ELLINT

Integration of image data in ellipses

jan 21, 1996

1. Purpose

ELLINT integrates image data from a GIPSY set (*INSET=*) in elliptical rings, it can be used to find the radial intensity distributions in galaxies or, for example, to find the mean intensity of instrumental rings in maps. The ellipses are projected circles, viewed at an inclination i (*INCL=*). Further, the rings are characterized by a major axis (*RADII=*), a width (*WIDTH=*), the position angle of the major axis (*PA=*) and a central position (*POS=*). *ELLINT* has three options. It calculates:

1. Statistics in a ring or segment, like the sum, mean etc.
2. The projected and face-on surface brightness in a ring or segment
3. The (scaled) mass surface density Σ_M in a ring

1. Options

Option 1 (sum, mean, median, rms, area): Circles in the plane of an object (e.g. galaxy) are projected onto the sky as ellipses. The ratio between major and minor axis of an ellipse depends on the inclination at which we view the object. Two ellipses with different major/minor axes enclose a certain number of image pixels in a ring. In *ELLINT*, a pixel belongs to such a

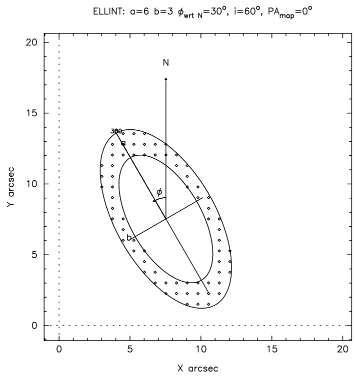

Figure 1: Example of plot generated by *ELLINT*

ring if its center is positioned between the two ellipse boundaries or on the boundary of the inner ellipse. Pixels in that ring either have an image value or are undefined (*blank*). If a non-blank pixel with index k has image value I_k and there are N such pixels in a ring, then the sum of the image values is:

$$S = \sum_{k=1}^{N} I_k \tag{1}$$

The units of the sum are the units of the image values (e.g. W.U.). The second ring parameter is the mean image value per pixel:

$$\bar{I} = \frac{S}{N} \tag{2}$$

Option 2 (surface brightness): The calculation of the surface brightness is necessary for option 2 and option 3. For optical data the symbol μ is used. The next table summarizes the meaning of the symbols:

μ = surface brightness per pixel, projected on the sky
μ_0 = face-on surface brightness (in plane of galaxy) per pixel
$\bar{\mu}$ = mean projected surface brightness in a ring averaged over non blank area
$\bar{\mu_0}$ = mean face-on surface brightness in a ring averaged over non blank area
$\bar{\mu_t}$ = mean projected surface brightness in a ring averaged over total area
$\bar{\mu_{0t}}$ = mean face-on surface brightness in a ring averaged over total area

If a pixel has size $dxdy$ (dx, dy in seconds of arc) then the projected surface brightness per pixel is

$$\mu_k = \frac{I_k}{dxdy} \tag{3}$$

If N is the number of non-blank pixels and M is the total number of pixels, then *ELLINT* distinguishes two mean surface brightnesses. The first is an average over the non-blank area and is written as

$$\bar{\mu} = \frac{1}{N}\sum_{k=1}^{N}\mu_k = \frac{1}{N}\sum_{k=1}^{N}\frac{I_k}{dxdy} = \frac{\bar{I}}{dxdy} \tag{4}$$

The second is an average over the total area and can be written as

$$\bar{\mu_t} = \frac{1}{M}\sum_{k=1}^{N}\mu_k = \frac{N}{M}\frac{1}{N}\sum_{k=1}^{N}\frac{I_k}{dxdy} = \frac{N}{M}\frac{\bar{I}}{dxdy} = \frac{N}{M}\bar{\mu} \tag{5}$$

If there are no blank pixels in your rings then $\bar{\mu}$ and $\bar{\mu_t}$ are equal. Otherwise, you have to think about what the blanks in your map actually represent.

The face-on area of a pixel is increased by a factor $cos(i)$. Then a pixel has a surface brightness:

$$\mu_{0_k} = \frac{I_k}{\frac{dxdy}{cos(i)}} = cos(i)\mu_k \tag{6}$$

and therefore the mean face-on surface brightness in a ring is:

$$\bar{\mu_0} = cos(i)\bar{\mu} \tag{7}$$

$$\bar{\mu_{0t}} = cos(i)\bar{\mu_t} \tag{8}$$

Option 3 (mass surface density Σ_M): For optical thin HI data, the mass in a ring is a linear function of the total flux S. If you use this option, the program does not calculate masses directly, but it scales a given mass M_{tot} (in $M_\odot$ entered by *MASS=*) proportional to the surface densities of the rings and converts the results to a mass surface density Σ_M ($\frac{M_\odot}{pc^2}$). The surface densities σ_0 are calculated in the same way as the surface brightness in option 2, and again, we distinguish two values σ_0 and σ_{0t}. Before scaling any mass, *ELLINT* assumes that the entire area contributes to the total flux whether there are blank pixels in it or not. If we know the mean face-on surface density $\bar{\sigma_0}$ in a ring with inner major axis R_1 and outer major axis R_2 then the sum in a (face-on) ring can be written as:

$$S_0 = \bar{\sigma_0}\pi(R_2^2 - R_1^2) \tag{9}$$

Note that if there are no blank pixels in the ring and that $dxdy$ is small compared to the area of the ring, then it is justified to make the approximation:

$$S_0 \approx \bar{\sigma_0}N\frac{dxdy}{cos(i)} = \bar{\sigma}Ndxdy = \sum_{k=1}^{N}\sigma_k dxdy = \sum_{k=1}^{N} I_k = S$$

which implicates that the total face-on and projected mass, are the same. In *ELLINT* we use the geometrical expression for S_0. To scale the mass we calculate S_0 for each ring and then do the summation

$$S_{tot} = \sum^{rings} S_0$$

If we divide M_{tot} over all rings then the mass in each ring is

$$M = M_{tot}\frac{S_0}{S_{tot}} = M_{tot}\frac{\bar{\sigma_0}\pi(R_2^2 - R_1^2)}{S_{tot}} \tag{10}$$

Suppose an object has distance $D(pc)$ (*DISTANCE=*), size $d(pc)$ and is viewed at an angle $\alpha(radians)$ then the relation between these parameters is:

$$d(pc) = D(pc).\tan(\alpha) \approx D(pc).\alpha(rad)$$

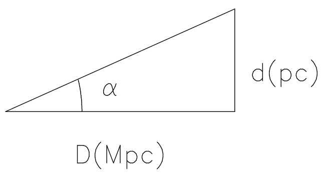

Figure 2: $d(pc) \approx D(pc).\alpha(rad)$

If we want to express the distance D in Mpc and the angle in seconds of arc, then use the conversions 1 $Mpc = 10^6\ pc$ and 1 $arcsec = \frac{2\pi}{360.3600}\ radians$ to obtain the relation:

$$d(pc) = 4.8481 D(Mpc)\alpha('') \tag{11}$$

The previous expression for the mass in a ring can then be written as:

$$M = M_{tot} \frac{\bar{\sigma_0}}{S_{tot}} \frac{\pi(R_2^2 - R_1^2)}{(4.848D)^2} \tag{12}$$

The mass surface density Σ_M has units $\frac{M_\odot}{pc^2}$, i.e. a mass divided by the enclosed area in pc^2 and therefore:

$$\Sigma_M = M_{tot} \frac{\bar{\sigma_0}}{S_{tot}} \frac{1}{(4.848D)^2} \tag{13}$$

ELLINT lists two columns with mass densities. The first column is a density derived from σ_0 and the second is derived from σ_{0t}. In radio data a blank usually represents a zero image value. Then the second column is a better approximation of the true densities than the first column.