Red Arrows in Orbit

Phil Palmer
# Satellites in Orbit

<table>
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<tr>
<th>Mission</th>
<th>Year</th>
<th>Launch</th>
<th>Mass</th>
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# Future of Satellites

<table>
<thead>
<tr>
<th>OLD MODEL</th>
<th>FUTURE?</th>
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<tbody>
<tr>
<td>Tonnes</td>
<td>&lt;10 - 100’s kg</td>
</tr>
<tr>
<td>Multi-sensor</td>
<td>Single or Multi sensor</td>
</tr>
<tr>
<td>Expensive (£$10^5$)</td>
<td>Cheap (£$10^4$)</td>
</tr>
<tr>
<td>Lead time (10-15 yrs)</td>
<td>6-18 months</td>
</tr>
<tr>
<td>Few sats</td>
<td>Many sats</td>
</tr>
<tr>
<td>Competition for data</td>
<td>High data rates</td>
</tr>
<tr>
<td>High risk</td>
<td>Low risk</td>
</tr>
<tr>
<td>State of art +10 years</td>
<td>COTS +1 year</td>
</tr>
</tbody>
</table>

Bernard’s Cosmic Stories
DMC Images

- DMC Images 600 km swath
- Whole UK in 4 images
- Resolution 32 metres
- Landsat requires 5 years
- SPOT = 30 x DMC Sats
Asian Tsunami

Bernard’s Cosmic Stories
**Formation Flying: Pros & Cons**

- ✔ Redundancy - robust to single point failure
- ✔ Flexibility - adaptable to changes in mission
- ✔ Synthesise more complex satellites
- ✗ Collision and Contamination risk
- ✗ Complexity of command and control
- ✗ Distributed knowledge - inter-satellite communication
Relative Motion Models

- Hills Equations (Hill, Clohessy & Wiltshire)
  - valid only over short timescales

- Nonlinear Models (Karlgaard & Lutz)
  - incorporates curvature of circular orbit
  - ignores other perturbations

- Inclusion of $J_2$ (Schweighart & Sedgwick, Alfriend & Schaub)
  - considers both secular evolution and periodic variations
    - much greater complexity and valid for few orbital periods

- Eccentric Orbits (Melton et al)
  - employs variable rotating frame
    - not time explicit and provides little insight
\[ \ddot{x} - 2\Omega \dot{y} - 3\Omega^2 x = T_x \]
\[ \ddot{y} + 2\Omega \dot{x} = T_y \]
\[ \ddot{z} + \Omega^2 z = T_z \]

\[ T_x(t) = \frac{2}{3} T_1 + \frac{\Lambda}{2} \sin(\Omega t - \Phi) \]
\[ T_y(t) = T_0 - T_1(\Omega t) + \Lambda \cos(\Omega t - \Phi) \]
\[ T_z(t) = \Gamma \cos(\Omega t - \zeta) \]

where \((T_0, T_1, \Lambda, \Gamma, \zeta, \Phi)\) are all functions of the boundary conditions and \(t_F\).
\[ r = a(1 + \rho) - ae \cos(\alpha - \alpha_P) + a\chi \sin \alpha + \Delta r_2^1 \cos 2\alpha \]
\[ \lambda = \alpha(1 + \kappa) + 2e \left[ \sin(\alpha - \alpha_P) + \sin \alpha_P \right] - 2\chi \left[ 1 - \cos \alpha \right] + \Delta \lambda_2^1 \sin 2\alpha \]
\[ I = I_0 + \Delta I_2^1 (1 - \cos 2\alpha) \]
\[ \Omega = \Omega_0 + \theta \alpha + \Delta \Omega_2^1 \sin 2\alpha \]
\[ \alpha = \eta(t - t_e) \]
Model Accuracy

ALONG-TRACK ERROR

error (m)

orbital phase

- 32.8 km motion
- 23.8 km motion
Relative Keplerian Motion

The difference of the Hamiltonians for two satellites at coordinates \((r \pm \frac{1}{2} \delta r, v \pm \frac{1}{2} \delta v)\) expanded around \((r, v)\):

\[
H_R = H_1 - H_2 = v.\delta v + \frac{\mu}{|r|^3} (r.\delta r)
\]

Equations of motion from Hamilton’s extended equations:

\[
\dot{r} = \frac{\partial H_R}{\partial \delta v} \quad \dot{v} = -\frac{\partial H_R}{\partial \delta r} \quad (1)
\]

\[
\delta \dot{r} = \frac{\partial H_R}{\partial v} \quad \delta \dot{v} = -\frac{\partial H_R}{\partial r} \quad (2)
\]

The secular drift due to energy differences can be separated:

\[
\delta r = \delta r_p - \frac{H_R}{H} (r - \frac{3}{2} vt)
\]
Solving the Relative Motion

<table>
<thead>
<tr>
<th>a(km)</th>
<th>e</th>
<th>l</th>
<th>( \Omega )</th>
<th>( \omega )</th>
<th>( \theta )</th>
<th>H</th>
<th>L</th>
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<tbody>
<tr>
<td>Sat1</td>
<td>15945.8</td>
<td>0.45</td>
<td>60</td>
<td>40.03</td>
<td>20</td>
<td>73</td>
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<tr>
<td>Sat2</td>
<td>-1.15</td>
<td>0.0001</td>
<td>0.03</td>
<td>0.03</td>
<td>-0.05</td>
<td>0.05</td>
<td>-1.881e-6</td>
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![Graph showing relative position error and time separation](image)

Bernard’s Cosmic Stories
Positional Accuracy

\[ a = 7065.3 \text{ km}, \delta a = 1200 \text{ m}, e = 0.055 \text{ include } 36 \times 36 \text{ model.} \]
Conclusions

✔ Formation Acquisition

✔ optimal Hill’s Solutions for continuous thrust

✔ Describe relative motion for near circular formations

✔ exploits analytic solutions to equations of motion
✔ separates secular, rigid formation motion and inter-satellite motions
✔ accurate to approx 1 metre.

✔ Formation Design

✔ Invert from required relative motion into orbital elements
✔ design multiple baslines

✔ Symplectic Formation Propagation

✔ long term evolution accurate to < 2% over 1 month
✔ conserves relative energy and angular momentum exactly