Sixth set of problems for the course on Galaxies, 2007

1. Chemical evolution

   In this problem, we will investigate a more elaborate version of the accretion model discussed in class. We assume that initially all of the mass is in metal-free gas, and that there is gas infall at a rate $f(t) = dM_t/dt$. We also assume that a constant fraction $(1-q)$ of each infalling gas parcel δM_t is locked up by star formation. Thus the corresponding change in gas mass is $\delta M_g = q\delta M_t$.

   (a) Using that

   $$dZ = \frac{dM_t}{M_g}\left[p - Z - pq\right] \tag{1}$$

   show that a solution for $Z(M_g)$ is $Z = p(1-q)(1-e^{-u})$, where $u = 1/q \ln\left(M_g/M_g(0)\right)$, where $M_g(0)$ is the initial gas mass.

   (b) Show that the ratio of the stellar mass at t_1 to the mass in gas at the present time t_0 is

   $$\frac{M_s(u_1)}{M_g(u_0)} = \frac{1-q}{q}(e^{qu_1} - 1)e^{-qu_0} \tag{2}$$

   where u_i is the value of the parameter u at time t_i. *Hint:* use $M_g(u) = M_g(0)e^{qu}$.

   (c) Now consider the case $u_0 >> 1$, and let u_1 be an epoch at which the metallicity Z_1 was substantially lower than Z_0. Show that (i) the present metallicity is $Z_0 \sim p(1-q)$; (ii) that $u_1 \sim -\ln(1 - Z_1/Z_0) << 1$; (iii) that

   $$\frac{M_s(u_1)}{M_g(u_0)} \sim -\ln\left(1 - \frac{Z_1}{Z_0}\right)e^{-qu_0}(1-q) \tag{3}$$

   This formula differs from that obtained for the simple accretion model only by the presence of the factor $(1-q)e^{-qu_0}$. Since this factor can take any value between 0 and 1 depending on q and u_0, it follows that in this modified accretion model, the fraction of low metallicity stars can be made arbitrarily small.

2. Structure of Elliptical galaxies.

   (a) If the luminosity density of stars in a galaxy is $j(r) = j_0(r_0/r)^\alpha$, show that the surface brightness at distance R from the center is

   $$I(R) = I_0(r_0/R)^{\alpha-1} \tag{4}$$

   as long as $\alpha > 1$. What happens if $\alpha < 1$? Compute the total luminosity of the system.

   (b) The fraction of galaxies with apparent axis ratios $(q_0, q_0 + dq_0)$ observed from a random direction and true axis ratios $q = \beta/\alpha$, is

   $$f(q_0)dq_0 = \frac{q_0 dq_0}{\sqrt{(1-q^2)(q_0^2 - q^2)}}. \tag{5}$$

Show that if we view these galaxies from random directions, the fraction of oblate elliptical galaxies with true axis ratio q that appear more flattened than q_0 is

$$F(< q_0) = \int_q^{q_0} f(q_0')dq_0' = \sqrt{\frac{q_0^2 - q^2}{1 - q^2}} \tag{6}$$

If these galaxies have $q = 0.8$, show that the number seen in the range $0.95 < q_0 < 1$ should be about one-third of those with $0.8 < q_0 < 0.85$. Show that for smaller values of q, an even higher proportion of the images are nearly circular, with $0.95 < q_0 < 1$. Based on these results and on the figure shown in page 25 of the lecture notes, explain why it is unlikely that all elliptical galaxies have oblate shapes.

(c) The virial theorem relates the internal potential energy W and the kinetic energy K of a system in equilibrium through: $2K + W = 0$. Assuming that both the velocity dispersion σ and the mass-to-light ratio M/L are constant throughout a galaxy, and that no dark matter is present, use the virial theorem to show that

- Since the potential energy $W \propto -GM^2/R_e$, where M is the total mass of the galaxy and R_e its effective radius, and the kinetic energy $K \sim M\sigma^2/2$, so the mass of the galaxy should be $M \propto \sigma^2 R_e$.
- If the surface brightness $I(R)$ of all elliptical galaxies could be described by Sersic's law $I(R) = I_e \exp[-b(R/R_e)^{1/n} - 1]$ with the same value of n, explain why their total luminosity L should follow $L \propto I_e R_e^2$.
- If all elliptical galaxies had the same mass-to-light ratio M/L and surface brightness at the effective radius I_e, the Faber-Jackson relation is expected.