Climbing Mount Olympus : Geometry as Pathway to the Universe

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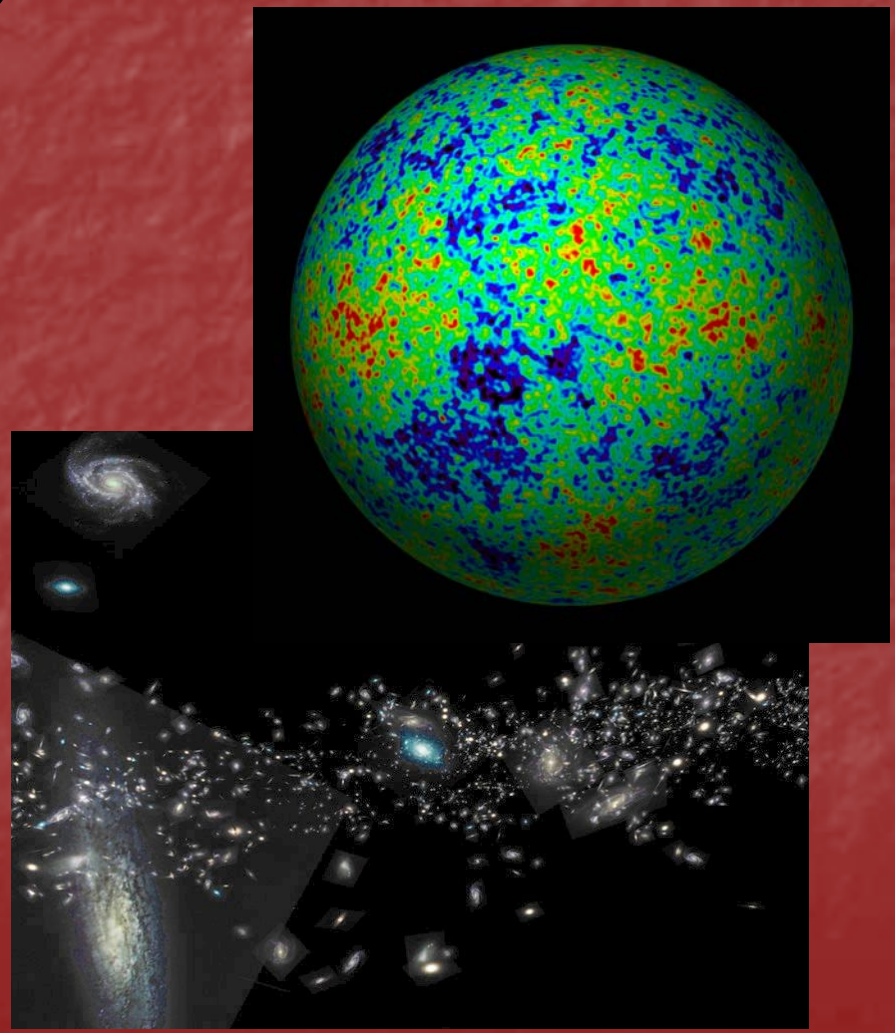
Cosmology: the Science of the Universe

What is Cosmology?

- Formally:
  Astronomical science or theory of the Universe as an ordered unity; study of the structure and evolution of the Universe.
- Broadest meaning:
  human enterprise, joining science, philosophy, theology and the arts to seek to gain understanding of what unifies and is fundamental to our world
- Scientific:
  Study of large-scale structure and infrastructure of the Universe



Cosmology:

the key questions and issues

- What is the structure of the Universe ?
- How big is the world ? (finite, infinite,...)
- How did the world begin ? When did the world begin ? Did it begin at all ?
- What is the fate of the Universe ?
- What is the world made of ? What does it contain ?
- Are these questions meaningful at all ?

The Universe:

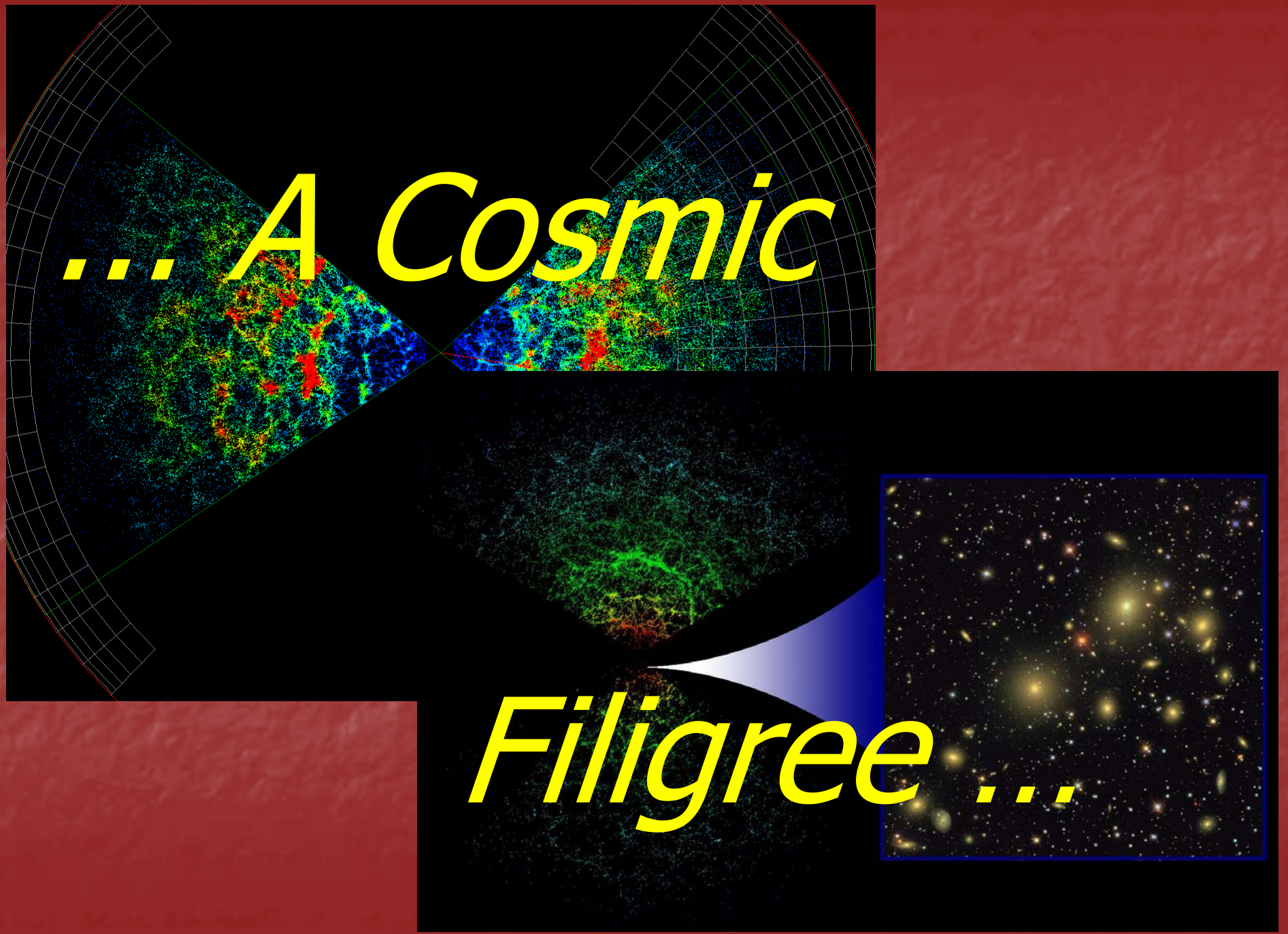
... a journey ...

...The Stars ...

Our Galaxy ...

... the galaxies ...

... galaxy
assemblies ...
...groups &
clusters ...

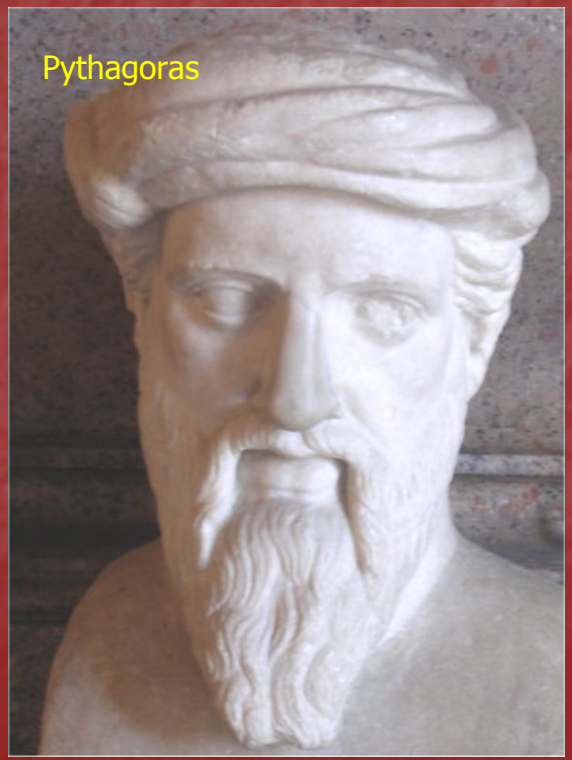

... A Cosmic
Filigree ...

... Out to the deepest realms of the visible Universe ...

Ionia, 6th century B.C.

A phase transition in human history: the mythical world obsolete

… the Ionian coast, 6th century B.C., *regularities and symmetries* in nature recognized as keys to the cosmos …


Pythagoras

➔ *Mathematics* as natural language of cosmos

➔ Physical cosmos modelled after *ideal form*, encrypted in concepts of *geometry*

… Anaximander of Miletus: the Apeiron

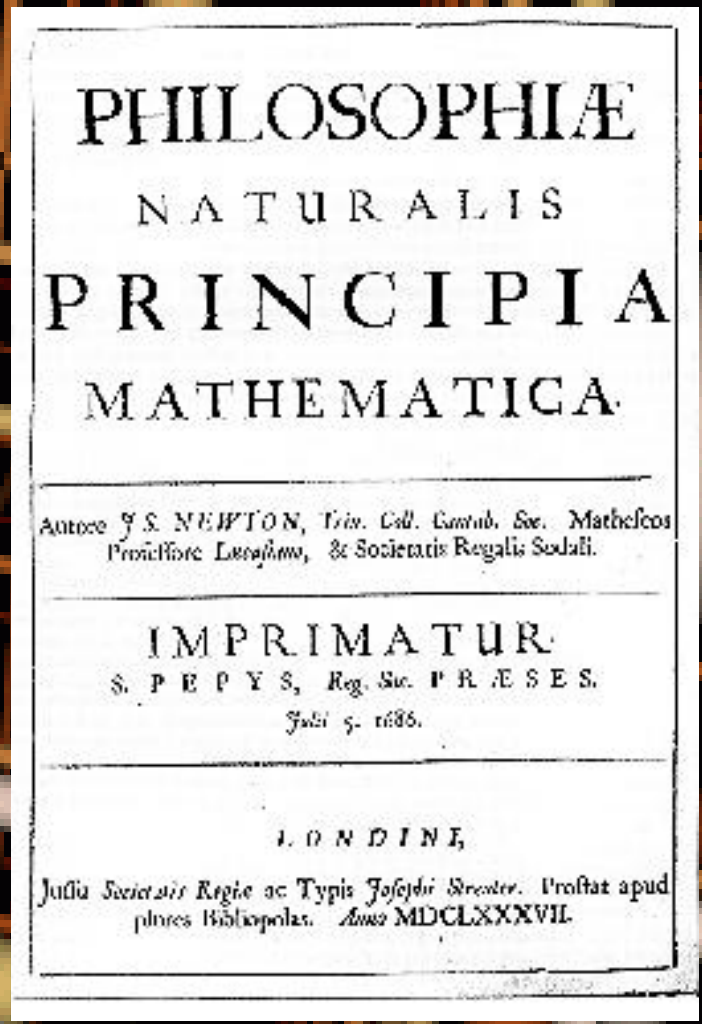
Pythagoras of Samos: music of spheres

Plato: Platonic solids

Renaissance of Western Science

In footsteps of Copernicus, Galilei & Kepler, *Isaac Newton* (1687) in his *Principia* formulated a comprehensive model of the world. Cosmologically, it meant

- absolute, static & infinite space
- absolute and uniform time
- space & time independent of matter
- dynamics: - action at distance
  - instantaneous
- Universe edgeless, centerless & infinite
- Cosmological Principle:
  Universe looks the same at every place in space, every moment in time



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

Autore J. S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Juſſu Societatis Regiæ ac Typis Joſephi Streater. Proſtat apud plures Bibliopolas. Anno MDCLXXXVII.

Einstein's Field Equations

... Spacetime becomes a dynamic continuum,
integral part of the structure of the cosmos ...
curved spacetime becomes force of gravity

$$R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R = -\frac{8\pi G}{c^4} T^{\alpha\beta}$$

... its geometry rules the world,
the world rules its geometry...

Einstein's Field Equations

... Spacetime becomes a dynamic continuum,
integral part of the structure of the cosmos ...
curved spacetime becomes force of gravity

But, no longer

Euclides suffices ...

... its geometry rules the world,
the world rules its geometry...

Geometry of the Universe: Cosmological Principle

"God is an infinite sphere whose centre is everywhere and its circumference nowhere"
Empedocles, 5th cent BC

Cosmological Principle:

Describes the symmetries in global appearance of the Universe:

- Homogeneous → The Universe is the same everywhere:
  - physical quantities (density, T,p,…)
- Isotropic → The Universe looks the same in every direction

- Universality → Physical Laws same everywhere
- Uniformly Expanding → The Universe "grows" with same rate in
  - every direction
  - at every location

"all places in the Universe are alike"
Einstein, 1931

Geometry of the Universe

Fundamental Tenet

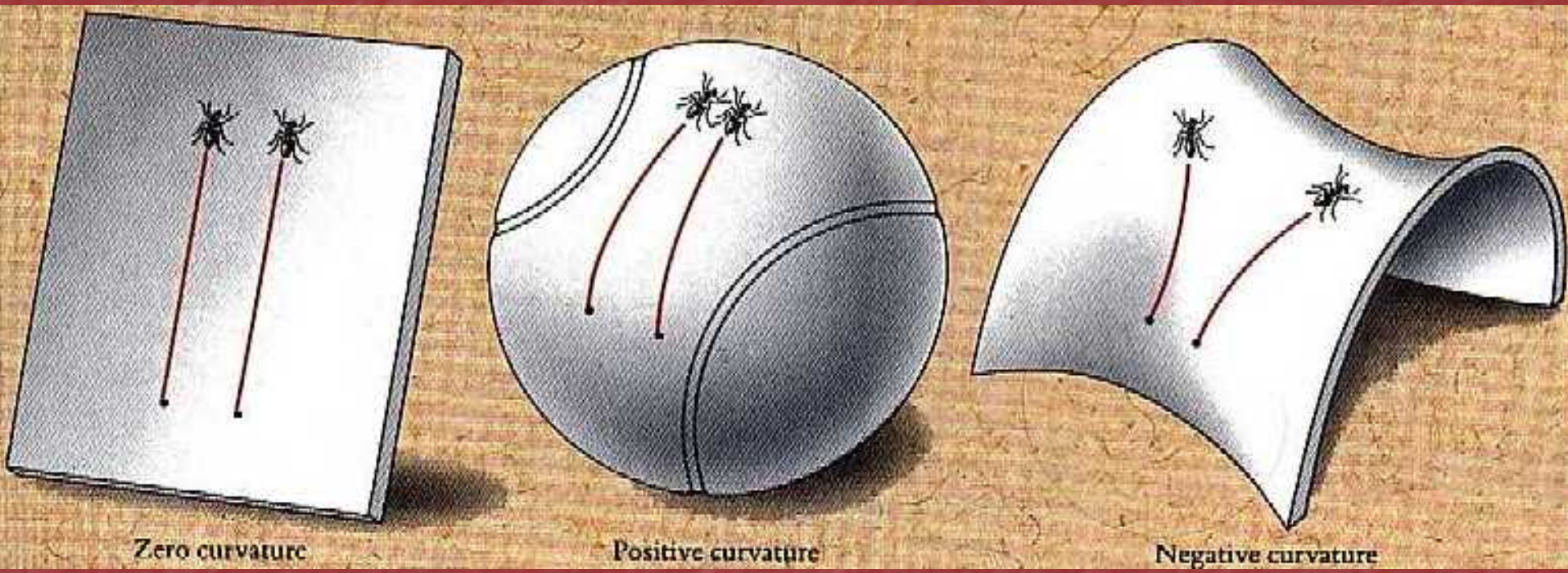
(Non-Euclidian = Riemannian) Geometry:

- there exist no more than THREE uniform spaces:

1)	Euclidian (flat) Geometry	Euclides
2)	Hyperbolic Geometry	Gauß, Lobachevski, Bolyai
3)	Spherical Geometry	Riemann

uniform=
homogeneous & isotropic
(cosmological principle)

Geometry of the Universe



The three possible geometries of the Universe

Geometry & Dynamics:

Friedmann-Robertson-Walker-Lemaitre Universe

Having confined the Universe to the highly symmetric geometries corresponding the Cosmological Principle, the Einstein field equations are reduced tenfold to

- TWO equations, for
- ONE scale factor R(t), the uniform growth factor of the Universe

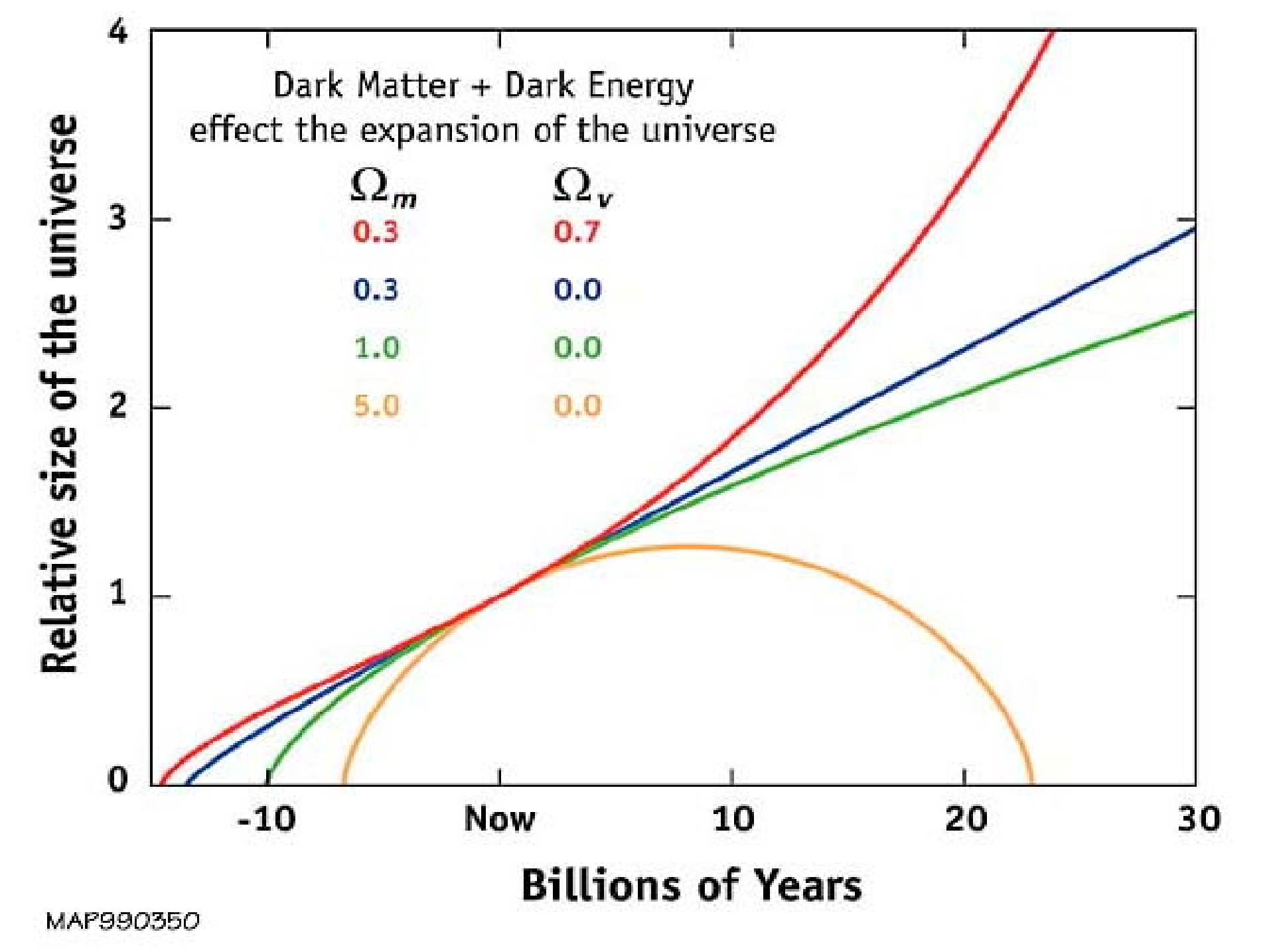
The equations are called the *Friedmann-Robertson-Walker-Lemaitre equations*, and are the fundamental solutions for dynamics and evolution of the Universe.

Geometry & Dynamics:

Friedmann-Robertson-Walker-Lemaitre Universe

$$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) R + \frac{\Lambda}{3} R$$

$$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2 + \frac{\Lambda}{3} R^2 - kc^2$$


Dark Matter + Dark Energy
effect the expansion of the universe
Ω_m
Ω_v
0.3
0.7
0.3
0.0
1.0
0.0
5.0
0.0
Relative size of the universe
0
1
2
3
4
-10
Now
10
20
30
Billions of Years
MAP990350

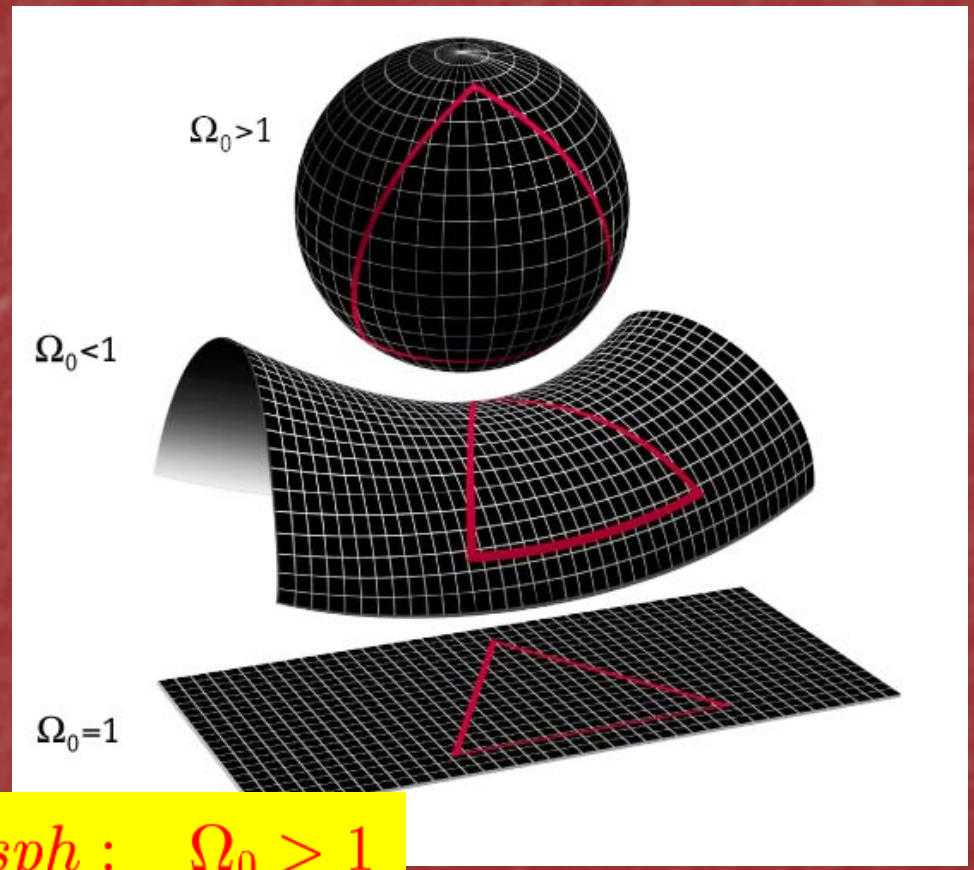
Geometry & Dynamics:

Friedmann-Robertson-Lemaitre Universe

… for an *expanding Universe* with

- matter density ρ(t)
- pressure p(t)
- cosmological constant Λ(t)

  [or, the elusive dark energy ρ_v]



… whose dynamics is ultimately set by the geometrical curvature term "k" :

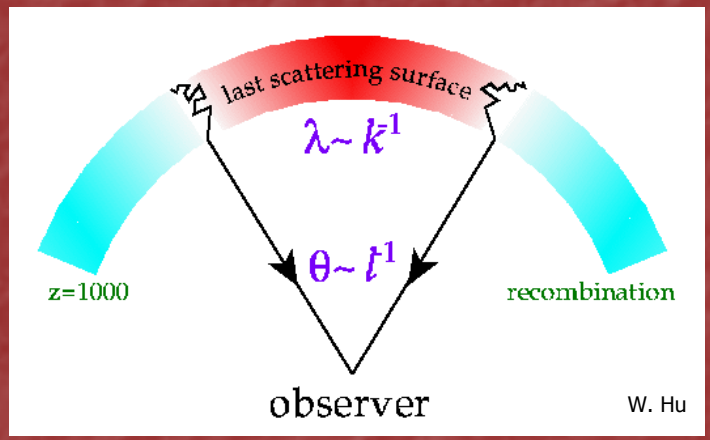
$$k = \frac{H_0^2 R_0^2}{c^2}(\Omega_0 - 1) = \begin{cases} 1 & sph: \quad \Omega_0 > 1 \\ 0 & flat: \quad \Omega_0 = 1 \\ -1 & hyp: \quad \Omega_0 < 1 \end{cases}$$

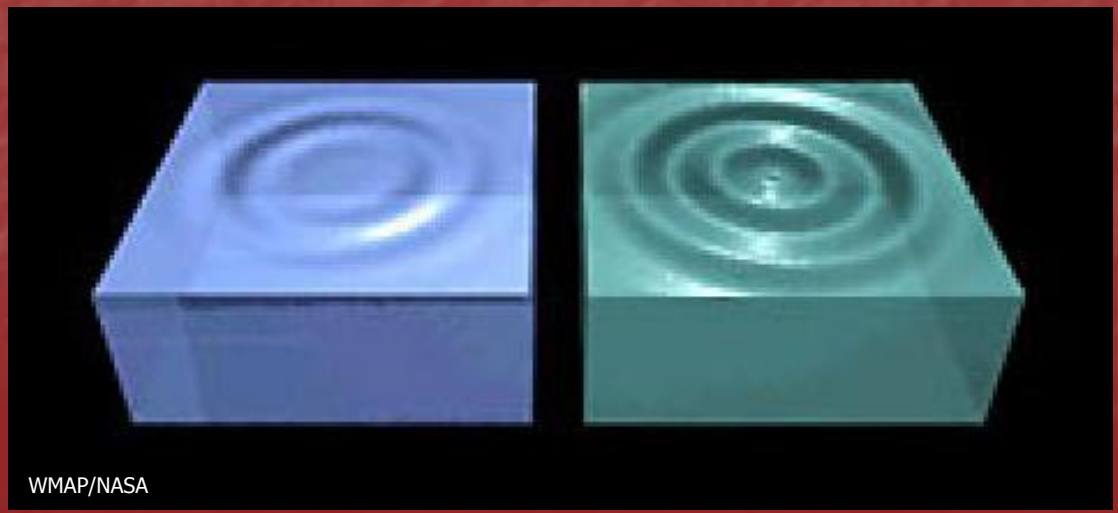
Geometry of the Universe: Music of the Spheres

Measuring the Geometry of the Universe:

- Object with known physical size, at large cosmological distance
- Measure angular extent on sky
- Comparison yields light path

→ *Geometry of space*





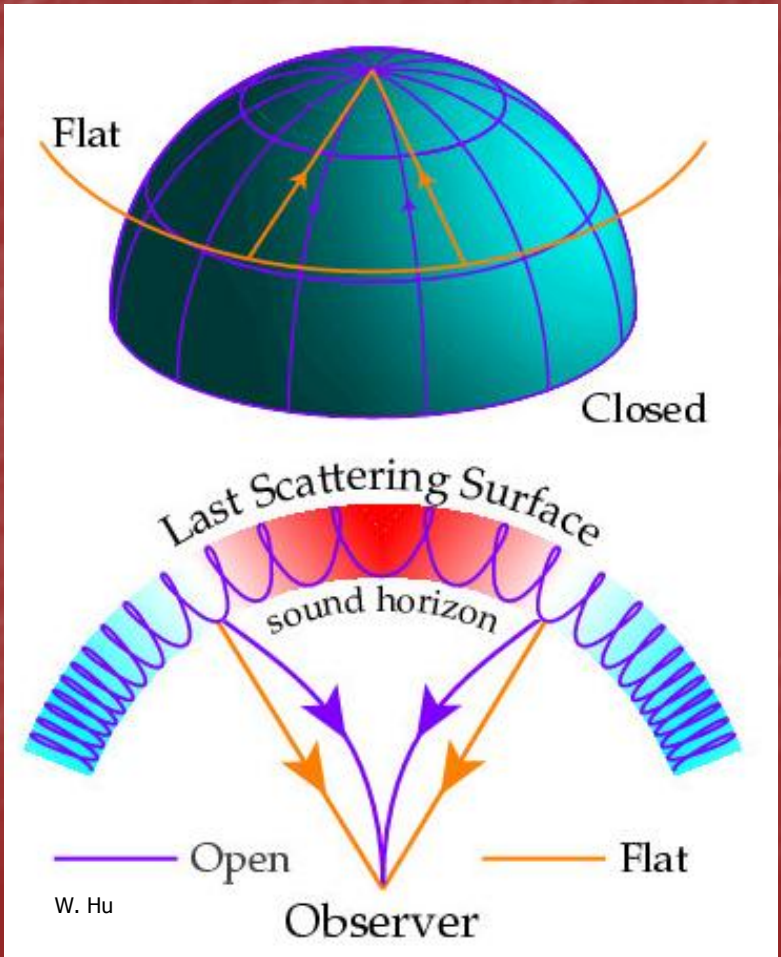
"Physical Object":

- Sound waves in primordial matter-radiation plasma: wavelength λ_s

- observable at surface of epoch recombination, at which photons were last scattered

Music of the Spheres

Primordial Soundwaves

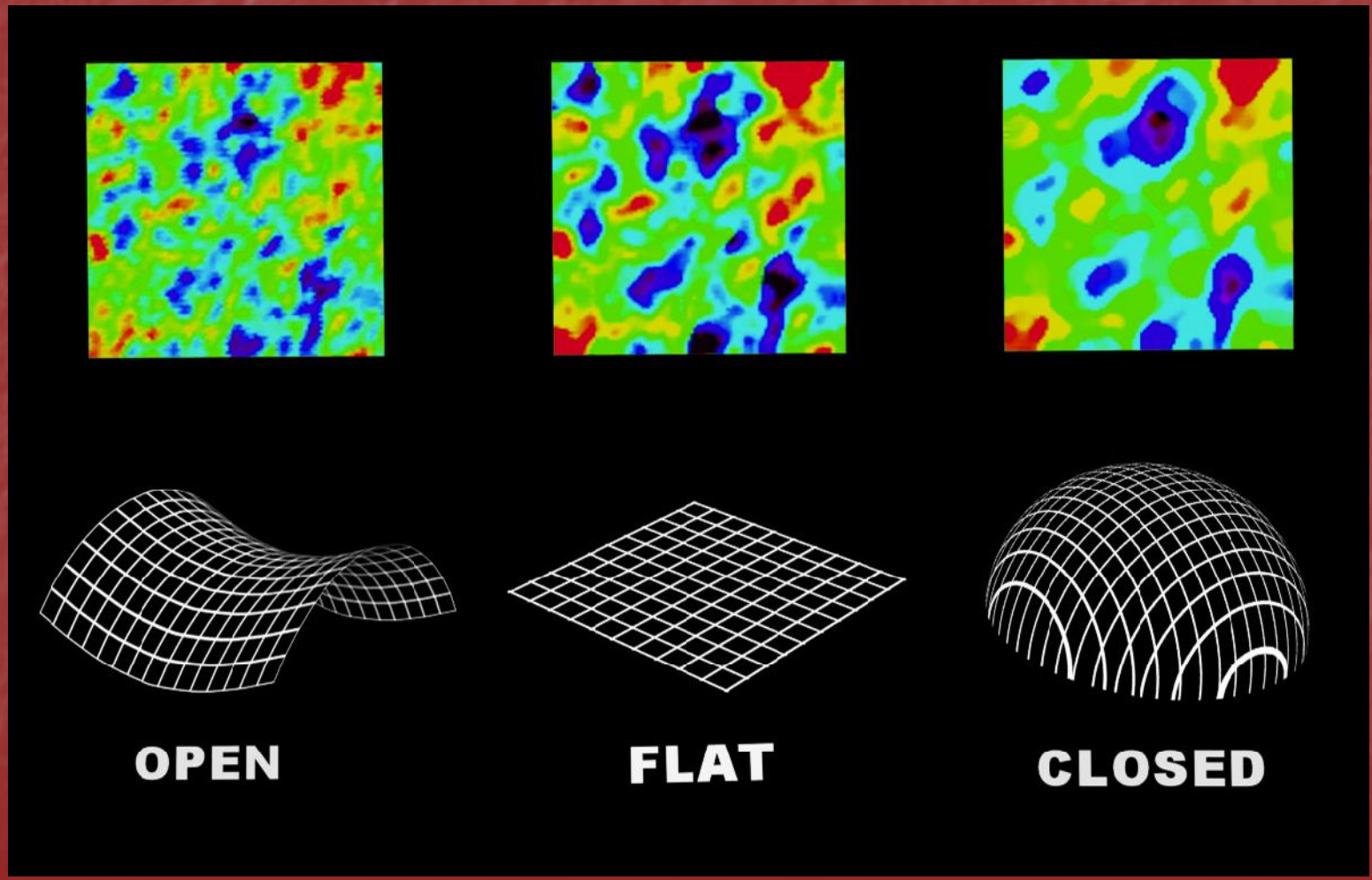


- small ripples in primordial matter & photon distribution
- gravity tries to compress primordial photon gas, photon pressure resists:
- compressions and rarefactions in photon gas:
  → → → *sound waves*
- sound waves not heard, but seen:
  - compressions: (photon) temperature higher
  - rarefactions: lower
- fundamental mode sound spectrum
  - size of *"instrument":*
  - (sound) horizon size Universe last scattering

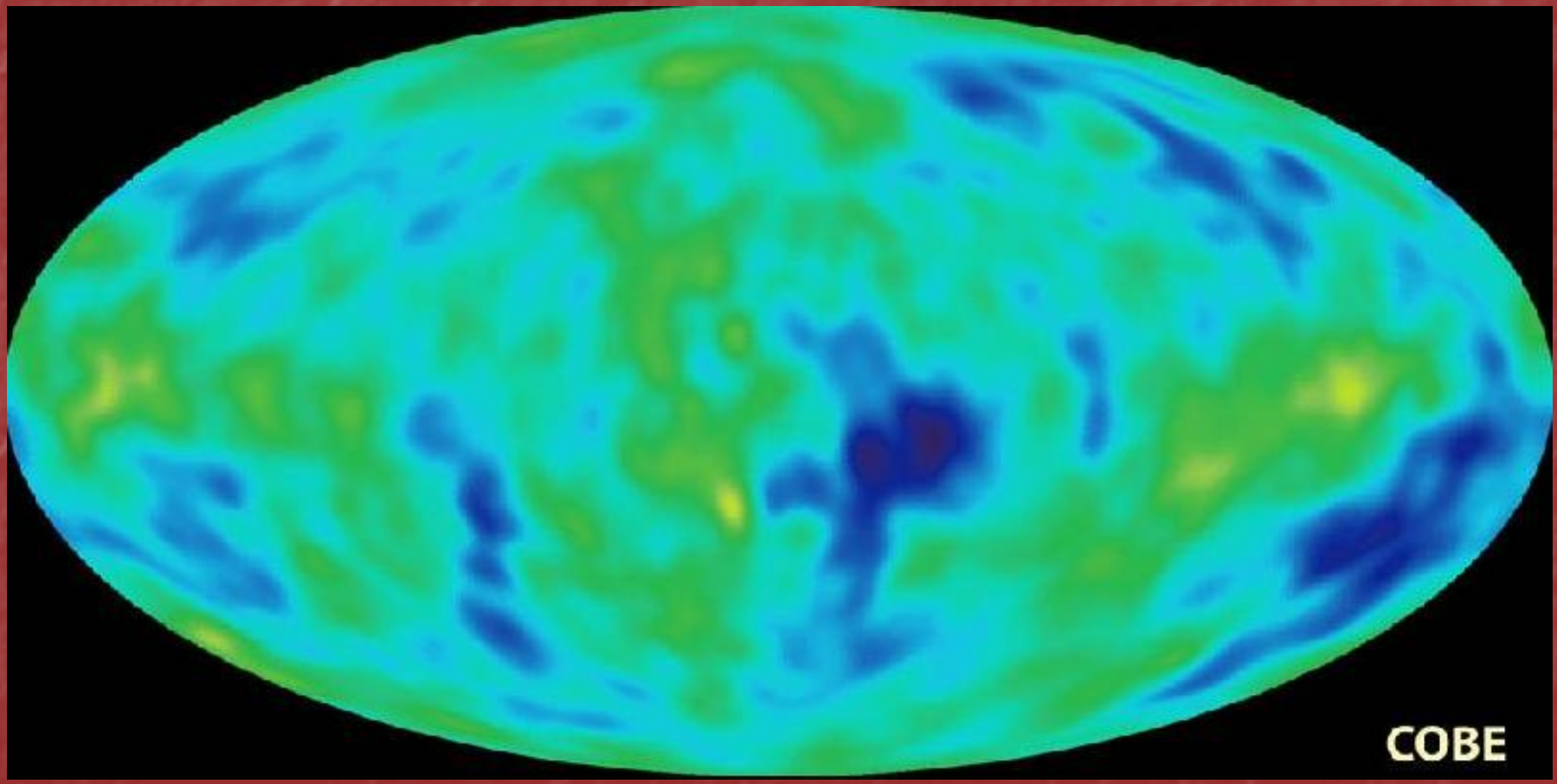
- Observed, angular size: θ~1º
  - exact scale maximum compression, the *"cosmic fundamental mode of music":*

→ → → Curvature Universe ← ← ←

Music of the Spheres

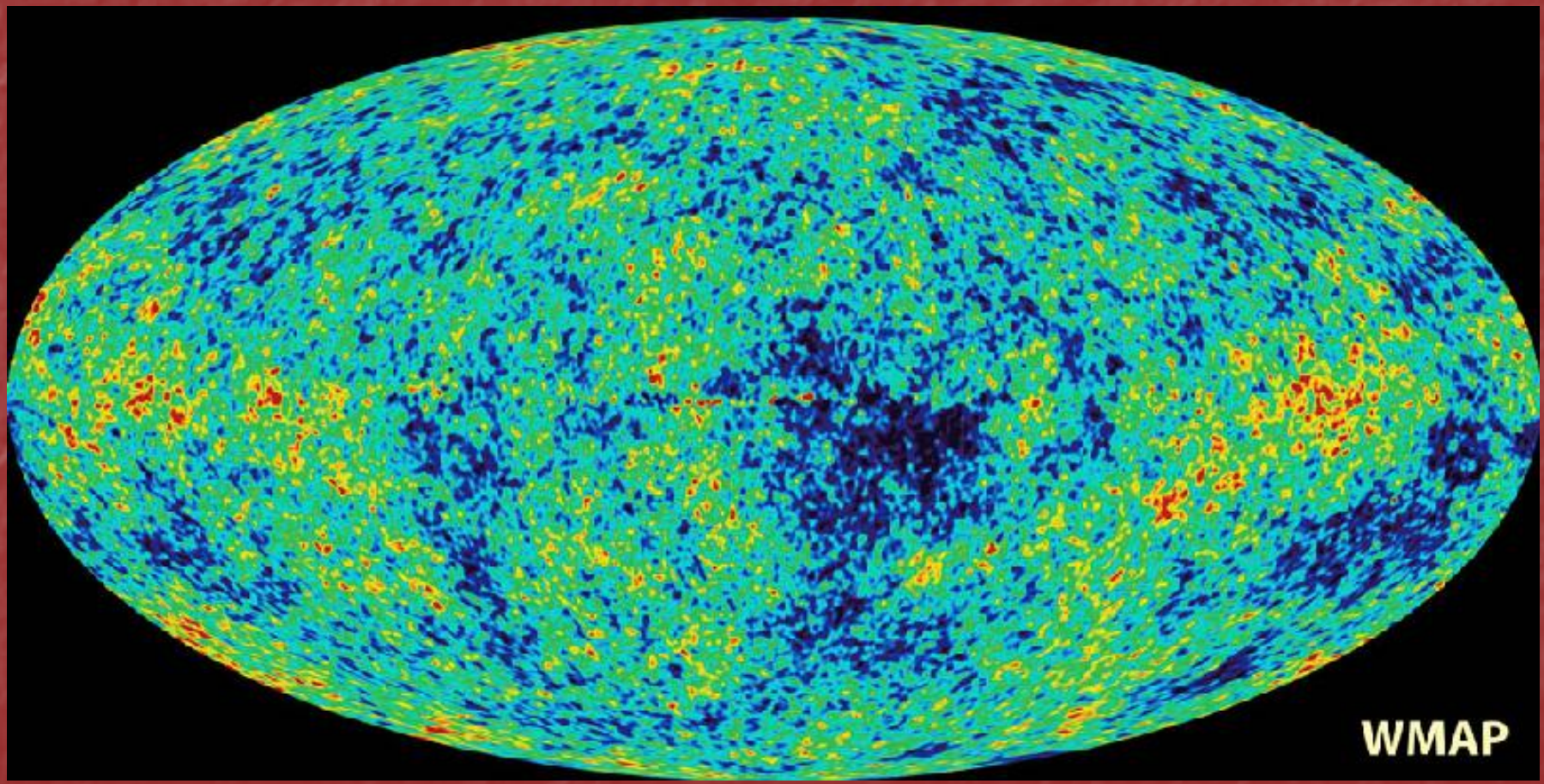


Music of the Spheres



The Cosmic Microwave Background Temperature Anisotropies: the Embryonic Universe

Music of the Spheres



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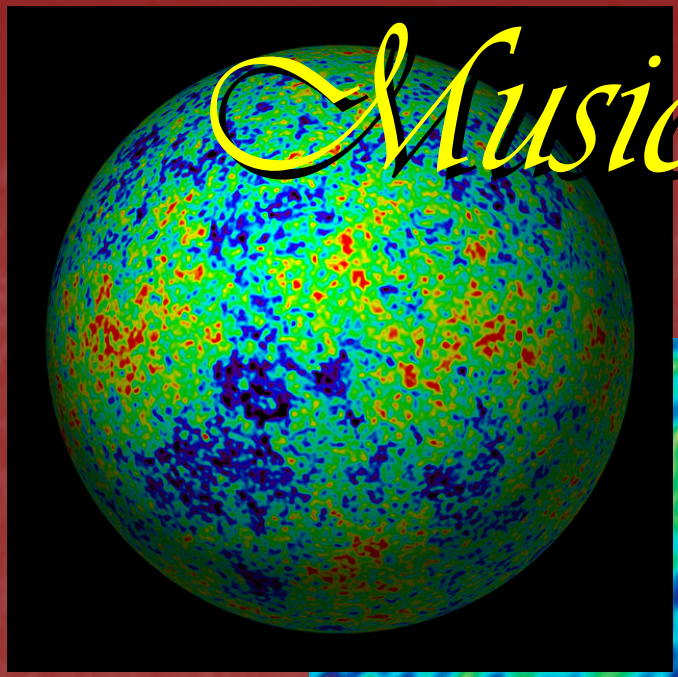
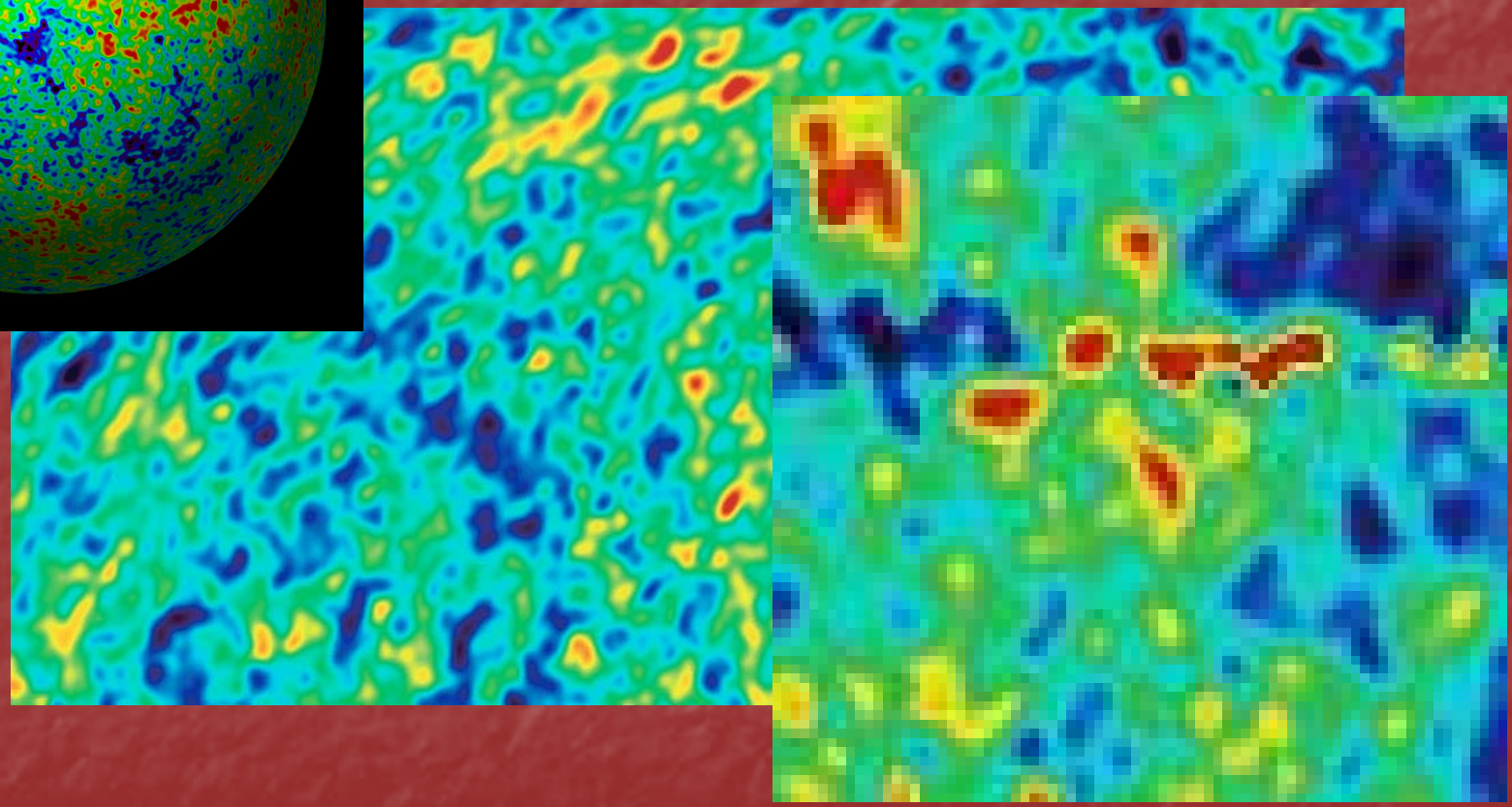
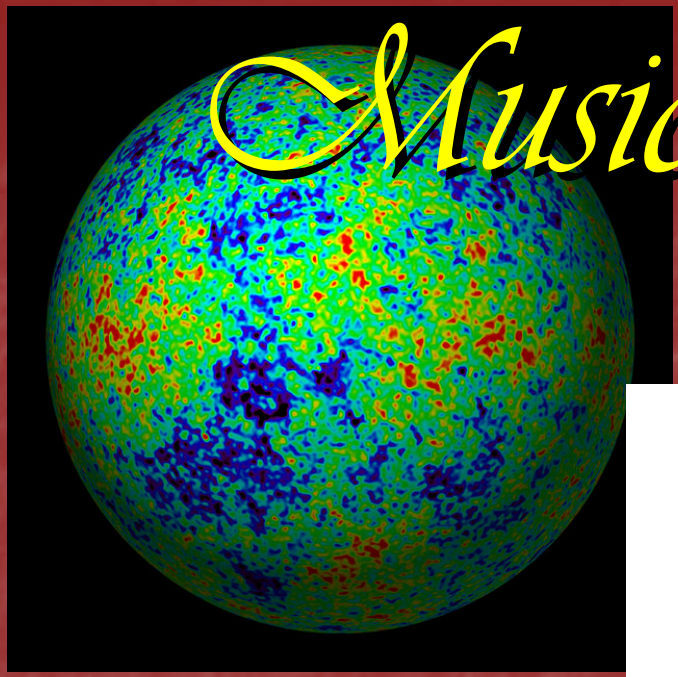




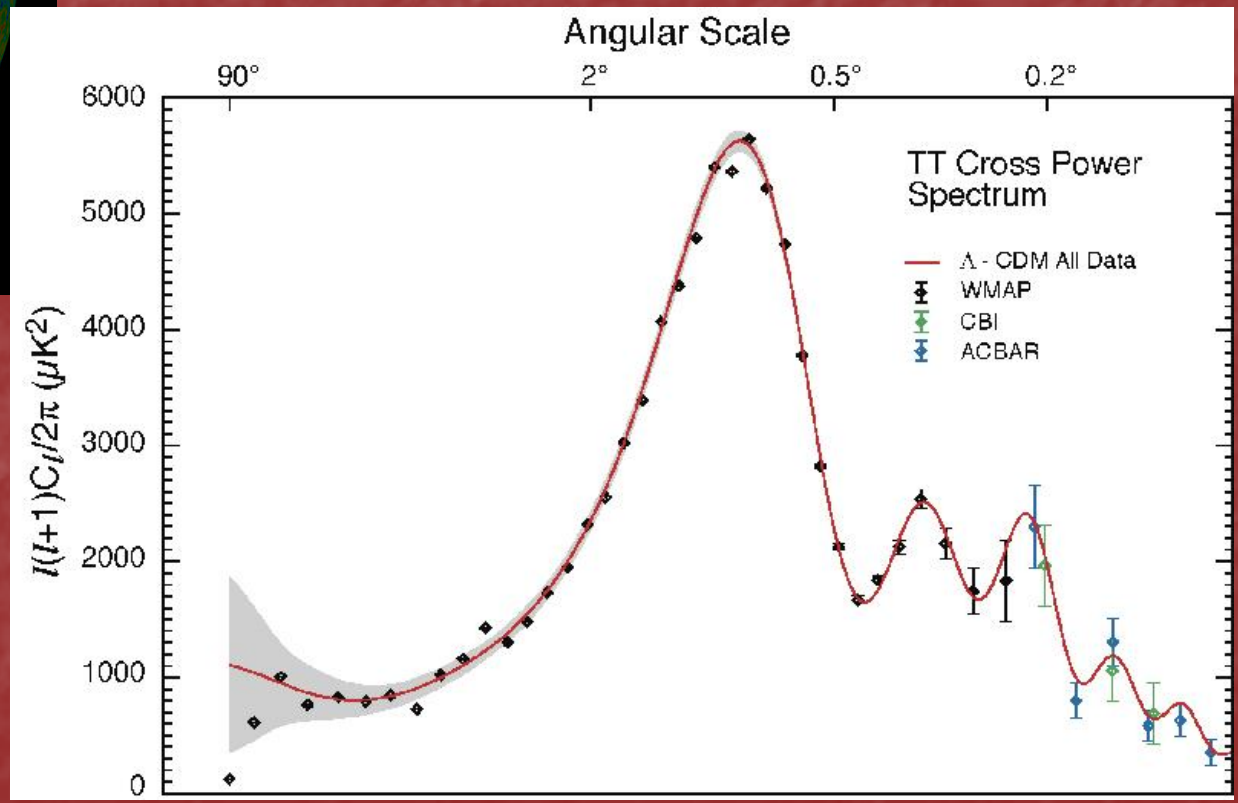
The Cosmic Microwave Background Temperature Anisotropies: the Embryonic Universe

Music of the Spheres

The Cosmic Tonal Ladder



The WMAP CMB temperature power spectrum



The Cosmic Microwave Background Temperature Anisotropies: the Embryonic Universe

Music of the Spheres

and, indeed ...

the Universe appears to be flat,

perfectly so ...

Euclid was right after all ...

Euclid's Universe

Music of the Spheres

Euclid's Universe

Music of the Spheres

Old Universe, *New* Numbers

$\Omega_{tot} = 1.02^{+0.02}_{-0.02}$

$w < -0.78$ (95% CL)

$\Omega_\Lambda = 0.73^{+0.04}_{-0.04}$

$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$

$\Omega_b = 0.044^{+0.004}_{-0.004}$

$n_b = 2.5 \times 10^{-7}{}^{+0.1\times10^{-7}}_{-0.1\times10^{-7}}$ cm^{-3}

$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$

$\Omega_m = 0.27^{+0.04}_{-0.04}$

$\Omega_\nu h^2 < 0.0076$ (95%CL)

$m_\nu < 0.23$ eV (95% CL)

$T_{cmb} = 2.725^{+0.002}_{-0.002}$ K

$n_\gamma = 410.4^{+0.9}_{-0.9}$ cm^{-3}

$\eta = 6.1 \times 10^{-10}{}^{+0.3\times10^{-10}}_{-0.2\times10^{-10}}$

$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$

$\sigma_8 = 0.84^{+0.04}_{-0.04}$ Mpc

$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$

$A = 0.833^{+0.086}_{-0.083}$

$n_s = 0.93^{+0.03}_{-0.03}$

$dn_s/d\ln k = -0.031^{+0.016}_{-0.018}$

$r < 0.71$ (95% CL)

$z_{dec} = 1089^{+1}_{-1}$

$\Delta z_{dec} = 195^{+2}_{-2}$

$h = 0.71^{+0.04}_{-0.03}$

$t_0 = 13.7^{+0.2}_{-0.2}$ Gyr

$t_{dec} = 379^{+8}_{-7}$ kyr

$t_r = 180^{+220}_{-80}$ Myr (95% CL)

$\Delta t_{dec} = 118^{+3}_{-2}$ kyr

$z_{eq} = 3233^{+194}_{-210}$

$\tau = 0.17^{+0.04}_{-0.04}$

$z_r = 20^{+10}_{-9}$ (95% CL)

$\theta_A = 0.^\circ598^{+0.002}_{-0.002}$

$d_A = 14.0^{+0.2}_{-0.3}$ Gpc

$l_A = 301^{+1}_{-1}$

$r_s = 147^{+2}_{-2}$ Mpc

...and at high precision,

we have its numbers...

The Apeiron

With the measured *flatness* of the Universe,
an intriguing issue surfaces:

Curvature ⟺ *Matter-Energy*

$$k\,c^2 = H_0^2\,R_0^2\,(\Omega_{tot} - 1) \approx 0$$

$$\Longrightarrow$$

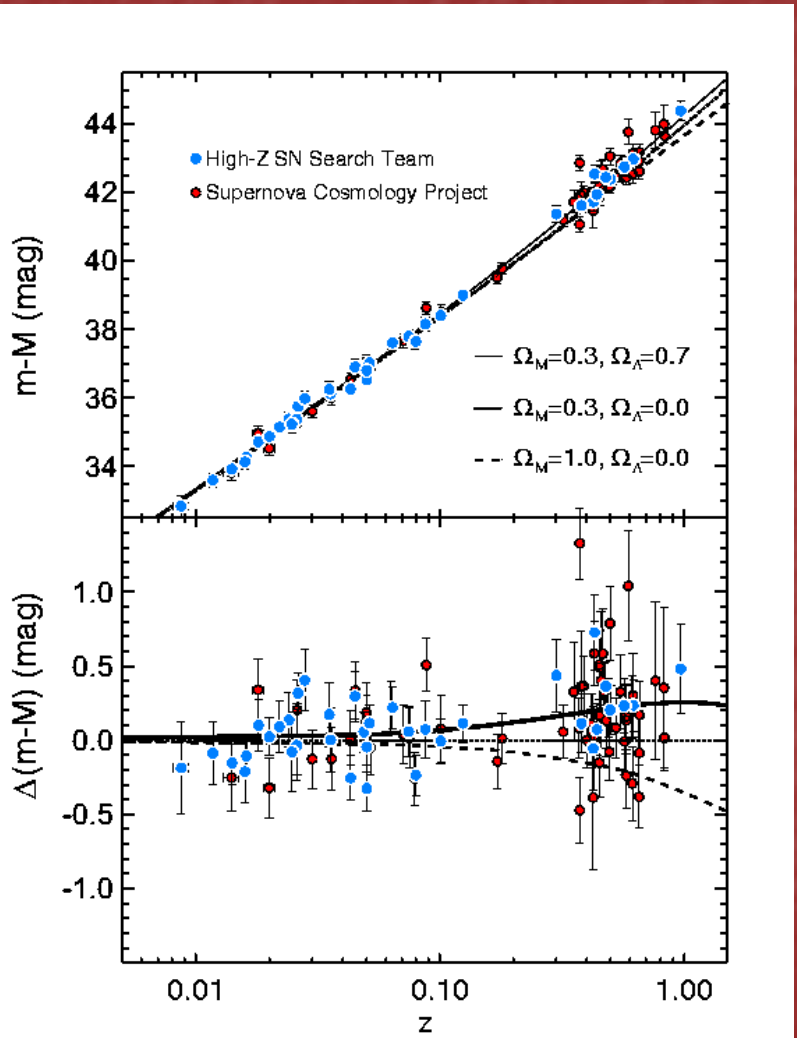
$$\Omega_{tot} = \Omega_{matter} + \Omega_{rad} + \Omega_{\Lambda} \approx 1$$

$$\Omega_{rad} \approx 10^{-5}$$

$$\Omega_{matter} \approx 0.3$$

$$\Rightarrow \quad \Omega_{\Lambda} \approx 0.7 \quad \Leftarrow$$

The Apeiron



Measuring the brightness of supernova explosions, and their apparent dimming due to distance:

distance dependent on cosmology

$$\Omega_{rad} \approx 10^{-5}$$
$$\Omega_{matter} \approx 0.3$$
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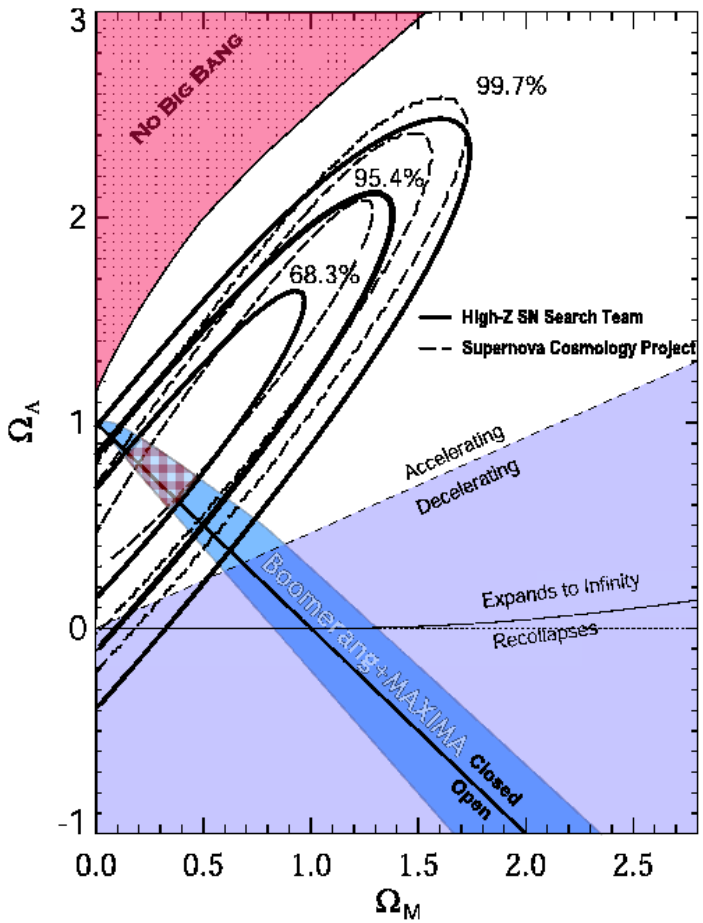
The Apeiron



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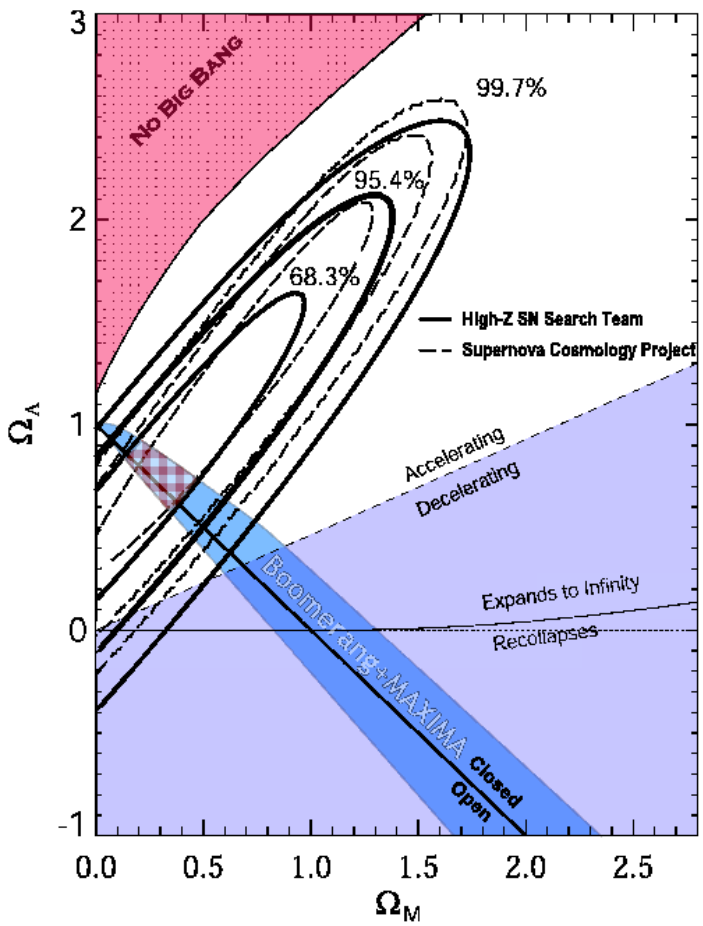


Conclusion, seemingly inescapable:

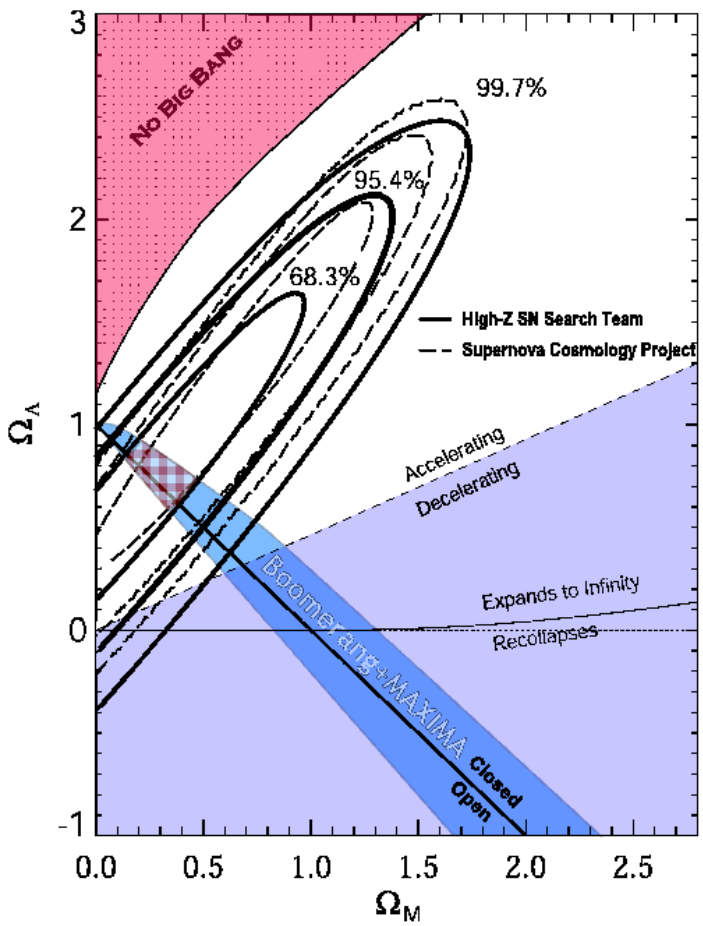
- Expansion Universe accelerating !!!
  - ➔ Cosmos forever
- Dynamics Universe dominated by:
  - elusive vacuum energy ρ_v :
  - = Cosmological Constant Λ
  - = Dark Energy
- Totally unclear what it is …

$$\Omega_{rad} \approx 10^{-5}$$

$$\Omega_{matter} \approx 0.3$$

$$\Rightarrow \quad \Omega_\Lambda \approx 0.7 \quad \Leftarrow$$

The Apeiron



Is this Anaximander ?

The Apeiron ...
from which the elements
[are formed]
is something that is different

$$\Omega_{rad} \approx 10^{-5}$$
$$\Omega_{matter} \approx 0.3$$
$$\Rightarrow \quad \Omega_\Lambda \approx 0.7 \quad \Leftarrow$$

Anaximander of Miletus

Anaximander

Miletus, 610 BC-546/545 BC

Founder of astronomy and cosmology as science

- prized symmetry
  introduced geometry and mathematical proportions to map and understand heavens

- Origin of the World: *the Apeiron*
  - nonperceptible substance preceding *"separation"* into contrasting qualities, such as *hot, cold, wet, dry …*
  - primitive unity all phenomena

"The Apeiron, from which the elements [are formed], is something that is different"

Famous last words: finite or infinite ?

Most cosmologists think that the Universe is infinite in all directions …

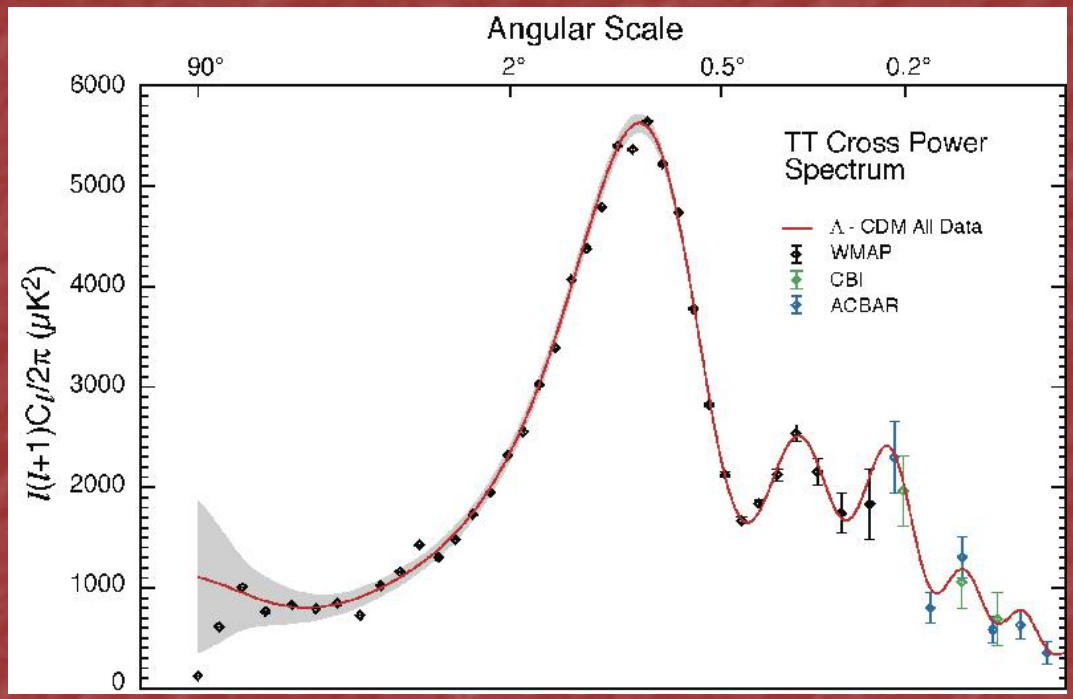
… yet, recent work by Weeks, Luminet et al. (2003), suggests

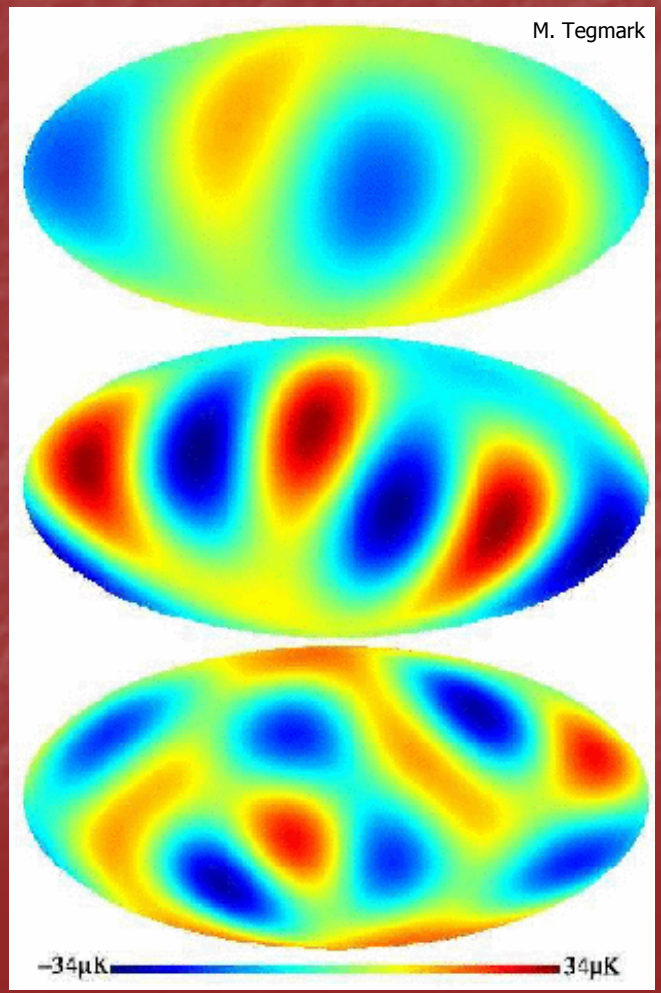
- it may be finite
- and 12-sides, a dodecahedron

Three major questions for any cosmological model:

- What is its curvature ? (positive, flat, negative)
- Is it finite or infinite ? (open: infinite amount of matter
  closed: finite amount of matter)
- What is its topology ? (multiply connected ?)

The Cosmos: a Dodecahedron ?



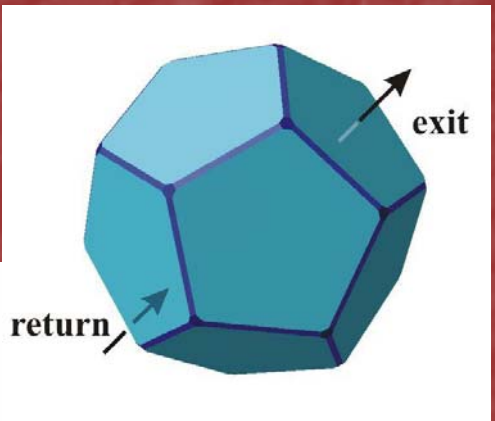


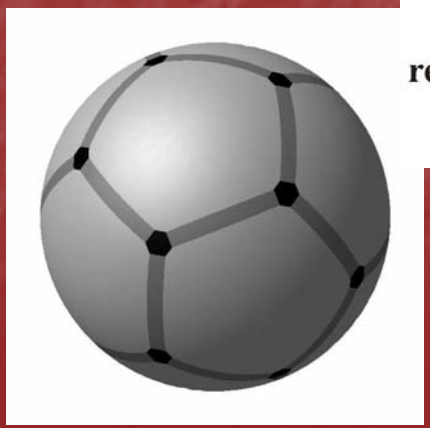
Major problem:
large-scale CMB quadrupole anisotropy
has very low amplitude … too low …

The Cosmos: a Dodecahedron ?

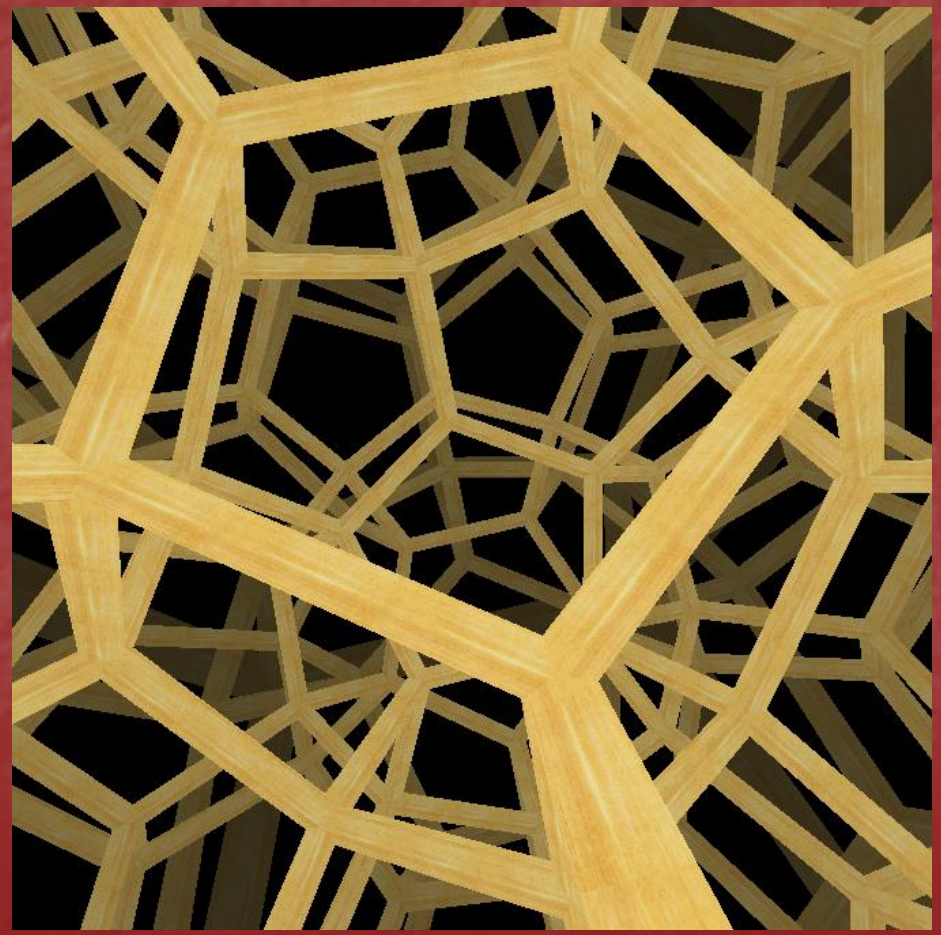
Suggestion Luminet, Weeks et al.:

- Universe finite in extent …
- and shape of dodecahedron …





The cosmic soundwaves would resonate in cosmic cavity at lower amplitude

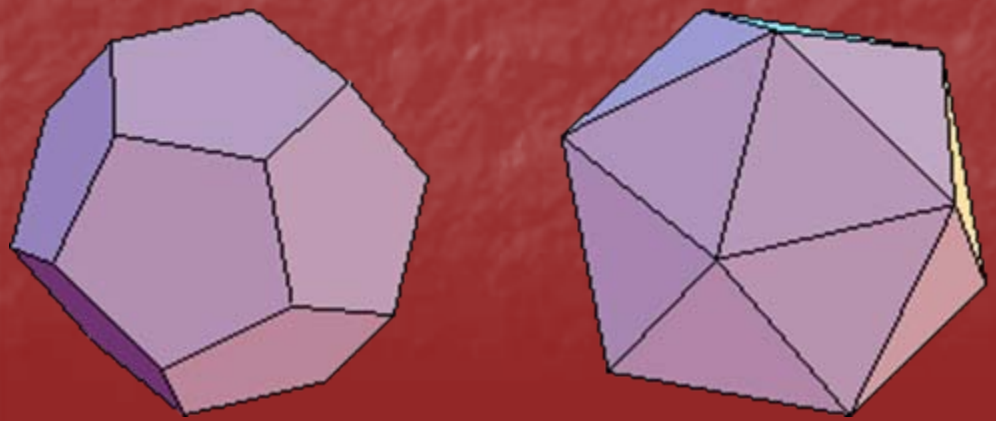


The Platonic Solids

Plato's Cosmic Scheme (Timaeus):

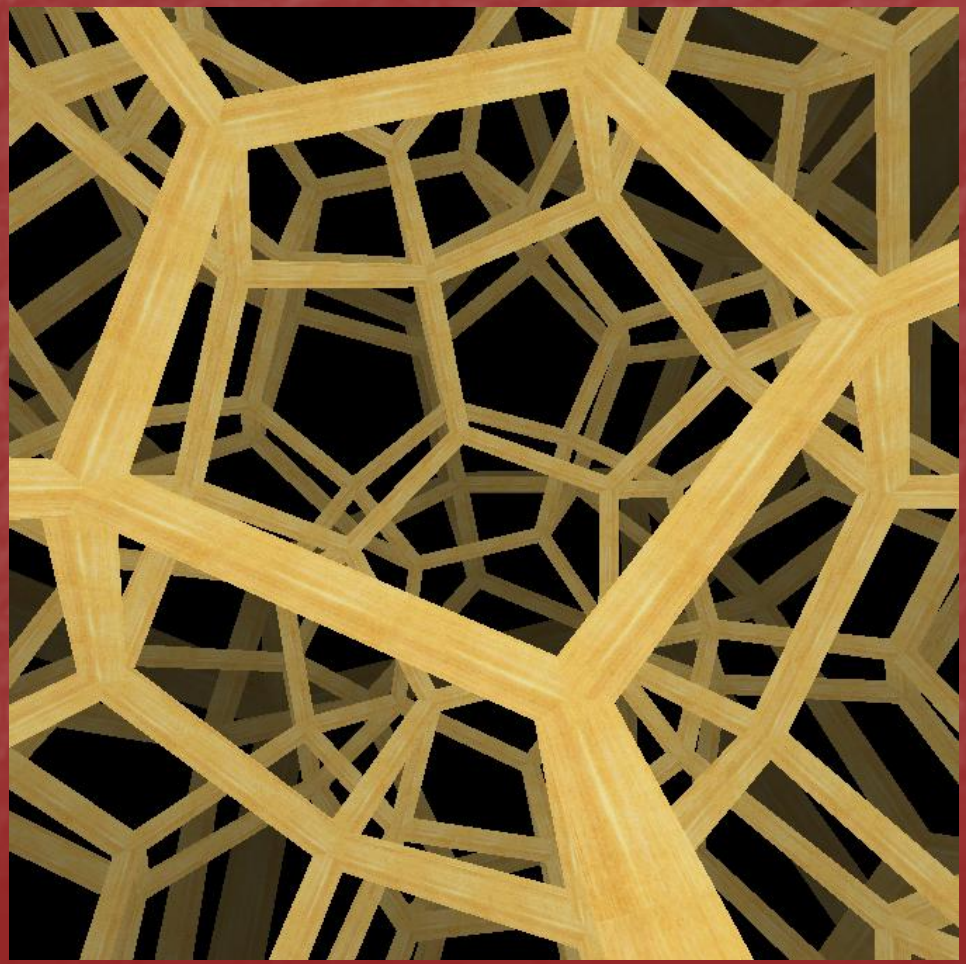
- ➔ *Demiurge*, divine craftsman, is a *mathematician*:
- ➔ *Universe constructed according to geometric principles*

- ➔ *the Five Platonic solids*
  - there are only five convex regular polyhedra !
  - Plato identified them with the cosmos and its constituents



- Four basic constituents of nature:
  - fire — Pyramid
  - air — Octahedron
  - earth — Cube
  - water — Icosahedron
- The Cosmos itself:
  - the stuff used for `embroidering the constellations on the heavens' — Dodecahedron

The Cosmos: a Dodecahedron ?



Plato:

said so all along in Timaeus

"Let no one unversed in geometry enter here"

Academia of Plato, Athens