Formation and Evolution of Galaxies: Lecture 3

We begin with a quick overview of the timetable of structure formation in the Universe, a quick review of cosmology, and the basic equations of structure formation.

Timetable for Structure Formation

Gravitational potential fluctuations	$z \gtrsim 10^3$
The first stars	$z \sim 20$
Dark halos of galaxies	$z \gtrsim 20$
Intergalactic medium	$z \sim 10$
First AGN engines	$z \sim 10$
Angular momentum of galaxy rotation	$z \sim 5$
Spheroids of galaxies	$z \gtrsim 5$
Thick disks of galaxies	$z \gtrsim 5$
First 10% of heavy elements	$z \gtrsim 3$
Rich clusters of galaxies	$z \sim 1$–2
Thin disks of galaxies	$z \gtrsim 1$
Superclusters, walls, and voids	$z \sim 1$

We'll discuss several of these topics in this course, and we'll see where these numbers come from.

Basic Cosmology for Galaxy Formation

The Homogeneous, Isotropic Universe

Hubble's law: $v = H(t)r$ (isotropic, uniform expansion)

Redshift:

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} \quad (1)$$

For nonrelativistic expansion velocities, $v \approx cz$ ($z \ll 1$).

Today, $t = t_0$ and $H(t_0) = H_0 = 100\,h$ km s^{-1} Mpc^{-1}, with $h \approx 0.7$ (more on this later).

In a homogeneous and isotropic universe in which a universal time can be defined, the metric g_{ij} is given by the Robertson-Walker metric:

$$ds^2 = c^2dt^2 - \frac{a^2(t)}{(1+kr^2/4)^2} \times \left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \quad (2)$$

where $k = -1, 0, 1$ measures the spatial curvature, and $a(t)$ is the *scale factor*, which defines the *relative* size of the Universe at time t. We set $a(t_0) = 1$ for convenience.

Let $r(t) = r_0 a(t)$ define the *proper* distance between a pair of well-separated galaxies (to minimize gravitational interaction). Then

$$v = \frac{dr}{dt} = r_0\frac{da}{dt} = r_0\dot{a} = r\frac{\dot{a}}{a} = Hr$$

and therefore

$$H = \frac{\dot{a}}{a}. \quad (3)$$

Let's consider the radial propagation of light toward an observer (us). Since light travels on world lines with $ds = 0$, we find that

$$c\,dt = -\frac{a(t)dr}{1 + kr^2/4}$$

If a photon emitted at position r at time t arrives here ($r = 0$) at time t_0, then

$$\int_t^{t_0} \frac{c\,dt}{a(t)} = \int_0^r \frac{dr}{1 + kr^2/4}$$

If a second photon is emitted at the same position at time $t + \Delta t$, then

$$\int_{t+\Delta t}^{t_0+\Delta t_0} \frac{c\,dt}{a(t)} = \int_0^r \frac{dr}{1 + kr^2/4}$$

For sufficiently small Δt, then, $\Delta t/\Delta t_0 = a(t)/a(t_0)$. The frequency of light is just the reciprocal of the interval between wave crests $\nu = 1/\Delta t$, so

$$1 + z = \frac{\nu}{\nu_0} = \frac{a(t_0)}{a(t)} = \frac{\lambda_0}{\lambda} = a^{-1}. \qquad (4)$$

Dynamics of the Universe

In general relativity, the geometry of space-time is described by the metric tensor g_{ij}, which is determined by the Einstein field equation

$$R_{ij} - \frac{1}{2} g_{ij} R + \Lambda g_{ij} = -\frac{8\pi G}{c^2} T_{ij} \qquad (5)$$

(including the cosmological constant Λ). The Reimann tensor R_{ij} and R are functions of g_{ij} and its first two derivatives and T_{ij} is the energy-momentum stress tensor.

For an ideal fluid with density ρ and pressure p (in the rest frame of the fluid), the energy-momentum tensor can be written, in the frame in which the fluid is instantaneously at rest,

$$T_{ij} = \begin{bmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix}$$

Subsistuting the Robertson-Walker metric (Eq. 2) into the Einstein field equation (Eq. 5), we get the dynamical equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3} \qquad (6)$$

and

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho + \frac{\Lambda}{3} - \frac{k}{a^2} \qquad (7)$$

Note that pressure exerts three times the gravitational force as does mass or energy and therefore radiation exerts twice as much gravitational force than cold matter for equal amounts of energy density (because $p_{\text{gas}} = 3u/2$ and $p_{\text{rad}} = 3u$).

For the rest mass density, conservation of mass (or particles) requires that

$$\rho_{\text{matter}} \propto a^{-3} \propto (1+z)^3. \tag{8}$$

We can use the first law of thermodynamics, $dU = -p\,dV$ to show that

$$\rho_\gamma \propto a^{-4} \propto (1+z)^4 \tag{9}$$

(see Peebles, PPC, Chaps. 4, 5, 6). We can understand this by considering the conservation of photons (three powers of a) and the fact that photon energy redshifts with time (one power of a). We'll leave it as an exercise to show that this can also be derived from Louiville's Theorem. There is an epoch in which $\rho_{\text{matter}} > \rho_\gamma$; we live in that epoch today ($\rho_\gamma \sim 10^{-4}\rho_{\text{matter}}$).

We can also show that

$$T_{\text{CMB}}(z) = \frac{T_{\text{CMB}}(0)}{a} = 2.73(1+z)\text{ K} \tag{10}$$

Let's assume a matter-dominated universe with no cosmological constant ($\Lambda = 0$). (This likely isn't correct, as we'll soon see, but you can find the more complicated equations including Λ in Peebles, PPC, Chap. 13.) Then our dynamical equations (Eqs. 6–7) are the equations for a uniform, self-gravitating sphere of matter. We can rewrite those equations as

$$\dot{a}^2 = \frac{8\pi G \rho_0}{3a} + \frac{8\pi G}{3}(\rho_{\rm crit} - \rho_0), \qquad (11)$$

where the critical density

$$\rho_{\rm crit} = \frac{3H_0^2}{8\pi G} = 3 \times 10^{11} h^2\, {\rm M_\odot\, Mpc^{-3}}. \quad (12)$$

We also define the present density parameter $\Omega_0 = \rho_0/\rho_{\rm crit}$ and use the last two equations to find

$$\Omega(z) = \frac{\Omega_0(1+z)}{1+\Omega_0 z}, \qquad (13)$$

so that in the early Universe ($z \gg 1$), $\Omega(z) \approx 1$ for any value of Ω_0.

Major Epochs in the History of the Universe

	$(1+z)$	T_{CMB}
Now	1	2.725 ± 0.002 K
Decoupling	1089 ± 1	2790 K
(Universe becomes neutral, CMB let loose)		
Matter–radiation	$2 \times 10^4 \Omega_0$	$6 \times 10^4 \Omega_0$ K
equality ($\rho_{\mathrm{matter}} = \rho_\gamma$)		
Nucleosynthesis	3×10^8	10^9 K
Inflation	3×10^{26}	10^{27} K

Our collapse solutions will be valid at redshifts lower than matter–radiation equality.

Timescales when $\Omega_0 = 1$:

$$a \propto \begin{cases} t^{2/3} & \text{when } \rho_{\mathrm{matter}} > \rho_\gamma \\ t^{1/2} & \text{when } \rho_{\mathrm{matter}} < \rho_\gamma \end{cases} \tag{14}$$

Current Estimated Cosmological Parameters

Description	Symbol	Value	$\pm$
Total density	Ω_{tot}	1.02	0.02
Eqn. state quint.	w	< -0.78	
Dark energy density	Ω_Λ	0.73	0.04
Baryon density	Ω_b	0.044	0.004
Matter density	Ω_m	0.27	0.04
Light neutrino density	Ω_ν	< 0.015	
Fluctuation amplitude	σ_8	0.84	0.04
Power spectrum index	n_s	0.93	0.03
Hubble constant	h	0.71	0.04
Age of Universe (Gyr)	t_0	13.7	0.2

Taken from *WMAP* results (Bennett et al. 2003)