Physics of Galaxies 2019/2020

Problem Set 6

1. Let us examine the properties of the spiral galaxy NGC 7331.

a) The radial velocity of this galaxy is $v_r = 820\,\mathrm{km\,s^{-1}}$. What is its distance? Assume $H_0 = 60\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

b) The galaxy integrated V-band magnitude is $V = 8.75$ mag. Use the distance that you determined in a) to show that its V-band luminosity is $L_V \approx 5 \times 10^{10}\,\mathrm{L_{V,\odot}}$.

c) Is NGC 7331 more or less luminous than the Milky Way (MW)?

d) Use Fig. 5.4 in Sparke & Gallagher (see below) and the distance you determined above to show that $h_R \approx 3.6\,\mathrm{kpc}$.

e) Determine the disk radius R_{25} in kpc. How does this compare with the MW's radius?

f) Use Figure 5.4 in Sparke & Gallagher and the fact that a surface brightness of $\mu_I = 15\,\mathrm{magarcsec^{-2}}$ corresponds to $I_I \approx 1.8 \times 10^4\,\mathrm{L_\odot\,pc^{-2}}$ to show that:

f.1 - NGC 7331's (extrapolated) central surface brightness is $I_I(0) \approx 315\,\mathrm{L_\odot pc^{-2}}$.

f.2 - the disc's total luminosity is $L_D \approx 3 \times 10^{10}\,\mathrm{L_{I,\odot}}$.

g) Use the equation for the mass enclosed within a radius R, i.e., $M(< R) = v^2 R/G$, to show that NGC 7331 has $M/L \approx 15$ in solar units within 37 kpc. Read the velocity at this radius from the figure below.

h) Use the calibrated Tully-Fisher relation in Eq. 5.6 of Sparke & Gallagher

$$\frac{L_I}{4 \times 10^{10}\,\mathrm{L_{I,\odot}}} \approx \left(\frac{v_{\max}}{200\,\mathrm{kms^{-1}}}\right)^4 \tag{1}$$

to infer the total luminosity L_I of NGC 7331. In addition, answer the following:

h.1 - what is its absolute magnitude in the I band?

h.2 - If this galaxy has an I-band apparent magnitude $I = 7.92$ mag, what is its distance?

h.3 - Why is the distance obtained in h.2 different to the distance obtained in part a) above?

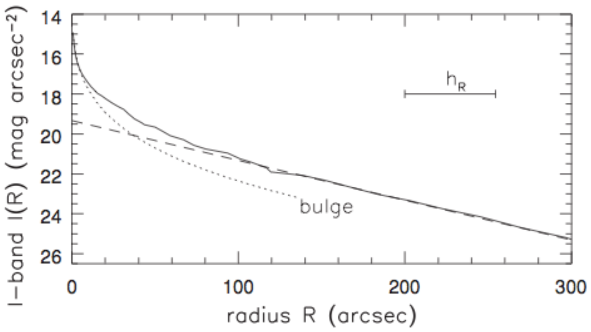

Fig. 5.4. NGC 7331: the solid line shows surface brightness in the I band, near 8000 Å. The dashed line is an exponential with $h_R = 55''$; the dotted line represents additional light – R. Peletier.

2. Use the following equation

$$\frac{v^2(r)}{r} = -F_r(r) = \frac{GM(<r)}{r^2} \tag{2}$$

a) to explain why we might expect the mass M of a spiral galaxy to follow approximately $M \propto v_{max}^2 h_R$. Ignore the presence of a bulge in all this analysis.

b) Use $I(R) = I(0)\,exp(-R/h_R)$ to show that $L = 2\pi I(0) h_R^2$, and hence that (if M/L and $I(0)$ are constants) then $L \propto v_{max}^4$.

c) In fact, $I(0)$ is lower in low surface-brightness galaxies. So, show that if spirals of different surface-brightnesses follow the same Tully-Fisher relation, then they must have higher mass-to-light ratios, with approximately $M/L \propto 1/\sqrt{I(0)}$.

3. In this problem you will derive the Fundamental Plane of early-type galaxies and one of its projections, known as the Faber-Jackson relation. Assume an elliptical galaxy with a constant and isotropic velocity dispersion σ_r and a constant mass-to-light ratio M/L throughout the galaxy, with no

dark matter.

a) Show that the kinetic energy is $KE = 3M\,\sigma_r^2/2$.

b) Assume that the galaxy is a uniform sphere of radius R_e. Consider that the potential energy is given by $PE = -3GM^2/(5R_e)$. Use the virial theorem, which states than on average the kinetic and potential energy should be in balance, to show that $M = 5\sigma_r^2\,R_e/G$.

c) If all elliptical galaxies follow the Sérsic law with the same value of n, show that their luminosities must scale as $L \propto I_e R_e^2$ and their mass-to-light ratios as $M/L \propto \sigma_r^2/(I_e R_e)$.

d) Show that the Faber-Jackson relation, $L \propto \sigma_r^4$, only holds if M/L and I_e are constant for *all* elliptical galaxies (which in fact is not the case).

e) Show that the observed Fundamental Plane, $R_e \propto \sigma^{1.2} I_e^{-0.8}$, implies $I_e \propto R_e^{-1.25}\sigma_r^{1.5}$ and hence that $M/L \propto \sigma^{0.5} R_e^{0.25} \propto M^{0.25}$, so that the mass-to-light ratio increases with increasing galaxy mass.