

The motions of stars in the Galaxy

- The various Galactic components not only differ in their spatial distribution but also in their kinematics
- The dominant motion of stars (and gas) in the Galactic disk is rotation (around the centre of the Galaxy), and these motions occur on nearly circular orbits.
- The stars in the thick disk, rotate more slowly than those in the thin disk. Their random motions are slightly larger.
- The stars in the halo, however, do not rotate in an orderly fashion, their random motions are large, and their orbits are rather elongated.

Questions of the day

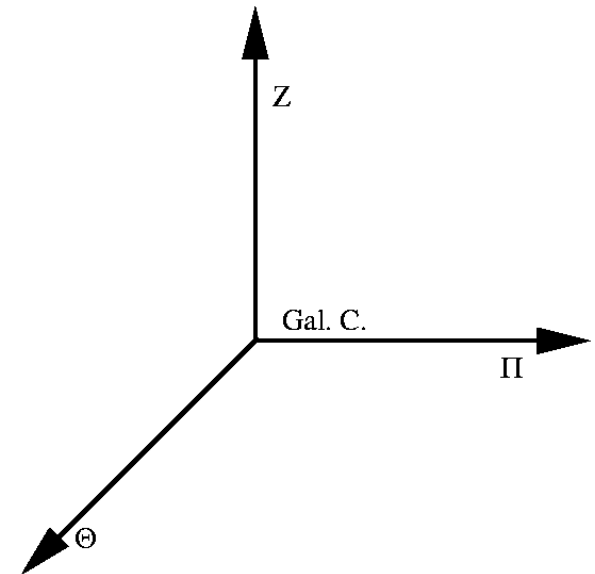
- How do we know this?
- How can we determine the rotational speed of the nearby disk?
- How can we best describe the motions of the stars in the different components?
- What are the motions of the gas in the *Galaxy*

Reference frames

The **fundamental Galactic reference frame** is centered on the galaxy's centre of mass.

The velocity of a star is often given in cylindrical coordinates (Π, Θ, Z)
(or (V_R, V_ϕ, V_z))

- Π : is along the radial direction (in the Galactic plane), and positive outwards ($l=180, b=0$)
- Θ : is in the tangential direction (in the Galactic plane), positive in the direction of galactic rotation ($l=90, b=0$)
- Z : is perpendicular to the galactic plane, and positive northwards



The local standard of rest

- Reference system on the Galactic plane that is moving on a circular orbit around the Galactic centre
- **“The” local standard of rest** is that located in the solar neighbourhood

The SN is a sphere of negligible size centered on the Sun, containing an adequate sample of stars. For example, for disk stars: radius 50-100 pc (1% of the disk size); for stellar halo stars ~ 1 kpc radius (1% of the halo extent).

• It makes sense and it is possible to define such a coordinate system, because a star moving on a circular orbit in the Galactic plane will continue to do so because

- the Galaxy is axisymmetric, $\Phi = \Phi(R, z)$
- symmetric with respect to the Galactic plane, $\Phi = \Phi(R, z^{2m})$
- in steady state (i.e. it is not evolving in time).

• At each point on the Galactic plane, the force that a star feels while moving in the Galactic disk is exclusively radial: $\mathbf{F}(R, z) = -\nabla\Phi(R, z)$

$$\mathbf{F} = -\frac{\partial\Phi}{\partial R}_{z=0} \boldsymbol{\varepsilon}_R - \frac{\partial\Phi}{\partial z}_{z=0} \boldsymbol{\varepsilon}_z = -\frac{\partial\Phi}{\partial R}(R) \boldsymbol{\varepsilon}_R$$

which implies that the angular momentum is conserved ($\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \mathbf{0}$).

Galactic rotation

- The concept of LSR is useful because disk stars move on nearly circular orbits
- There are essentially two ways in which a disk can rotate:
 - all stars move with the same angular velocity (rigid body rotation)
 - the angular velocity depends on radius: stars closer to the centre complete their orbits in less time than those farther out. This is known as differential rotation.
- Galaxies show differential rotation.

Differential rotation of the disk

- Consider a nearby disk star moving on a perfectly circular orbit. The velocity of this star with respect to the Galactic centre is $\mathbf{V} = \boldsymbol{\Omega} \times \mathbf{R}$, while the velocity of the LSR is $\mathbf{V}_0 = \boldsymbol{\Omega}_0 \times \mathbf{R}_0$.
- The **line of sight velocity** of this star with respect to the LSR is

$$v_{\text{los}} = (\mathbf{V} - \mathbf{V}_0) \cdot (\mathbf{R} - \mathbf{R}_0) / |\mathbf{R} - \mathbf{R}_0|$$

or in terms of the angular velocity $\boldsymbol{\Omega}$ at \mathbf{R} and that at \mathbf{R}_0

$$v_{\text{los}} = (\boldsymbol{\Omega} \times \mathbf{R} - \boldsymbol{\Omega}_0 \times \mathbf{R}_0) \cdot (\mathbf{R} - \mathbf{R}_0) / |\mathbf{R} - \mathbf{R}_0|$$

or

$$v_{\text{los}} = (\Omega - \Omega_0) \cdot (\mathbf{R}_0 \times \mathbf{R}) / |\mathbf{R} - \mathbf{R}_0|$$

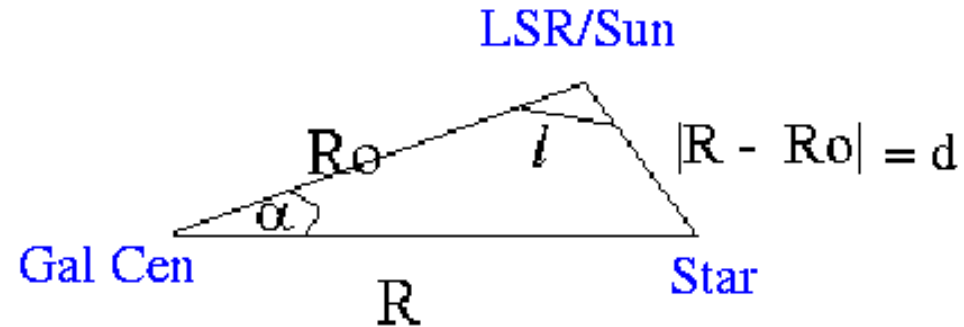
(using that $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0$ and $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})$)

Note that

$$\Omega - \Omega_0 = -(\Omega - \Omega_0) \mathbf{k} = -d\Omega/dR |_{R_0} (R - R_0) \mathbf{k}$$

$$\begin{aligned} \text{and } \mathbf{R}_0 \times \mathbf{R} &= -R R_0 \sin \alpha \mathbf{k} \\ &= -\mathbf{k} R_0 |\mathbf{R} - \mathbf{R}_0| \sin l \end{aligned}$$

(after using the law of sines)



Therefore

$$v_{\text{los}} = d\Omega/dR |_{R_0} (R - R_0) R_0 \sin l$$

Since $(R - R_0) \sim -d \cos l$, then

because $R^2 = R_0^2 + d^2 - 2 d R_0 \cos l \sim R_0^2 - 2 d R_0 \cos l$, and $R^2 - R_0^2 = (R - R_0) (R + R_0) \sim (R - R_0) 2 R_0$,

$$v_{\text{los}} = -d\Omega/dR |_{R_0} R_0 d \sin l \cos l,$$

$$v_{\text{los}} = A d \sin 2l \quad \text{where} \quad A = -0.5 * R \frac{d\Omega}{dR} \Big|_{R_0}$$

or, in terms of the velocity of a circular orbit $V_c = \Omega R$

$$A = 0.5 * (V_c/R - dV_c/dR) \Big|_{R_0}$$

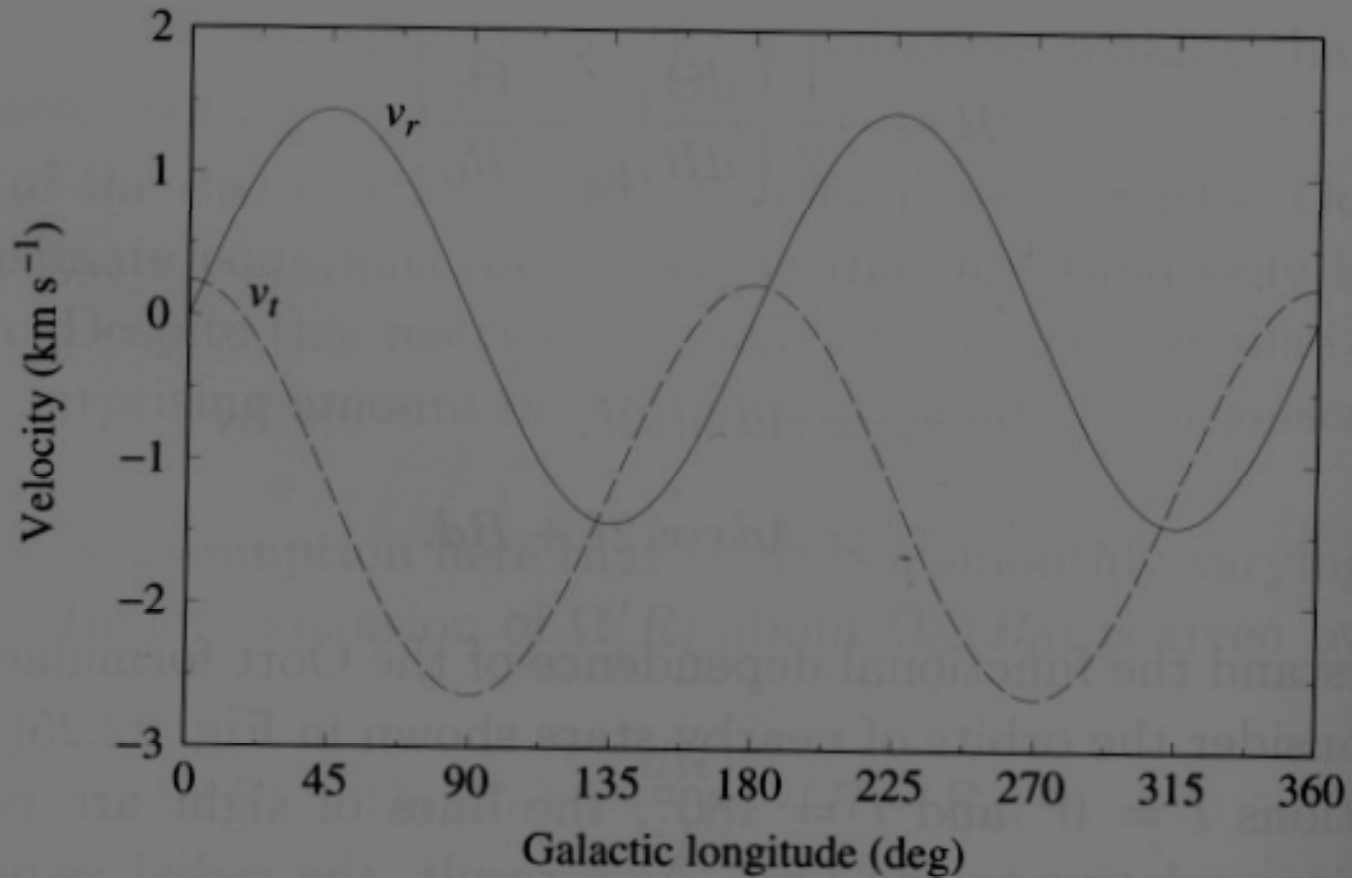
In the same way, the proper motion with respect to the LSR of a nearby star moving on a circular orbit is:

$$\mu = B + A \cos 2l$$

where

$$B = -(\Omega + 0.5 * R \frac{d\Omega}{dR}) \Big|_{R_0} = -0.5 * (V_c/R + dV_c/dR) \Big|_{R_0}$$

Variation of the line of sight and tangential velocities as function of Galactic longitude for disk stars moving on circular orbits



(b)

A and B are the Oort constants:

- **A measures the shear in the disk**

- the deviation from rigid body rotation because it depends on $d\Omega/dR$ (for the rigid body case $\Omega = \text{const.}$ and $A = 0$).

- **B measures the vorticity:** tendency of stars to circulate about a point

Jan Oort discovered that the motion of stars near the Sun varied with longitude, as described above, and he correctly interpreted as this being due to the differential rotation of the Galactic disk.

Results

- The most recent determinations of the Oort constants:

$$A = 14 \pm 1 \text{ km/s kpc}^{-1} \quad B = -12 \pm 1 \text{ km/s kpc}^{-1}$$

- The circular velocity and its gradient near the Sun are:

$$V_c = R_o (A - B)$$
$$dV_c/dR |_{R_o} = - (A + B)$$

Using the numerical values quoted above, one finds

$$V_c(R_o) = 218 (R_o/8 \text{ kpc}) \text{ km/s}$$

- Knowledge of the circular velocity and its variation as a function of distance are extremely important.
 - An object orbiting around a point mass (e.g. Earth - Sun system) has an acceleration
$$V_c^2/r = G M/r^2, \text{ or } M = r V_c^2/G$$
- For a spatially extended spherical system, a similar equation holds, where M is the mass within the radius of the circular orbit.
 - For a flattened system, a similar relation holds.
- By mapping $V_c(r)$ it is possible to derive:
 - amount of mass inside the orbit
 - how this mass is distributed $M = M(r)$.

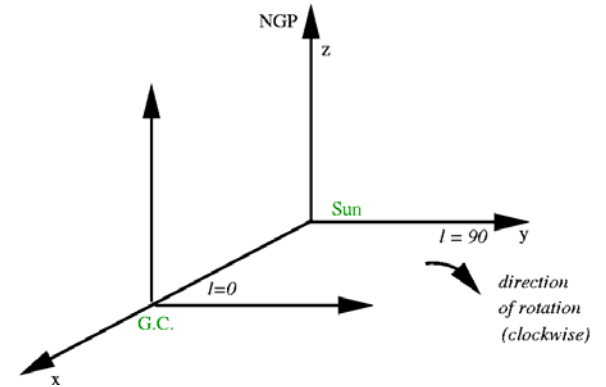
The solar motion

- The stars in the disk do not move on perfectly circular orbits
- The motion of the Sun with respect to the LSR is known as the "Solar motion".
- The solar motion is measured with respect to the mean velocity of stars of the same spectral type (e.g. gK, dM, etc).
 - This is because the LSR is "fiducial"
 - Essentially, we are defining the LSR to have the mean motion of stars (of similar spectral type) in the SN.
- We use radial velocities or proper motions

The solar motion: results

In the LSR frame, the velocity components of a star are:

$$(U, V, W) = (v_R, v_\phi - \Theta_{\text{LSR}}, v_z)$$

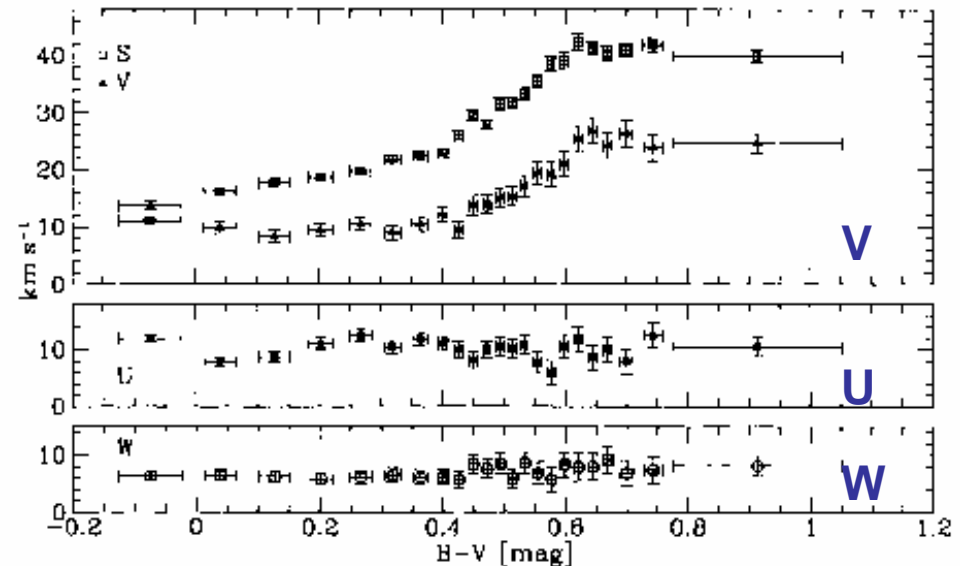


- Nearby disk stars observed with Hipparcos
- Their mean velocities (wrt Sun) should be a reflection of the solar motion:

$$\langle \mathbf{v} \rangle = - \langle \mathbf{v}_{\text{sun}} \rangle$$

$$U_{\text{sun}} = 10 \pm 0.4 \text{ km/s}$$

$$W_{\text{sun}} = 7.2 \pm 0.4 \text{ km/s}$$

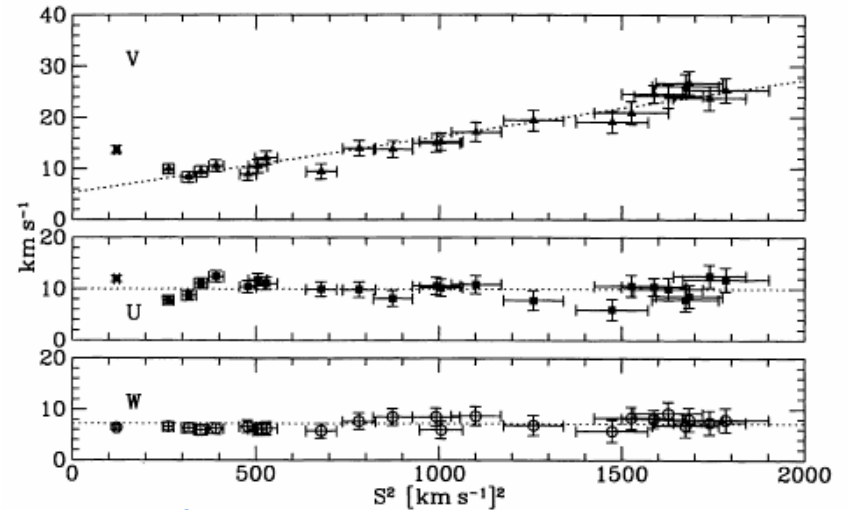


Notice that these are independent of $(B-V)$, while V_{sun} varies with color

V_{sun} correlates with the random velocities of stars in the SN.

The fit to this relation (in the limit of zero velocity dispersion) gives the actual solar motion

$$V_{\text{sun}} = 5.2 \pm 0.6 \text{ km/s}$$



S^2 : measure of random vel

The Sun is moving towards the Galactic centre, upwards (away from the plane) and faster than if it was moving on a perfectly circular orbit

Random velocities of stars

Stars in the Galactic disk have two types of motions:

- “ordered” on nearly circular orbits
- “random”
 - described by velocity dispersions

$$\sigma_k = \langle (v_k - \langle v_k \rangle)^2 \rangle^{1/2},$$

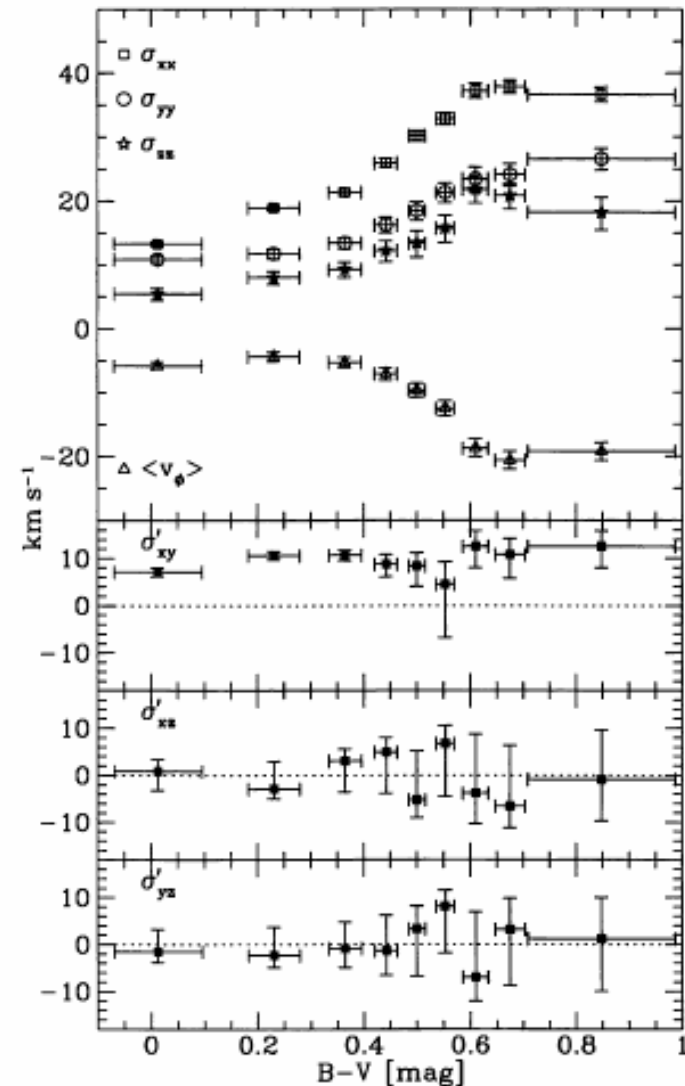
$\langle v_k \rangle$ is the mean velocity in the k -direction ($k=x,y,z$).

All dispersions increase with color up to $(B-V) \sim 0.6$

The increase of σ with $(B-V)$ points at a physical mechanism that operates progressively in time:

perturbations on the orbit

- graininess of the potential of the Galaxy,
- molecular clouds, spiral arms...



Schwarzschild distribution

- Used to describe velocities of nearby stars
- The probability that the velocity of a star lies in $d^3\mathbf{v} = dv_1 dv_2 dv_3$ is:

$$f(\mathbf{v}) d^3\mathbf{v} = d^3\mathbf{v} / [(2\pi)^{3/2} \sigma_1 \sigma_2 \sigma_3] \exp\{-\sum v_i^2 / (2\sigma_i^2)\}$$

$f(\mathbf{v})$ is constant on ellipsoids in velocity space

Different from the Maxwell-Boltzmann distribution function to describe the motions of gas particles:

$$f(v) = 1/(2\pi s^2)^{1/2} \exp(-v^2/2s^2)$$

because there is no distinction between the different directions of motion (only speed ($v = |\mathbf{v}|$) is important)

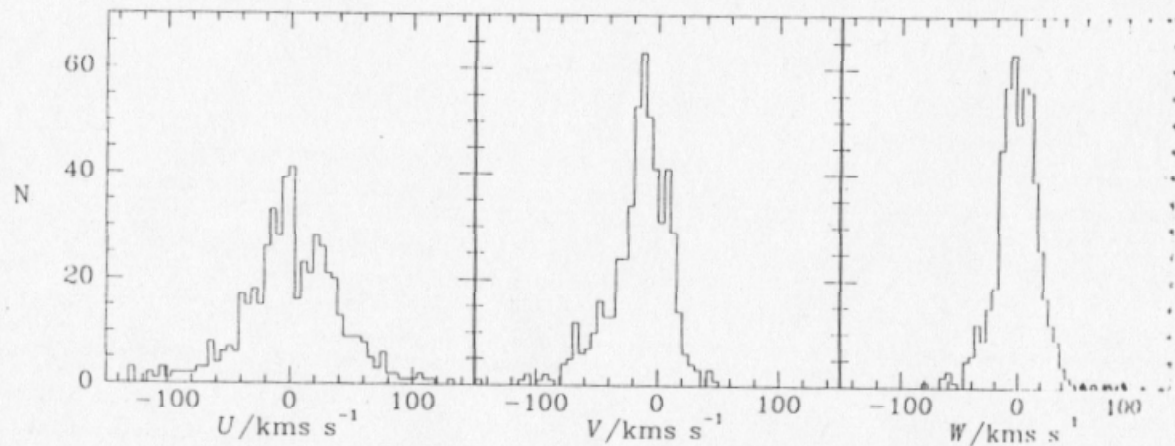
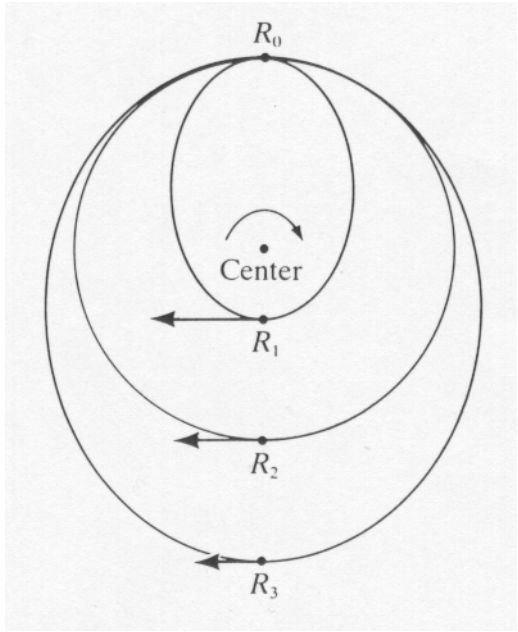


Figure 10.14 Upper panels: histograms of U , V and W for a sample of 323 nearby M_0 stars of MK type F1 and earlier. Lower panels: similar data for 510 K and M dwarf stars. Velocities are with respect to the LSR that is defined by equations (10.11). [From data kindly supplied by H. Jahreiss]

- The velocity distribution of stars in U and W is close to Gaussian.
- The distribution in the V -direction is skewed towards negative velocities.

• Stars with $V < 0$ km/s have orbits at smaller radii:

Consider a star on an elliptical orbit inside the solar circle.
 At the pericenter R_1 , its velocity is purely tangential and is larger than the circular velocity at that point (it needs to reach larger radii): $\Theta(R_1) > \Theta_c(R_1)$.
 At the apocenter R_0 , its velocity is again purely tangential, but now it has to be smaller than the circular velocity $\Theta_c(R_0)$ (since it reaches a smaller radius): $\Theta(R_0 = R_{\max}) < \Theta_c(R_0)$

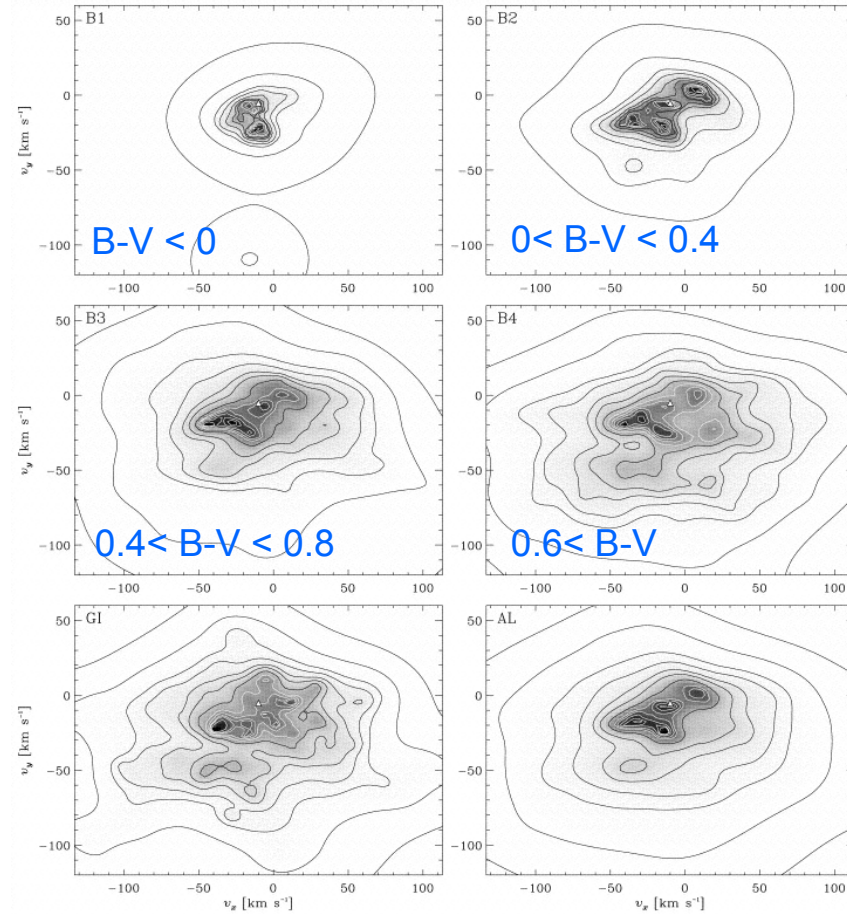


* The density of stars increases exponentially towards the center:
 there are more stars with $V < 0$ than with positive V

* The velocity dispersion also increases exponentially towards the center:
 the probability that a star from $R < R_0$ visits the SN is larger than for a star with $R > R_0$

Star streams

- The distribution of stars in velocity space is not smooth.
- U vs V , V vs W plots show substructures/ moving groups/ streams



Origin:

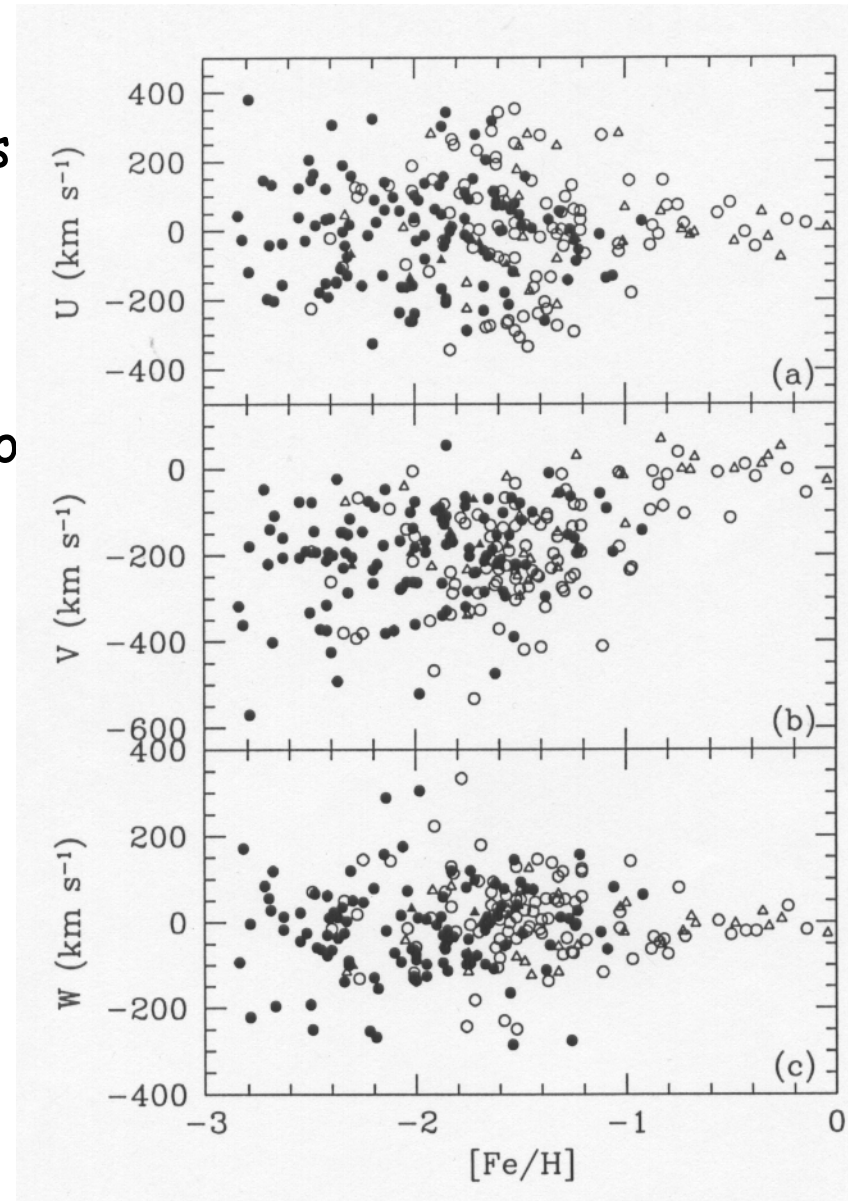
- Groups of stars born together: open clusters / associations / also spatial structure
- Dynamical origin (like due perturbations by spiral arms, bar)

The thick disk

- Thick disk stars have different kinematics from thin disk stars
- The **velocity dispersions are larger** (they have to be if the stars are to reach higher distances above the Galactic plane)
 - $\sigma_R = 61$ km/s; $\sigma_\phi = 58$ km/s; $\sigma_z = 39$ km/s
- The **rotational velocity is lower**: in the SN typically $V_\phi \sim 160$ km/s

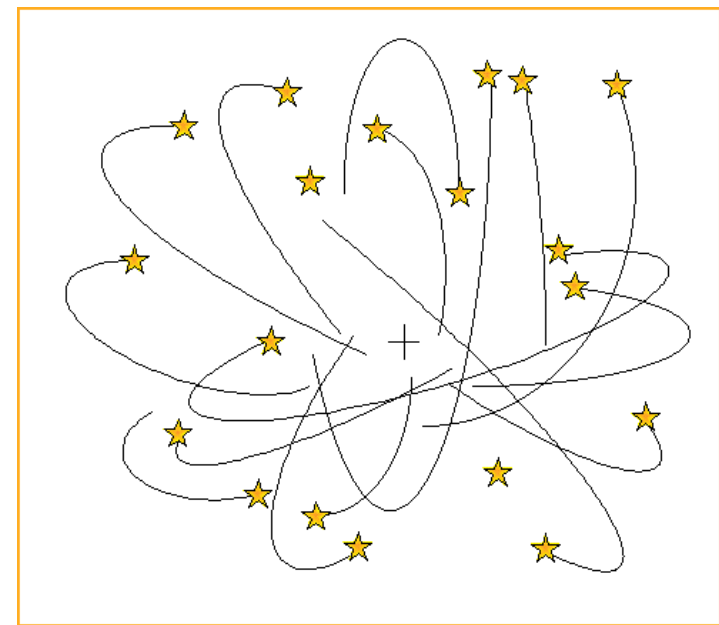
Kinematics of halo stars

- The velocity distribution of halo stars is close to Schwarzschild distribution
- The principal axes are closely aligned to the (Π, Θ, Z) directions, and the halo does not rotate
- The velocity dispersions are $(140, 105, 95)$ km/s



- The velocity ellipsoid is aligned with the radial direction:
 - halo stars are preferentially moving on very eccentric orbits
- Halo stars can be easily identified in proper motion surveys
 - distinct orbits from disk stars in the disk.
 - Their velocities wrt Sun are very large
- The large speeds imply they travel far into the halo of our Galaxy.

They can be used to estimate the escape velocity (and hence the mass of the Milky Way).



Kinematics of the bulge

- The bulge is not the extension of the stellar halo towards the center of the Galaxy:
 - Besides having a different spatial distribution, its stars have different metallicities (closer to those of the disk)
 - The kinematics are different: **bulge stars rotate** with a mean velocity of ~ 100 km/s
 - Their **velocity dispersions are slightly smaller** than those of the stellar halo

Surveys of disk, thick disk, stellar halo and bulge: what criteria?

- We want to **define sets of criteria** to preferentially select stars from a given Galactic component
- Criteria can be based on:
 - **spatial distribution** (location of fields, etc),
 - **color selection** (blue, red stars?),
 - **kinematics** (radial velocities, proper motions)
 - **metallicities** (high, low, etc)
- No criterion is perfect:
 - **Some contamination** is unavoidable
 - Try to keep this to a **minimum**

Gas in the Galaxy

- Nearby disk stars moving on perfectly circular orbits have
$$v_{\text{los}} = R_o (\Omega - \Omega_o) \sin l = R_o \sin l (V/R - V_o/R_o)$$
- This characteristic pattern also observed in the motion of disk gas

No gas with positive vels
in the 2nd quadrant
or with negative vels in
the 3rd quadrant

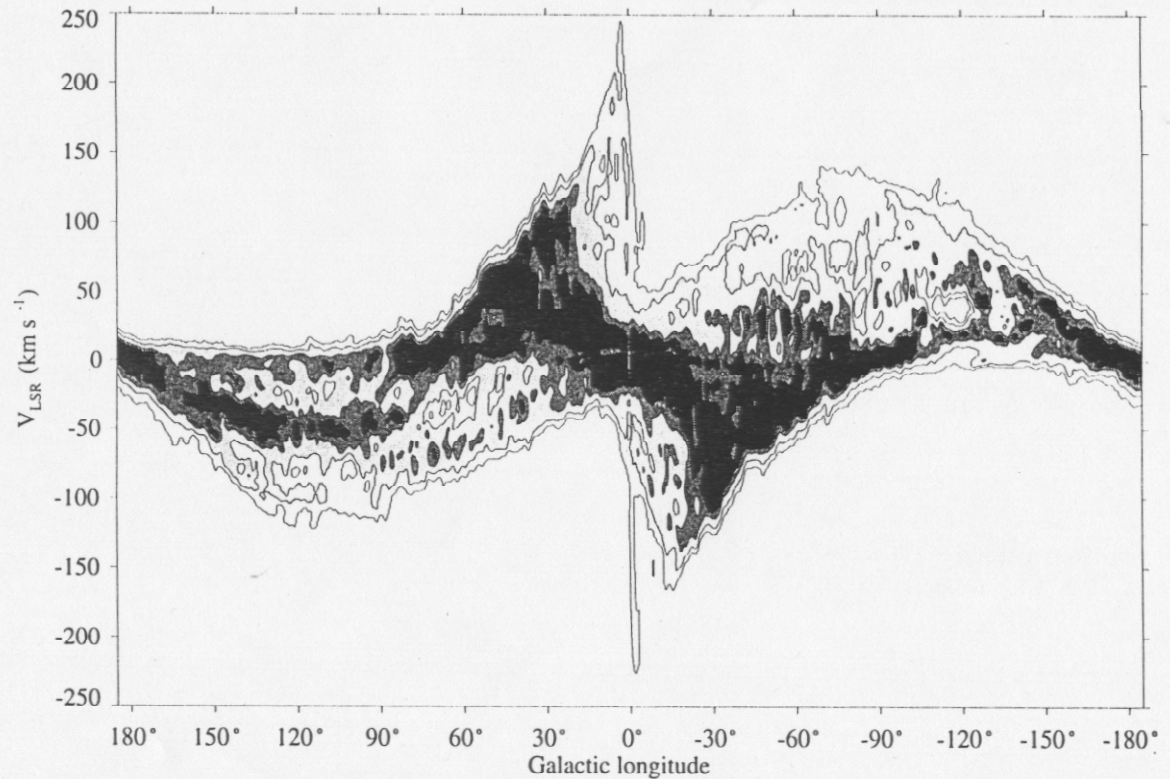


Figure 2.18 In the plane of the disk, the intensity of 21 cm emission from neutral hydrogen gas moving toward or away from us with velocity V_{LSR} , measured relative to the local standard of rest – D. Hartmann, W. Burton.

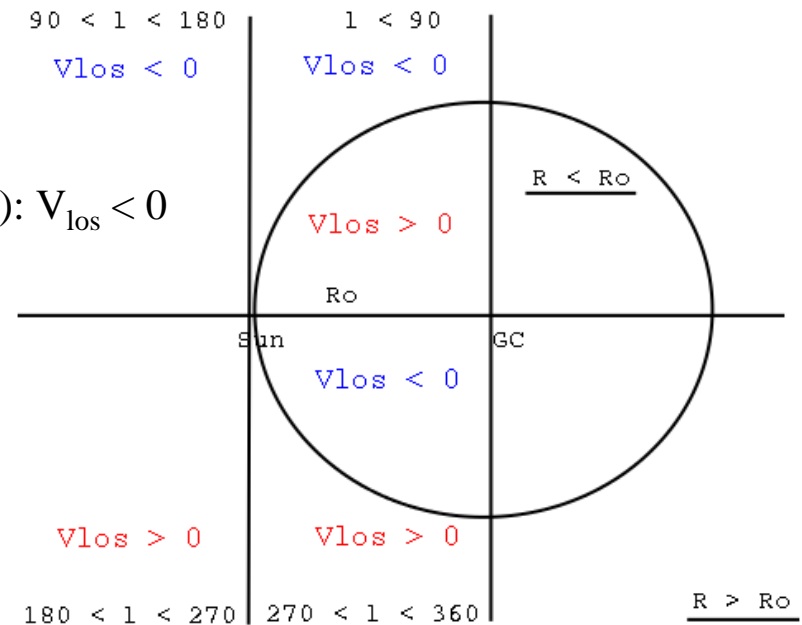
- $$v_{\text{los}} = R_o (\Omega - \Omega_o) \sin l = R_o \sin l (V/R - V_o/R_o)$$

- Typically, the angular speed drops with radius

- in the Kepler problem, $V = (GM/R)^{1/2}$, so that $V/R \sim R^{-3/2}$.

- This implies that

- $0 < l < 90$ and $R < R_o$ (e.g. nearby) : $V_{\text{los}} > 0$
- $0 < l < 90$ and $R > R_o$ (external to the solar circle): $V_{\text{los}} < 0$
- $90 < l < 180$ ($R > R_o$ always) : $V_{\text{los}} < 0$
- $180 < l < 270$ ($R > R_o$ always): $V_{\text{los}} > 0$
- $270 < l < 360$ and $R < R_o$: $V_{\text{los}} < 0$,
- $270 < l < 360$ and $R > R_o$: $V_{\text{los}} > 0$



The rotation curve of the Galaxy

- A dynamically interesting quantity
- Very hard to measure from radial velocities and proper motions of stars because their light is strongly absorbed by dust.
- HI gas which emits in the radio and is not affected by dust.
- The problem in this case is that it is usually impossible to know the distance to the emitting gas.

Tangent-point method

To determine the circular velocity of gas in the inner Galaxy ($R < R_0$).
Based on angular speed decreases with radius.

In the direction $0 < l < 90$, the l.o.s.
velocity is greatest at the tangent point T

Here the line of sight direction is perpendicular
to the vector to the Galactic centre, and hence this
line of sight is parallel to the tangential velocity
at that point, which is just the circular velocity.

For the tangent point T:

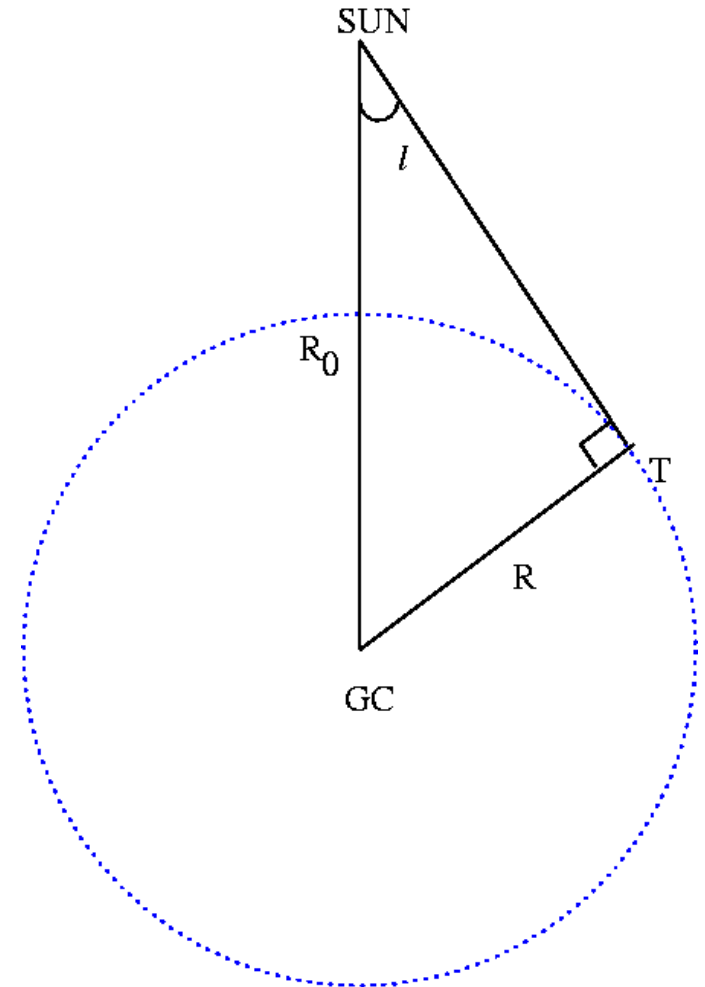
$$R_T = R_0 \sin l$$

and

$$V_{\text{los}} = R_0 \sin l (V_T/R_T - V_0/R_0) = V_T - V_0 \sin l$$

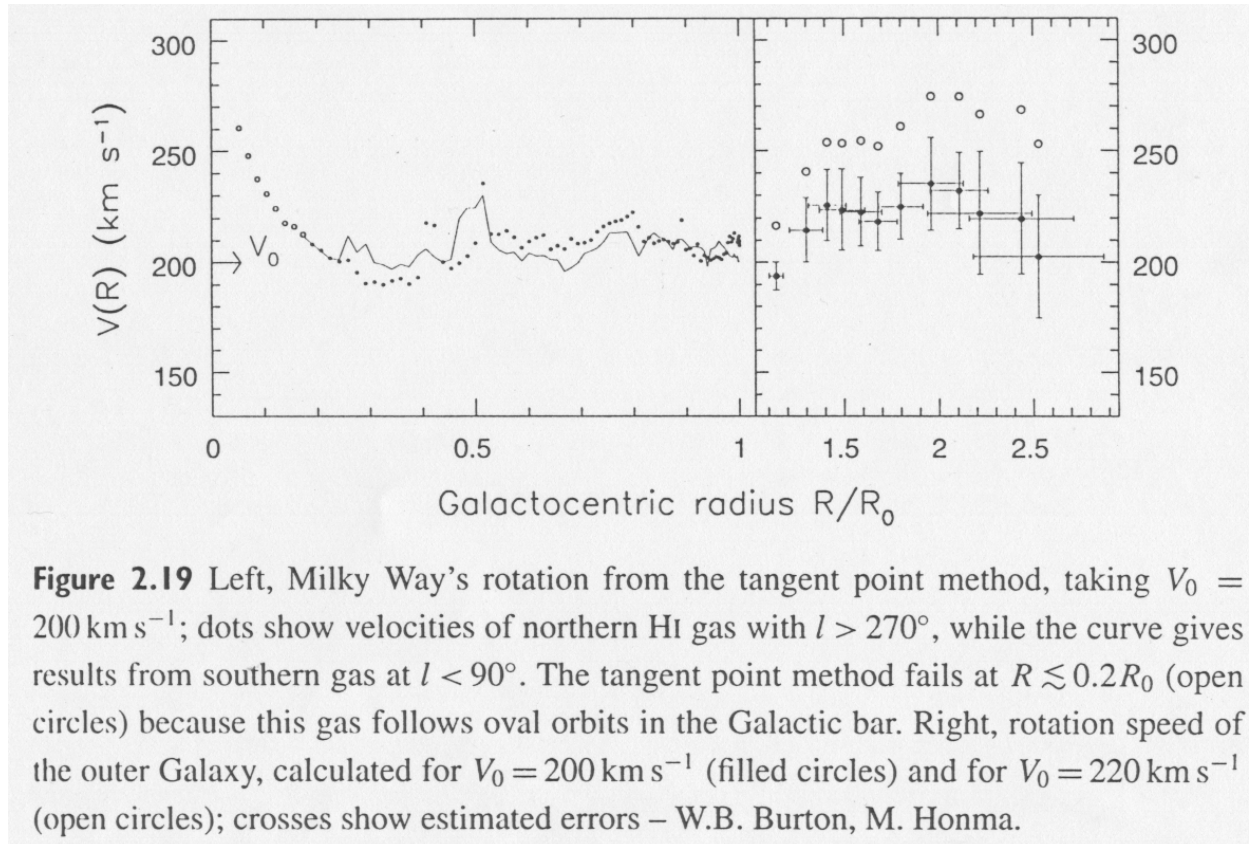
therefore

$$\Theta_c = V_T = V_0 \sin l + V_{\text{los}}$$



The rotation curve of the Galaxy: results

We can thus derive the circular velocity by measuring the largest velocity where emission from HI is observed for each longitude.



$V(R)$ is not completely smooth. This is due to the presence of spiral arms: they induce velocity changes of the order of 10-20 km/s. Thus if the tangent point is close to a spiral arm, the velocity measured will differ from the average speed of a circular orbit at that radius.

- More difficult to determine the circular velocity in outer Galaxy
 - since the distance to the gas is unknown

- Possible to use distances to cepheids, O-B associations, and measure their radial velocity from the emission lines of cold or hot gas around these stars.

- Sufficiently accurate to show that
 - the rotation speed $V(R)$ does not decline in the outer Galaxy
 - it may be rising.

- The circular velocity at a given radius $V(R)$ is related to the mass interior to that radius $M(<R)$ by

$$M(< R) = R V^2 / G$$

- Since $V(R)$ does not decline, this means that the mass of the Milky Way must increase almost linearly with radius
 - Even in the outer Galaxy where there are many fewer stars observed.
- Discrepancy between the light and mass is common in spiral galaxies.
- Galaxies contain a large amount of matter that does not emit any light: this is the infamous dark-matter.

The gas distribution in the Galaxy

- To measure how the HI is distributed in the disk distances are needed.
- But not possible to derive distances to individual gas clouds.
- However, if the rotation curve of the Galaxy is known, we can use the relation between the V_{los} and $V(R)$ to derive the distance R .
- This is called a **kinematic distance**.
- The next plot shows the surface density distribution of HI and of H_2
 - derived assuming that CO traces it; the problem is that H_2 has no transitions in the radio or sub-millimeter which would make it directly detectable.

The distributions of atomic and molecular H are quite different

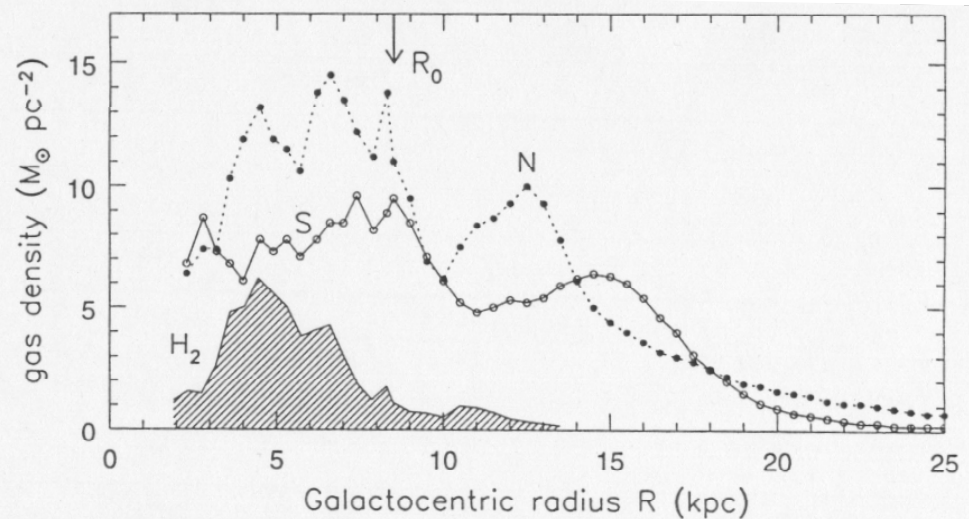


Figure 2.20 Surface density of neutral hydrogen, as estimated separately for the northern ($0 < l < 180^\circ$; filled dots) and southern ($180^\circ < l < 360^\circ$; open circles) half of the Galaxy. Within the solar circle, the density is sensitive to corrections for optical thickness; outside, it depends on what is assumed for $V(R)$. The shaded region shows surface density of molecular hydrogen, as estimated from the intensity of CO emission – W. Burton, T. Dame.

- almost all CO seems to lie inside the solar circle
- only 20% of all HI in the disk is inside R_{sun}
- CO concentrated in a ring at ~ 4 kpc and has a central hole
- HI spreads out farther than the stars in the disk, and also seems to have a central hole.
- The north and southern distributions of HI are not exactly the same.
 - HI warp towards $b > 0$ $l = 90$, and it bends southwards near $l = 270$