Appendix A

Sky Projections used in GIPSY

GIPSY knows about 4 Sky coordinate types. The kind of coordinate is conveyed into the first characters of the axis type as 'RA...', 'DEC...' etc. The Sky coordinates known are:

'RA'	Equatorial, Right Ascension
'DEC'	Equatorial, Declination
'GLON'	Galactic Longitude
'GLAT'	Galactic Latitude
'ELON'	Ecliptic Longitude
'ELAT'	Ecliptic Latitude
'SLON'	Supergalactic Longitude
'SLAT'	Supergalactic Latitude

GIPSY knows of 10 different projections from rectangular grids to sky coordinates and vice versa. This chapter discusses all these projections. The type of geometry is conveyed in the last characters of the axis type, as '...TAN', '...NCP', etc. The known projections are:

'AIT'	Aitoff equal area projection
'CYL'	Equivalent Cylindrical projection
'FLT'	Flat projection
'TAN'	Gnomonic projection
'SIN'	Orthographic projection
'ARC'	Rectangular projection
'GLS'	Transversal projection
'NCP'	Nort Celestial Pole projection
'STG'	Stereographic projection
'MER'	Mercator projection

To separate the coordinate type from the projection type at least one minus-sign (−) must be present ('RA−NCP', 'DEC−NCP').
In a projected plane, the position of a point (x, y) with respect to the coordinate reference point in an arbitrary linear system may be represented as

$$x = Lcos(\rho) + Msin(\rho)$$

$$y = M\cos(\rho) - L\sin(\rho)$$

where ρ is a rotation, L is the direction cosine parallel to latitude at the reference pixel, and M is the direction cosine parallel to longitude at the reference pixel. Both the (x, y) and (L, M) systems are simple linear, perpendicular systems. If we represent longitude and latitude with the symbols α and δ and the projection centres with α_0 and δ_0, we get the conversion routines shown in the following sections.

A.1 Aitoff equal area projection

$$L = \frac{2f_\alpha \cos(\delta)\sin(\frac{\alpha-\alpha_0}{2})}{Z}$$

$$M = \frac{f_\delta \sin(\delta)}{Z} - M_0$$

where

$$Z = \sqrt{\frac{1 + \cos(\delta)\cos(\frac{\alpha-\alpha_0}{2})}{2}}$$

$$M_0 = \frac{f_\delta \sin(\delta_0)}{\sqrt{\frac{1+\cos(\delta_0)}{2}}}$$

A.2 Equivalent Cylindrical projection

$$L = \alpha - \alpha_0$$

$$M = \sin(\delta - \delta_0)$$

A.3 Flat projection

$$L = (\alpha - \alpha_0)$$

$$M = (\delta - \delta_0)$$

A.4 Gnomonic projection

The Gnomonic projection is common in optical astronomy.

$$L = \frac{\cos(\delta)\sin(\alpha - \alpha_0)}{\sin(\delta)\sin\delta_0 + \cos(\delta)\cos(\delta_0)\cos(\alpha - \alpha_0)}$$

$$M = \frac{\sin(\delta)\cos(\delta_0) - \cos(\delta)\sin(\delta_0)\cos(\alpha - \alpha_0)}{\sin(\delta)\sin\delta_0 + \cos(\delta)\cos(\delta_0)\cos(\alpha - \alpha_0)}$$

A.5 Orthographic projection

The Orthographic projection is used by the VLA.

$$L = cos(\delta)sin(\alpha - \alpha_0)$$

$$M = sin(\delta)cos(\delta_0) - cos(\delta)sin(\delta_0)cos(\alpha - \alpha_0)$$

A.6 Rectangular projection

The Rectangular projection is used in Schmidt telecopes.

$$L = \frac{\theta}{sin(\theta)}cos(\delta)sin(\alpha - \alpha_0)$$

$$M = \frac{\theta}{sin(\theta)}\Big(sin(\delta)cos(\delta_0) - cos(\delta)sin(\delta_0)cos(\alpha - \alpha_0)\Big)$$

where

$$cos(\theta) = sin(\delta)sin(\delta_0) + cos(\delta)cos(\delta_0)cos(\alpha - \alpha_0)$$

A.7 Global Sinusoidal projection

$$L = (\alpha - \alpha_0)cos(\delta)$$

$$M = (\delta - \delta_0)$$

A.8 North Celestial Pole projection

The North Celestial Pole projection is used by the WSRT.

$$L = cos(\delta)sin(\alpha - \alpha_0)$$

$$M = \frac{cos(\delta_0) - cos(\delta)cos(\alpha - \alpha_0)}{sin(\delta_0)}$$

A.9 Stereographic projection

$$L = 2\frac{cos(\delta)sin(\alpha - \alpha_0)}{1 + sin(\delta)sin(\delta_0) + cos(\delta)cos(\delta_0)cos(\alpha - \alpha_0)}$$

$$M = 2\frac{sin(\delta)cos(\delta_0) - cos(\delta)sin(\delta_0)cos(\alpha - \alpha_0)}{1 + sin(\delta)sin(\delta_0) + cos(\delta)cos(\delta_0)cos(\alpha - \alpha_0)}$$

A.10 Mercator projection

$$L = f_\alpha(\alpha - \alpha_0)$$

$$M = f_\delta ln\Big(tan(\frac{\delta}{2} + \frac{\pi}{2})\Big) - M_0$$

where

$$f_\alpha = cos(\delta_0)$$

$$f_\delta = \frac{\Delta_\delta}{ln\Big(tan(\frac{\delta_0+\Delta_\delta}{2} + \frac{\pi}{2})\Big) - ln\Big(tan(\frac{\delta_0}{2} + \frac{\pi}{2})\Big)}$$

$$M_0 = f_\delta ln\Big(tan(\frac{\delta_0}{2} + \frac{\pi}{4})\Big)$$