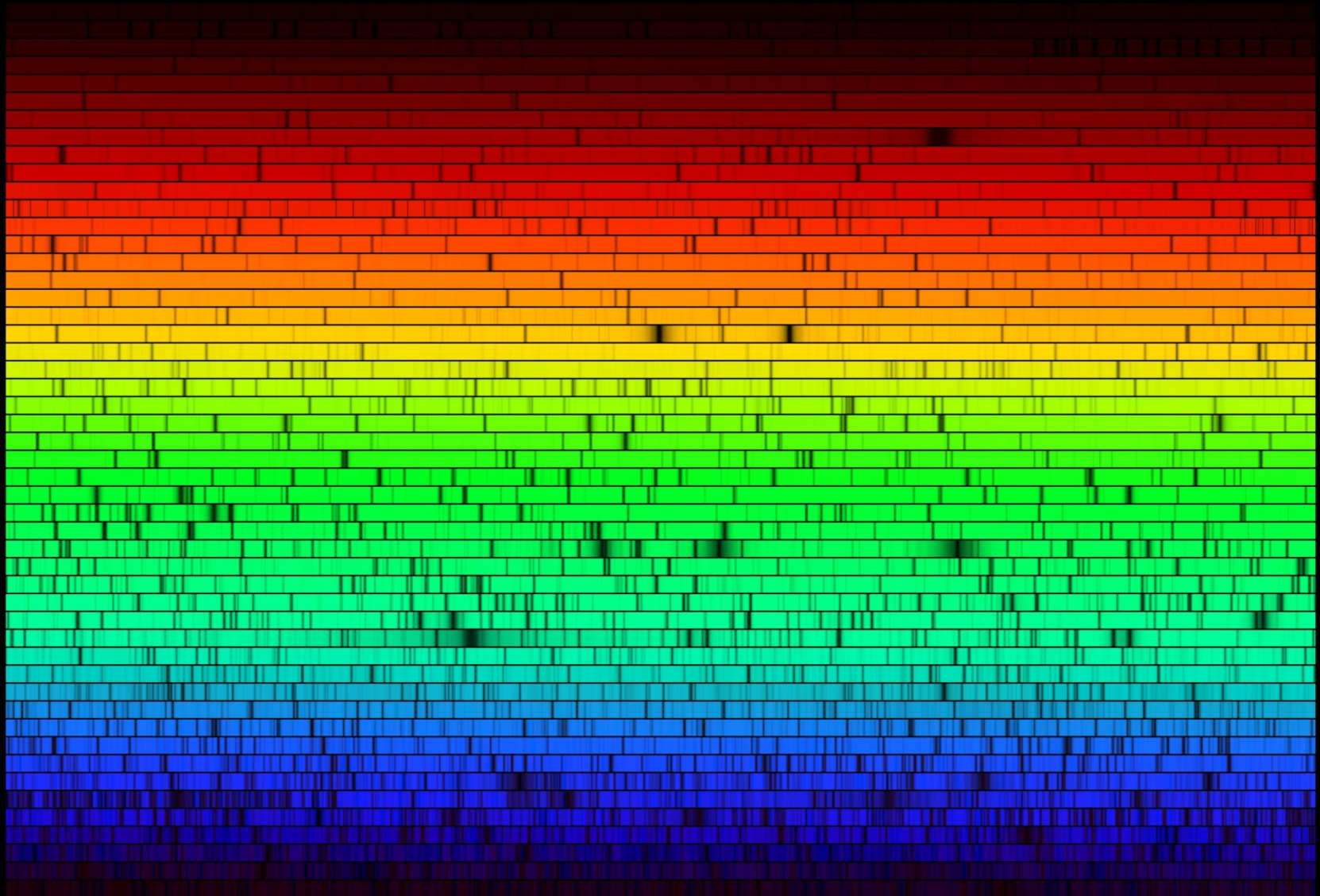
Visual portion of stellar spectrogram

Hartmann/The Cosmic Journey, 4th ed., Fig. 16-5; The Cosmic Voyage, Fig. 16-3

© 1991 Wadsworth, Inc.



Solar Spectrum



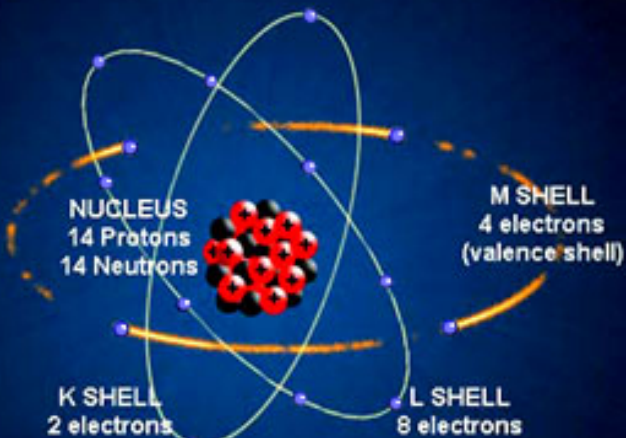
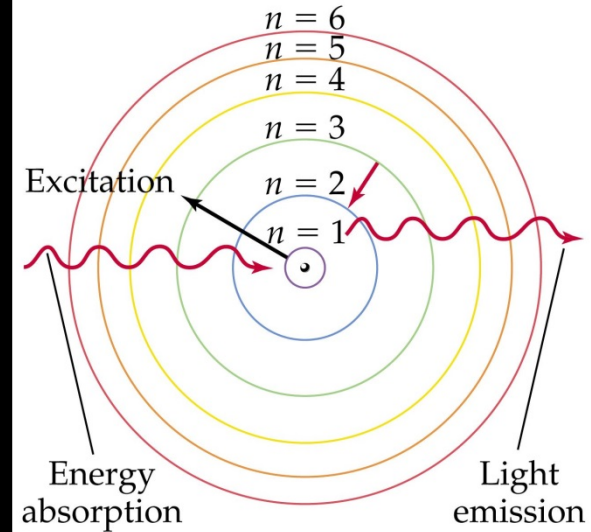
Notice the signatures/absorption lines of atoms (and molecules) in the atmosphere of the Sun

Atoms & Spectral Lines

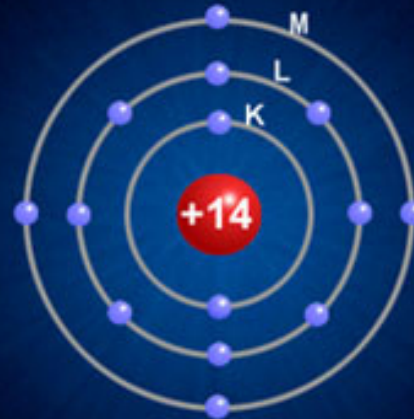
$$E = h\nu = \frac{hc}{\lambda}$$

The energy of a photon is directly proportional to

- directly proportional to its frequency (ie. colour) ☑
 - inversely proportional to its wavelength ☑
- in this: c - velocity of light; h - Planck constant



(a) Pictorial view



(b) Schematic View

The energy transitions go along with

- towards *higher* level: *absorption* of photon with that specific energy
- towards *lower* level: *emission* of photon with that specific energy

Energy of photon = frequency light

Galaxy Spectra & Cosmic Redshift

- Galaxy spectra:
 - the combined light of 100s billions of stars
 - absorption lines mark the frequencies at which the atmospheres of the stars in the galaxy have absorbed light emitted by the stars
- Galaxy redshift determination:
 - identify (well-known and strong) spectral lines
 - compare to rest wavelength, then determine z

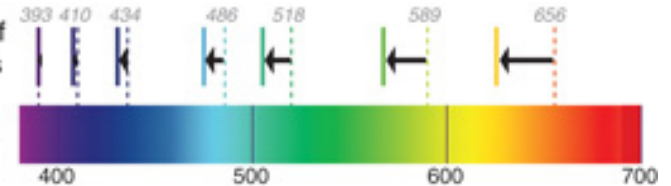
$$z = \frac{\lambda - \lambda_0}{\lambda_0}$$



← if a galaxy is moving **towards** an observer on Earth



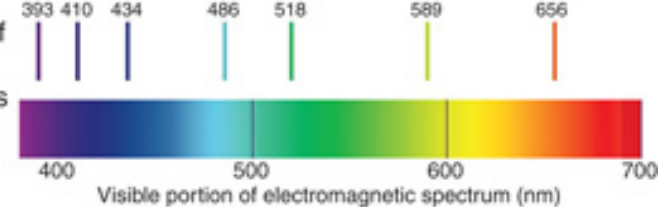
Emission spectrum of approaching galaxies will shift towards shorter wavelengths: **blueshift**



if a galaxy is **not moving** relative to an observer on Earth



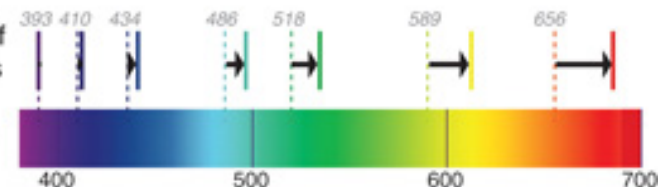
Emission spectrum of stationary galaxies will be at wavelengths of component gases like Ca, H, Mg, Na



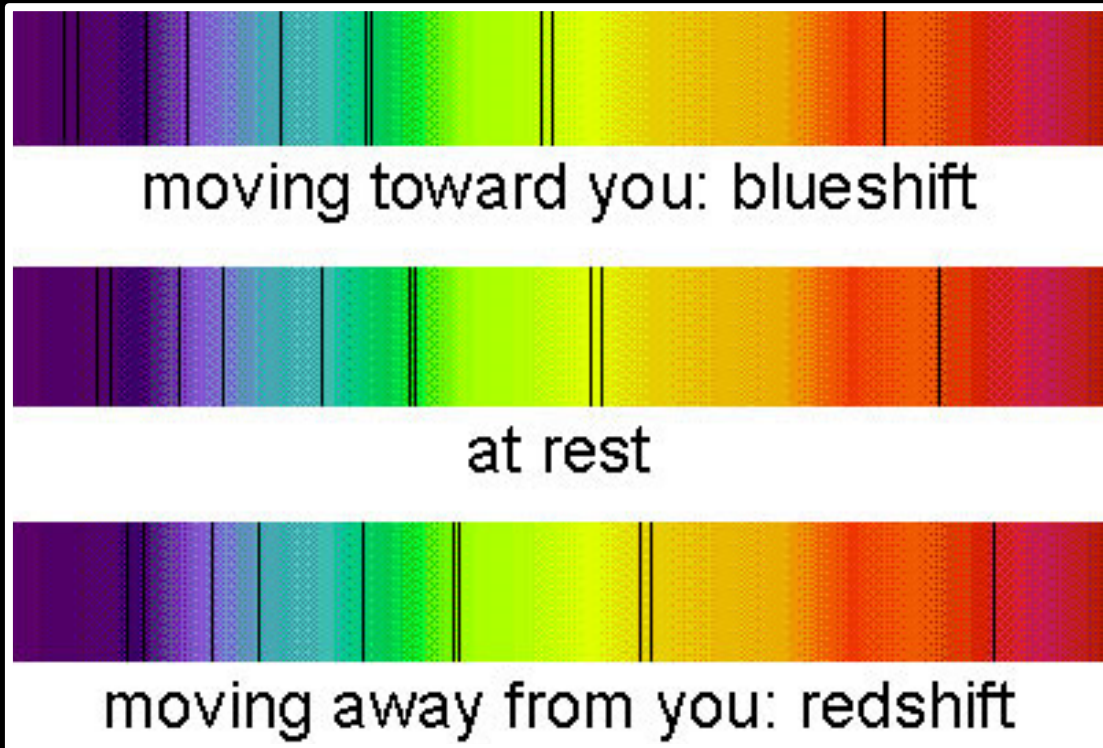
→ if a galaxy is moving **away from** an observer on Earth



Emission spectrum of receding galaxies will shift towards longer wavelengths: **redshift**



Redshifted Galaxy



Slipher & Galaxy Redshifts



Vesto Slipher
(1875-1969)

US astronomer who was the first to measure redshifts of galaxies

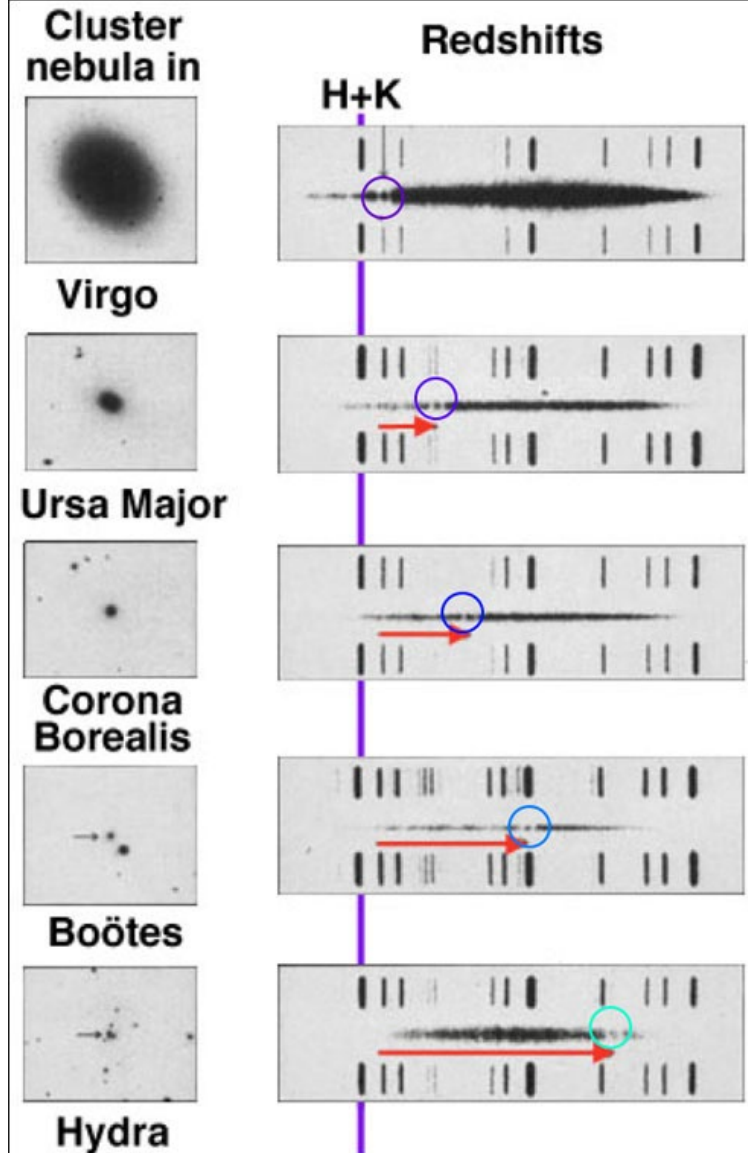
For a major part of his career he was director of

Lowell Observatory,
Flagstaff, Arizona, USA

1913: Slipher finds that the spectrum of M31 is shifted to blue, corresponding to a velocity of ~ 300 km/s

Note: and, indeed, M31 is belonging with our Galaxy to a dense group of galaxies, the Local Group, and is moving towards us.
M31 and the Galaxy will collide in 4.5 billion years

1914: additional redshifts of 14 spirals, some blueshifted (approaching), some redshifted (moving away)



Slipher & Galaxy Redshifts



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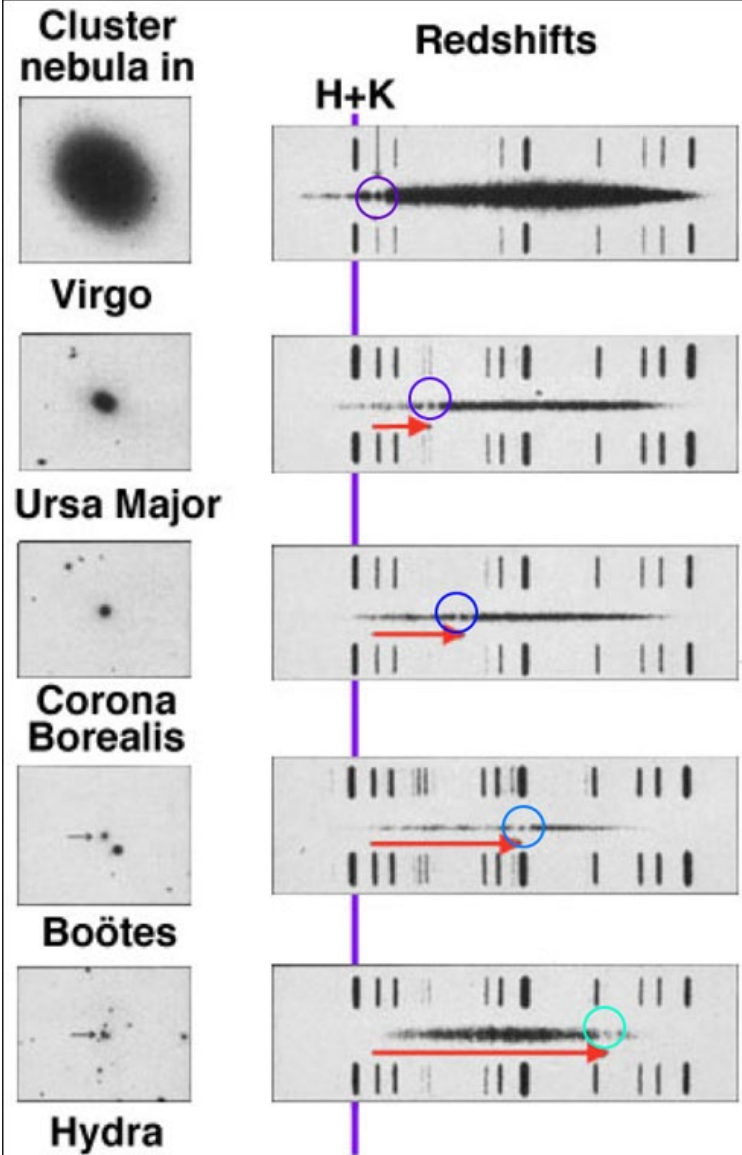
1917: Slipher measures more galaxy redshifts:

- more and more galaxies are redshifted
- proportion of redshifted galaxies such that it is no longer in accordance with random galaxy motions

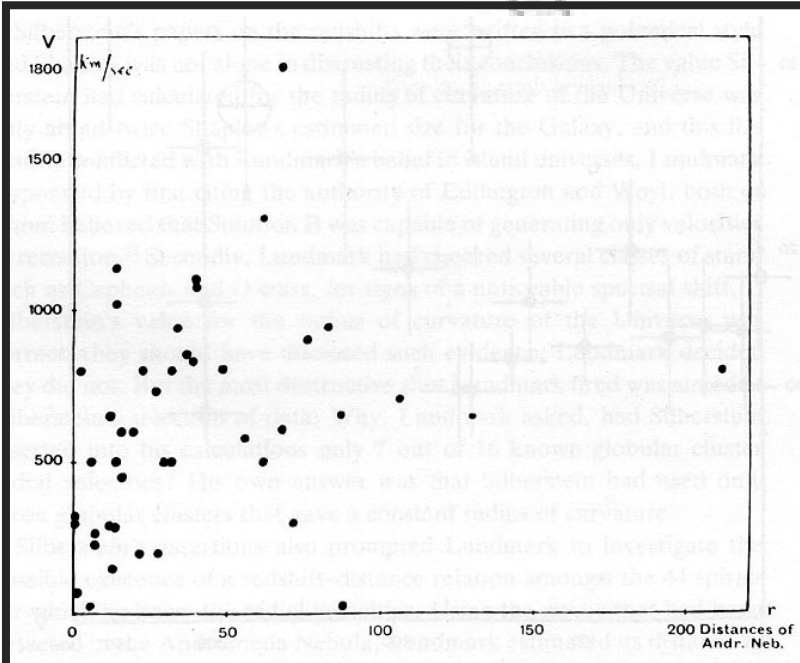
AND

- redshift on average larger as galaxy is smaller (ie. seems further away) !!!!!

Is there a physical relationship between Radial Velocity and Distance of a galaxy ???



Cosmic Expansion: first indications



1925: Lundmark, Swedish astronomer (1889-1958)

- radial velocity 44 galaxies
- rough distance estimates, comparing distances and brightnesses
- comparing to M31, estimated to be at 650,000 ly (in fact $\sim 2,000,000$ ly).

Lundmark concluded that there may be a relationship between galactic redshift and distance, but “not a very definitive one”

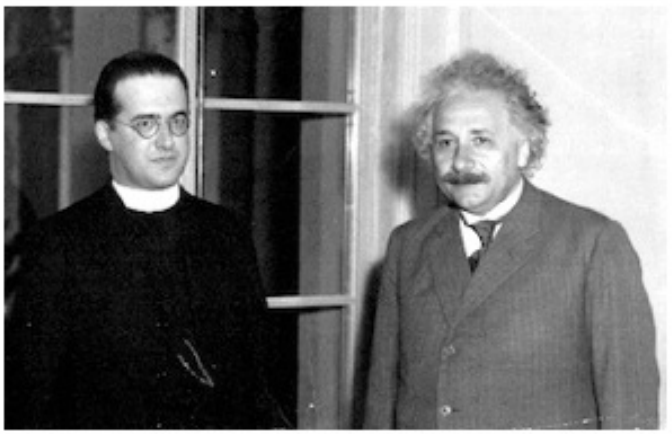
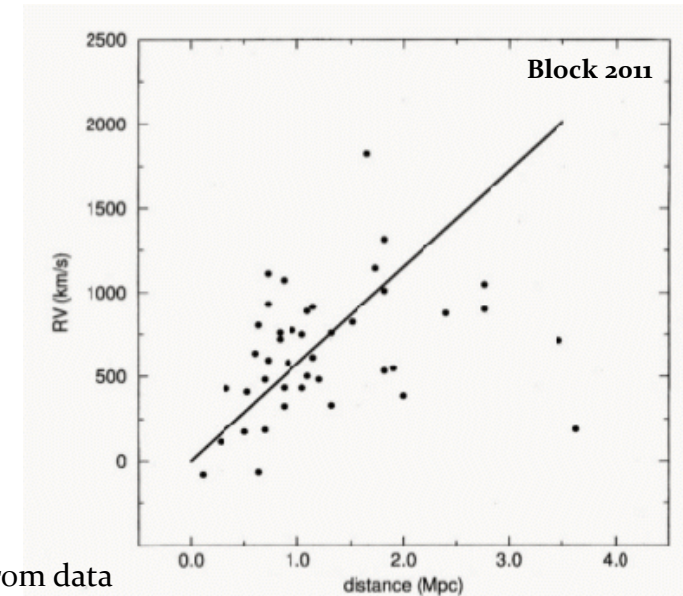
1927:
Georges Lemaitre
(1894-1966)

Belgian priest

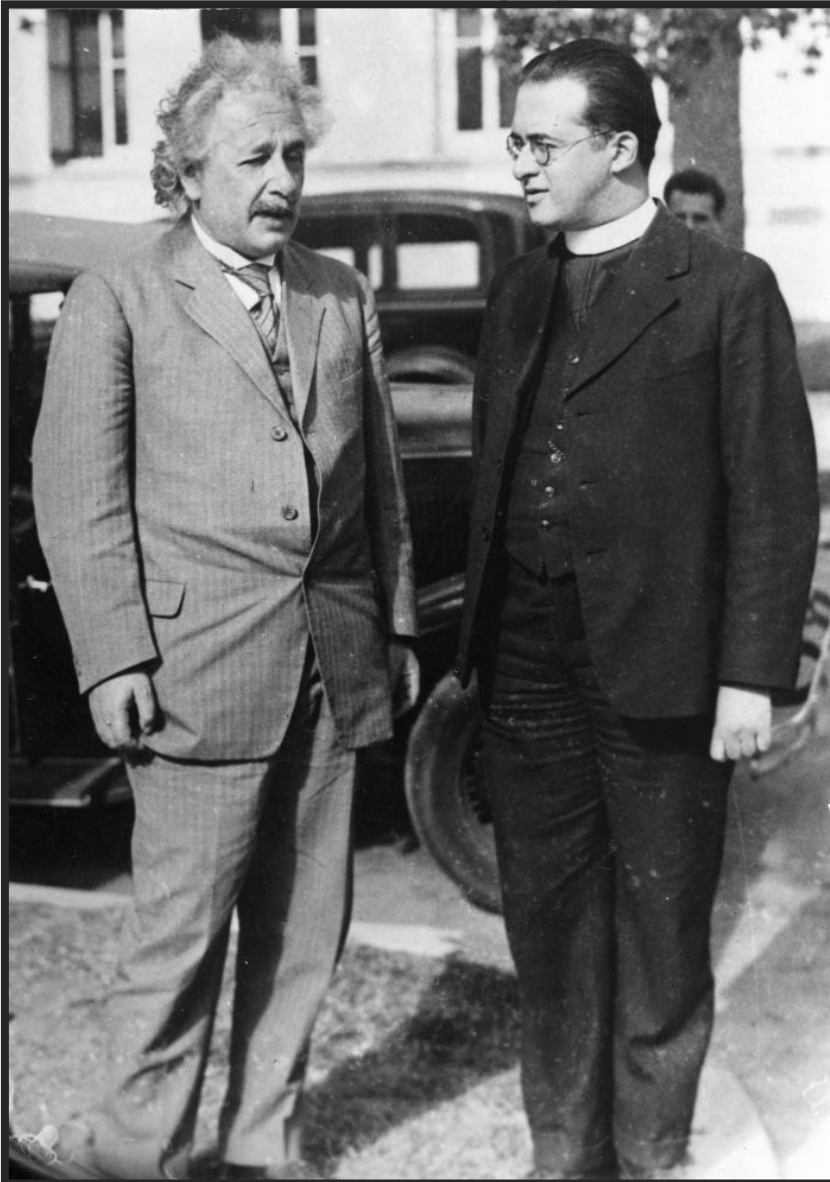
One of few who by 1920s understood General Relativity,

Predicted linear relationship redshift - distance

and ... inferred it from data



Lemaitre Expansion ?



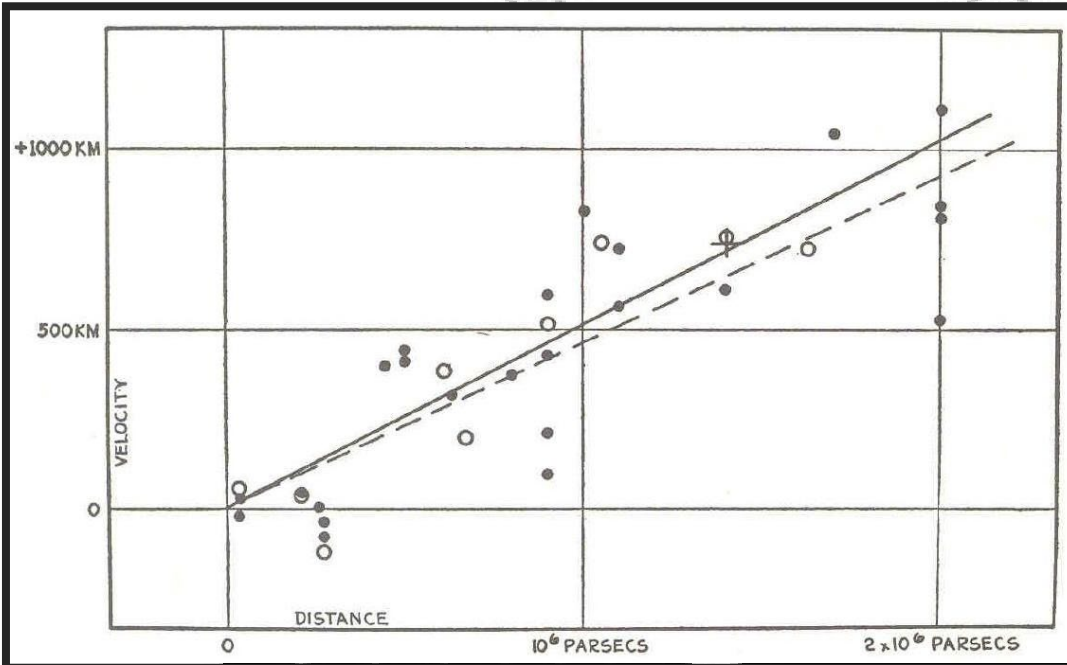
Georges Lemaitre (1894-1966)

• On the basis of the General Theory of Relativity, Lemaitre derived the equations describing the expansion of the Universe:

Friedmann-Robertson-Walker-Lemaitre equations

- He then went on to show that this predicted a linear relation between redshift/recession velocity and distance.
- In a remarkable paper, in an “obscure” French-language journal, 1927, *Annales de la Societe Scientifique de Bruxelles*, A47, 49 he then used redshifts and distances of 42 galaxies to show that it seems indeed there is such a relation, and inferred the slope of the relation, now known as the “Hubble constant”
- He assumed that the absolute brightness of galaxies can be used as standard candle, and thus inferred distances on the basis of galaxy brightnesses.
- Strangely enough, when the paper got later translated into English, the passage in which the expansion constant was determined got omitted.
- Had Hubble tried to cover up the earlier finding of expansion by Lemaitre ? A few years it was found Lemaitre himself who had translated the paper.
- Note: the scatter of the distance estimates on the basis of intrinsic brightness has a large scatter.. Significance of result was not very strong.

Hubble Expansion – Hubble Law



Finally, the ultimate evidence for an expanding Universe follows in 1929, when Edwin Hubble (1889-1953) describes his finding of a

linear recession velocity – distance relation

This relation is now known as the **Hubble Law**.

A relation between distance and radial velocity among extragalactic nebulae
E. Hubble, Proc. Nat. Acad. Sciences, 1929, 15, 168-173

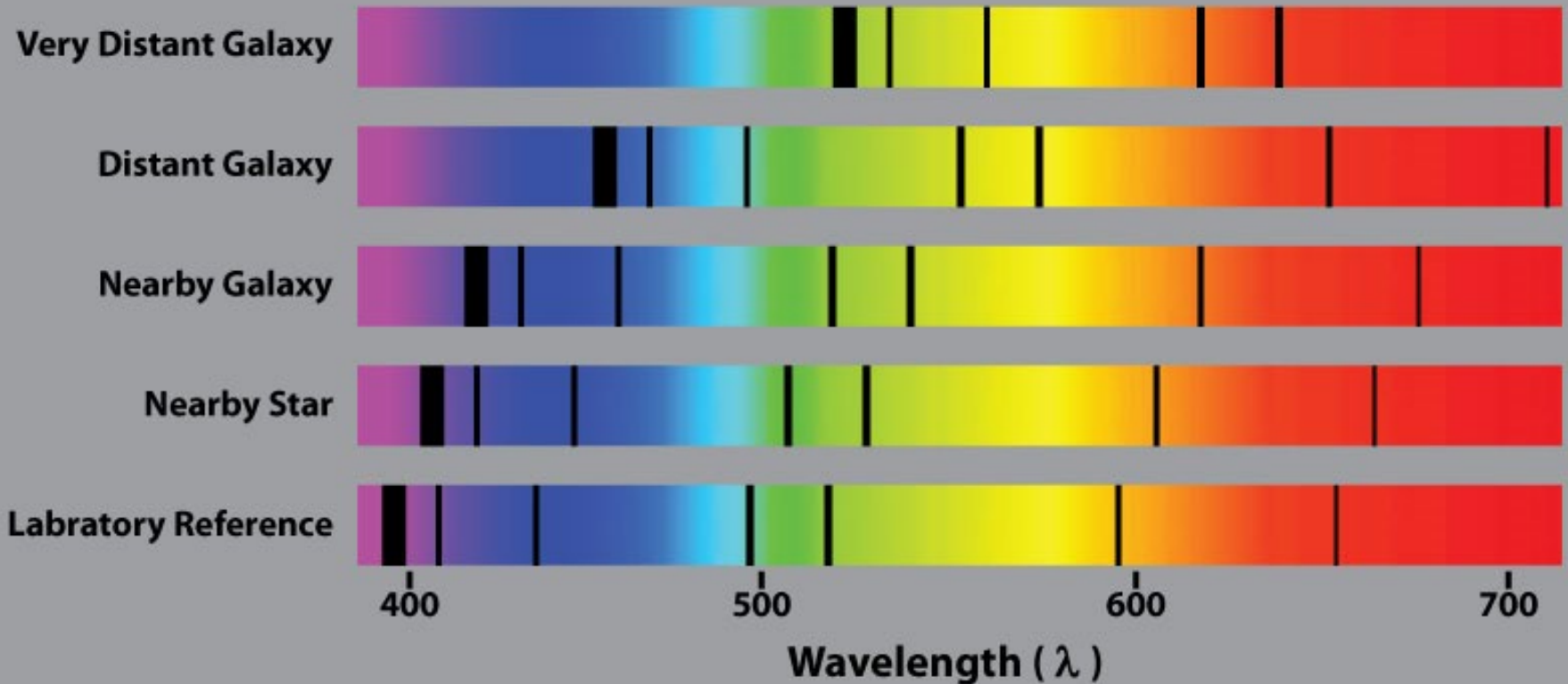
Note: Hubble himself never grasped that this was the evidence for an expanding Universe as described by the Friedmann-Lemaitre equations i.e. as implied by Einstein's theory of General Relativity.

$$v_{rad} = cz = H_0 r$$

H_0 : Hubble constant

specifies expansion rate
of the Universe

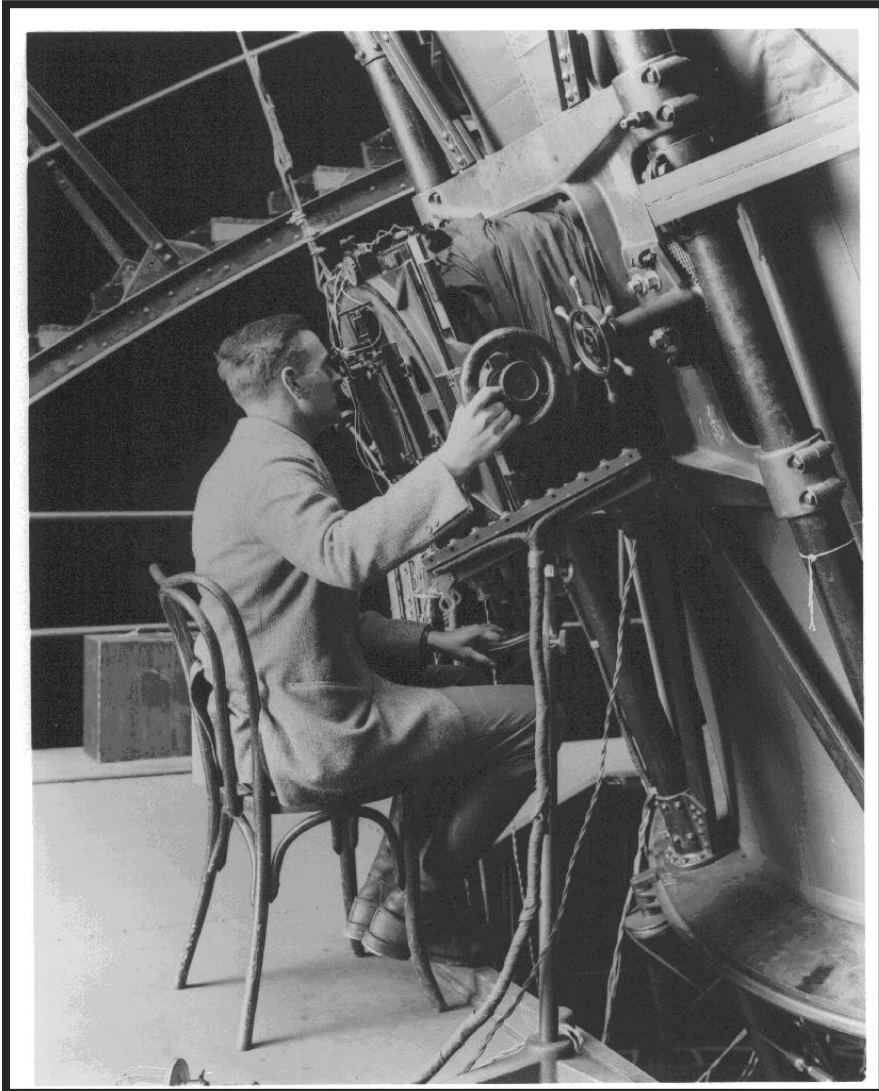
Hubble Expansion & Galaxy Redshift



$$v_{rad} = cz = H_0 r$$

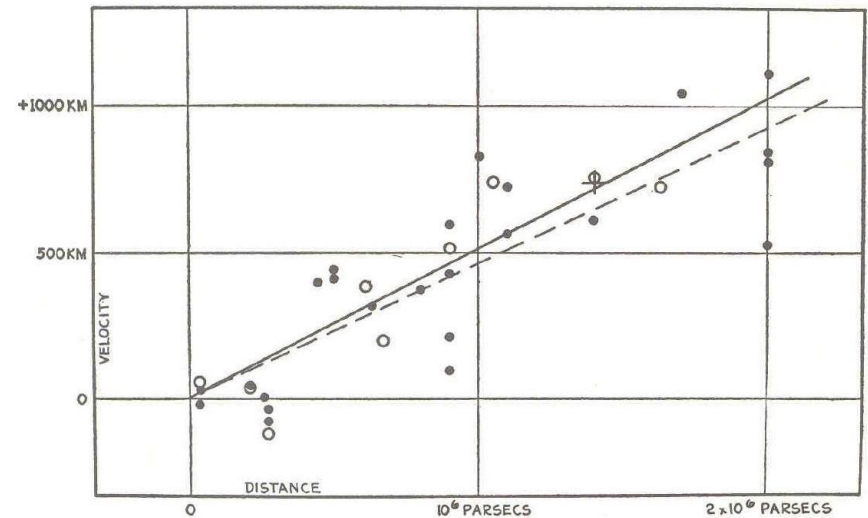
The Hubble law tells us that the further a galaxy is, the more redshifted it is. Moreover, because this is a linear relation, we can even estimate distances to galaxies once we know the value of the Hubble constant !

Hubble Expansion



Edwin Hubble

(1889-1953)



$$v = H r$$

Hubble Expansion

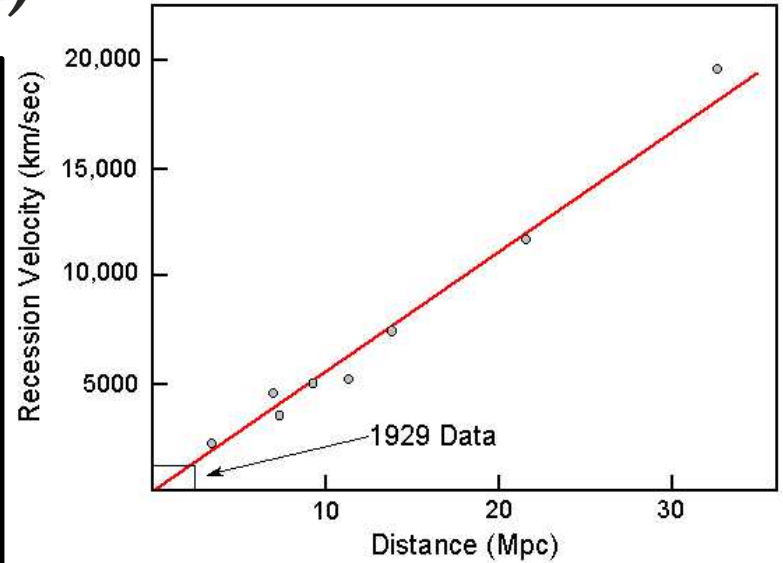
Hubble – Humason (1931)

It was in the additional publication by Hubble & Humason (1931) that the linear Hubble relation was firmly established to far larger depths into the Universe:

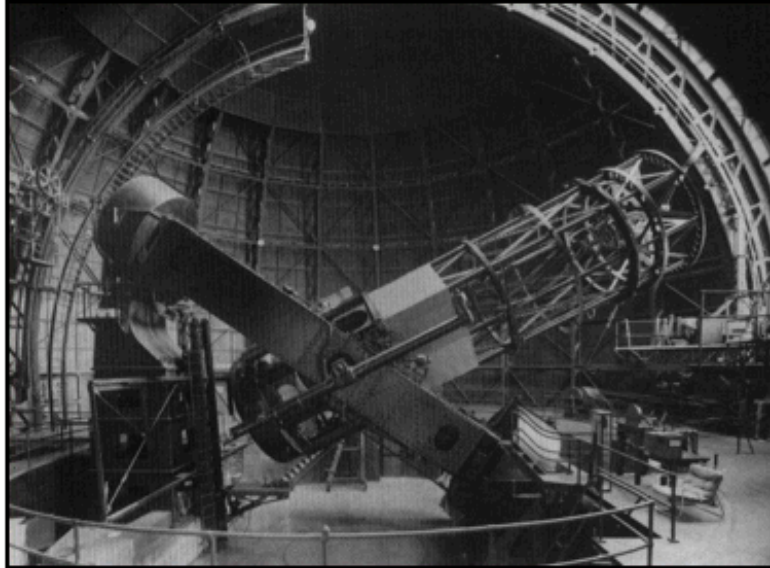
Hubble, Humason, 1931, *Astrophys. J.*, **74**, 43

Humason (1891-1972) assisted Hubble, and did most of the work on world's most powerful telescope at the time, the 100 inch Mt. Wilson telescope.

Humason did not have a PhD, left school at 14, and was hired as janitor at Mt. Wilson Observatory.
His role in the discovery of the expansion of the Universe was seminal.



Edwin Hubble
1889 – 1953



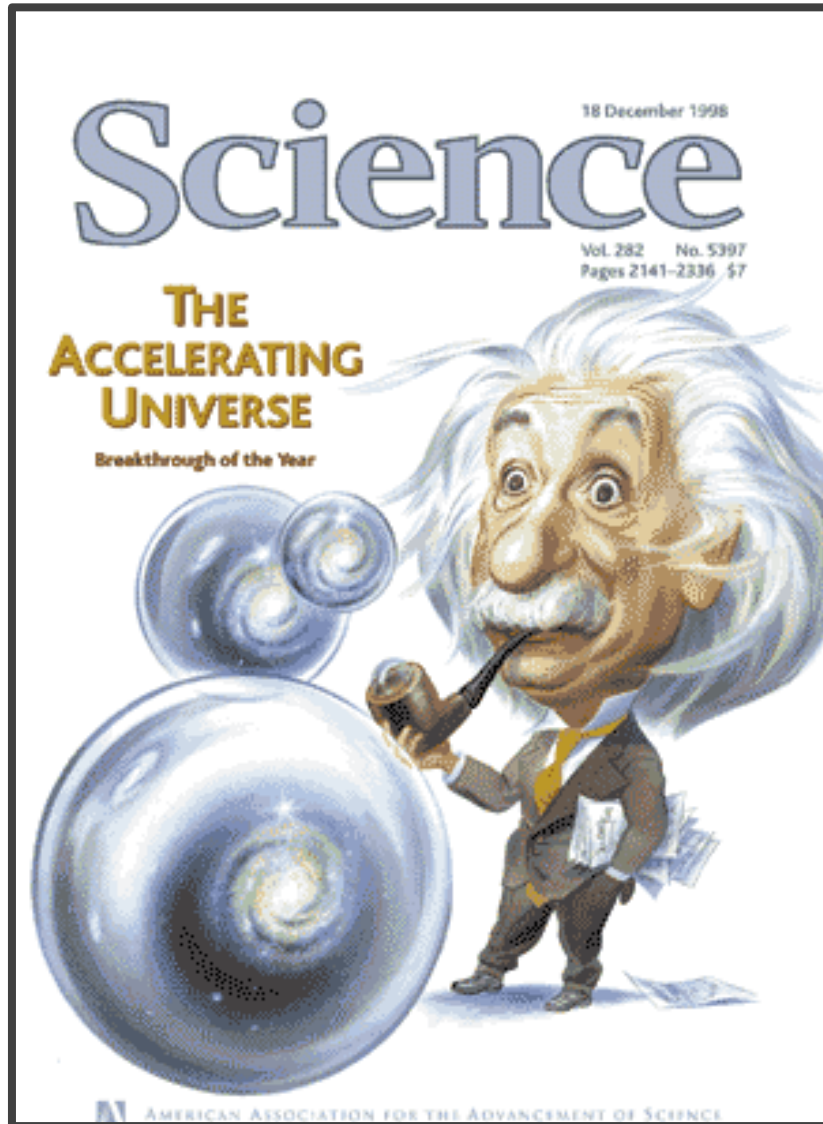
100 inch Mt Wilson Telescope



Milton Humason
1891 – 1972

Cosmic Acceleration

Science Magazine 1998



**Science
Breakthrough of the Year
1998**



Λ

Einstein's Biggest Blunder

Type Ia Supernovae

Supernova Explosion & Host Galaxy



M51 supernovae

Type Ia Supernova Explosion



Type Ia Supernova

- Amongst the most energetic explosions in our Universe:
 $E \sim 10^{54}$ ergs
- During explosion the star is as bright as entire galaxy ! (ie. 10^{11} stars)

- Violent explosion Carbon-Oxygen white dwarfs:
- Embedded in binary, mass accretion from companion star
- When nearing Chandrasekhar Limit ($1.38 M_{\odot}$), electron degeneracy pressure can no longer sustain star.
- while contracting under its weight, carbon fusion sets in, powering a catastrophic deflagration or detonation wave,
- leading to a violent explosion, ripping apart entire star

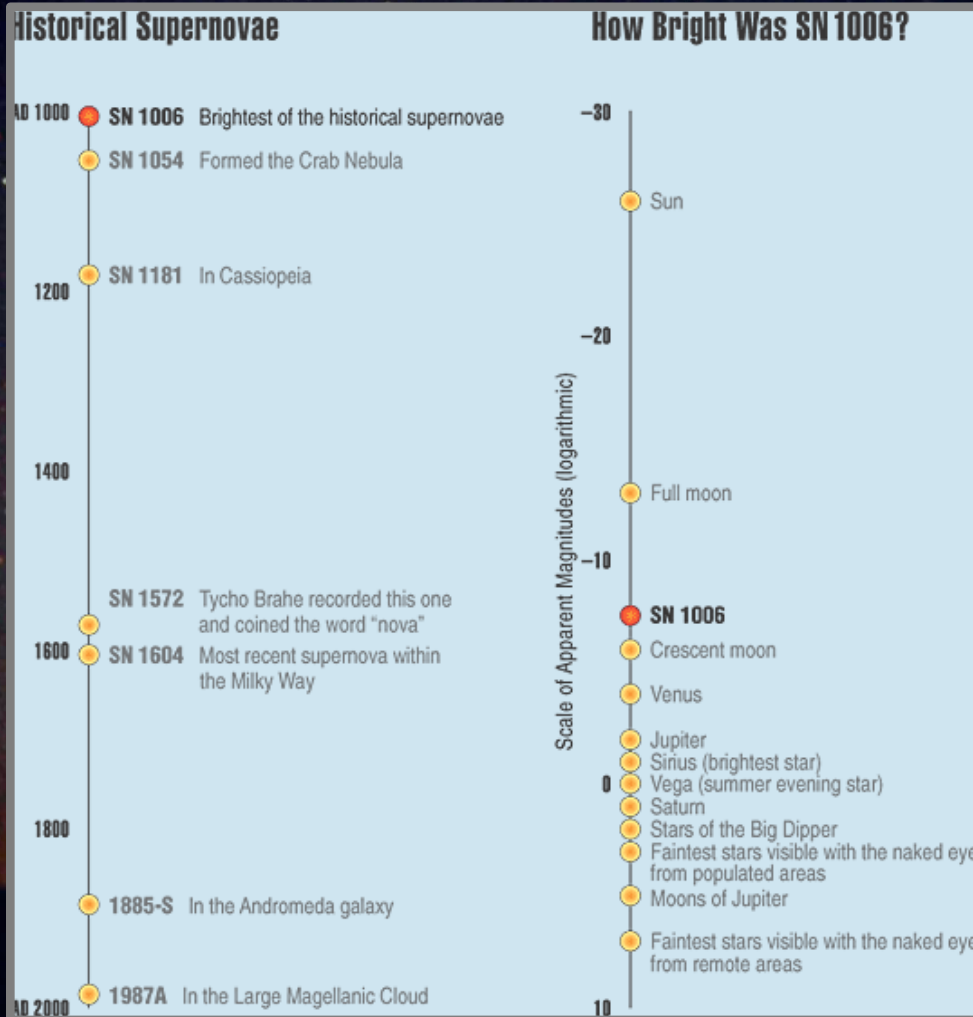
- Because exploding stars have nearly uniform progenitor ($\sim 1.38 M_{\odot}$ white dwarf), their luminosity is almost the same:
 $M \sim -19.3$
Standard Candle

SN1006



**Supernova SN1006:
brightest stellar event recorded in history**

SN1006

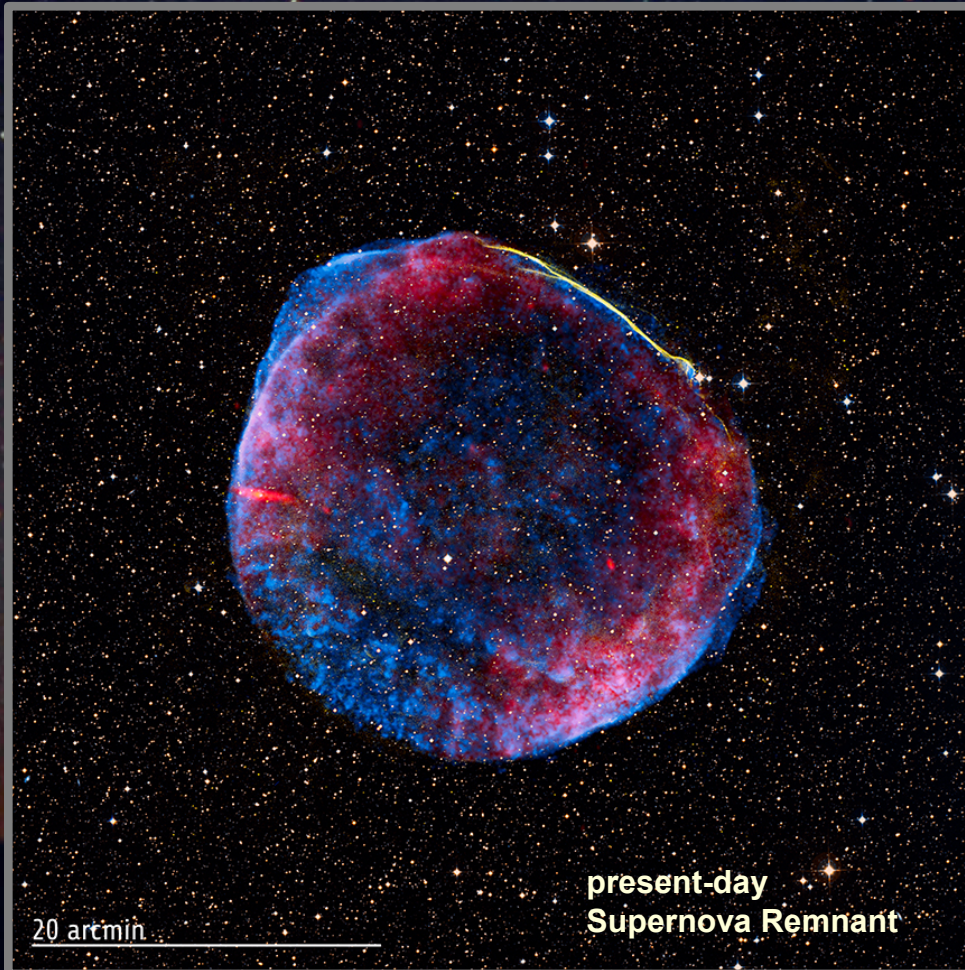


Supernova SN1006:

- **brightness:** $m = -7.5$
- **distance:** $d=2.2$ kpc
- **recorded:** China, Egypt, Iraq, Japan, Switzerland, North America

Supernova SN1006:
brightest stellar event recorded in history

SN1006



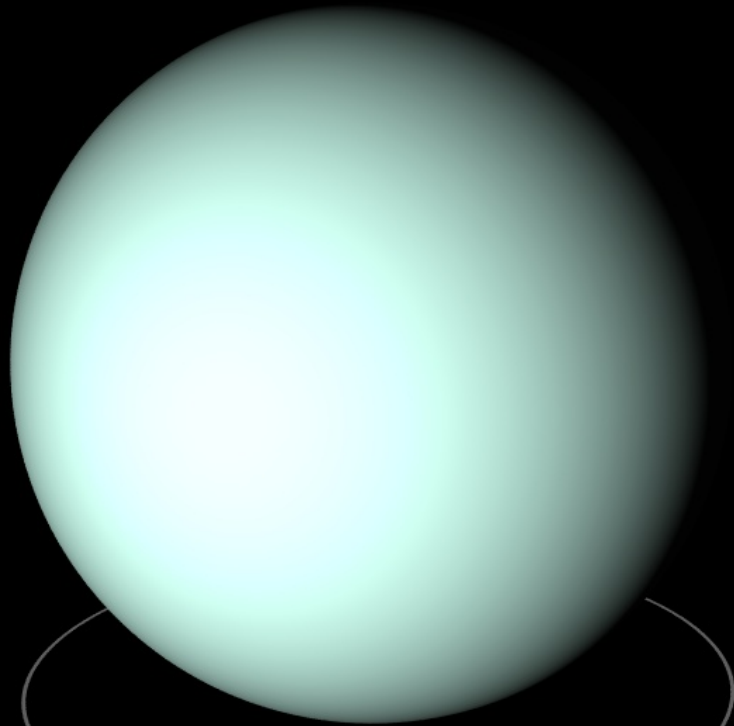
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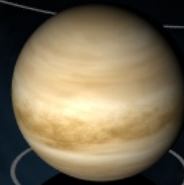
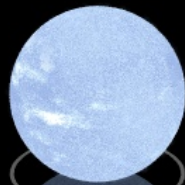
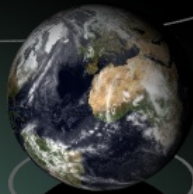
Supernova SN1006:
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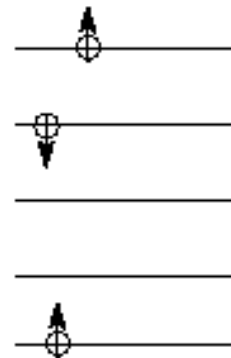
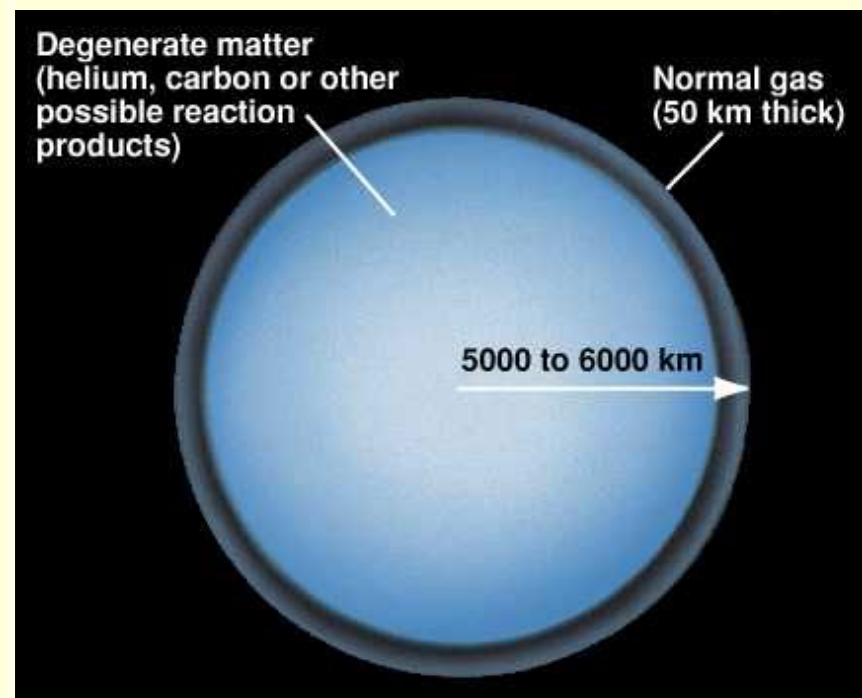
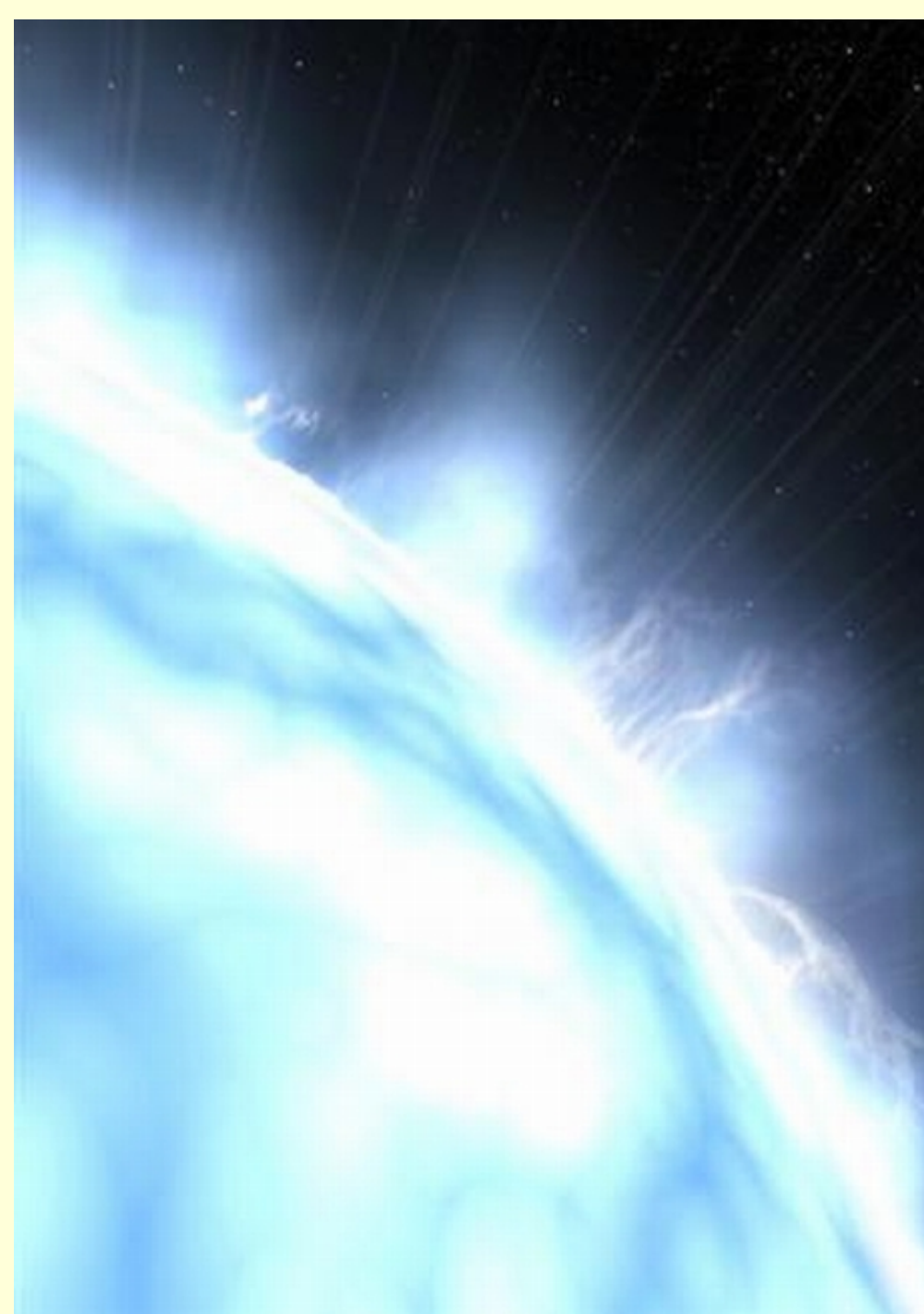
White Dwarfs



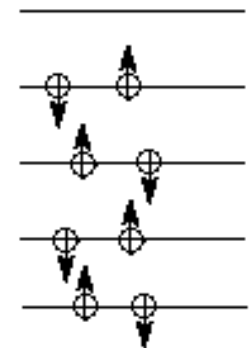


Sirius B





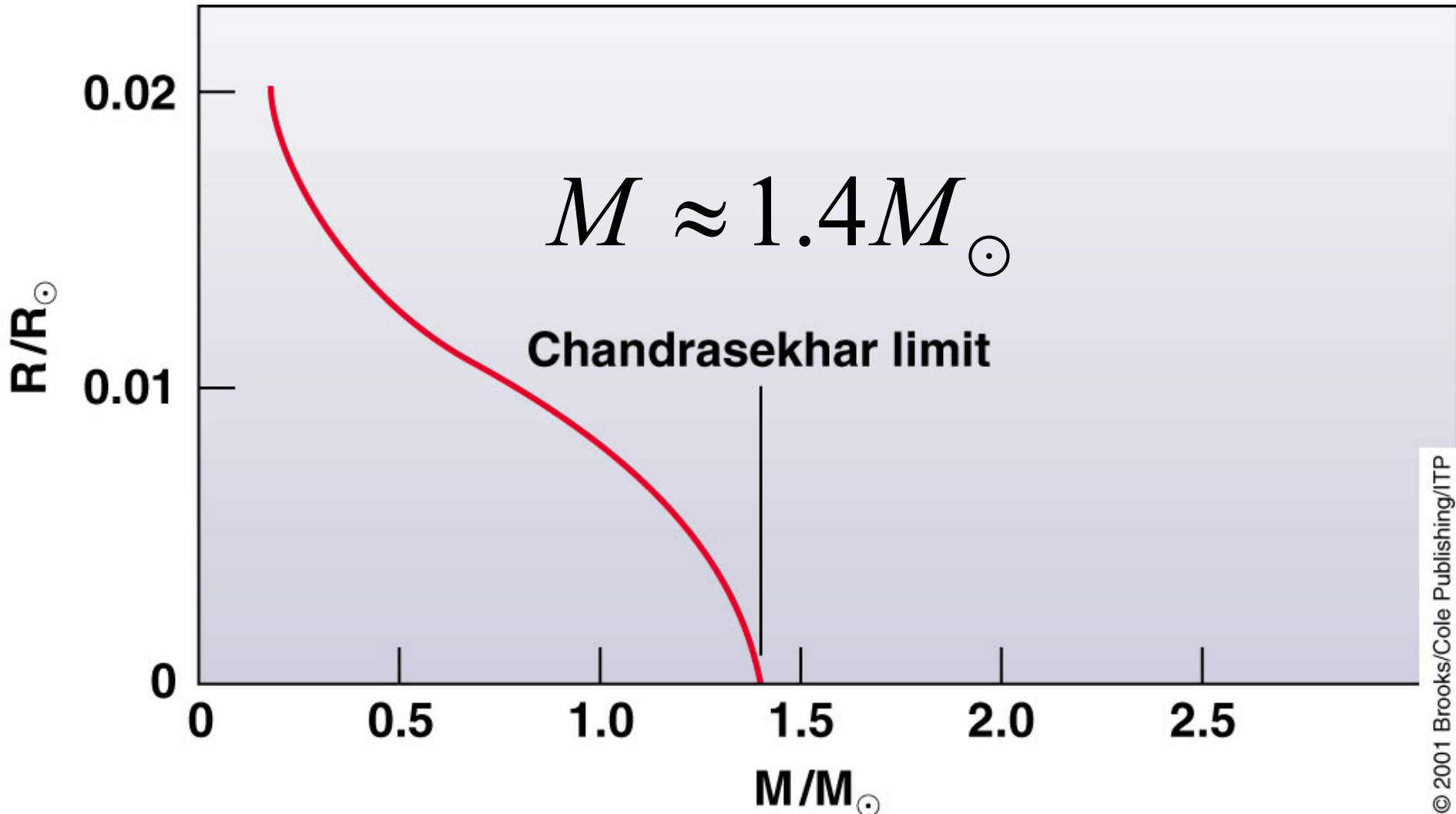
Regular gas: many unfilled energy levels. Particles free to move about and change energy levels.



Degenerate gas: all lower energy levels filled with two particles each (opposite spins). Particles **locked** in place.

Chandrasekhar Mass Limit

What is the maximum mass that can be supported by the dense compact material of a white dwarf star?



Supernova Lightcurves

SN 2007uy

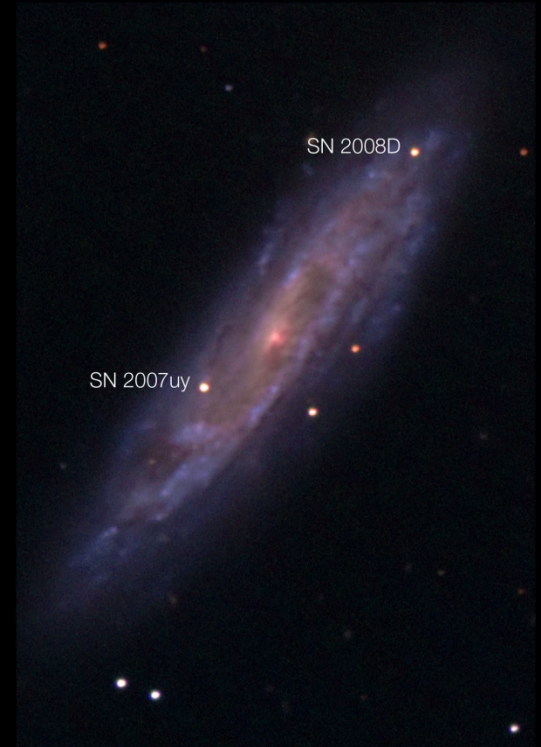
6 January 2008



12 January 2008



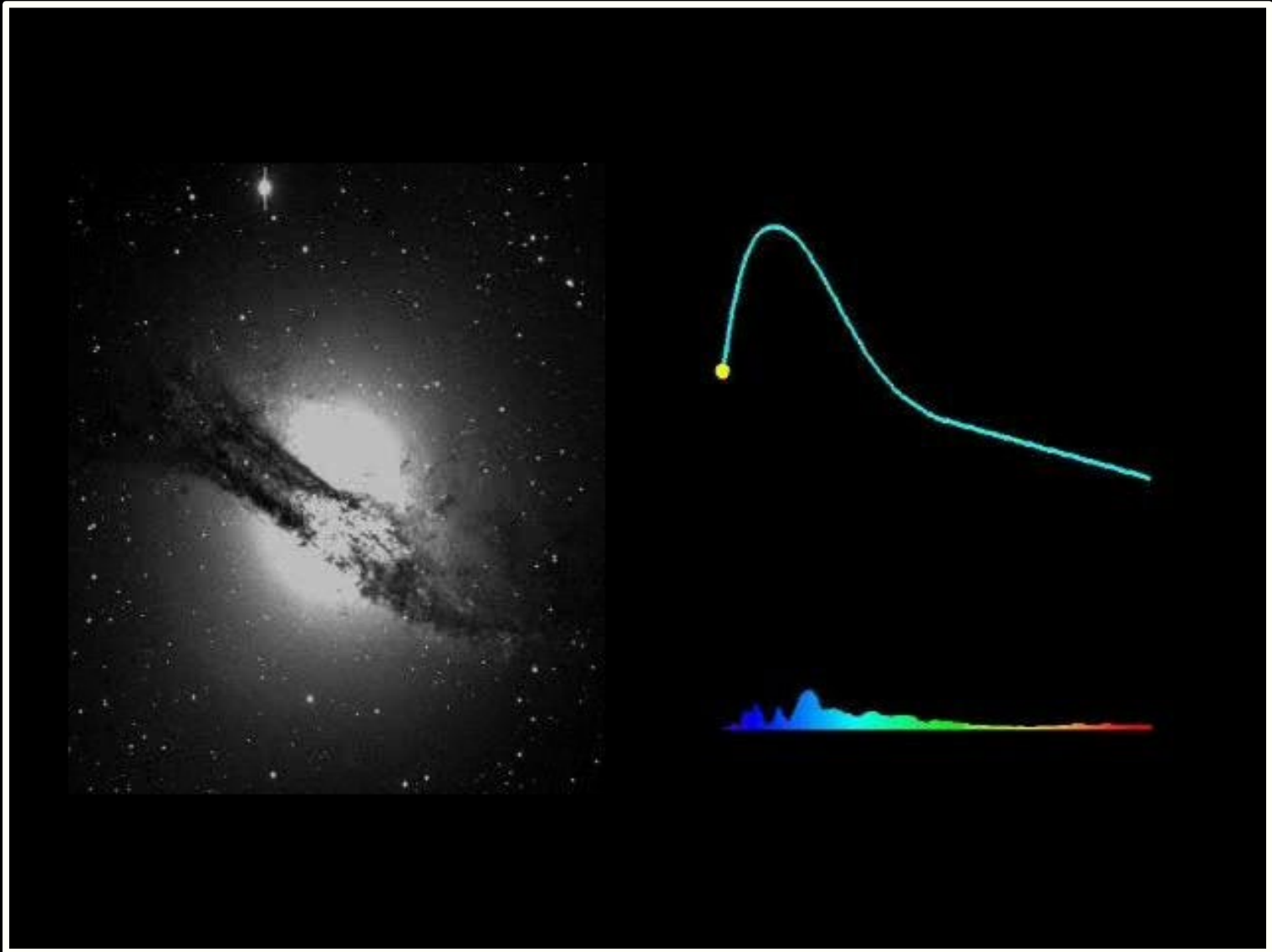
10 February 2008



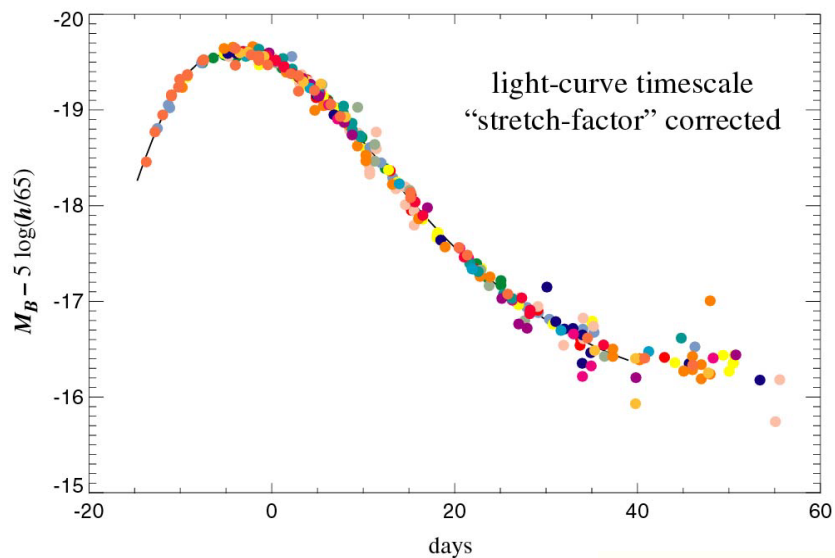
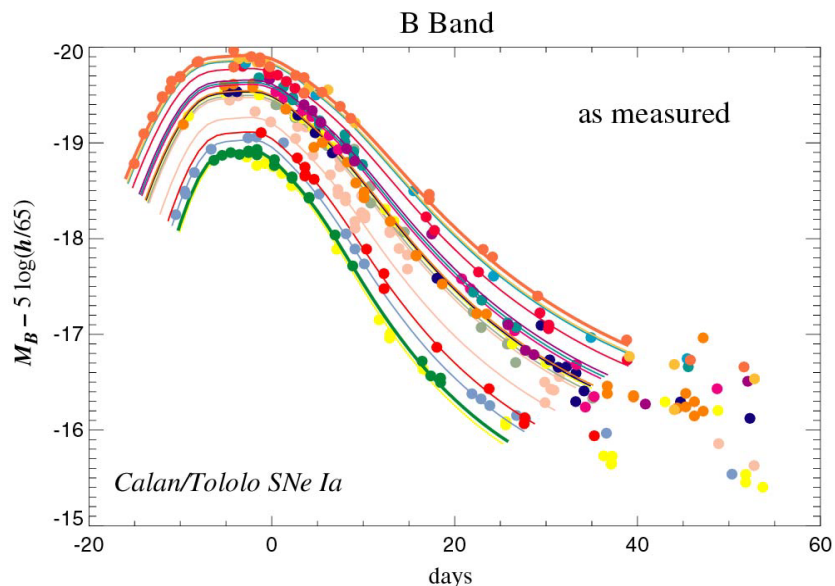
Supernova SN 2007uy in NGC2770

while fading, another supernova, SN2008D, went off in same galaxy

Supernova Lightcurve & Spectrum

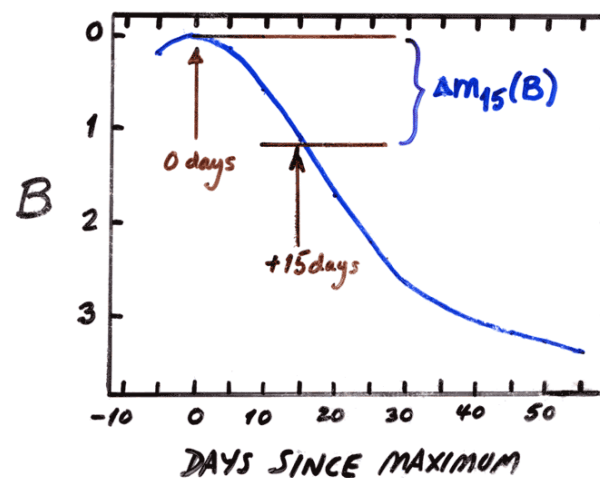


the Phillips Relation



Relationship between

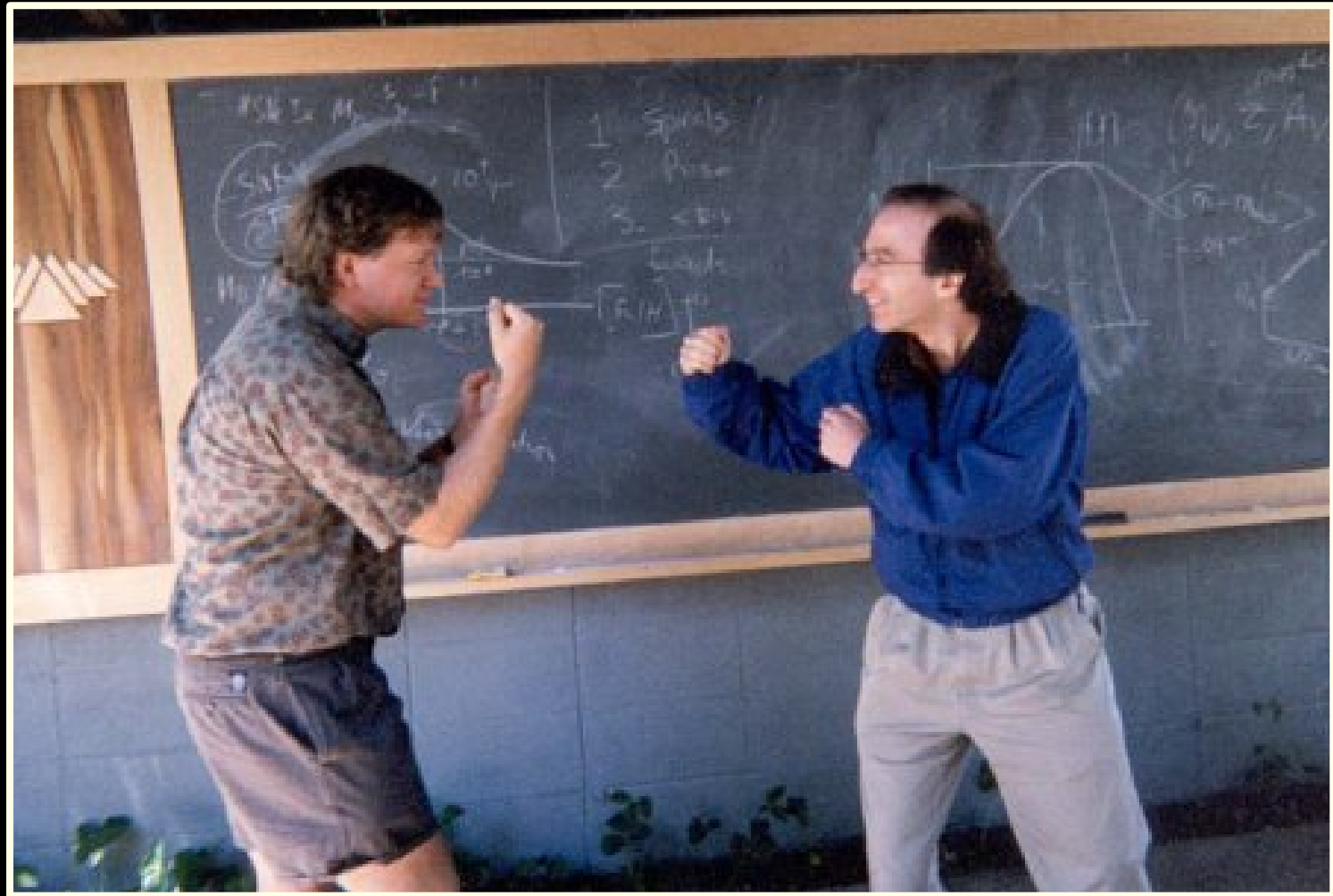
- peak luminosity of a Type Ia supernova
- speed of luminosity evolution after maximum light.



Mark Phillips (1993):

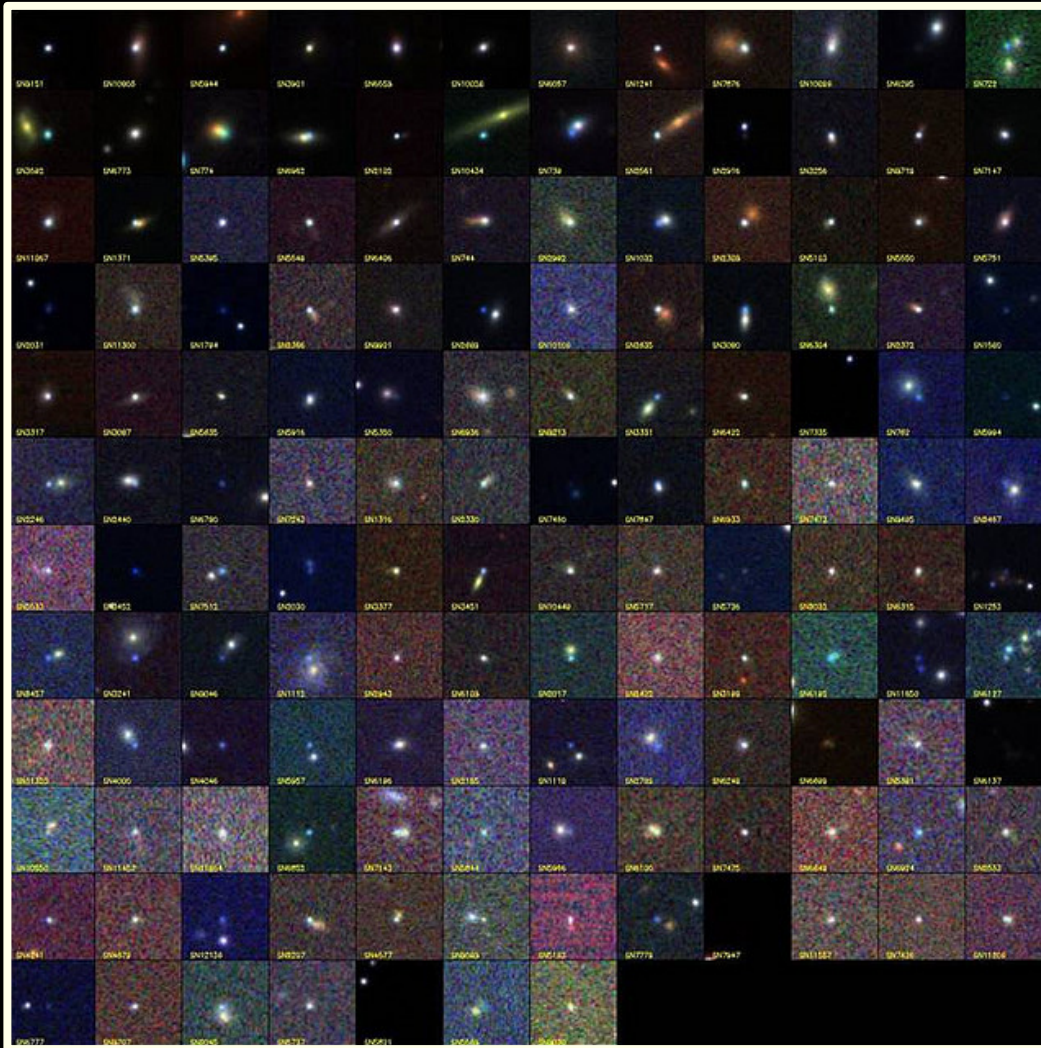
- on the basis of Calan/Tololo Supernova Survey
- the faster a supernova fades after peak,
the fainter its intrinsic peak luminosity
- reduces scatter in Hubble diagram to $\sigma < 0.2$ mag
- heuristic relationship, as yet not theoretically "understood"

Supernova Cosmology Project



High-z Supernova Search Team

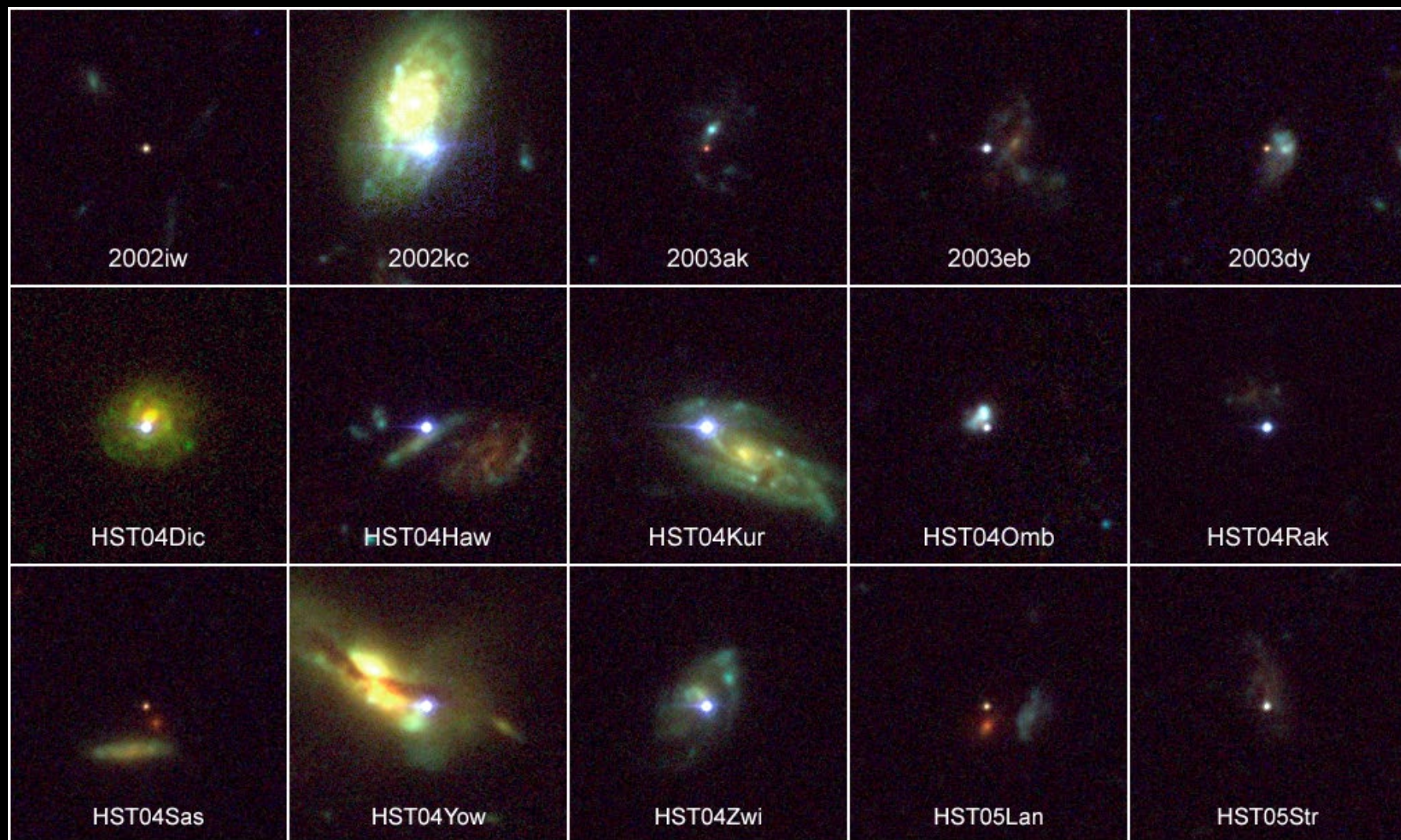
Supernova Cosmology Project



diligently monitoring millions of galaxies, in search for that one explosion ...

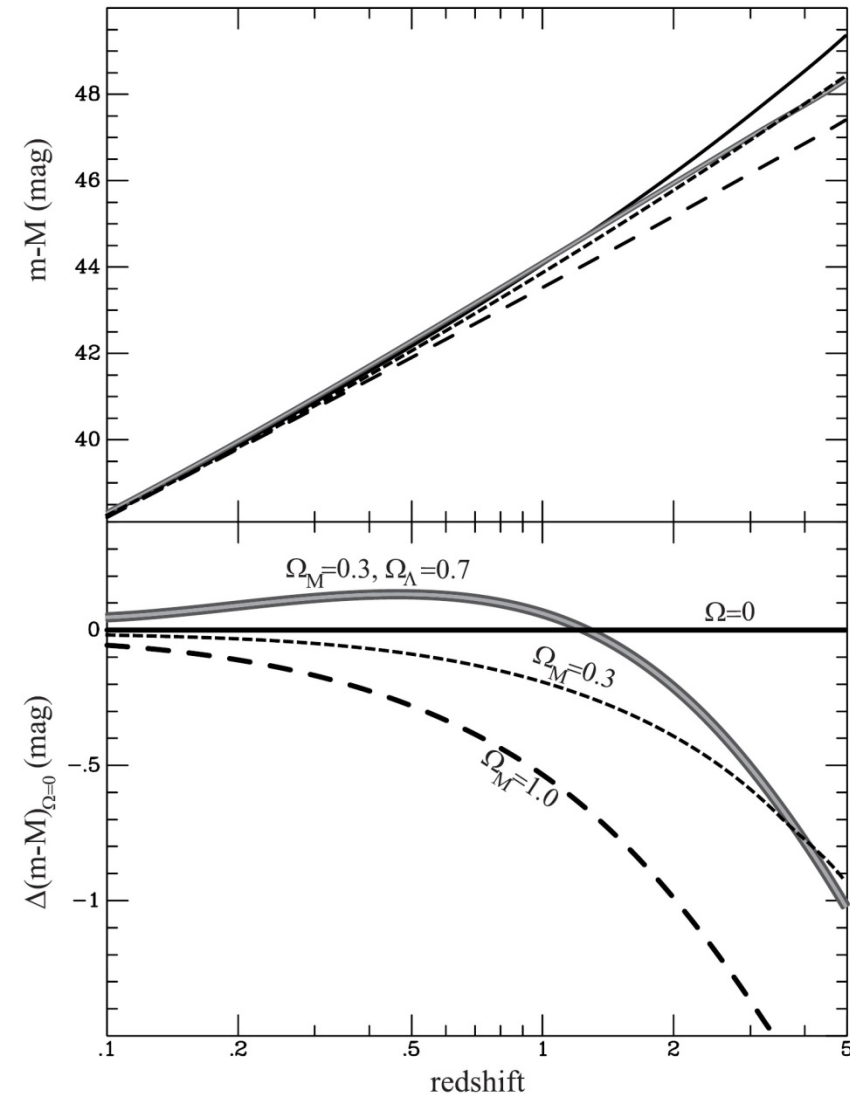
High-z Supernova Search Team

High-z SNIa: sample



Cosmic Acceleration & Cosmic Density

Cosmic Acceleration



Hubble Diagram high-z SNIa

- distance vs. redshift z
m-M vs. redshift z
- determine:
 - absolute brightness of supernova Ia
 - from dimming rate (Phillips relation)
- measure:
 - apparent brightness of explosion
- translates into:
 - luminosity distance of supernova
 - dependent on acceleration param. q

Luminosity Distance

For all general FRW Universe, the second-order luminosity distance-redshift relation, only depends on the *deceleration parameter* q_0 :

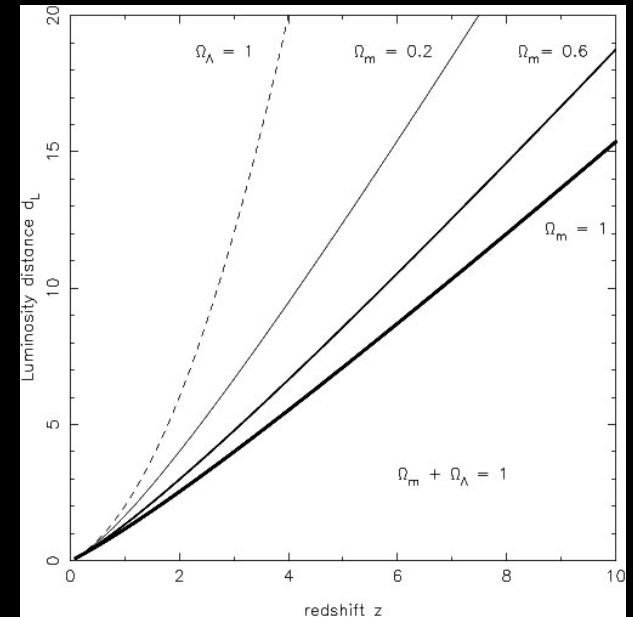
$$D_L(z) = (1+z)D(z) = (1+z)R_c S_k \left(\frac{r}{R_c} \right)$$
$$\simeq \frac{c}{H_0} (1+z) \left(z - \frac{1}{2} (1+q_0) z^2 \right)$$

q_0 can be related to Ω_0 once the *equation of state* is known.

Luminosity Distance

matter-dominated FRW Universe

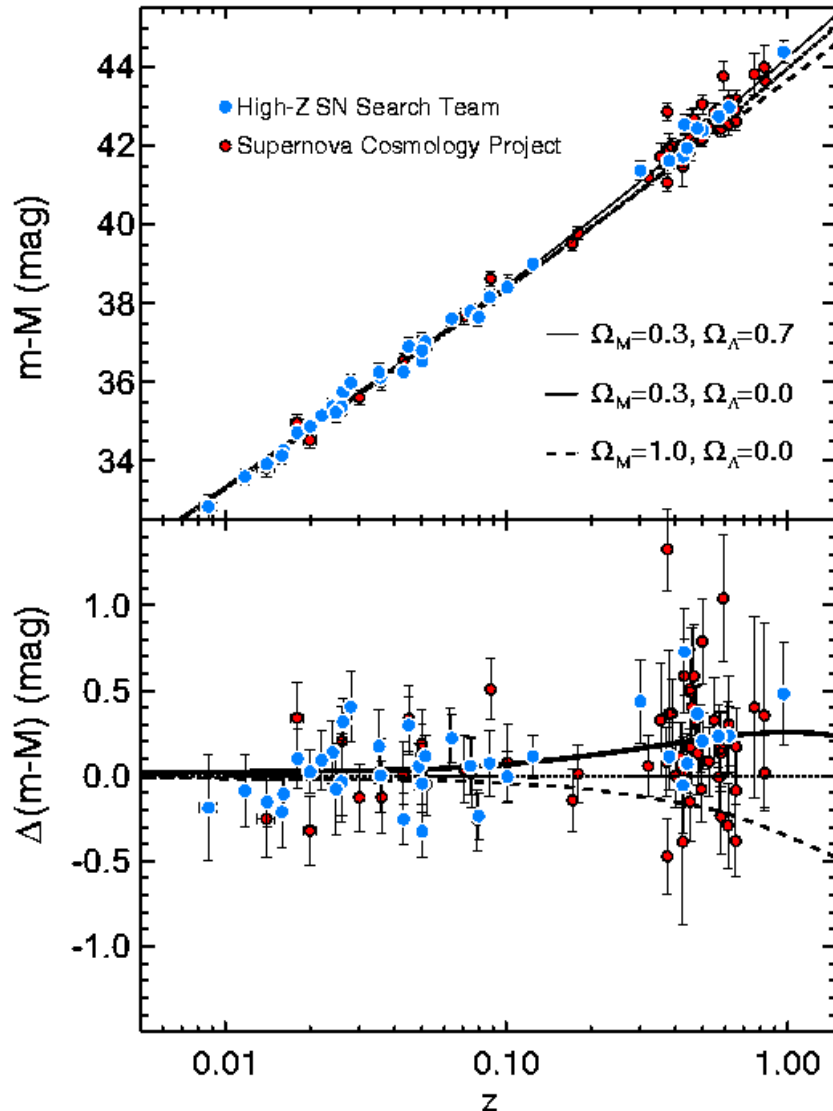
$$D_L = D(1+z) = (1+z)R_c S_k \left(\frac{r}{R_c} \right)$$



In a matter-dominated Universe, the luminosity distance as function of redshift is given by:

$$D_L(z) = (1+z)R_c S_k \left(\frac{r}{R_c} \right) = \frac{2c}{\Omega_0^2 H_0} \left\{ \Omega_0 z + (\Omega_0 - 2) \left(\sqrt{1 + \Omega_0 z} - 1 \right) \right\}$$

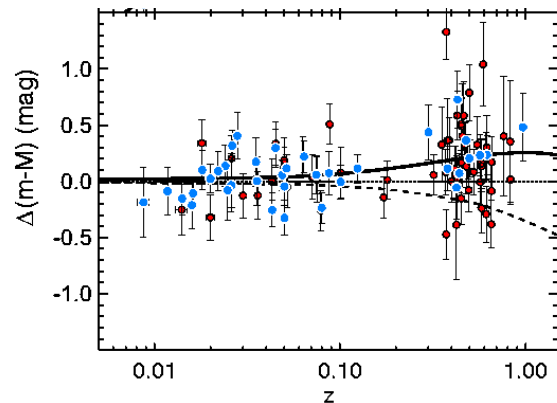
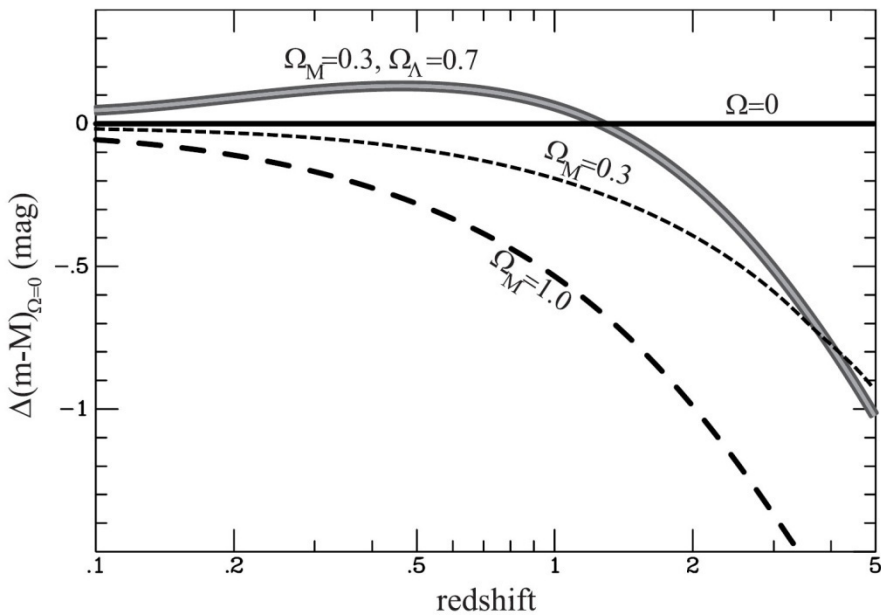
Cosmic Acceleration



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 - apparent brightness of explosion
- translates into:
 - luminosity distance of supernova
 - dependent on acceleration param. q

Cosmic Acceleration



Relative Hubble Diagram

$\Delta(m-M)$ vs. Redshift z

with Hubble diagram for empty Universe

$$\Omega_m=0.0, \Omega_\Lambda=0.0$$

as reference.

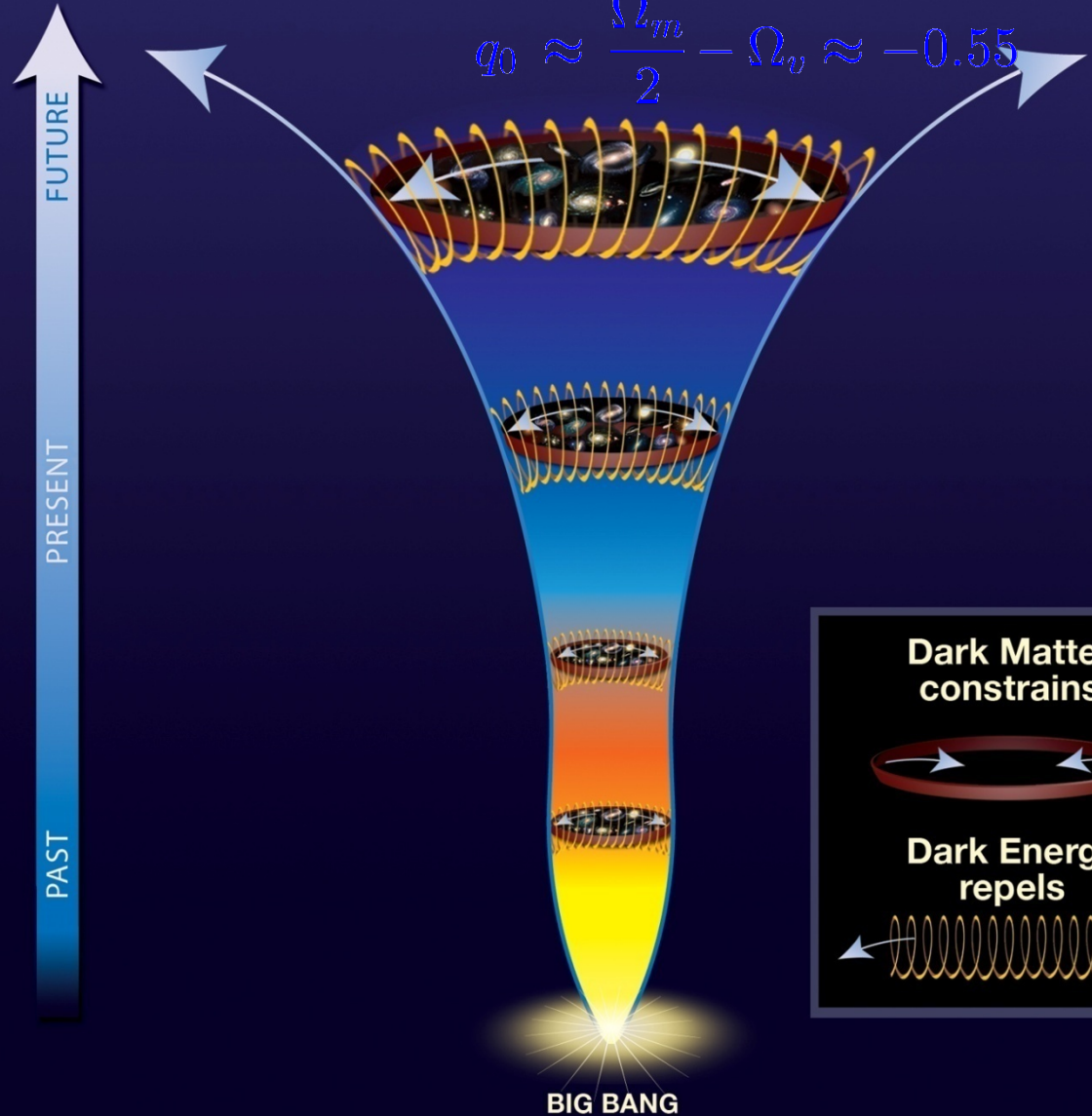
Acceleration of the Universe:

$$q_0 \approx \frac{\Omega_m}{2} - \Omega_v \approx -0.55$$

Cosmic tug of war

The force of dark energy surpasses that of dark matter as time progresses.

$$q_0 \approx \frac{\Omega_m}{2} - \Omega_\nu \approx -0.55$$

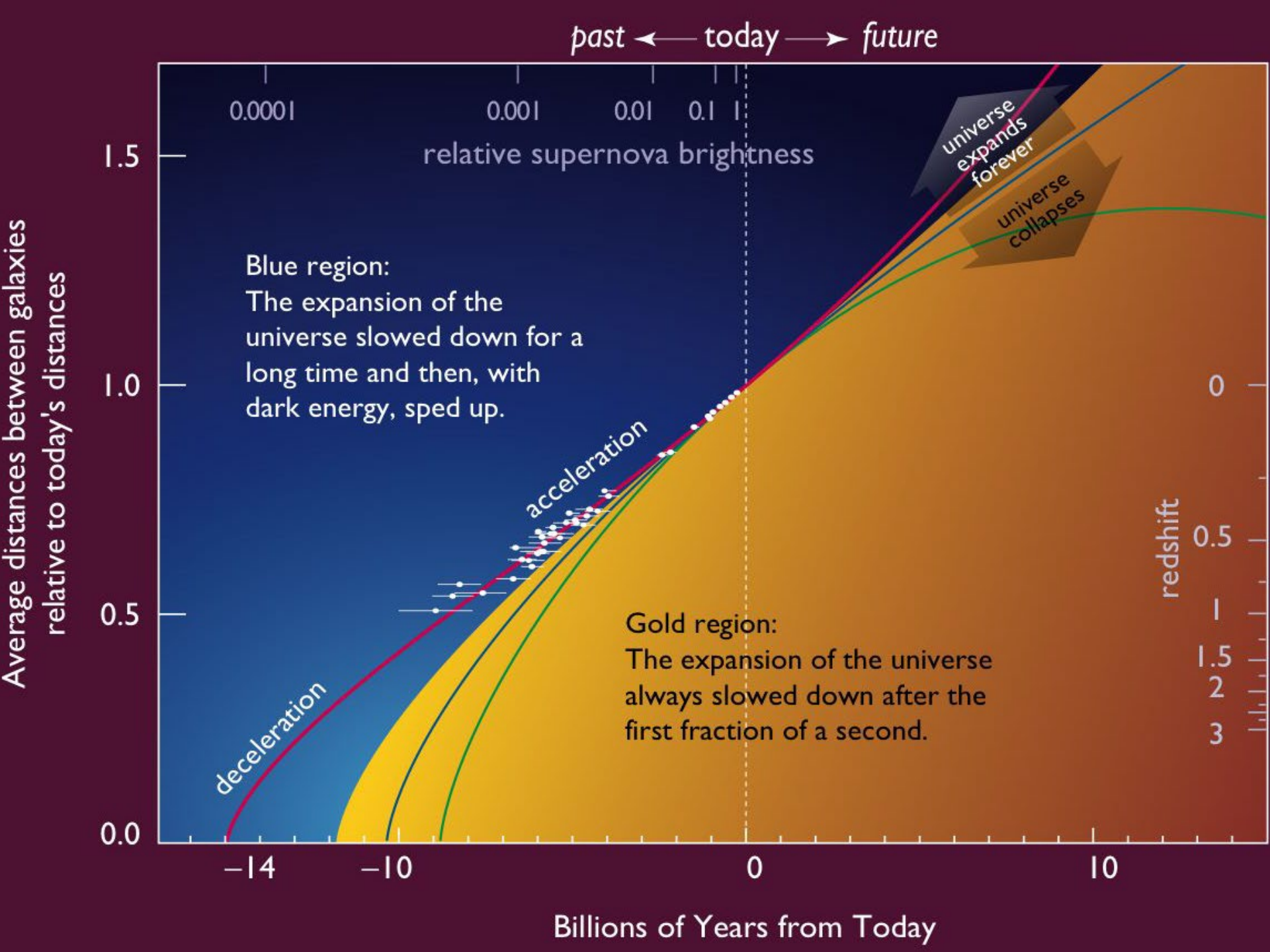


Dark Matter constrains

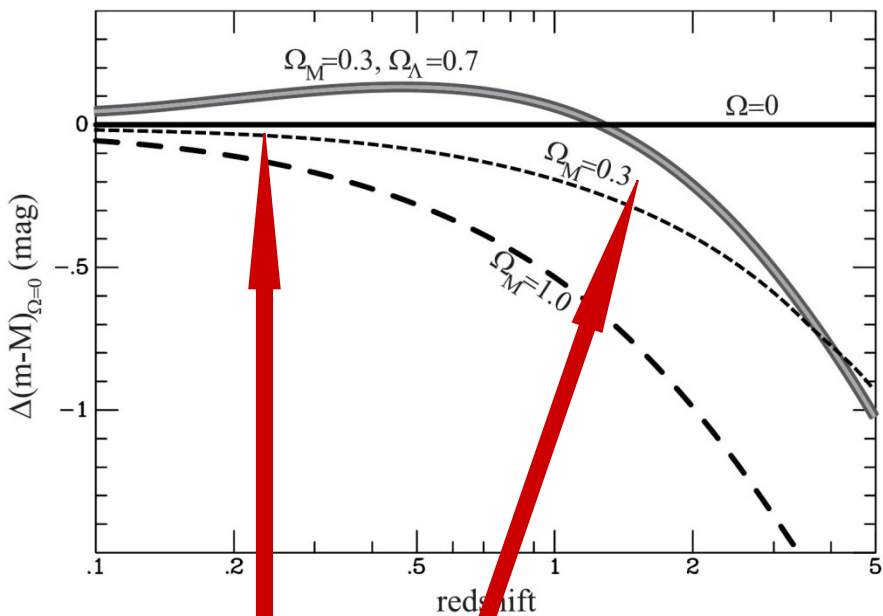
Dark Energy repels

Present:
ACCELERATION

Past:
DECELERATION



Cosmic Deceleration



**Cosmic deceleration:
SNIa brighter**

**Cosmic acceleration:
SNIa fainter**

Before current Dark Energy epoch

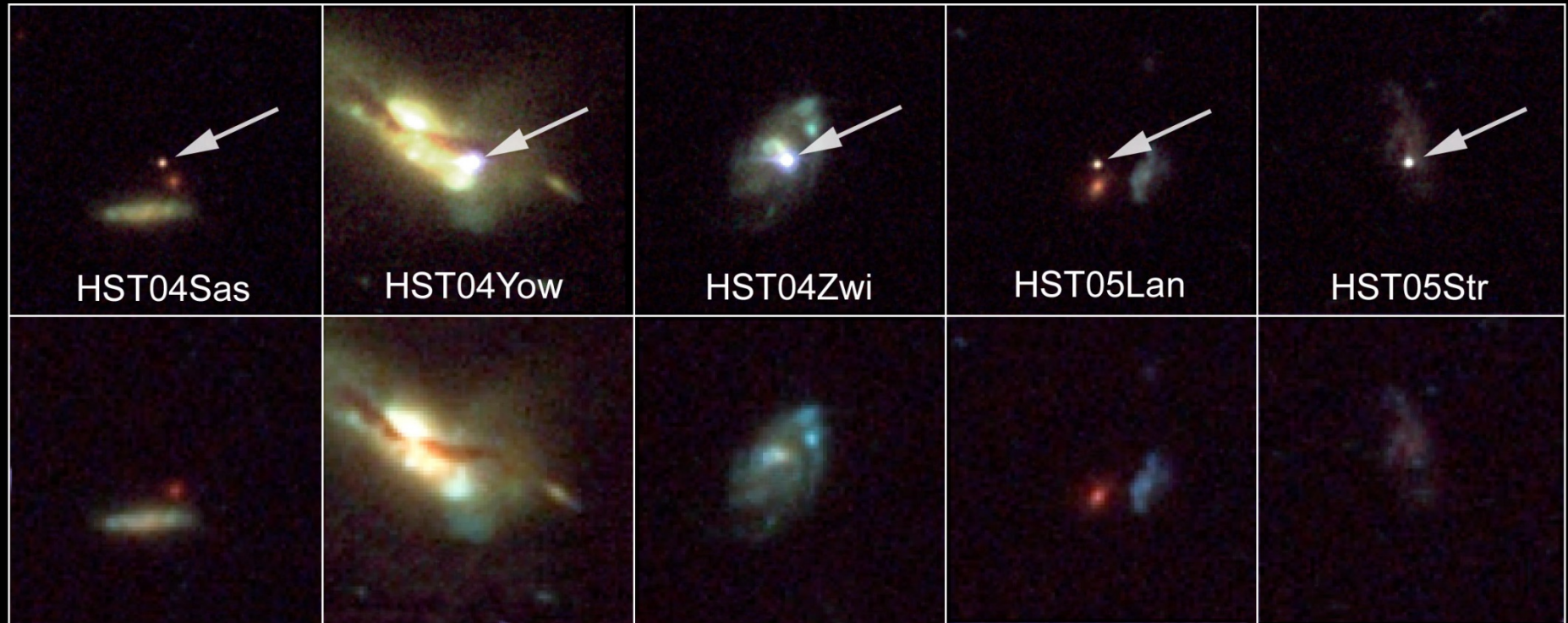
☐ Universe dominated by matter:

Decelerating Expansion

☐ Observable in SNIa at very high z :

$$z > 0.73$$

Beyond Acceleration: SNe Ia at $z > 0.7$



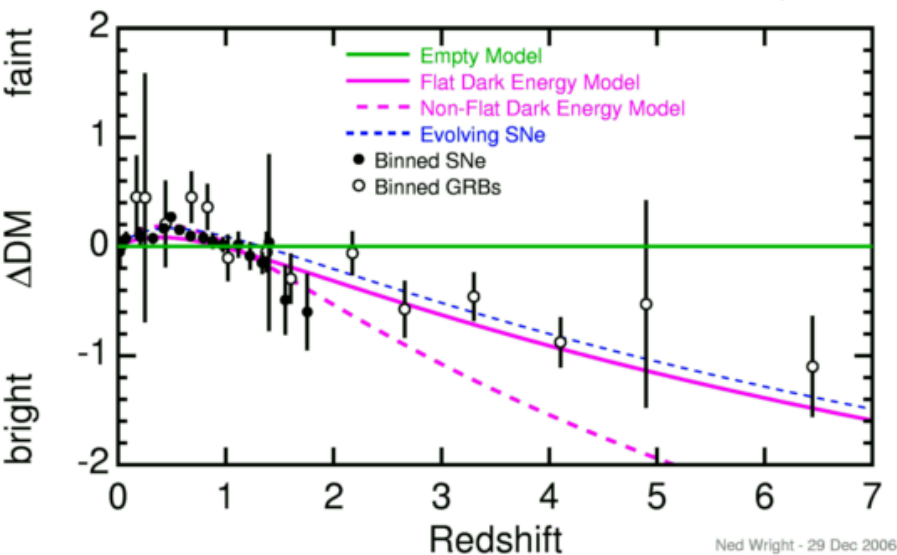
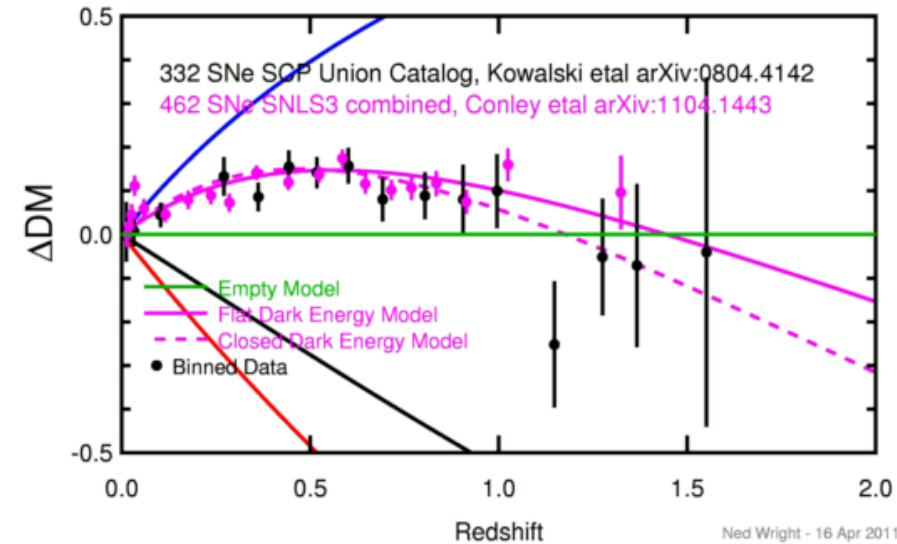
Five high- z SNIa, images HST-ACS camera

SNIa and host galaxies

lower panel: before

top panel: after explosion)

Cosmic Deceleration



Before current Dark Energy epoch

- Universe dominated by matter:

Decelerating Expansion
("Einstein-de Sitter phase")

- Observable in SNIa at very high z :

$$z > 0.73$$

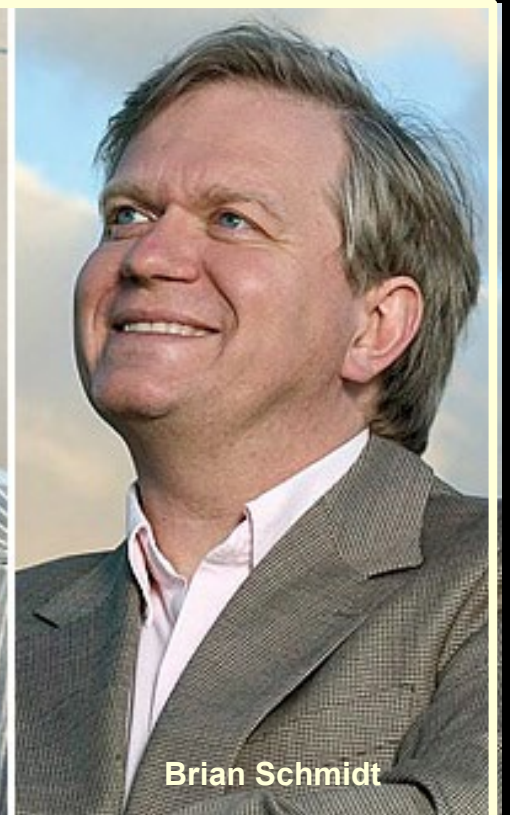
Nobel Prize Laureates



Adam Riess



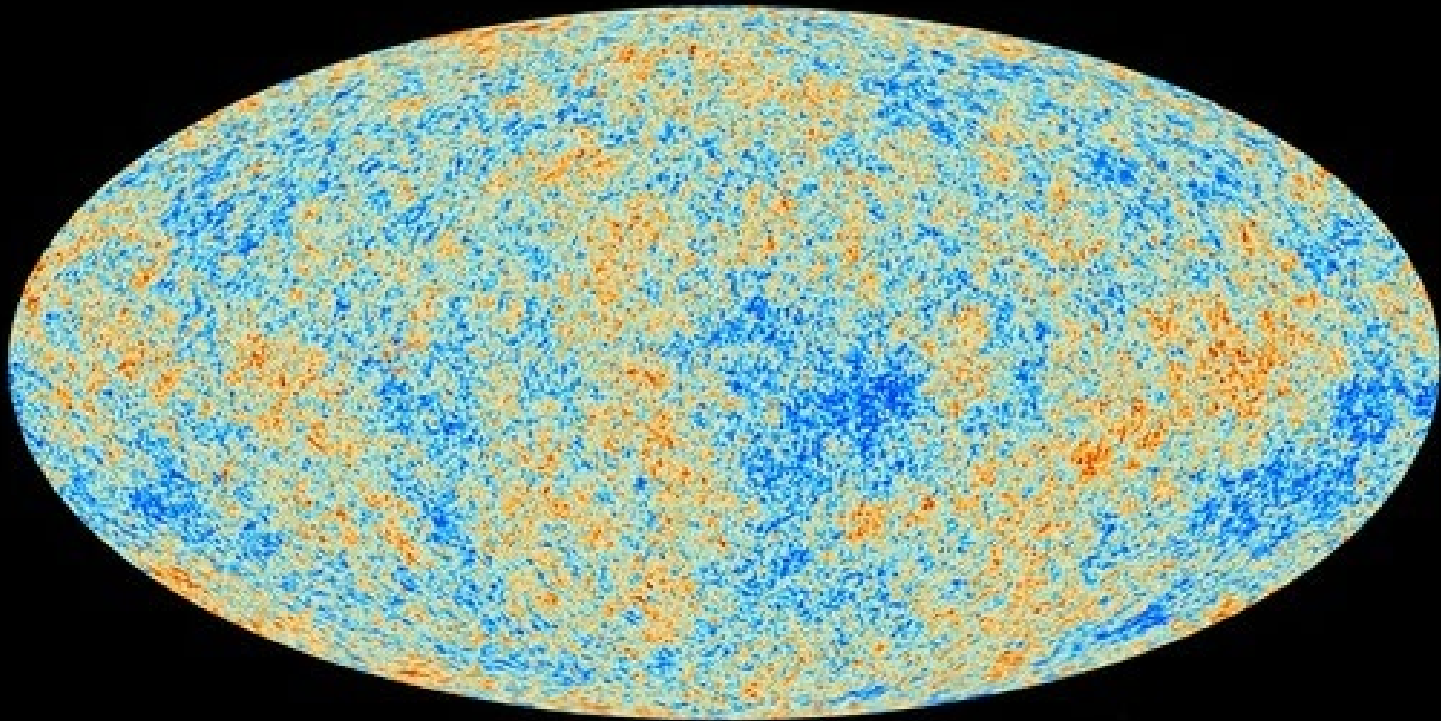
Saul Perlmutter



Brian Schmidt

Cosmic Curvature Measured

Cosmic Microwave Background



Map of the Universe at Recombination Epoch (Planck, 2013):

▣ **379,000 years after Big Bang**

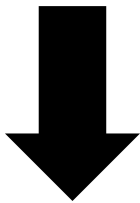
▣ **Subhorizon perturbations: primordial sound waves**

▣ **$\Delta T/T < 10^{-5}$**

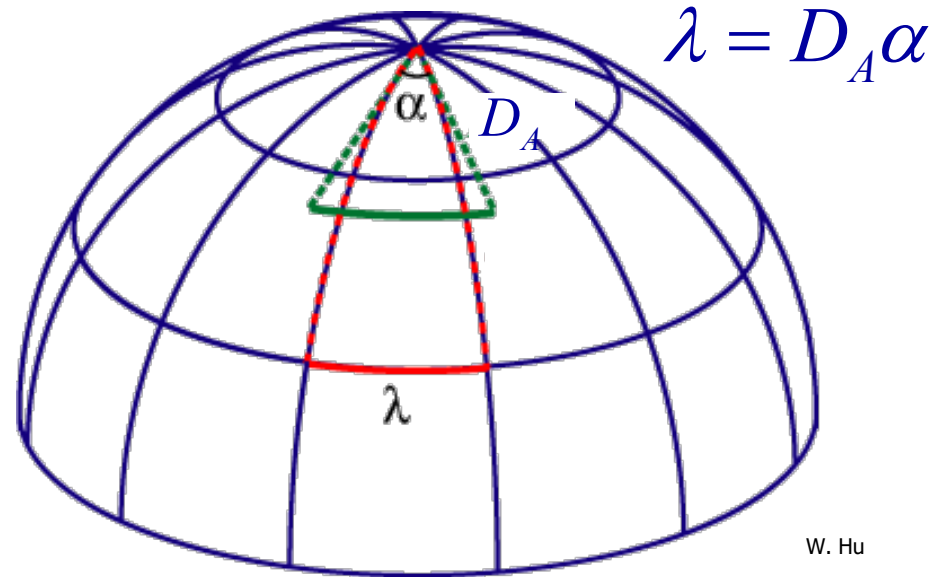
Measuring Curvature

Measuring the Geometry of the Universe:

- Object with known physical size, at large cosmological distance
- Measure angular extent on sky
- Comparison yields light path, and from this the curvature of space



Geometry of Space



FRW Universe:

lightpaths described by Robertson-Walker metric

$$ds^2 = c^2 dt^2 - a(t)^2 \left\{ dr^2 + R_c^2 S_k^2 \left(\frac{r}{R_c} \right) \left[d\theta^2 + \sin^2 \theta d\phi^2 \right] \right\}$$

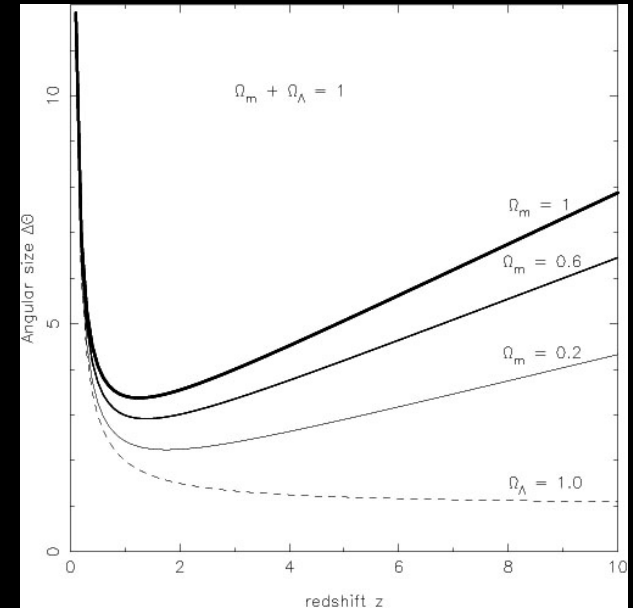
Here: angular diameter distance D_A : $\lambda = D_A \alpha$

Angular Size - Redshift

FRW Universe

$$\theta(z) = \frac{\ell}{D_A}$$

The angular size $\theta(z)$ of an object of physical size ℓ at a redshift z displays an interesting behaviour. In most FRW universes it has a minimum at a medium range redshift – $z=1.25$ in an $\Omega_m=1$ EdS universe – and increases again at higher redshifts.

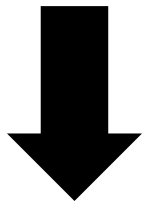


In a matter-dominated Universe, the angular diameter distance as function of redshift is given by:

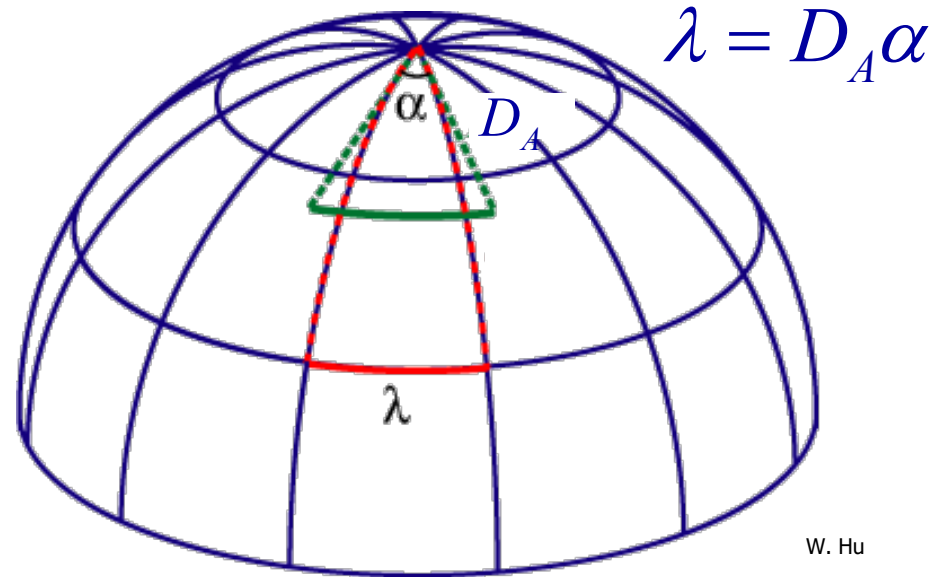
$$D_A(z) = \frac{1}{1+z} R_c S_k \left(\frac{r}{R_c} \right) = \frac{2c}{H_0} \frac{1}{\Omega_0^2 (1+z)^2} \left\{ \Omega_0 z + (\Omega_0 - 2) \left(\sqrt{1 + \Omega_0 z} - 1 \right) \right\}$$

Measuring Curvature

- Object with known physical size, at large cosmological distance:
- Sound Waves in the Early Universe !!!!



**Temperature Fluctuations
CMB**

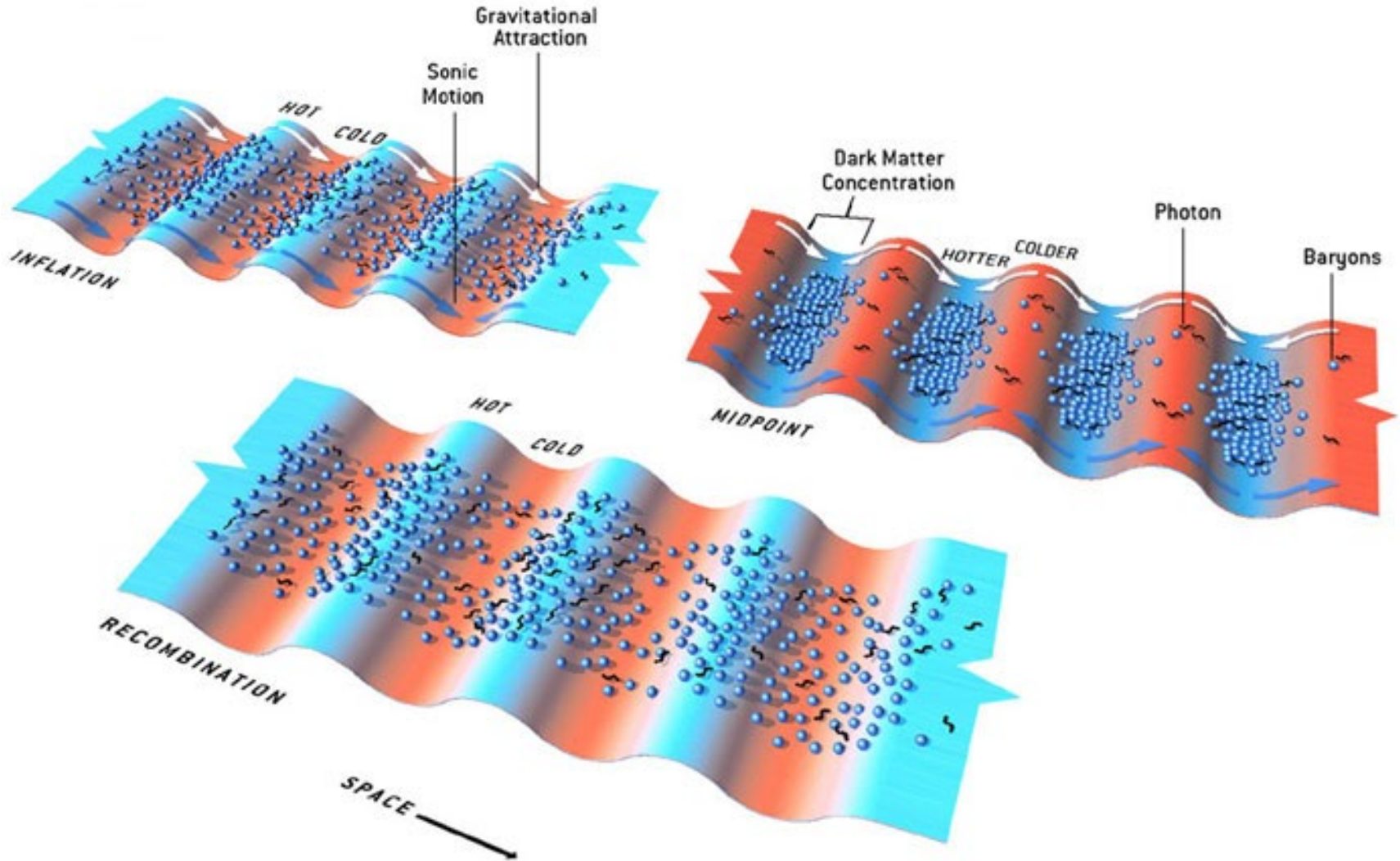


FRW Universe:
lightpaths described by Robertson-Walker metric

$$ds^2 = c^2 dt^2 - a(t)^2 \left\{ dr^2 + R_c^2 S_k^2 \left(\frac{r}{R_c} \right) \left[d\theta^2 + \sin^2 \theta d\phi^2 \right] \right\}$$

Here: angular diameter distance D_A : $\lambda = D_A \alpha$

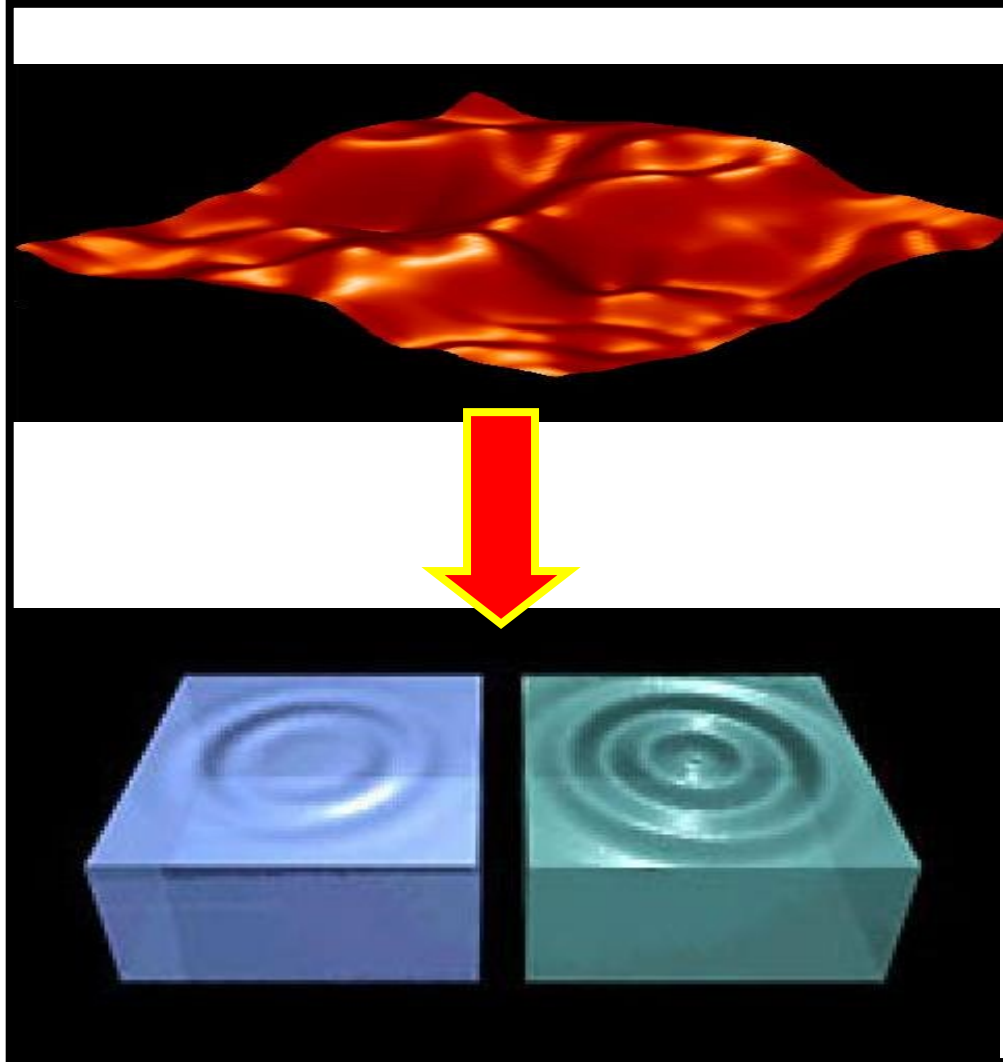
Fluctuations-Origin



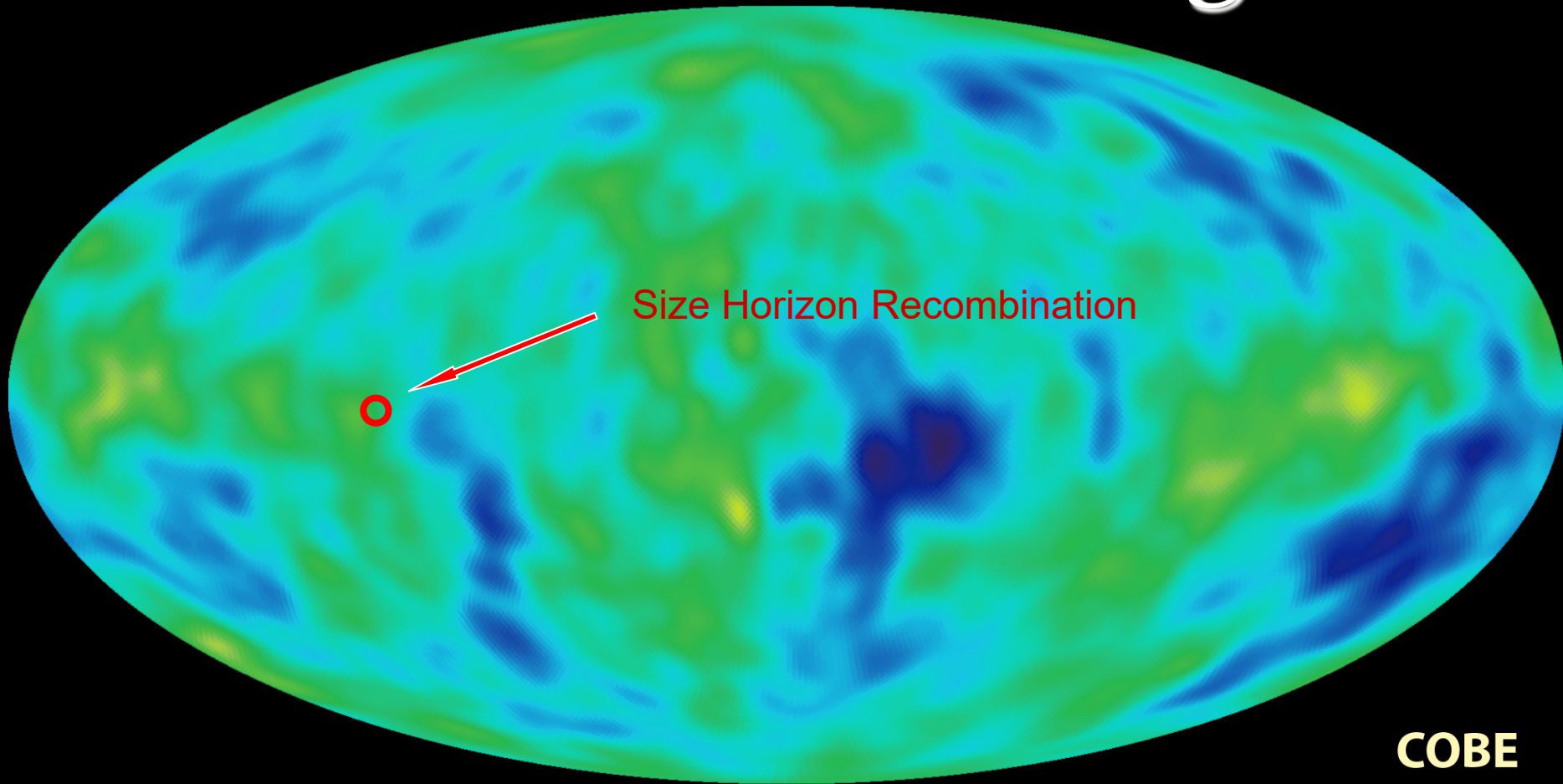
Music of the Spheres

- small ripples in primordial matter & photon distribution
- gravity:
 - compression primordial photon gas
 - photon pressure resists
- compressions and rarefactions in photon gas: sound waves
- sound waves not heard, but seen:
 - compressions: (photon) T higher
 - rarefactions: lower
- fundamental mode sound spectrum
 - size of “instrument”:
 - (sound) horizon size last scattering
- Observed, angular size: $\theta \sim 1^\circ$
 - exact scale maximum compression, the “cosmic fundamental mode of music”

W. Hu



Cosmic Microwave Background

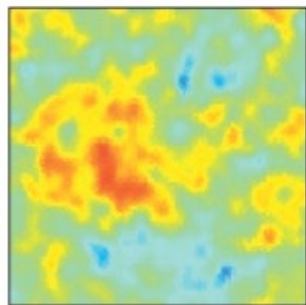
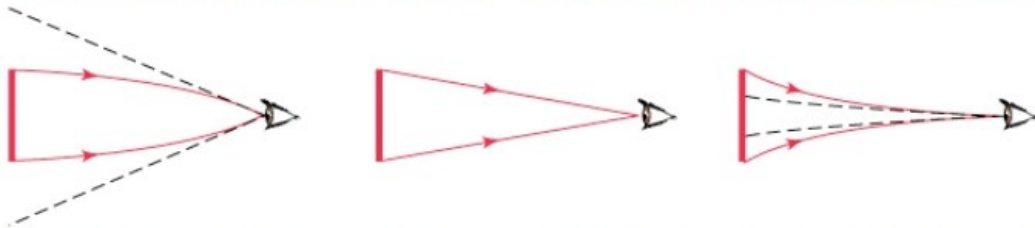
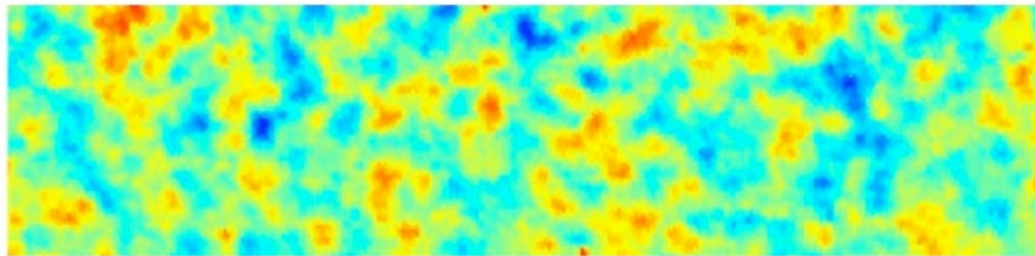


COBE measured fluctuations: $> 7^\circ$
Size Horizon at Recombination spans angle $\sim 1^\circ$

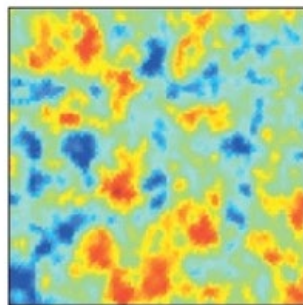
COBE proved that superhorizon fluctuations do exist: prediction Inflation !!!!!

Flat universe from CMB

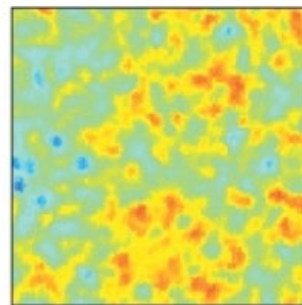
- **First peak: flat universe**



Closed:
hot spots
appear larger



Flat:
appear as big
as they are



Open:
spots appear
smaller

We know the redshift and the time it took for the light to reach us:

from this we know the

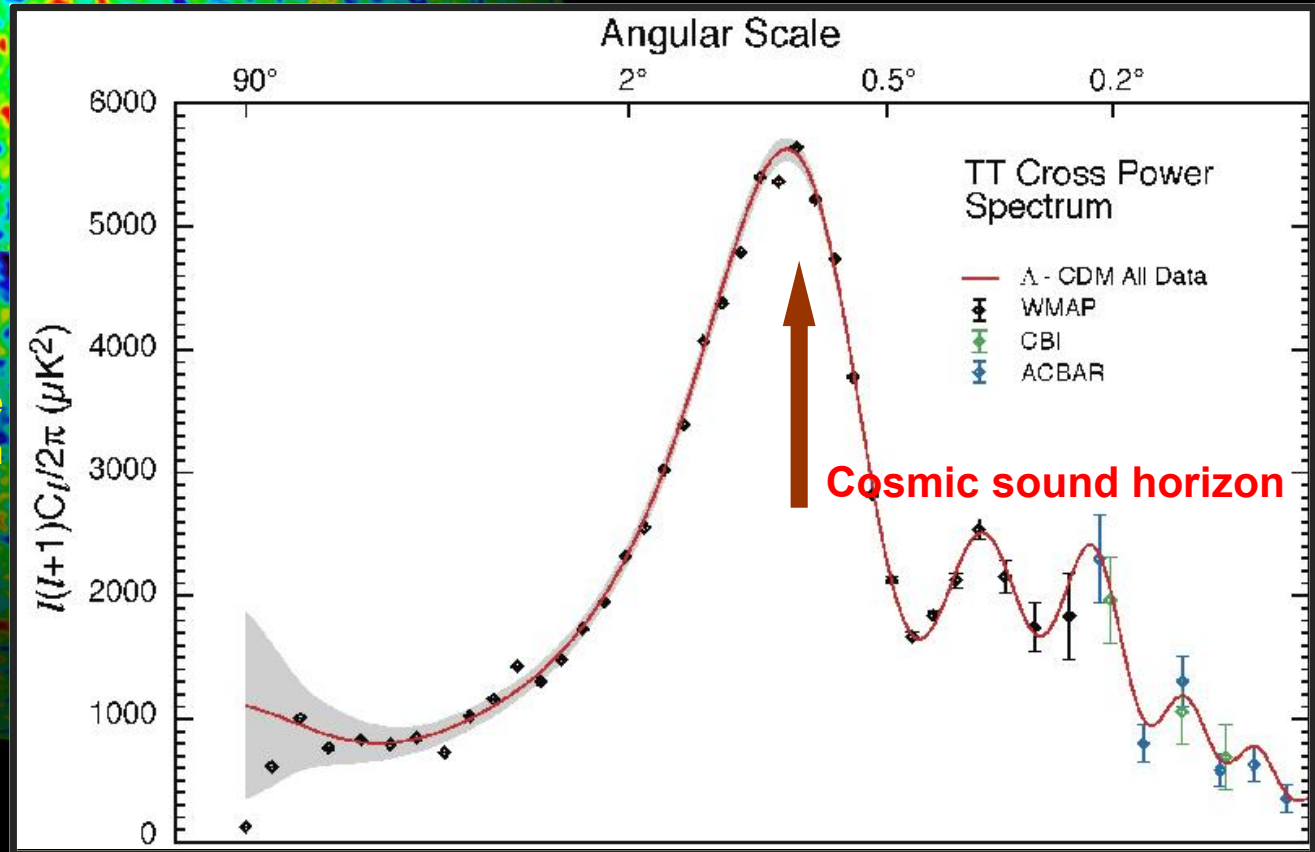
- length of the legs of the triangle
- the angle at which we are measuring the sound horizon.

$$v \approx \frac{c}{\sqrt{3}}$$

$$l \approx 200 / \sqrt{1 - \Omega_k}$$

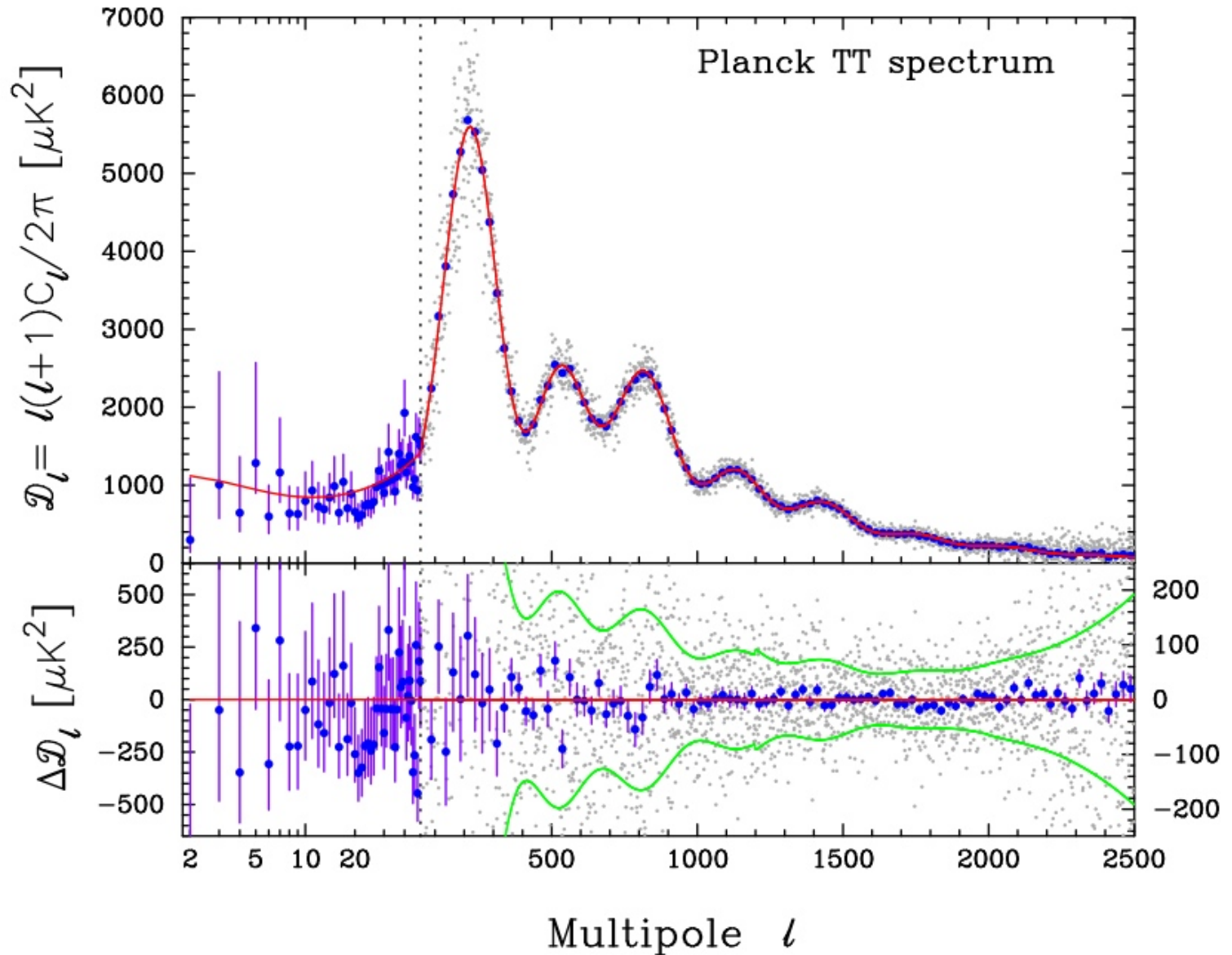
The Cosmic Tonal Ladder

The WMAP CMB temperature power spectrum

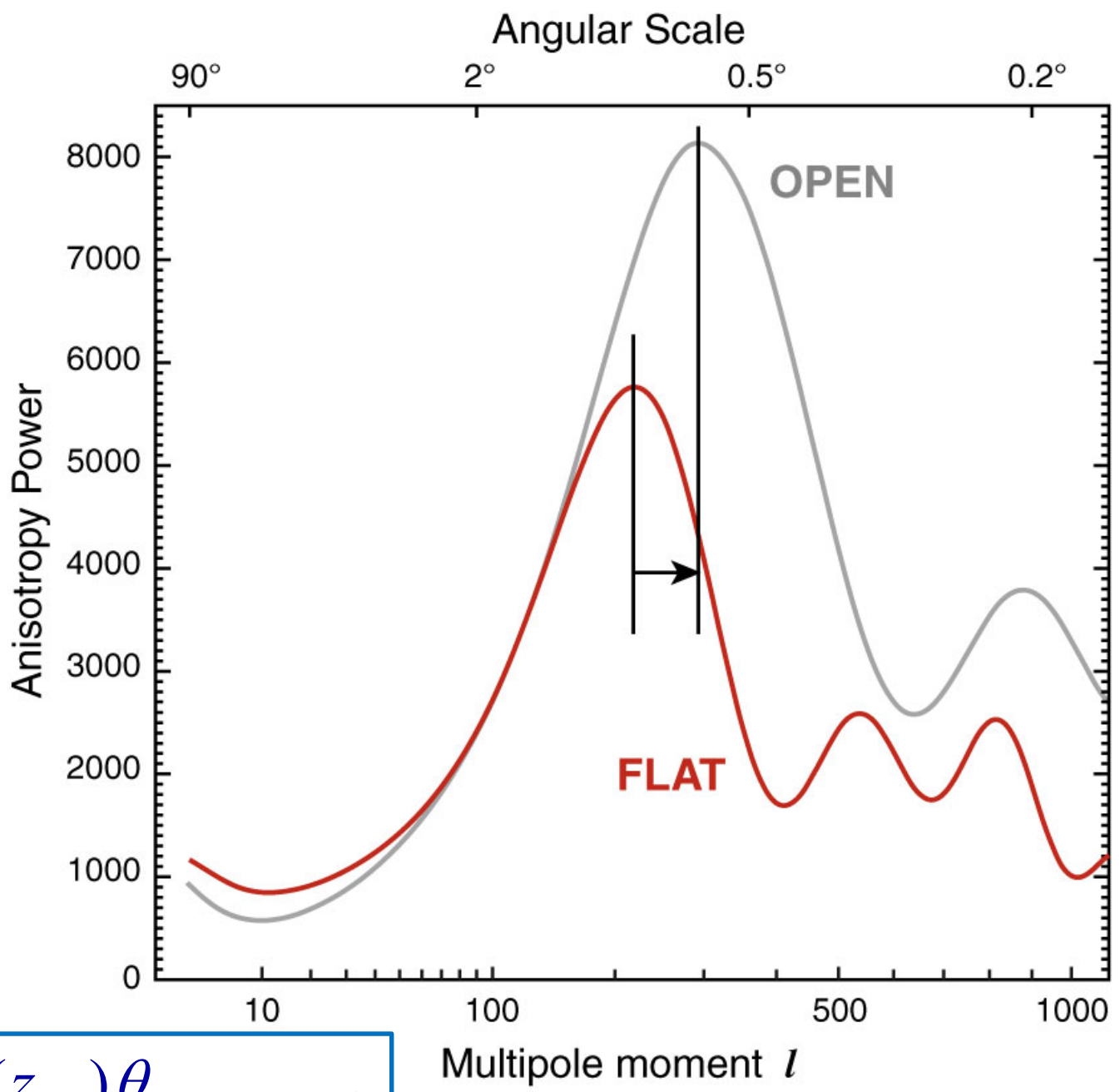
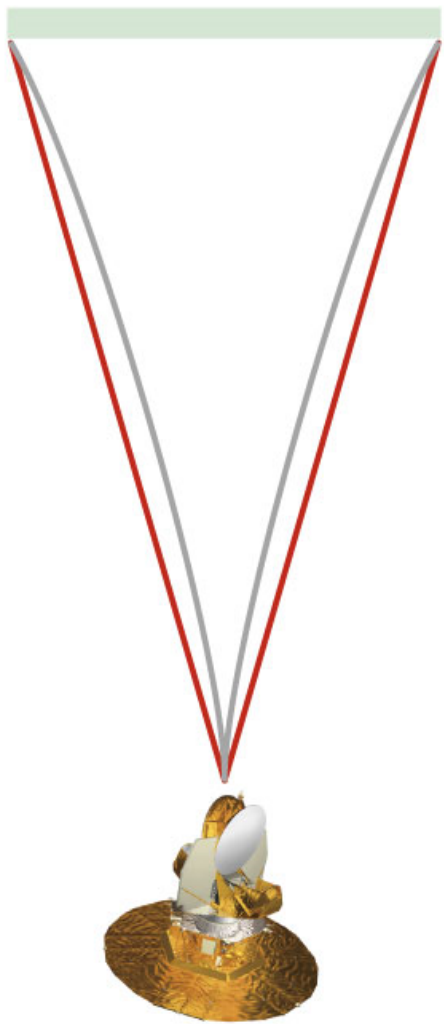


The Cosmic Microwave Background Temperature Anisotropies:
Universe is almost perfectly FLAT !!!!

Planck CMB Temperature Fluctuations



Standard Ruler:
 1° arc measurement of
 dominant energy spike



$$\lambda_{\text{sound horizon}} = D_A(z_{\text{rec}}) \theta_{\text{CMB, 1st peak}}$$

FRW Universe: Curvature

There is a 1-1 relation between the total energy content of the Universe and its curvature. From FRW equations:

$$k = \frac{H^2 R^2}{c^2} (\Omega - 1)$$

$$\Omega = \Omega_{rad} + \Omega_m + \Omega_\Lambda$$

$\Omega < 1$ $k = -1$ *Hyperbolic* *Open Universe*

$\Omega = 1$ $k = 0$ *Flat* *Critical Universe*

$\Omega > 1$ $k = +1$ *Spherical* *Close Universe*

Cosmic Curvature & Cosmic Density

$$q \approx \frac{\Omega_m}{2} - \Omega_\Lambda$$

$$k = \frac{H^2 R^2}{c^2} (\Omega_m + \Omega_\Lambda - 1)$$

SCP Union2 constraints (2010)

on values of matter density Ω_m
dark energy density Ω_Λ

Supernova Cosmology Project
Amanullah, et al., *Ap.J.* (2010)

