Contents

Chapter 1

Structure Formation in the Universe

On small scales the Universe today is very lumpy. For example, the density of a galaxy is about 10^5 times the average density of the Universe and that within galaxy cluster is 10^{2-3} times the average density of the Universe. Of course on very large scales the Universe is smooth and isotropic. In this section we want to deal with the growth of structure in the Universe which can evolve in a very nonlinear manner and lead to the formation of the radiation emitting objects we see around us. Such objects are obviously very important for reionization as the UV radiation that the first generation(s) of these objects emit is the *prime suspect* in reionizing the Universe.

The purpose of this chapter is the review the physics of structure formation in cosmological context. We first discuss the background cosmological models. The theory of linear growth of perturbations is then presents. We also discuss the spherical collapse model and show its main results. The Press-Schechter formalism is developed. Finally we discuss the linear and non-linear evolution of the baryonic component of the fluctuations.

1.1 Background Cosmological Model

1.1.1 The Expanding Universe

The modern physical description of the Universe as a whole can be traced back to Einstein, who argued theoretically for the so-called cosmological principle: that the distribution of matter and energy must be homogeneous and isotropic on the largest scales. To date the isotropy assumption is observationally well established (mainly from the CMB measurement) and the homogeneity assumption has reasonable observational support.

Assuming General Relativity and homogeneity and isotropy of the Universe one can reach the so called Friedman-Robertson-Walker (hereafter FRW) metric, which can be written in the form,

$$ds^2 = dt^2 - a(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right], \tag{1.1}$$

where (t, r, θ, ϕ) are the comoving coordinates and $a(t)$ is the cosmic scale factor, and with an appropriate rescaling of the coordinates, k can be chosen to be +1, -1 or 0 for spaces of constant positive, negative or zero curvature, respectively. Observers at rest (fixed r, θ, ϕ) remain at rest with the distance between them increasing in time proportionally to $a(t)$. The distance, D, between two observers increases as a function of time as $H(t)D$ where $H(t)$ is the Hubble constant at time t and is given by $H(t) = \frac{da/dt}{a}$. For convenience we set $a(t_0) = 1$ where t_0 is the edge of the current age of the Universe. Light emitted at time t is seen by an observer at t_0 redshifted by $z = 1/a(t) - 1$.

Following the discussion in Kolb & Turner (1990) the conservation of energy and the Einstein equations yield the equation

$$d(\rho a^3) = -pd(a^3) \tag{1.2}$$

where ρ and p are the time dependent energy density and pressure, respectively. The physical significance of Equation 1.2 is clear: the change in energy in a comoving volume, $d(\rho a^3)$, is equal to minus the pressure time the change in volume, $-pd(a^3)$. For a simple equation of state, $p = w\rho$, where w is independent of time, the energy density evolves as $\rho \propto a^{-3(1+w)}$. Examples of interesting cases include:

$$\begin{array}{rcl} Radiation & (p = \frac{1}{3}\rho) & \Longrightarrow \quad \rho \propto a^{-4} \\ Matter & (p = 0) & \Longrightarrow \quad \rho \propto a^{-3} \\ Vacuum\ Energy & (p = -\rho) & \Longrightarrow \quad \rho \propto constant. \end{array}$$

The Einstein field equations for FRW model yields the so called Friedmann equation which relates the expansion of the Universe to its matter-energy content.

$$H^2(t) = \frac{8\pi G}{3}\rho - \frac{k}{a^2}. \tag{1.3}$$

Notice that the critical density of the Universe is defined using this equation (i.e., assuming $k = 0$) which gives $\rho_c = \frac{3H^2}{8\pi G}$. Using this definition one can recast the

Friedman equation in terms,

$$1 = \Omega - \frac{k}{H^2 a^2} \tag{1.4}$$

where $\Omega \equiv \rho/\rho_c$ is the total (matter, radiation and vacuum) energy density at a given time.

Equation 1.3 can also be rewritten in terms of the current day energy density components Ω_r, Ω_m and Ω_Λ as

$$\frac{H(t)}{H_0} = \left[\frac{\Omega_m}{a^3} + \Omega_\Lambda + \frac{\Omega_r}{a^4} + \frac{\Omega_k}{a^2}\right]^{1/2} \tag{1.5}$$

where $\Omega_k \equiv -k/H_0^2$. The current total energy density given by, $\Omega_0 = \Omega_r + \Omega_m + \Omega_\Lambda$.

The solution of these equations is particularly simple in Einstein de-Sitter (EdS) Universe, i.e., $\Omega_m = 1$ and $\Omega_r = \Omega_\Lambda = \Omega_k = 0$, where $a \propto t^{\frac{1}{2}}$. Also for $\Omega_k = 0$ models, which our Universe seem to belong to, the Universe behaves at high redshifts like an EdS Universe which simplifies our background geometry significantly.

The evolution of the Universe is terms of its geometry also implies certain temperature evolution. This is mainly dependent on how the various components of the Universes energy density evolve with its expansion. For example, radiation energy density evolves like a^{-4} and matter density evolves like a^{-3} whereas vacuum energy density remains constant. This evolution together with baryonic physics dictates certain sequence of stages that represent different phases of the Universe evolution. To our story in this course, an important stage is the so called decoupling stage which demarcates a point in time in which the Universe transformed from ionized and opaque to neutral and transparent. This transition defines the so called last scattering surface from which the Cosmic Microwave Background (CMB) radiation arrive and which .

1.2 Linear Growth of Fluctuations

Consider the standard Newtonian equations for the evolution of the density, ρ, and velocity, $\vec{u}$, of a fluid under the influence of gravitational field with potential Φ:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0 \tag{1.6}$$

$$\rho \frac{d\vec{u}}{dt} = -\vec{\nabla} p - \rho \vec{\nabla} \Phi. \tag{1.7}$$

This equation must be supplemented by Poisson's equation to relate the gravitational field to the density of the fluid, and by an equation of state to specify the pressure p. To obtain equation suitable for structure formation in the Universe it is better to take into account the mean properties of the Universe which in this context are given by the scale factor $a(t)$ and the mean density of the Universe, $\bar{\rho}$. This amount to changing the coordinates to comoving positions through the transformation $\vec{r} = \vec{x}/a(t)$ and to peculiar velocity $\vec{v} = a\, d\vec{r}/dt = \vec{u} - (da/dt)\vec{r}$. It is also easier to express the density in terms of the dimensionless over-density, $\delta = \rho/\bar{\rho} - 1$ and to use the conformal time $d\tau = dt/a$ instead of proper time. The fluid equations and Poisson equation then become:

$$\dot{\delta} + \vec{\nabla} \cdot [(1+\delta)\vec{v}] = 0 \tag{1.8}$$

$$\dot{\vec{v}} + (\vec{v} \cdot \vec{\nabla})\vec{v} + \frac{\dot{a}}{a}\vec{v} = -\frac{\vec{\nabla} p}{\rho} - \vec{\nabla}\Phi \tag{1.9}$$

$$\nabla^2 \Phi = 4\pi G \bar{\rho} a^2 \delta. \tag{1.10}$$

where $\vec{v} \equiv \dot{\vec{r}}$ and $\vec{\nabla}$ and $\dot{}$ operators are differentiations with respect to $\vec{r}$ and τ, respectively. Note that is the absence of gravitational and pressure forces Euler's equation becomes $d(a\vec{v})/d\tau$, showing that the peculiar velocity decays with the Universe expansion which extends to all vortical modes.

Linearizing the fluid equation and substituting both the continuity and Poisson's equations into the divergence of Euler's equation one obtains,

$$\ddot{\delta} + \frac{\dot{a}}{a}\dot{\delta} = \frac{\nabla^2 p}{\rho} + 4\pi G \bar{\rho} a^2 \delta. \tag{1.11}$$

In case of a pressure free Universe (like the dark matter case) the equation involves no spatial derivatives. This equation could be recast as,

$$\ddot{\delta} + \frac{\dot{a}}{a}\dot{\delta} - \frac{3}{2}\Omega_m \left(\frac{\dot{a}}{a}\right)^2 \delta = 0. \tag{1.12}$$

It is easy to show that this equation has a general solution of the following form:

$$\delta(\vec{r}, \tau) = \delta_+(\vec{r}) D_+(\tau) + \delta_-(\vec{r}) D_-(\tau). \tag{1.13}$$

This generic solution has two modes, a growing mode and a decaying mode. The decaying mode gets suppressed very quickly and, except in certain cases, could be ignored. The growing on the other hand is the one that has the seeds of the nonlinear structures that inevitably evolve to form galaxies and galaxy clusters. The main feature that one should notice is the self similar manner on which the linear growing mode evolves with time. Since we will focus of the growing mode we rename $\delta_-(\vec{r})$ to $\delta(\vec{r})$ and $D_+(\tau)$ to $D(\tau)$.

In the case of $\Omega_m = 1$ (EdS Universe) an analytical solution for equation 1.12 could be easily obtained where $a \propto t^{2/3} \propto \tau^2$ and $\dot{a}/a = 2/\tau$. In this case the growing mode $\delta \propto D_+(\tau) \propto \tau^2 \propto t^{2/3}$, while the decaying mode is $\delta \propto D_-(\tau) \propto \tau^{-3} \propto 1/t$.

1.3 Statistical properties of the linear density field

The initial (linear) cosmological density field was most likely produced by some stochastic process, therefore, it is essential to use statistical description to depict its propoerties. Indeed the leading model for the production of the Universes initial density fluctuations, the theory of cosmological inflation, predicts that the density field fluctuations (deviations from homogeneity and isotropy) could be (mostly) described as a Gaussian Random Field (GRF) that is characterized by its two point correlation function or its Fourier conjugate, the power spectrum. Notice that such Gaussian random field must have a mean of zero and should be small on large scales as to not disrupt the assumptions of the Big Bang model. Another property of this field is that it should be statistically homogeneous and isotropic, which means that its statistical properties are independent of position and orientation. Given all of these properties one can describe the GRF either in configuration (real) or Fourier spaces.

The spatial Fourier transform of $\delta(\vec{x})$ is

$$\tilde{\delta}(\vec{k}) = \int \delta(\vec{x}) e^{-i\vec{k}\cdot\vec{x}} \mathrm{d}^3\mathrm{x} \tag{1.14}$$

The correlation properties of the density Fourier modes are,

$$\langle \tilde{\delta}(\vec{k})\tilde{\delta}^{\dagger}(\vec{k}')\rangle = \frac{1}{(2\pi)^3} P(k)\delta^D(\vec{k} - \vec{k}^{\dagger}), \tag{1.15}$$

where δ^D is the Dirac delta function which reflects the symmetry of the statistical properties under translation, whereas isotropy is reflected by $P(\vec{k}) = P(k)$. Notice that there is no mode coupling is Fourier space and the power spectrum is only dependent on k. The real space analogy is the so called two point correlation function of $\delta(\vec{x})$:

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2)\rangle = \xi(|\vec{x}_2 - \vec{x}_1|) = \xi(r). \tag{1.16}$$

Here the statistical homogeneity and isotropy are reflected in the fact that the correlation is a function of the amplitude of difference between the position vectors (distance). Notice that due the well know Wiener-Khinchin theorem $P(k)$ and $\xi(r)$ are Fourier pair,

$$\xi(r) = \frac{1}{(2\pi)^3}\int P(k)\exp(-i\vec{k}\cdot\vec{r})dk^3 = \frac{1}{2\pi^2}\int P(k)\frac{\sin kr}{kr}k^2 dk. \tag{1.17}$$

Based on similarity arguments Harrison (1970), Zel'dovich (1970) and Peebles & Yu (1970) have argued that the original fluctuations should have a scale independent power spectrum, namely a power spectrum of the form

$$P(k) = Ak^n T^2(k; \text{cosmological parameters}) \tag{1.18}$$

Where A is a normalization factor, n is the power law index which has a value around 1 (the so called Harrison-Zel'dovich power law index) and $T(k)$ is a transfer function that reflects the fact that during the radiation dominated era in the Universe the fluctuations on small scales have been modified. The transfer function $T(k)$ behaves as a constant at very large scale (small k's) and as k^{-4} at small scales (large k's). A number of analytical fits have been proposed in the literature for this function, e.g., Bardeen et al. (1986), Sugiyama (1995), however most accurate calculations are nowadays made using numerical codes like CMBFast. In order to estimate, for example, the expected rms fluctuations of the mass on a certain scale one must use a filter function with which to smooth the power spectrum. The mass variance for a given filter function is given

$$\sigma^2(M = \frac{4\pi}{3}\bar{\rho}R^3) = \frac{1}{2\pi^2}\int P(k)W^2(kR)k^2 dk, \tag{1.19}$$

where W is the window function which could be identified as the Fourier transform of the filter function and R is the scale associate with the filter function, F. The most used filter functions are the Gaussian and top-hat filter functions respectively defined as,

$$\begin{aligned} F_G(r; R_G) &= \frac{1}{(2\pi R_G^2)^{3/2}} \exp\left(-\frac{r^2}{2R_G^2}\right), \\ F_{TH}(r; R_{TH}) &= \frac{1}{(\frac{4\pi}{3}R_{TH}^3)^{3/2}} \Theta\left(1 - \frac{r}{R_G}\right) \end{aligned} \tag{1.20}$$

with Θ is the Heaviside step function. The Fourier space window functions for these two filtered are,

$$\begin{aligned} W_G &= \exp\left(-\frac{1}{2}(kR_G)^2\right), \\ W_{TH} &= \frac{3(\sin kR_{TH} - kR_{TH}\cos kR_{TH})}{(kR_{TH})^3}, \end{aligned} \tag{1.21}$$

respectively. In fact, normally the power spectrum is normalized in term of the so called σ_8 which is given in terms of equation 1.19 for a top hat filter with $R_{TH} = 8\,Mpc/h$ with h being the Hubble constant in units of 100 km/s/Mpc. The top hat filter will be important for us also in developing the Press-Schechter formalism.

1.4 Simple Non-linear model: Spherical collapse model

We now move to consider the simplest possible model describing the formation of an object, the so call, spherical-collapse model. Assume a spherical region with a uniform overdensity $\bar{\delta}$ and a physical radius R in an otherwise uniform Universe. Birkhoff's Theorem (from GR) states that the contribution of the exterior material to spherically symmetric solution must be given by the Schwarzschild metric, in other words within the sphere the only thing that matters is the material inside the sphere. Hence one can write

$$\frac{d^2R}{dt^2} = -\frac{GM}{R^2} = -\frac{4\pi G}{3}\bar{\rho}(1+\bar{\delta})R \tag{1.22}$$

This is exactly the same equation that governs the evolution of the scale factor in a Universe with a different density but with the same initial time and initial expansion rate. The first intergal of this equation is

$$\frac{1}{2}\left(\frac{dR}{dt}\right)^2 - \frac{GM}{R} = E. \tag{1.23}$$

For $E < 0$ the solution for this equation has the parametric form

$$R/R_m = \frac{1}{2}(1-\cos\eta);\;\; t/t_m = (\eta - \sin\eta)/\pi, \tag{1.24}$$

where R_m is the maximum radius of the sphere and t_m is the time it reaches it. For a small η in EdS Universe one can show that the mean overdensity as a function of time is

$$\bar{\delta} = \frac{3}{20}\left(6\pi t/t_m\right)^{2/3} \propto a_{EdS}. \tag{1.25}$$

The collapse of the sphere to $R = 0$ occurs at $t = 2t_m$, and at this time the extrapolated *linear* overdensity is

$$\delta_{collapse} = \bar{\delta}(2t_m) = \frac{3}{20}(12\pi)^{2/3} = 1.686. \tag{1.26}$$

This simple model for collapse of a uniform overdense region is clearly unrealistic, but notice that the assumption of uniformity has not been used directly in the above analysis. As long as the different mass shells do not cross each other, we can parametrize them in terms of the (constant) mass they enclose and write $E(M)$, $t_m(M)$ and $R_m(M)$ in al the above equations, which then describe the evolution of any spherical perturbation in which $\bar{\delta}$ is a decreasing function of M. This model will be used in the Press-Schechter formalism we'll develop next.

1.5 Press-Schechter formalism

Press and Schechter (1974) have proposed, based on heuristic arguments, a simple but plausible analytic form for the mass distribution of nonlinear objects present at any given redshift. Subsequent work by a number of groups in the 90s have improved the model and put on a much more firmer statistical grounds. Although the physical justification of it remains somewhat weak, some of its main predictions agree remarkably well with simulations, especially insofar as dark mass assembly is concerned. This is still the only available basis for a full treatment of galaxy formation in a hierarchically clustering Universe.

Here we will only derive the simple Press-Schechter model and not its more modern excursion set based derivation. Let us rewrite equation 1.19 for a top-hat window function of radius R and redshift

$$\sigma_z^2(R) = \frac{1}{2\pi^2}\int P(k;z)W_{TH}^2(kR)k^2dk = D^2(z)\sigma_0^2(R), \tag{1.27}$$

where $P(k;z) = D^2(z)P(k;z=0)$ and D is the linear growing mode growth function. Because the overdensity δ is a random Gaussian field we can easily calculate the field fraction that lies above a given value δ_c. This is obviously a question of scale as one can imagine a case in which the same density field smoothed over a certain scale have a different fraction above δ_c when smoothed over a different scale. Therefore such a fraction could be defined as,

$$F(R,z) = \int_{\delta_c}^{\infty} d\delta \frac{1}{\sqrt{2\pi}D(z)\sigma_0(R)}\exp\left[-\frac{\delta^2}{2D^2(z)\sigma_0^2(R)}\right]. \tag{1.28}$$

Press and Schechter (1974) suggested the assumption that this fraction id identified with the fraction of mass within a nonlinear lump with mass that exceeds $M = 4\pi\bar{\rho}a^3R^3/3$. An obvious value to take for δ_c is 1.686 the linear overdensity value at the collapse of a spherical perturbation.

Press and Schechter realized that there is a problem with this formula that arises in the case of $M \to 0$ where $\sigma_0 \to \infty$ and $F \to \frac{1}{2}$. Hence this formula suggests that only half the Universe can belong to lump of any mass. Press and Schechter solved this problem, arbitrarily, by multiplying the mass function by a factor of 2. The mass distribution of nonlinear lumps is then

$$n(M,z)dM = -2\frac{\bar{\rho}}{M}\frac{\partial F}{\partial R}\frac{dR}{dM}dM \tag{1.29}$$

$$= -\sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M}\frac{\delta_c}{D\sigma_0^2}\frac{d\sigma_0}{dM}\exp\left[\frac{-\delta_c^2}{2D^2\sigma_0^2}\right]dM. \tag{1.30}$$

One can also define a characteristic mass M_* which is given by solving the equation $\sigma_0(R_*) = \delta_c/D(z)$ where $M_* = 4\pi/3\bar{\rho}R_*^3$. Notice that redshift (time) enters equation 1.30 only through $D(z)$ and that the mass enters only through $\sigma_0(M)$ and its derivatives. We therefore could define the fraction of the Universe in objects with $\sigma_0(M)$ in the range $(\sigma_0, \sigma_0 + d\sigma_0)$ is

$$f(\sigma_0, D)d\sigma_0 = \sqrt{\frac{2}{\pi}}\frac{\delta_c}{D\sigma_0^2}\exp\left[-\frac{\delta_c^2}{2D^2\sigma_0^2}\right]d\sigma_0. \tag{1.31}$$

The relevance of this formalism to reionization is that it allows us to conduct simple calculations which allow us to determine the number density of objects for a given mass as a function of redshift. It also tells us what is the typical mass that is collapsing at a given redshift.

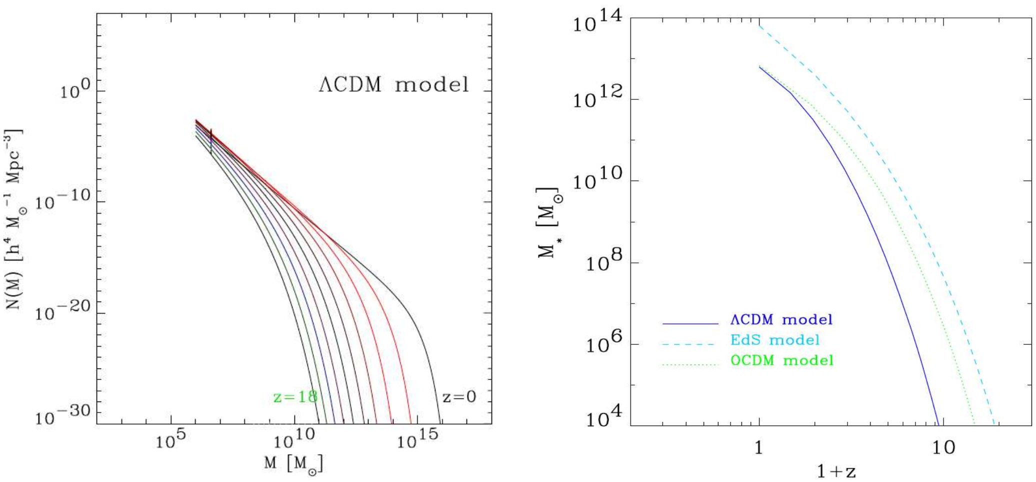

Figure 1.1: **Left panel: The number of objects per unit mass per unit comoving volume in a ΛCDM Universe as a function of redshift as calculated from the Press-Schechter formalism. Right panel: The Evolution of the the typical mass as a function of redshift for different cosmological models.**

1.6 Baryonic temp., fluctuations, etc.:

1.6.1 The evolution of baryonic temperature

Until recombination baryons are tightly coupled to the photons, however, after recombination and decoupling the baryons start falling into the potential wells that has grown due to dark matter fluctuations evolution and quickly thereafter they follow the distribution of dark matter. However, the temperature of the gas does not decouple as quickly from the CMB temperature. The coupling is caused by Compton scattering of CMB photon off residual electron leftover from recombination which in turn collisionally heat the rest of the gas to T_{CMB}. This coupling remains until redshift of about 200 where the universe expands enough and cools enough to render Compton heating insufficient to keep the gas temperature equal to that of the CMB.

After $z \approx 200$ the gas cools adiabatically and its pressure and density will have the following relationship,

$$P\rho_{gas}^{-\gamma} = \text{const.}, \tag{1.32}$$

where P is the pressure and γ is the adiabatic index which, for monoatomic gas, has the value of 5/3. Now if one substitutes this relation in the ideal gas equation, $P \propto \rho_{gas} T_{gas}$, one gets $T_{gas} \propto \rho_{gas}^{\gamma-1} = \rho_{gas}^{2/3}$. Remembering that the gas density evolves with the expansion of the Universe as $\rho_{gas} \propto (1+z)^3$ then one obtains that the gas temperature due to adiabatic expansion evolves as $T_{gas} \propto (1+z)^2$. This is of course in contrast to the CMB temperature which evolves linearly with (1+z). The adiabatic conditions are obviously not valid at $z > 200$ nor at low redshifts when substantial amount of entropy is injected into the IGM due to the formation of first objects. Figure 1.2 shows the expected global evolution of the various temperatures as a function of redshift. The blue solid line represents T_{CMB}, which drops as $1+z$. The green line shows the gas temperature as a function of redshift, whereas the red lines show the spin temperature evolution which we will discuss later in the course.

1.6.2 Cosmological Jean Mass

The Jeans length is the scale at which pressure support makes the gas stable against the growth of linear uctuations due to self-gravitation ; the Jeans mass is the amount of mass contained within a sphere of diameter the Jeans length. The Jeans length λ_J and mass M_J are defined by the well-known formulae,

$$\lambda_J = \left(\frac{\pi c_s^2}{G\rho}\right)^{\frac{1}{2}}, \tag{1.33}$$

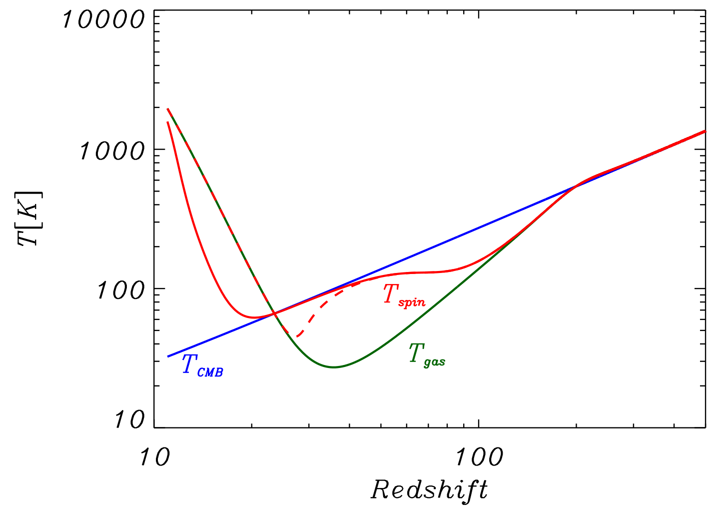

Figure 1.2: The global evolution of the CMB (blue line), gas (green line) and spin (red solid line and red dashed line) temperatures as a function of redshift. The CMB temperature evolves steadily as 1+z whereas the gas and spin temperatures evolve in a more complicated manner (see text for detail).

$$M_J = \frac{4\pi}{3}\rho\left(\frac{\lambda_J}{2}\right)^3 = \frac{\pi\rho}{6}\left(\frac{\pi c_s^2}{G\rho}\right)^{\frac{3}{2}} \tag{1.34}$$

(Binney & Tremaine 1987), where ρ is the mass density and c_s is the sound speed. Obviously, in cosmological perturbation gravity is dominated by the dark matter components whereas the pressure is dominated by the baryonic component.

Homework: Calculate the Jeans mass before and after recombination as a function of redshift.

1.6.3 Cooling processes

Gas of a given temperature emit photons which carry away the thermal energy of the gas: this process is called radiative cooling. In galaxies/groups/clusters, this happens mainly through collisions between atoms, during which these atoms get excited/ionized and hence radiate away photons when they return to an unexcited state and/or when the ions capture back their electrons. These processes are efficient down to temperature of around $T \approx 10^4$K, and it is worth noting that at temperatures above 10^6 K the main source of cooling is radiation from Bremsstrahlung. One can estimate the time it will take for gas at a given temperature to cool down completely, i.e. radiate away all of its thermal energy:

$$t_{cool} = \frac{E}{\dot{E}} = \frac{3}{2}\frac{NkT}{n_e n_i \Lambda(Z,T)}, \tag{1.35}$$

where E is the total thermal energy density of the gas, N the number density of particles in the gas, n_e the number density of electrons, n_i the number density of ions and $\Lambda(Z,T)$ is the cooling function of the gas which depends on the temperature and the metallicity, of the gas. The cooling function is the net sum of all possible cooling channels for the gas (all the possible transitions of atoms and ions) in units of $erg\ cm^3 s^{-1}$ and, in general, needs to be numerically calculated using quantum mechanics. For primordial gas which has "only" hydrogen and helium (zero metallicity gas) one can simplify the previous equation.

The zero metallicity atomic cooling function shown in figure 1.3 clearly shows the problem with primordial atomic gas, there is no efficient cooling at temperatures below 10^4K. This is because the first excitation energy level is the Lyman-α transition which has an energy of 10.2 eV. For collisional energy of gas to reach these energy levels the gas temperature should be of the order of a few times 10^4K. This is a very high temperature which could be reached through virialization only if the initial halo mass is of the order of $\approx 10^9 M.$ which is much beyond the typical mass of collapsing halos (M_* in Press-Schechter formalism) at redshifts > 10. We will return to this point in more detail later in the course.

Furthermore, the issue of cooling influences the fragmentation of the collapsing proto-galaxies into stars, namely, the mass function of stars. More efficient cooling leads to the formation of more massive stars whereas less efficient cooling leads to the formation of very massive ones. Hence it is generally believed that the collapse of primordial clouds will inevitably lead to formation of very massive stars.

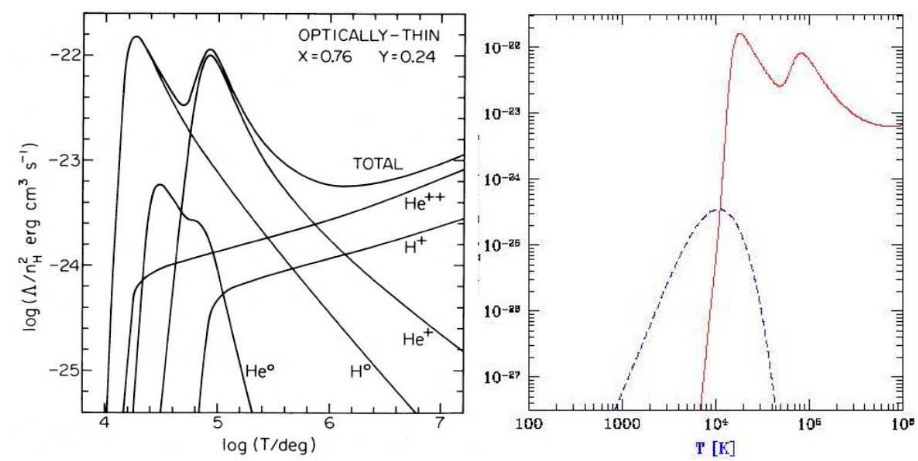

Figure 1.3: Cooling function of an optically thin plasma with a primordial composition with ionization equilibrium (Fall & Rees 1985). The left panel shows the atomic zero metallicity atomic cooling function and the processes that contribute to it. The right panel shows the atomic cooling function together with the H_2 cooling which is more efficient at lower temperature. The molecular cooling is driven by the rotational and vibrational excitation levels that H_2 has. The uncertainty in the molecular case however is its abundance which is not clear at all. Notice the different axis scales in both panels.