From Initial Perturbations to Dark Matter Haloes

Karina Caputi

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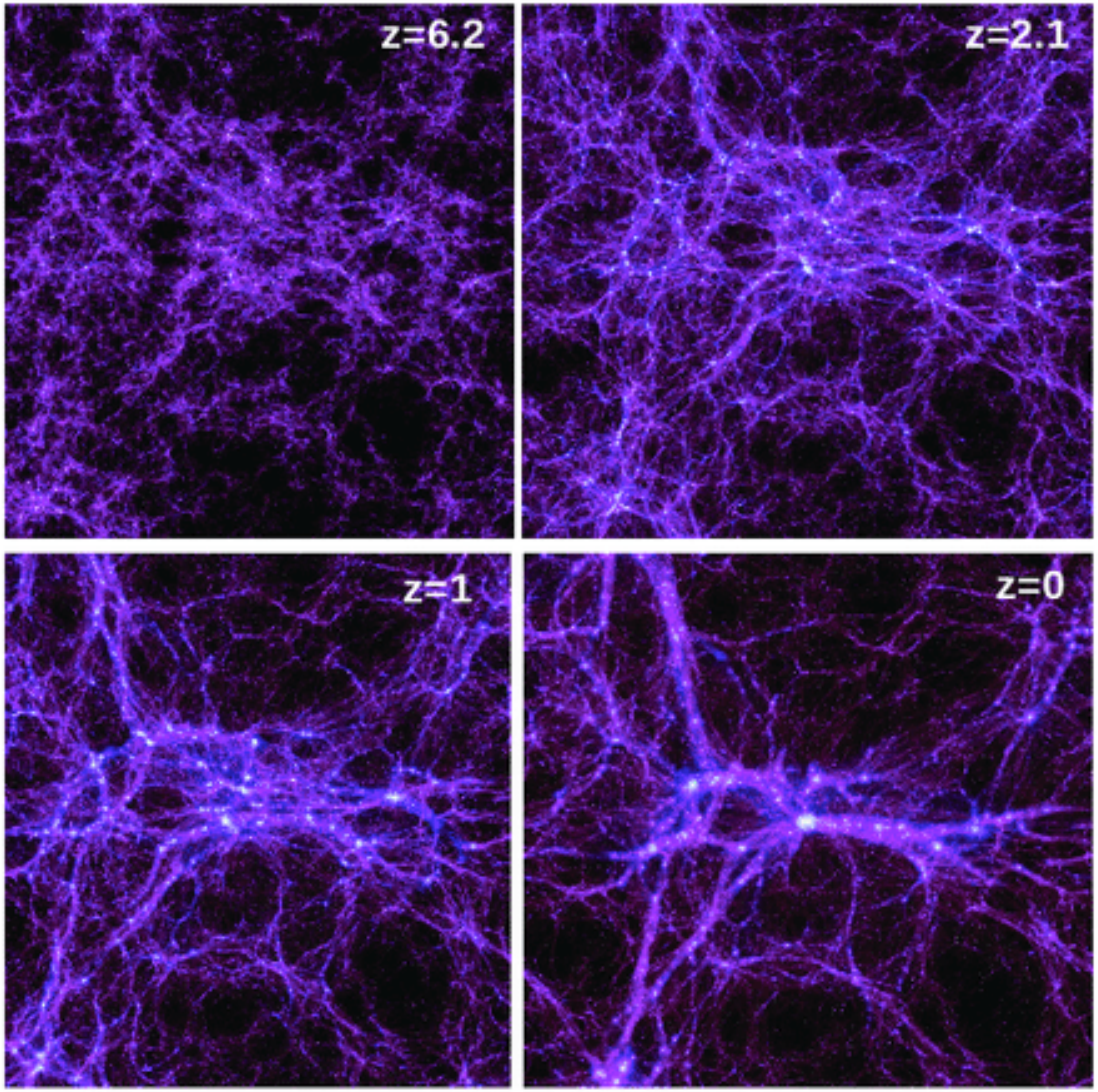
Rijksuniversiteit Groningen

Growth of structures in ΛCDM



Linear growth of perturbations **(valid before recombination)**

The key equations of motion for a fluid in a gravitational field (in Lagrangian co-ordinates) are:

Non-relativistic regime for small perturbations

$$\frac{d\rho}{dt} = -\rho(\nabla.\mathbf{v}) \;\; (1)$$

Continuity equation (conservation of mass)

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\nabla p - \nabla\phi \;\; (2)$$

Equation of motion (Euler's equation)

$$\nabla^2\phi = 4\pi G\rho \;\; (3)$$

Gravitational potential (Poisson's equation)

Here, $\rho, p \,\mathrm{and}\, \mathbf{v}$ represent the density, pressure and velocity of the fluid.

In comoving co-ordinates, **x** = a(t) **r,** where r is co-moving co-ordinate distance and a(t) is the scale factor.

$$\Rightarrow \delta x = \delta[a(t)\mathbf{r}] = \mathbf{r}\delta a(t) + a(t)\delta\mathbf{r}$$

$$\Rightarrow \mathbf{v} = \frac{\delta\mathbf{x}}{\delta t} = \mathbf{r}\frac{da}{dt} + a\frac{d\mathbf{r}}{dt}$$

Uniform and isotropic Universe

Linear growth of perturbations

Now perturb these equations by a very small amount

$$\mathbf{v} = \mathbf{v_0} + \delta\mathbf{v}, \rho = \rho_0 + \delta\rho, \phi = \phi_0 + \delta\phi, p = p_0 + \delta p$$

Taking co-moving divergence of Euler's equation and time-derivative of continuity equation, combining these and using Poisson's equation, we get a wave equation. Assuming adiabatic perturbations, the pressure and density are related via sound speed as $\delta p = c_s^2 \delta\rho$

Using $\Delta = \delta\rho/\rho$, we get

density contrast

$$\frac{d^2\Delta}{dt^2} + 2\frac{\dot{a}}{a}\frac{d\Delta}{dt} = \frac{c_s^2}{\rho_0 a^2}\nabla_c^2 \delta\rho + 4\pi G \delta\rho$$

see Longair's book (ch.11) for a complete derivation

We then seek wave solutions of the form $\Delta \propto exp\, i(\mathbf{k_c}.\mathbf{r} - \omega t)$

where k_c is wave vector in co-moving co-ordinates which yields the wave equation:

$$\boxed{\frac{d^2\Delta}{dt^2} + 2\frac{\dot{a}}{a}\frac{d\Delta}{dt} = \Delta(4\pi G\rho_0 - k^2 c_s^2)}$$

This linear regime is valid when density contrast is small (redshift z>1000)

Jeans' instability: static case

If we set *da/dt = 0*, then the solution is $\Delta = \Delta_0 \exp i(\mathbf{k}\cdot\mathbf{r} - \omega t)$

where $\omega^2 = k^2 c_s^2 - 4\pi G\rho$

Static medium

Jeans criterion: If $w^2 > 0$: we have an oscillating solution

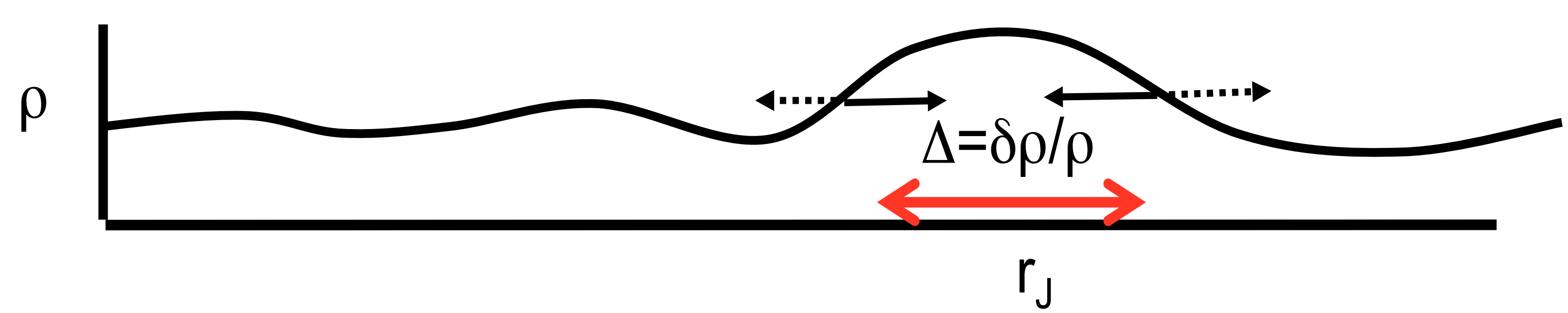
(the pressure gradient is sufficient to support the region, $p = c_s^2 \rho$)

If $w^2 < 0$: we have an exponentially growing (or decaying) solution

(the gravitational attraction is stronger than the pressure.)

In terms of wavelength: $\lambda > \lambda_J = \frac{2\pi}{k_J} = c_s\left(\frac{\pi}{G\rho}\right)^{1/2}$

No expansion considered here!



Jeans' instability in an expanding Universe

$$\boxed{\frac{d^2\Delta}{dt^2} + 2\left(\frac{\dot{a}}{a}\right)\frac{d\Delta}{dt} = \Delta(4\pi G\varrho_0 - k^2 c_s^2)}$$

Wave equation for Δ

Suppose we can neglect the pressure term, and assume for simplicity Ω= 1 and Ω_Λ= 0, then $4\pi G\varrho = \frac{2}{3t^2}$ and $\frac{\dot{a}}{a} = \frac{2}{3t}$.

Einstein-de Sitter model

Therefore

$$\boxed{\frac{d^2\Delta}{dt^2} + \frac{4}{3t}\frac{d\Delta}{dt} - \frac{2}{3t^2}\Delta = 0\,.}$$

If we seek solutions of power-law form → $\Delta = at^n$ then n = - 1, 2/3. The growing solution is: $\Delta \propto t^{2/3} \propto a = (1+z)^{-1}$

This implies that

$$\boxed{\Delta = \frac{\delta\varrho}{\varrho} \propto (1+z)^{-1}\,.}$$

Growth of perturbations is not exponential.

↓

Problem for forming galaxies by gravitational collapse within this model

Evolution of perturbations

fluctuations grow in a matter dominated universe like: (after recombination)

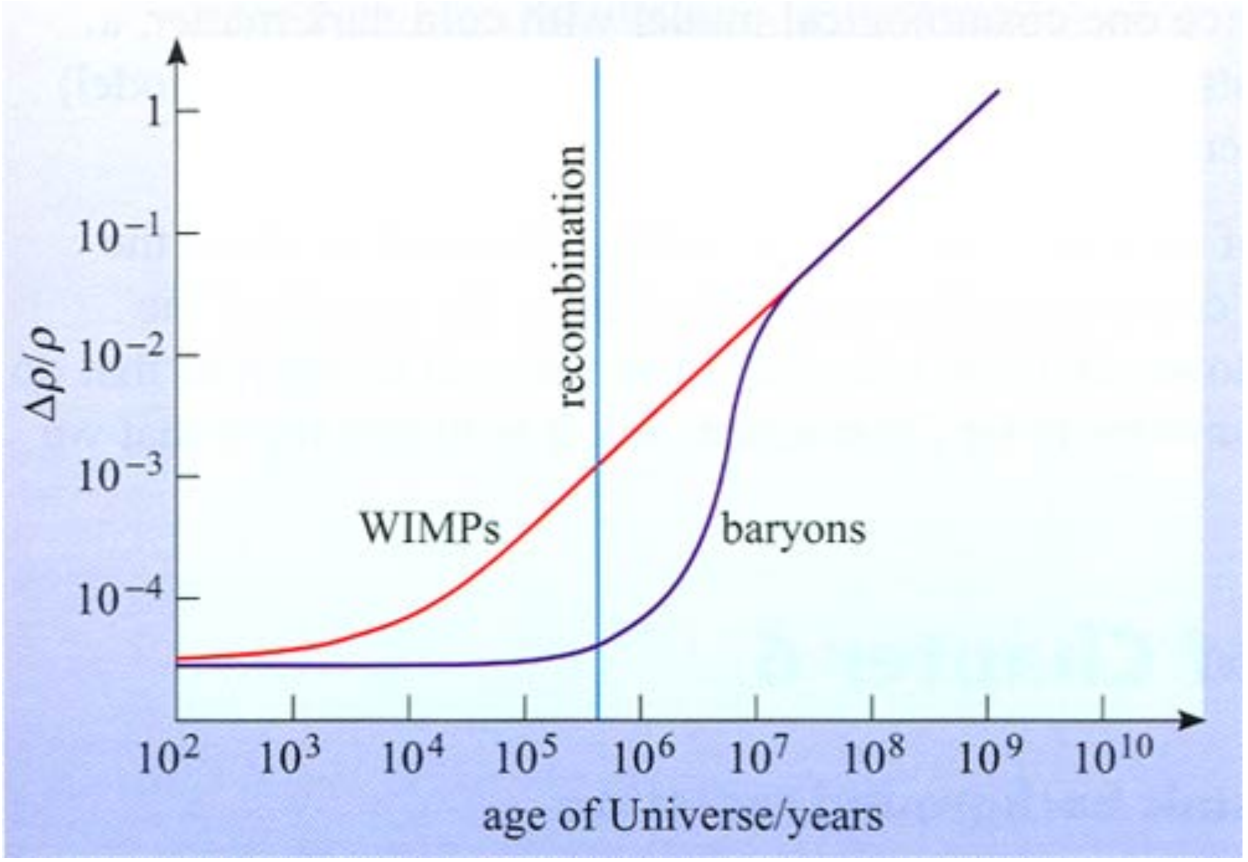
$$\delta \sim (1+z)^{-1} \quad \text{if } \Omega = 1$$
$$\delta \sim \text{const} \quad \text{if } \Omega \sim 0$$

One can show with a similar calculation that in a radiation dominated universe ($z>z_{eq}\sim z_{CMB}$):

$$\delta \sim (1+z)^{-2} \quad \Omega = 1$$

This applies only for fluctuations that fulfilled the Jeans criterion and could grow, i.e. fluctuations of large enough scales. On small scales the density fluctuations only oscillate.

Dark Matter and baryons



Baryons follow Dark Matter

Since DM does not interact through any force other than gravity, DM density contrasts keep growing steadily. On the other hand, matter and radiation are coupled till recombination - it is only after this baryons can start collapsing into potential wells and then quickly match DM contrast.

Cold dark matter

- Devised to explain rotation curves and missing mass in clusters.
- Also required to explain large-scale structure and CMB.

  → Numerical simulations on cosmological scales

- Power law of initial fluctuations set at CMB surface.
- Assumed non-interacting except via gravity.
- Growth via gravity alone.

- Halo build-up via hierarchical merging.
- Numerical simulations can now predict dark matter distributions very well
- Robust prediction of Large Scale Structure.
- Testable under assumption light traces matter

Standard ΛCDM model

Basic assumptions:

Universe is homogeneous & isotropic on large scales (Friedmann-Robertson-Walker metric)

Geometry of Universe is flat, as predicted by inflation

Dark matter is cold

(gravit. potential is independent of k)

Cosmological parameter values: H_0=67.8 (km/s)/Mpc ; $\Omega_m = 0.31$; $\Omega_\Lambda = 0.69$; $\Omega_b = 0.048$

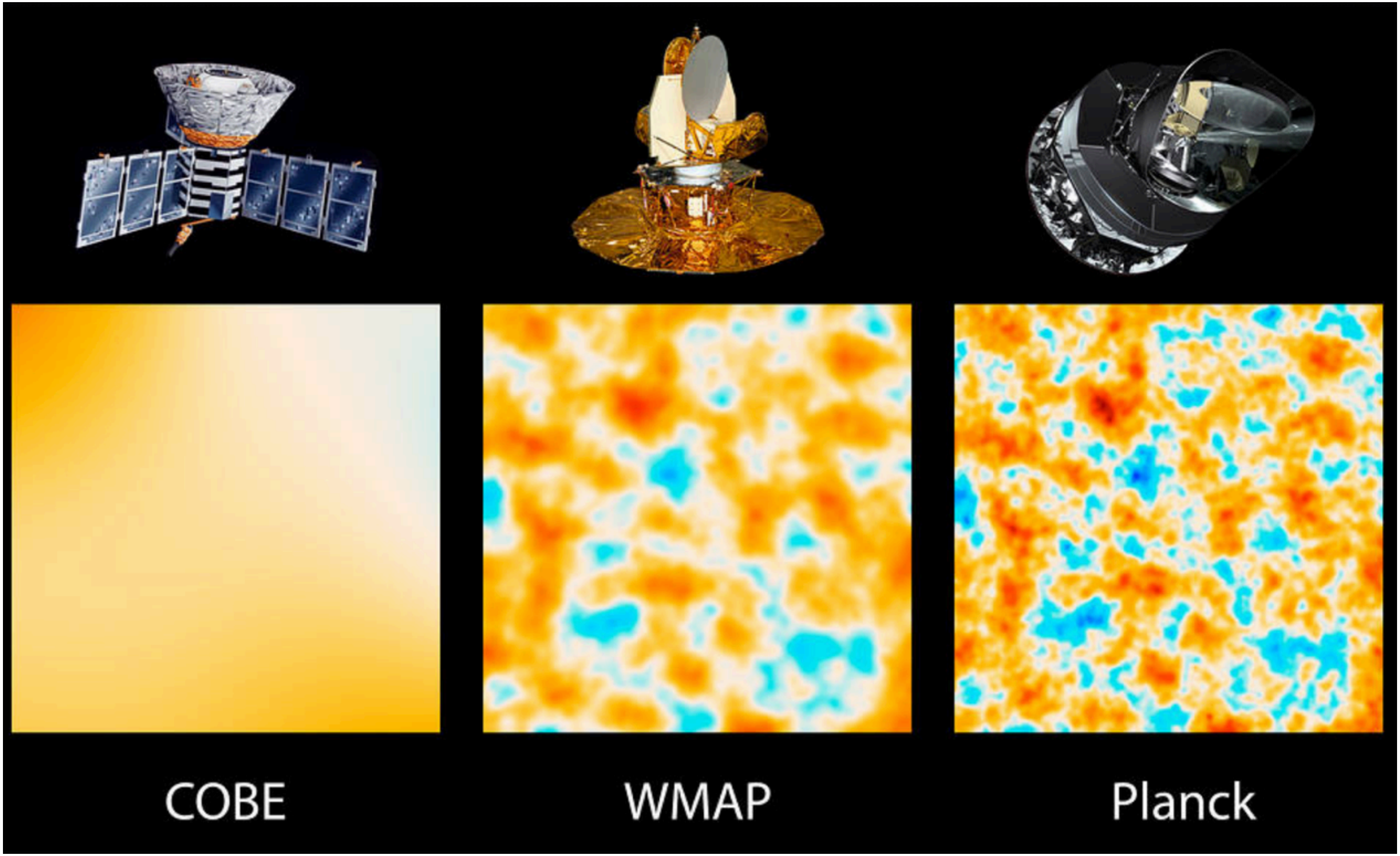
Initial density fluctuations in density were very small ($|\delta| << 1$) and described by random Gaussian field

Initial power spectrum of density fluctuations was approx. $P(k) \sim k^n$, with n=1

Harrison - Zel'dovich spectrum

Scale-invariant

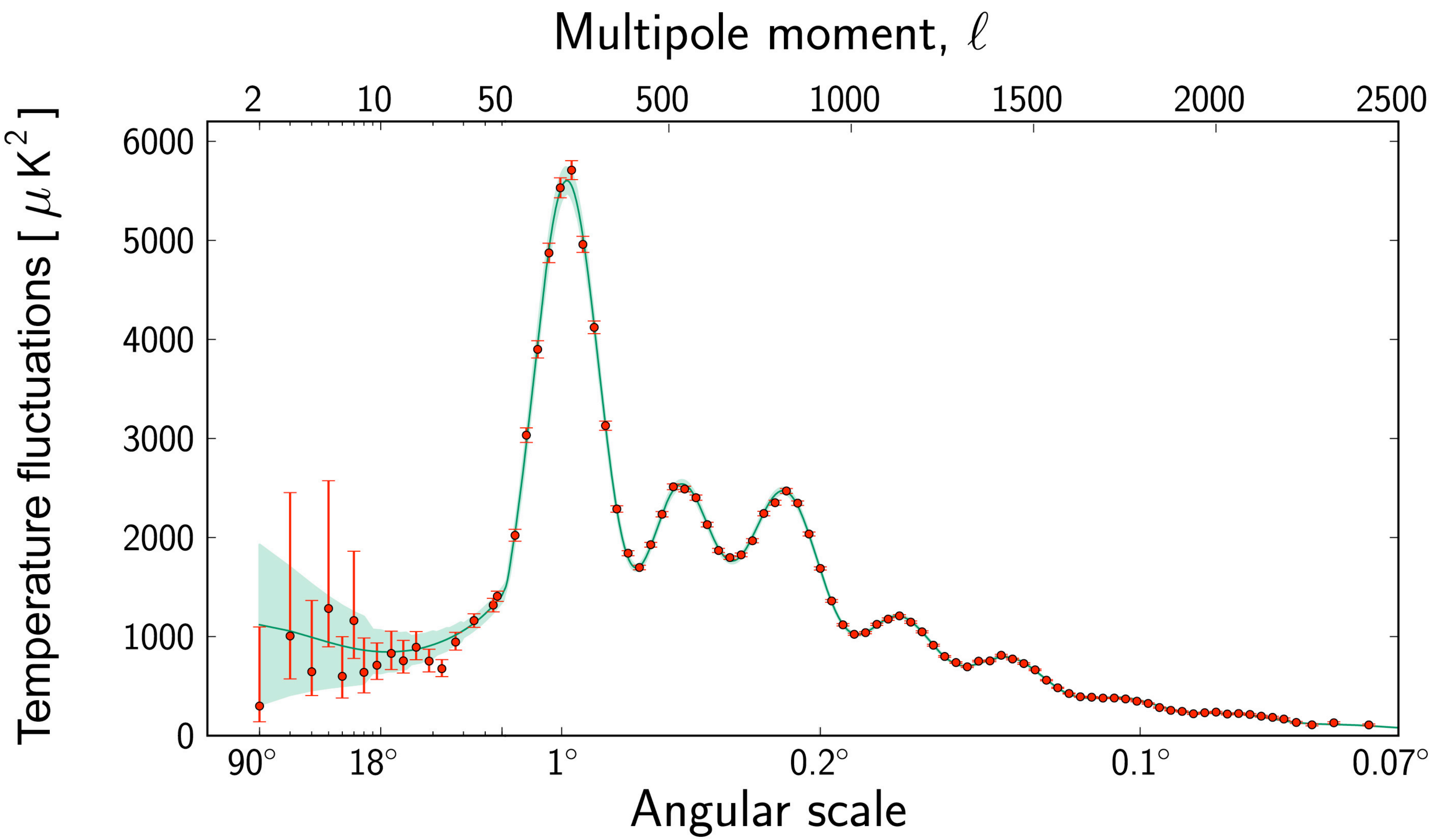
The CMB



$dT/T \sim 10^{-4}$

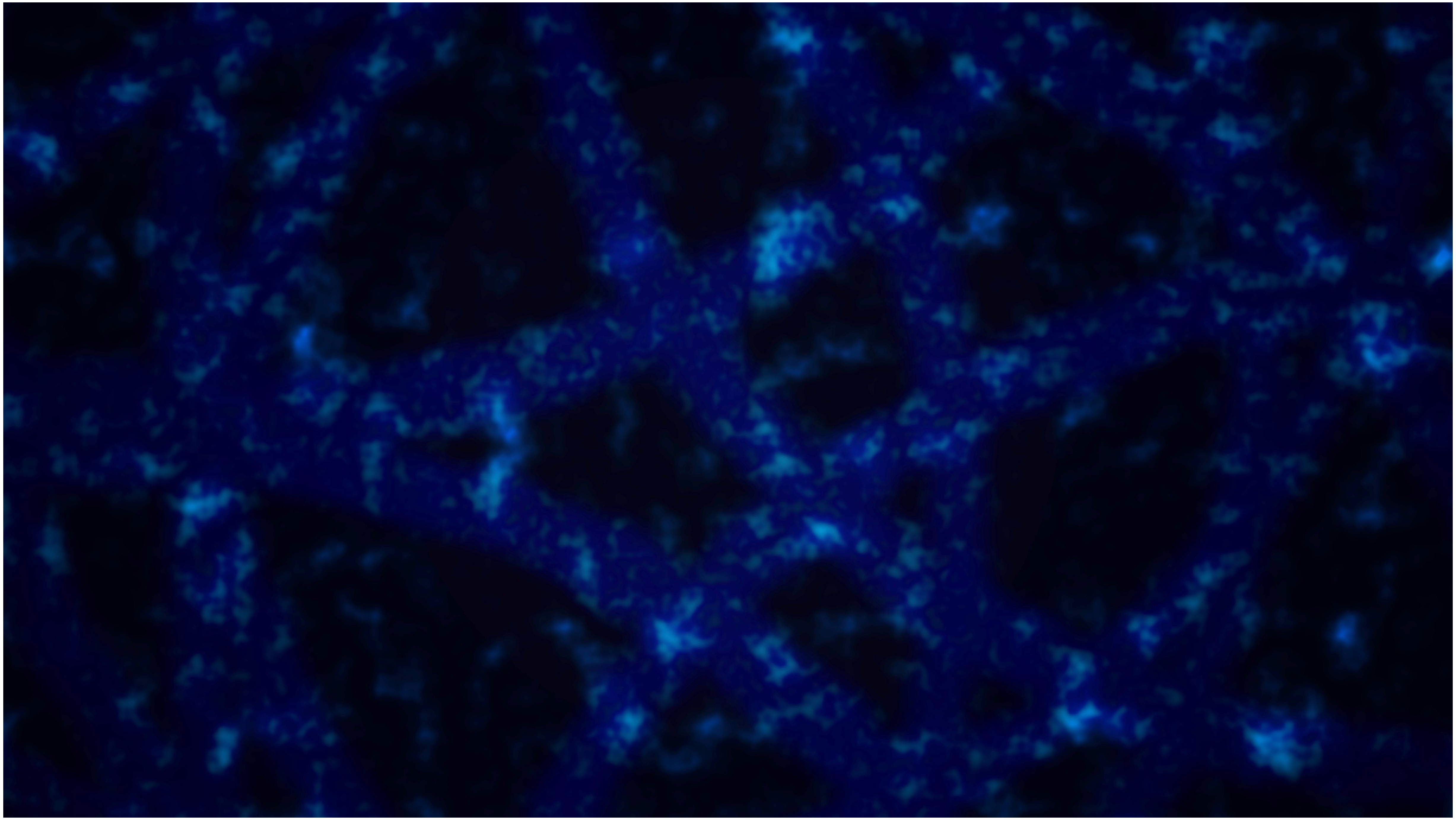
Credit: Frank van den Bosch's lectures

Power spectrum measurement by Planck and LCDM "fit"



see http://background.uchicago.edu/~whu/Presentations/warnerprint.pdf

Baryon Acoustic Oscillations
(BOE)



Tiny variations in density excited sound waves that rippled through the fluid. When the universe was about 400,000 years old, the waves froze where they were. Slightly more galaxies formed along the ripples. These frozen ripples stretched as the universe expanded, increasing the distance between galaxies. Astronomers

Credit: NASA's Goddard Space Flight Center

See video explaining BAO at

https://svs.gsfc.nasa.gov/13768

This animation explains how BAOs arose in the early universe and how astronomers can study the faint imprint they made on galaxy distribution to probe dark energy's effects over time. In the beginning, the cosmos was filled with a hot, dense fluid called plasma. Tiny variations in density excited sound waves that rippled through the fluid. When the universe was about 400,000 years old, the waves froze where they were. Slightly more galaxies formed along the ripples. These frozen ripples stretched as the universe expanded, increasing the distance between galaxies. Astronomers can study this preferred distance between galaxies in different cosmic ages to understand the expansion history of the universe.

Credit: NASA's Goddard Space Flight Center

The post-recombination era

The post-recombination era

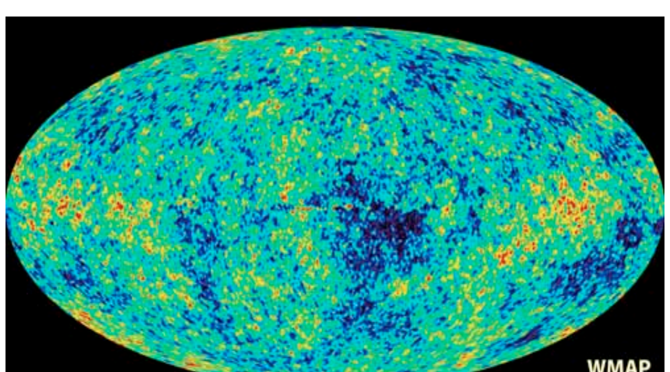
Growth of perturbations is not linear any more at z<1000

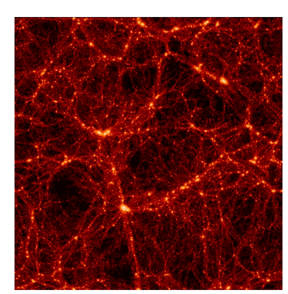
<u>Three main periods:</u>

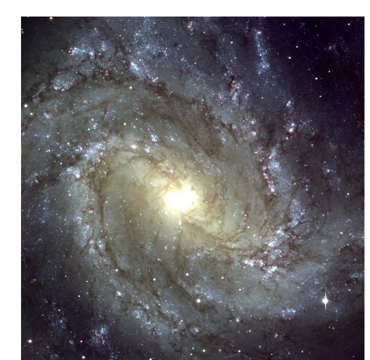
* between z=1000 (recombination) and start of reionisation **Dark ages**

* reionisation period (between z=? and z=7) **Stars and galaxies starting to form and evolve**

* after reionisation (z<7) **Galaxy evolution is ‘easily’ observable**







Non-linear collapse of density perturbations

The density of a luminous galaxy at a radius of a few kpc is ~10^5 times larger than critical density, ρ_c (=$1.3599 \times 10^{11}\ h_7^{-2}\ M_\odot \mathrm{Mpc}^{-3}$).

Thus, galaxy formation involves highly non-linear density fluctuations, and our linear formalism must be supplemented by approximate analytic arguments and numerical simulations to follow structure formation into the non-linear regime.

Although full development of gravitational instability cannot be solved exactly without N-body techniques there are some very useful special cases and approximations that help to understand the general case

Dark matter halos: mass function

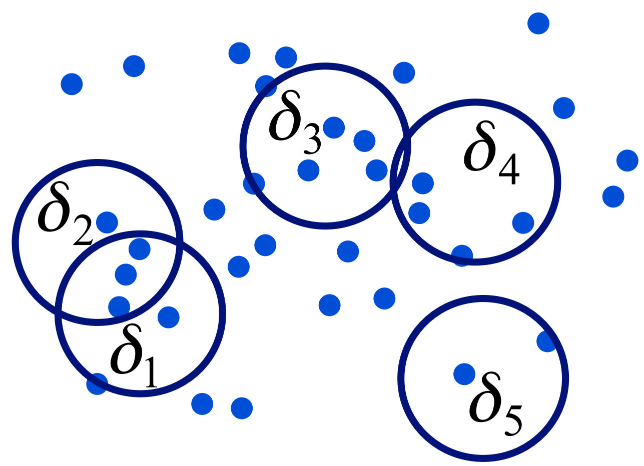
Sec. 16.3.; Longair book

White & Rees (1978) were the first to suggest that the formation process of galaxies must be in two stages – baryons condense within the potential wells defined by the collisionless collapse of dark matter haloes.

This simplifies the problem in many ways – since the complex fluid-mechanical and radiative behaviour of the gas can be initially ignored.

Smoothly fluctuating density field; randomly scattered equal volume spheres, each has some over density δ. Some of these volumes will have a large enough over-density that they will eventually collapse and form gravitationally bound objects.

What is the mass function of these objects at any given cosmic epoch?



Press-Schechter Mass Function

Analytic estimate of the mass function F(M), of bound objects in the Universe, defined such that N(M)dM is the co-moving number density of objects between M and M+dM.

Objective was to provide an analytic formalism for the process of structure formation once the density perturbations had reached an amplitude that they could be considered to have formed bound objects.

Assumption that primordial density perturbations were **Gaussian fluctuations.** Thus, the probability distribution of the amplitudes of the perturbations could be described by Gaussian function:

$$p(\Delta) = \frac{1}{\sqrt{2\pi}\sigma(M)} exp\Big[- \frac{\Delta^2}{2\sigma^2(M)} \Big]$$

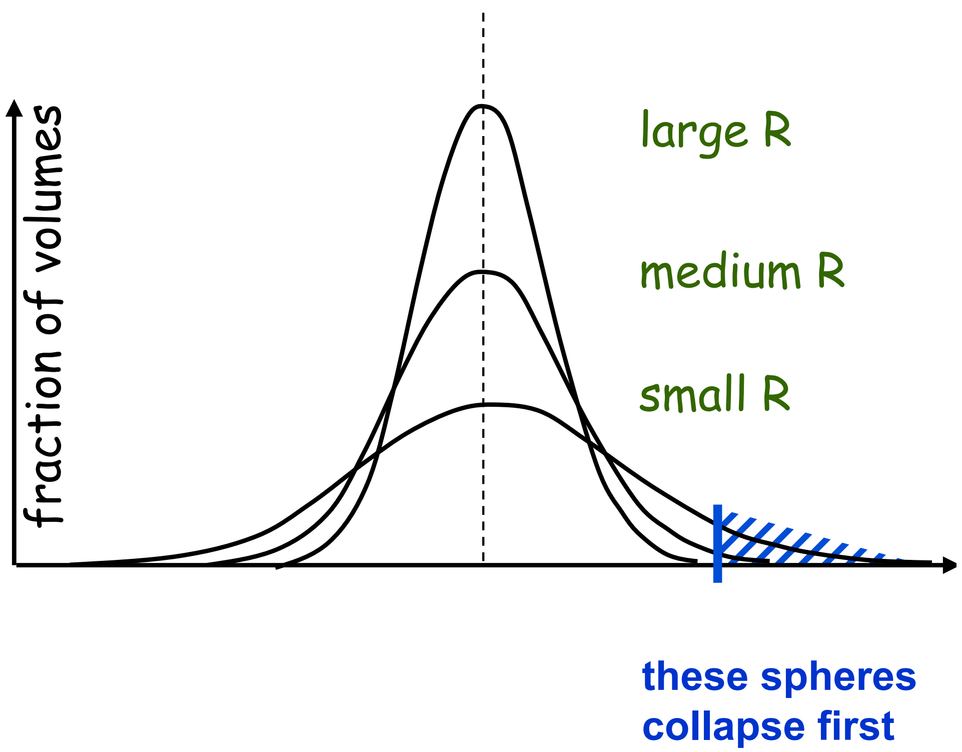
$$\Delta = \delta\rho/\rho$$

is the density contrast associated with perturbations of mass M.

Being a gaussian distribution, the mean value is zero, with a finite variance $\sigma^2(M)$

$$\langle \Delta^2 \rangle = \Big\langle \Big(\frac{\delta\rho}{\rho}\Big)^2 \Big\rangle = \sigma^2(M)$$

This is exact statistical description of the perturbations implicit in the analysis of early universe.



Press-Schechter Mass Function

assumption is that when the perturbations have developed an amplitude greater than some critical value Δ_c, they evolved rapidly into bound objects with mass M.

The problem is thus completely defined, assume perturbations had power law spectrum $P(k) = k^n$ and we know the rules that describe the growth of the perturbations with cosmic epoch. Press & Schechter assumed that the background world model was critical Einstein-deSitter model ($\Omega_0=1$ $\Omega_\Lambda=0$) so that the perturbations developed as $\Delta \propto a \propto t^{2/3}$ right up to present epoch. Straightforward extension to LCDM.

$$\mathrm{F(M)} = \frac{1}{\sqrt{2\pi}\sigma(\mathrm{M})}\int_{\Delta_c}^{\infty}\exp\left[-\frac{\Delta^2}{2\sigma^2(\mathrm{M})}\right]\mathrm{d}\Delta = \frac{1}{2}[1-\Phi(\mathrm{t_c})]$$

$$t_c = \Delta_c/\sqrt{2}\sigma$$

$\Phi(x)$ is the probabilty integral, defined by

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$$

The mean square density perturbations on mass scale M are defined as being related to the power spectrum in this form:

$$\sigma^2(M) = \left\langle\left(\frac{\delta\rho}{\rho}\right)^2\right\rangle = \langle\Delta^2\rangle = AM^{-(3+n)/3}$$

A, constant

Can now express t_c in terms of mass distribution

$$t_c = \frac{\Delta_c}{\sqrt{2}\sigma(M)} = \frac{\Delta_c}{\sqrt{2}A^{1/2}}M^{(3+n)/6} = \left(\frac{M}{M^\star}\right)^{(3+n)/6}$$

Here we introduce a reference mass: $M^\star = (2A/\Delta_c^2)^{3/(3+n)}$

Press-Schechter Mass Function

Since the amplitude of the perturbation $\Delta(M)$ grew as $\Delta(M) \propto a \propto t^{2/3}$ it follows that

$$\sigma^2(M) = \Delta^2(M) \propto t^{4/3} \;\; \text{thus, A} \propto \text{t}^{4/3} \;\; \text{and} \quad M^\star \propto A^{3/(3+n)} \propto t^{4/(3+n)}$$

$$M^\star = M_0^\star \left(\frac{t}{t_0}\right)^{4/(3+n)}$$

value at present epoch

The fraction of perturbations with masses in the range M to M+dM is:

$$dF = \frac{\partial F}{\partial M} dM$$

In the linear regime, the mass of the perturbation is $M = \bar{\rho} V$

mean density of background model

Once the perturbation became non-linear, collapse begins and the result is a bound object of mass M,

The space density N(M)dM of these masses is 1/V:

$$N(M)dM = \frac{1}{V} = -\frac{\bar{\rho}}{M}\frac{\partial F}{\partial M} dM$$

F decreasing fn of increasing M

Combining with:

$$\mathrm{F(M)} = \frac{1}{\sqrt{2\pi}\sigma(\mathrm{M})} \int_{\Delta_\mathrm{c}}^{\infty} \exp\left[-\frac{\Delta^2}{2\sigma^2(\mathrm{M})}\right] \mathrm{d}\Delta = \frac{1}{2}[1 - \Phi(\mathrm{t_c})]$$

given that

$$\frac{d\Phi}{dx} = \frac{2}{\sqrt{\pi}} e^{-x^2}$$

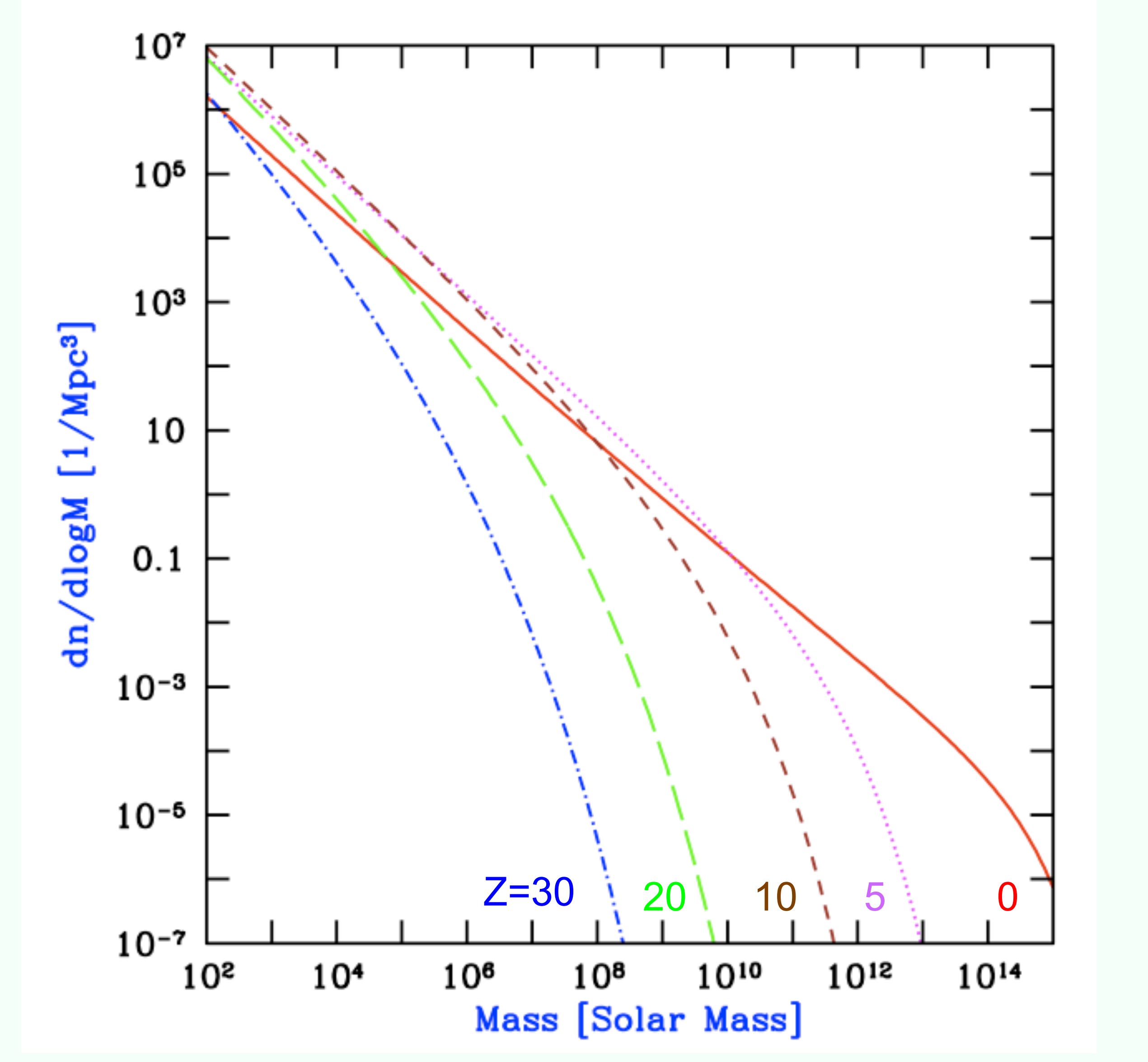
Mass distribution and how it evolved with time

$$\boxed{\mathrm{N(M)} = \frac{1}{2\sqrt{\pi}}\left(1 + \frac{\mathrm{n}}{3}\right)\underbrace{\frac{\bar{\rho}}{\mathrm{M}^2}\left(\frac{\mathrm{M}}{\mathrm{M}^*}\right)^{(3+\mathrm{n})/6}}_{\text{power law}} \underbrace{\exp\left[-\left(\frac{\mathrm{M}}{\mathrm{M}^*}\right)^{(3+\mathrm{n})/3}\right]}_{\text{exponential}}}$$

$$M^\star = M_0^\star \left(\frac{t}{t_0}\right)^{4/(3+n)}$$

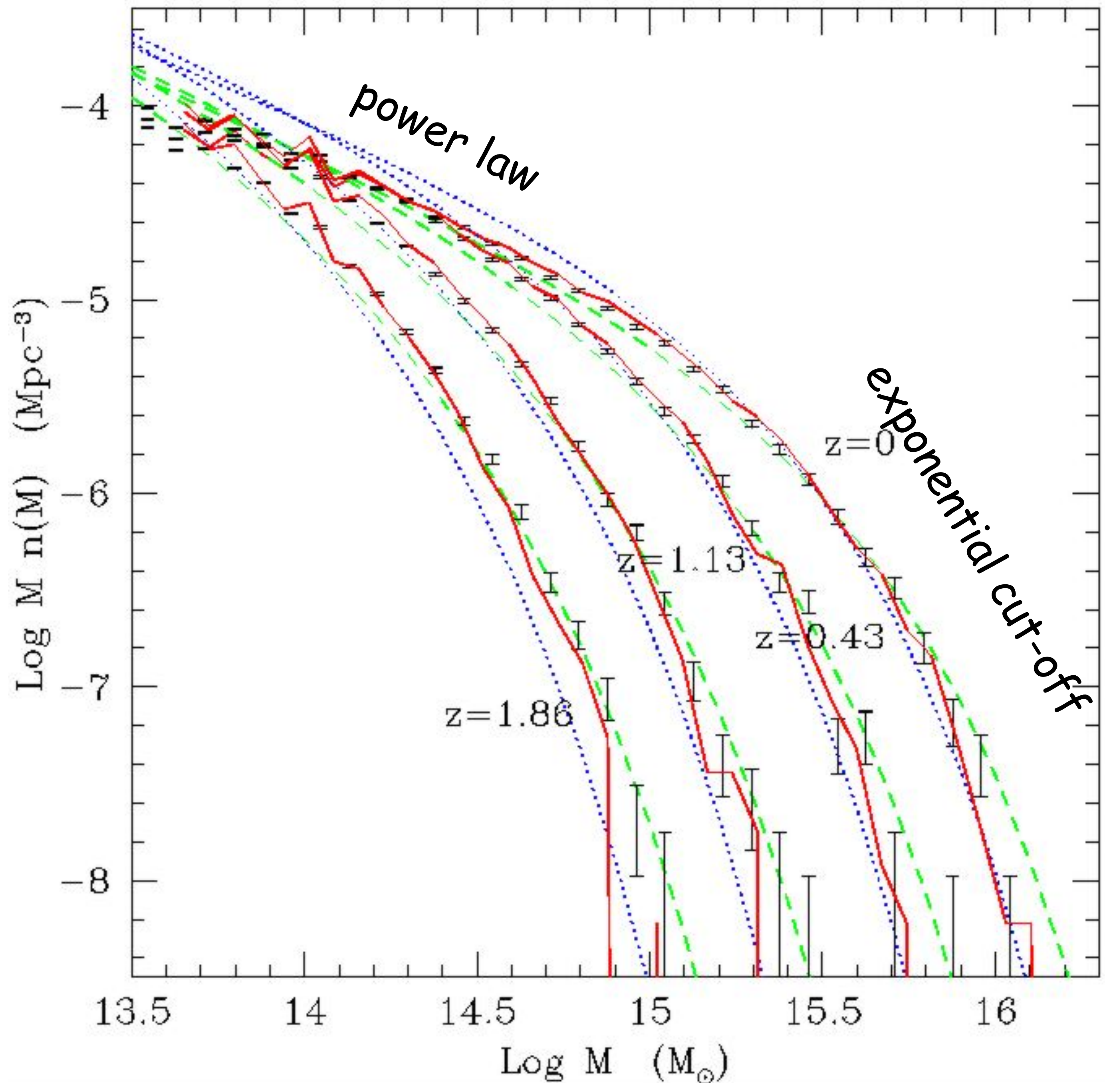
This formalism only gives half the mass density being condensed into bound objects - because only the positive density fluctuations of a symmetric gaussian develop into bound systems, and linear approximations are used. Press & Schechter suggested that their mass spectrum should be multiplied by a factor 2 to take account of accretion during non-linear stage.

The halo mass function



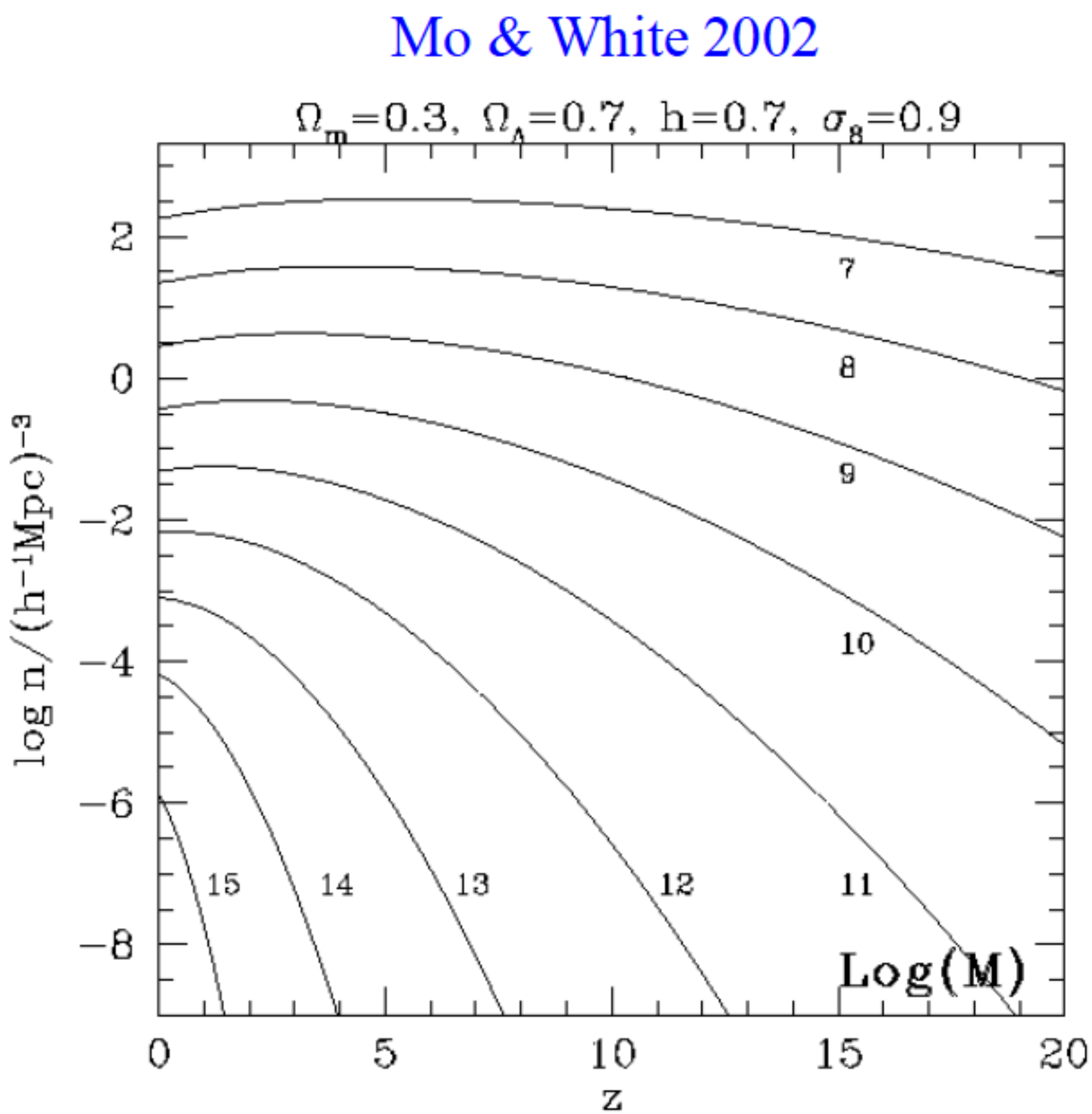
the halo mass function is the number density of collapsed, bound, virialised structures per unit mass, as a function of mass and redshift

Press-Schechter compared to simulations



solid red lines: simulations
blue dotted: Press-Schechter
green dashed: extended Press-Schechter
(takes into account non-sphericity
of proto halos.)

Evolution of halo abundance



Abundance of rich cluster halos drops rapidly with z

Abundance of Milky Way mass halos drops by less than a factor of 10 to z=5

$10^9 M_\odot$ halos are almost as common at z=10 as at z=0

White, IMPRS Lecture Notes 2013

Dark halo density profiles

The halo profiles for different masses and cosmologies have the same "universal" functional form: $r \sim r^{-1}$ and $r \sim r^{-3}$ at small/large radii

$$\rho(r) = \frac{\rho_c \delta_c}{(r/R_s)(1 + r/R_s)^2}$$

NFW profile

Navarro, Frenk and White 1996 ApJ, 462, 563

Navarro, Frenk and White 1997 ApJ, 490, 493

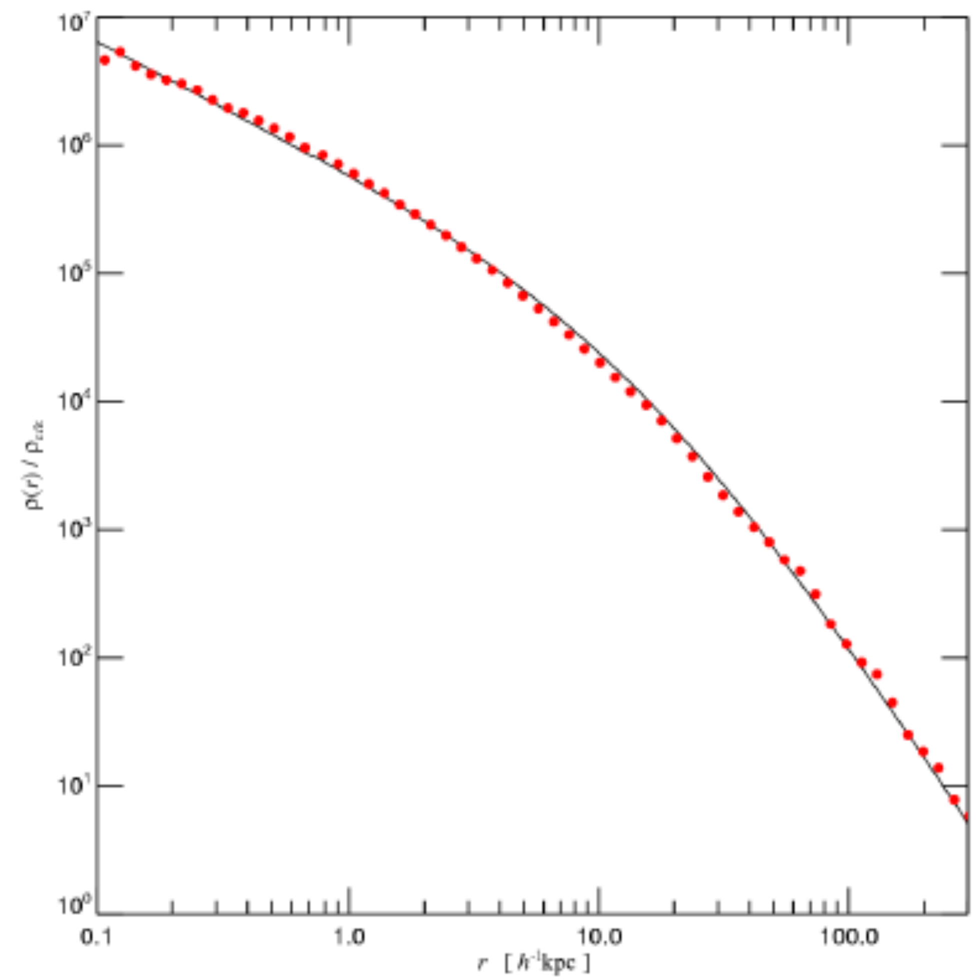
$$\delta_c = \frac{200}{3} \frac{c^3}{[ln(1+c) - c/(1+c)]}$$

The concentration parameter $c = R_{vir}/R_s$

Scaled density profiles of most and least massive galaxies look very similar.

Simple NFW works extremely well...

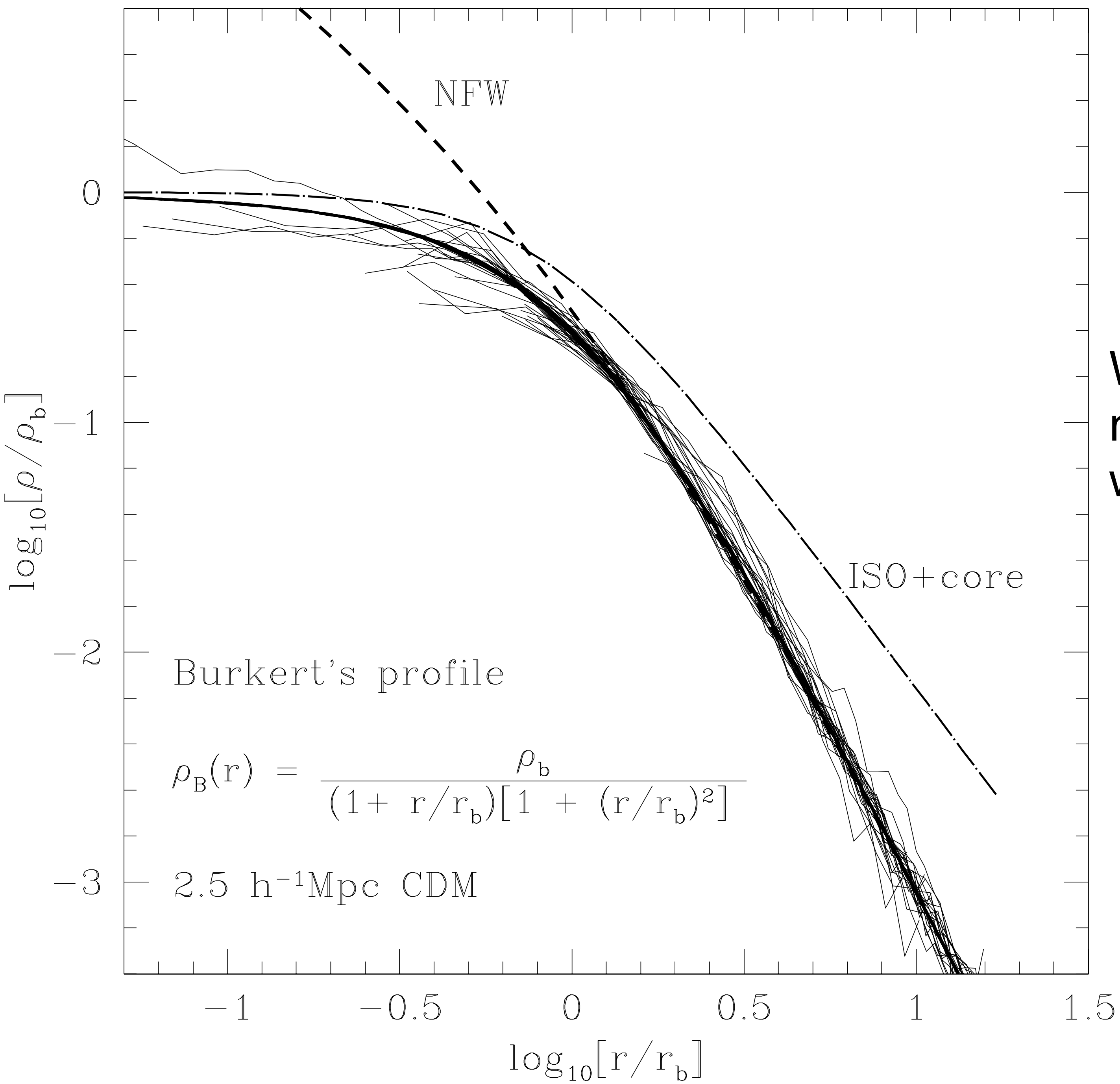
600 kpc



$N_{200} \sim 3 \times 10^7$

Navarro et al 2006

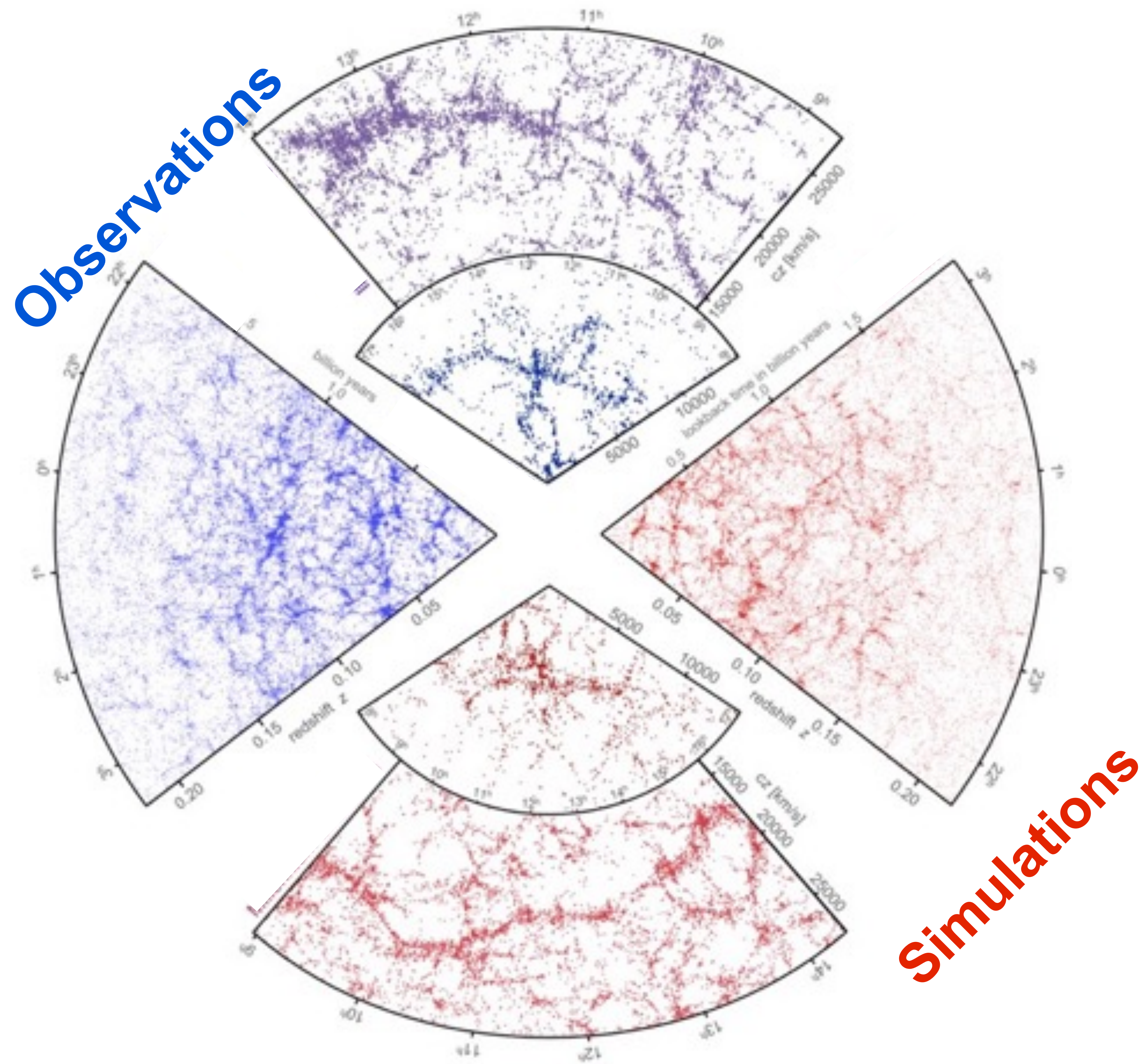
Other density profiles..



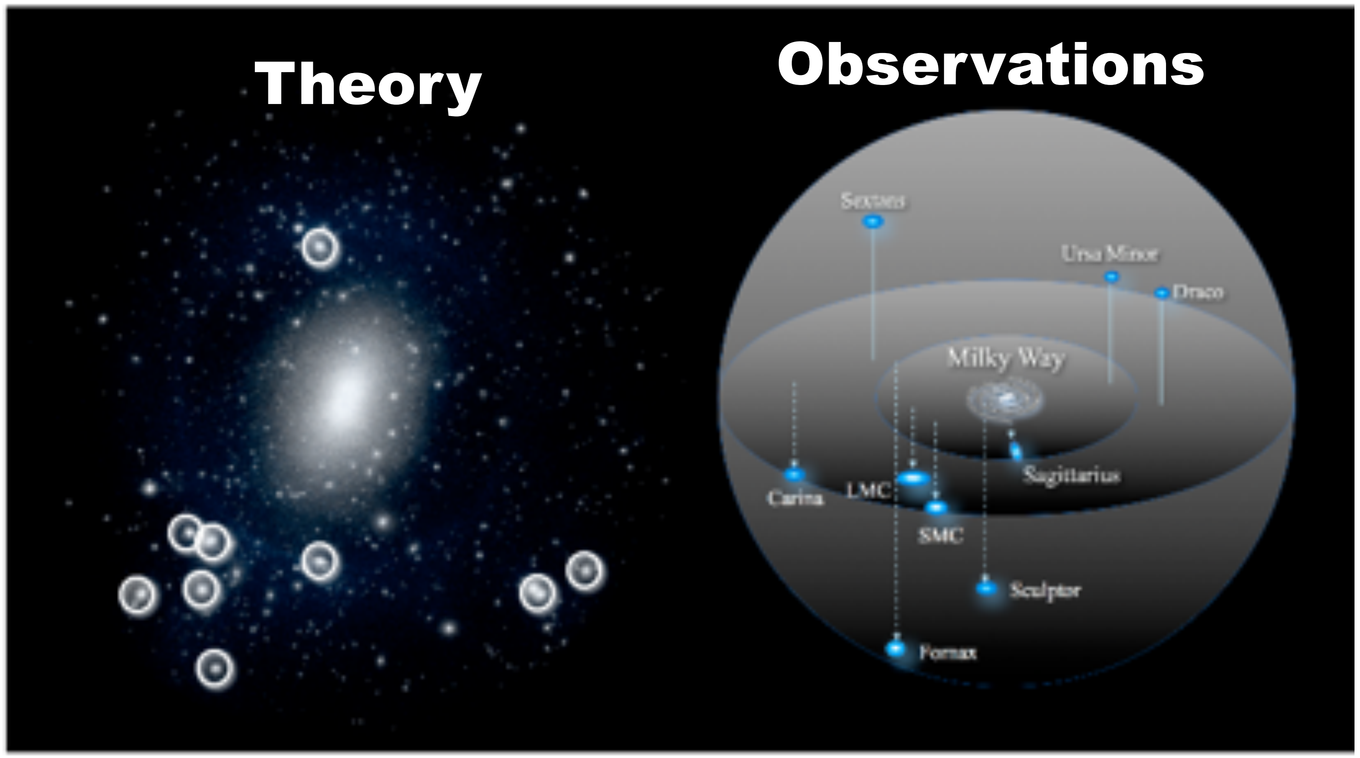
While Burkert core better for low mass dwarfs, NFW generically works better

https://cds.cern.ch/record/384623/files/9904159.pdf

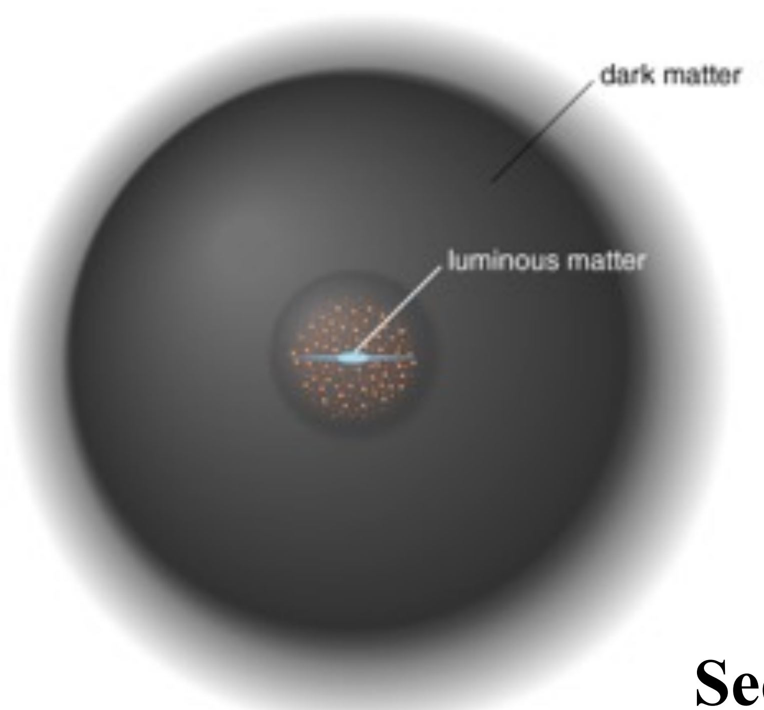
At large scales CDM theory & observations in excellent agreement..



However, CDM has a number of problems at small (galaxy) scales



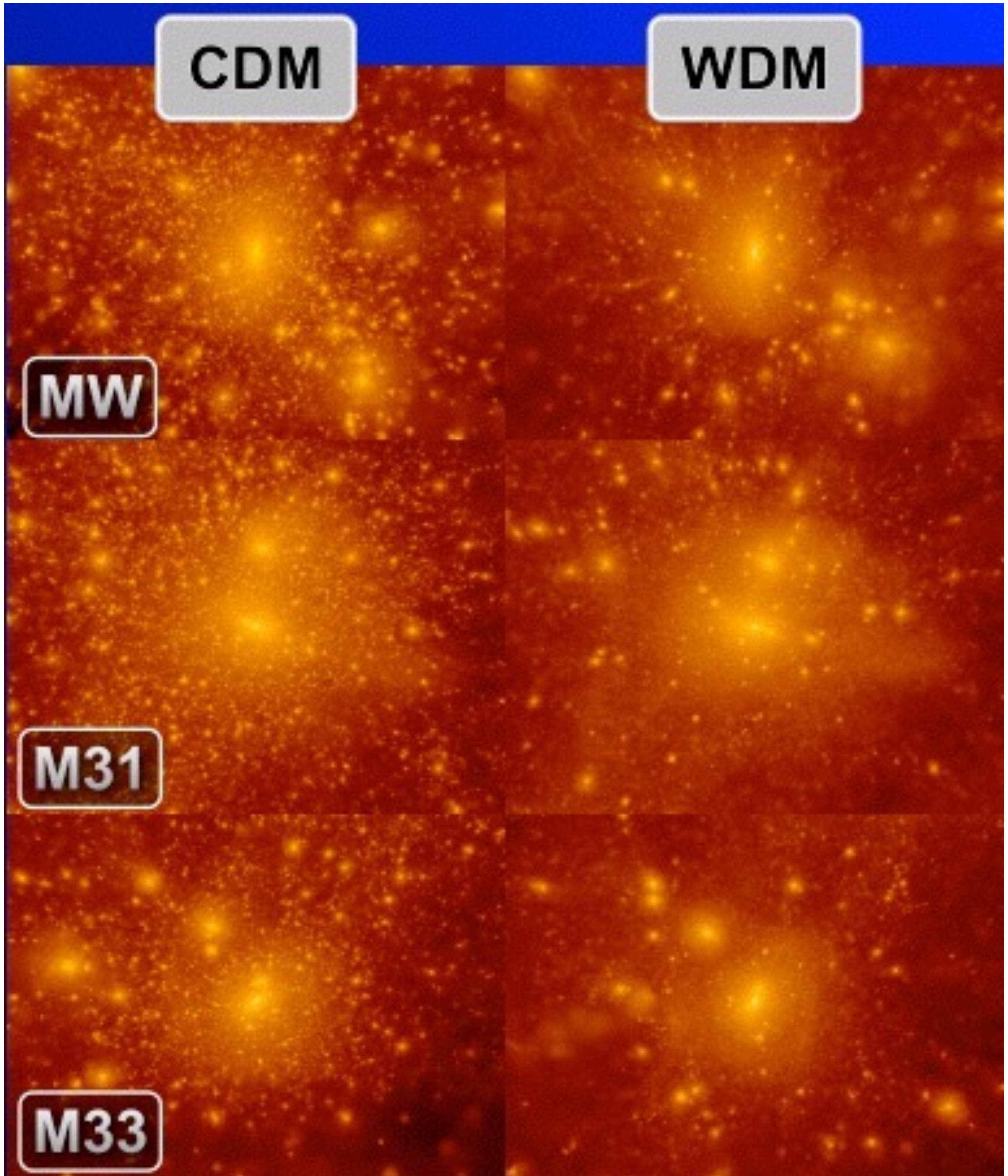
1. **"Missing satellite problem" - where are all the massive satellites theory is predicting? (Klypin+1999; Morre+1999)**
2. **"Too big to fail problem" - where are massive satellites that could not have been quenched by feedback? (Boylan-Kolchin+2012)**



"Cuspy halo problem" - halos too concentrated compared to observations (Navarro+1997; Morre+1999)

See Bullock & Boylan-Kolchin (2017) for a review

WDM leads to a lack of small scale structure, solving the "satellite" problem



Nature of DM still an open question

courtsey: Mark Lovell