A solution of the cusp problem in virialized DM halos in standard cosmology

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A short list from Bernard’s Stories (advanced studies in cosmology today)

- Analysis of CMB and its polarization
- Investigation of Ly-α forest
- Properties of earlier galaxies and quasars at large redshifts
- Properties of dwarf galaxies and the internal structure of galaxies (black holes, rotation curves, etc)

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Cusp problem in galaxy halos
(came from simulations)

$\rho \sim r^{-\alpha}$
$\alpha \in (1, 3/2)$

Diemand et al. 2004
The cusp problem is considered as main problem of standard model (non-interacting particles).

Does nature create divergent cusps?

We are not convinced: can be solved in the framework of standard $ΛCDM$.
Cusp realization needs low entropy particles

- How to transport the low energy particles into the cusp (the compression factor is limited)?
- Where to take low entropy particles in real density fields?

CDM is cold, but there are perturbations at small scale

CDM preheated by primordial perturbations in collapsing protohalos

N-body: Is there a refrigerator at small scales?

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Cores instead of cusps

Idea: take into account the small scale part of initial background perturbations that transforms into random velocities of DM particles in the process of relaxation.

Method: total entropy = initial (given) + generated (gained during relaxation)

⇒ entropy profiles related to density profiles in DM halos

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Equilibrium DM halos

- **Adiabatic and irreversible processes**

- **Entropy function:** \( F = \frac{T}{n^{2/3}} = \frac{p}{n^{5/3}} \)

- **Hydrostatic equilibrium:**
  \[
  \frac{1}{\rho} \frac{dp}{dr} = -\frac{GM(r)}{r^2}
  \]

- **Initial (background) entropy:**
  \( F \sim M^{1/3-2/3} \)

- **Hierarchical (mergers) and violent relaxation entropy of compressed matter:**
  \( F \sim M^{5/6-4/3} \)

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Entropy function of DM particles for isotropic velocity distribution

\[ p = \rho \langle v^2 \rangle = nT = Fn^{5/3} \]

one-dimensional peculiar velocity

Power-law density profiles: \( \alpha \in (0, 2) \)

\[ \rho(r) \propto r^{-\alpha} \quad M \propto r^{3-\alpha}, \quad p = C_1 + C_2 r^{2(1-\alpha)} \]

\( \alpha < 1 \) - finite pressure in the centre (core)
\( \alpha \geq 1 \) - infinite pressure at \( r \to 0 \) (cusp)

conserving both for initial and relaxed matter fields

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\[ F(M) \propto C_1 M^{\beta_1} + C_2 M^{\beta_2} \propto M^\beta \]

\[ \beta \in (\beta_1, \beta_2) , \quad \beta_{1,2} = \frac{1 + 2\alpha / 3}{3 - \alpha} \pm \frac{\frac{\alpha - 1}{\alpha - 3}}{3 - \alpha} \]

\[ \beta_{cr} = \beta_1 = \beta_2 = \frac{5}{6} \quad \Rightarrow \quad \alpha_{cr} = 1 \]

core (\( \beta < \frac{5}{6} \)): \( \beta_1 \in \left( 0, \frac{5}{6} \right) \), \( \beta_2 \in \left( \frac{2}{3}, \frac{5}{6} \right) \)

cusp (\( \beta \geq \frac{5}{6} \)): \( \beta_1 \in \left( \frac{5}{6}, \frac{4}{3} \right) \), \( \beta_2 \in \left( \frac{5}{6}, \frac{10}{3} \right) \)

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48 low brightness galaxies
(LBG, de Blok et al. 2001)
15 low brightness galaxies
(Swaters et al. 2003)
DM halo formation
(Zel’dovitch approximation)

\( \tilde{r}(z, \tilde{x}) = (1 + z)^{-1}[\tilde{x} - g \cdot \tilde{S}(\tilde{x})] \), \( \sigma = \sqrt{\int P(k)dk} = 11 \, h^{-1}\text{Mpc} \)

\( \tilde{V}(z, \tilde{x}) \equiv H_0 (1 + z)^{1/2}[\tilde{x} / 2 - g(z)\tilde{S}(\tilde{x})] \), \quad \delta(\tilde{x}) = \text{div} \tilde{S} \)

\( \tilde{S}(\tilde{x}) = \tilde{S}_R (\tilde{x}) + \tilde{S}_* (\tilde{x}) \), \quad \delta(\tilde{x}) = \delta_R (\tilde{x}) + \delta_* (\tilde{x}) \)

local background – protohalo with linear scale \( R \) [collapsing into virialized halo by \( z_0 \) with compression factor \( \sim 5(1+z_0) \)]

conditional perturbations [transform (adiabatically at least) into microscopic particles’ motion in halo]

\( |\tilde{x} - \tilde{x}_0| < R : \quad \delta_R \equiv 3(1 + z_0)/2 \), \quad \tilde{S}_R \equiv \tilde{S}_0 + (1 + z_0)(\tilde{x} - \tilde{x}_0)/2 \)

\[ \langle \tilde{S}_* \rangle = \langle \delta_* \rangle = 0 \], \quad \langle \tilde{S}_*^2 \rangle \equiv \sigma_*^2 (R) = \int P(k)[1 - W(kR)]^2 \, dk \]

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Conditional variance of peculiar velocity inside halo of mass $M$

$$\vec{v}_*(z, \vec{x}) \equiv H_0 (1 + z)^{-1/2} \vec{S}_* (\vec{x})$$

$$\sigma_{v*}^2 \equiv \sigma_v^2 (z, M) \equiv H_0^2 (1 + z)^{-1} \sigma_*^2 (R)$$

$$\sigma_* / \sigma \approx M_{13}^{1/3} \ln[1 + M_{13}^{-1/3}]$$

$$R \equiv \ell_v M_{16}^{1/3} , \quad \ell_v = 31 \text{ h}^{-1} \text{Mpc} , \quad M_n = M / 10^n M_\odot$$

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Background entropy function

\[ \langle F(M) \rangle = \frac{m_{DM} \sigma_v^2(z_0, M)}{n^{2/3}(z_0)} \approx F_0 z_5^{-3} M_{10}^{1/3} \ln[1 + M_{13}^{-1/3}] \]

\[ F_0 = \mu^{5/3} \text{keV cm}^2, \quad z_5 \equiv (1 + z_0)/5, \quad \mu \equiv m_{DM}/m_p \]

Probability distribution function

\[ dW(f) = e^{-f/2} \frac{df}{\sqrt{2\pi f}}, \quad f = \frac{F(M)}{\langle F(M) \rangle} \]

\[ \langle f^2 \rangle = 3\langle f \rangle^2 = 3 \]

large variations of F from mean value

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Background entropy slopes

\[ \beta \equiv \frac{dF(M)}{d \ln M} = 2 \left( \frac{1}{3} + \frac{1}{\ln M_{13}} \right) \]

\[ n \equiv \log \left( \frac{M}{M_\odot} \right) \]

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Entropy $\Rightarrow$ lost of phase $\Rightarrow$ assemble

**ideal gas**

$R >$ particle separation  \quad \Rightarrow \quad$ perturbation size

$t > R/v_{par}$  \quad \Rightarrow \quad$ collapse time, $t_0$

**collapsing cloud**

at $t < t_0$ we average over assemble of clouds collapsing by the given $t_0$

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Violent relaxation entropy (special analytical models)

**Isotermal shere**

(Fillmore & Goldreich 1984)

\[ \rho \sim r^{-2}, \quad M \sim r, \quad F \sim M^{4/3} \]

**Collapse of ellipsoide**

(Gurevich, Zybin 1988)

\[ \alpha \sim 1.7 - 1.9 \]

Generated entropy in the central region is negligible in comparison with background one!

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Hierarchical clustering/merging
(N-body simulations)

- Density and entropy slopes in halos:
  \[ \rho \sim r^{-\alpha}, \quad F \sim M^\beta \]

- Universal NFW profile:
  \[ \rho \sim x^{-1}(1+x)^{-2} \]
  \[ x = r/r_s, \quad \alpha = 1, \quad \beta = 5/6 \]

- Empirical Burkert profile:
  \[ \rho \sim (1+x)^{-1}(1+x^2)^{-1} \]
  \[ \alpha = 0, \quad \beta = 0 \]

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Rotation curves (Marchesini 2002)
Rotation curves (Marchesini 2002)
Galaxy clusters profiles
(Pointecouteau et al., 2005)
Analytically modeled halos

\[ F_b(M) \sim M^{\beta_b}, \quad \beta_b < \frac{5}{6} \]

\[ F_r(M) \sim M^{\beta_r}, \quad \beta_r \geq \frac{5}{6} \]

\[ F(M) = \sqrt{C_1 M^{2\beta_b} + C_2 M^{2\beta_r}} \]

\[ \kappa = \frac{F_b}{F} \in (0, 1) \]

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Generated rotation curves

$\beta_r = \frac{5}{6}$

$\alpha = 0.333$

$\alpha = 0.567$

$\alpha = 0.667$

$\frac{r}{r_{\text{max}}} = \frac{5}{6}$

$\kappa \sim 1$ – solid lines, $\kappa \ll 1$ – dashed lines

NFW – “stars”, Burkert – “dots”

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Conclusions

* The background entropy can prevent the cusp formation for halos with

$$10^6 \, M_\odot < M < 10^{12} \, M_\odot$$

* For smaller and larger galaxies and for clusters of galaxies the impact of background entropy is attenuated

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Conclusions - II

* The impact of the background entropy allows to reproduce the observed rotation curves

* N-body slopes:
  - high entropy at large radii (mergers, clust.)
  - low entropy in inner region - cold particles!

Underestimation of initial perturbations? Result depends on simulations (Moore, NFW Klypin)

* Where do cusps saturate to cores?

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Happy birthday Bernard!

We need you Bernard for solving exciting problems of astronomy!