Appendix A

Kinematic Extraction Methods

The spectrum extracted from a galaxy is the superposition of the individual spectra of each star in that region of the galaxy, along the line of sight. As the line of sight of each star is different , the spectra of the stars with respect to the others will be shifted by a certain amount in wavelength. The net effect is a broadening in the galaxy spectrum owing to these shifts. In other words. the galaxy spectrum is the convolution of the spectrum of the stars with an appropriate velocity distribution along the line of sight.

In order to quantify these offsets and broadening *Line of Sight Velocity Distribution* (LOSVD, $F(v_{los})$) has been defined as the fraction of stars contributing to the spectrum with a line of sight between v_{los} and v_{los}+dv_{los}. Assuming that all the stellar spectra are identical ($S(u)$), with u the stellar velocity, the galaxy spectrum can be given by the equation:

$$G(u) \approx \int F(v_{los}) \cdot S(u - v_{los}) \cdot dv_{los} \tag{A.1}$$

This is the most important equation in the analysis of galactic kinematics. In order to obtain the kinematics of a particular galaxy we need to make use of a '*stellar template*', whose spectrum is required to be a good approximation of the galaxy's spectrum. From this point, the better selection of the template star, the more accurate the fit to the galaxy spectrum and therefore the better kinematics we will get. The simplest properties of the LOSVD are its mean velocity and velocity dispersion:

$$\overline{v} = \int F(v_{los}) \cdot v_{los} \cdot dv_{los} \tag{A.2}$$

$$\sigma^2_{los} = \int F(v_{los}) \cdot (v_{los} - \overline{v}_{los})^2 \cdot dv_{los} \tag{A.3}$$

To first approximation, it is assumed that the LOSVD of the stars is Gaussian. However the improvements of the detectors has made it possible to measure deviations from Gaussian shape through a new set of algorithms. We review the most relevant methods in the literature in the following paragraphs. All the methods shown here take advantage of the fact that the convolution shown in A.1 becomes a multiplication in '*Fourier space*' and therefore it is easier to extract and analyse the LOSVD there. Note that there is no distinction between a function and its Fourier transform. The main advantage of working if Fourier space is the possibility of *filtering* residual low-order and high frequency components in the original spectrum that are due to an imperfect continuum subtraction and Poisson noise respectively. Some computer codes have been developed to fit the LOSVD in '*Real space*' (Rix & White 1992; van der Marel & Franx 1993), although they won't be discussed here.

• Fourier Quotient (Sargent et al. 1977)
Sargent et al. (1977) based their algorithm on the assumption that the $F(v_{los})$ has a Gaussian shape. Under this consideration they fit a Gaussian to the quotient between the galaxy spectra and the template spectra in Fourier space:

$$F(k) = \frac{G(k)}{S(k)} \approx \gamma \cdot exp\Big[-\frac{1}{2}\frac{2\pi k\sigma}{N} + \frac{2\pi vki}{N}\Big] \tag{A.4}$$

where $F(k), G(k)$ and $S(k)$ are the fourier transforms of the $F(v_{los}), G(v)$ and $S(v)$ respectively, γ is the mean relative line-strength, σ is the velocity dispersion and v is the mean radial velocity of the assumed Gaussian. N is the number of pixels in the input spectra.

• Cross-Correlation Method (Simkin 1974; Tonry & Davis 1979)
The Cross-Correlation approach, initiated by Simkin (1974) and improved by Tonry & Davis (1979), is a simple method based on the fitting of the cross-correlation peak defined by the functions: $G(k)$ and $S(k)$

$$C(k) = \frac{1}{N\Delta_g\Delta_t}\, G(k) \cdot S^*(k) \tag{A.5}$$

where Δ_g and Δ_t are the rms of the galaxy and template respectively and * indicates complex conjugation. Cross-correlating a template spectrum with the galaxy spectrum then produces a function $C(k)$ with a peak at the redshift of the galaxy with a width related to the dispersion of the galaxy. The resultant function is then fitted with a Gaussian in Fourier space.

• Fourier Correlation Quotient (Bender 1990)
The two previous methods were based on the assumption that the broadening function was a Gaussian. The Fourier Correlation Quotient (FCQ) is an improvement of the two and a generalization of the problem because it provides the full LOSVD function instead of the results of a specific parametrisation. The main advantage of this procedure lies in the fact that one can choose the most appropriate function to fit the LOSVD *a posteriori* rather than *a priori*. This approach is based on the deconvolution of the peak of the template-galaxy correlation function with the autocorrelation function of the template star:

$$F(k) = \frac{G(k) \cdot S^*(k)}{S(k) \cdot S^*(k)} \tag{A.6}$$

Compared to previous methods (i.e. Fourier Quotient) FCQ is less sensitive to template mismatch because the LOSVD comes from the ratio of template-galaxy correlation and template autocorrelation functions which are smooth functions, instead of the division of 2 spectra.

• Unresolved Gaussian Decomposition (Kuijken & Merrifield 1993)
This method assumes deviations from pure gaussian are due to the composition of several gaussians uniformly distributed in mean velocity and velocity dispersion, but not in amplitude:

$$F(k) \approx \sum_{k=1}^{N} a_k \cdot exp\Big[-\frac{(v_{los} - v_k)^2}{2\sigma_k} \Big] \tag{A.7}$$

where a_k, v_k, σ_k represents the amplitude, mean velocity and velocity dispersion of each gaussian. The algorithm, under certain physical constraints, determines the different amplitudes a_k for each component that best fit the galaxy spectrum in the least square sense. Those physical constraints are:

- The LOSVD must be non-negative everywhere
- The LOSVD is expected to be smoothly varying on small scales
- The LOSVD is expected to be non-zero over only a fairly small range in velocity

The Rayleigh criterium is also required to define the separation between 2 adjacent peaks. This should be less than 2σ in order to avoid dips between 2 peaks.

• Gauss-Hermite Expansion (van der Marel & Franx 1993)
This approach defines the LOSVD as the product of a normal Gaussian and a truncated expansion of the Hermite polynomials:

$$F(v_{los}) \approx \Big[\frac{\gamma\alpha(w)}{\sigma}\Big] \cdot \Big[1 + \sum_{j=3}^{n} h_j H_j(w)\Big] \tag{A.8}$$

$$w = \frac{(v_{los} - v)}{\sigma}$$

where γ, v, σ characterise the line-strength, mean velocity and velocity dispersion of the traditional Gaussian and the Hermite coefficients (h_j) describe deviations from the gaussian shape (i.e. h_3 measures asymmetries and h_4 symmetries) of the LOSVD. $\alpha(w)$ is the standard Gaussian.

For most of the work presented in this thesis we have made use of FOURFIT, a computer code developed by van der Marel & Franx (1993) (chapters 2, 3, 4). In chapter 5 we also use an adapted implementation of Bender (1990) that deals with SAURON data.