Statistical Signal Processing
Assignment 3

Submission deadline: 4th, December, 2015

Note: **C**: Computer Assignment, **W**: Workout Problem. For computer assignments, please submit both the code (properly indented) and the results in form of plots (properly labelled).

1. (**W**) The data $\{x[1], x[2], ..., x[N]\}$ are observed, where $x[n]$ are independent and identically distributed (IID) as $\mathcal{N}(0, \sigma^2)$. We wish to estimate the variance σ^2 as

$$\hat{\sigma^2} = \frac{1}{N}\sum_{n=1}^{N} x^2[n]$$

   Is this an unbiased estimator? Find the variance of $\hat{\sigma^2}$ and examine what happens as $N \to \infty$.

2. (**W**) Consider the data points $\{x_1, x_2, ..., x_N\}$, where $x_i = A + w_i$ and w_i are IID random variables drawn from $\mathcal{N}(0, \sigma^2)$. If we choose to estimate $\theta = A^2$, the estimater $\hat{\theta}$ can be written as

$$\hat{\theta} = \left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)^2$$

   Can we say that the estimator is unbiased? What happens as $N \to \infty$

3. (**W**) Consider a parameter model given by

$$x_i = Ar^i + w_i, \quad i = [1, 2, ...N]$$

   where A is the parameter to be estimated, $r > 0$ is already known and w_i are the IID gaussian random variables drawn from $\mathcal{N}(0, \sigma^2)$. Find the CRLB for A. Also, show that an efficient estimator exists and find its variance. What happens to the variance as $N \to \infty$ for various values of r.

4. (**C+W**) The data $x[n] = A + w[n]$ $(n = 1, 2, ..., N)$ are observed, where A is the parameter to be estimated and $w[n]$ is the gaussian noise with distribution $\mathcal{N}(0, \sigma^2)$.

   (a) Analytically compute CRLB for A.

(b) Consider the estimator given by

$$\hat{A} = \frac{1}{N}\sum_{n=1}^{N} x[n]$$

Generate mock data with $A = 15, \sigma = 2$, and $N = 100$. Now use the estimator given above to compute $\hat{A}$. Repeat this 10000 times with different realizations of noise to get 10000 values of $\hat{A}$. Plot the PDF of $\hat{A}$. Compute the variance of $\hat{A}$ values and compare it with the CRLB computed in (a).

5. (**C+W**) Consider the two parameter model for a straight line fit given by

$$x_i = A + Bi + w_i, \quad i = [1, 2, ...N]$$

where A, and B are parameters to be estimated, and w_i are IID random variables drawn from $\mathcal{N}(0, \sigma^2)$.

(a) Analytically compute the Fisher information matrix F_{ij} for A and B.

(b) Generate mock data with $A = 3,\ B = 2,\ \sigma = 1$, and $N = 10$. Use the built-in routines for curve-fitting as estimator for A and B. Repeat this 10000 times with different realizations of noise to get 10000 values of $\hat{A}$ and $\hat{B}$. Compute and plot(colormap preferably) the joint PDF of $\hat{A}$ and $\hat{B}$.