Gravitational Lensing

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Some publicly available references:

- **M. Meneghetti notes**
  - www.ita.uni-heidelberg.de/~massimo/sub/Lectures/gl_all.pdf
- J. Wambsganss, 1998, Gravitational Lensing in Astronomy, Living Reviews in Relativity
  - https://arxiv.org/abs/astro-ph/9812021
- C.S. Kochanek, 2004, The Saas Fee Lectures on Strong Gravitational Lensing
  - https://arxiv.org/abs/astro-ph/0407232
- M. Bartelmann, 2010, Gravitational Lensing
  - https://arxiv.org/abs/1010.3829

Some books:

- Schneider, Ehlers & Falco, 1992, Gravitational Lenses, ISBN 9783540665069
- Mollerach & Roulet, 2001, Gravitational Lensing And Microlensing, ISBN 9789810248529
- Petters, Levine & Wambsganss, 2001, Singularity Theory and Gravitational Lensing, ISBN 9780817636685
- Congdon & Keeton, 2018, Principles of Gravitational Lensing: Light Deflection as a Probe of Astrophysics and Cosmology, ISBN 9783030021214

Special lecture for the course "Physics of Galaxies"; May-2020

Outline

General concepts (2/3 of the time)

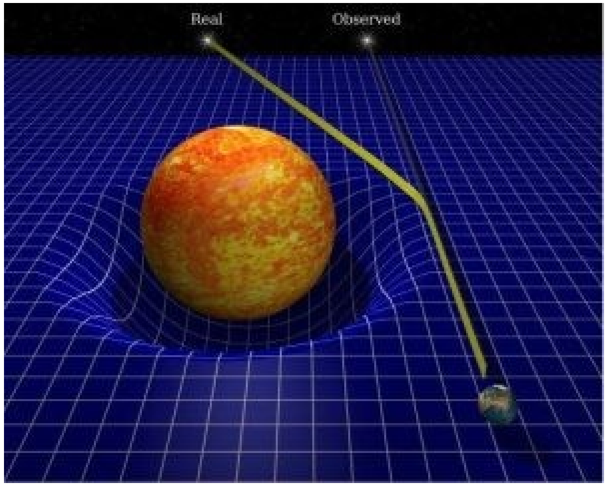
- A little of history
- First observations
- Lens equation and potential
- Magnification and distortion
- Gravitational lensing by stars, microlensing
- Gravitational lensing by galaxies and galaxy clusters
  - Strong lensing
  - Weak lensing

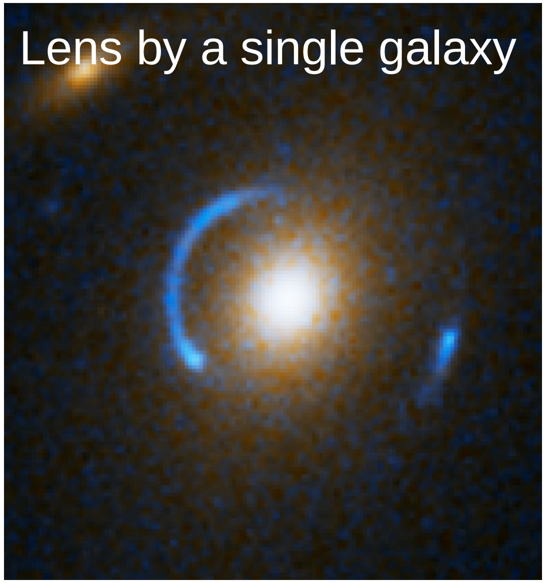
Applications (1/3 of the time)

- Mass distribution of galaxies and galaxy clusters
- Large scale structure of the Universe
- The distant and faint Universe

What is gravitational lensing?

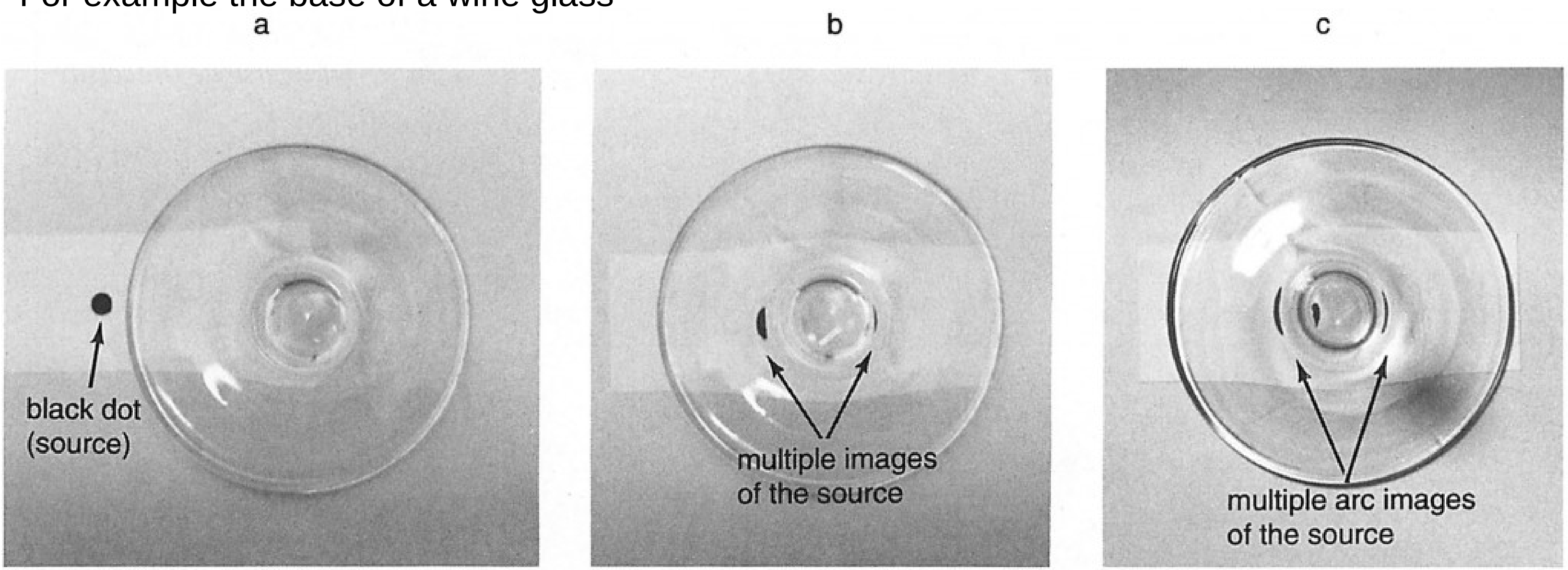
The deflection of the light of a background source (galaxy) by a massive object





Credit: NASA / ESA

It is possible to emulate some gravitational lensing effect with optical lenses
For example the base of a wine glass



Credit: https://www.um.edu.mt/think/the-einstein-enigma/

What is gravitational lensing?

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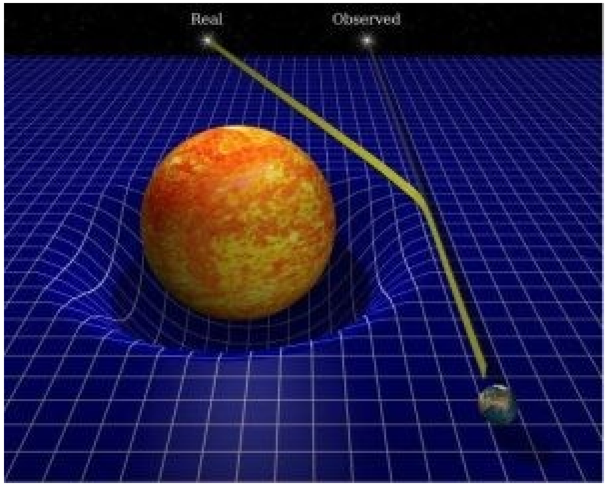
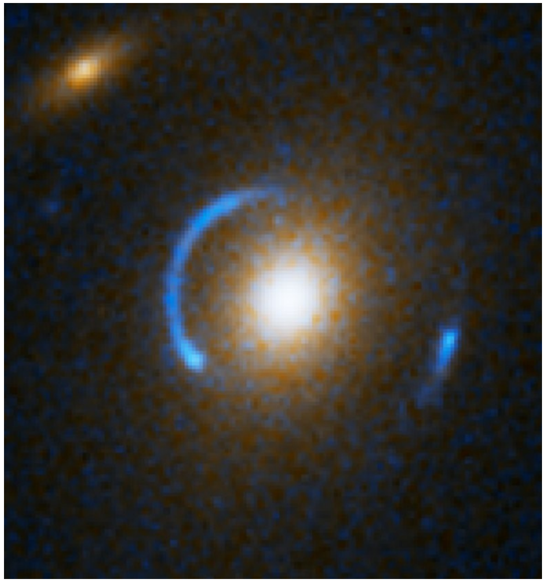


Lens by a massive galaxy cluster, Abell 370

Lens by a single galaxy



Credit: NASA / ESA

History

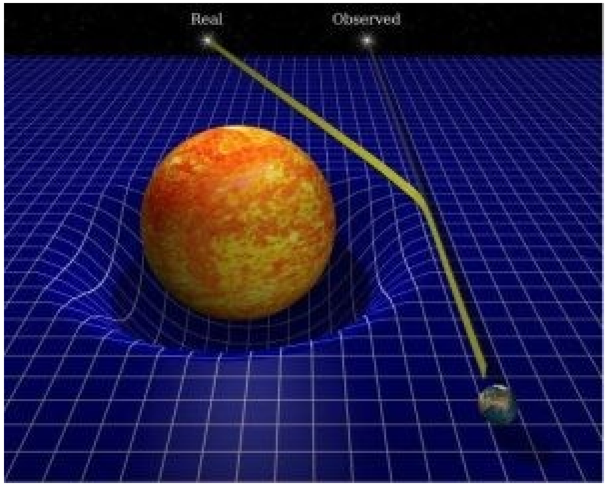
1783 – Letters between John Mitchell and Henry Cavendish discussing about the effects of a massive star on the light.

- First attempt to compute the **light deflection** by a gravitational field using Newtonian physics, wrong value

1919 – Arthur Eddington and his collaborators observations of the light deflection of distant stars by the Sun during an total eclipse.

- The most notorious test of **general relativity** was performed using gravitational lensing (May 29 1919)

$$\alpha = \frac{4GM}{c^2}\frac{1}{\xi}$$

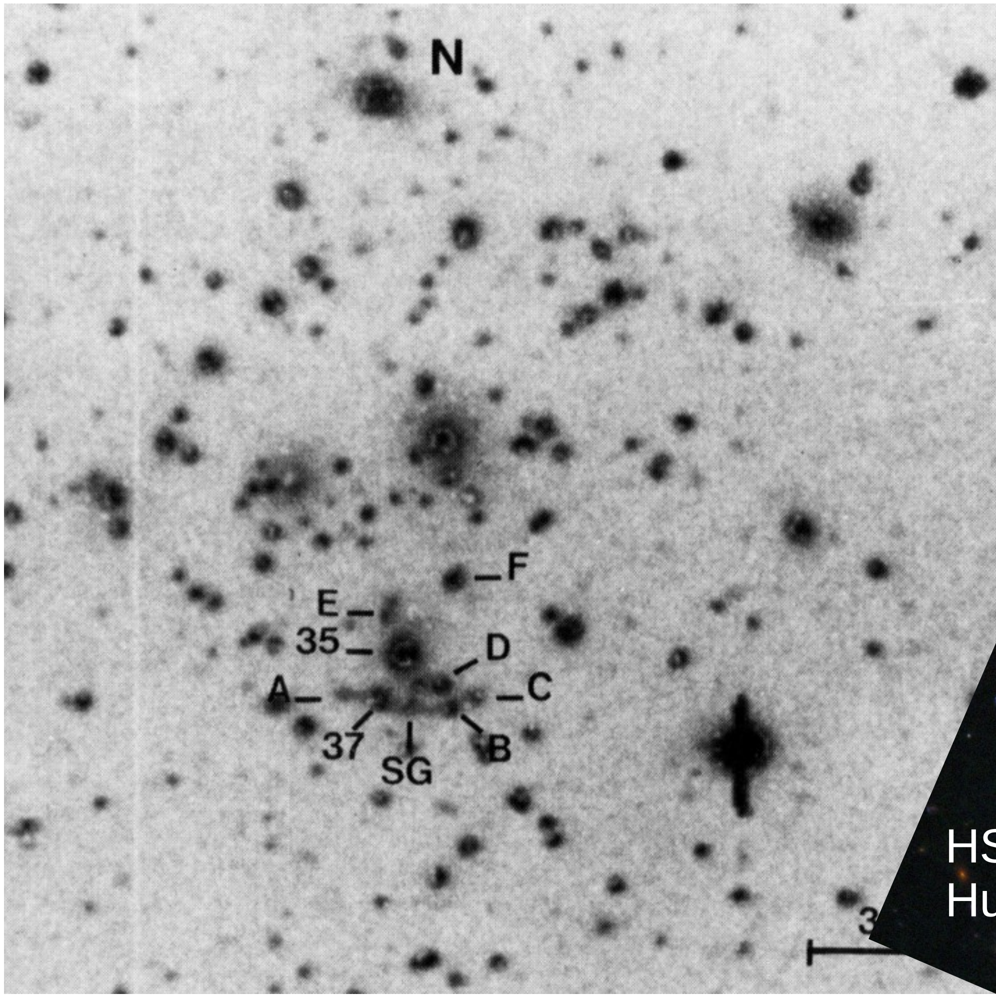


1924 – Orest Chwolson (Astronomische Nachrichten, 1924, 221, 329) **multiple images** of background stars could be formed. Moreover, with a perfect alignment a **image of a ring** will be created.

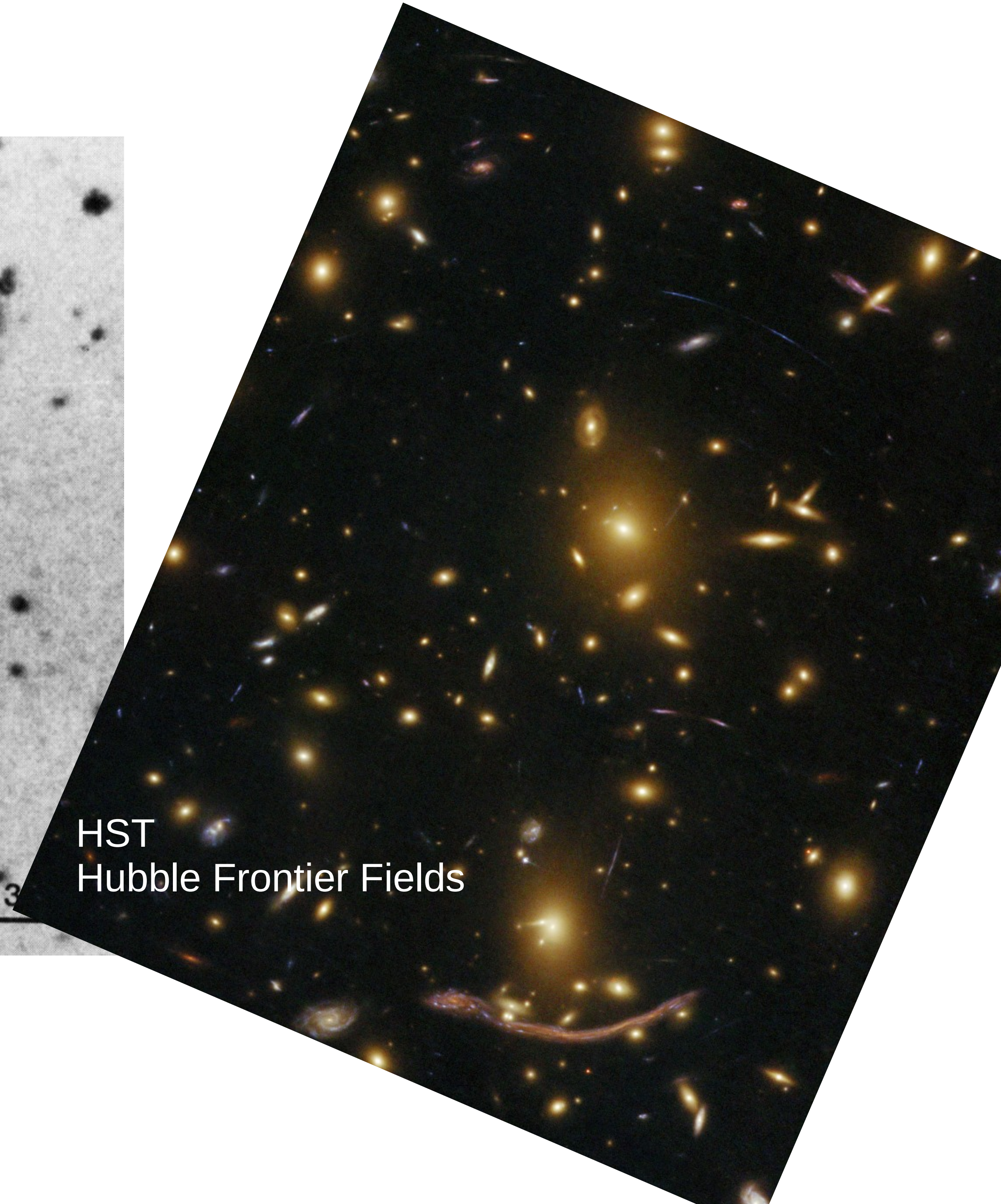
1936 – Einstein makes his own calculations (Science, 1936, 84, 56), and in addition finds that the apparent brightness of a background star can be **magnified** by the “lens-like action of the gravitational field”.

1987-89 – Discovery of elongated, curved features around galaxy clusters. First “detection” of gravitational arcs (Soucail+1987 and Petrosian+1986)

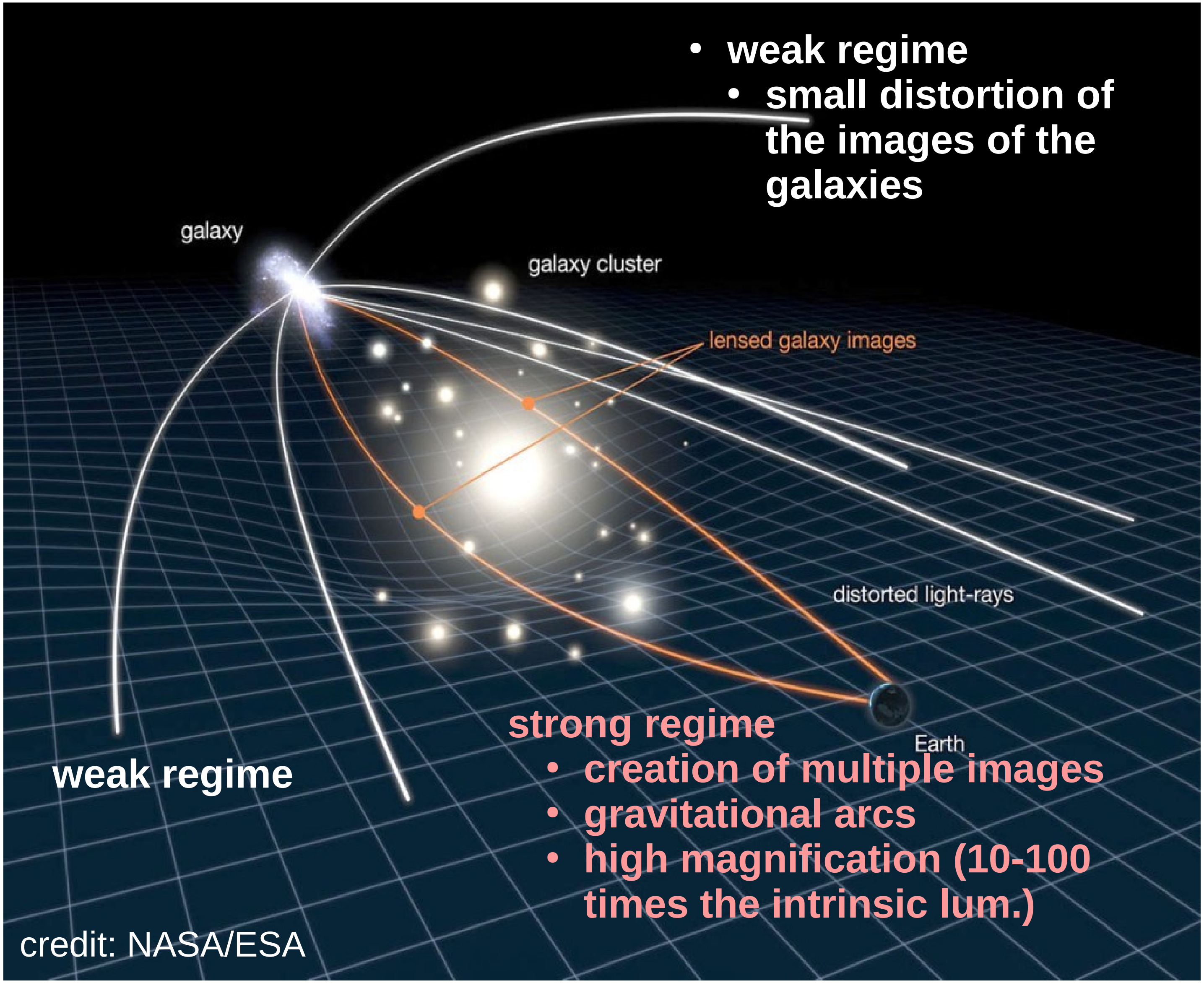
History



Soucail+1987 (CFH Telescope)



Gravitational lensing



Deflection of the light of a background galaxy by a massive object: galaxies, galaxy clusters, stars

Measure the mass distribution of these systems

- Does not depend on any assumption of the dynamical state or baryonic processes of the object acting as lens

Detect/measure substructures of the Dark Matter distribution

Magnifications preserves surface brightness:

$$\mu \equiv \frac{A_{\rm observed}}{A_{\rm intrinsic}}$$

↓

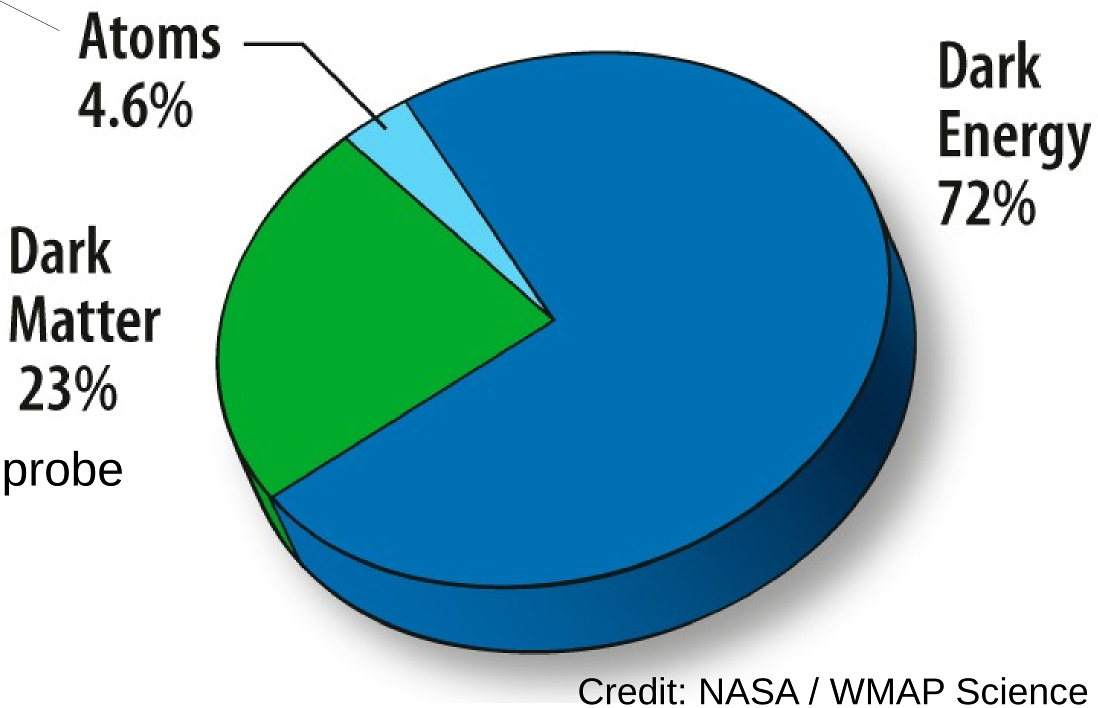
Allow us to observe very faint and distant object

The content of the Universe

Stars, 0.5%
Free Hydrogen and Helium 4%
Neutrinos and heavy elements ~0.1%

We know little about Dark Matter, and even less about Dark Energy

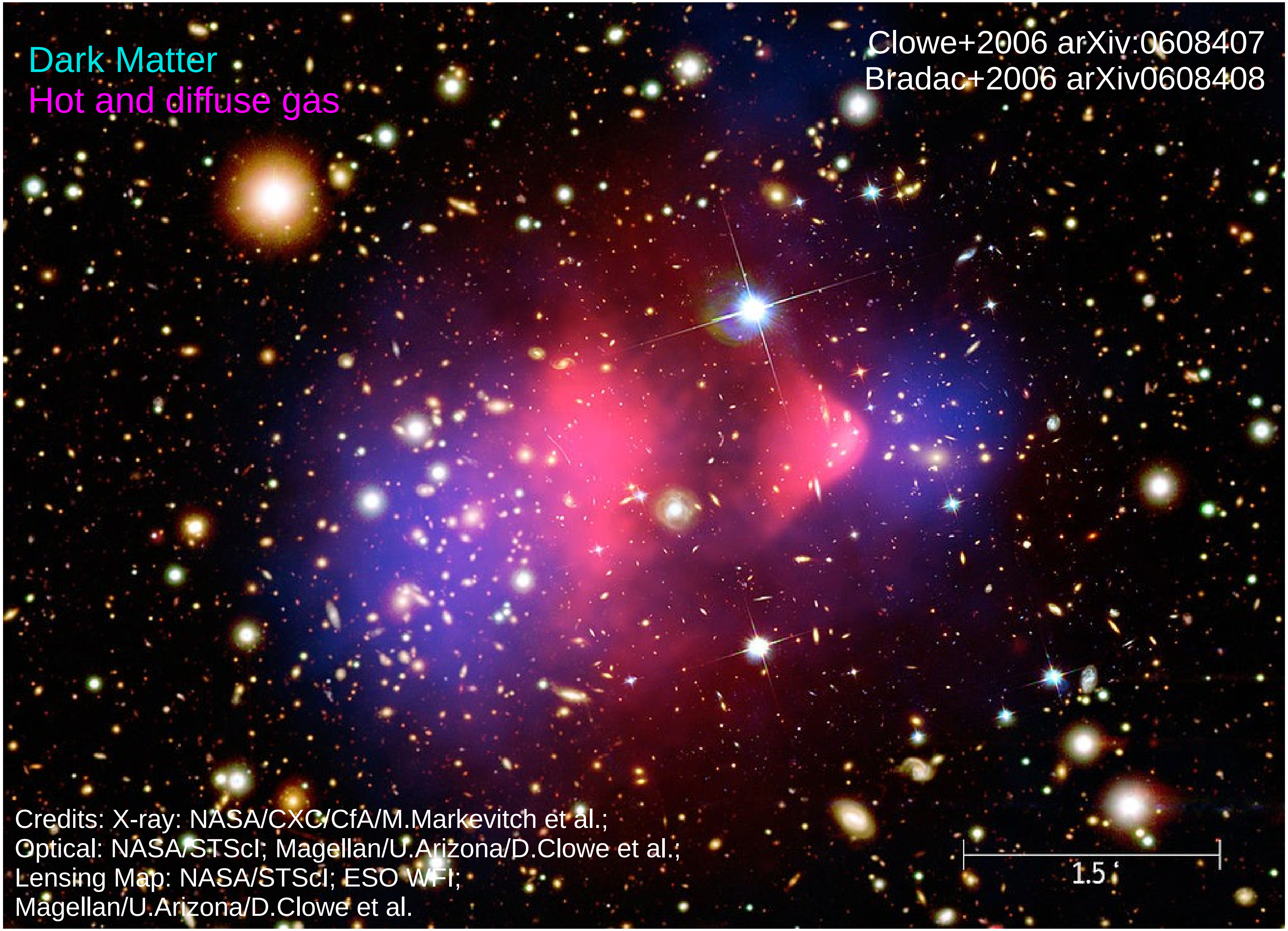
With gravitational lensing we can probe the dark sector of the Universe



Credit: NASA / WMAP Science Team

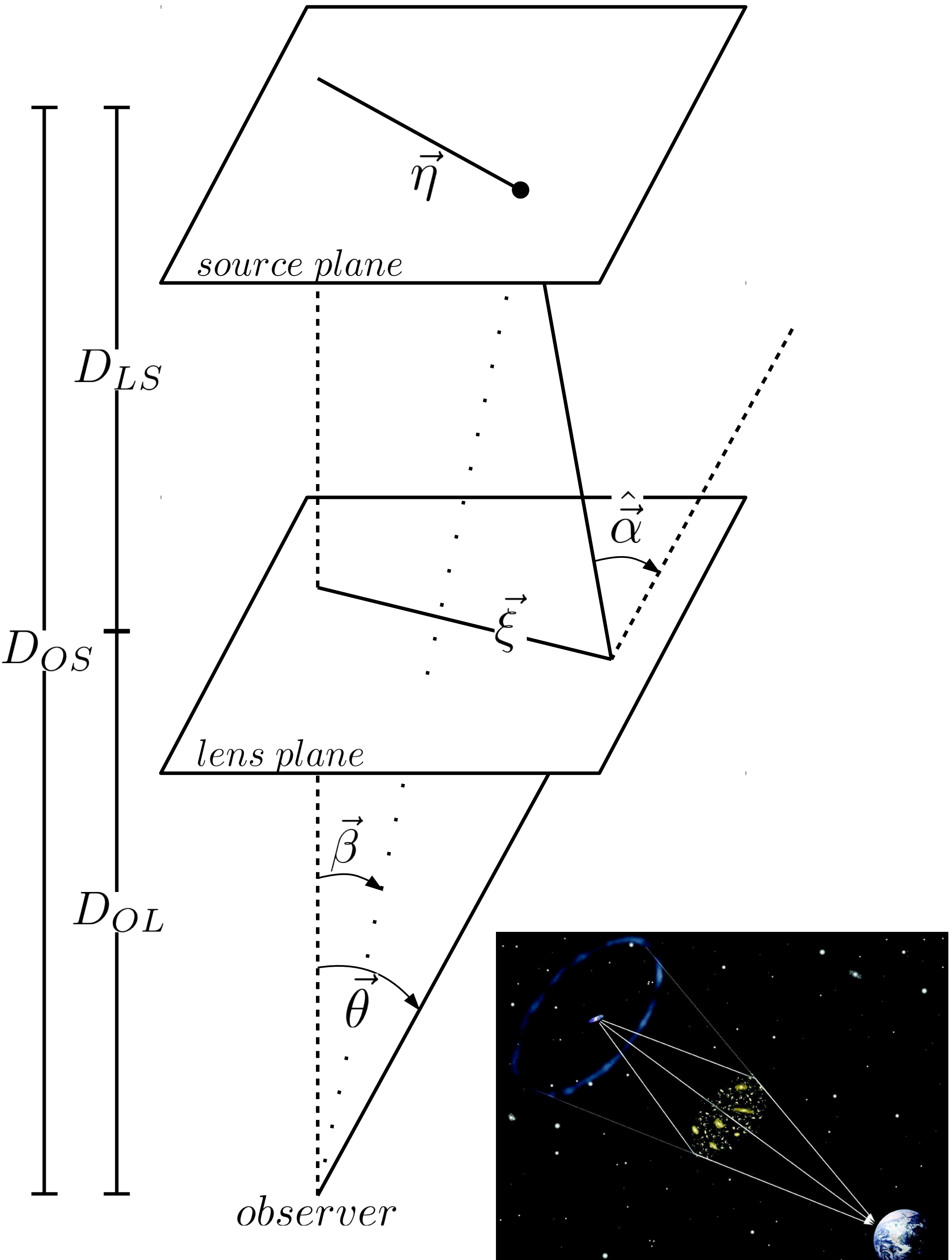
The Bullet Cluster

Mass distribution of a massive galaxy cluster
Image: **visible light, galaxies**; **Red: Hot gass**; **Blue: Dark Matter**
Dark Matter does not interacts directly with baryons and light, only **gravitationally**



Credits: X-ray: NASA/CXC/CfA/M.Markevitch et al.;
Optical: NASA/STScI; Magellan/U.Arizona/D.Clowe et al.;
Lensing Map: NASA/STScI; ESO WFI;
Magellan/U.Arizona/D.Clowe et al.

Lens equation



Angular diameter distances

Lens equation:

$$\vec{\beta} D_{OS} + \hat{\vec{\alpha}}(\vec{\theta}) D_{LS} = \vec{\theta} D_{OS}$$

Observed position – lens plane

Real position – source plane

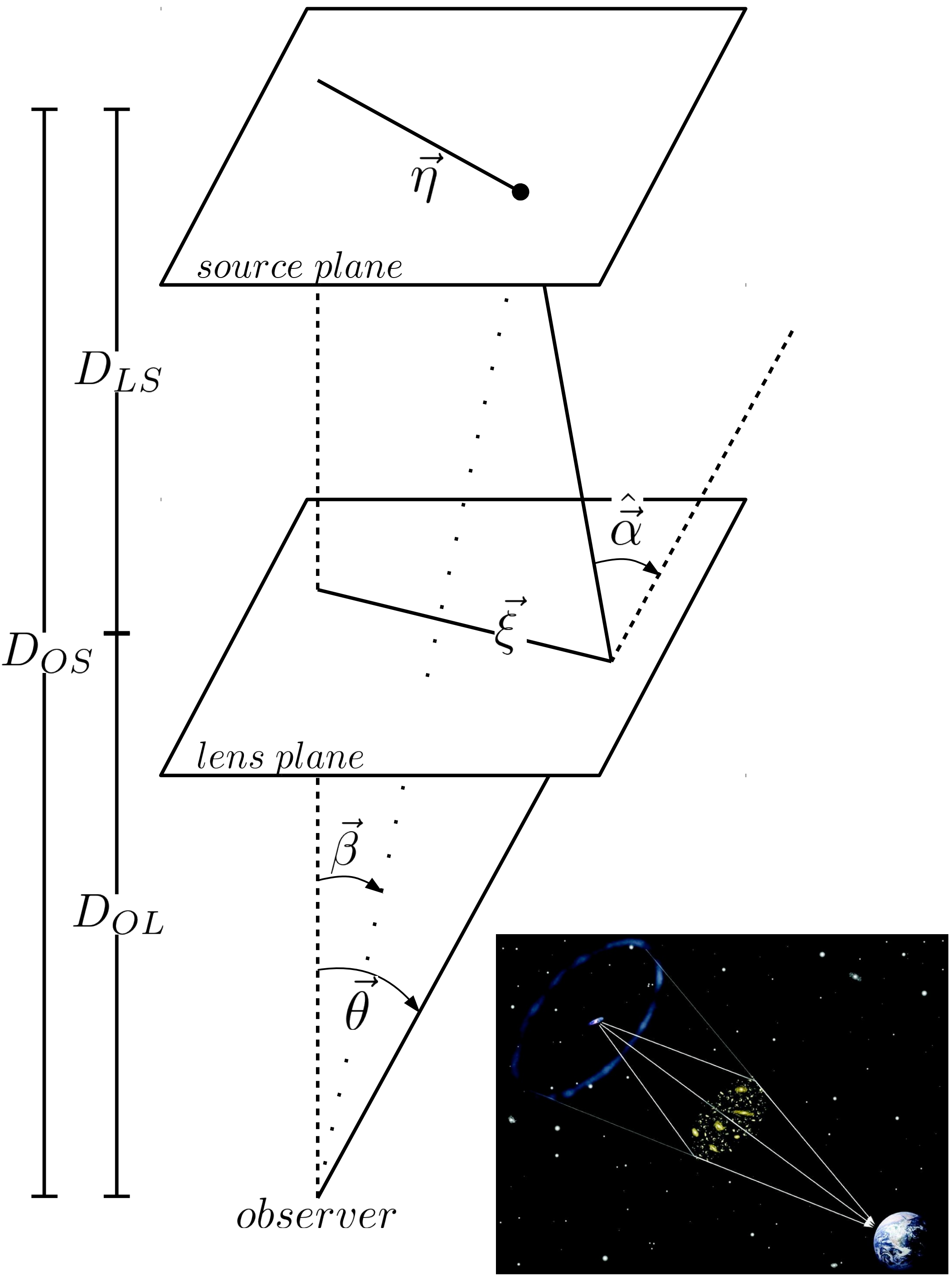
The deflection angle $\hat{\vec{\alpha}}$ depends on the “lens” mass distribution, and also on the observed position

For one position on the source plane $\vec{\beta}$, we can have multiple solutions for $\vec{\theta}$ in some cases.

$$\vec{\xi} = D_{OL} \vec{\theta}$$

$$\vec{\eta} = D_{OS} \vec{\beta}$$

Lens equation



Angular diameter distances

Lens equation:

$$\vec{\beta} = \vec{\theta} - \hat{\vec{\alpha}}(\vec{\theta})\frac{D_{LS}}{D_{OS}}$$

Observed position – lens plane

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For one position on the source plane $\vec{\beta}$, we can have multiple solutions for $\vec{\theta}$ in some cases.

$$\vec{\xi} = D_{OL}\vec{\theta}$$

$$\vec{\eta} = D_{OS}\vec{\beta}$$

Deflection angle

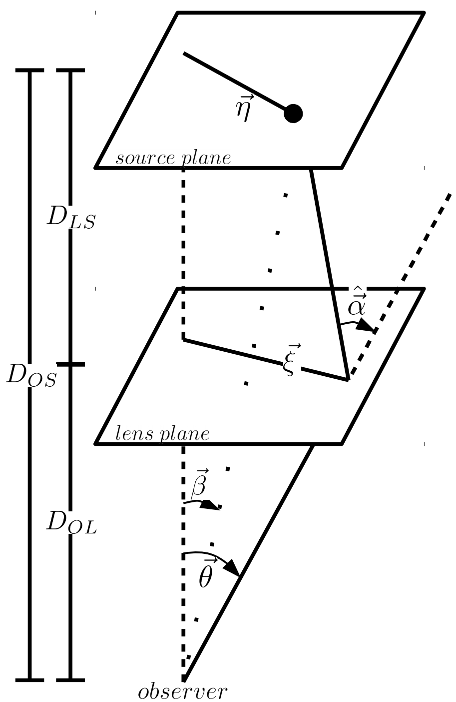
From General Relativity and assuming a weak gravitational field (satisfied by galaxies, clusters and stars), the deflection angle of a “point mass” object is given by:

$$\hat{\vec{\alpha}} = \frac{4GM}{c^2}\frac{\vec{\xi}}{|\vec{\xi}|^2}$$

OBS: if you are observing regions very close to a black hole, this equation is not valid

For a extended lens we can “sum” the deflection angles of its mass density distribution ρ

$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4G}{c^2}\int d^2\xi' \underbrace{\int dz\rho(\vec{\xi'}, z)}_{\equiv \Sigma(\vec{\xi'})}\frac{\vec{\xi} - \vec{\xi'}}{|\vec{\xi} - \vec{\xi'}|^2}$$



Projected mass density profile
We are not considering perturbers along the line of sight

$$\vec{\alpha}(\vec{\theta}) \equiv \hat{\vec{\alpha}}(D_{OL}\vec{\theta})\frac{D_{LS}}{D_{OS}}$$

Gravitational lens definitions

$$\vec{\alpha}(\vec{\theta}) = \frac{1}{\pi} \int d^2\theta' \kappa(\vec{\theta'}) \frac{\vec{\theta} - \vec{\theta'}}{|\vec{\theta} - \vec{\theta'}|^2}$$

$$\vec{\alpha}(\vec{\theta}) \equiv \hat{\vec{\alpha}}(D_{OL}\vec{\theta}) \frac{D_{LS}}{D_{OS}}$$

$$\kappa(\vec{\theta}) \equiv \Sigma(D_{OL}\vec{\theta})/\Sigma_{crit}$$

Dimensionless projected mass distribution, named convergence

$$\Sigma_{crit} \equiv \frac{c^2}{4\pi G} \frac{D_{OS}}{D_{OL} D_{LS}}$$

Critical density

Lens potential $\psi(\vec{\xi}) \propto \int dz \Psi(\vec{\xi}, z)$, projection of the 3D gravitational potential

The important relations are:

$$\nabla^2 \psi(\vec{\theta}) = 2\kappa(\vec{\theta}) \longrightarrow \vec{\alpha}(\vec{\theta}) = \vec{\nabla}\psi(\vec{\theta})$$

$$\vec{\beta} = \vec{\theta} - \hat{\vec{\alpha}}(\vec{\theta}) \frac{D_{LS}}{D_{OS}}$$

Magnification

In gravitational lensing, the surface brightness (= flux/solid angle) is conserved
The magnification is defined as the ration between the observed and intrinsic fluxes

$$\mu \equiv \frac{\mathrm{d}\Omega_{\mathrm{observed}}}{\mathrm{d}\Omega_{\mathrm{intrinsic}}}$$

For infinitesimal quantities, we can use the Jacobian matrix:

$$\left(\vec{\alpha}(\vec{\theta}) = \vec{\nabla}\psi(\vec{\theta})\right)$$

$$A_{i,j} = \frac{\partial \beta_i}{\partial \theta_j}$$

$$\left(\vec{\beta} = \vec{\theta} - \hat{\vec{\alpha}}(\vec{\theta})\frac{D_{LS}}{D_{OS}}\right)$$

Information about the mapping between the source plane and lensing (observed) plane

$$\mu(\vec{\theta}) = \det A(\vec{\theta})^{-1}$$

Rewriting the lens equation as: $\vec{\beta} = \vec{\theta} - \vec{\nabla}\psi$

$$A_{i,j} = \delta_{i,j} - \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}$$

Short notation:

$$\psi_{ij} \equiv \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}$$

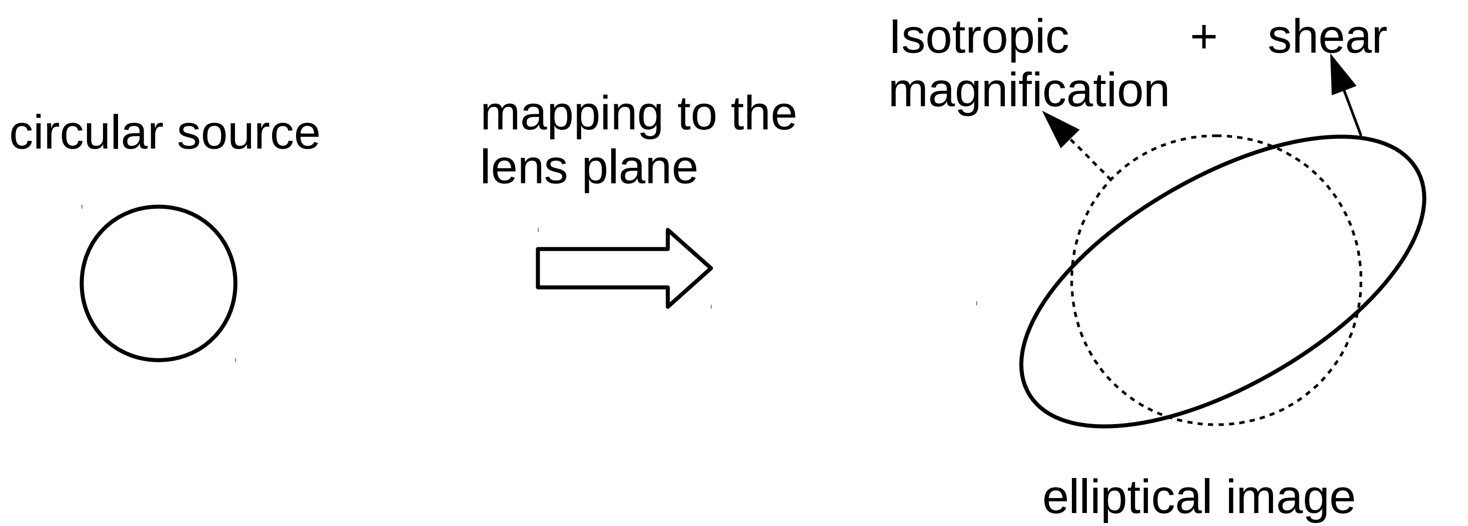
Magnification

$$A = \begin{bmatrix} 1-\psi_{11} & \psi_{12} \\ \psi_{21} & 1-\psi_{22} \end{bmatrix}$$

We can split this matrix in two parts, one isotropic and other traceless

$$A = (1-\kappa)I - \begin{vmatrix} \frac{1}{2}(\psi_{11}-\psi_{22}) & \psi_{12} \\ \psi_{12} & -\frac{1}{2}(\psi_{11}-\psi_{22}) \end{vmatrix}$$

We now define the shear $\vec{\gamma} = (\gamma_1, \gamma_2)$, with $\gamma_1 = \dfrac{1}{2}(\psi_{11} - \psi_{22}) \quad \gamma_2 = \psi_{12}$



Magnification

The total magnification is given by: $(\mathrm{Flux}_{\mathrm{observed}} = \mu \times \mathrm{Flux}_{\mathrm{intrinsic}})$

$$\mu = \frac{1}{\det A} = \frac{1}{(1-\kappa)^2 - \gamma^2}$$

The two eigenvalues are related to the magnification in the tangential and radial directions:

$$\mu_t = \frac{1}{1-\kappa-\gamma} \qquad \mu_r = \frac{1}{1-\kappa+\gamma}$$

Example for an elliptical mass distribution

Demagnified central images $|\mu| < 1$

Source plane - Caustic

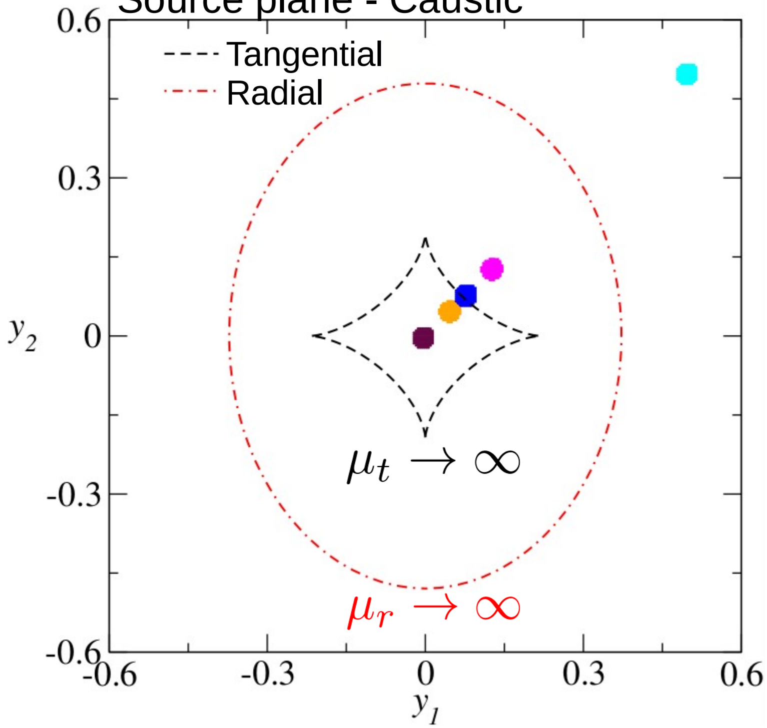
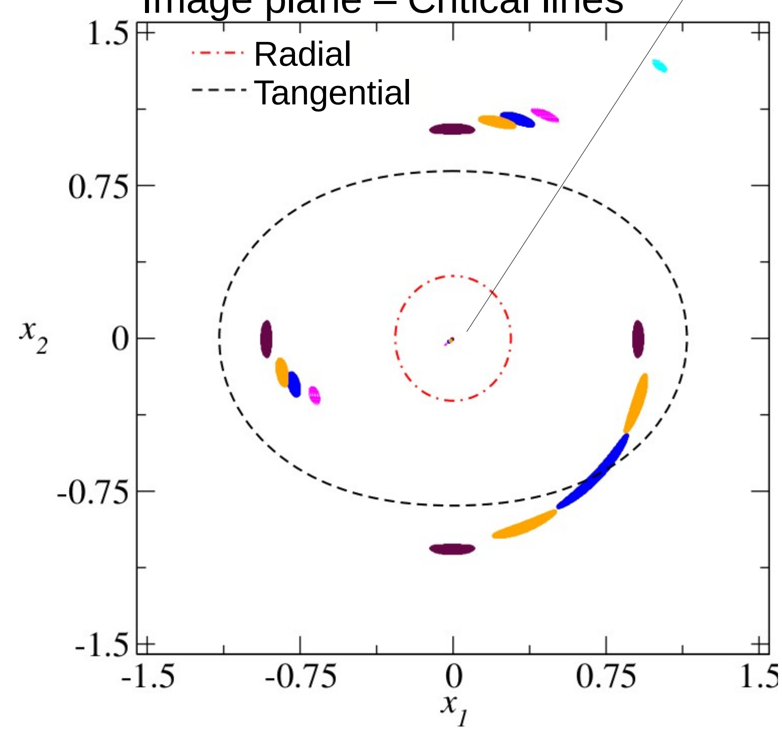


Image plane – Critical lines



Time delay

The time delay has two contributors, a geometrical and a gravitational

$$t - t_0 \equiv \delta t = \frac{1 + z_{lens}}{c} \frac{D_{OS} D_{OL}}{D_{LS}} \left[\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi \right]$$

Difference between the actual arrival time and the arrival time with no deflector

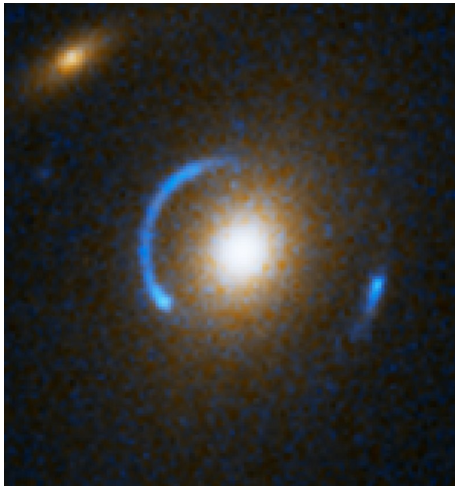
From the lens equation we have:

$$\vec{\nabla} \left[\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi \right] = \vec{\theta} - \vec{\beta} - \vec{\nabla}\psi = 0$$

Therefore the lens equation can be rewritten as simply:

$$\vec{\nabla}(\delta t) = 0$$

The time delay defines a two-dimensional surface and the multiple images are formed in the stationary points, i.e. minima, maxima or saddle points



Credit: NASA / ESA

Example of a time delay surface

The point mass distribution is singular in the origin, the surface is not smooth, the central point diverges to infinity. Each source position has one surface.
Time delay surface can be used to study the occurrence of multiple images

Figure from Mollerach & Roulet, 2001, Gravitational Lensing And Microlensing

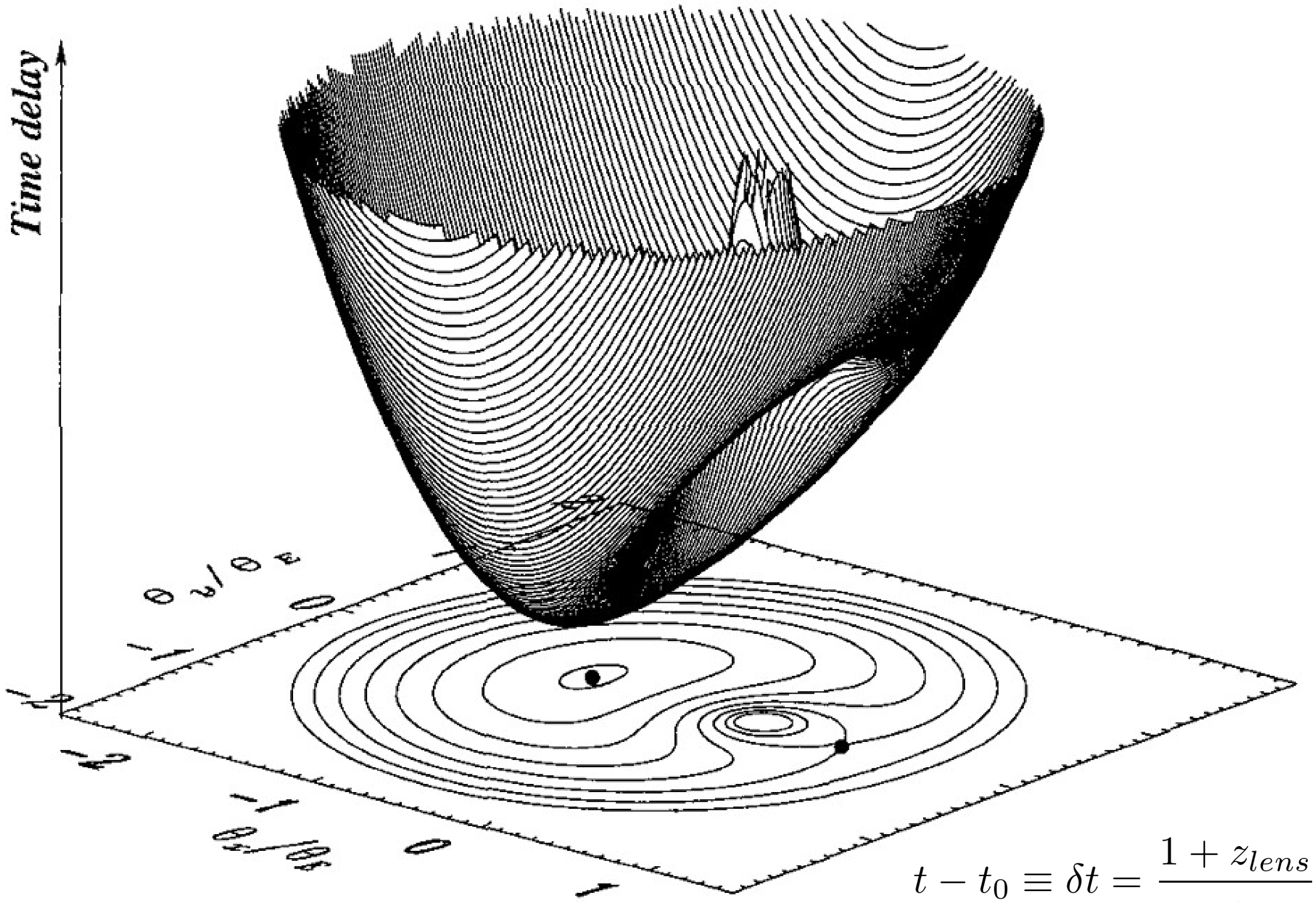


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Fig. 3.7 Time delay surface for a point-like lens not perfectly aligned with the source ($\beta \neq 0$). Contour levels are shown in the bottom plane and the dots indicate the location of the two images.

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The time delay has two contributors, a geometrical and a gravitational

$$t - t_0 \equiv \delta t = \frac{1 + z_{lens}}{c} \frac{D_{OS} D_{OL}}{D_{LS}} \left[\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi \right]$$

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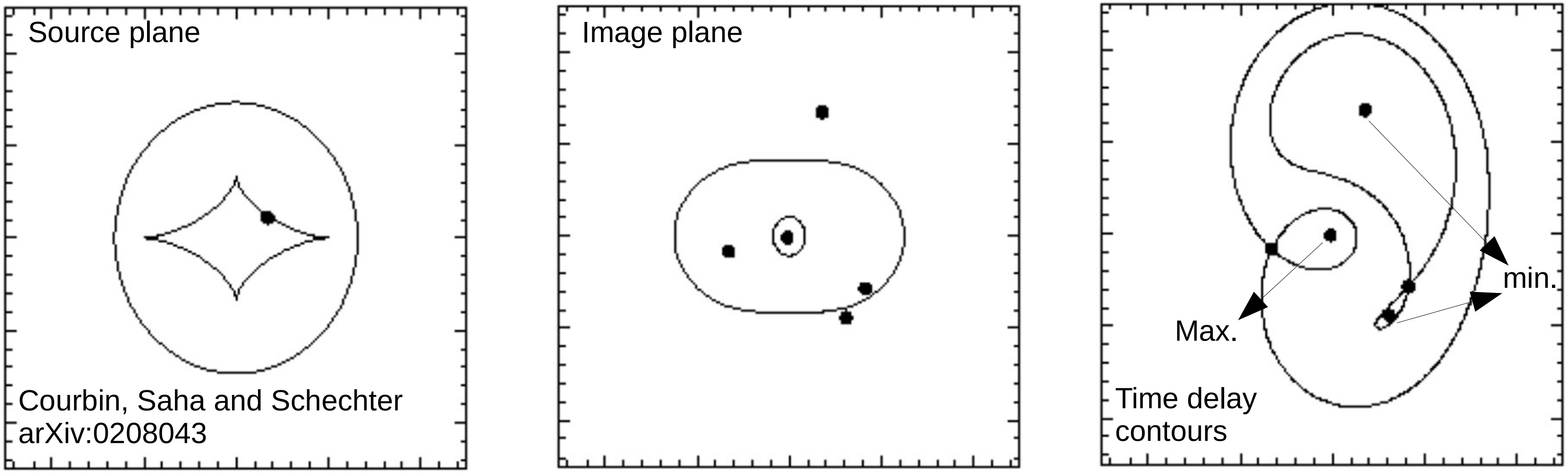


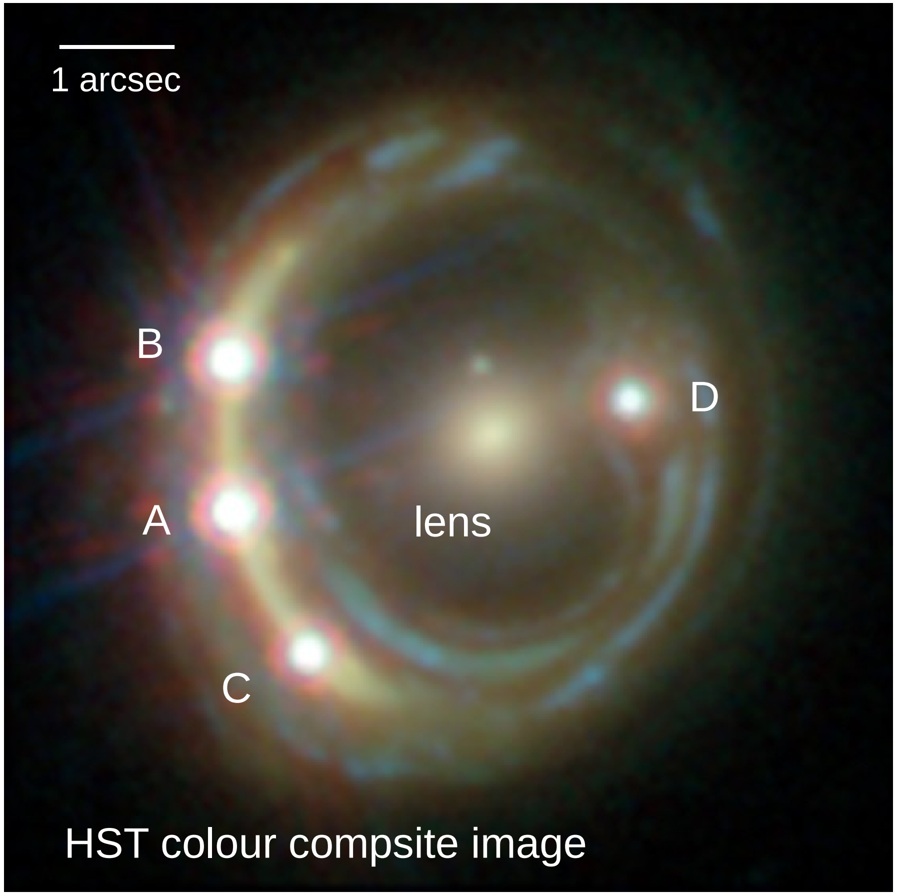
Time delay

If we measure the time delay between two images from the same source we can compute

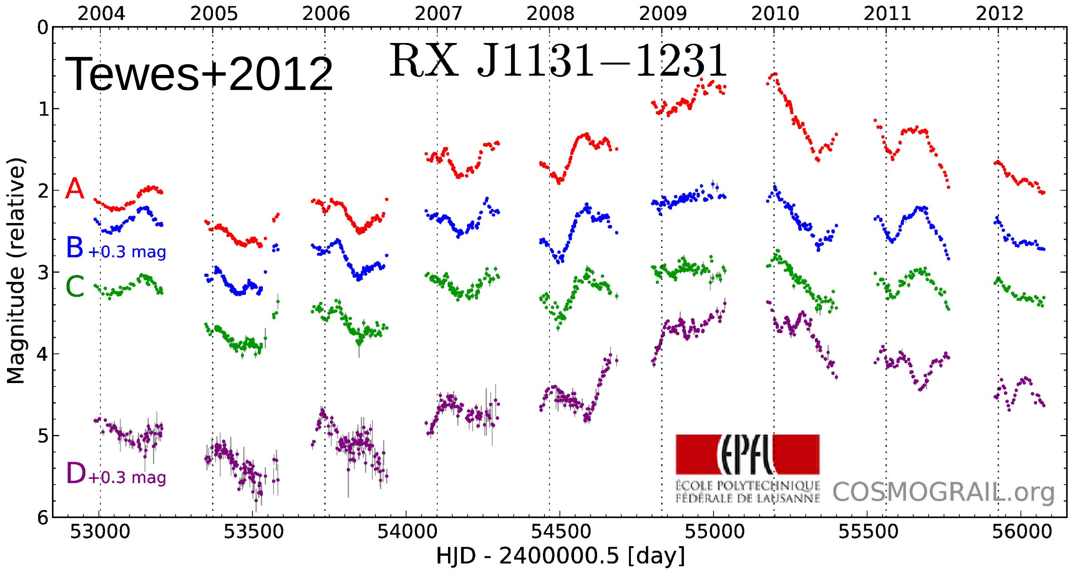
$$\delta t_1 - \delta t_2 = t_1 - t_2 \propto H_0^{-1} \times F(\psi)$$

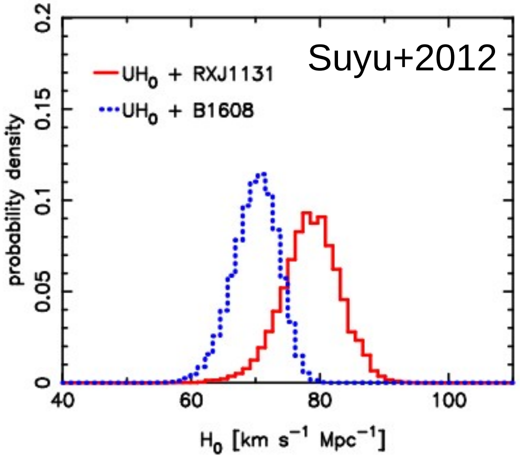
F depends on the lens potential (mass distribution).
Can be used to constraint the Hubble parameter

- see e.g. Suyu+2012
- (chap. 1 and 2, arXiv:1208.6010)



Light curve of the four multiple images





Time delay in galaxy scale systems is few days – few hundred days (< year)

$$H_0 = 75 \pm 5$$

Caustics and critical lines

Source plane

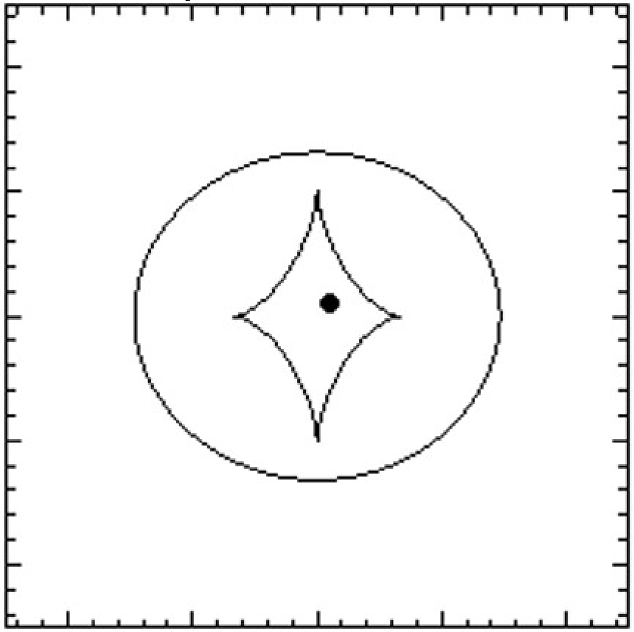
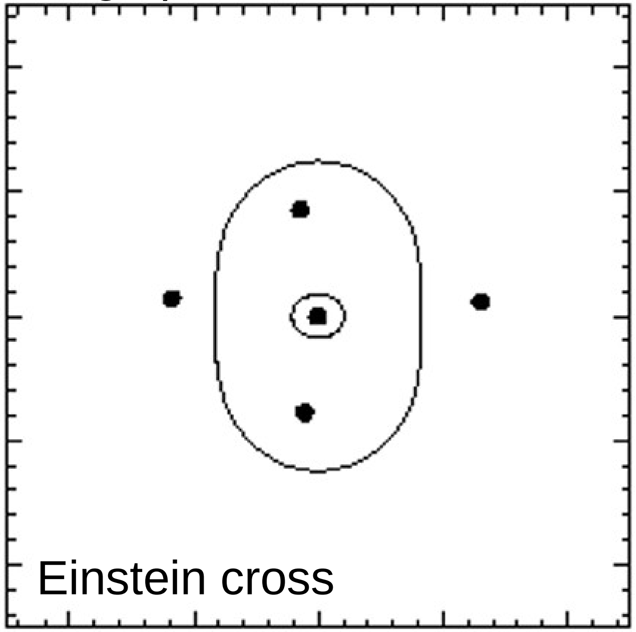
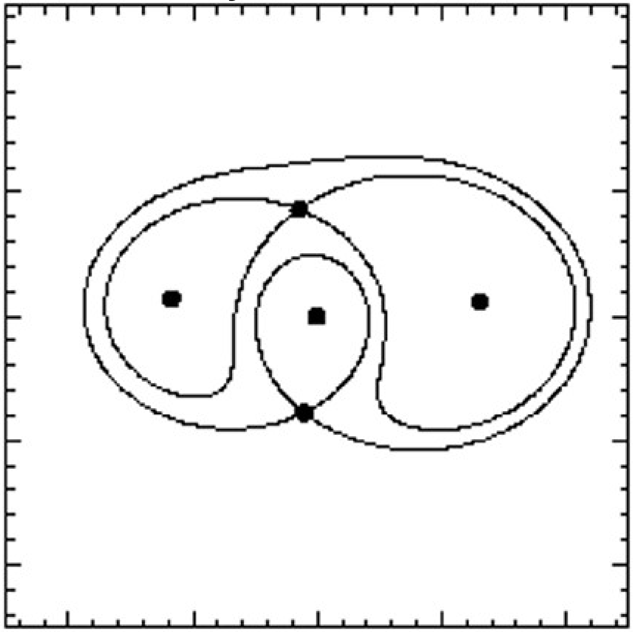
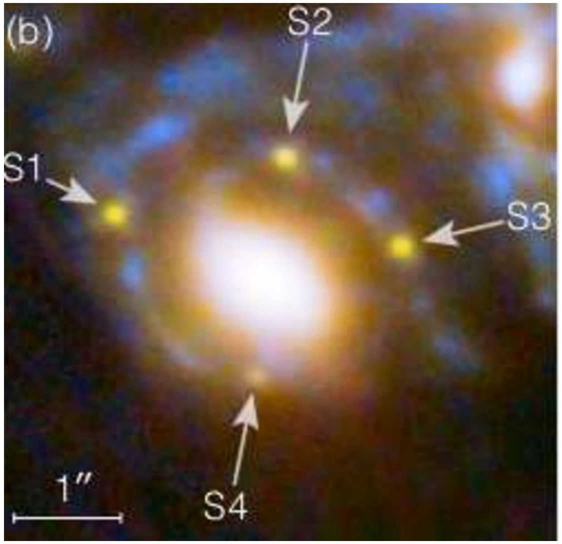


Image plane



Time delay contours

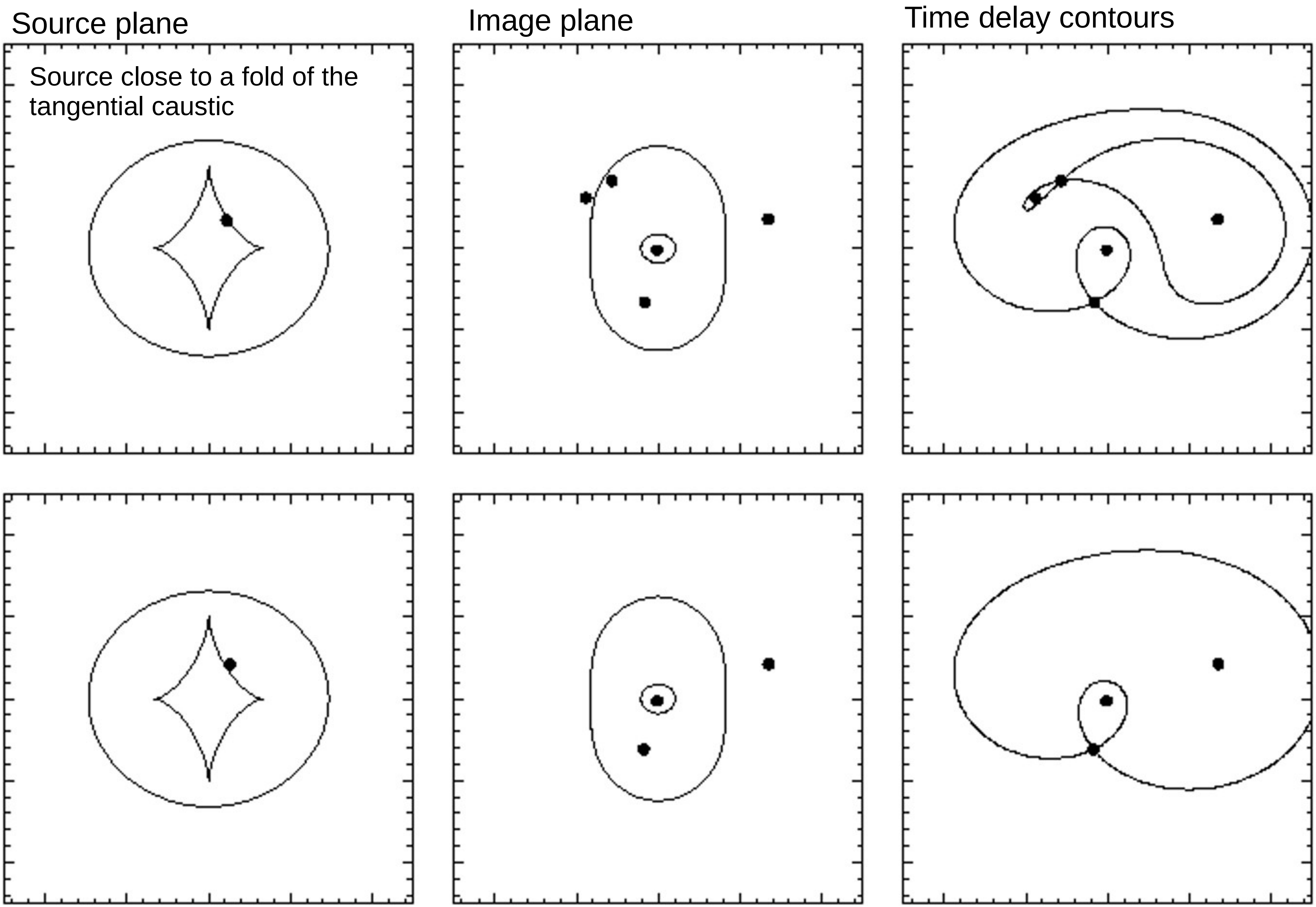




Real image from Grillo+2018
S1-4 are multiple images from one source
The bright galaxy in the acting as a gravitational lens

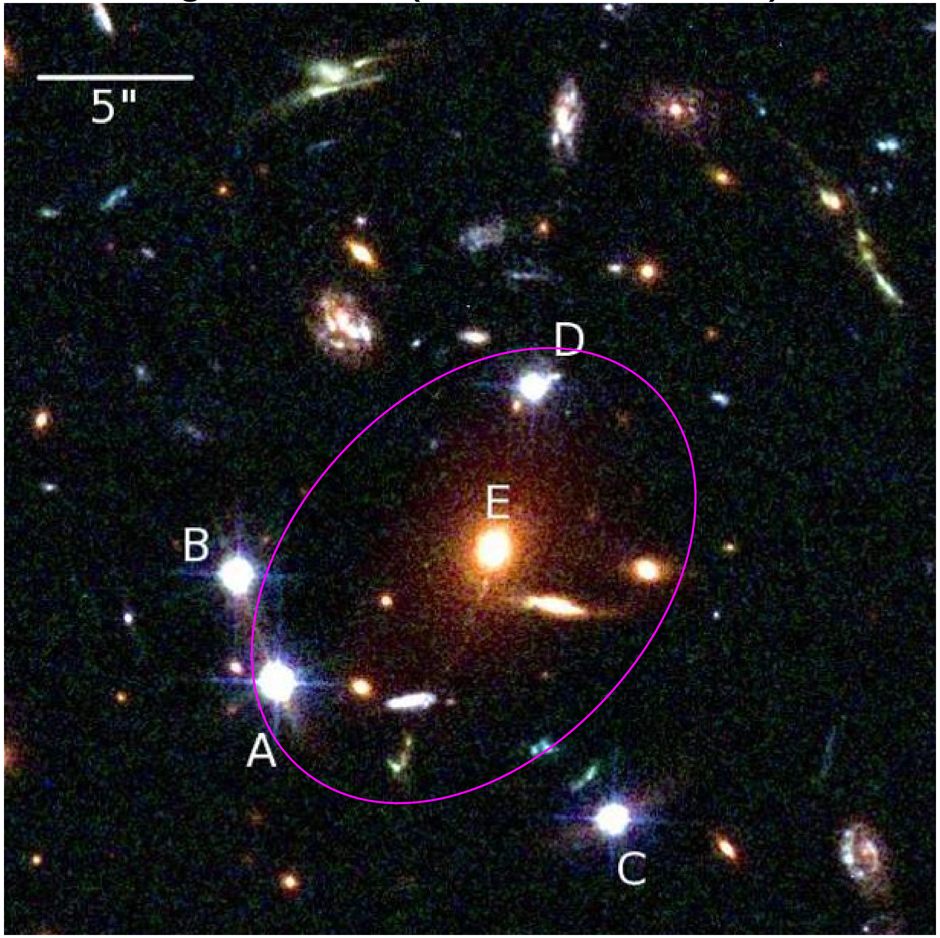
Caustics and critical lines

Lens is elongated along the y-direction. Every time the source passes through a caustic, two extra images are formed



Example of a fold system

From Oguri+2010 (arXiv:1005.3103)



OBS: the critical line is much more complicated than this simple representation

General relation for smooth mass distribution

$$\mu_{\rm A} + \mu_{\rm B} = 0$$

Images B and C are outside the critical line

- They have positive magnifications

A and D are inside the critical line

- Negative magnifications

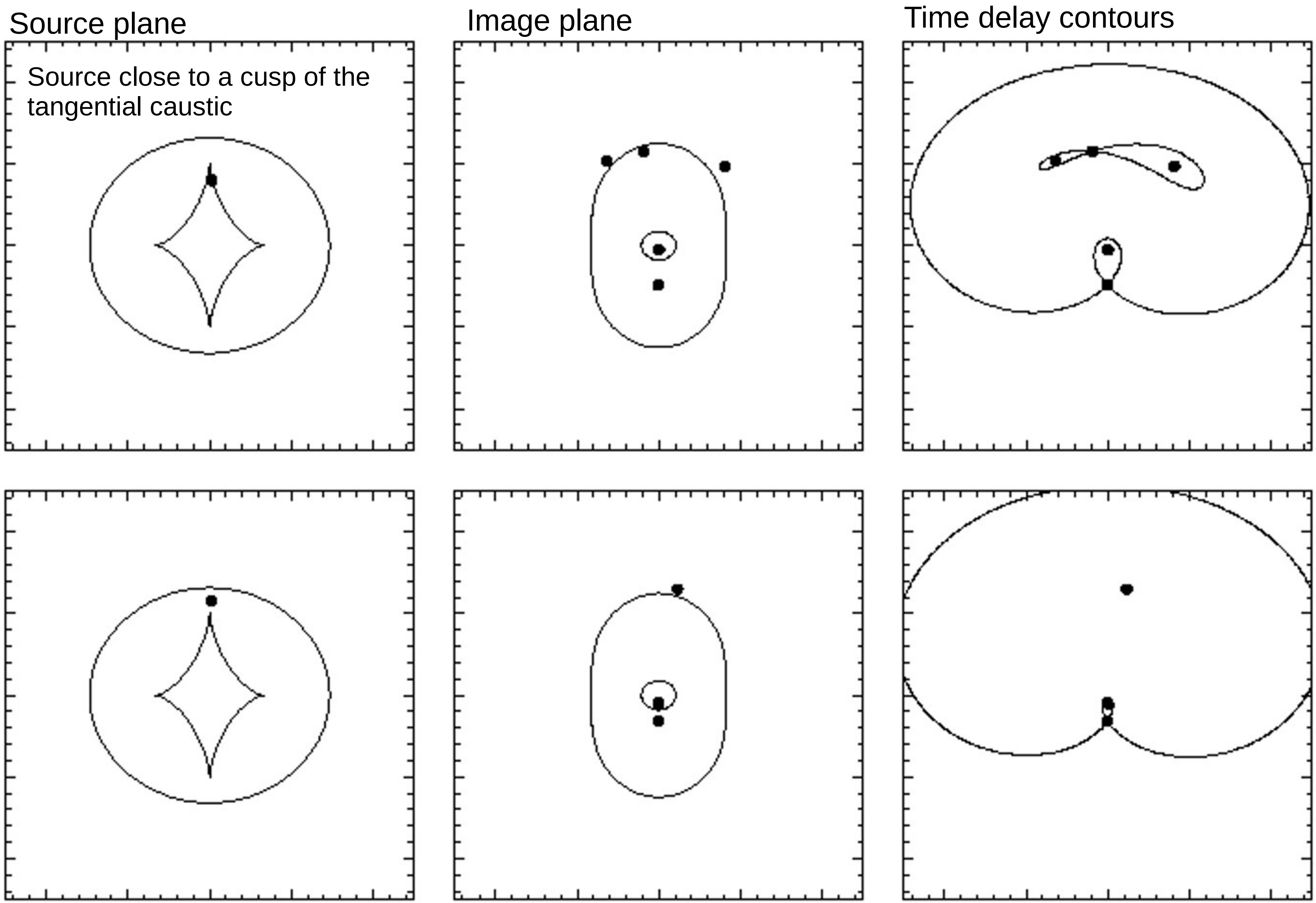
There is a central image E, with positive magnification but de-magnified

$$|\mu_E| < 1$$

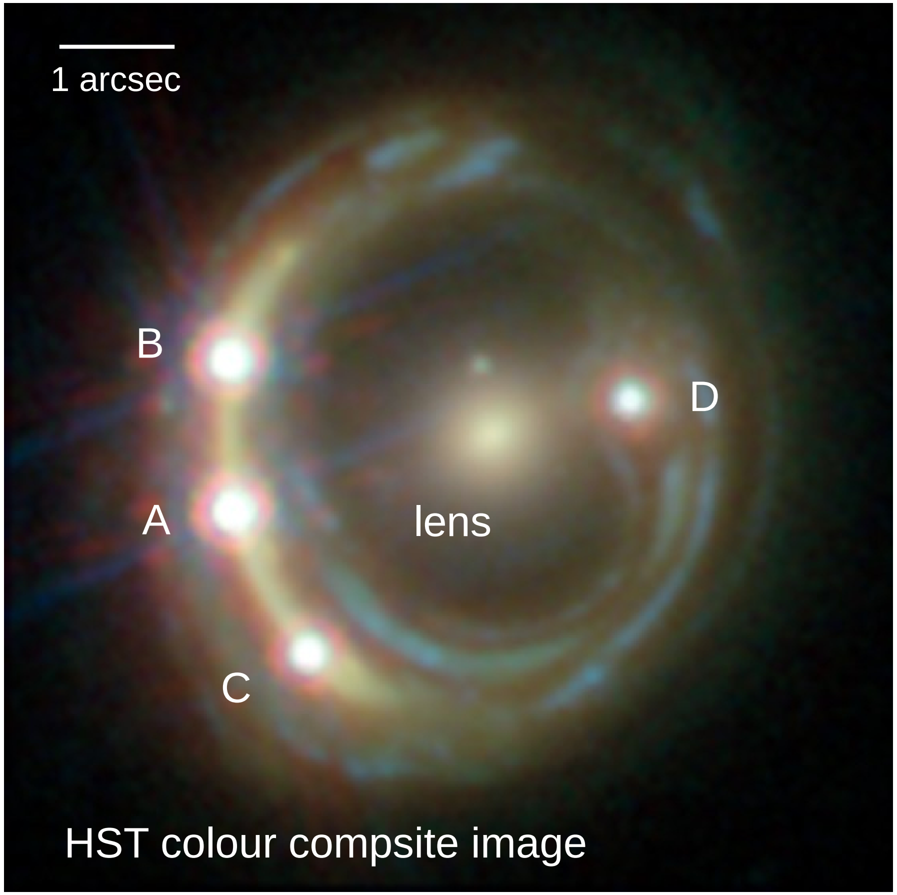
Usually difficult to observe because its low luminosity and contamination from the central galaxy

Caustics and critical lines

Lens is elongated along the y-direction. Every time the source passes through a caustic, two extra images are formed



Example of a cusp system



General relation for smooth mass distribution

$$|\mu_{\mathrm{A}}| = |\mu_{\mathrm{B}}| + |\mu_{\mathrm{C}}|$$

Images A and D are inside the critical line

- They have negative magnifications

B and C are outside the critical line

- Positive magnifications

No Central image is detected in this system

If we know well the mass distribution and have control on some systematics, we can use these relations to detect dark matter substructures

Lens model 1: Point mass

Point mass:

$$\hat{\vec{\alpha}} = \frac{4GM}{c^2}\frac{\vec{\xi}}{|\vec{\xi}|^2}$$

This is an antisymmetric case, all vectors have the same direction, so we can reduce to one dimension. The lens equation is:

$$\beta = \theta - \frac{4GM}{c^2}\frac{D_{LS}}{D_{OL}D_{OS}}\frac{1}{\theta} \qquad \theta_E^2 \equiv \frac{4GM}{c^2}\frac{D_{LS}}{D_{OL}D_{OS}}$$

In this simple case we can solve for θ

$$\theta_{\pm} = \frac{\beta \pm \sqrt{\beta^2 + 4\theta_E^2}}{2}$$

Two different images from the same source position (the time delay surface is not smooth)

$$\beta = 0\,,\ \theta = \pm\theta_E \qquad \beta \to \infty\,,\ \theta_+ \to \beta\,,\ \theta_- \to 0$$

Deviations from the axial symmetry will “break” the ring

Formally there is a central image, but this is an artefact because the point mass is singular

The magnification is given by:

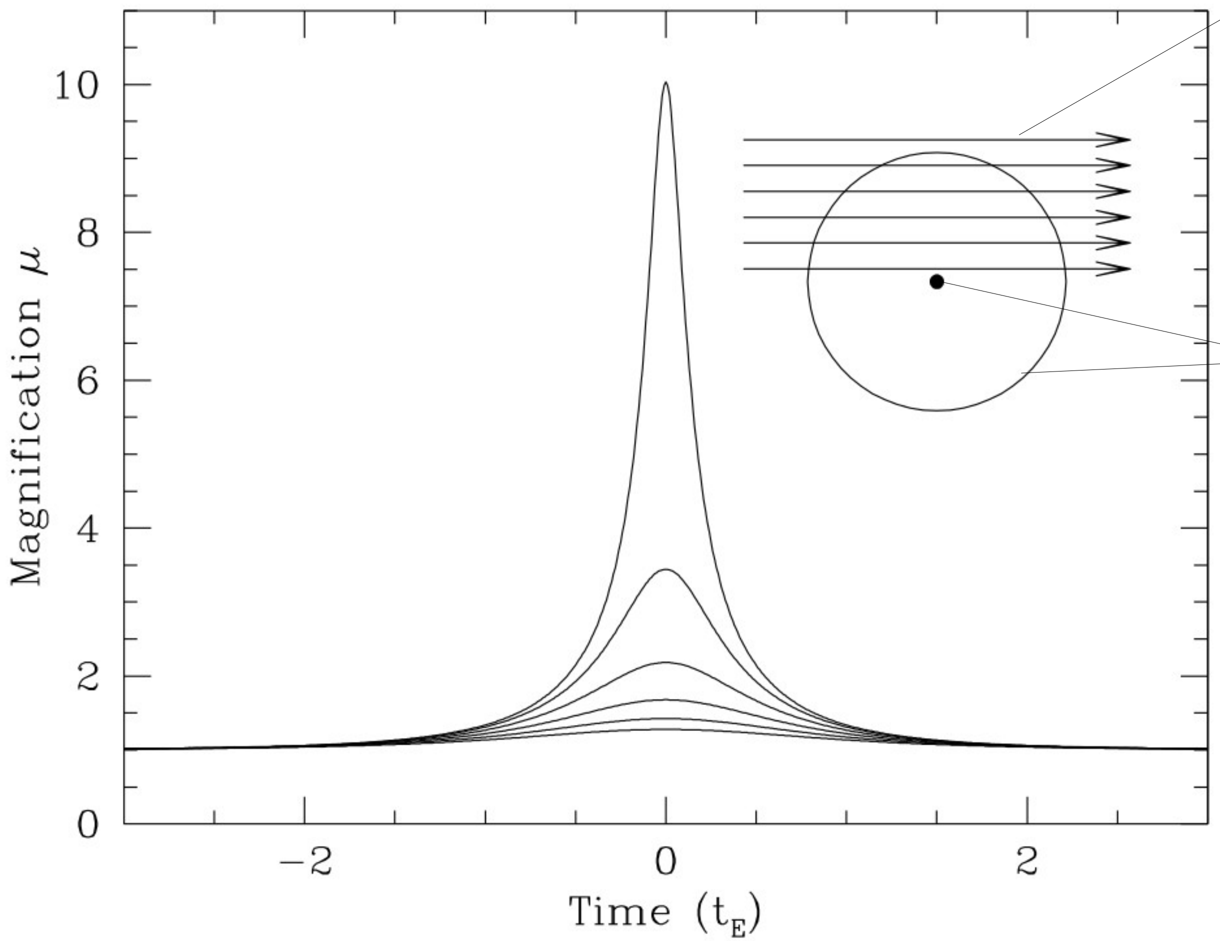
$$\mu = \left[1 - \left(\frac{\theta_E}{\theta}\right)^4\right]^{-1}$$

Microlensing

Point mass lenses are good approximations for stars

Microlensing refers to very small deflection angles (<0.01 arcsec) that we can not resolve, caused by compact objects on other sources

Observed by monitoring stars during several days



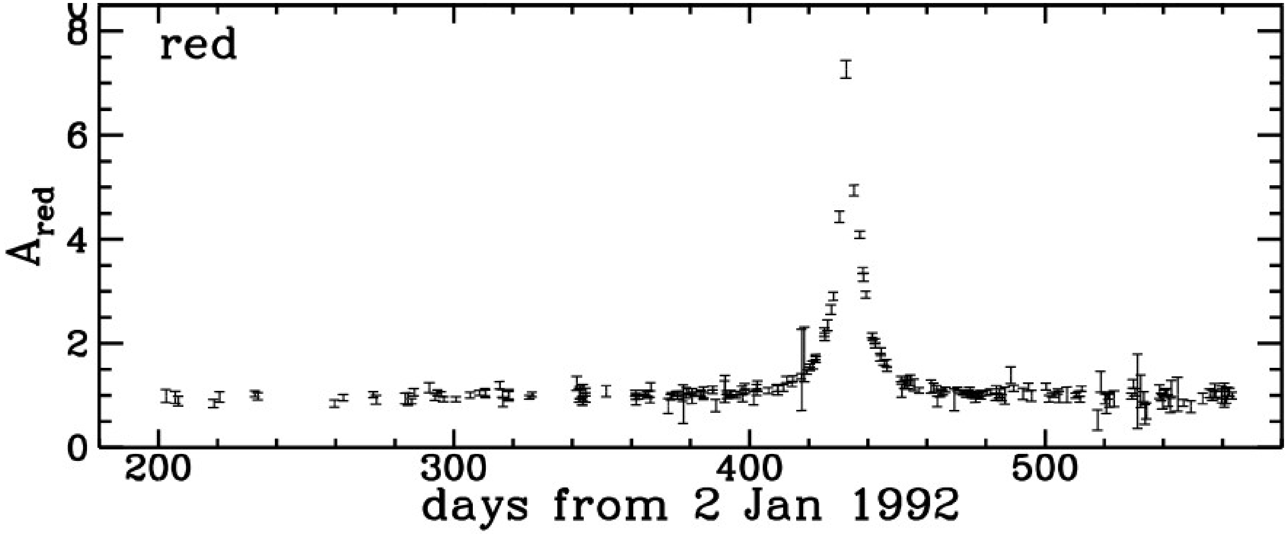
Trajectory of a background star with different impact parameters β_0

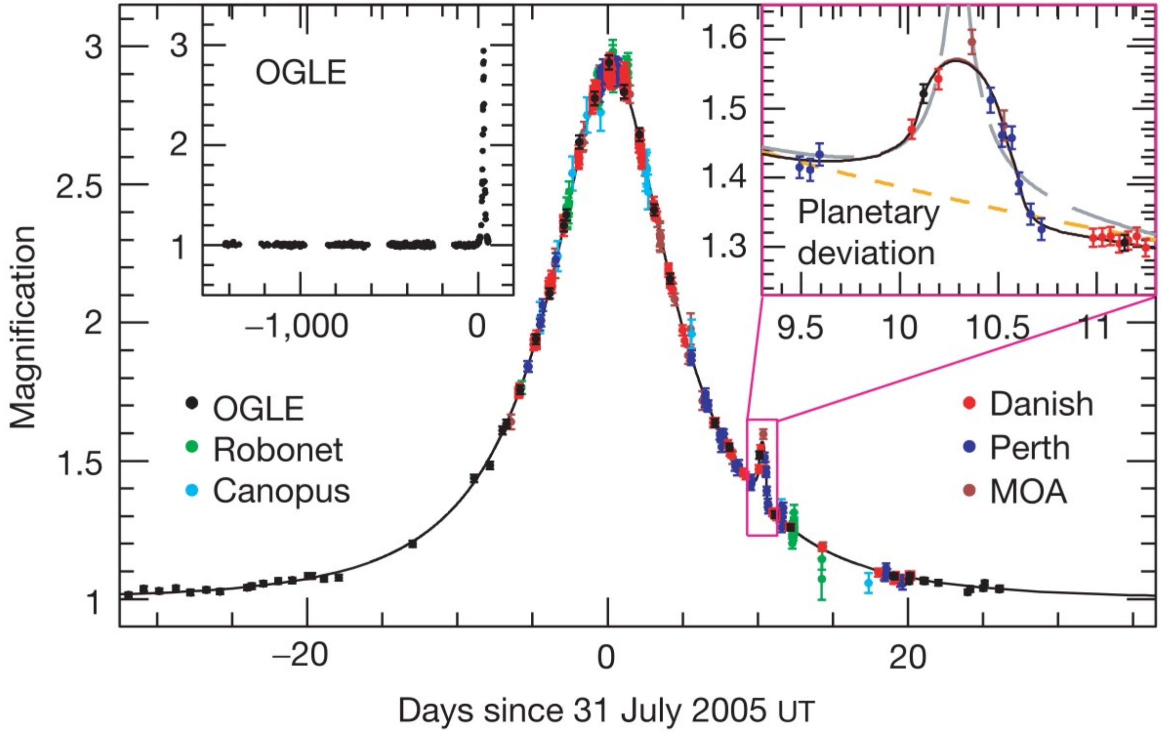
Caustics for of a point mass lens. The tangential caustic is degenerated in a central point

Figure from Wambsganss+2006 (arXiv:0604278)

Detection of a planet using microlensing

First detected microlensing effect, Alcock+1993 (arXiv:9309052)





Beaulieu+2006 (arXiv:0601563)
Perturbation caused by a object with ~5 times the Earth mass

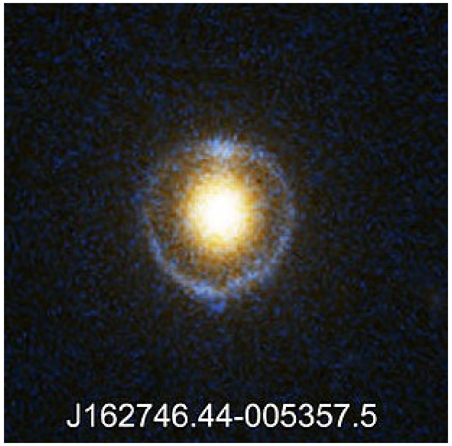
Lens model 2: Axisymmetric mass distribution

In this case, the deflection angle depends on the mass enclosed inside the image position

$$\hat{\vec{\alpha}} = \frac{4GM(<\xi)}{c^2}\frac{\vec{\xi}}{|\vec{\xi}|^2}$$

Similar to the point mass lens, if we have an perfect alignment between observed, lens and source, we can compute:

$$M(<\theta_E) = 1.1\times 10^{14} M_\odot \left(\frac{\theta_E}{30''}\right)^2 \frac{D_{OL}D_{OS}}{\mathrm{Gpc} D_{LS}}$$



For example:
Mass of a lens at z=0.5 and source at z=2.0, forming an Einstein ring with 2 arcseconds

$$M(<2'') \approx 10^{12} M_\odot$$

If we have two different sources we can compute the projected mass within two radii, and measure the slope of the mass distribution!

Lens model 3

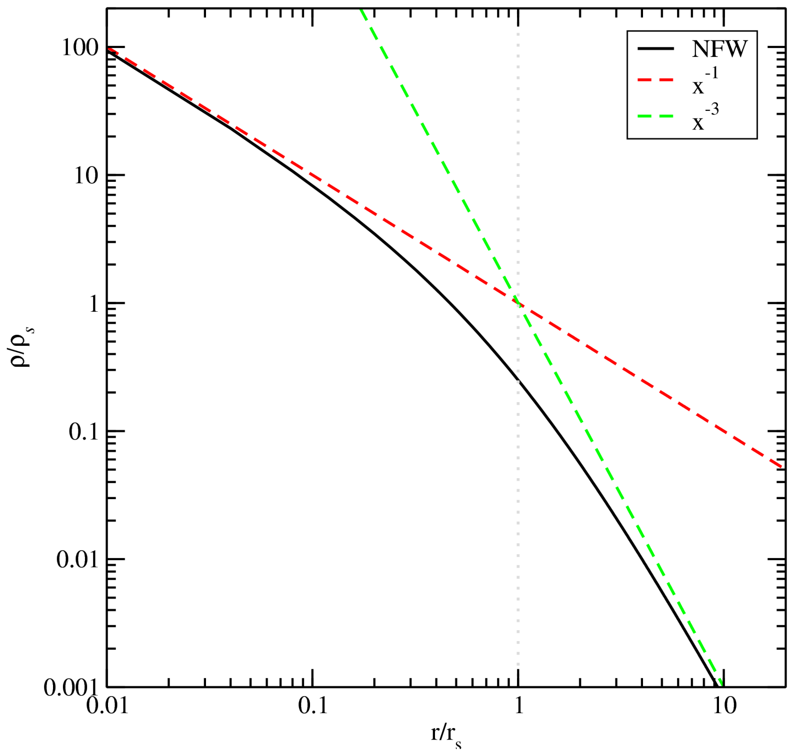
Singular Isothermal Sphere:
Obtained by assuming that the matter distribution behaves as an ideal gas, thermal and hydrostatic equilibrium.

$$\rho(r) = \frac{\sigma_v}{2\pi G r^2}$$

Central velocity dispersion

Navarro, Frenk and White
Obtained from numerical simulations. The shape is universal regardless of the mass range

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$



See for example Meneghetti notes for the calculation of the deflection angle and other quantities.

Lenses with elliptical symmetry

We can construct projected mass distribution with elliptical symmetry by replacing the radial coordinate by an elliptical one:

$$\kappa(x) \rightarrow \kappa(x_e)$$

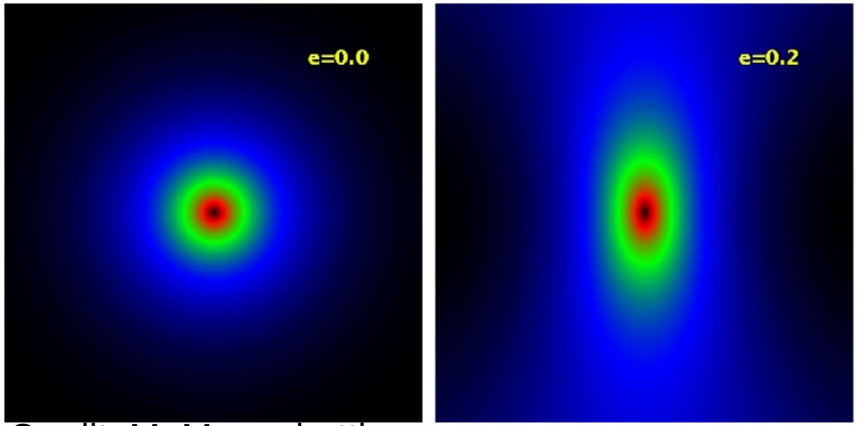
Where:

$$x_e = \sqrt{\frac{x^2}{1-e} + y^2(1-e)}$$

Ellipticity = 1 - b/a

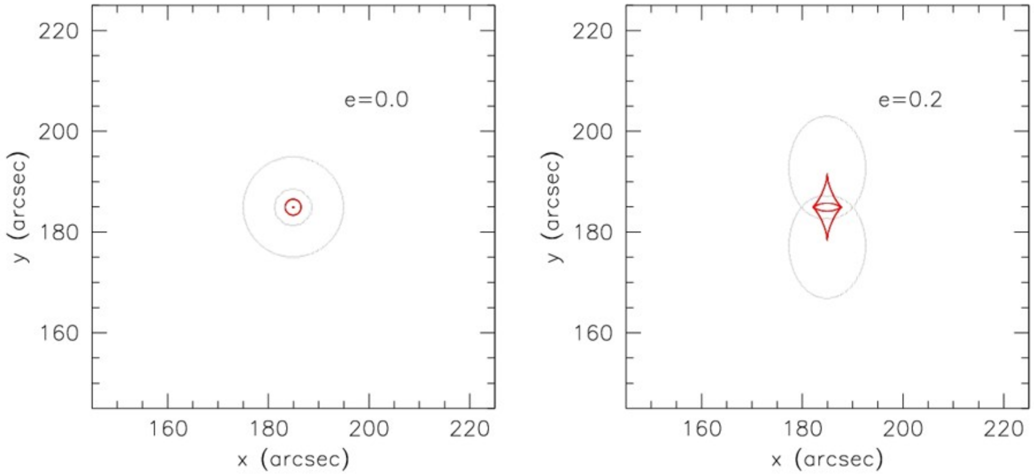
In many mass models it is not possible to obtain analytical solution for the lens quantities, but we can solve numerically

Example of a NFW model



Credit: M. Meneghetti

Critical lines and caustics



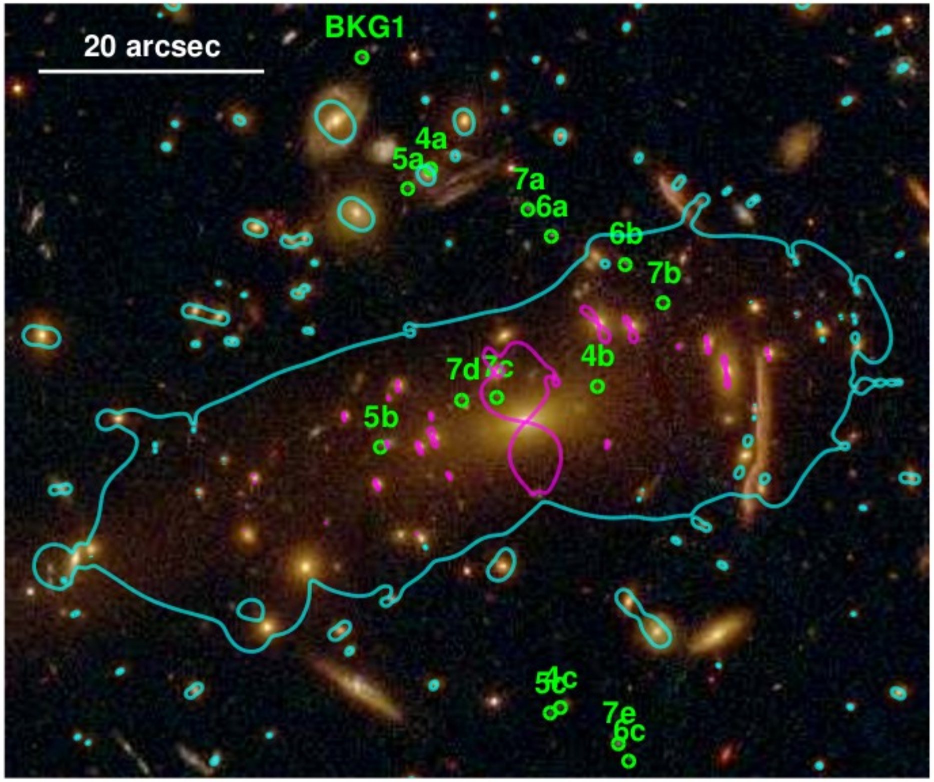
Lenses with elliptical symmetry can produce most of the observed configurations of multiple images. But still very simplistic for real systems

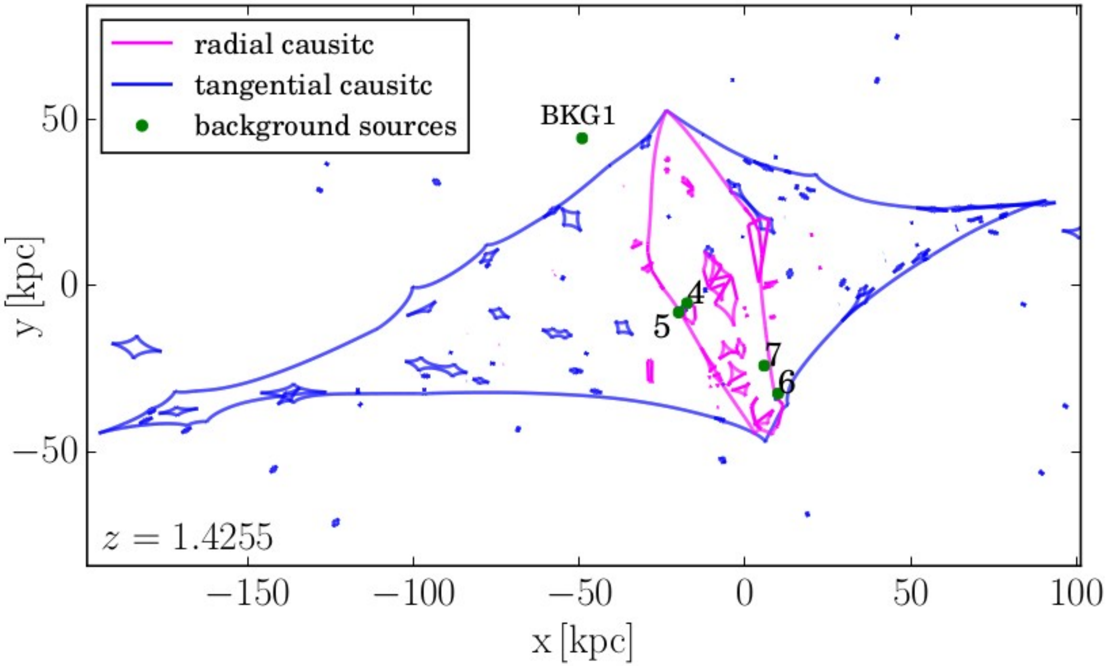
Example of multi component lens

Sum of several mass components

$$\kappa(\vec{x}) = \sum_{i}^{N} \kappa_i(\vec{x})$$

Each galaxy member is described as a singular isothermal sphere, and the Dark Matter extended component by an elliptical mass distribution NFW or a cored IS





z cluster = 0.44; z sources = 1.4

Weak lensing regime

Regions where the lensing effect weakly distorts the back ground galaxies

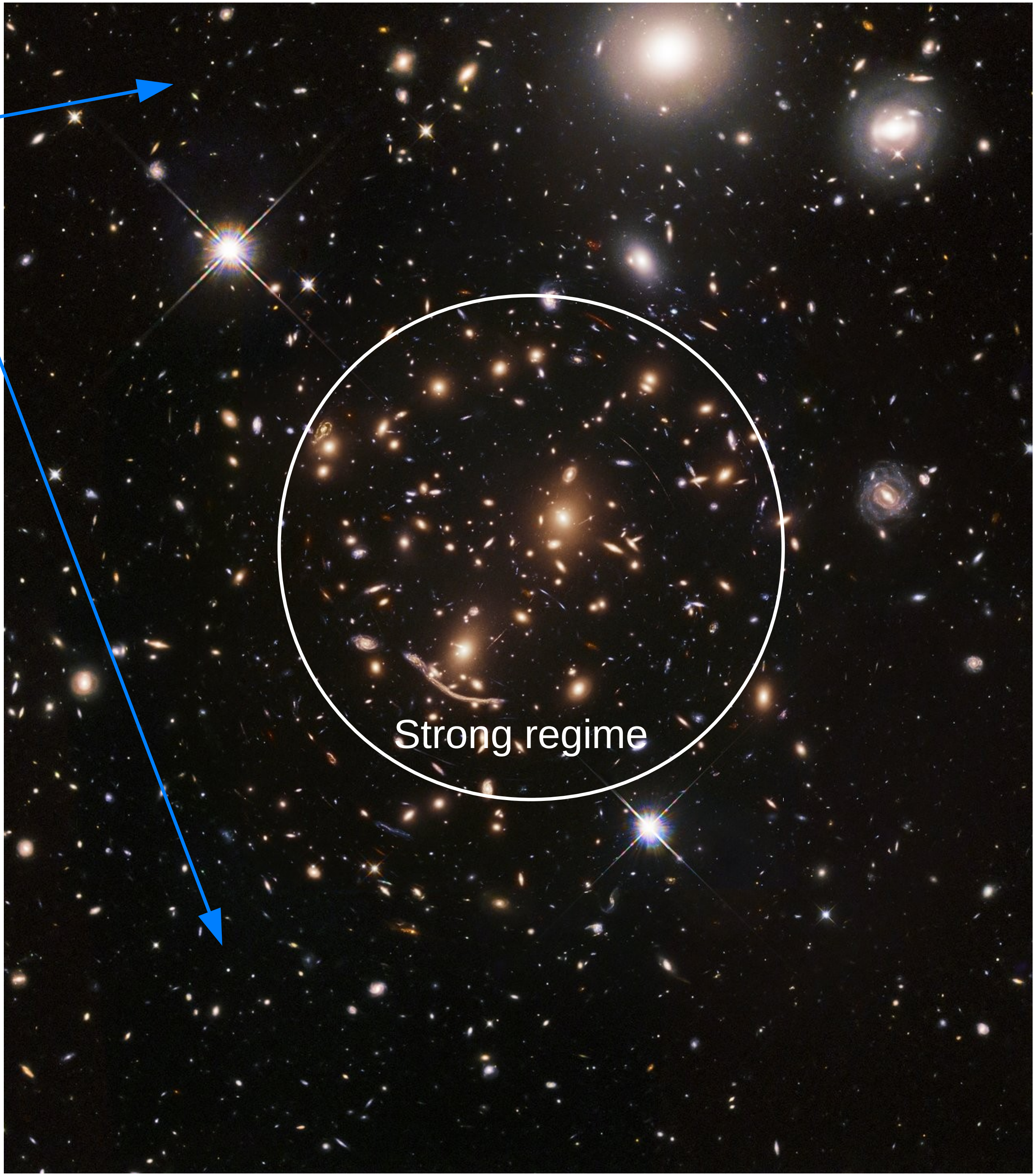
- Larger distances from the core

Assumes that background sources are randomly oriented → average out the intrinsic orientations of a sample of galaxies, obtaining the lensing signal.

Different systematics:

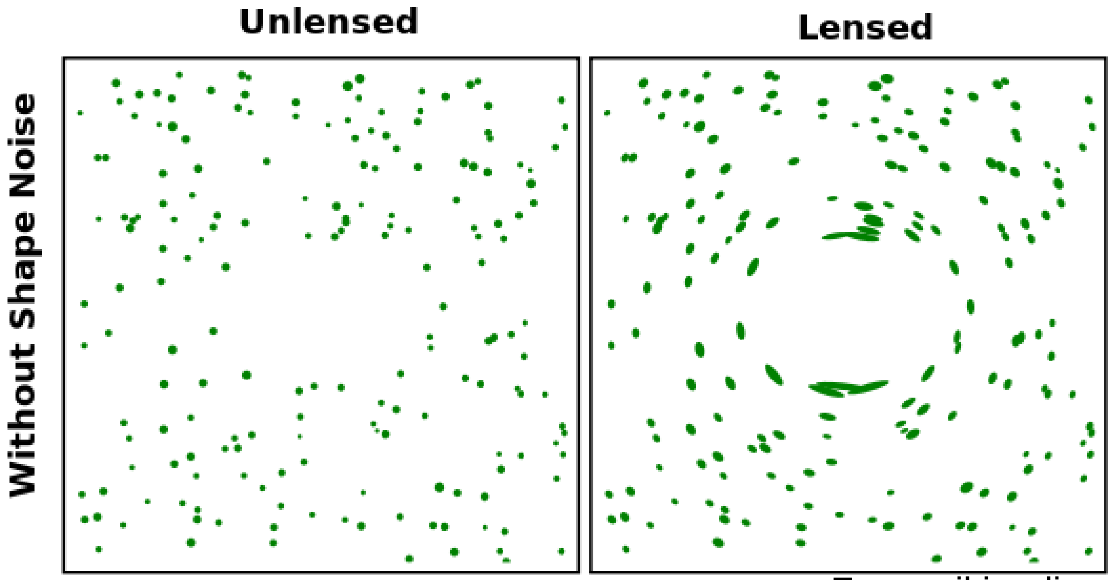
- Observational seeing
- Instrumental distortions
- Intrinsic alignments, etc.

See Schneider+2005 (arXiv:0509252) for a review



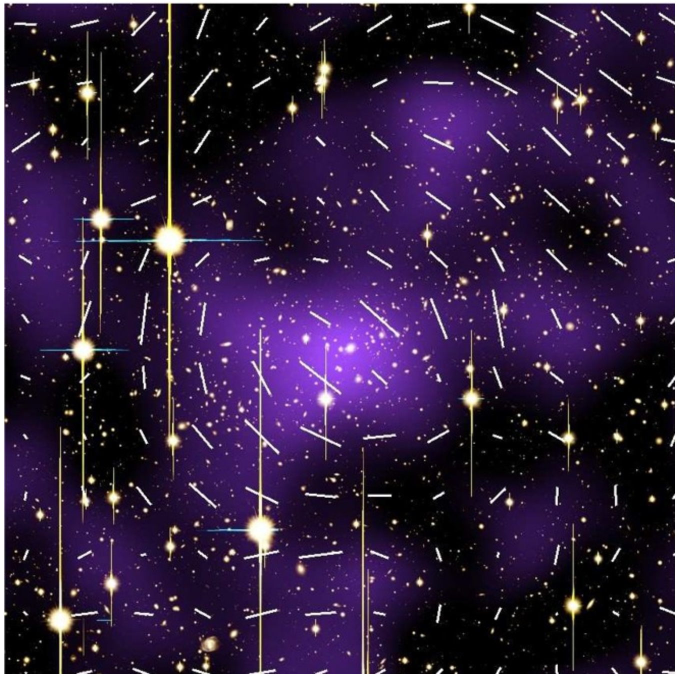
Weak lensing regime

Effect on the shape and sizes of background sources:

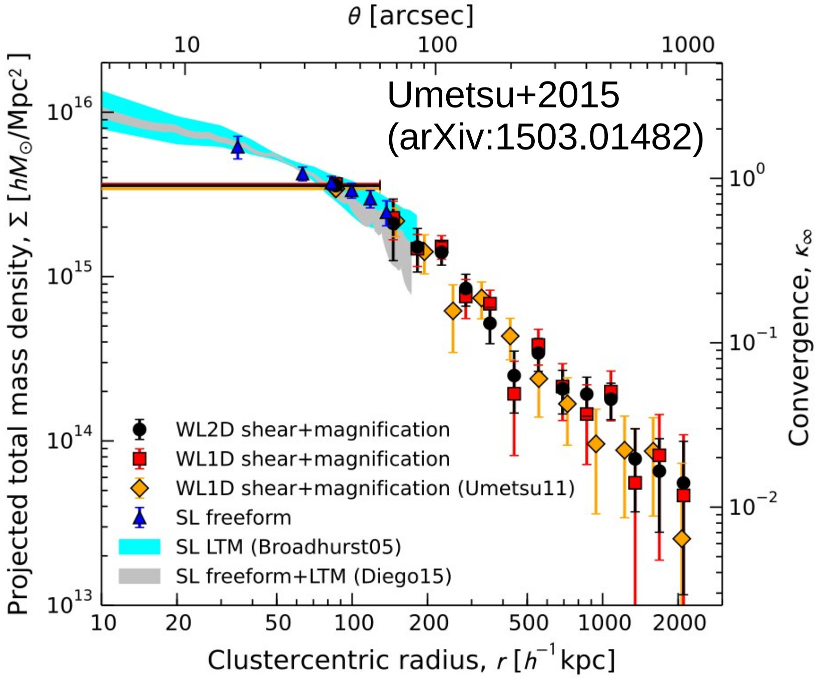


From wikipedia

Oguri+2010(arXiv:1004.4214)



You can measure the mass of galaxy clusters at large radii
Stack the signal of several single galaxies to obtain an average profile

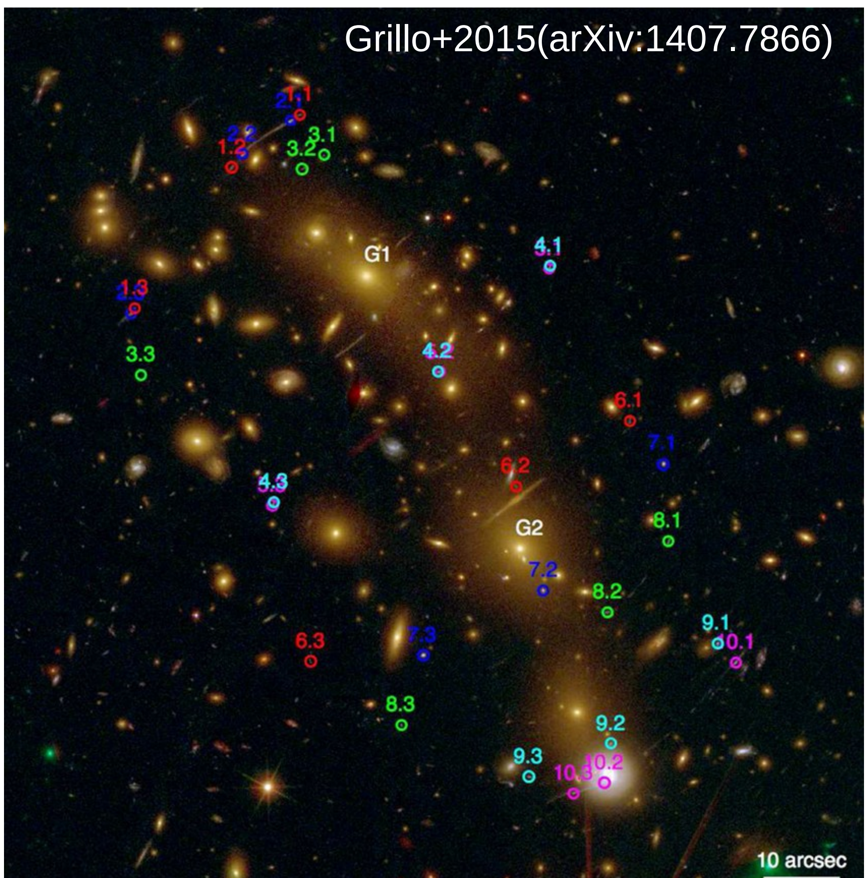


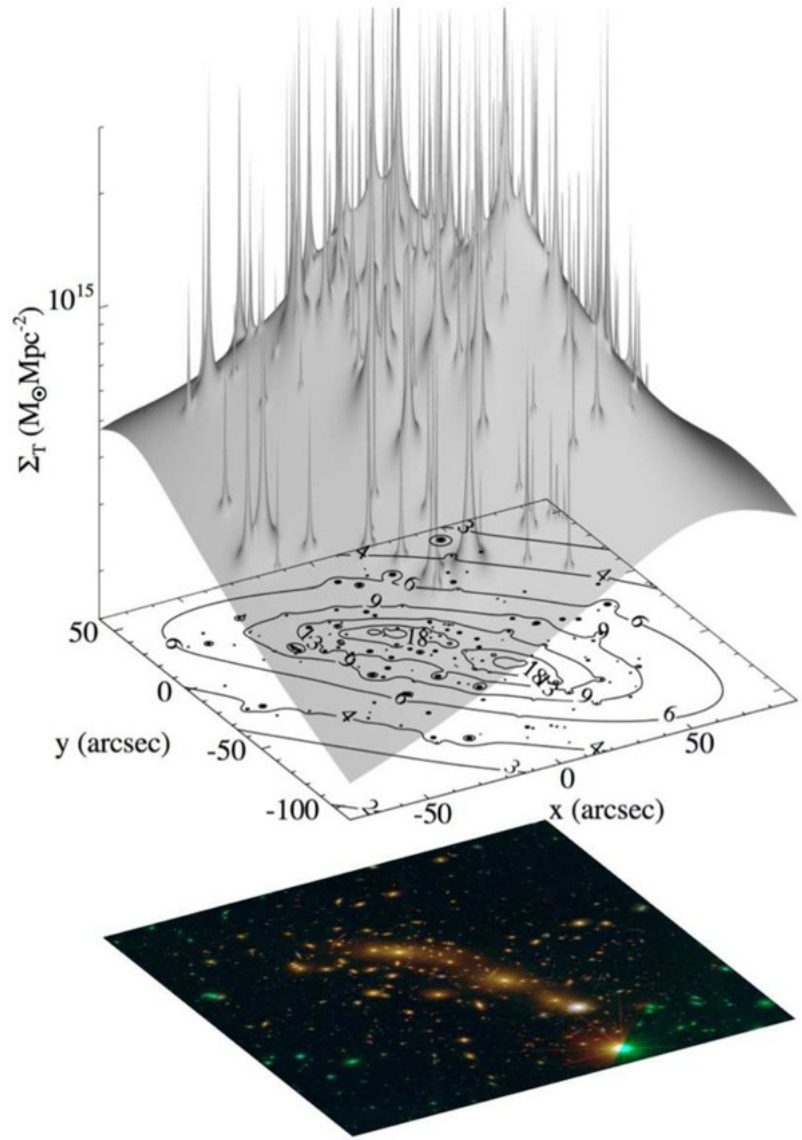
Projected mass distribution of a galaxy cluster

The real world

Systems with Einstein rings are rare → we use the multiple images or distortion to measure the mass distribution of galaxies
The mass distribution is **never** axisymmetric → we have to consider at least elliptical symmetry and substructures
We can model the mass distribution of cluster and galaxies using multiple images

$$\chi^2_{\mathrm{lens}} := \sum_{j=1}^{N_{\mathrm{sys}}} \sum_{i=1}^{N^j_{\mathrm{im}}} \left(\frac{\vec{\theta}^{\mathrm{obs}}_{i,j} - \vec{\theta}^{\mathrm{mod}}\left(\vec{\beta}, \vec{\Pi}\right)}{\sigma^{\mathrm{obs}}_{i,j}} \right)^2$$





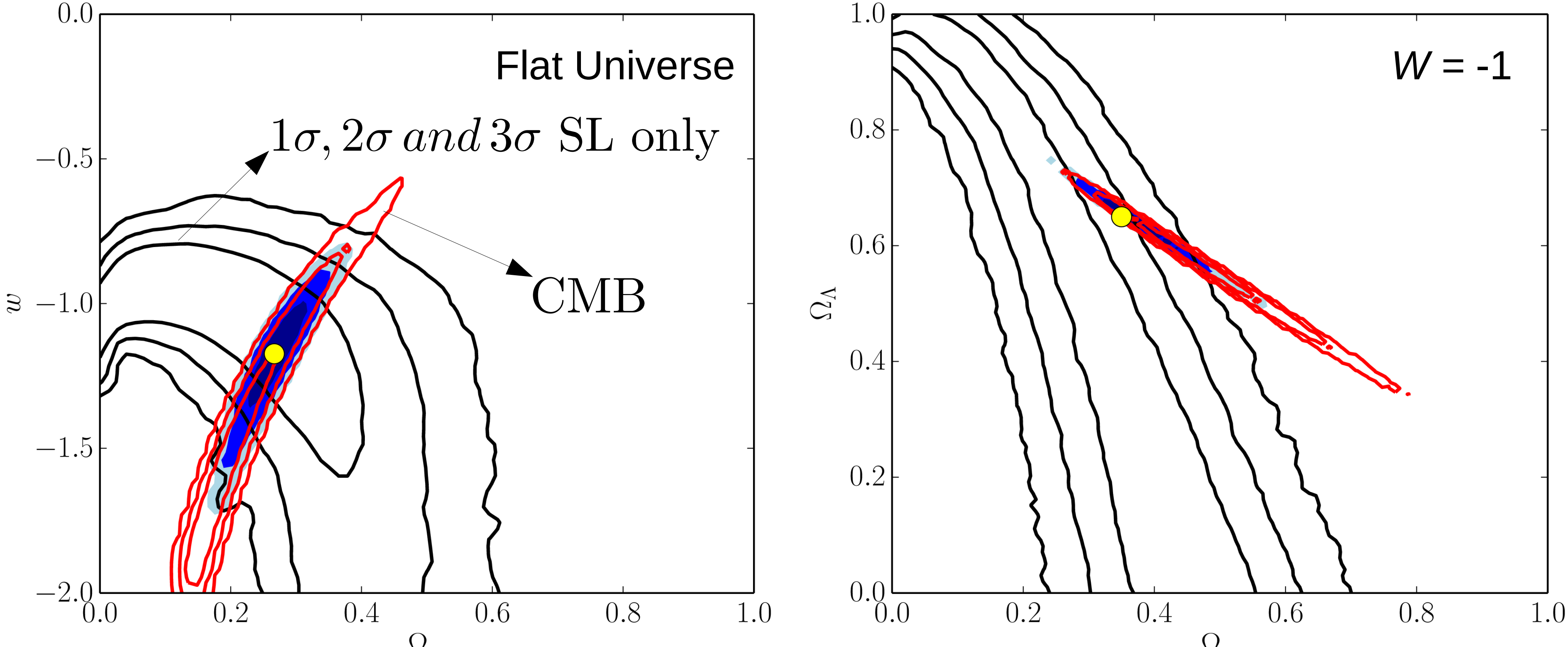
Probe the geometry of the universe

$$\vec{\beta} = \vec{\theta} - \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}}$$

The angular diameter distances depend on the cosmological model

$$D \propto \int_{z1}^{z2} dz \left[\Omega_m (1+z)^3 + \Omega_\Lambda (1+z)^{3(w+1)} \right]$$

Caminha+2016 arXiv:1512.04555



$w = -1.07^{+0.16}_{-0.42}$ $\Omega_m = 0.25^{+0.13}_{-0.16}$ $\Omega_m = 0.35^{+0.11}_{-0.14}$ $\Omega_\Lambda = 0.38^{+0.40}_{-0.26}$

Final remarks

Applications in many fields of Astronomy

- Mass distribution of galaxies and galaxy clusters
- Cosmological measurements, Hubble constant and geometry of the Universe
- Distribution of mass substructures
- Studies of faint and distant galaxies using systems with strong magnifications
- etc

Mathematical formalism/theory (from General Relativity) is well established, but many effects are challenging to consider:

- Structures along the line of sight
- Realistic mass distributions – higher order asymmetries (beyond elliptical symmetry)
- Intrinsic alignments in weak lensing studies
- etc

Perspectives:

- Many observational programs using the best observatories
- Hubble Space Telescope, ESO- Very Large Telescope
- Approved programs with the James Webb Telescope to observe lensing systems
- Euclid satellite will provide a very large sample (100x more than know today) of lenses for robust statistical studeis