Fourth set of problems for the course on Galaxies, 2007
Dynamics

1. In a spherical galaxy, the gravitational potential $\Phi(r)$ is

$$\Phi(r) = -\frac{GM(<r)}{r} - 4\pi G \int_r^\infty \rho(r')r'dr' \tag{1}$$

Check that by differentiating this expression with respect to r you recover Newton's second theorem.

2. Show that at radius r inside a uniform sphere of density ρ, the radial force $F_r = -4\pi G\rho r/3$. If the density is zero for $r > a$, show that

$$\Phi(r) = -2\pi G\rho(a^2 - \frac{r^2}{3}), \tag{2}$$

3. The collision-timescale is

$$t_{coll} \sim \frac{V^3}{4\pi G^2 m^2 n} \tag{3}$$

where n is the number density of particles in a system, m is their mass, and V is their typical (relative) speed, i.e $V^2 = GNm/R$ where R is a characteristic scale of the system, and N the total number of particles (for a spherical object $N = 4\pi nR^3/3$). The relaxation timescale is

$$t_{relax} \sim \frac{N}{8\log\Lambda} t_{cross} \tag{4}$$

where $\Lambda = R/R_i$ with R_i the characteristic size of the particles, and $t_{cross} = R/V$ the crossing time. Compute t_{coll} and t_{relax} for a globular cluster, an elliptical galaxy and for a cluster of galaxies using the parameters below. What do you conclude from the comparison? Which process are important for stars in a cluster? Which for galaxies in a cluster?

object	N	R_i	m	R
globular cluster	10^5	7×10^5 km	1 M_{sun}	4 pc
elliptical galaxy	10^{11}	7×10^5 km	1 M_{sun}	10 kpc
galaxy cluster	10^3	100 kpc	$10^{12} M_{sun}$	3 Mpc

4. Use the definition of the epyciclic frequency to show that within a spherical galaxy of constant density, $\kappa = 2\Omega$ and the Oort constants $A = 0$ and $B = \Omega$.