A New Boundary Correction And Multifractal Analysis For Galaxy Surveys

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Abstract

A key issue in the statistical analysis of galaxy surveys is how to deal with the presence of boundaries and masked regions, which reduce the observed number counts of galaxies at large distances. The impact of these selection effects must be addressed, and corrected for, if galaxy counts are to be an effective tool for describing galaxy clustering. In this poster we present a new, improved, boundary correction method and investigate its performance.

Multifractal Formalism

The universe has already been shown to be well described using a multifractal framework [3]. In this analysis we will adopt the procedure laid out in Hentschel et al. [2] to determine the Rényi (Generalised) dimensions of a point set embedded in a three-dimensional Euclidean space. The probability of a galaxy, j, being within a sphere of radius r centred on galaxy i, is,

$$p_i(r) = \frac{n_i(r)}{N} = \frac{1}{N}\sum_{j=1}^{N}\Theta(|r_i - r_j| - r) \qquad (1)$$

Here $n_i(r)$ is the number of galaxies within radius r, N is the total number of galaxies and Θ is the Heaviside step function. Eq.1 can then be related to the partition sum [1],

$$Z(q,r) = \frac{1}{M}\sum_{i=1}^{M}[p_i(r)]^{q-1} \propto r^{\tau(q)} \qquad (2)$$

In this case M is the number of counting spheres and q defines the generalised dimension we are investigating. $\tau(q)$ is the scaling exponent, which is then related to the infinite set of dimensions through,

$$D_q = \frac{\tau(q)}{q-1}, \quad q \neq 1 \qquad (3)$$

Clearly the special case of $q = 1$, the information dimension, cannot be determined using the above expression but can be found approximately in the limit $q \to 1$. This is an important dimension to calculate as it gives equal weighting to voids and clusters. Voids are enhanced for $q < 1$ and clusters are enhanced for $q > 1$, so $q = 1$ is the most unbiased dimension in the set.

Boundary Corrections

Various methods have been proposed to correct for the 'missing' galaxies outside the boundary of a survey.

Angular Correction

The angular correction (Pan & Coles [4]) assumes that the universe is isotropic, and is illustrated via a 2-D analogy in Figure 1(a,b). Here the slice AOB with angle θ_1 is discarded and its contribution to the galaxy count replaced by the mean galaxy count averaged over the remaining slice that lies entirely within the survey volume. This leads to a complicated and slow algorithm which, moreover, throws away useful data, R2 in figure 1(b).

Volume Correction

Consider now our *volume* correction. In figure 1(a) the counting sphere extends beyond the geometrical boundary of the survey. The number of galaxies counted in the sphere of radius r is therefor depleted. To correct for this problem we could either add galaxies to the missing region or we could somehow modify the volume. We can recast Eq.1 as,

$$p_i(r) = \frac{V_i(r)\rho^\star(r)}{N} = \frac{V_i(r)}{V_i^\star(r)}.\frac{n_i^\star(r)}{N} \qquad (4)$$

Here $V^\star$ is the reduced volume. On its own this method can be visualised in figure 1(a), as assigning to the missing region 2 the same density as region 1. This would be wrong if density varies with distance, so that $\rho_{R1} \neq \rho_{R2}$. To overcome this problem we assume *only* that the density does not vary with θ or ϕ i.e. the universe is isotropic and hence Eq.4 will hold for fixed r.

So to apply our method to a galaxy survey we must count in spherical shells, correcting in each shell and then integrating over radius. This method is shown in figure 1(c). The shells are individually corrected and summed according to,

$$p_i(r) = \sum_{r=0}^{r}\alpha_i(r)\frac{n_i^\star(r)}{N} \qquad (5)$$

Where $\alpha_i(r) \equiv \frac{V}{V^\star}$, this is the enhancement factor of the i^{th} shell at radius r and has value ≥ 1.

FIGURES

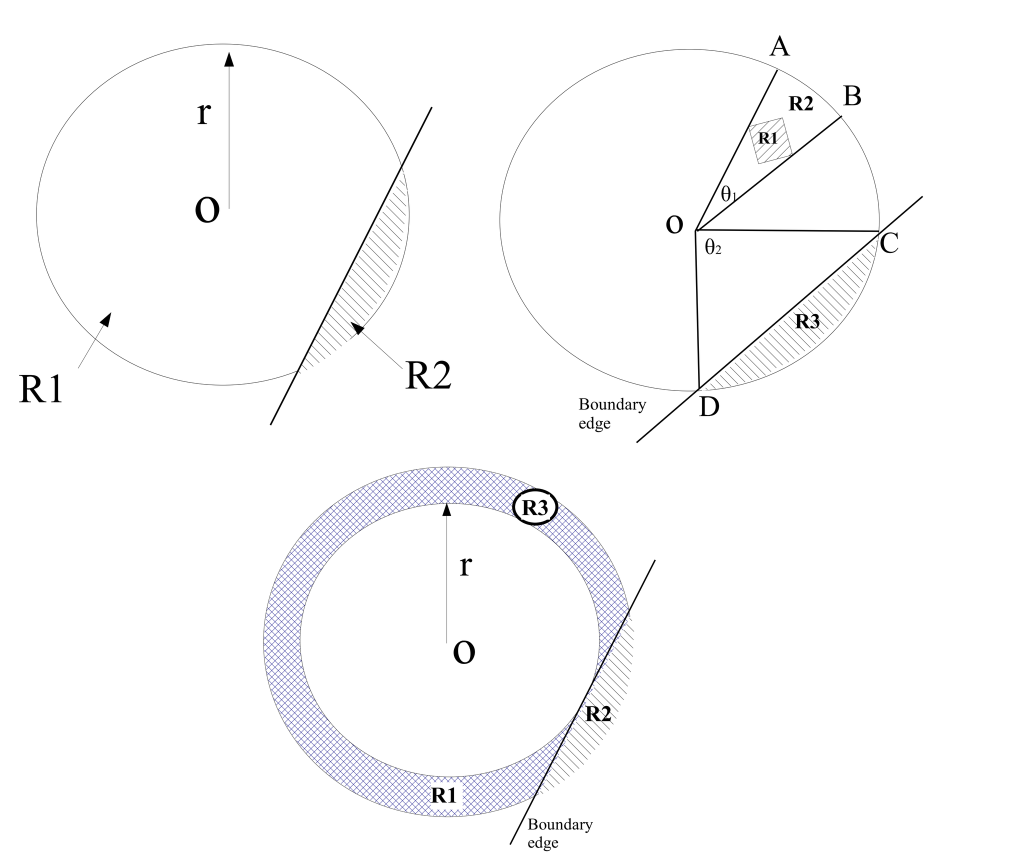

Figure 1: **Starting top left clockwise. a) A counting sphere centred on galaxy O with radius r. Regions R1 & R2 are inside and outside the survey respectively. b) This counting cell has a masked region R1 and a missing portion due to the intersection with the boundary R3. The slices AOB and COD encompass both of these missing parts. c) A counting shell centred on galaxy O with radius r. Region R1 is inside the survey, R2 & R3 are outside the survey. The missing parts of the shell R2 & R3 are replaced by the average over the rest of the shell**

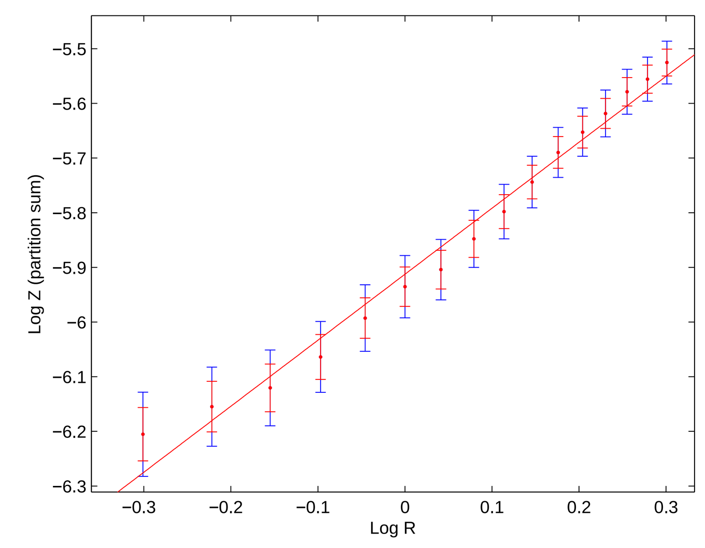

Figure 2: **This is a plot of the Partition sum varying with distance. The error bars are from 100 bootstrap re-samples (red) and from our own error analysis (blue), as described the Error section**

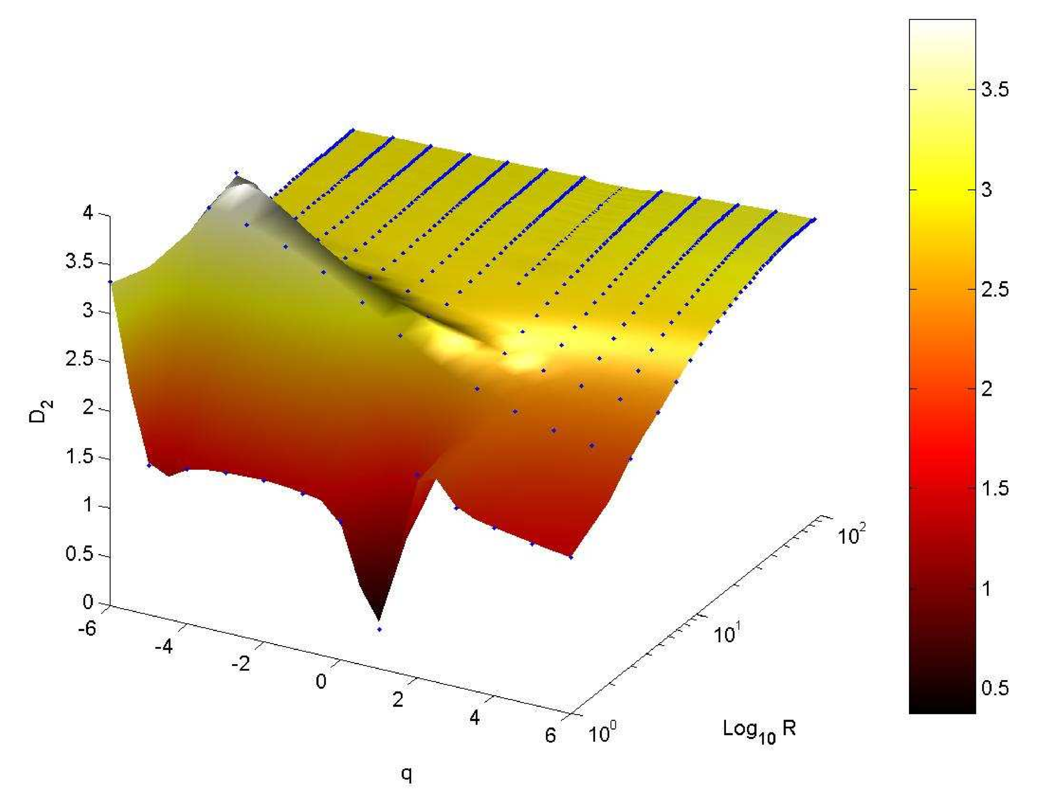

Figure 3: D_q **varying with both distance and moment, q. This is a unique descriptor of a discrete point set. The data used is the PSCz Mocks which have a ΛCDM cosmology.**

Error Analysis

We begin with Eq.2 to estimate the generalised dimensions of a set of N galaxies. By construction the mean of the partition function is positive as there are always cells with galaxies within. However, redshift surveys have a finite number of objects in a finite volume. This can lead to configurations where none of the cells have galaxies other than the central one when $\lim r \to 0$. We can then construct the second moment of the distribution and rearrange as below,

$$\langle Z(q)Z(q')\rangle = \langle\sum_i p_i^{q-1}\sum_j p_j^{q'-1}\rangle \qquad (6)$$

$$= \langle\sum_{i,j} p_i^{q-1}p_j^{q'-1}\rangle = \sum_{i,j}\langle p_i^{q-1}p_j^{q'-1}\rangle \qquad (7)$$

$$= \frac{1}{N^{q+q'-2}}\sum_{i,j}\langle n_i^{q-1}n_j^{q'-1}\rangle \qquad (8)$$

$$= \frac{1}{N^{q+q'-2}}\sum_{i=1}^{M}\langle n_i^{q+q'-2}\rangle + \frac{2}{N^{q+q'-2}}\sum_{i=1}^{M}\sum_{j=i+1}^{M}\langle n_i^{q-1}n_j^{q'-1}\rangle \qquad (9)$$

where $\langle\cdots\rangle$ represents an ensemble average. Applying the *Cosmological Ergodic Theorem* to the above equation leads to sums over the cells.

Thus, for $q = q'$ the $\langle Z(q)Z(q')\rangle$ can be cast as

$$\langle Z(q)Z(q)\rangle = \sum_i\langle p_i^{2(q-1)}\rangle + \text{cross terms} \qquad (10)$$

$$= \langle Z(2q-1)\rangle + \text{cross terms} \qquad (11)$$

$$= \sum_i\langle\left(p_i^2\right)^{(q-1)}\rangle + \text{cross terms} \qquad (12)$$

$$\leq 2\sum_i\langle\left(p_i^2\right)^{(q-1)}\rangle. \qquad (13)$$

We can then measure the standard deviation directly from,

$$\sigma_q^2 \simeq 2.Z(2q-1) - Z(q)^2 \qquad (14)$$

Results

Here we compare the *angular* and *volume* corrections using two distributions. Firstly we use a monofractal known as the Lévy Flight, the construction of this fractal is similar to a Brownian Random Walk. Then we consider a multifractal, Cantor/Sierpinski, The fractal dimensions can be predicted in advance. Our results to date are:

	Lévy Flight	Cantor/Sierpinski	PSCz Mock
Prediction	1.20	-	-
Angular Correction	1.24 ± 0.11	tbc	tbc
Volume Correction	1.18 ± 0.08	tbc	tbc

Conclusion and extensions

In this poster we have:

- Introduced a new *volume* correction as a very efficient method to apply boundary corrections, under the assumption of isotropy, when calculating correlation functions.
- Extended the idea of a multifractal to vary with distance, thus condensing a wealth of information from a discrete point set into a 2-d surface (see figure 3).
- Derived a new, simple, expression for calculating the errors from the data.

We are currently applying this methodology to compare simulations with real surveys. In future work we propose to expand our treatment to velocity space.

Acknowledgements

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References

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