0.1 Chapter 2

0.1.1 2.4 Conceptual questions

1. Use the virial theorem to explain why stars are hot, i.e. have a high internal temperature and therefore radiate energy.

**Answer:** Gravitational bound spheres must be hot to maintain hydrostatic equilibrium: heat provides the pressure required to balance gravity. The more compact such a sphere, the more strongly bound and therefore the hotter it must be. Because a hot sphere of gas is hotter than the surroundings, it must radiate energy away because nature wants to restore thermal differences.

2. What are the consequences of energy loss for the star, especially for its temperature?

**Answer:**

\[ E_{tot} = E_{int} + E_{GR} = -E_{int} = \frac{1}{2}E_{GR} \]

\[ -\dot{E}_{tot} = \frac{1}{2}\dot{E}_{GR} = -\dot{E}_{int} \]

\[ \dot{E}_{GR} = -2L < 0 \]

\[ \dot{E}_{int} = L > 0 \]

A consequence of losing energy is that a star contracts, becomes more strongly bound and that the star’s internal energy increases, meaning that the star becomes hotter, so the temperature increases. If a star contracts, half the energy is used for heating, the other half is radiated away.

3. Most stars are in thermal equilibrium. What is compensating for the energy loss?

**Answer:** When a star is in thermal equilibrium, the star is in a stationary state, and energy is radiated away with the same rate at which it is produced by nuclear reactions in the interior, so the compensation for the energy loss is thermal nuclear reactions.

4. What happens to a star in thermal equilibrium (and in hydrostatic equilibrium) if the energy production by nuclear reactions in a star drops (slowly enough to maintain hydrostatic equilibrium)?

**Answer:** There will be a disturbed thermal equilibrium, which means that \( L_{nuc} < L \), so the total energy will decrease, and as a consequence the star will contract and become hotter.

5. Why does this have a stabilizing effect? On what timescale does the change take place?

**Answer:** The disturbed thermal equilibrium will become stable again. Because it becomes hotter, other reactions are able to take place, keeping the thermal equilibrium. The timescale on which this event takes place is the thermal or Kelvin-Helmholtz timescale, \( \tau_{KH} = \frac{E_{nu}}{L} \).

6. What happens if hydrostatic equilibrium is violated, e.g. by a sudden increase of the pressure.

**Answer:** If hydrostatic equilibrium is violated, the star either increases (decrease in temperature) or decrease (increase in temperature) in size, to restore the hydrostatic equilibrium. Since the pressure is directly related to temperature, this has an immediate effect. When the pressure increases, for example, the star will expand.

7. On which timescale does the change take place? Can you give examples of processes in stars that take place on this timescale.

**Answer:** These events take place on the free-fall, or dynamical time scale. \( \tau_{dyn} \approx \sqrt{\frac{R^3}{GM}} \). Examples of processes which take place on this timescale are supernovas (especially type II core collapse supernovas) or pulsations of variable stars.


0.2 Chapter 3

0.2.1 3.1 Conceptual questions

1. What do we mean by local thermodynamic equilibrium (LTE)? Why is this a good assumption for stellar interiors? What is the difference between LTE and thermal equilibrium (as treated in Chapter 2)?

**Answer:** By local thermodynamic equilibrium, we mean that a region of a star with size much smaller than the radius of that star, but larger than the mean free path of photons, there is a well defined local temperature. This means that in the star there is a well defined temperature distribution. The local thermal equilibrium says something about the local temperature (that they are approximately the same), while TE says that there is no net change in energy; the same amount of energy that is produced, is radiated away.

2. In what type of stars does degeneracy become important? Is it important in main-sequence stars? Is it more important in high mass or low mass MS stars?

**Answer:** Degeneracy becomes most important in neutron stars, white dwarfs, brown dwarfs, and mostly in red giants right before the helium flash. Degeneracy is not particularly important for MS stars. However, it does become important in the (later) life of high mass MS stars.

3. Explain qualitatively why for degenerate matter, the pressure increases with the density.

**Answer:** When the density increases and more particles are in a certain volume with a too low amount of quantum states with low momenta, particles will go to higher momenta. By adding more particles, the pressure thus increases. Pressure is independent of temperature because further decrease of temperature will not affect the momentum distribution.

4. Why do electrons become relativistic when they are compressed into a smaller volume? Why does the pressure increase less steeply with the density in this case?

**Answer:** As the particles are getting higher momenta, their density will be able to keep rising. Their velocity can then increase to relativistic heights. In this relativistic case, the pressure rises less steeply with density because the velocity is not dependent on the momentum anymore.

5. In the central region of a star we find free electrons and ions. Why do the electrons become degenerate first? Why do the ions never become degenerate in practice?

**Answer:** Electrons become degenerate first because they have a lower mass, and thus a lower momentum than ions. In practice, ions will never become degenerate, because the atoms will capture free electrons, making them neutron rich.

0.3 Chapter 5

0.3.1 5.4 Conceptual questions: Convection

1. Why does convection lead to a net heat flux upwards, even though there is no net mass flux (upwards and downwards bubbles carry equal amounts of mass)?

**Answer:** The temperature of the upwards bubbles is higher than the downwards bubbles, resulting in a net flux upwards.

2. Explain the Schwarzschild criterion,

\[
\left( \frac{d \ln T}{d \ln P} \right)_{\text{rad}} > \left( \frac{d \ln T}{d \ln P} \right)_{\text{ad}}
\]

in simple physical terms (using Archimedes law) by drawing a schematic picture. Consider both cases \( \nabla_{\text{rad}} > \nabla_{\text{ad}} \) and \( \nabla_{\text{rad}} < \nabla_{\text{ad}} \). Which case leads to convection?
\[ \frac{d \log \rho}{d \log \mathcal{P}} \approx - \frac{d \log \rho}{d \log T} \]
\[ \log \rho = \log \mathcal{P} + \log \mu m_u - \log T - \log k_B \]

Stable:
\[ (\frac{d \log \rho}{d \log \mathcal{P}})_{ad} < (\frac{d \log \rho}{d \log \mathcal{P}})_{rad} \]
\[ (\frac{d \log \mathcal{P}}{d \log T})_{ad} > (\frac{d \log \mathcal{P}}{d \log T})_{rad} \]

Unstable:
\[ (\frac{d \log \rho}{d \log \mathcal{P}})_{ad} > (\frac{d \log \rho}{d \log \mathcal{P}})_{rad} \]
\[ (\frac{d \log \mathcal{P}}{d \log T})_{ad} < (\frac{d \log \mathcal{P}}{d \log T})_{rad} \]

This last one leads to convection.

3. What is meant by the superadiabaticity of a convective region? How is it related to the convective energy flux (qualitatively)? Why is it very small in the interior of a star, but can be large near the surface?

**Answer:** Superadiabaticity, \( \nabla - \nabla_{ad} \), is the degree to which the actual temperature gradient \( \nabla \) exceeds the adiabatic value.

The convective flux is related to the superadiabaticity via \( F_{\text{conv}} \propto (\nabla - \nabla_{ad})^{3/2} \); higher superadiabaticity means higher convective flux.

The superadiabaticity is smaller in the interior of the star, because the temperature gradient here is nearly adiabatic. At the surface it is much larger because the temperature and density are much lower here, so the difference between \( \nabla \) and \( \nabla_{ad} \) becomes much larger.

### 0.4 Chapter 6

#### 0.4.1 6.1 Conceptual question: Gamow peak

you saw that the reaction rate is proportional to
\[
\langle \sigma v \rangle = \left( \frac{8}{m^3 \pi} \right)^{1/2} \frac{S(E_0)}{(kT)^{3/2}} \int_0^\infty e^{-E/kT} e^{-b/E^{1/2}} dE
\]

where the factor \( b = \pi(2m)^{1/2} Z_1 Z_2 e^2 / \hbar \), and \( m = m_1 m_2 / (m_1 + m_2) \) is the reduced mass.

1. Explain in general terms the meaning of the terms \( e^{-E/kT} \) and \( e^{-b/E^{1/2}} \)

**Answer:** \( e^{-E/kT} \) is related to the Coulomb repulsion and means that there are more particles with a low velocity than with a high velocity.

\( e^{-b/E^{1/2}} \) represents the tunnel probability, which means that at high energies, the tunnel probability is higher.

2. Sketch both terms as function of \( E \). Also sketch the product of both terms.

**Answer:**

[Diagram showing Maxwelian distribution, Gamow peak, and tunnelling probability]
3. The reaction rate is proportional to the area under the product of the two terms. Draw a similar sketch as in question (b) but now for a higher temperature. Explain why and how the reaction rate depends on the temperature.

**Answer:** For a higher temperature the graph is shifted a bit to the right and has a bigger area, the reaction rate depends on the temperature according to $T^\nu$, the increase of the peak depends strongly on which kind of reaction, so the charge of the particles and mass are very important.

4. Explain why hydrogen burning can take place at lower temperatures than helium burning.

**Answer:** The Coulomb repulsion for hydrogen burning is lower than for helium burning, so hydrogen nucleons have a higher tunnel probability at lower temperatures.

5. Elements more massive than iron, can be produced by neutron captures. Neutron captures can take place at low temperatures (even at terrestrial temperatures). Can you explain why?

**Answer:** For neutron capture, there is no Coulomb potential, since they’re neutral.

### 0.5 Chapter 7

#### 0.5.1 7.1 General understanding of the stellar evolution equations

The differential equations (7.1–7.5) describe, for a certain location in the star at mass coordinate $m$, the behaviour of and relations between radius coordinate $r$, the pressure $P$, the temperature $T$, the luminosity $l$ and the mass fractions $X_i$ of the various elements $i$.

1. Which of these equations describe the mechanical structure, which describe the thermal-energetic structure and which describe the composition?

**Answer:** Mechanical structure:

\[
\begin{align*}
\frac{\partial r}{\partial m} &= \frac{1}{4\pi r^2 \rho} \\
\frac{\partial P}{\partial m} &= -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2} \frac{\partial^2 r}{\partial t^2}
\end{align*}
\]

Thermal-energetic structure:

\[
\begin{align*}
\frac{\partial l}{\partial m} &= \epsilon_{\text{nuc}} - \epsilon_{\nu} - T \frac{\partial s}{\partial t} \\
\frac{\partial T}{\partial m} &= -\frac{Gm}{4\pi r^4} T \nabla
\end{align*}
\]

Composition:

\[
\frac{\partial X_i}{\partial t} = \frac{A_i}{\rho} \left( -\sum_j 1 + \delta_{ij} + \sum_k l_i r_{k,i}\right)[+\text{mixing}]
\]

2. What does $\nabla$ represent? Which two cases do we distinguish?

**Answer:** $\nabla$ represents the temperature gradient over the pressure, $\frac{d\log T}{d\log P}$.

We distinguish the following two cases:

- $\nabla_{\text{rad}} \leq \nabla_{\text{ad}}$, in which case we can use $\nabla = \nabla_{\text{rad}}$
- $\nabla_{\text{rad}} > \nabla_{\text{ad}}$, in which case we have to use the MLT.

3. How does the set of equations simplify when we assume hydrostatic equilibrium (HE)? If we assume HE, which equation introduces a time dependence? Which physical effect does this time dependence represent?

**Answer:** In hydrostatic equilibrium, we have $\frac{\partial^2 r}{\partial t^2} = 0$, so the second differential equation becomes:

\[
\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}
\]

The third differential equation introduces a time-dependence: $\frac{\partial s}{\partial t}$. This represents the gravitational radiation.

The equation for the chemical composition also introduces a time-dependence.

4. What do the terms $\epsilon_{\text{nuc}}$ and $T \partial s/\partial t$ represent?
Answer: The $\epsilon_{\text{nuc}}$ represents the generated nuclear energy, and $T\frac{\partial s}{\partial t}$ represents the gravitational radiation, due to the contraction of the star.

5. How does the set of equations simplify if we also assume thermal equilibrium (TE)? Which equation introduces a time dependence in TE?

Answer: In thermal equilibrium, the term $\frac{\partial s}{\partial t} = 0$, so we get:

$$\frac{\partial l}{\partial m} = \epsilon_{\text{nuc}} - \epsilon_{\nu}$$

The equation introducing time-dependence is now only:

$$\frac{\partial X_i}{\partial t} = \frac{A_i}{\rho}(-\sum_i 1 + \delta_{ij} + \sum_{k,l} r_{kl,i})[+\text{mixing}]$$

6. Equation (7.5) describes the changes in the composition. In principle we need one equation for every possible isotope. In most stellar evolution codes, the nuclear network is simplified. This reduces the number of differential equations and therefore increases speed of stellar evolution codes. The STARS code behind Window to the Stars only takes into account seven isotopes. Which do you think are most important?

Answer: $^1H, ^4He, ^12C, ^14N, ^16O, ^28Si, ^56Fe$

0.6 Other conceptual questions

1. Draw the different effects on the equation of state in a log $T$ vs log $\rho$.

Answer:

2. What is the Chandrasekhar mass and what does it mean and why is it important?

Answer: The Chandrasekhar mass is 1.44 solar masses, and it is the maximum mass of a white dwarf. The Chandrasekhar mass is important because the supernova type Ia always have this mass and thus are a standard candle.

3. How are cross sections interpolated and what is the disadvantage of this method?

Answer: For the interpolation of the cross section the function $\sigma(E)$ is used, called the astrophysical cross-section factor. This equation gives a straightline see figure 6.3 page 81. The disadvantage of this method is that you don’t know if there is a resonance at low temperatures, you assume that the cross section can be linear interpolated, but if there is a resonance this is not possible.

4. Name the burning cycles.
**Answer:** pp-cycle, triple alpha, CNO-cycle, carbon burning, oxygen burning, neon burning and silicon burning

5. What is the Hertzsprung-gap?

**Answer:** The Hertzsprung-gap is the place in the Hertzsprung-Russel diagram between the main sequence and the red giant branch, because most stars live a short time in this area, it is called a gap.

6. What is a polytropic stellar model?

**Answer:** A polytropic stellar model is a stellar model where, the equation of state is independent of temperature and only depends on the density.

7. What is an adiabatic process?

**Answer:** A process where there is no heat exchange

8. What is the influence of a partly ionised gas on the adiabatic gradient?

**Answer:**

9. What is the Lane-Emden equation and when can you solve it?

**Answer:** A differential equation that describes the density for a polytropic stellar model. You can solve it for $n = 0, 1$ or $5$.

10. Explain the different ways of energy transport in stars

**Answer:** Radiative diffusion, particle diffusion, convection, conductive energy transport (only for degenerate stars).

11. What is the Rosseland mean opacity

**Answer:** An equation to find the mean opacity over every frequency.

12. What is the mixing length theorem?

**Answer:** It describes how the chemical composition of stars mixes due to convection.

13. In what kind of process are neutrinos emitted?

**Answer:** Nuclear fusion (and pair creation).

14. What are the boundary conditions for stellar models?

**Answer:** $T, P = 0$ at the surface. $M, L, R = 0$ at the core.
15. How can you change between $\frac{\partial}{\partial r}$ and $\frac{\partial}{\partial m}$?

**Answer:**

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial m} \frac{\partial m}{\partial r} = \frac{\partial}{\partial m} 4\pi R^2$$

16. What are barotropes?

**Answer:** Polytropes: stars for which the pressure only depends on the density.

17. When are stars marginal bounded? bounded? and unstable?

**Answer:** Marginal bounded: between bounded and unstable. Phi = 3 or a gas is completely dominated by radiation pressure. If phi > 3 then the star is bounded, so ideal gas is important. If phi < 3 then the star is unstable. This almost never happens.

18. How can you calculate the mean molecular weight if you know the mean molecular weight of the ions and the electrons?

**Answer:**

$$\mu = \frac{1}{\mu_e} + \frac{1}{\mu_{ion}}$$

19. Which kind of momenta distribution do bosons and fermions follow?

**Answer:** At low temperatures, bosons follow the Bose-Einstein distribution, and fermions the Fermi-Dirac. At high temperatures, both follow the Maxwell-Boltzmann distribution.

20. What is the planck function and what does it mean?

**Answer:** The function for black bodies that describes how radiation depends on the frequency.

21. Which equation becomes important by partial ionised matter?

**Answer:** The Saha equation, this equation lets you calculate the ionisation fraction.

22. What are the sources of opacity?

**Answer:** Electron scattering, free-free absorption, free-bound and bound-bound absorption, negative hydrogen ion, molecules and dust (for very low T), conductive opacities (only for degenerate stars).

23. What is the eddington luminosity?

**Answer:** The maximum possible luminosity in hydrostatic equilibrium.

24. In which case can hydrostatic equilibrium and radiative equilibrium not hold simultaneously?

**Answer:** In the outer layers of the star, because thermal equilibrium doesn’t hold there.

25. What is convective overshooting?

**Answer:** When a convective blob goes up higher than expected in the area where convection happens. The blob ‘overshoots’ the convection area.

26. What is a resonance interaction?

**Answer:** Resonance between particles, so the fusion probability increases significantly. This effect can usually be neglected in calculations.

27. Which interaction has the highest cross-section, the lowest and the intermediate one?

**Answer:** The strong force has the highest cross-section, the electromagnetic force the intermediate one, and the weak force the lowest one.

28. What is a compound nucleus?

**Answer:** An excited nucleus that can split between two other nuclei or can fall back to the ground state.
29. What is electron screening and how does this increase the tunnel probability?

   **Answer:** When an electron is close to a nucleus, it reduces the net charge of the nucleus, which means that the tunnel probability increases, because the coulomb barrier is reduced.

30. What are pycno nuclear reactions?

   **Answer:** Nuclear reactions that can happen because of the high density.