Exercises for the course on Galaxies

- Generalities

  1. Show that if two stars of the same luminosity form a close binary pair, the apparent magnitude of the pair measured together is $\sim$ 0.75 mag brighter than each star individually.
  2. Show that an error or uncertainty of 0.1 magnitudes in the distance modulus is equivalent to ~ 5 % error in distance.

- Stellar populations and photometric evolution

  3. Integrate the Salpeter mass function between a lower mass limit M_l and upper limit $M_u \gg M_l$ to find (a) the number of stars formed in the population; (b) their total mass; (c) their total luminosity. Show that the number and mass of stars depends mainly on the mass M_l of the smallest stars, while the luminosity depends on M_u, the mass of the largest stars. Taking $M_l = 0.3 M_\odot$ and $M_u \gg 5 M_\odot$, what fraction of all stars have $M > 5 M_\odot$? What fraction have $M > 1 M_\odot$?
  4. The star formation rate measures the number of solar masses in stars formed per unit time.
     (a) In the Solar neighborhood the SFR has been found to be close to constant: $\Psi(t) = A$. Show that in this case, the initial luminosity function $\Phi_0(M_V)$ is equal to the observed luminosity function for main sequence stars alone $\Phi_{MS}(M_V)$ for $t < T_{MS}(M_V)$, while for $t > T_{MS}(M_V)$ it is given by $\Phi_0(M_V) = \Phi_{MS}(M_V) \times t/T_{MS}$.
     (b) Suppose that the star formation rate in a population has decayed exponentially in time $\Psi(t) = \mathrm{e}^{-\alpha t}$, and let the oldest star in the population have formed at $t = 0$. Show that the initial luminosity function $\Phi_0(M_V)$ is related to the luminosity function for main sequence stars only that is observed today $\Phi_{MS}(M_V)$ as

$$\Phi_0(M_V) = \frac{\mathrm{e}^{\alpha t}(1 - \mathrm{e}^{-\alpha t})}{\mathrm{e}^{\alpha T_{MS}} - 1} \Phi_{MS}(M_V) \qquad \text{for } t > T_{MS}(M_V) \tag{1}$$

5. Use the (in-class) derived evolution of the luminosity of a single stellar population $L(t)$ to:
   (a) Show that single stellar populations become fainter in time
   (b) Compute the color evolution of a single stellar population as a function of time: $(U–B)$ and $(B–V)$. Use that $U_\odot = 5.40$, $B_\odot = 5.25$, $V_\odot = 4.70$. Compute the colors at t=0.1, 1 and 5 Gyr.