Physics of Galaxies 2019/2020

Problem Set 4

1. Consider a star (or any other object) moving under gravity within a galaxy.

a) Balance the centripetal force against the gravitational force to determine the angular frequency $\Omega(r) = v(r)/r$. Show that, if the density ρ of the galaxy is constant, then Ω is also constant.

b) Show that a star moving on a radial orbit, i.e., in a straight line through the center, will oscillate harmonically in radius with period

$$P = \sqrt{\frac{3\pi}{G\rho}} \approx 3\, t_{\mathrm{ff}}, \tag{1}$$

where t_{ff} is the free-fall time

$$t_{\mathrm{ff}} = \sqrt{\frac{1}{G\rho}}. \tag{2}$$

c) Show that if you bored a hole through the center of the Earth to the other side and dropped a golf ball down it, then (ignoring air resistance, and all other practical problems, and assuming that the Earth has a constant density) you could return about an hour and a half later to retrieve the golf ball when it would return to its starting point.

Tip: consider that the mass of the Earth is $M \approx 6 \times 10^{27}\,\mathrm{g}$ and its radius is $R \approx 6.4 \times 10^{8}\,\mathrm{cm}$.

2. Let us consider the relaxation time of globular clusters and the importance of collisions in these dense systems.
Assume that a typical star in the globular cluster 47 Tucanae (47 Tuc) has a mass $M = 0.5\,\mathrm{M}_{\odot}$ and that the Coloumb parameter for this cluster is $\Lambda = r_c/1\,\mathrm{AU}$. The Coulomb parameter Λ is the ratio between the maximum

and minimum values of the impact parameters $\Lambda = b_{\rm max}/b_{\rm min}$ (see lecture notes).

a) Use Table 3.1 in Sparke & Gallagher (see below) to determine the relaxation time in this cluster. Note that the density ρ given in this table is actually log10(ρ), not ρ itself! 47 Tuc is about 12 Gyr old – is it relaxed?

b) Now compute the collision time for a star in this cluster. How many strong encounters has a single star had (on average) in the lifetime of 47 Tuc?

c) How many encounters in total have occured in 47 Tuc during its lifetime?

Table 3.1 Dynamical quantities for globular and open clusters in the Milky Way

Cluster		σ_r (km s^{-1})	$\log_{10}\rho_c$ ($\mathcal{M}_\odot$ pc^{-3})	r_c (pc)	$t_{\rm relax,c}$ (Myr)	*Mass* ($10^3\mathcal{M}_\odot$)	$\mathcal{M}/L_V$ ($\mathcal{M}_\odot/L_\odot$)
NGC 5139	ω Cen	20	3.1	4	5000	2600	2.5
NGC 104	47 Tuc	11	4.9	0.7	50	800	1.5
NGC 7078	M15	12	>7	<0.1	<1	900	2
NGC 6341	M92	5	5.2	0.5	2	200	1
NGC 6121	M4	4	4–5	0.5	30	60	1
	Pal 13	~0.8	2	1.7	10	3	3–7
NGC 1049	Fornax 3	9	3.5	1.6	600	400	~3
Open cluster	Pleiades	0.5	0.5	3	100	0.8	0.2

Note: σ_r is the dispersion in radial velocity V_r in the cluster core; ρ_c is central density; $t_{\rm relax,c}$ is the relaxation time at the cluster's center found using Equation 3.55 with $V = \sqrt{3}\sigma_r$, $\langle m_* \rangle = 0.3\mathcal{M}_\odot$, and $\Lambda = r_c/1$ AU. Clusters with upper limits to r_c probably have collapsed cores.

3. In cylindrical coordinates centered on the Galactic Center (R, ϕ, z), the double-exponential profile for the distribution of stars in the disk is written (Sparke & Gallagher Eq. 2.8):

$$n(R, z, S) = n(0, 0, S)\, exp[-R/h_R(S)]\, exp[-|z|/h_z(S)], \qquad (3)$$

where S represents some population of stars with scale length $h_R(S)$ and scale height $h_z(S)$.

a) Show that at radius R, the number per unit area (i.e., the surface density) of stars of type S is $\Sigma(R, S) = 2\, n(0, 0, S)\, h_z(S)\, exp[-R/h_R(S)]$.

b) If each star has luminosity $L(S)$, then the surface brightness is $I(R, S) = L(S)\, \Sigma(R, S)$. Assuming that h_R and h_z are the same for all S, show that the disk's total luminosity is $L_D = 2\pi\, h_R^2\, I(R = 0)$.

c) For the Milky Way, $L_D \approx 1.5 \times 10^{10}\,\mathrm{L}_\odot$ in the V band and $h_R \approx 4\,$kpc.

Show that the disk's surface brightness at the Sun's position, which is 8 kpc from the Galactic Center, is $\sim 20\,\mathrm{L}_\odot\,\mathrm{pc}^{-2}$.

d) The stellar mass density in the disk is $40-60\,\mathrm{M}_\odot\,\mathrm{pc}^{-2}$, so $M/L_V \sim 2-3$ (in solar units). Why is this larger than $M/L_V \approx 0.67$ for stars within 100 pc from the Sun?