Physics of Galaxies 2019/2020

Problem Set 5

1. This problem is intended to help you interpret $l - v$ diagrams like that in Figure 2.20 in Sparke & Gallagher. Inspect Fig. 2.19 in the book (or see below). Assume that the Galaxy's disc has a constant velocity everywhere, with $v(R) \approx 220\,\mathrm{km/s}$, and the Sun is located at $R_0 = 8\,\mathrm{kpc}$.

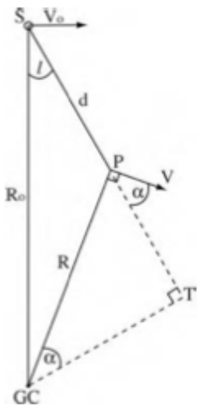

Fig. 2.19. Galactic rotation: a star or gas cloud at P with longitude l and Galactocentric radius R, at distance d from the Sun, orbits with speed $V(R)$. The line of sight to P is closest to the Galactic center at the *tangent point* T.

a) Find $v_r(l)$, which is the radial velocity at longitude l, for gas on circular orbits in the Galaxy at $R = 4, 6, 10$ and 12 kpc. Do this by varying the Galactocentric azimuth ϕ around each ring: find d for each (ϕ, R), and hence the longitude l and v_R (see geometry in the figure below which corresponds to Fig. 2.19 in Sparke & Gallagher).

b) Make a plot showing v_r as a function of l as seen from the *local standard of rest (LSR)*. Where is the gas in the Galaxy at the following longitudes and velocities: ($l \sim 50°$, $v_r > 0\,\mathrm{km/s}$); ($l \sim 50°$, $v_r < 0\,\mathrm{km/s}$); ($l \sim 120°$, $v_r < 0\,\mathrm{km/s}$); ($l \sim 240°$, $v_r > 0\,\mathrm{km/s}$); ($l \sim 300°$, $v_r > 0\,\mathrm{km/s}$); and

($l \sim 300°$, $v_r < 0\,\mathrm{km/s}$). And where is the gas at ($l \sim 120°$, $v_r > 0\,\mathrm{km/s}$) ?

2. The Virial Theorem tells us that the mass $M(< R)$ of a spherical system in equilibrium enclosed in a sphere of radius R is

$$M(< R) = \frac{v^2 R}{G} \tag{1}$$

where v is some characteristic velocity of the system. Let us consider this for the Milky Way.

a) Assuming that the circular velocity at $R = R_0 = 8\,\mathrm{kpc}$ is $v(R) = 200\,\mathrm{km\,s^{-1}}$, what is $M(< R)$?

b) What is the average density of the sphere with radius R? Note that the critical density of the Universe is

$$\rho = \frac{3H_0^2}{8\pi G}. \tag{2}$$

c) How many times larger is the average density of the Galaxy than the critical density of the Universe?

d) Now assume that the MW disc has an exponential profile with a scale length $h_R = 4\,\mathrm{kpc}$ at $R = R_0$. Show that 60% of the MW's disc mass lies at distances smaller that the distance from the Sun to the Galactic centre.

e) Assume $L_V = 5 \times 10^9\,\mathrm{L_\odot}$ for the bulge and $L_V = 15 \times 10^9\,\mathrm{L_\odot}$ for the disc. Show that the V-band mass-to-light ratio inside the Sun's orbit is $M/L_V(< R_0) \approx 5$ in solar units.

3. Consider a disc galaxy with stellar mass $M = 10^{11}\,\mathrm{M_\odot}$.

a) Derive an estimate of its rotational velocity v at a radius $r = 8.5\,\mathrm{kpc}$, assuming an ideal point-like mass distribution at the centre. Compare your result with the rotational velocity of $220\,\mathrm{km\,s^{-1}}$ observed near the Sun.

b) Now derive the rotational velocity v at the same distance assuming that the mass density profile varies as $\rho(r) \propto exp[-r/(5\,\mathrm{kpc})]$

c) idem to b) but for $\rho(r) \propto exp[-r/(10\,\mathrm{kpc})]$.

d) Find the rotational velocity within a spherical dark matter halo whose density is given by $\rho(r) = \rho_s\,(r/R_s)^n$, where n, ρ and R_s are constants. Find n such that the rotation curve is flat, as observed, i.e., v is idnependent of r.

Note: more recent dark matter halo models have

$$\rho(r) = \frac{\rho_s}{(r/R_s)(1 + r/R_s)^2}, \tag{3}$$

which is the so-called NFW profile (Navarro, Frenk & White, ApJ, 490, 493, 1997).