
The giant elliptical galaxy ESO 325-G004. HST

The HI-H_2 Transition in CO-Rich Early-Type Galaxies

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Motivation

- It is now well known that many early-type galaxies (E and S0) contain significant amounts of cold gas. A large number of HI and CO maps exist for several representative surveys (e.g. ALFALFA, HIPASS, ATLAS^{3D}).
  - (e.g. Wiklind et al. 1995; Young 2002; Welch & Sage 2003; Sage & Welch 2006; Morganti et al. 2006; Sage et al. 2007; Combes et al. 2007; Osterloo et al. 2007; Alighieri et al. 2007; Welch et al. 2010; Serra et al. 2011, Young et al 2011).
- The evolutionary pathway of early-type galaxies is thought to be driven in part by the acquisition and transformation of this cold gas into new stars. To the best of our knowledge stars form only from the molecular gas phase.
  - ➢ Need a reservoir of HI and the right conditions to form H_2.
  - ➢ Clearly, knowledge of the amount of cold gas in the molecular phase versus the atomic phase is an important constraint for theoretical models of star formation and early-type galaxy evolution.

Disk Galaxies as Tests of Empirical and Theoretical Models of Molecule Formation

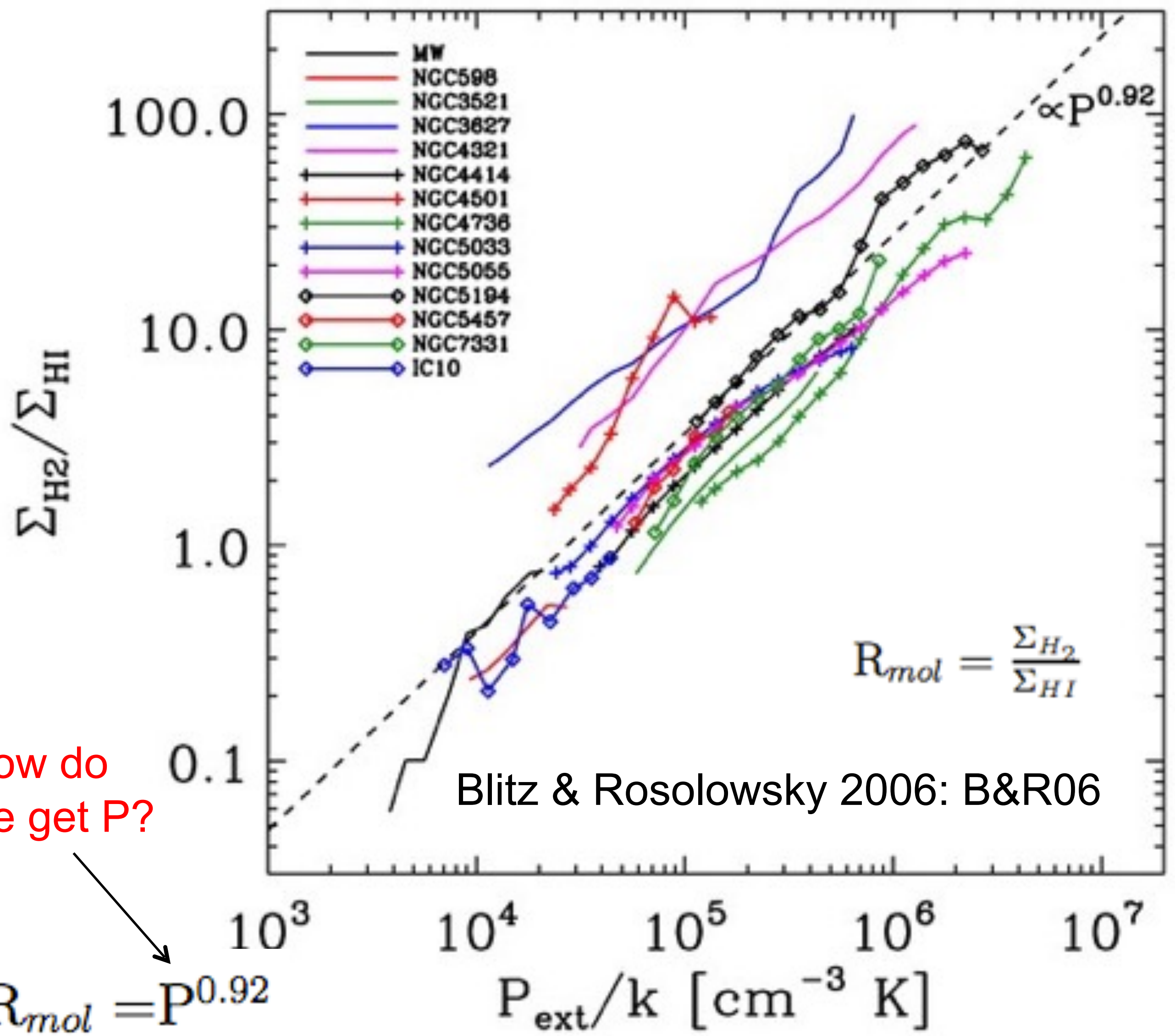
- The question of molecule formation has been thoroughly studied in the literature mostly through two different methods:

  - Empirical studies using resolved CO and HI maps of nearby disk galaxies (e.g. Blitz & Rosolowsky 2006; Bigiel et al. 2008; Leroy et al. 2008).
    - Infer $\mathrm{R}_{mol} = \frac{\Sigma_{H_2}}{\Sigma_{HI}}$ is a function of the hydrostatic mid-plane pressure P_{HSMP}, which in turn is a function of the stellar as gas volume densities.

  - Comparisons of observations (CO and HI) to physical models of the chemical and physics processes that regulate the balance between the formation and dissociation of molecules (e.g. Elmegreen 1993; Krumholz et al. 2010)

- Both approaches predict values of the molecular fraction which are roughly consistent with molecular and atomic observations in nearby disk galaxies.

R_{mol}–Pressure Relation: Disks



Blitz & Rosolowsky 2006: B&R06

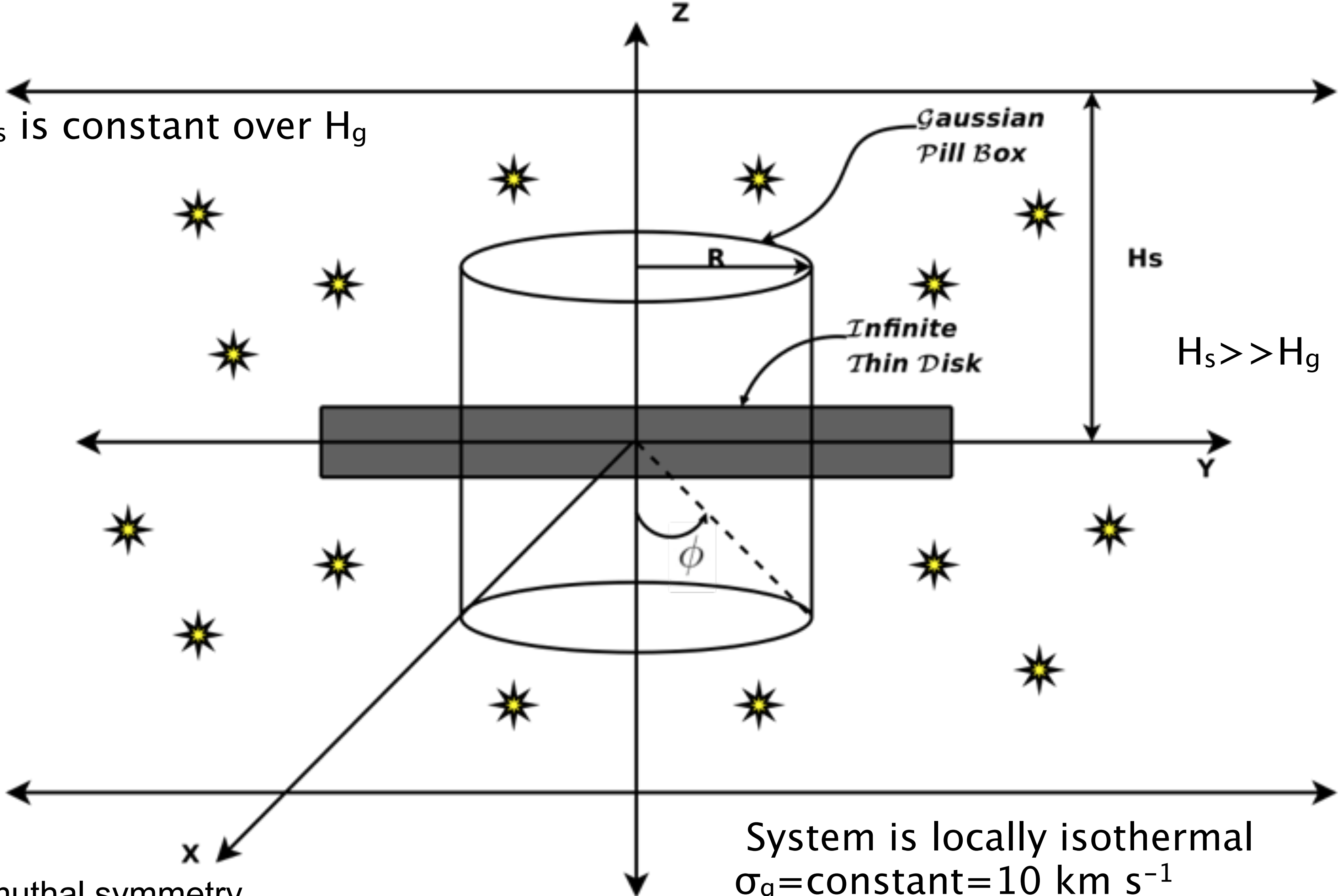
How do we get P?

$R_{mol} = P^{0.92}$

Hydrostatic equilibrium

Elmegreen 1993; Blitz & Rosolowsky 2006

ρ_s is constant over H_g



$H_s >> H_g$

Azimuthal symmetry

System is locally isothermal

σ_g=constant=10 km s^{-1}

Non-thermal pressures are neglected.

APPENDIX

DERIVATION OF THE HYDROSTATIC MIDPLANE PRESSURE

In this section we derive a self consistent expression for the midplane pressure of a disk galaxy using the formalism of Blitz & Roslowsky (2006) and Koyama and Ostriker (2009). A disk galaxy can be modeled as as an infinite, two fluid (gas and stars) azimuthally symmetric disk in hydrostatic equilibrium (Figure 23; see section 2.3 of Binney & Tremaine 2008). The gas pressure is assumed to come from the gravitational potential of the stars, gas, and a dark matter halo. The potential due to the dark matter is assumed to closely follow that of the stars. Components of the midplane pressure in galaxies include thermal (gas motions), and non-thermal (magnetic fields, cosmic rays, and micro turbulence) pressures. For simplicity non-thermal pressures due to magnetic fields, and cosmic rays are excluded. Additionally, it is assumed that the stellar velocity dispersion is constant, the stellar density is constant over the scale height of the gas disk, and the scale height of the stars is much larger than the scale height of the gas disk.

The equation of hydrostatic equilibrium is:

$$\vec{\nabla} P(R,z) = -\rho_g(R,z)\vec{\nabla}\Phi(R,z) \tag{A1}$$

where P is the gas pressure and $\Phi(R,z)$ is the total gravitational potential of the galaxy. The system is assumed to be locally isothermal such that the velocity dispersion of the gas is constant with height z above the midplane. For an isothermal gas the pressure is given by:

$$P(R,z) = \sigma_g^2(R)\rho_g(R,z). \tag{A2}$$

Now equation A1 becomes:

$$\sigma_g^2(R)\frac{\partial\rho_g(R,z)}{\partial z} = -\rho_g(R,z)\frac{\partial\Phi(R,z)}{\partial z}. \tag{A3}$$

Invoking azimuthal symmetry guarantees that the contributions from the derivatives with respect to R will be negligible (e.g. see Olling 1995).

Poisson's equation equation relating the gravitational potential to the volume density of the gravitating material is given by:

$$\vec{\nabla}^2\Phi = 4\pi G\rho(z,R)), \tag{A4}$$

where G is the gravitational constant. If both the stars and the gas contribute to the total gravitational potential Poisson's equation becomes:

$$\vec{\nabla}^2\Phi = 4\pi G(\rho_s(R) + \rho_g(z,R)). \tag{A5}$$

We assume that the density of the stars is constant over the scale height of the gas disk, which appears to be a good approximation for both spirals and early-type galaxies (see section 4.4). Combining equation A5 and the derivative of equation A3 gives:

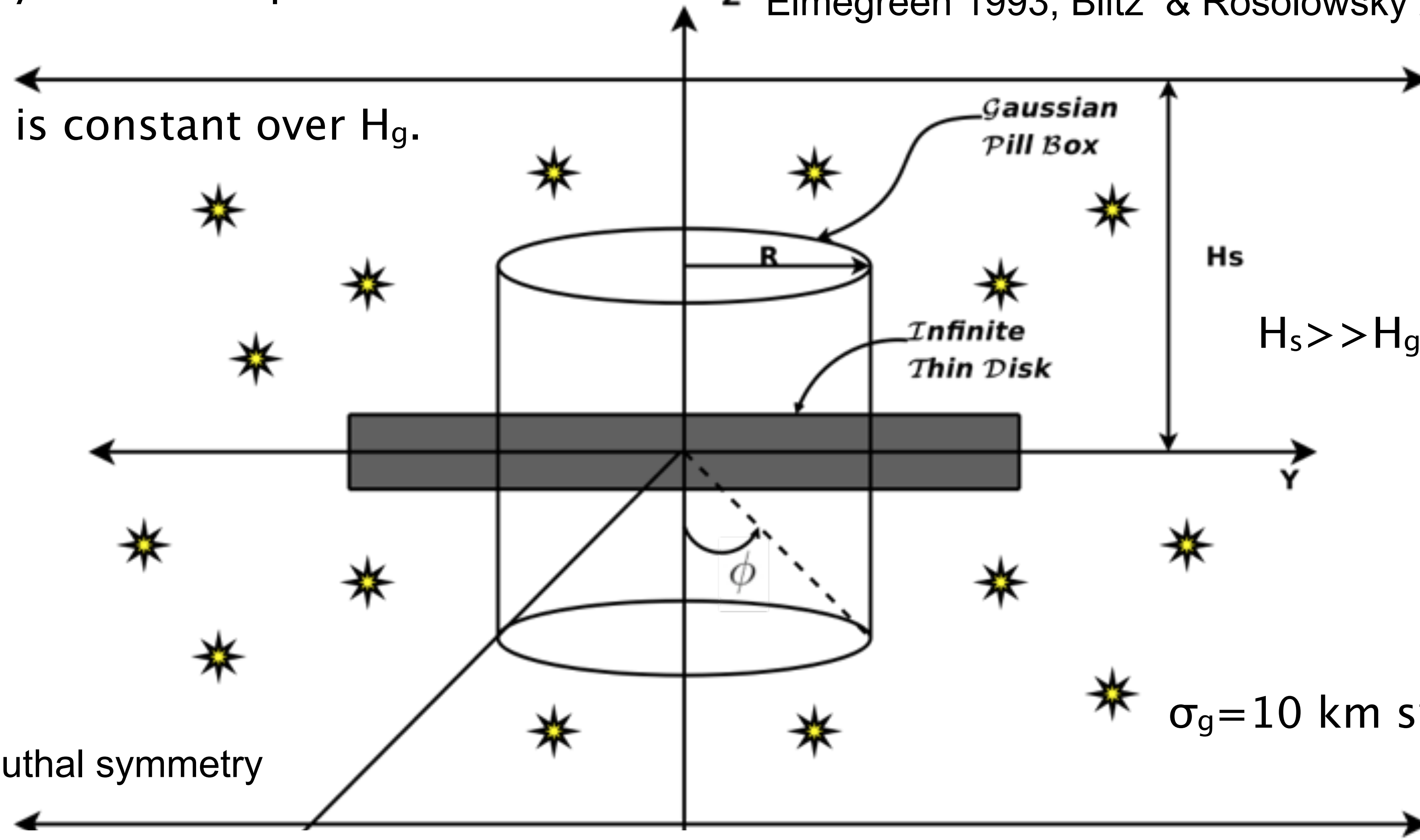
$$\frac{1}{\rho_g^2(z)}\left(\frac{d\rho_g(z)}{dz}\right)^2 - \frac{1}{\rho_g(z)}\frac{d^2\rho_g(z)}{dz^2} = \frac{4\pi G}{\sigma_g^2}(\rho_s + \rho_g(z)). \tag{A6}$$

This equation ignores the radial dependence for now. No analytical solution exists for the above differential equation. However, solutions do exist for the limiting cases where $\rho_s >> \rho_g$ and $\rho_g >> \rho_s$.

Hydrostatic equilibrium

Elmegreen 1993; Blitz & Rosolowsky 2006

ρ_s is constant over H_g.



$H_s >> H_g$.

$\sigma_g = 10$ km s^{-1}

Azimuthal symmetry

$$\frac{1}{\rho_g^2(z)}\left(\frac{d\rho_g(z)}{dz}\right)^2 - \frac{1}{\rho_g(z)}\frac{d^2\rho_g(z)}{dz^2} = \frac{4\pi G}{\sigma_g^2}(\rho_s + \rho_g(z)).$$

$\rho_s >> \rho_g$ or $\rho_s << \rho_g$

$$P(0) = \rho_g(0)\sigma_g^2 = (2G)^{0.5}\Sigma_g\sigma_g\left[\rho_s^{0.5} + (\frac{\pi}{4}\rho_g(0))^{0.5}\right].$$

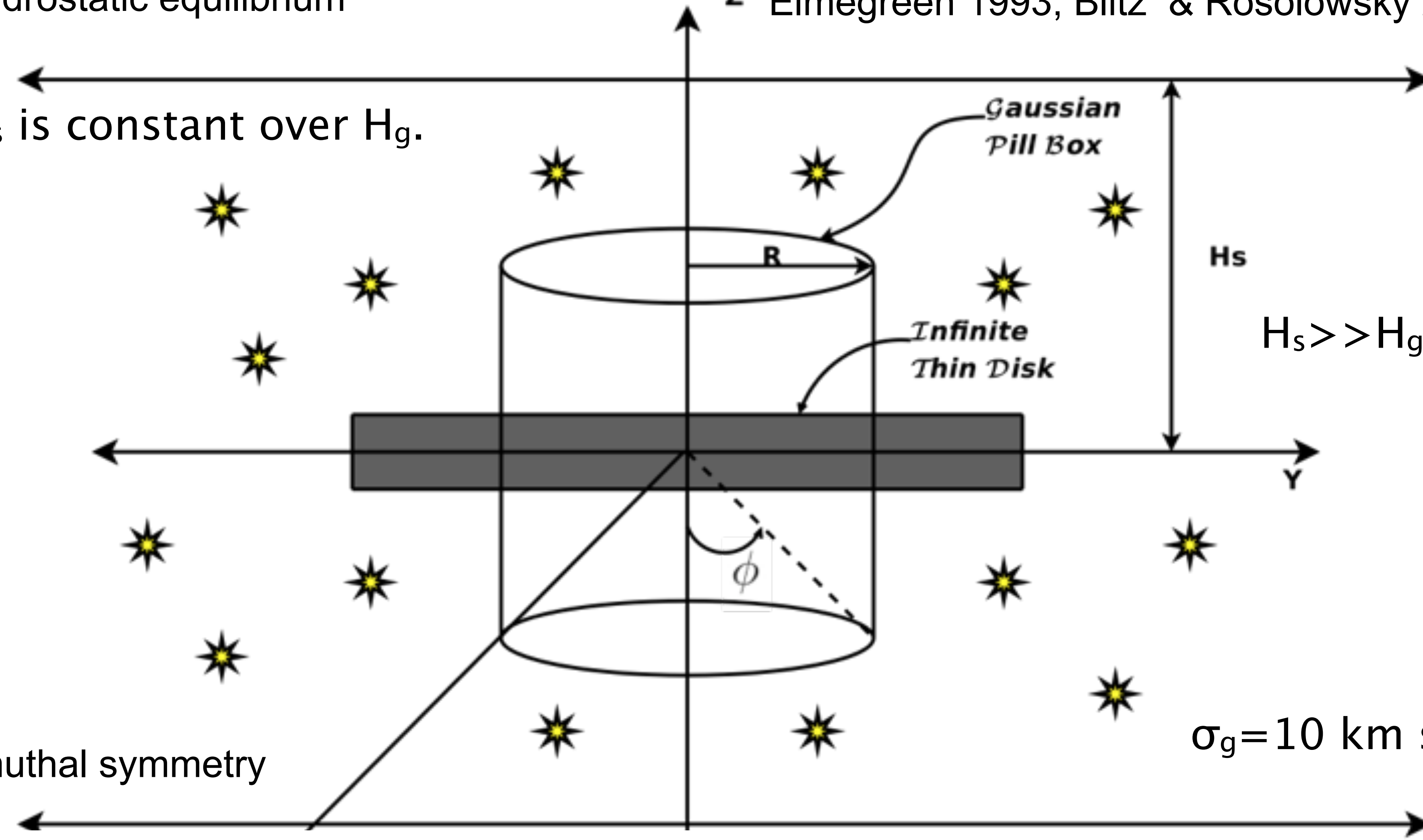
Bad Approximation for

$\rho_s = \rho_g$

Hydrostatic equilibrium

Elmegreen 1993; Blitz & Rosolowsky 2006

ρ_s is constant over H_g.



$H_s >> H_g$.

$\sigma_g = 10\ \mathrm{km\ s^{-1}}$

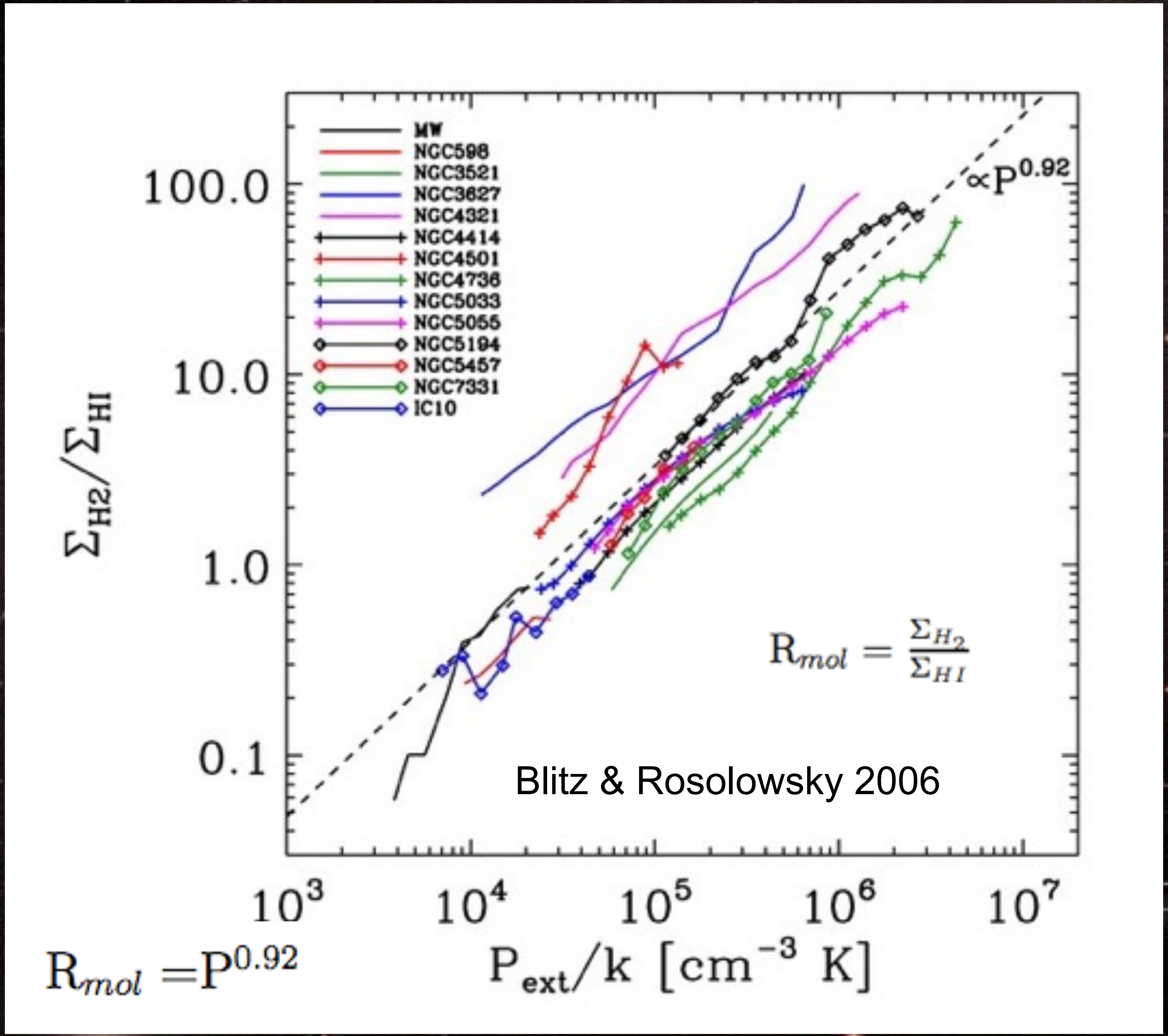
Azimuthal symmetry

$$\frac{1}{\rho_g^2(z)}\left(\frac{d\rho_g(z)}{dz}\right)^2 - \frac{1}{\rho_g(z)}\frac{d^2\rho_g(z)}{dz^2} = \frac{4\pi G}{\sigma_g^2}(\rho_s + \rho_g(z)).$$

$$P(0) = \rho_g(0)\sigma_g^2 = (2G)^{0.5}\Sigma_g\sigma_g\left[\rho_s^{0.5} + \cancel{\left(\frac{\pi}{4}\rho_g\right)^{0.5}}\right].$$

$\rho_s >> \rho_g$ for disk galaxies

R_{mol}–Pressure Relation: Disks



R_{mol}–Pressure Relation: Models

- These empirical relationship are already being used in theoretical models/ semi-analytic models of star formation and galaxy evolution (e.g. Lagos et al. 2014; Pallottini et al. 2016; Xie et al. 2016).

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The origin of the atomic and molecular gas contents of early-type galaxies – I. A new test of galaxy formation physics

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Uses Leroy et al. empirical relationship. $R_{mol}=P_{HSMP}^{0.8}$

ABSTRACT

We study the atomic (H I) and molecular hydrogen (H_2) contents of early-type galaxies (ETGs) and their gas sources using the GALFORM model of galaxy formation. This model uses a self-consistent calculation of the star formation rate, which depends on the H_2 content of galaxies. We first present a new analysis of H I Parkes All-Sky Survey and ATLAS^{3D} surveys, with special emphasis on ETGs. The model predicts H I and H_2 contents of ETGs in agreement with the observations from these surveys only if partial ram pressure stripping of the hot gas is included, showing that observations of neutral gas in 'quenched' galaxies place stringent constraints on the treatment of the hot gas in satellites. We find that ≈90 per cent of ETGs at $z = 0$ have neutral gas contents supplied by radiative cooling from their hot haloes, 8 per cent were supplied by gas accretion from minor mergers that took place in the last 1 Gyr, while 2 per cent were supplied by mass-loss from old stars. The model predicts neutral gas fractions strongly decreasing with increasing bulge fraction. This is due to the impeded disc regeneration in ETGs, resulting from both active galactic nuclei feedback and environmental quenching by partial ram pressure stripping of the hot gas.

Key words: stars: formation – galaxies: evolution – galaxies: formation – galaxies: ISM.

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- Reproduces HI and H_2 content of early-type galaxies from ATLAS^{3D}
  - ➡ Have to invoke partial ram pressure stripping of the hot gas.
- Reproduces kinematic misalignment distribution of molecular gas in early-type galaxies.
  - ➡ Have to invoke cooling from a misaligned hot gas halo.

regeneration in ETGs, resulting from both active galactic nuclei feedback and environmental quenching by partial ram pressure stripping of the hot gas.

Key words: stars: formation – galaxies: evolution – galaxies: formation – galaxies: ISM.

Can we use B&R06 for Early-Types?

- Very different gas contents, stellar populations, and evolutionary histories (Welch & Sage; Crocker et al. 2011).

- Wider range of HI/H_2 mass ratio than spirals (0.01 to 100; Welch et al. 2010; Lucero & Young 2012).

- Central gas and stellar densities are typically larger than that observed in disks. Large central gas and stellar densities translate to higher pressures.

- Higher metalicities (z ranges from 1-2.5 solar; Kuntschner et al. 2010) and lower UV field strengths.

- In the case of higher pressures and metalicities and lower UV field strengths, the photo-dissociation models predict a larger molecular fraction for early-types.

➢ For all of these reasons, the early-type galaxies provide a complementary test of HI/H_2 physics, and the spiral-derived relations should be tested in order to verify their applicability in the simulations.

Sample and Observations

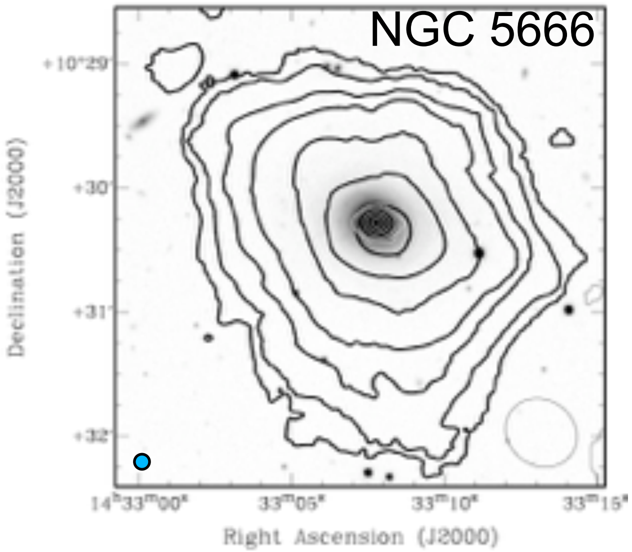
- Eleven of the most CO-rich early-types (M_{H2} $6x10^8$-$5x10^9$ M_{sol}) from Young 2002.

- A range of morphologies (6 S0, and 5 E), distances (up to 80 Mpc), environments (field/cluster/group), and optical luminosities ($-21.7 \leq M_B \geq -18.3$).

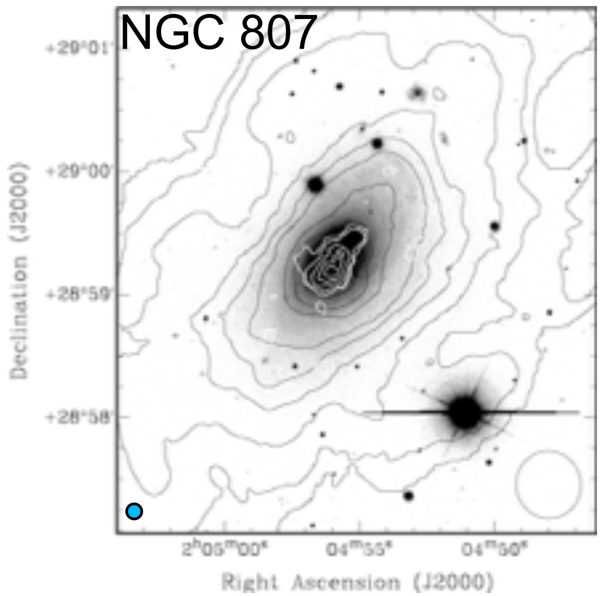
- All sample galaxies have $R^{1/4}$ classifications as E, E/S0 or S0 in several catalogs.

- Prior knowledge of amount and location of star formation for all sample galaxies (Young, Bendo, Lucero 2009; Lucero & Young 2007).

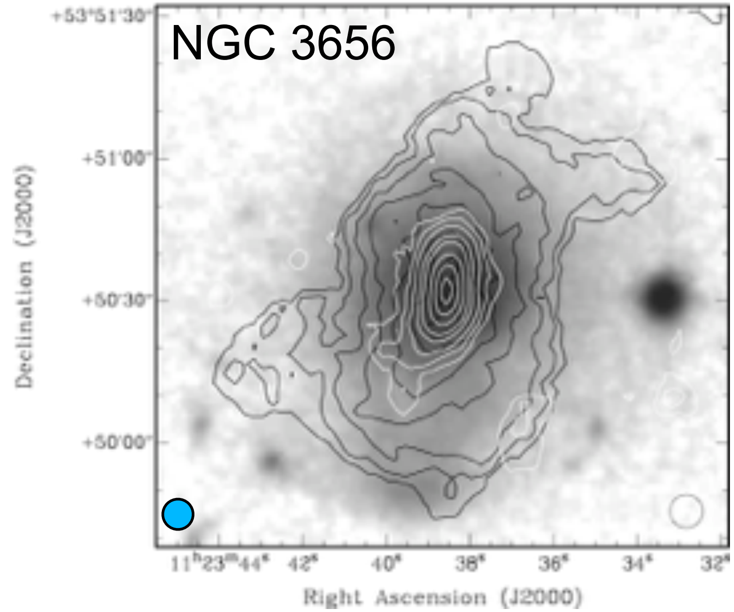
- HI VLA C array data (15'') are compared to the existing CO maps (7'') from BIMA. HI observations have 2 to 16 times better resolution than the current HI surveys (e.g. ATLAS 3D). Lucero & Young 2013.



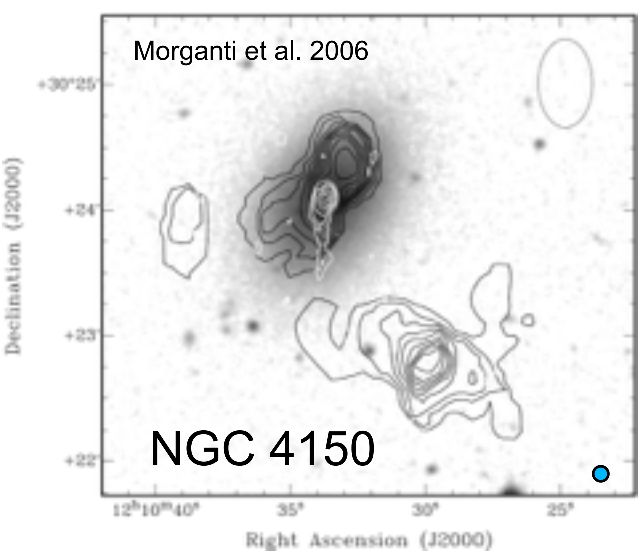
peak HI surface density is $9.1\mathrm{x}10^{20}$ cm^{-2}



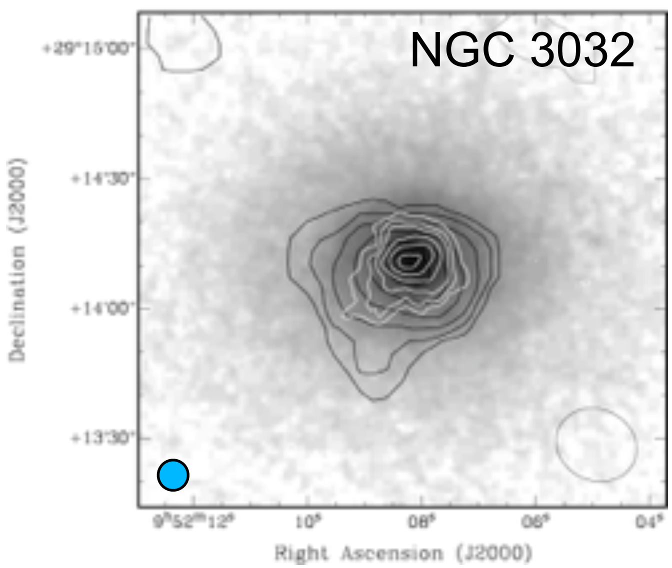
peak HI surface density is $7.7\mathrm{x}10^{20}$ cm^{-2}



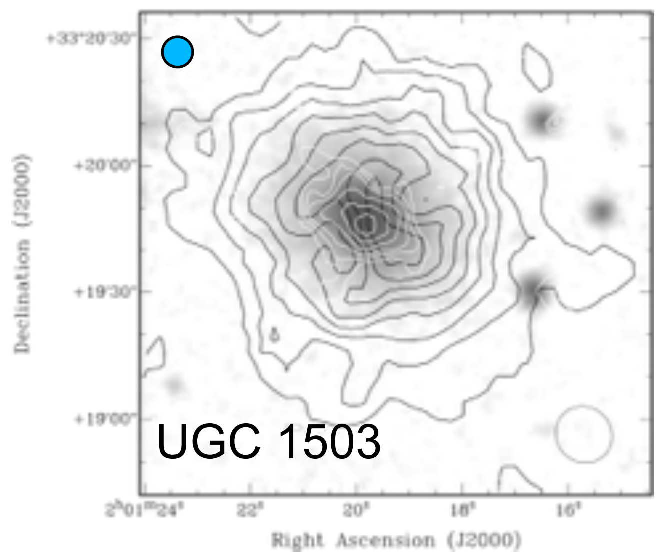
peak HI surface density is $7.2\mathrm{x}10^{21}$ cm^{-2}



peak HI surface density is $2.9\mathrm{x}10^{19}$ cm^{-2}



Peak HI surface density is $1.3\mathrm{x}10^{21}$ cm^{-2}

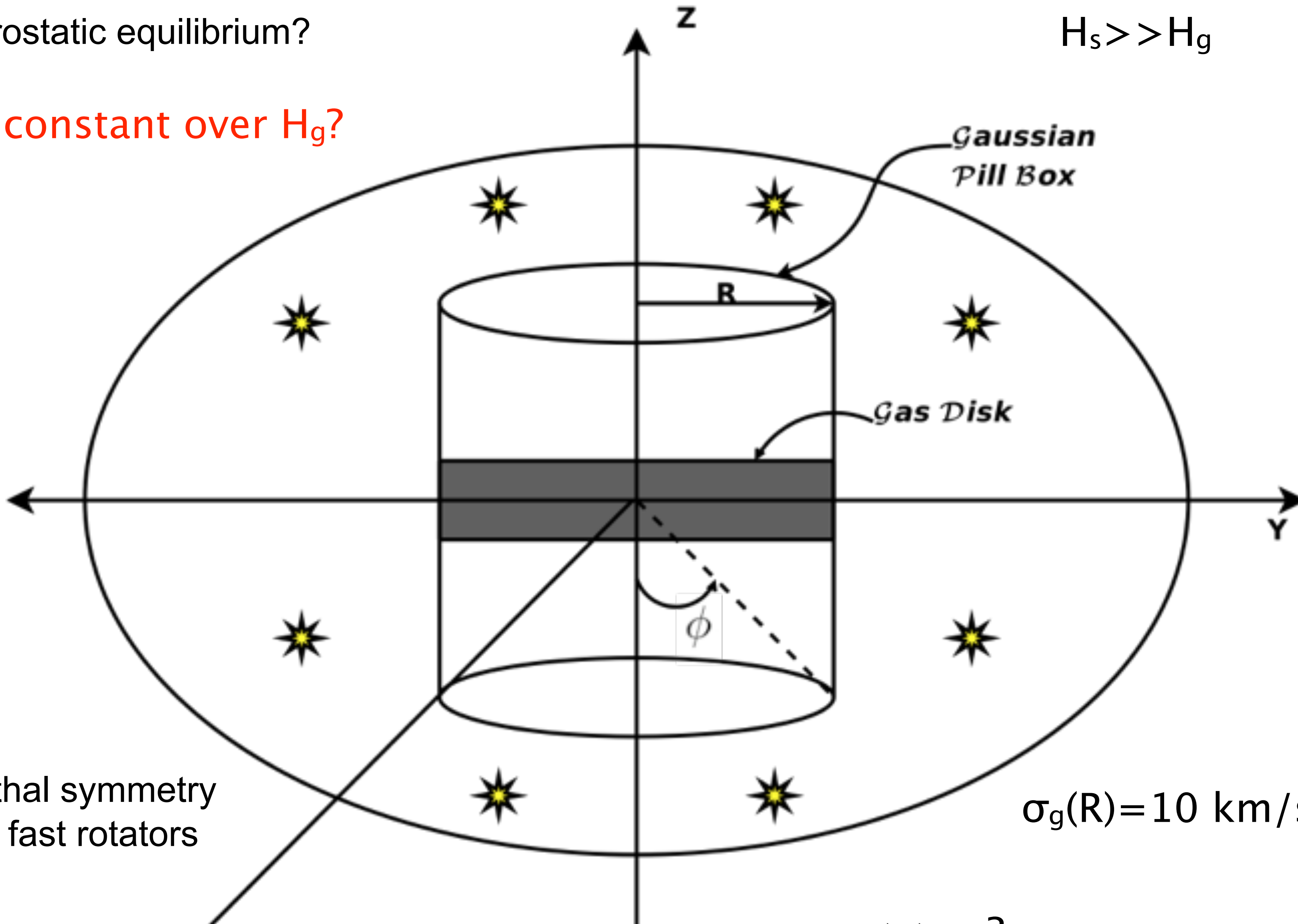


peak HI surface density is $9.9\mathrm{x}10^{20}$ cm^{-2}

Hydrostatic equilibrium?

$H_s >> H_g$

ρ_s is constant over H_g?



Azimuthal symmetry
Oblate fast rotators

$\sigma_g(R) = 10$ km/s

$\rho_s >> \rho_g$?

$$P(0) = \rho_g(0)\sigma_g^2 = (2G)^{0.5}\Sigma_g\sigma_g\left[\rho_s^{0.5} + (\frac{\pi}{4}\rho_g(0))^{0.5}\right].$$

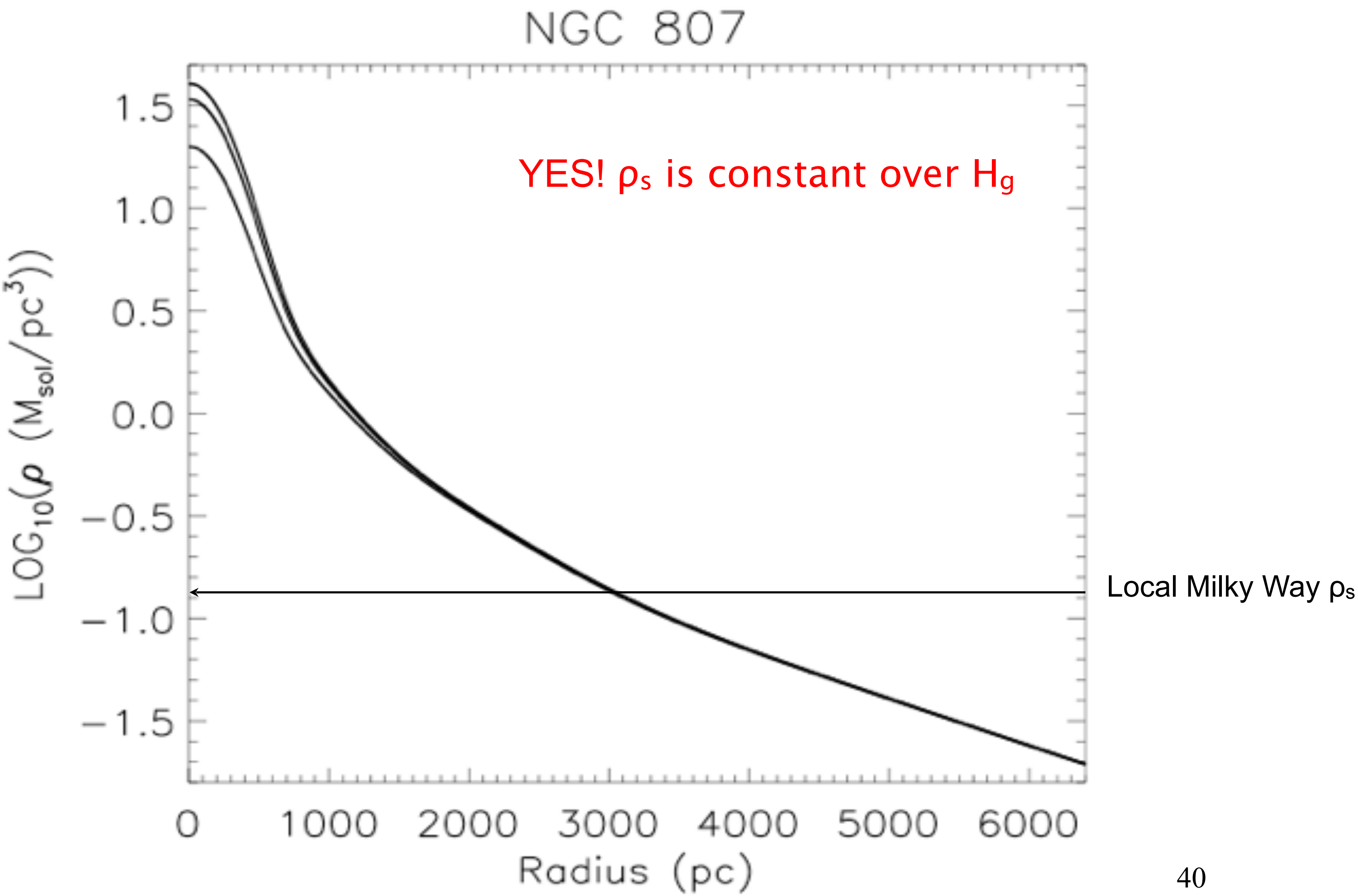
Is cold gas self gravity important?

Multi-Gaussian (MGE) Stellar Density

- The 3D stellar density, $\rho_s(R,0)$, is determined using a multi-Gaussian expansion (MGE) fit developed by Cappellari (2002) to 2μ All Sky Survey (2MASS) K_s-band ATLAS images (Skrutskie et al. 2006) or UKIDSS K-band images if available.
  - ➢ Characterizes the observed surface brightness as a sum of Gaussians, each Gaussian is deprojected assuming a constant inclination. This allows the photometry to be reproduced in detail.
- Requires an inclination for deprojection.
  - ➢ Estimated from optical dust images.
- Requires azimuthal symmetry.
  - ➢ Assume that sample galaxies are oblate/fast rotators.
- Doesn't require an assumed stellar scale height.

ρ_s Constant with H_g?

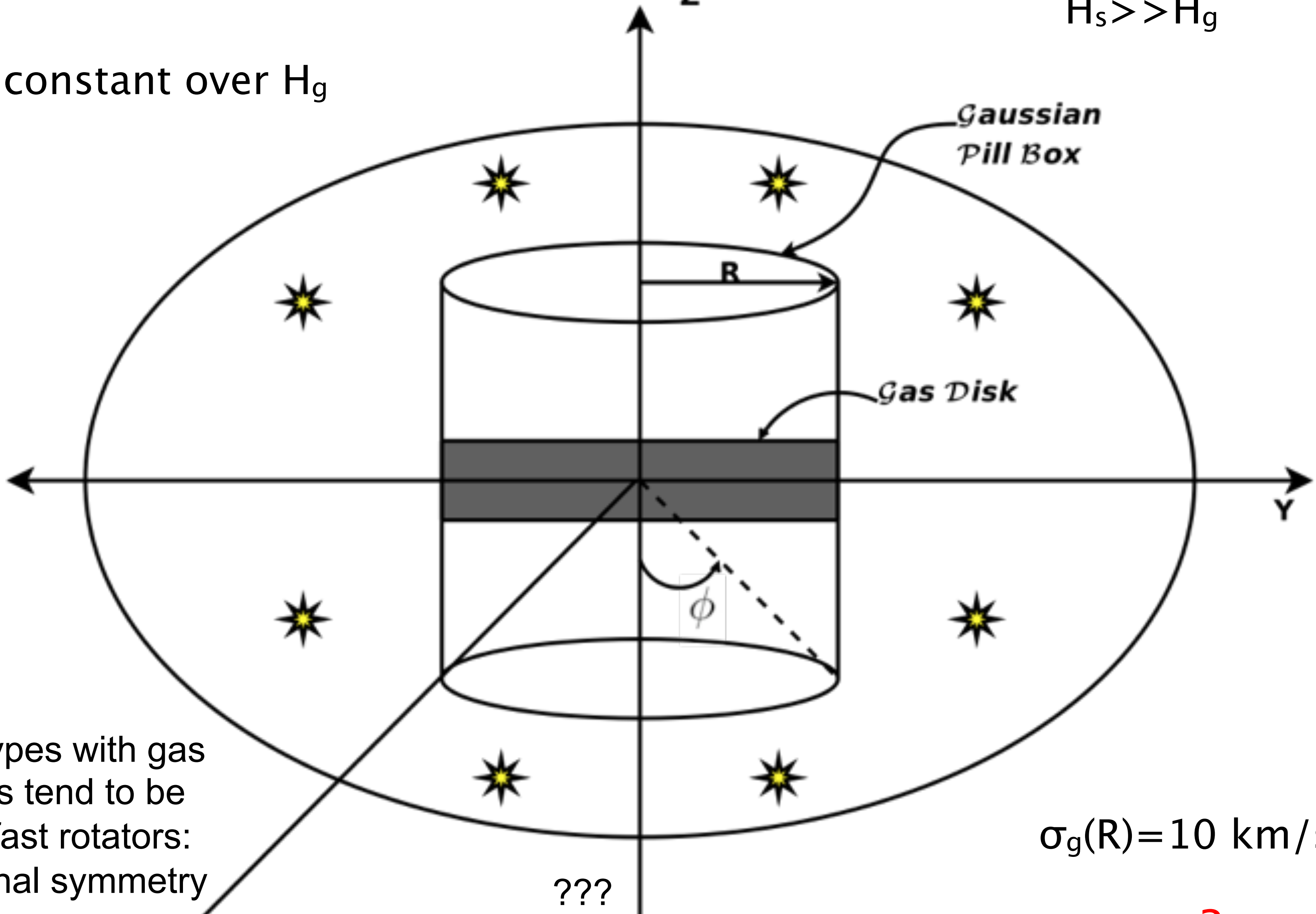
The MGE stellar density versus radius for several different scale heights.
H_s=0 pc, 150 pc, 300 pc above the midplane.



Hydrostatic equilibrium

ρ_s is constant over H_g

$H_s >> H_g$



Early-types with gas systems tend to be oblate fast rotators: Azimuthal symmetry

$\sigma_g(R) = 10$ km/s

???

$$P(0) = \rho_g(0)\sigma_g^2 = (2G)^{0.5}\Sigma_g\sigma_g\left[\rho_s^{0.5} + \cancel{\left(\frac{\pi}{4}\cdots\right)}^{0.5}\right].$$

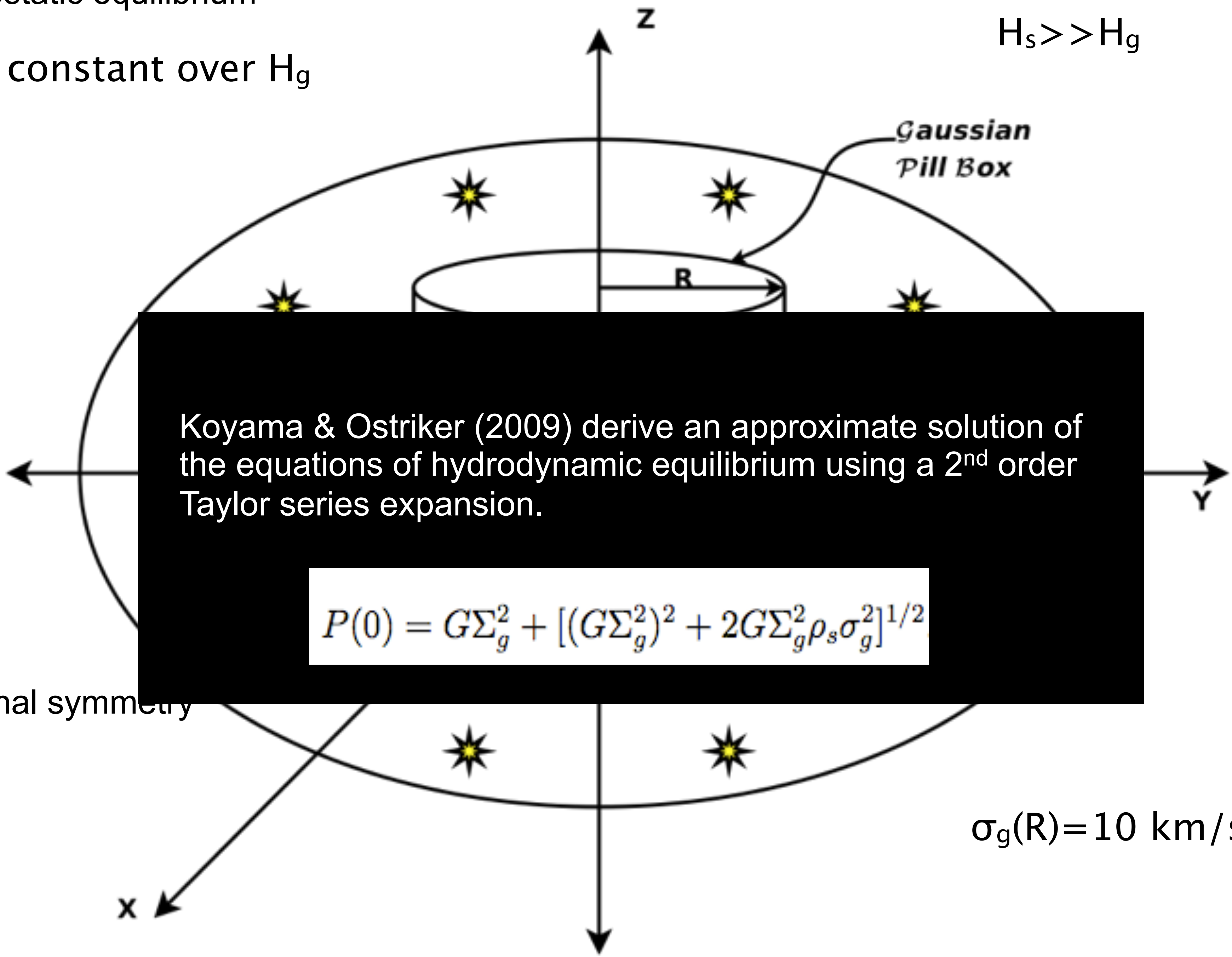
$\rho_s >> \rho_g$?

$\rho_s = \rho_g$?

Hydrostatic equilibrium

ρ_s is constant over H_g

$H_s >> H_g$



Koyama & Ostriker (2009) derive an approximate solution of the equations of hydrodynamic equilibrium using a 2nd order Taylor series expansion.

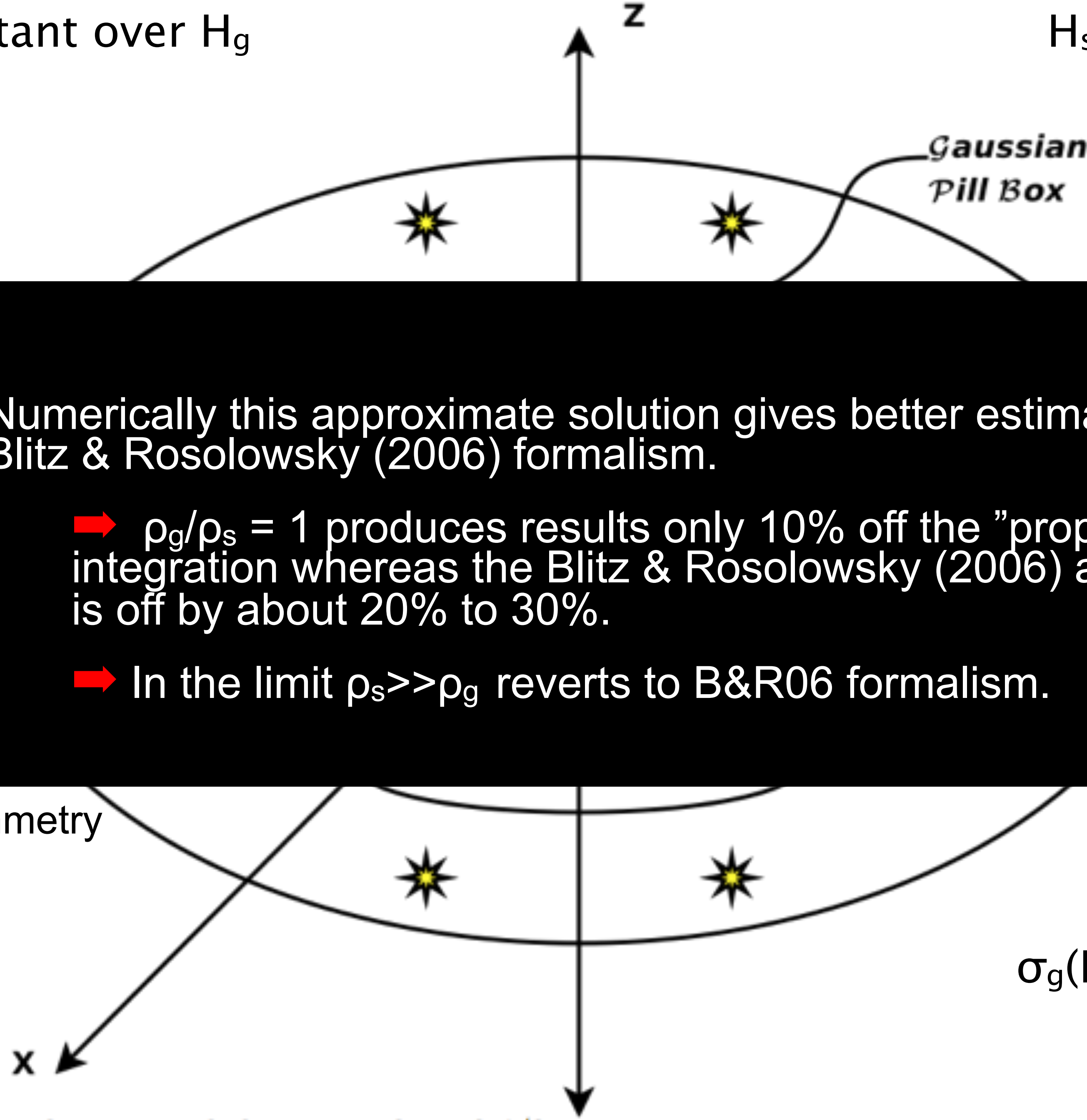
$$P(0) = G\Sigma_g^2 + [(G\Sigma_g^2)^2 + 2G\Sigma_g^2\rho_s\sigma_g^2]^{1/2}$$

Azimuthal symmetry

$\sigma_g(R)$=10 km/s

ρ_s is constant over H_g

$H_s >> H_g$



- Numerically this approximate solution gives better estimates than the Blitz & Rosolowsky (2006) formalism.
  - ➡ $\rho_g/\rho_s = 1$ produces results only 10% off the "proper" numerical integration whereas the Blitz & Rosolowsky (2006) approximation is off by about 20% to 30%.
  - ➡ In the limit $\rho_s >> \rho_g$ reverts to B&R06 formalism.

Azimuthal symmetry

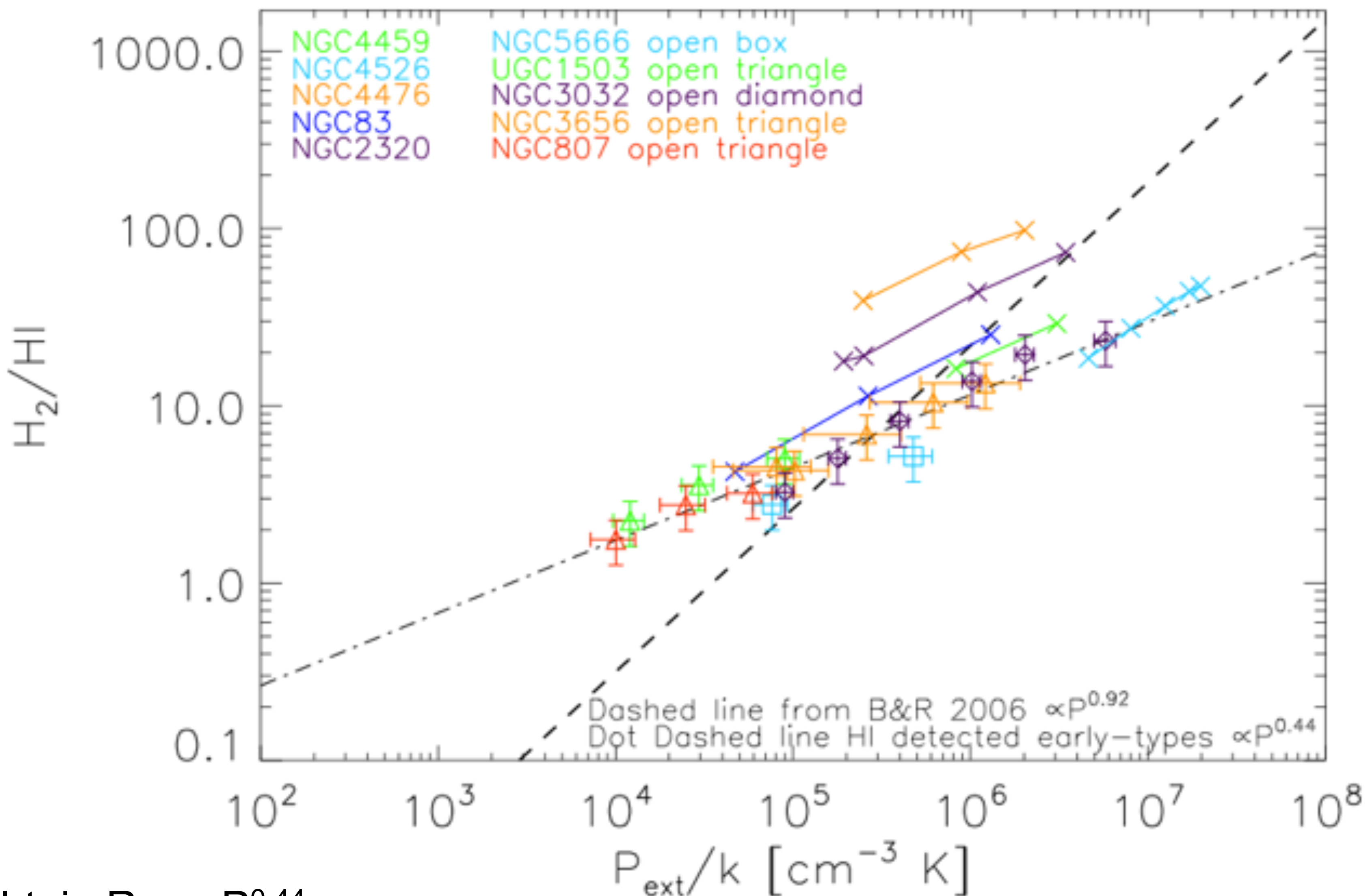
$\sigma_g(R) = 10$ km/s

$$P(0) = G\Sigma_g^2 + [(G\Sigma_g^2)^2 + 2G\Sigma_g^2\rho_s\sigma_g^2]^{1/2}$$

Koyama & Ostriker 2009

Koyama & Ostriker 2008 ---->

$$P(0) = G\Sigma_g^2 + [(G\Sigma_g^2)^2 + 2G\Sigma_g^2\rho_s\sigma_g^2]^{1/2}$$

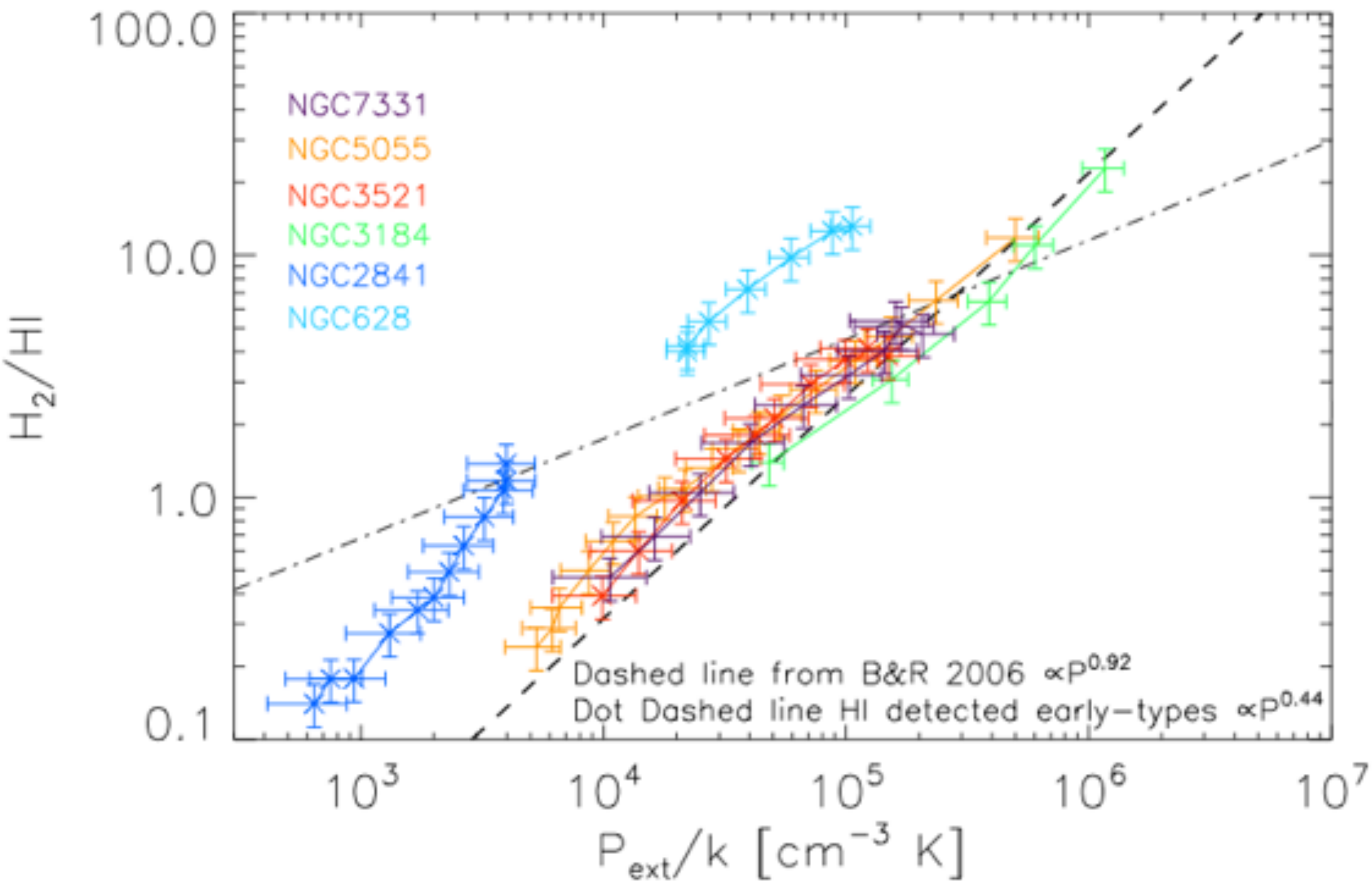


Obtain $R_{mol} = P^{0.44}$

Koyama & Ostriker 2008 ---->

$$P(0) = G\Sigma_g^2 + [(G\Sigma_g^2)^2 + 2G\Sigma_g^2\rho_s\sigma_g^2]^{1/2}$$

THINGS Galaxies



Main Conclusions

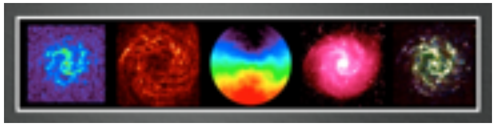
- Early-type galaxies are interesting! Not all are red and dead!
- An empirical relationship does exist between R_{mol} and the HSMP for early-type galaxies.
  - We find a slope of 0.44, much more shallow than observed in spiral galaxies.
    - Cold gas self gravity is important!
    - We can't simply use the B&R06 model for every morphological type as input into semi-analytical models of galaxy evolution.

Better Data is Needed!

- Biased Sample. Only very CO-rich early-types.
- Not much ancillary data.
- Poorly resolved gas distributions. Need JVLA B array, ALMA, SKA.
- Results are based on an assumed cold gas velocity dispersion. Most available HI and CO maps have velocity resolutions >>10 km/s, much too poor to measure cold gas velocity dispersions.
- Current CO data is not at sufficient resolution or sensitivity to probe the regime where $R_{mol}<1$.

R_{mol}-Pressure Relation: Early-Types

- Early-type empirical result: $P=[(P/k_B)/26\pm4\ K\ cm^{-3})]^{0.44\pm0.09}$
  - Much shallower slope than that for spirals!
  - Gas Self Gravity is important!
- Before we jump to a definitive conclusion:
  - Is there anything different I have done from the B&R06 formalism that could account for the difference in slope?
- Lets find out!



  - Test using disk galaxies from the THINGS (Walter et al. 2008) and Herecles survey (Leroy et al. 2009).
  - 34 nearby disk galaxy sample (3<D<15 Mpc). 7" resolution HI (VLA) and CO maps (IRAM).
  - I chose 6 that had good overlap between HI and CO.

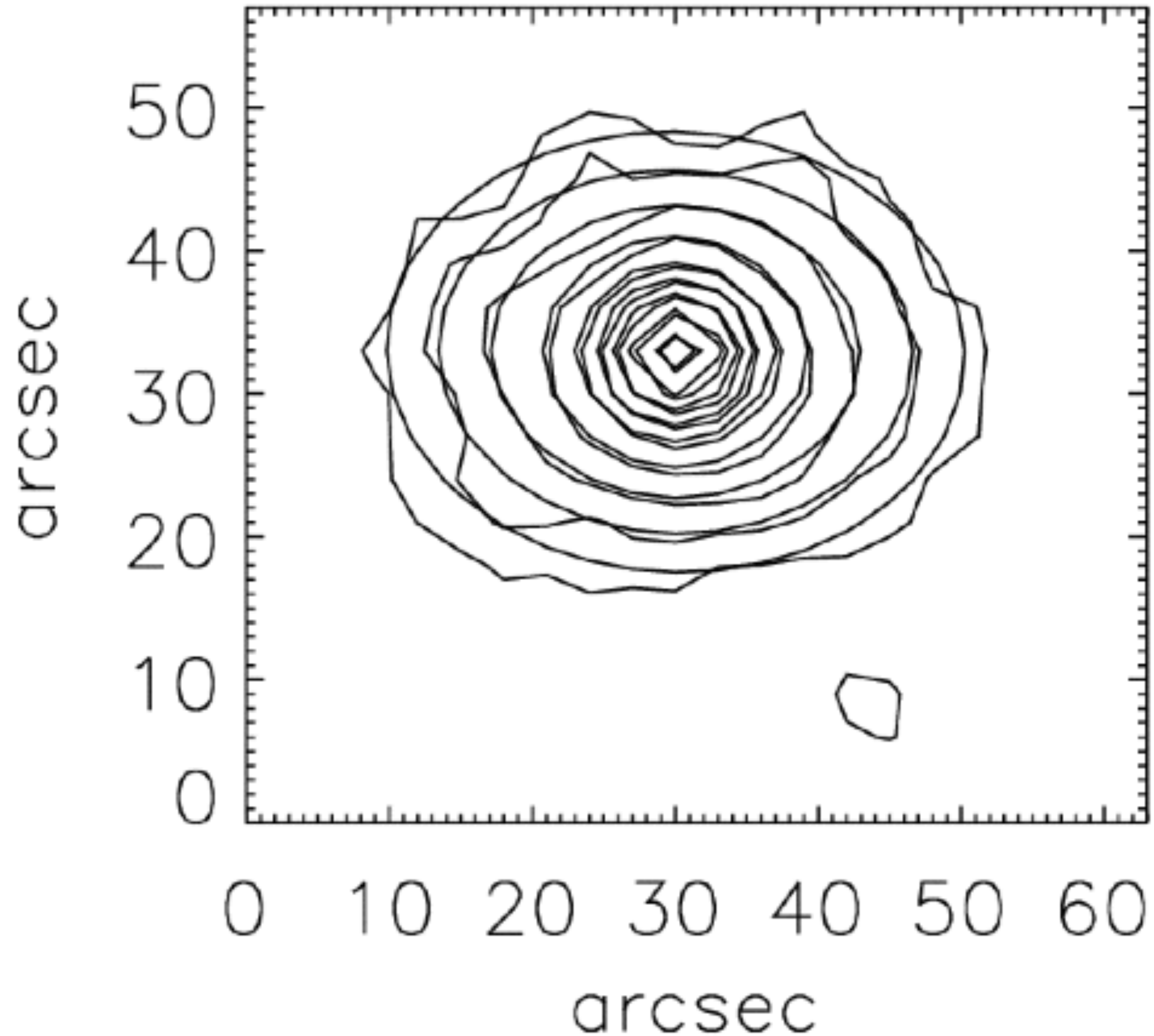


FIG. 24.— MGE fit to a 2MASS K-band image of NGC 3032. The jagged contours represent the 2MASS data and the smooth contours represent the MGE model. The pixel size and point spread function of the 2MASS images are 1″ and 2.57695, respectively.

Koyama & Ostriker 2008 ---->

$$P(0) = G\Sigma_g^2 + [(G\Sigma_g^2)^2 + 2G\Sigma_g^2\rho_s\sigma_g^2]^{1/2}$$

TABLE 4

Model Pressure Results: THINGS Galaxies

Galaxy	α	P_0/k ($10^4 cm^{-3} K$)	scatter[a]	Morphological class
N628	1.10±0.10	0.81	0.1	SA(s)c
N628[b]	0.76±0.05	0.32	0.03	SA(s)c
N5055	0.82±0.02	2.1	0.05	SA(rs)bc
N7331	0.79±0.03	3.5	0.04	SA(s)b
N3521	0.87±0.05	2.4	0.06	SAB(rs)bc
N2841	1.18±0.07	0.38	0.06	SA(r)b
N3184	1.10±0.07	7.7	0.10	SAB(rs)cd
mean (comb)	0.98±0.06	2.82	0.07	–
mean[c]	0.92±0.04	2.1	0.05	–

[a] Defined as the standard deviation of the residuals: $\Delta logR_{mol} = -\alpha(logP_{ext} - logP_0)$

[b] NGC 628 without points measured in the outer parts of the disk where the K-band image is too faint.

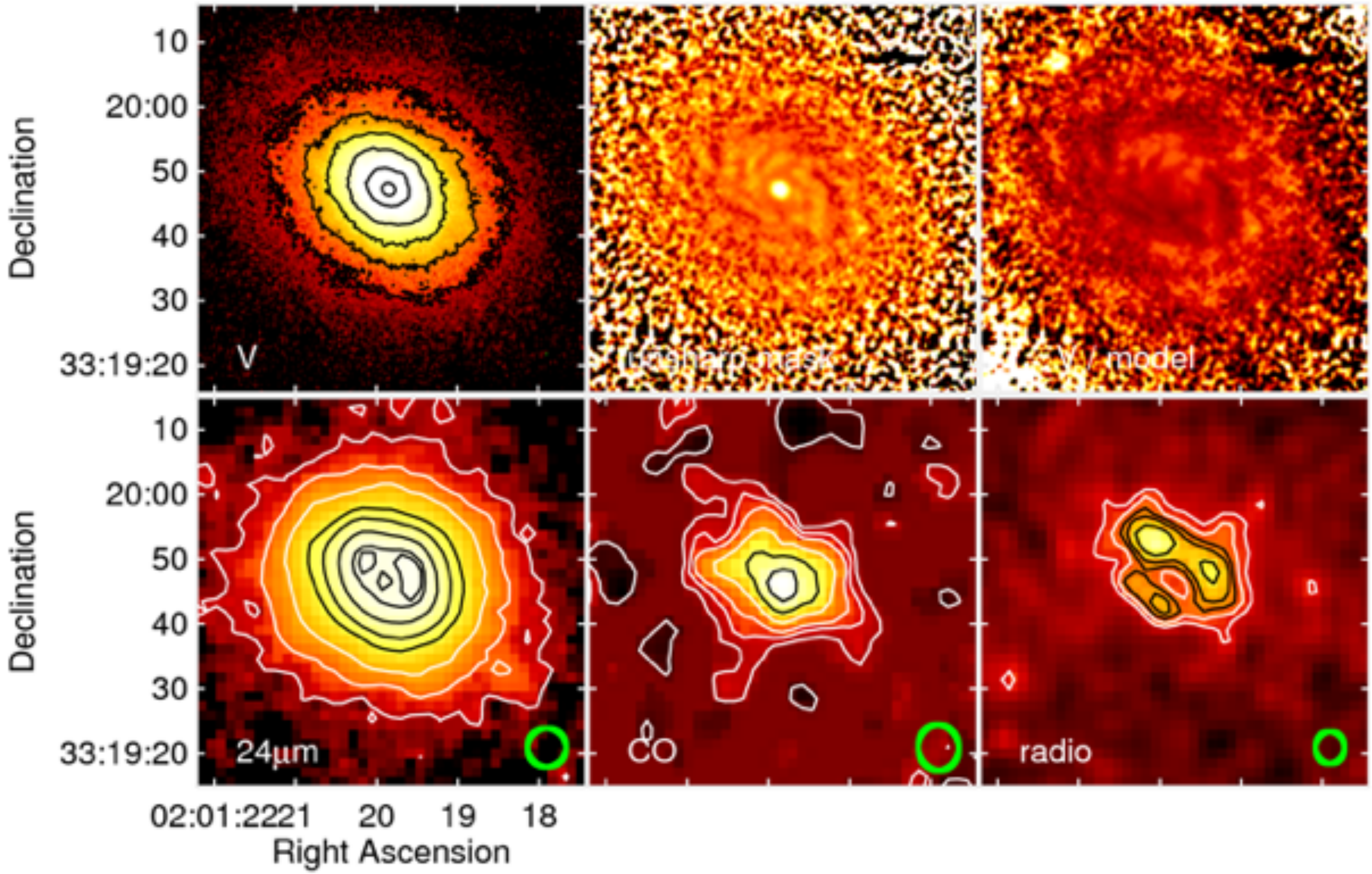
[c] Excludes data from NGC 628 and NGC 3184 completely.

Future Work

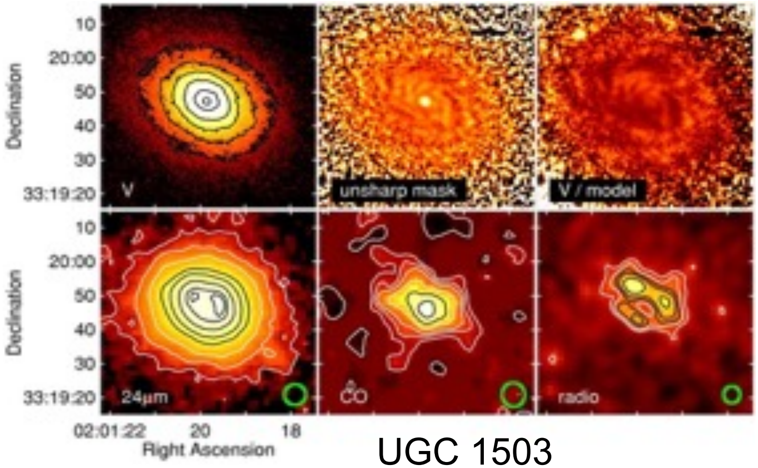
- Extend the analysis to a subsample of the ATLAS^{3D} survey.
  - A volume limited survey of 260 moderate and high mass early-type galaxies within 42 Mpc and $M_K \leq 21.5$.
    - Have existing CO (CARMA) and HI (WSRT) maps as well as a ton of ancillary data.
- JVLA study to get 5” resolution HI maps of a subsample of the ATLAS^{3D} survey. Data has been obtained and is being reduced.
- Cycle 5 ALMA proposal will be submitted to obtain better CO maps of the same subsample. I hope to obtain (1” res maps with <5 km/s velocity resolution). Data will be combined with the CARMA data.

Comparison of FIR, CO, Radio Continuum

UGC 1503: Elliptical



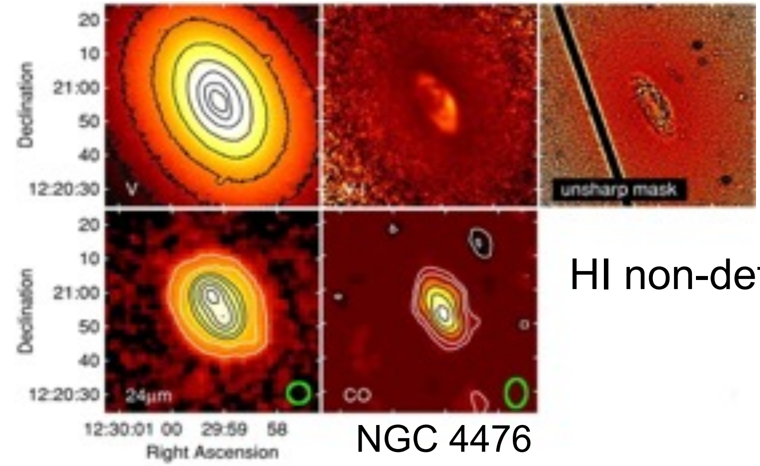
Young, Bendo, Lucero 2009



UGC 1503

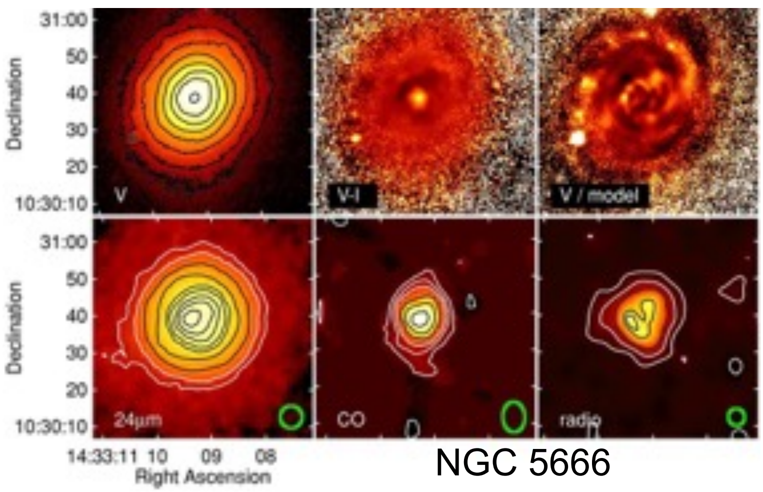
HI detected

Star Forming



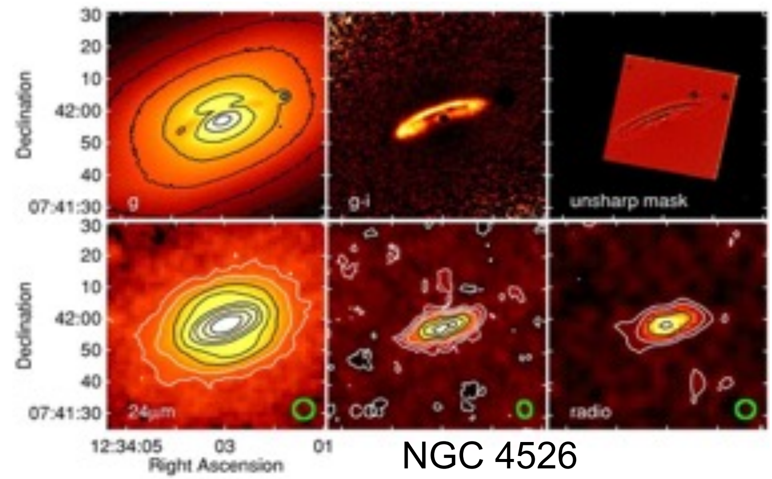
NGC 4476

HI non-detected

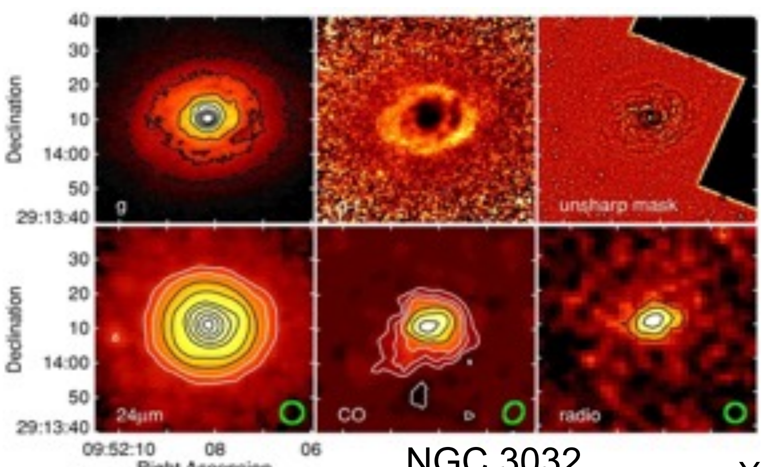


NGC 5666

Star Forming

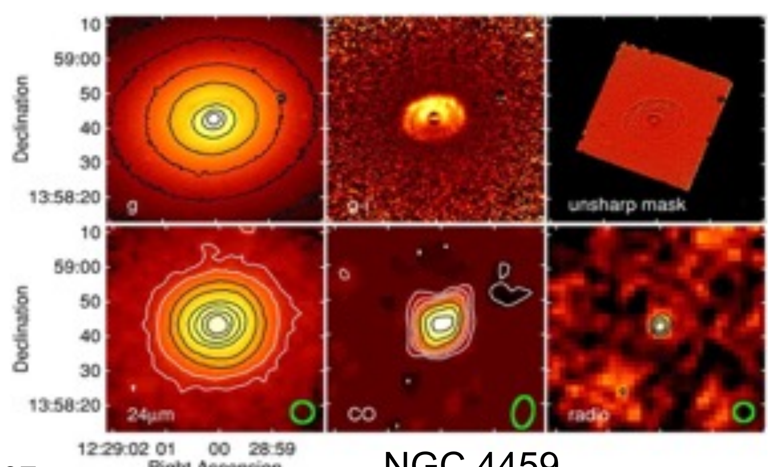


NGC 4526



NGC 3032

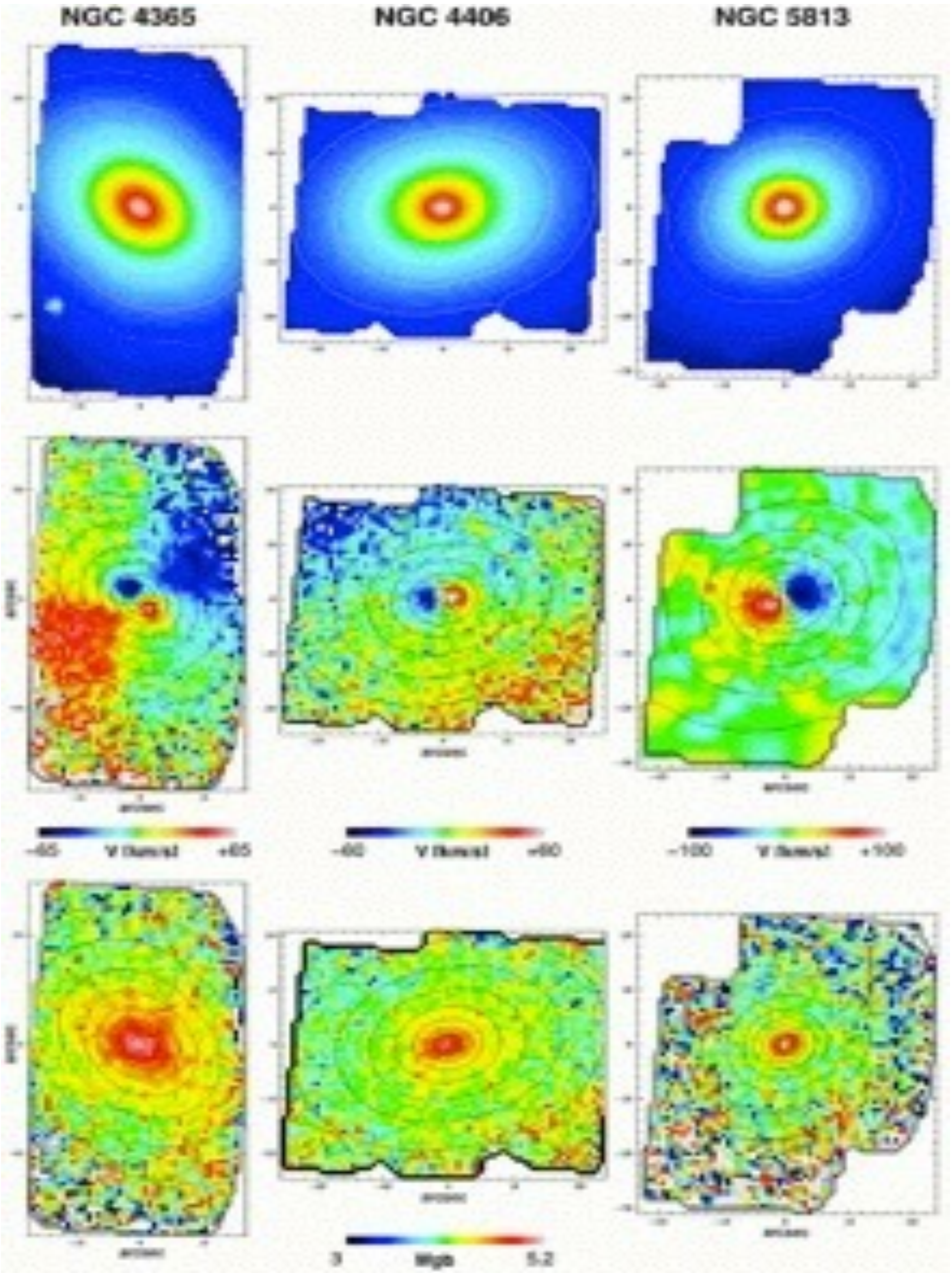
forming stars over only a portion of their disk.



NGC 4459

Young, Bendo, Lucero 2007

Kinematically Decoupled Cores



- Sauron Integral field optical observations of Mg II absorption lines (Emsellem 2004).
- Reveal kinematically decoupled cores.
- Stellar population age gradients.

Questions:

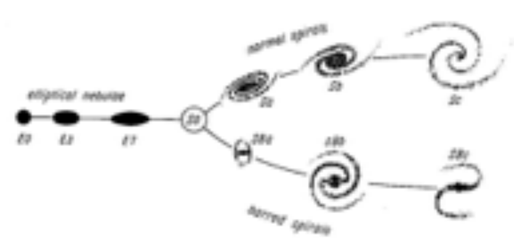
(1) How did these KDC form? → Gas accretion from major mergers.

Many Early-Types Do Contain Cold Gas!

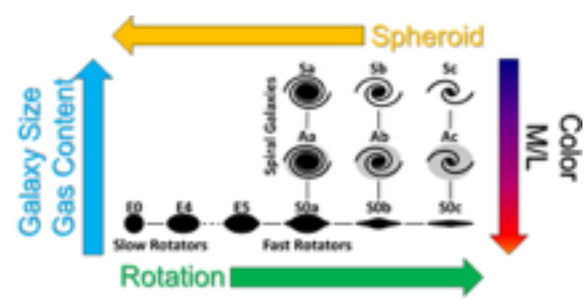
- It is now well known that many early-type galaxies (E and S0) contain significant amounts of cold gas. A large number of HI and CO maps exist for several representative surveys (e.g. ALFALFA, HIPASS, ATLAS^{3D}).

  - (e.g. Wiklind et al. 1995; Young 2002; Welch & Sage 2003; Sage & Welch 2006; Morganti et al. 2006; Sage et al. 2007; Combes et al. 2007; Osterloo et al. 2007; Alighieri et al. 2007; Welch et al. 2010; Serra et al. 2011, Young et al 2011).

- ATLAS^{3D}! ----> Multi-wavelength observations of a statistically well defined sample of 260 early-type galaxies derived from the 2MASS all sky survey (out to 42 Mpc; $M_K \leq 21.5$).

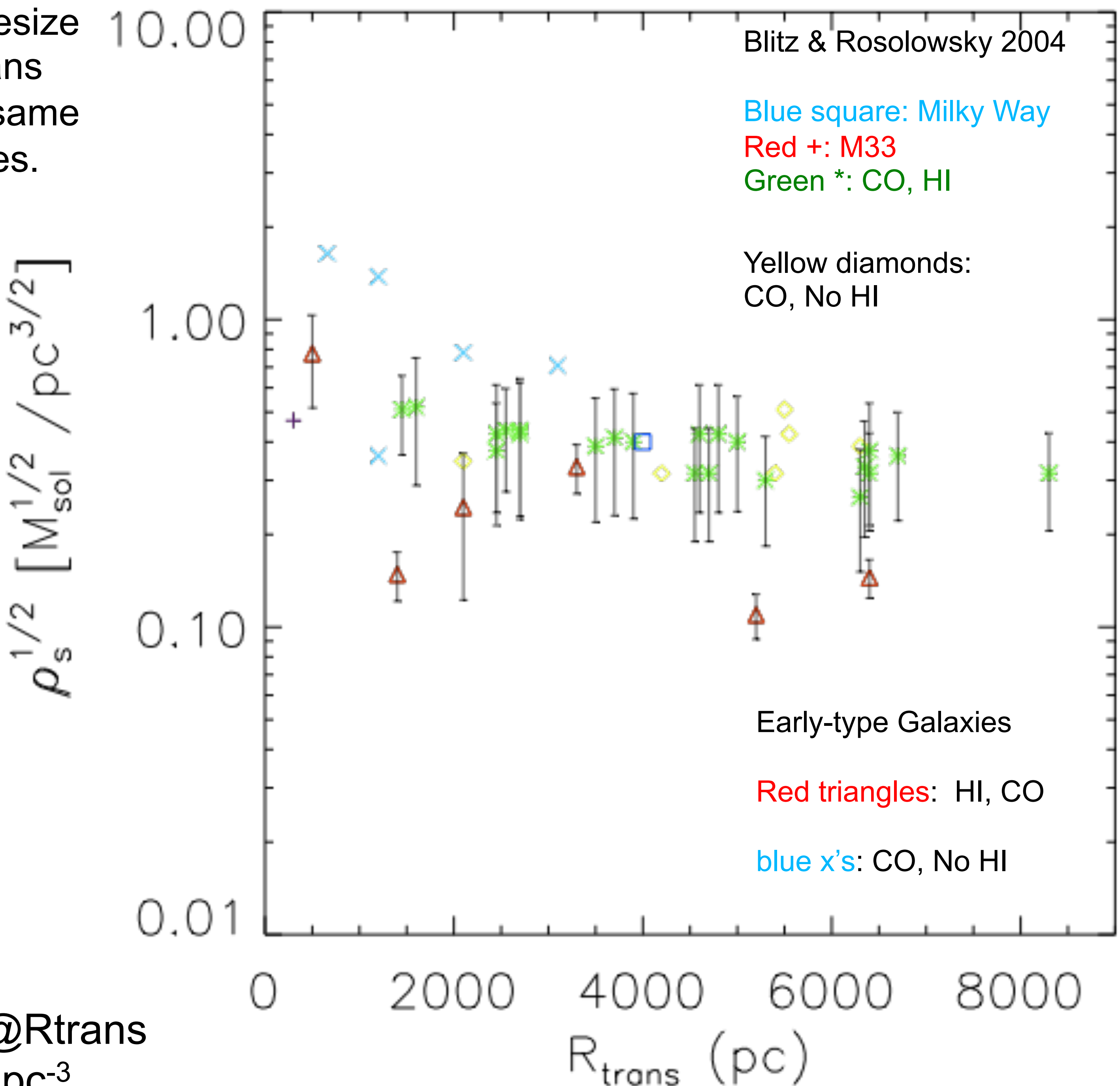


Hubble (1936) tuning fork



The ATLAS^{3D} comb (2011)

B&R04 hypothesize that ρ_s@Rtrans should be the same for all galaxies.



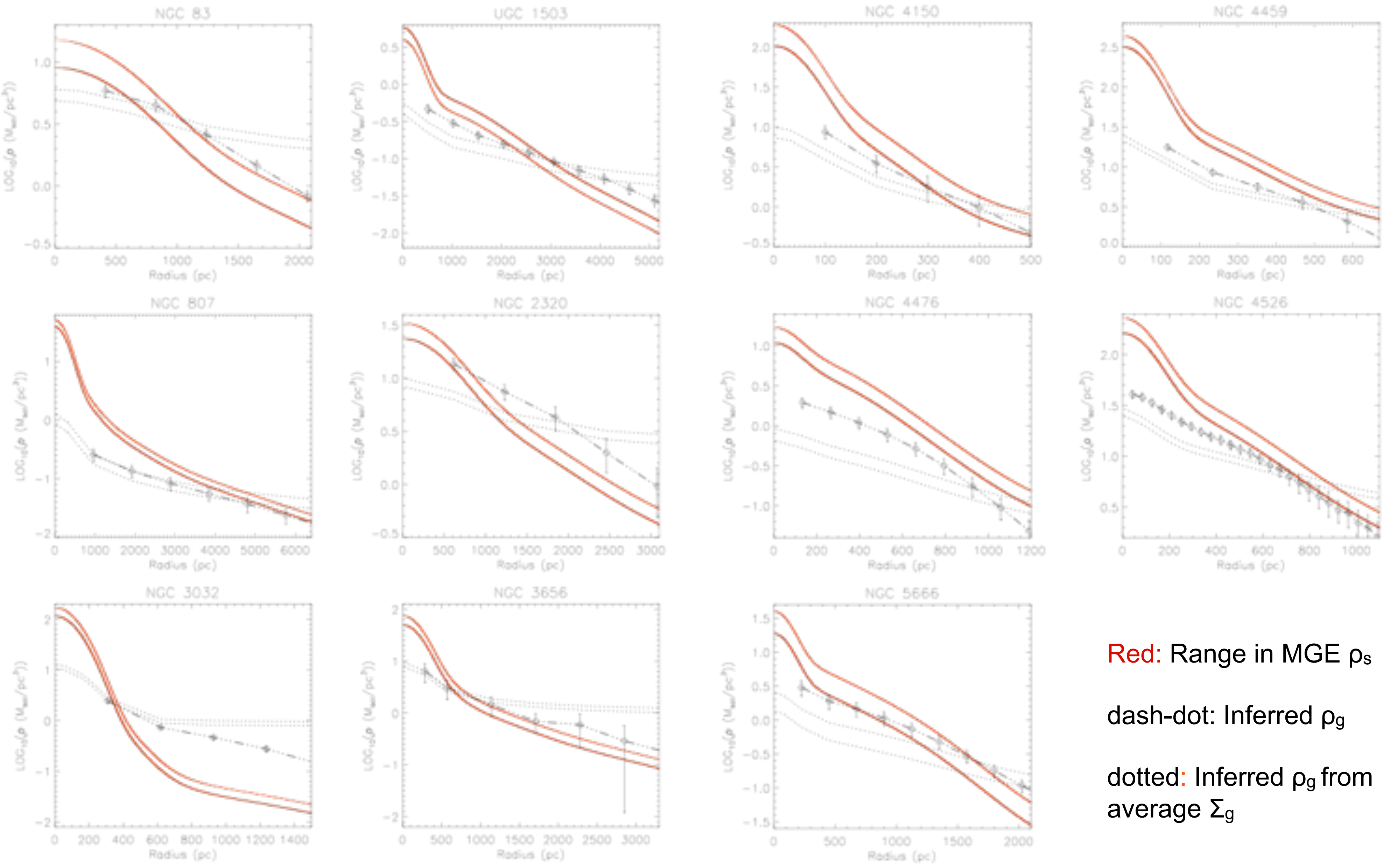
B&R04 ρ_s@Rtrans

$\langle\rho_s\rangle = 0.14\ M_\odot\ pc^{-3}$

$\langle\rho_s^{1/2}\rangle = 0.37\ M_\odot^{1/2}\ pc^{-3/2}$

Koyama & Ostriker 2008 ---->

$$\rho_g(0) = \frac{G\Sigma_g^2}{\sigma_g^2} + \left(\frac{G^2\Sigma_g^4}{\sigma_g^4} + \frac{2G\Sigma_g^2\rho_s}{\sigma_g^2}\right)^{1/2}$$



Red: Range in MGE ρ_s

dash-dot: Inferred ρ_g

dotted: Inferred ρ_g from average Σ_g

R_{mol}-Pressure Relation: History

Elmegreen (1989;1993) was the first to suggest that the amount of gas and gas cloud type (diffuse versus self gravitating) depends on the hydrostatic pressure as well as the cloud's ability to shield itself from photo-dissociation.

- Models a molecular cloud as an semi-infinite slab illuminated by a unidirectional beam of dissociating radiation. Assumptions include:
  - diffuse clouds have a constant internal density that scales with the ambient pressure.
  - self-gravitating clouds have internal densities which vary as $1/r^2$ and satisfy the pressure-dependent virial-theorem.
- Radiative transfer modeling gives the column densities of HI and H_2 in one cloud, and an ensemble of clouds is used to predict the relative amounts of HI and H_2 in a kpc-scale region of a disk galaxy.
- Predicts that the ratio of molecular to atomic gas in galactic disks is determined from the mean stellar volume emissivity, j (which is metallicity dependent), and the ambient hydrostatic mid-plane pressure, P or HSMP.

$$\mathrm{R}_{mol} = \frac{\Sigma_{H_2}}{\Sigma_{HI}} \qquad F_{mol} \propto P^{2.2} j(Z)^{-1} \qquad F_{mol} = \frac{\Sigma_{H_2}}{\Sigma_{H_2} + \Sigma_{HI}}$$