Multiscale Morphology Filters

We present a method based on multi-scale Hessian based filters in order to enhance structures in terms of their morphology and apply it to the Sloan Digital Sky Survey.

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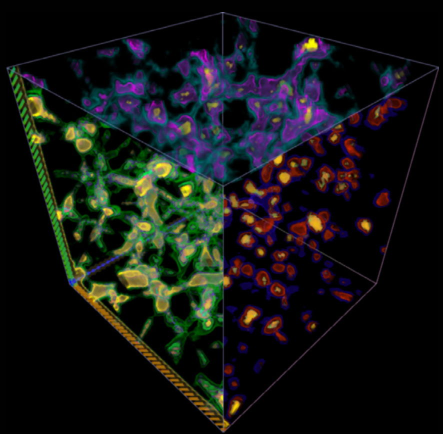
When we look at the distribution of galaxies from large scale surveys, the first thing that takes our attention is its peculiar texture. It is clearly non uniform with large associations of galaxies joined by stream-like patterns all surrounded by empty regions. This can be described as a mixture of blob, filament and wall structures interconnected as a cosmic web.



Three representations of the same dark matter distribution from a N-body simulation: raw Density Field obtained with the Delaunay Tesselation Field Estimator, filaments and blob structures enhanced with MMF.

It is possible to describe the morphology of a structure in terms of the local variations of the density field. For the variations in the neighborhood around point x_0 consider the Taylor expansion:

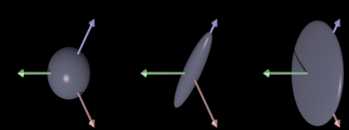
$$I(\boldsymbol{x}_0 + \delta\boldsymbol{x}_{0,\sigma}) = I(\boldsymbol{x}_0, \sigma) + \delta\boldsymbol{x}_0^T \nabla_{0,\sigma} + \delta\boldsymbol{x}_0^T H_{0,\sigma} \delta\boldsymbol{x}_0 + \ldots$$

where:

$$H = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$$

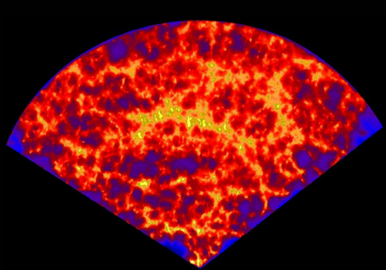
The eigenvalues of the Hessian describe the local variations of the density in the direction of the eigenvectors. These quantities can be used as basis for a set of morphology filters[1,2].

The Multiscale Morphology Filters (MMF) consist of two components, one purely geometrical and one dependent on the intensity of the field. The geometrical filter F(λ_1, λ_2, λ_3) is a function of the ratios of the eigenvalues. The choice of a balance between the two functions will determine the final performance of the MMF[3].

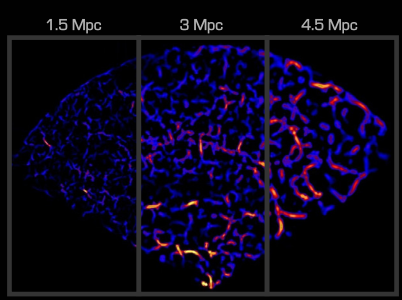


Ratios of the three eigenvalues of the Hessian matrix ($\lambda_3 >= \lambda_2 >= \lambda_1$) for blob, filament and wall structures.

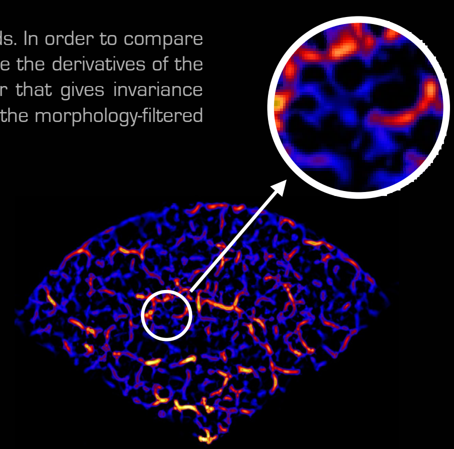
The morphology filtering is done on scale-space: a one-dimensional set of properly smoothed fields. In order to compare the scaled fields and select the most significant, we use the idea of *normalized derivatives*[4] where the derivatives of the field are weighted as function of the smoothing scale. For simplicity we choose a weight factor that gives invariance under rescaling. The final output is obtained by choosing the maximum response across scales of the morphology-filtered fields.



SDSS DR3 density field



Filament enhanced on three scales



Multiscale enhanced filaments

The results from applying the MMF are very promising. We are able to enhance and recover the embedded filamentary network as well as the blob-like structures in the LSS. The wall filters still need improvement since it seems that the eigenvectors alone are not enough as morphology filters for this case. The filter definition can be improved for better morphology selectivity and it is possible to extend the filters to include the eigenvectors. In that case we will be able to enhance (or diminish) structures based on their directional information like the fingers of God.

References:
1.- Y. Sato et al. 1998, 3D multi-scale line filter for segmentation and visualization of curvilinear structures in medical images.
2.- Frangi et al. 1998, Multiscale vessel enhancement filtering.
3.- Qiang Li 2003, Selective enhancement filters for nodules, vessels, and airway walls in two and three dimensional CT scans
4.- Lindeberg 1996, Edge detection and ridge detection with automatic scale selection.