Galaxy Clustering and the Large Scale Structure of the Universe

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2nd year BSc ‘Physics of Galaxies’ Guest Lecture

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Lecture Outline

- Galaxy Clustering
  - The two point correlation function
  - Galaxy clustering
- Large Scale Structure
  - Initial conditions
  - Linear perturbation theory
  - Evolution of density perturbations
  - Dark matter halos
  - Galaxy clusters
- Putting it all together

Recommended Texts (graduate level):
Coles & Lucchin
Peebles
The Large Scale Distribution of Galaxies

Galaxies not distributed at random

Cosmic web:

- Voids
- Filaments
- Halos
The two point correlation function $\xi(r)$

Joint probability $\delta^2 P_{12}$ of finding a galaxy in some volume $\delta V_1$ at $r_1$ and another in $\delta V_2$ at $r_2$ is

$$\delta^2 P_{12} = \bar{n}^2 [1 + \xi(r_{12})] \delta V_1 \delta V_2 \quad (1)$$

$\xi(r)$ is the two-point galaxy-galaxy correlation function
$\bar{n}$ is mean number density of galaxies
$r_{12} = |r_1 - r_2|$ (statistical homogeneity and isotropy)

$\xi(r) > 0 \implies$ galaxies are clustered
$\xi(r) = 0 \implies$ galaxies are randomly distributed
$\xi(r) < 0 \implies$ galaxies tend to avoid each other

NB: Integral constraint
NB: For $r > 0$, if $r = 0$ then $\xi(r)$ described by $\delta^D / \bar{n}$
Computing $\xi(r)$

A simple ‘estimator’ used to compute $\xi(r)$ is

$$\hat{\xi}(r) = \frac{DD(r)}{RR(r)} - 1$$  \hspace{1cm} (2)

$DD(r)$ are ‘data-data’ pair counts, number of galaxy pairs in bin of separation $r \pm \Delta r/2$

$RR(r)$ are ‘random-random’ pair counts

Can be computationally ‘expensive’ as for $N_d$ galaxies there are $N_d(N_d - 1)/2$ pairs!

Typically a random catalogue has $N_r \gtrsim 10 \times N_d$ points s.t.

$$\hat{\xi}(r) = \frac{N_r(N_r - 1)DD(r)}{N_d(N_d - 1)RR(r)} - 1$$  \hspace{1cm} (3)

NB: Randomly generated points must have same ‘selection function’ as data
Galaxy clustering by colour

A good description of $\xi(r)$ is (sometimes!) a powerlaw

$$\xi(r) = (r/r_0)^{-\gamma}$$

(4)

$r_0$ is correlation length

‘Red’ galaxies are more strongly clustered than ‘blue’ galaxies (have larger $r_0$)
The two-point correlation function $w(\theta)$

$\xi(r)$ has a two-dimensional (projected) analogue $w(\theta)$ s.t.

$$\delta^2 P_{12} = \bar{\eta}^2 \left[ 1 + w(\theta_{12}) \right] \delta \Omega_1 \delta \Omega_2$$

(5)

$\bar{\eta}$ is mean number of galaxies per unit solid angle $\Omega$

$\delta \Omega_i$ are elements of solid angle

$\theta_{12} = |\theta_2 - \theta_1|$

This is useful if you don’t have `accurate` redshift information!

$\xi(r)$ and $w(\theta)$ can be connected using the Limber (1953) equation (averaging $\xi(r)$ along the line of sight)
Initial Conditions

Gaussian random field of density perturbations

$$\rho = \bar{\rho} \left[ 1 + \delta \rho/\bar{\rho} \right] = \bar{\rho} \left[ 1 + \delta \right]$$  \hspace{1cm} (6)

$\bar{\rho}$ is mean matter density
$\delta(x, t)$ is fractional density perturbation

Gaussian because

$$P(\delta) \, d\delta = \frac{1}{(2\pi \sigma^2)^{1/2}} \exp \left( -\frac{\delta^2}{2\sigma^2} \right) \, d\delta$$  \hspace{1cm} (7)

Primordial power spectrum of density fluctuations $\delta$

$$P_k = A_k \, k^n$$  \hspace{1cm} (8)

$P(k)$ is the Fourier Transform of $\xi(r)$ ($k$ is ‘spatial frequency’)

$$P(k) = \int \xi(r) \exp(-ik \cdot r) \, dr = 4\pi \int \xi(r) \frac{\sin(kr)}{kr} \, dr$$  \hspace{1cm} (9)
Linear perturbation theory

Using fluid equations & treating CDM as a collisionless fluid we can derive the linear ($\delta \ll 1$) perturbation equation (for CDM)

$$\frac{\partial^2 \delta}{\partial t^2} + 2 \frac{\dot{a}}{a} \frac{\partial \delta}{\partial t} = 4\pi G \bar{\rho} \delta$$

(10)

$a(t)$ is cosmological scale factor
Can separate variables s.t. $\delta(x, t) = A(x) D(t)$

$$\frac{d^2 D}{dt^2} + \frac{\dot{a}}{a} \frac{dD}{dt} - 4\pi G \bar{\rho} D = 0$$

(11)

Has general solution

$$\delta(x, t) = A(x) D_{+}(t) + B(x) D_{-}(t)$$

(12)

$D_{+}(t)$ is growing mode
$D_{-}(t)$ is decaying mode (can often ignore)
$A(x), B(x)$ are arbitrary functions
Evolution of linear density perturbations

Given a growing mode, \( D_+(t) \), where \( \delta(x, t) = \delta(x, t_0)D(t) \), we can write down the linear evolution of \( P(k, t) \)

\[
P(k, t) = P(k, t_1) \left[ \frac{D_+(t)}{D_+(t_1)} \right]^2 = P_k T(k)^2 \left[ \frac{D_+(t)}{D_+(t_1)} \right]^2
\]

And thus calculate \( \xi(r, t) \) for matter

\( T(k) \) is ‘transfer function’ describes modification to primordial power spectrum the early Universe.

\( T(k) \) is different for hot, warm and cold dark matter

As density perturbations grow assumption linearity \( (\delta << 1) \) breaks down

Nonlinear evolution is (very!) complicated
Evolution of density perturbations in $\Lambda$CDM

$N$-body simulations such as the Millennium Simulations (Springel 2005) directly compute the (linear & non-linear) evolution of the cosmic density field.
Dark matter halos

The final stage of the nonlinear evolution of a density perturbation is a dark matter ‘halo’

They are ‘biased’ tracers of the matter distribution

\[ \xi(r)_{\text{matter}} = b^2 \xi(r)_{\text{halos}} \quad (14) \]

simulations show bias depends on halo mass

Halos can grow further through halo mergers and smooth accretion of matter
Formation of galaxy clusters

Many halo mergers in the densest (most biased) environments lead to formation of ‘clusters’
Putting it all together

Connecting galaxies & halos

- Measure $\xi(r)$ for a sample of galaxies
- Derive bias based on model for $\xi(r)_{\text{matter}}$ (based on perturbation theory)
- Compare to galaxy bias to halo bias in simulations

Galaxy colour & environment

- Most massive overdensities (clusters) develop hot gaseous atmospheres
- Ram pressure strips gas from infalling galaxies
- Limited gas supply inhibits star formation
- Red galaxies exhibit stronger clustering
Galaxies are not distributed at random
Their distribution traces large scale structure (Cosmic web)
Large scale structure result of evolution of density perturbations
Galaxy clustering can relate galaxies (we can observe) to dark matter
Red galaxies are more strongly clustered (environmental effects)

Thank you!