Excitation of the Hydrogen 21-CM Line*

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Summary—**The importance of spin temperature for 21-cm line studies is reviewed, and four mechanisms which affect it are studied. Two of the mechanisms, collisions with free electrons and interactions with light, are studied here in detail for the first time. The results are summarized in Table II of Section VI, in the form of certain *efficiencies* which can be used with (15) to calculate the spin temperature. In Section VI the results are applied to a variety of astronomical situations, and it is shown that in the usual situation collisions with H atoms are very effective in establishing the spin temperature equal to the kinetic temperature. Under conditions of low-density and/or high-radiation intensity, however, important deviations from the usual are noted. The significance of such deviations for absorption studies of radio sources and the galactic halo is discussed. In Section VII the deuterium line at 91.6 cm is considered in like fashion. It is shown that for deuterium also, the spin temperature probably is close to the kinetic temperature.**

I. Introduction

Great advances have been made in the understanding of the interstellar medium by studying the distribution of 21-cm emission. (See, for example, van de Hulst, Muller, and Oort,[1] and Muller and Westerhout.[2]) Absorption by interstellar clouds of radiation emitted by radio sources has also been observed, particularly by Williams and Davies,[3] Hagen, Lilley, and McClain,[4] and Muller.[5] The phenomenon of absorption, when connected with radio sources, has permitted studies of hydrogen distribution with the high-effective angular resolution resulting from the small diameters of the sources observed. The same phenomenon, when connected with emission features, has complicated interpretation of emission profiles as saturation is approached.

Several authors, including Hagen, Lilley, and McClain,[4] have discussed the theory of 21-cm absorption. For our purpose, the equation for a simple case is sufficient. Let a cloud (large compared to a beamwidth) of "spin temperature" T_S be placed in front of a radio source of antenna temperature T_B in the continuum. If the detector operates as a comparison system, the observed antenna temperature is

$$\Delta T_A = (T_S - T_B)(1 - e^{-\tau_\nu}), \tag{1}$$

where τ_ν is the frequency-dependent opacity of the cloud. ΔT_A is thus the sum of the diminished source radiation, $T_B e^{-\tau_\nu}$, and the self-absorbed cloud radiation, $T_S(1-e^{-\tau_\nu})$, less the undiminished source radiation, T_B, detected outside the line and hence subtracted from the total.

If the equation is applied to a radio source and to a point close enough to the source to assure that the beam is in the same cloud as when on the source, we see that both $(T_S - T_B)(1-e^{-\tau_\nu})$ and $T_S(1-e^{-\tau_\nu})$ may in principle be determined. Direct radiometry outside the line determines T_B, and hence we may obtain T_S and τ_ν separately. If the equation is applied to a point with little continuum emission, the extra data afforded by absorption of a source of known brightness are missing, and it is impossible to obtain T_S and τ_ν separately; only the combination $T_S(1-e^{-\tau_\nu})$ is known. It has been the practice to assume (*e.g.*, Muller, *et al.*[1]) that T_S is everywhere equal to its value determined toward the galactic center, *viz.*, 125°K, and to interpret all emission profiles on that basis.

If absorption observations are made and T_S determined, the relation of T_S to properties of the cloud is of interest. If emission observations are made, the reliability of assumptions concerning the value of T_S is important. In either case, it is worthwhile to relate T_S to physical properties of the cloud which may be predicted theoretically or observed in other ways. Previous considerations of this problem gave a simple answer; $T_S = T_K$, the gas kinetic temperature of the cloud.[6-8] In the present paper, this conclusion is substantiated for a wide variety of conditions, while in certain situations divergences are found.

II. Spin Temperature

The 21-cm line is a transition between the two hyperfine states of the $1^2S_{1/2}$ ground level of hydrogen. These states differ only in the value of the total spin angular momentum F, which is the sum of electron and proton spins:

$$F = S + I. \tag{2}$$

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[1] H. C. van de Hulst, C. A. Muller, and J. H. Oort, "The spiral structure of the outer part of the galactic system derived from hydrogen emission at 21-cm wavelength," *Bull. Astron. Inst. Neth.*, vol. 12, pp. 117–149; May, 1954.

[2] C. A. Muller, G. Westerhout, *et al.*, "A catalogue of 21-cm line profiles," *Bull. Astron. Inst. Neth.*, vol. 13, pp. 151–195; March, 1957, and the four papers following in the same number.

[3] D. R. W. Williams and R. D. Davies, "A method for the measurement of the distance of radio stars," *Nature*, vol. 173, pp. 1182–1183; June, 1954; also, *Phil. Mag.*, Ser. 8, vol. 1, p. 622; July, 1956.

[4] J. P. Hagen, A. E. Lilley, and E. F. McClain, "Absorption of 21-cm radiation by interstellar hydrogen," *Astrophys. J.*, vol. 122, pp. 361–375; November, 1955.

[5] C. A. Muller, "21-cm absorption effects in the spectra of two strong radio sources," *Astrophys. J.*, vol. 125, pp. 830–834; May, 1957.

[6] H. I. Ewen and E. M. Purcell, "Radiation from galactic hydrogen at 1420 mgc," *Nature*, vol. 168, p. 356; September, 1951.

[7] S. A. Wouthuysen, "On the excitation mechanism of the 21-cm interstellar hydrogen emission line," *Astrophys. J.*, vol. 57, pp. 31–32; April, 1952.

[8] E. M. Purcell and G. B. Field, "Influence of collisions upon population of hyperfine states in hydrogen," *Astrophys. J.*, vol. 124, pp. 542–549; November, 1956.

Since S and I are each $\frac{1}{2}$ (in units $\hbar$), F takes the value 0 or 1, splitting the ground level into a hyperfine doublet. In a magnetic field, the $F=1$ state is further split into three components and is therefore called a triplet; the $F=0$ state is a singlet. The triplet state lies above the singlet an amount $h\nu_{10}$ with $\nu_{10}=1420.405$ mc, so the $1\rightarrow 0$ transition results in the emission of a quantum of 21-cm radiation. The energy difference between the singlet and triplet is due to the different orientations of electron and proton magnetic moments (parallel in the singlet, antiparallel in the triplet) which results in a magnetic interaction analogous to that between two bar magnets. The $1\rightarrow 0$ transition then is equivalent to a "spin flip," the electron spin going from parallelism with the proton spin to antiparallelism.

The probability for a spontaneous $1\rightarrow 0$ transition is given by the Einstein A quoted by Wild:[9]

$$A_{10} = \frac{64\pi^4\beta^2}{3h\lambda^3} = 2.85\ 10^{-15}\ \text{sec}^{-1}, \tag{3}$$

where β is the Bohr magneton. Such an extremely small value for A corresponds to a lifetime of the triplet state of 1.1×10^7 years against spontaneous radiation and is responsible for the importance of collisions as against radiation in establishing the population of the triplet. Twenty-one-cm radiation incident on the atom can cause $0\rightarrow 1$ transitions (absorptions) and $1\rightarrow 0$ transitions (induced emissions). These processes have probabilities given by

$$I_\nu B_{01} = \frac{g_1}{g_0}\ B_{10} I_\nu = 3A_{10}\,\frac{\lambda^2 I_\nu}{2h\nu_{10}}, \tag{4}$$

and

$$I_\nu B_{10} = A_{10}\frac{\lambda^2 I_\nu}{2h\nu_{10}},$$

respectively. In these equations $I_\nu=$ the specific intensity of 21-cm radiation and $g=2F+1$ the statistical weight of the state.

A beam of 21-cm radiation, therefore, causes absorption and induced emission on passing through a hydrogen cloud. Absorptions subtract photons from the beam while induced emissions add them; consequently the induced emissions act like negative absorptions. Suppose that we know the absorption coefficient, per cm, calculated on the assumption that all atoms are in the singlet state. If we call this quantity K_{01}, the effective absorption coefficient is

$$K = K_{01}(1 - g_0 n_1/g_1 n_0), \tag{5}$$

when corrected for stimulated emission, as can be seen from the relative probabilities expressed in (4). If the cloud were in thermodynamic equilibrium at some temperature T, we could say that

$$g_0 n_1/g_1 n_0 = \exp\left(-h\nu_{10}/kT\right), \tag{6}$$

according to Boltzmann's law. Even if it is *not* in thermodynamic equilibrium, it is convenient to *define* a "spin temperature," T_S, by such a relation, so that

$$g_0 n_1/g_1 n_0 \equiv \exp\left(-h\nu_{10}/kT_S\right). \tag{7}$$

At this point a radical difference emerges between ordinary and radio astronomy. Interstellar temperatures which are directly observable are in the range 100°K to 10,000°K, with few exceptions. But $h\nu/k$ for most optical lines is 20,000°K or more. Hence in the optical case the exponential is small, most atoms are in the ground state, and a determination of K then refers to K_{01} and hence to practically all of the atoms. For the 21-cm line,

$$h\nu_{10}/k \equiv T_* = 0.0681°\text{K}, \tag{8}$$

and the exponential is close to unity. Therefore, K is determined by the small difference in populations of the singlet and triplet, this difference depending on the usually unknown spin temperature, T_S. It follows that determinations of opacity lead directly to densities in the optical realm, but not in the radio. On the other hand, because an assignable fraction of atoms is in the upper state in 21-cm work, emission is easily interpreted in terms of numbers of atoms.

It may be mentioned that since T_S is usually much greater than T_*,

$$K = 32{,}000\ n_0 f(\nu)/T_S \tag{9}$$

follows from (7), when we use $K_{01}=3n_0A_{10}\lambda^2 f(\nu)/8\pi$, where $f(\nu)$ is the line shape function. Eq. (9) reduces to one of Wild's[9] if we put the number of singlet atoms, n_0, equal to $g_0/(g_1+g_0)=\frac{1}{4}$ of the total. When Kl is substituted for τ_ν in (1) and assumed small; and if $T_B=0$ so that only emission is effective, we see that T_S cancels; the observed emission is independent of T_S as long as there is no saturation. Thus it is that T_S is of no consequence as long as we study optically thin clouds in emission. When we study absorption, however, it forms a vital link in the interpretation.

III. Statistical Equilibrium

The spin temperature, T_S, is merely a convenient shorthand for the relative population of the singlet and triplet. What then determines these populations physically? If the cloud were isolated from space by a wall, we could apply statistical mechanics to calculate the distribution of the available energy over the various energy levels. There would then be a unique temperature for all kinds of energy modes, including energy of translation, radiation, and discrete atomic levels including the hyperfine levels. Thus the spin temperature would be equal to this unique temperature, whatever it might be.

But the cloud is not isolated from space; it continu-

[9] J. P. Wild, "The radio-frequency line spectrum of atomic hydrogen and its applications in astronomy," *Astrophys. J.*, vol. 115, pp. 206–221; March, 1952.

ally receives radiation from the stars and emits it again into space. Our basic principle is to balance input and output. Given the rates of the various processes, it is then possible to compute the steady state populations. The final temperature which characterizes any mode of energy depends on the way that mode is coupled to other modes. There are temperatures appropriate to the translational energies of particles, to total radiation density, to the grains, to atomic levels, to the color of the light from stars. These can vary over wide limits depending on the degree to which the mode is coupled to the hot stars on the one hand and to free space on the other. In our case there are three temperatures of special interest to us; the brightness temperature of 21-cm radiation, averaged over all directions, T_R; the kinetic temperature of the atoms and electrons, T_K; and the color temperature of certain kinds of light, T_L. The brightness temperature of radio radiation at 21 cm is defined in the usual way:

$$T_R = \frac{\lambda^2}{2k} \bar{I}_\nu \tag{10}$$

with $\bar{I}_\nu$ the brightness at 21 cm averaged over the whole sky and over all Doppler shifts occurring in the group of H atoms. T_L will be defined later.

Evidently, there are fundamentally two kinds of excitation mechanisms, collisional and radiative. Each of these can excite the transition directly by exchange of a quantum of energy equal to $h\nu_{10}$. But conceivably, higher levels of the hydrogen atom can be excited from one hyperfine state with de-excitation occurring to the alternate hyperfine state. Since the $n=2$ level is the lowest level above the ground state, a transition $n=1$ to $n=2$, corresponding to the Lyman $-\alpha$ line in the spectrum of hydrogen (1216Å, and 10.2 ev.), is required to accomplish this. Certainly the scattering of Lyman $-\alpha$ should be taken into account, and possibly higher Lyman lines. But we need not consider high energy *collisions* because the fraction of particles with energies enough to excite $L\alpha$ collisionally is proportional to $10^{-(51{,}000/T_K)}$, while the fraction which can ionize the atom is proportional to $10^{-(68{,}000/T_K)}$. Hence if the value of T_K is such as to make the first value significant, the second factor will be only slightly less, and ionization will occur. Thus, we find that regions hot enough for high-energy collisions will be ionized and thus unobservable. We are left with the low energy collisions and both low- and high-energy radiative transitions as important mechanisms. We write $P^{C,R,L}$ for the rate of transitions (per second) caused by collisions, radio frequency, and light, respectively. Using (4), (8), and (10), we find

$$\left.\begin{aligned} P_{01}{}^R &= 3\frac{T_R}{T_*}A_{10}, \\ P_{10}{}^R &= \left(1+\frac{T_R}{T_*}\right)A_{10}, \end{aligned}\right\} \frac{P_{01}{}^R}{P_{10}{}^R} \simeq 3\left(1-\frac{T_*}{T_R}\right), \tag{11}$$

for the transitions caused by 21-cm radiation of average brightness, $I_\nu = 2kT_R/\lambda^2$.

Collisional transitions have probabilities which will be calculated in Section IV, but the *ratio* of $P_{01}{}^C$ to $P_{10}{}^C$ is easily derived. For any type of colliding particle, the rate is proportional to the number of such particles, times some function of the kinetic temperature. Hence, the ratio of the upward to downward rates is a function of the kinetic temperature alone and is, therefore, the same as in a closed system at the same temperature. The latter will be just that for thermodynamic equilibrium, and (6) holds in addition to the condition of steady state so we find that

$$n_0 P_{01}{}^C = n_1 P_{10}{}^C = \frac{g_1}{g_0} n_0 P_{10}{}^C \exp(-T_*/T_K),$$

so

$$\frac{P_{01}{}^C}{P_{10}{}^C} = \frac{g_1}{g_0}\exp(-T_*/T_K) \simeq 3\left(1-\frac{T_*}{T_K}\right). \tag{12}$$

(We shall always assume all T's$\gg T_*$.) Since the argument holds for each kind of collision, it holds for any combination of collisions.

We note that (12) for the ratio holds also for 21-cm radiation, with T_K replaced by T_R. Inasmuch as it follows from a thermodynamic argument involving the energy source, we may assume that the argument holds also for the case of excitation by Lyman radiation. In this case, the value of the "temperature of the light" follows from a detailed argument which will be considered in Section V. For the present let us *define* T_L, the light temperature, by

$$\frac{P_{01}{}^L}{P_{10}{}^L} = 3\left(1-\frac{T_*}{T_L}\right). \tag{13}$$

The hypothesis of equilibrium states that the populations of singlet and triplet do not change with time. Therefore, the number leaving the triplet equals the number entering the triplet, per second. This leads to

$$\begin{aligned} \frac{n_1}{n_0} &= \frac{g_1}{g_0}\exp(-T_*/T_S) \simeq 3\left(1-\frac{T_*}{T_s}\right) \\ &= 3\,\frac{\dfrac{T_R}{T_*}A_{10} + \left(1-\dfrac{T_*}{T_K}\right)P_{10}{}^C + \left(1-\dfrac{T_*}{T_L}\right)P_{10}{}^L}{A_{10}\left(1+\dfrac{T_R}{T_*}\right) + P_{10}{}^C + P_{10}{}^L} \end{aligned} \tag{14}$$

when (6), (11), (12), and (13) are applied. Upon rearranging, we find

$$T_S = \frac{T_R + y_C T_K + y_L T_L}{1 + y_C + y_L},$$

if

$$y_C = \frac{T_*}{T_K}\;\frac{P_{10}{}^C}{A_{10}}, \qquad y_L = \frac{T_*}{T_L}\;\frac{P_{10}{}^L}{A_{10}} \tag{15}$$

are normalized probabilities or "efficiencies." Eq. (15) shows that T_S is a weighted mean of the three temperatures T_R, T_K, and T_L; when any one of the efficiencies is very large, T_S takes on the corresponding temperature value. Our next step is to discuss the efficiencies and to show how they are related to certain properties of the interstellar medium.

IV. Collisions

We have calculated in the previous section the *ratio* of the de-excitation to the excitation probability, which holds for any type of collision. Let us now consider what the rate of de-excitation is. Since y_C is proportional to $P_{10}{}^C$, we must calculate the probability for a given triplet atom that a de-excitation takes place via a collision of that atom with another particle. Evidently, since the large mass of the proton makes it sluggish compared to the electron, the forces induced in the atom by a collision are much more likely to cause the electron spin to flip than the proton spin to do so. An electron spin flip can be caused magnetically by the action of magnetic forces on the magnetic moment of the electron, or electrostatically by forcing out the original electron and replacing it with one of opposite spin.

The latter process of substitution of an "up" for a "down" electron, called spin-exchange, can occur whenever the colliding particle contains electrons and, in particular, if the colliding particle is a hydrogen atom or a free electron. The total density of H atoms and electrons is always greater than that of any other species since the number of H atoms plus electrons is greater than the number of hydrogen nuclei (protons) by the number of free electrons originating in the ionization of other elements, while the proton density usually exceeds the helium density by a factor 10 and other atoms by a factor 1000. It is clear that H atoms and free electrons will always play a dominant role then; the latter assumes relative importance whenever the ionization of hydrogen is more than a few per cent. It may be added that the spin-exchange process for collisions with H atoms and free electrons is much more probable than the magnetic spin flip alluded to above, because the magnetic interaction induced by the atomic or free electron, as the case may be, is less than the electrostatic interaction by roughly v^2/c^2. This quantity is 5×10^{-5} for atomic electrons and only 4×10^{-6} for free electrons at 10^4°K.

It seems very probable, therefore, that by far the most frequent collisions will be with other H atoms and free electrons. The H atom case is treated in detail by Purcell and Field.[8] They showed that the atomic encounter can be treated semiclassically, and that the strong dependence of the interatomic forces on the separation makes the collision cross section relatively independent of energy; it was found to vary as $T_K{}^{-0.27}$ over the range 1° to 10^4°K. The collision effectiveness, y_H, was computed and presented in Table I of their paper. Since it is proportional to n_H, the density of hydrogen atoms, y_H/n_H was actually given. The resulting values of y_H/n_H are listed in Table II of the present paper.

The treatment of the free electron spin exchange is of necessity quantum mechanical, the angular momenta of the important collisions being less than $\hbar$ because of the small electron mass. Although this particular problem is not solved in the literature, the problem of scattering of a polarized beam of electrons by a ground state hydrogen atom has been treated by many authors, in various energy ranges and using various approximations. An excellent discussion of this work is given by Seaton.[10]

Our problem can be related to the scattering problem as follows. (See Appendix for a rigorous demonstration.) The scattering problem results in "scattering amplitudes," $T(\theta)$ and $S(\theta)$, which are functions of the polar angle of the scattered beam with the incident beam, and which refer to the cases in which the incident and atomic electrons are in the triplet state $[T(\theta)]$, or singlet state $[S(\theta)]$. The *differential scattering cross section* in direction θ is well known to be $\frac{3}{4}|T|^2+\frac{1}{4}|S|^2$ for an unpolarized beam. The analogous *differential cross section for electron exchange* is $\frac{1}{4}|T-S|^2$. Since, however, only half of the exchanges are with electrons having opposite spin, the *electron spin exchange cross section* is half of this quantity. Furthermore, half of the electron spin exchanges with a hyperfine triplet result only in a transition to another component of the triplet, so the *de-excitation cross section* is only half of the spin exchange cross section. Thus, the differential de-excitation cross section for which we seek is

$$\frac{d\sigma}{d\omega}=\frac{1}{16}\,|T-S|^2, \tag{16}$$

where $d\omega$ is an element of solid angle in direction θ. Now all of the calculations we shall use are based on S-wave scattering, in which T and S are independent of θ. (That this is a good approximation up to 10,000°K follows from the study of P waves by Chandrasekhar and Breen.[11]) Consequently we have the total de-excitation cross section for triplet atoms by free electrons:

$$\sigma=\frac{\pi}{4}\,|T-S|^2. \tag{17}$$

Naturally, the scattering amplitudes and σ depend on the energy of the incident electron. Here we may divide the energy range into two domains: one, right around zero energy, the other extending up to perhaps one volt. The higher energies were treated by two variational

[10] M. J. Seaton, "The application of variational methods to atomic scattering problems. V. The zero energy limit of the cross-section for elastic scattering of electrons by hydrogen atoms," *Proc. Roy. Soc. A*, vol. 241, pp. 522–530; September, 1957.

[11] S. Chandrasekhar and F. H. Breen, "The motion of an electron in the Hartree field of a hydrogen atom," *Astrophys. J.*, vol. 103, pp. 41–70; January, 1946.

principles by Massey and Moiseiwitsch.[12] The results using the two variational principles were in good agreement for the triplet amplitude, and we can use them directly. For the singlet amplitude the results disagreed by 10 per cent, but the authors preferred the Hulthen (H) method with a free parameter (b). Thus we may use the H-b results for the singlet. Seaton[10] feels that the H-b results are accurate to about 20 per cent up to energies at which P waves become important (presumably 10,000°K). On the other hand, the low-energy data for the singlet are considered dubious by Seaton, who introduces a different trial function and finds agreement between his results using the two variational methods for zero energy of the incident electron. However, he found a 6 per cent discrepancy in the zero energy singlet amplitude which he had calculated, and the H-b singlet amplitude of Massey and Moiseiwitsch extrapolated to zero energy. I find, however, that the two agree within a per cent or so.

Borowitz and Greenberg[13] also calculated scattering amplitudes near zero energy (up to 0.04 ev). Their results are presented in Table I in terms of the scattering amplitudes for singlet and triplet at zero energy in units $a_0 = 5.3 \times 10^{-9}$ cm. These amplitudes are called "scattering lengths." A second parameter which gives the dependence on energy near zero energy, called the "effective range," is also tabulated. For comparison, we also give the same quantities as calculated by Seaton[10] and deduced from Massey and Moiseiwitsch.[12]

We note that the zero energy triplet amplitudes agree well, but while Seaton agrees with Massey and Moiseiwitsch for the singlet, they both disagree with Borowitz and Greenberg. Furthermore, the form of the variation near zero energy is quite different as determined by the effective range in the two papers for both the singlet and triplet. Inasmuch as Borowitz and Greenberg do not attain high energies anyway and the Massey and Moiseiwitsch data agree with Seaton's at zero energy, it seems advisable to adopt Massey and Moiseiwitsch's results over the entire energy range.

Their data are given in terms of phase shifts, δ, which are related to the amplitudes by

$$T \text{ or } S = k^{-1} \sin \delta_{t,s} e^{j\delta_{t,s}}, \tag{18}$$

where k is the momentum of the incident electron divided by $\hbar$. When these definitions are inserted in (17) for the cross section, we find

$$\sigma = \frac{\pi}{4k^2} \sin^2 (\delta_t - \delta_s). \tag{19}$$

[12] H. S. W. Massey and B. L. Moiseiwitsch, "The application of variational methods to atomic scattering problems. I. The elastic scattering of electrons by hydrogen atoms," *Proc. Roy. Soc. A.*, vol. 205, pp. 483–496; March, 1951.

[13] S. Borowitz and H. Greenberg, "Variational calculation of the scattering of electrons by hydrogen atoms at near zero energy," *Bull. Amer. Phys. Soc.*, pt. II, vol. 2, p. 172; April, 1957.

TABLE I
SCATTERING PARAMETERS FOR ZERO ENERGY ELECTRONS

Author	Scattering Length		Effective Range	
	Singlet	Triplet	Singlet	Triplet
MM	7.00	2.34	2.46	1.40
S	7.01	2.33	—	—
BG	7.75	2.35	3.80	0.81

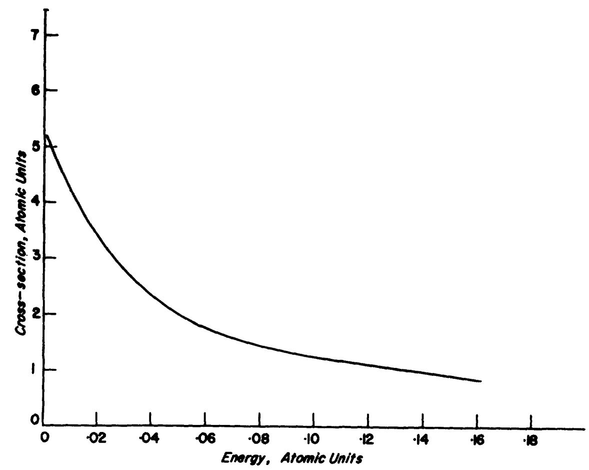

Fig. 1—Cross section for de-excitation of the hyperfine triplet by electrons. Energies are in Rydbergs (13.6 ev) and cross sections in units $\pi a_0^2 = 8.8 \times 10^{-17}$ cm^2.

The resulting cross section in units πa_0^2 using Massey and Moiseiwitsch phase shifts is plotted vs the electron energy in units of 13.6 ev in Fig. 1.

The cross section may be averaged over a Maxwellian distribution of electron speeds, using the formula due to Jeans:[14]

$$P_{10}^e = n_e \sqrt{\frac{8kT_K}{\pi\mu}}\, \bar{\sigma}(T_K),$$

$$\bar{\sigma}(T_K) = \int_0^\infty u e^{-u} \sigma(u) du, \tag{20}$$

$$u = m_e E / \mu k T_K.$$

E is the incident electron energy, and μ the reduced mass of the electron-atom system. Since μ is about equal to m_e, u is very close to E/kT_K, and we have for the value of y_e, defined by (15),

$$y_e = \frac{2h\nu_{10}}{kA_{10}} \sqrt{\frac{2k}{\pi m_e}}\, n_e \bar{\sigma}(T_K) T_K^{-1/2}. \tag{21}$$

The ratio between this equation and the analogous one of Purcell and Field[8] is seen to be

$$\frac{8}{\sqrt{2}} \left(\frac{m_H}{m_e}\right)^{1/2}.$$

[14] J. Jeans, "Dynamical Theory of Gases," Cambridge University Press, Cambridge, Eng., p. 36; 1925.

The factor 8 arises from our use here of $\bar{\sigma}$ as the de-excitation cross section and there as the strong collision cross section, and the $\sqrt{2}$ arises from the fact that here the relative velocity is just the electron velocity, not $\sqrt{2}$ times the atom velocity. The results of averaging the cross section over electron speeds are shown in Fig. 2, and the values of y_e/n_e are given in Table II.

V. Light

In Section III we pointed out that light could cause hyperfine transitions in the ground level via the intermediate step of transitions to higher optical levels in the atom. The lowest of these levels is $n=2$, for which the excitation energy is 10.2 volts, or 1216 Å, the $L\alpha$ line. In a complete theory, $n=3, 4, \cdots$ should be considered, as well as the continuum of levels for the ionized state. The principles, however, can be made clear by discussing only $L\alpha$ light. Furthermore, there is every reason to suspect that $L\alpha$ will be particularly important in the interstellar medium, because radiation at this wavelength has a very short mean free path in hydrogen gas and yet endures many scatterings since the possibility of splitting into two photons via permitted transitions to an intermediate level is closed to it.[15] In consequence, every $L\alpha$ photon is enormously more effective than an $L\beta$, for example, in scattering from ground level hydrogen atoms and hence in causing hyperfine transitions since the $L\beta$ quickly splits into $H\alpha$ and $L\alpha$.

We consider a triplet atom moving at speed v through an isotropic radiation field which has intensity $I_\nu(\nu)$ in the neighborhood of $L\alpha$. We see that if the atom is initially in the ground level hyperfine triplet it may absorb a photon and thus reach the $n=2$ level, returning however to the hyperfine singlet by emission of a more energetic photon. The probability of doing so depends on $I_\nu(\nu)$, where here ν is the precise frequency corresponding to the upward transition in question, Doppler shifted by the speed v.

Similarly, a singlet atom can absorb a slightly higher frequency to reach the same upper state, so that the rates of leaving the singlet and triplet depend on the relative intensities at the two frequencies in question, while the rate of return is independent of the latter. Consequently, if there are more red photons than blue, the singlet becomes more heavily populated than the triplet.

More precisely, refer to the energy level diagram of Fig. 3, p. 246. $2S$ level is omitted, as it is connected with $1S$ only by a forbidden transition. $2P$ is split into $J=\frac{1}{2}$ and 3/2 by spin-orbit interaction; each of these levels is again doubled by hyperfine interaction with $F=J\pm\frac{1}{2}$. The $L\alpha$ line consists of the totality of transitions labelled in the figure. However, the transitions

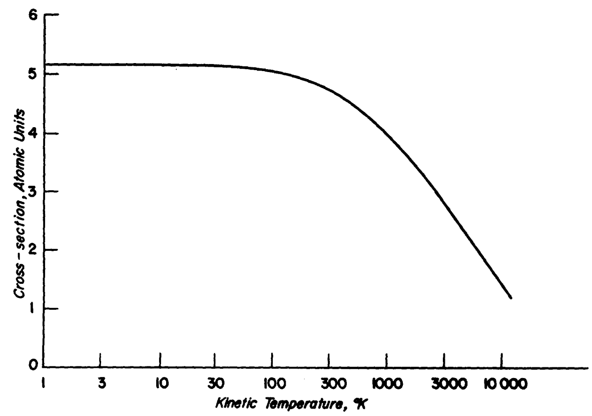

Fig. 2—Cross section for de-excitation, averaged over electron speeds at temperature T_K.

TABLE II

Efficiencies for Collisions and $L\alpha$, and Critical Ionization

(1) T_K(°K)	(2) y_H/n_H	(3) y_e/n_e	(4) y_α/n_α	(5) $(n_e/n_H)_c$ per cent
1	1200	6700	5.9×10^{11}	18
3	490	3900	1.1×10^{11}	13
10	190	2100	1.9×10^{10}	9
30	85	1200	3.6×10^{9}	7
100	35	650	5.9×10^{8}	5
300	16	350	1.1×10^{8}	5
1000	6.7	130	1.9×10^{7}	5
3000	3.3	66	3.6×10^{6}	5
10000	1.3	18	5.9×10^{5}	7

${}_2P_{3/2}\rightarrow{}_0S_{1/2}$ and ${}_0P_{1/2}\rightarrow{}_0S_{1/2}$ are forbidden since they involve $\Delta F=2$ and $0\rightarrow0$, respectively. The relative strengths for the downward transitions are noted on the diagram. They were found from the usual rules for LS coupling of the hyperfine doublets given by Condon and Shortley[16] together with the observation that the relative strengths of the $P_{3/2}$ and $P_{1/2}$ lines are 2:1 in accordance with the LS coupling to form J. In the following we shall discuss only the transitions to and from the ${}_1P$ states since the transitions to and from ${}_2P$ and ${}_0P$ via the "complementary transitions" of Fig. 3 cannot mix the ground state populations as they are paired with forbidden transitions.

Let us call ν_0 and ν_1 the frequencies, Doppler shifted by a speed v (which we hold constant for the present) from the ground level singlet and triplet up to the ${}_1P_{1/2}$, and ν_0' and ν_1' similarly up to ${}_1P_{3/2}$. If $I(\nu)$ is the intensity at those frequencies and B the corresponding Einstein probability, the rate at which a singlet atom is excited to ${}_1P_{1/2}$ is $B_0I(\nu_0)$. It has a chance $A_1/(A_0+A_1)$ of returning to the triplet (if we ignore stimulated emission in the $L\alpha$ line as is legitimate in interstellar space) thus effectively exciting the triplet state. Since the same can happen with ${}_1P_{3/2}$ the rate of excitation is, for v atoms

[15] Although two photon processes destroy even $L\alpha$ after many scatterings, via transitions through a *virtual* level.

[16] E. U. Condon and G. H. Shortley, "The Theory of Atomic Spectra," Cambridge University Press, Cambridge, Eng. p. 243; 1953.

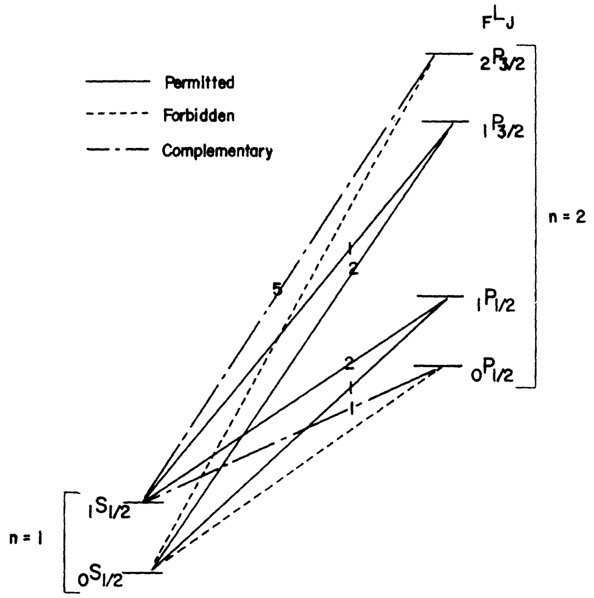

Fig. 3—The 1S and 2P levels of hydrogen, showing the particular transitions in the $L\alpha$ line which excite the triplet (dark lines). The numbers in the center are relative strengths.

$$P_{10}{}^{L} = B_0 I(\nu_0) \frac{A_1}{A_0 + A_1} + B_0' I(\nu_0') \frac{A_1'}{A_0' + A_1'}; \quad (22)$$

similarly,

$$P_{10}{}^{L} = B_1 I(\nu_1) \frac{A_0}{A_0 + A_1} + B_1' I(\nu_1') \frac{A_0'}{A_0' + A_1'} \cdot \quad (22')$$

With the relations between B's and A's, and the relative strengths of Fig. 3, one can show that

$$\begin{aligned} P_{01}{}^{L} &= \frac{B_0 A_1}{A_1 + A_0} [\bar{I}(\nu_0) + \bar{I}(\nu_0')], \\ P_{10}{}^{L} &= \frac{g_0}{g_1} \frac{B_0 A_1}{A_1 + A_0} [\bar{I}(\nu_1) + \bar{I}(\nu_1')]. \end{aligned} \quad (23)$$

Each expression of (23) is correct for any speed v. The bar signifies averaging over all v's, $I(\nu)$ being a function of v through the dependence of ν on v in each case.

From (23), it is clear that the probabilities depend on the intensity profile of the radiation near $L\alpha$, and hence on the solution of a difficult scattering problem, the exact premises of which depend on physical conditions. For example, the radiation near a star would have a profile which depended on the atmosphere of the star, while radiation which has been scattered many times in the interstellar medium may have a profile dependent on the Doppler effects in the medium. It is the latter case which is of greatest interest: $L\alpha$ is generated by recombination in an HII region and, scattering an enormous number of times, penetrates an HI region where it causes observable effects on 21-cm excitation. In fact, Wouthuysen[7] originally called attention to this possibility, and while it was dropped for a time in view of the high efficiencies of collisions, it now seems that the $L\alpha$ mechanism can be of import when the gas is dilute enough. In a forthcoming paper[17] the question of a $L\alpha$ profile within a cloud will be treated. The work so far shows that if a portion of photons is scattered t times on the average by hydrogen atoms in a cloud having Doppler width ν_D, the line will progressively broaden to $\pm\nu_D(2 \ln t)^{1/2}$, while flattening in the center between those limits. That is, the line profile approaches an ever widening "mesa" function. However, the flat portion slopes slightly down toward the blue as a consequence of photon energy losses through recoil, this slope being constant to a first approximation and equal in slope to the Planck curve corresponding to the atom kinetic temperature, T_K. Thus the radiation via recoil comes to quasi-equilibrium with the matter in a narrow range around the center of $L\alpha$, as predicted on general grounds by Wouthuysen.[7]

From the detailed scattered profile discussed above, it follows that we may put

$$\begin{aligned} \bar{I}(\nu_1) + \bar{I}(\nu_1') &= 2 \frac{h\nu_\alpha}{4\pi} \frac{n_\alpha c}{2\nu_D}, \\ \bar{I}(\nu_1) - \bar{I}(\nu_0) = \bar{I}(\nu_1') - \bar{I}(\nu_0') &= \frac{h\nu_\alpha}{4\pi} \frac{n_\alpha c}{2\nu_D} \frac{h\nu_{10}}{kT_K}, \end{aligned} \quad (24)$$

where there are n_α $L\alpha$ photons per cm^3 within $\pm\nu_D$ of the line center. The error in this procedure comes from those scattering processes outside the straight line part of the line profile, *i.e.*, at distances greater than $\nu_D(2 \ln t)^{1/2}$ from the center. But the fraction of scatterings at such frequencies is only $\exp(-\nu^2/2\nu_D{}^2) = t^{-1}$, which we may take to be a very small number. Using (13), (23), and (24), we see that

$$\frac{T_*}{T_L} \equiv \frac{h\nu_{10}}{kT_L} = \frac{h\nu_{10}}{kT_K}; \qquad T_L = T_K. \quad (25)$$

This result, first pointed out by Wouthuysen,[7] follows, for the special case of radiation trapped in a cloud, from the detailed calculations summarized here. Using it, we have also

$$y_\alpha = \frac{h(\lambda c)^2}{36\pi k} \sqrt{\frac{m_H}{k}} \frac{A_\alpha}{A_{10}} \frac{\nu_{10}}{\nu_\alpha} \frac{n_\alpha}{T_K{}^{3/2}}, \quad (26)$$

when the various constants are inserted in (23). Note from (15), that we may not, however, draw Wouthuysen's conclusion that T_S depends only on T_K (which describes the "color" of the $L\alpha$ radiation) but not on $P_{10}{}^{L}$, the frequency of $L\alpha$ collisions. Clearly, this conclusion is valid only if $L\alpha$ collisions dominate, *i.e.*, if $y_\alpha \rightarrow \infty$. In any case it is of interest that $L\alpha$ does serve to couple T_S even more strongly to T_K. The values of y_α/n_α are given in Table II.

[17] G. B. Field, "The time dependent scattering of a resonance line by a dilute gas," to be published.

TABLE III

FIVE EXAMPLES OF SPIN TEMPERATURE

Cloud	T_K	T_R	Atoms		Electrons		Photons		Contributions to T_S				T_S
			n_H	y_H	n_e	y_e	n_α	y_α	Radio	Atoms	Electrons	Photons	
1) Remote	10^2	10	1	35	10^{-3}	0.65	$<10^{-13}$	$<10^{-4}$	0.27	95.30	1.78	0	97.35
2) Near HII (Orion)	10^2	20	1	35	10^{-3}	0.65	4×10^{-3}	2.4×10^{6}	$<10^{-5}$	10^{-3}	$<10^{-4}$	100	100.0
3) Source: Cygnus A	10^2	2×10^5	1	35	10^{-3}	0.65	0	0	5450	95.3	1.8	0	5547
Virgo A	10^2	2×10^3	1	35	10^{-3}	0.65	0	0	54.5	95.3	1.8	0	151.6
4) Galactic Halo	10^4	10	10^{-2}	1.3×10^{-2}	10^{-2}	0.18	4×10^{-5}	2.4×10^{4}	$<10^{-3}$	$<10^{-2}$	0.12	10^4	10^4
5) Intergalactic	10^3	1	10^{-5}	6.7×10^{-5}	10^{-5}	0.0013	—	1.6	0.38	0.03	0.49	615	616

VI. APPLICATIONS

In Table II we have collected the efficiencies for collisions with H atoms, electrons, and $L\alpha$ quanta. We see that y_e/y_H is about 6 to 20 on a per particle basis. This implies that a small ionization will make electrons more important than atoms. The critical ratio at which this occurs, $(n_e/n_H)_c$, is listed in column 5. It indicates that in an HI region, electrons are unimportant since $(n_e/n_H)_c \simeq 10^{-3}$. In the galactic halo and in the intergalactic medium electrons may be important because coexistence of roughly equal numbers of atoms and electrons is possible at low densities.

y_α/n_α exceeds the other efficiencies by large factors, mostly because c/v is large while the cross sections are not so different. We may thus be led to expect that Wouthuysen was correct in his original estimate of the importance of $L\alpha$. Obviously what matters is n_α, and we shall see that the range of n_α is such as to make $L\alpha$ important under some conditions, while not in others.

We consider five examples of the application of (15) and Table II:

1) A remote cloud,
2) A cloud near a source of $L\alpha$ (HII region),
3) A cloud near or in a radio source,
4) A cloud in the galactic halo,
5) An intergalactic cloud.

In Table III are estimates of T_K, T_R, n_H, n_e, and n_α for all five cases. Although we shall to some extent justify these estimates, in many cases they are mere guesses. Table III serves only to *illustrate* various possibilities for, using the parameters therein, we may compute the contributions to the spin temperature implied by (15) and summing the results, compare T_S with T_R and T_K. It should be remembered that while we have used $T_L = T_K$ here in accordance with (25), one could use a more general light temperature defined by

$$\frac{T_*}{T_L} = -\frac{d \log \bar{I}_\nu}{d(\nu/\nu_{10})}, \tag{27}$$

in which case,

$$y_\alpha = 5.9 \times 10^{11} n_\alpha / T_L T_K^{1/2}. \tag{28}$$

Cases 1) and 2) are meant to differ only in the amount of $L\alpha$. In a remote cloud, T_R may be estimated by averaging 21-cm profiles in the vicinity of the plane[1] over a band width of 15 km/sec and allowing a few degrees for radiation near the poles and continuous emission. Case 2) may have a somewhat larger T_R owing to the radio emission of the HII region.

Case 3) is meant to demonstrate the effect of large T_R—the other parameters can be taken as equal to those of a remote interstellar cloud, although it is of course not certain what the real parameters are, of a cloud imbedded in high-velocity material. Both Cygnus A and Virgo A are treated, the former being the brightest, and the latter, the faintest among the five bright sources at 21 cm. T_R was estimated from the measures of Hagen, McClain, and Hepburn[18] together with angular dimensions at lower frequencies from Pawsey's[19] list.

Case 4) is based on Pickelner and Shklovsky's[20] model of the galactic halo, in which the hydrogen is about half-ionized. Case 5) adopts speculative parameters for the intergalactic medium. T_K is no more than a guess, while $T_R = 1°$K is a reasonable extrapolation of results at lower frequencies near the galactic pole. The densities approximate the smoothed out densities of galactic matter, and the ionization may be 50 per cent.

We have not yet mentioned $L\alpha$. Of course what is needed is a detailed theory of the transfer of $L\alpha$ from stars into the interstellar medium. Lacking this, we may calculate n_α roughly by various means. First of all, HI regions such as 1) and 2) must receive their $L\alpha$ from stars. Near a hot star, there will be an HII region where $L\alpha$ is generated copiously by recombination after photoionization of the neutral hydrogen by stellar ultraviolet.

[18] J. P. Hagen, E. F. McClain, and N. Hepburn, "Detection of discrete radio sources at 21-cm wavelength," PROC. IRE, vol. 42, p. 1811; December, 1954.

[19] J. L. Pawsey, "A catalogue of reliably known discrete sources of cosmic radio waves," *Astrophys. J.*, vol. 121, pp. 1–5; January, 1955.

[20] S. B. Pickelner and I. S. Shklovsky, "An investigation of the properties and energy dissipation of the galactic halo," *Astrophys. J. USSR*, vol. 34, pp. 145–158; March–April, 1957. (In Russian, English abstract.)

This process has been studied by Zanstra,[21] who shows that

$$n_\alpha(\text{center}) = \frac{R}{c}\frac{dn_\alpha}{dt} = \frac{R}{c}\alpha n_e^2, \tag{29a}$$

$$n_\alpha(\text{edge}) = S_m^{-1} n_\alpha(\text{center}), \tag{29b}$$

where S_m is the optical depth of $L\alpha$ at the line center corresponding to the radius, R, of the nebula. αn_e^2 is the number of recombinations per cm^3 per sec in the nebula. S_m for a classical HII region may be taken as 10^4, while for the Orion nebula,[22] $n_e = 10^3$ cm^{-3}, $\alpha = 4\times10^{-13}$, and $R = 1$ psc. Consequently, for the Orion nebula

$$n_\alpha(\text{edge}) = 4 \times 10^{-3}\ \text{cm}^{-3}. \tag{30}$$

On the other hand, these photons are scattered very frequently on their way into the HI region, and thus traverse large distances in order to make radial progress. It can easily be shown that the effective mean free path in the radial direction is reduced, because of absorption in the interstellar dust, to

$$l_e = (l_a l_s)^{1/2} = 0.04\ \text{psc}, \tag{31}$$

for a mean free path against scattering, $l_s = 5\times10^{-6}$ psc, and a mean free path against absorption, $l_a = 300$ psc. It follows that the density of $L\alpha$ given by (30) holds only a minute distance from the edge of the Orion nebula and at larger distances will be reduced by e^{-25d}, where d is the distance to the HII region in parsecs. Since the typical remote cloud will be a parsec or more from an HII region,

$$n_\alpha(\text{remote cloud}) < 10^{-13}\ \text{cm}^{-3}, \tag{32}$$

and hence is entirely negligible. For the same reasons we put $n_\alpha = 0$ in case 3).

In the halo, however, $L\alpha$ is being generated locally by recombination in the partially ionized gas; furthermore, the absorption by dust is probably very low. In this case, (29a) will be appropriate, giving $n_\alpha = 4\times10^{-5}$ cm^{-3} for $n_e = 10^{-2}$, $\alpha = 4\times10^{-13}$, and $R = 10$ kpc. The case of the intergalactic medium again needs special treatment because of the peculiar transfer effects in an expanding medium. Here we find that the situation is dominated by the escape of red $L\alpha$ photons from the local region; this will be shown elsewhere[17] to give $P_{10}{}^L = 6.5\ 10^{-11}$ and hence $y_\alpha = 1.6$ for the adopted model.

Although we must emphasize that Table III is based on speculative values of the parameters in many cases, we may draw some conclusions of a general sort. First of all, the contribution of the radio transitions to T_S is small in every case but near a radio source. There, T_S is increased 50 per cent in the weak source, while it is multiplied by 50 in the stronger one over what it would be in the absence of radio frequency emission. We conclude that absorption in the vicinity of strong sources can not be safely interpreted[23] in terms of mass of HI at the present time, in view of (9) and Table III, as even the T_S of Table III is still dependent on the unknown parameters of the absorbing cloud.

A comparison of cases 1) and 2) shows that while $L\alpha$ can be very important in the immediate vicinity of an HII region, it is of no consequence at remote points. Even when it is important, it drives T_S toward T_K just as collisions would. On the other hand, 3), the T_S in the galactic halo is apparently completely determined by its own $L\alpha$. Should $L\alpha$ not act as we have supposed, T_S in the halo would drop from 10^4 to something nearer 10^{3}°K. In any case, it seems quite hopeless to observe absorption in the halo until very large antennas are available, as the opacity can be expected to be between 10^{-3} and 10^{-4}. It is interesting to note that in the halo electrons become important, contributing 10 times the temperature that the atoms do, albeit a small fraction of the $L\alpha$ contribution.

The intergalactic cloud depends almost entirely on $L\alpha$; were it not present in the estimated quantity, T_S would fall to 1°K. Hence, the presence of $L\alpha$ will make the intergalactic medium rather transparent if the temperature is high.

VII. Application to the Deuterium Line

Recently there have been attempts[24-26] to observe absorption at 327 mc which could be attributed to interstellar deuterium. Inasmuch as the absorption coefficient for such a line depends inversely on the spin temperature, T_S', of the deuterium (D) atoms, it is of interest to see whether $T_S' = T_K$ as it does for the hydrogen (H) in remote interstellar clouds. Radio-frequency, collisional, and optical excitations will be considered, using the same notation as previously, but with primes on the parameters for D.

The D hyperfine structure results from an I value of 1 for the deuteron, giving rise to a hyperfine doublet $F = 3/2$ (upper) and $\frac{1}{2}$ (lower). The separation is 327 mc or 91.6-cm wavelength. The partial sum line strength $S(aB)$ for any one of the four levels, a, of the quartet is found to be $(4/3)\beta^2$ rather than simply β^2 as in H. Conse-

[21] H. Zanstra, "On scattering with redistribution and radiation pressure in a stationary benula," *Bull. Astron. Inst. Neth.*, vol. 11, pp. 1–10; March, 1947.

[22] D. E. Osterbrock, "Electron densities in the Orion nebula," *Astrophys. J.*, vol. 122, pp. 235–239; September, 1955.

[23] A. E. Lilley and E. F. McClain, "The hydrogen-line red shift of radio source Cygnus A," *Astrophys. J.*, vol. 123, pp. 172–175; January, 1956.

[24] G. G. Getmanzev, K. S. Stankevitch, and V. S. Troitzky, "Detection of the spectral line of deuterium from the center of the galaxy on the wavelength of 91.6 cm." IAU Symposium No. 4, Cambridge University Press, Cambridge, Eng., pp. 90–91; 1957. Also *Dok. Akad. Nauk. USSR*, vol. 103, pp. 783–786; August, 1957.

[25] G. J. Stanley and R. Price, "An investigation of monochromatic radio emission of deuterium from the galaxy," *Nature*, vol. 177, pp. 1221–1222; June, 1956.

[26] R. L. Adgie and J. S. Hey, "Intensity of the radio line of galactic deuterium," *Nature*, vol. 179, pp. 370–371; February, 1957.

quently, the spontaneous transition probability, A', which is proportional to $S(aB)$ and to λ'^{-3}, is

$$A' = \frac{4}{3}\left(\frac{\lambda}{\lambda'}\right)^3 A = 0.0163A = 4.65 \times 10^{-17}\ \mathrm{sec}^{-1}. \quad (33)$$

As a result, radio-frequency transitions are weak relative to those in H, and this is reflected in increased values of the efficiencies for collisions and light. Radio-frequency transitions in our formulas are specified by the quantity T_R', which is the average brightness temperature over the sky at 91.6-cm wavelength. An estimate based on Allen and Gum's[27] observations at 200 mc, corrected by a $\lambda^{-2.5}$ law, yields $T_R'=40°$K for interstellar clouds near the sun.

Collisions will occur at very nearly the same rate as for H, differing only by reduced-mass factors near unity. Remembering the definition of y_c (15), we have, neglecting such effects,

$$y_c' = \frac{T_*'}{T_*}\frac{A}{A'}y_c = 14.1\ y_c. \quad (34)$$

We may put T_K equal to that taken previously for a remote cloud, 100°K. Then $y_c=35$ if $n_H=1$ and $y_c'=494$ and $y_c'T_K=49{,}400°$K. We have ignored electron collisions, as is usual for HI regions.

Finally optical transitions take place at the rate[28]

$$P_{10}'^L = A_\alpha \frac{\lambda^4}{18\pi h\nu}\frac{dE}{d\lambda}, \quad (35)$$

where $dE/d\lambda$ is the spectral energy density at $L'\alpha$. In the case of H, we wrote (27) in a form suited to a profile drastically affected by scattering in the interstellar gas. Here the situation is quite different owing to the isotope shift of $L'\alpha$ 0.33Å shortwards of $L\alpha$, together with the small ratio $n(\mathrm{D})/n(\mathrm{H})\simeq 1/7000$. The blue shift is about 7 Doppler widths for $L\alpha$ with an rms velocity of 8.5 km, so that the interstellar scattering due to H at the wavelength of $L'\alpha$ is due to the $L\alpha$ damping wings and has a coefficient of about one parsec^{-1}, if $n_H=1$ cm^{-3}. On the other hand, D itself exhibits a scattering coefficient of about 27 psc^{-1}, and hence a mean free path, l_s, of 0.037 psc in the line center. The effective mean free path against absorption in the dust (defined in the previous section) with $l_a=300$ psc is then

$$l_e = (l_a l_s)^{1/2} = 3\tfrac{1}{3}\ \mathrm{psc}, \quad (36)$$

and a considerable amount of $L'\alpha$ will find its way to remote clouds. The precise amount which does so depends on a detailed calculation of the scattering, as outlined above. Since, however, $3\frac{1}{3}$ parsec is not small compared to distances between the stars, we may consider $dE/d\lambda$ in (35) as given by dilution only. The best estimate is probably that of Lambrecht and Zimmerman,[29] who found

$$\left(\frac{dE}{d\lambda}\right)_{1215\text{Å}} = 1.2 \times 10^{-16}\ \mathrm{erg\ cm^{-3}(A)^{-1}}, \quad (37)$$

which, when put in (35) yields

$$P_{10}'^L = 1.310^{-10}\ \mathrm{sec}^{-1},$$
$$y_\alpha' = \frac{T_*'}{T_L'}\frac{P_{10}'^L}{A'} = \frac{45{,}000}{T_L'}. \quad (38)$$

This time it may not be wise to put $T_L'=T_K$, which is true only when the $L\alpha$ profile is shaped by many scatterings. The scattering coefficients already calculated indicate that the situation is marginal, so that T_L' may approach T_K or, on the other hand, it may more nearly reflect the spectral gradient to the blue of $L\alpha$ in the starlight itself. Even in this case we cannot specify T_L', for if the spectral gradient approximated that of a black body of temperature T_c, T_L' would be near T_c, or perhaps 50,000°K.[29] On the other hand, T_L' may reflect primarily the structure of the stellar $L\alpha$ line. It seems reasonable to suppose, however, that T_L' will be between T_K and T_c. We conclude that if $T_L'=T_K$,

$$y_\alpha' = 450; \qquad \nu_\alpha' T_L' = 45{,}000°\mathrm{K}, \quad (39)$$

while if $T_L'=T_c=50{,}000°$K,

$$y_\alpha' = 0.90; \qquad y_\alpha' T_L' = 45{,}000°\mathrm{K}. \quad (40)$$

Putting the results together we find that T_R' is negligible, as expected, and

$$T_s' = \begin{cases} 100°\mathrm{K} \text{ if } T_L' = T_K, \\ 191°\mathrm{K} \text{ if } T_L' = T_c, \end{cases} \quad (41)$$

which indicates that an extreme assumption concerning the effectiveness of light at most only doubles the spin temperature. Therefore, it seems justified to put $T_s'=T_K$ in discussions of the deuterium line, although a different result for the interstellar radiation field near $L\alpha$ would upset this conclusion.

Appendix

Electron De-excitation Cross Section

Let a be an up proton spin function and α and β up and down electron spin functions. One component of the hyperfine triplet is $u(r_2)a\alpha(2)$, where (2) refers to the atomic electron and u is the hydrogen ground state. An unpolarized beam of electrons of unit flux is $2^{-1/2}[e^{i\gamma}\alpha(1)+\beta(1)]e^{ikz_1}$, aside from a constant velocity factor, where (1) refers to the incident electron and k its wave number. γ is a random phase. Since the part of the beam containing $\alpha(1)$ cannot possibly cause a spin flip, the initial wave function of the system hyperfine triplet

[27] C. W. Allen and C. S. Gum, "Survey of glactic radio-noise at 200 mgc," *Aust. J. Sci. Res., A*, vol. 3, pp. 224–233; June, 1950.

[28] This rate is taken to be that for H; there may be factors of order unity resulting from the exact calculation.

[29] H. Lambrecht and H. Zimmermann, "New calculation of the interstellar radiation field, II," *Wiss. Z. Fr. Schiller Univ. (Jena)*, vol. 5, pp. 217–220; 1955.

plus incident unpolarized beam is $2^{-1/2}u(r_2)e^{ikz_1}a\alpha(2)\beta(1)$, which is written decomposed into an electron triplet and singlet as

$$2^{-3/2}u(r_2)ae^{ikz_1} \times [\{\alpha(2)\beta(1) + \alpha(1)\beta(2)\} + \{\alpha(2)\beta(1) - \alpha(1)\beta(2)\}].$$

Now as long as we ignore magnetic interactions, the spin functions do not change in the scattering. Nevertheless, the singlet and triplet scatter with different amplitudes because the requirement of over-all antisymmetry when r_1 and r_2 are comparable leads to different electrostatic interactions for the two cases. Hence the scattered wave is, in general,

$$2^{-3/2}u(r_2)a\frac{e^{ikr_1}}{r_1}[\{\alpha(2)\beta(1) + \alpha(1)\beta(2)\}T(\theta_1) + \{\alpha(2)\beta(1) - \alpha(1)\beta(2)\}S(\theta_1)],$$

where $T(\theta_1)$ and $S(\theta_1)$ are the triplet and singlet scattering amplitudes. The final spin state we are interested in is the hyperfine singlet $2^{-1/2}\times[a\beta(2) - b\alpha(2)]$, and the product of this with the scattered wave, or

$$2^{-2}u(r_2)\frac{e^{ikr_1}}{r_1}\alpha(1) \times [T(\theta_1) - S(\theta_1)],$$

is the amplitude corresponding to de-excitation. The differential cross section for the process of de-excitation is thus r_1^2 times the absolute square of this amplitude, summed over r_2, or

$$\frac{d\sigma}{d\omega} = 2^{-4}|T - S|^2,$$

agreeing with (16).

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Spectral Lines in Radio Astronomy*

A. H. BARRETT†

Summary—**A review of the elements of microwave spectroscopy as it concerns, or is likely to concern, radio astronomy is presented. This review attempts to show how atomic and molecular resonance lines arise in the radio frequency spectrum and to discuss the important parameters of spectral lines. Finally, it is shown how the intensity of radio emission received from the galaxy is modified by the presence of an interstellar atomic gas having a resonance transition at the frequency of observation.**

I. Introduction

THE frequency spectrum of radio frequency radiation received from a typical region of the galaxy is without discontinuities or irregularities for the most part. However, in a relatively narrow frequency interval centered about 1420.4 mc ($\lambda = 21$ cm) the intensity is approximately double that of adjacent portions of the spectrum. It was predicted in 1944 by van de Hulst, a Dutch astronomer, that the galactic radiation at 1420.4 mc might be different from the neighboring frequencies. This was shown to be the case by Ewen and Purcell in 1951[1] and was verified immediately thereafter, by workers in the Netherlands[2] and in Australia.[3] Subsequently, this frequency band has become the most studied part of the spectrum and will most likely continue to be. Undoubtedly there are similar such increases, or decreases, in the level of galactic radiation at other frequencies many times less intense which so far have escaped detection. It is the purpose of this paper to review briefly the origin of the radiation at 1420.4 mc and at other frequencies which might exhibit similar behavior. Although this is not a new subject, it is intended here to present more background material than has usually been given in the more detailed papers on this subject.[4–9] Accordingly, this paper

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