Physics of Galaxies 2019/2020

Problem Set 3

1. Assume a galaxy at a distance of d Mpc from us.

a) What would be the apparent B-band magnitude of a star like our Sun contained in this galaxy?

b) For that galaxy, show that $1''$ on the sky corresponds to $5d$ pc.

c) If the surface brightness is $I_B = 27\,\mathrm{mag\,arcsec^{-2}}$, how much B-band light does 1 arcsec^2 of the galaxy emit, compared with a star like the Sun? Show that this is equivalent to $1\,\mathrm{L_\odot\,pc^{-2}}$ in the B band, but that, in the I band, having $I_I = 27\,\mathrm{mag\,arcsec^{-2}}$ corresponds to only $0.3\,\mathrm{L_\odot\,pc^{-2}}$. Explain.

Tips: for the Sun, consider that the B-band absolute magnitude is $M_B^\odot = 5.43$ and that it has a colour $(B-I)_\odot = 1.37$.

2. Integrate the Salpeter initial mass function (IMF) between a lower mass limit Ml and an upper mass limit $Mu \gg Ml$ to find

a) the total number of stars formed in the population

b) the total mass of the population

c) the total luminosity of the population

d) Show that the number and mass of stars formed depend on Ml while the luminosity depends on Mu.

e) Now assume that $Ml = 0.3\,\mathrm{M_\odot}$ and $Mu = 100\,\mathrm{M_\odot}$. What fraction of stars have $M > 5\,\mathrm{M_\odot}$? And what fraction of stars have $M < 1\,\mathrm{M_\odot}$?

3. Show that an error or uncertainty of 0.1 magnitudes in the distance modulus is roughly equivalent to a 5% error in the luminosity distance d_L.

4. The Malmquist bias, which can affect all measurements (not just distance measurements!) when using samples that are defined in terms of a flux limit (i.e., an apparent magnitude limit). If we observe stars down to a fixed apparent brightness, we do not get a fair mixture of all of the stars in the sky (or observed volume), but we include more of the most luminous stars simply because we miss stars which are fainter than the depth of our observations.

a) Consider a model of sky with G stars (i.e., like the Sun) in three regions: A ($70 < d(\mathrm{pc}) < 90$) , B ($90 < d(\mathrm{pc}) < 110$), and C ($110 < d(\mathrm{pc}) < 130$). If the number density of stars is uniform, and you have 100 stars in region B, how many stars are there in regions A and C (round to the nearest integer)?

b) Not all G stars have the same luminosity: the variation corresponds to about 0.3 mag around the solar value ($M_V^{\odot} = 4.78\,\mathrm{mag}$). What fractional change in luminosity is this?

c) Now write a piece of code that generates a star sample as the one described above, with random absolute V-band magnitudes around the solar value, considering the dispersion of 0.3 mag. Compute the apparent magnitude for each star. For simplicity, assume that all the stars in region A are at a distance of 80 pc, those in B at 100 pc, and those in C at 120 pc. In your problem set hand in, include a table showing only the first ten entries of your random star list in each region A, B and C, quoting (M_V, m_V) for each star.

d) Now *observe* your sky. Assume that you can only see stars with apparent magnitude $m_V \leq 10\,\mathrm{mag}$. How many of these do you have in your randomly generated sample in c)? Consider that these stars with $m_V \leq 10\,\mathrm{mag}$ are your *sample*. What is the mean absolute magnitude of your sample? How does this compare to the mean absolute magnitude of *all* the stars? Why?

e) What is the average distance of the stars in your sample? Assume that your sample stars have the same average absolute magnitude as the full set of stars in (a,b,c). Now calculate the distances from their apparent magnitudes: what is their average distance? Why is this different from the true average distance of the sample?

5. Consider the ring around supernova 1987A and assume that it is circular. The first light signal from the ring is received 83 days after the supernova light appearance (at time t_0), and the last signal from the ring appears 395 days after t_0.

a) Find the radius R of the ring in light days and its inclination angle i in degrees.

b) Now assume that the ring measures $1.62'' \times 1.23''$ on the sky. Find the distance to the supernova.

c) At its brightest, the supernova had apparent magnitude $m_V \approx 3$. Show that its luminosity was $L_V \approx 1.4 \times 10^8\,\mathrm{L}_\odot$.

Tip: the Sun's V-band absolute magnitude is $M_V^\odot = 4.78$.