# Statistical Signal Processing
# Assignment 2

Submission deadline: 25th, November, 2015

Note: For computer assignments, please submit both the code (properly indented) and the results in form of plots (properly labelled).

1. (Workout Problem) Let $\mathbf{X}$ and $\mathbf{Y}$ be random variables, and

$$p_{x|y}(\mathbf{x}|\mathbf{y}) = \begin{cases} \left(\dfrac{1}{e-1}\right) e^{-(x-2y)}, & \text{if } y \leq x < \infty. \\ 0, & \text{otherwise.} \end{cases}$$

   and

$$p_y(\mathbf{y}) = \begin{cases} 1, & \text{if } 0 \leq y < 1. \\ 0, & \text{otherwise.} \end{cases}$$

   Where $p_{x|y}(\mathbf{x}|\mathbf{y})$ is the conditional PDF of $\mathbf{X}$ given $\mathbf{Y}$ and $p_y(\mathbf{y})$ is the marginal density of $\mathbf{Y}$.

   (a) Find the joint PDF $p_{x,y}(\mathbf{x}, \mathbf{y})$ and specify the region where joint probability of $\mathbf{X}$ and $\mathbf{Y}$ is non-zero. Check the vailidity of the limits by using the normalization equation and integrating over the region. Also, sketch this region on x-y plane where joint probability of $\mathbf{X}$ and $\mathbf{Y}$ is non-zero.

   (b) Find the marginal density $p_x(\mathbf{x})$ and specify the regions where it is non-zero.

2. (Workout Problem) The skewness of a random variable $\mathbf{X}$ is defined as

$$S_{\mathbf{x}} = E\left\{ \left(\frac{x-\mu}{\sigma}\right)^3 \right\}$$

   Skewness is a measure of the 'asymmetry' in the shape of a distribution. Solve analytically for the skewness of the distribution given by

$$p_x(\mathbf{x}) = \begin{cases} \sqrt{\dfrac{2}{\pi}} x^2 e^{-\frac{x^2}{2}}, & \text{if } 0 \leq x < \infty. \\ 0, & x < 0. \end{cases}$$

   **Miscellaneous**: You may also try to solve for the 'Kurtosis' ($\mathcal{K}_{\mathbf{x}}$) of the distribution. Kurtosis is a measure of how 'peaked' a distribution is. Kurtosis is defined as

$$\mathcal{K}_{\mathbf{x}} = E\left\{ \left(\frac{x-\mu}{\sigma}\right)^4 \right\}$$

3. (Computer Assignment) If $\mathbf{X}$ and $\mathbf{Y}$ are Gaussian random variables with zero mean and unit variance, then $\mathbf{Z} = \sqrt{\mathbf{X}^2 + \mathbf{Y}^2}$ has a standard Rayleigh distribution. For zero mean and unit variance, PDF of standard rayleigh distribution is given by

$$p(x) = xe^{-\frac{x^2}{2}}$$

   (a) Use the above mentioned transformation to draw $N$ samples from a standard rayleigh distribution and plot the histogram of the PDF.

   (b) An estimator for the skewness may be written as

$$\hat{S}_x = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^3}{\left(\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2\right)^{3/2}}$$

   where $\bar{x}$ is the sample mean. Numerically compute and plot the sample mean, sample variance, and sample skewness for $N = 10^n$ where $n$ is an integer and $n \epsilon [1, 6]$ as a function of $n$.

4. (Computer Assignment + Workout) Find the fourier transform of the function $f(x)$, given by

$$f(x) = \begin{cases} 1, & \text{if } -1 \leq x \leq 1. \\ 0, & \text{otherwise.} \end{cases}$$

   Also, compute the Fast Fourier Transform (FFT) of the function and compare it with the analytical result by plotting them on same graph.

5. (Workout Problem) Consider two functions $f(x)$ and $g(x)$ given by

$$f(x) = \delta(x - a), \; a \neq 0 \text{ and } g(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

   Find the convolution $f \otimes g$ of the two functions using the convolution theorem.

**Important formulae:**

$$\int_0^\infty x^2 e^{-x^2} = \frac{\sqrt{\pi}}{4}$$

$$\int_0^\infty x^4 e^{-x^2} = \frac{3\sqrt{\pi}}{8}$$

Convolution theorem: $\mathcal{F}(f(x) \otimes g(x)) = \mathcal{F}(f(x))\mathcal{F}(g(x))$