Elliptical galaxies

- The brightest and the faintest galaxies in the Universe

- Elliptical galaxies appear simple:

    - roundish on the sky,
    - light is smoothly distributed
    - lack star formation patches
    - lack strong internal obscuration by dust.

- But detailed studies reveal great complexity:

    - shapes (from oblate to triaxial);
    - large range of luminosity and light concentration;
    - fast and slowly rotating;
    - cuspy and cored …

Classification by luminosity

- Giant ellipticals have $L > L_*$
  L_* is luminosity of a large galaxy, $L_* = 2 \times 10^{10} L_\odot$ or $M_B = -20$ (MW)

- Midsized ellipticals are less luminous, with $L > 3 \times 10^9 L_\odot$ or $M_B < -18$

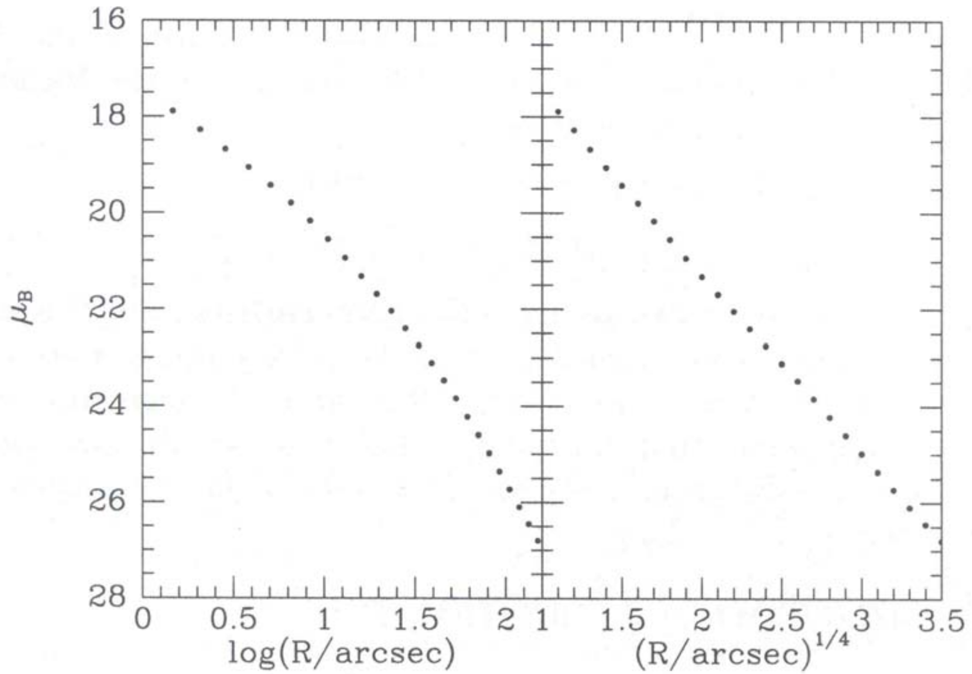
- Dwarf ellipticals have luminosities $L < 3 \times 10^9 L_\odot$

- These luminosity classes also useful to describe other properties
  - in contrast to disk galaxies (each class Sa … Sd contains wide range of luminosities)

- Ellipticals are one-size sequence:
  - internal properties correlate with their total luminosity (or mass)

Surface brightness profiles

Surface brightness profile μ for giant elliptical galaxy: as f(R) and f($R^{1/4}$).



Very good fit over 2 decades in radius

The surface brightness falls 9 magnitudes from centre to outskirts: 10^9 fall-off in projected luminosity!

The light in elliptical galaxies is quite centrally concentrated

De Vaucouleurs profile

The light distribution of many E can be fitted with:

$$I(R) = I_e \, e^{\{-7.67[(R/R_e)^{1/4} - 1]\}}$$

- R_e the effective radius (half of the total light is emitted inside R_e)
- I_e is the surface brightness at $R = R_e$
- the central brightness of the galaxy is $I_0 \sim 2000\ I_e$.

This profile is a particularly good description of the surface brightness of giant and midsized elliptical galaxies.

Other common profiles

Sersic law:

$$I(R) = I_e \, \mathrm{e}^{\{-b_n[(R/R_e)^{1/n}-1]\}}$$

- b_n is chosen such that half the luminosity comes from R < R_e.
- Recover de Vaucouleurs for n=4, and exponential for n=1 (dwarf E)

Hubble-Oemler law:

$$I(R) = \frac{I_0 \mathrm{e}^{-R^2/R_t^2}}{(1+R/r_0)^2}$$

- I_0 the central surface brightness
- R < r_0: surface brightness profile is approx. constant.
- r_0 < R < R_t: the surface brightness: $I \sim R^{-2}$.
- R > R_t :the surface brightness profile decays very quickly

- In the limit $R_t \rightarrow \infty$ this reduces to the Hubble law: $I(R) = \dfrac{I_0}{(1+R/r_0)^2}$

Central regions: the effect of seeing

- Giant E follow closely de Vaucouleurs profiles except inside the core
- Here atmospheric turbulence (seeing) blurs the image.

At small radii, the light is suppressed and redistributed at large radii

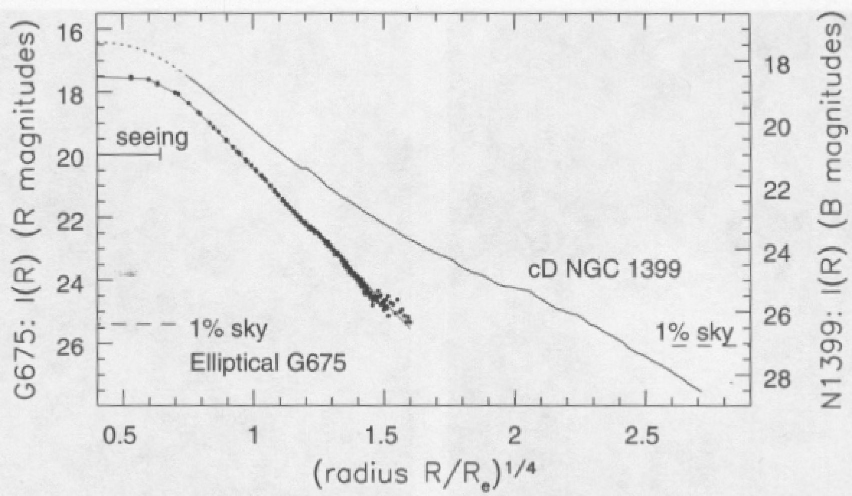


Figure 6.3 Surface brightness of two luminous ellipticals: an $R^{1/4}$ law corresponds to a straight line. Dots show the measured R-band surface brightness for galaxy G675, in cluster Abell 2572. It has $L_V \approx 2 \times 10^{10} L_\odot$, and $R_e = 4.95''$ or 3.8 kpc. The curve gives an $R^{1/4}$ profile, smoothed by atmospheric seeing: the horizontal bar shows 1.67″, the half-width of a stellar image. The upper curve shows the measured B-band profile of the cD galaxy NGC 1399, about twice as luminous as G675. For it, $R_e = 15.7'' \approx 1.4$ kpc, so measurements cover $R \lesssim 850''$ or 75 kpc. Between the dotted region where seeing has affected the measurement, and $R \sim 2^4 R_e$, $I(R)$ follows the $R^{1/4}$ profile closely – R. Saglia, N. Caon.

Seeing transforms a power-law profile into a profile with a core (a central region with nearly constant surface brightness).

Effect of Seeing - PSF

- Due to seeing, stars have finite extent (not observed as point sources).
- The light profile of stars is known as the PSF: Point Spread Function.

- The effect of the seeing is to blur an otherwise sharp image.

If in absence of seeing the surface brightness of an object at a position R′ is I_t(R′), the measured brightness at a location R will be:

$$I_{\text{app}}(\mathbf{R}) = \int d^2R' P(\mathbf{R}-\mathbf{R}') I_t(\mathbf{R}')$$

where P(d = |**R** - **R**'|) is the PSF.

In the absence of seeing, P is the Dirac δ-function: δ(R – R′), and one recovers the original (true) profile.

PSF

In the simplest case, the PSF can be treated as a circularly symmetric Gaussian

$$P(d) = \frac{1}{2\pi\sigma^2} e^{-d^2/2\sigma^2}$$

(This is often used to characterize stars; σ is the radius at which the light has fallen by $e^{1/2}$)

For a circularly symmetric surface brightness distribution $I_t(R')$:

$$I_{\mathrm{app}}(R) = \int_0^\infty dR'\, R'\, I_t(R')\, I_0(\frac{RR'}{\sigma^2})$$

where I_0 is the modified Bessel function.

Cores and cusps

•Space observations, where seeing does not play any role, show that midsized galaxies have central cusps, and not cores.

The measured central surface brightness from the ground is only a lower bound to the true value.

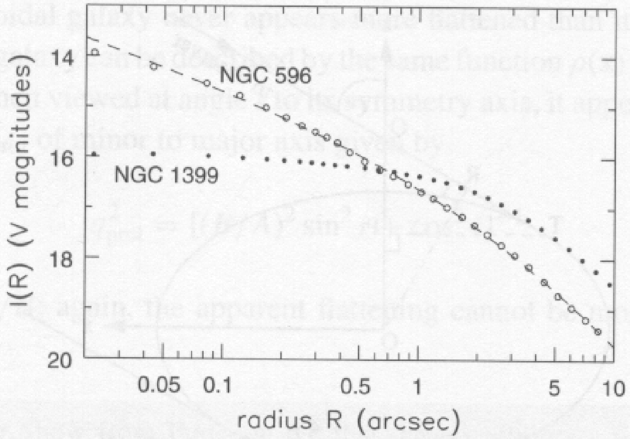


Figure 6.7 Surface brightness $I_V(R)$ in the V-band at the centers of two elliptical galaxies. The cD galaxy NGC 1399 ($M_V = -21.7$) has a *core* at $R \lesssim 1''$, where $I(R)$ is nearly constant. NGC 596 ($M_V = -20.9$) is half as luminous; the surface brightness continues to rise as a *cusp*. The dashed line shows $I(R) \propto R^{-0.55}$ – T. Lauer.

•Midsize ellipticals are usually cuspy, giant ellipticals tend to have cores

Photometric properties of E galaxies

•The central surface brightness is tightly correlated with total luminosity.

•The plot shows the central brightness $I_V(0)$, the core radius r_c (the radius at which the surface brightness has dropped to half its central value) as functions of the total luminosity or absolute magnitude

•for the giant and midsized E, the more luminous the galaxy: the lower its central brightness I(0), and the larger its core r_c.

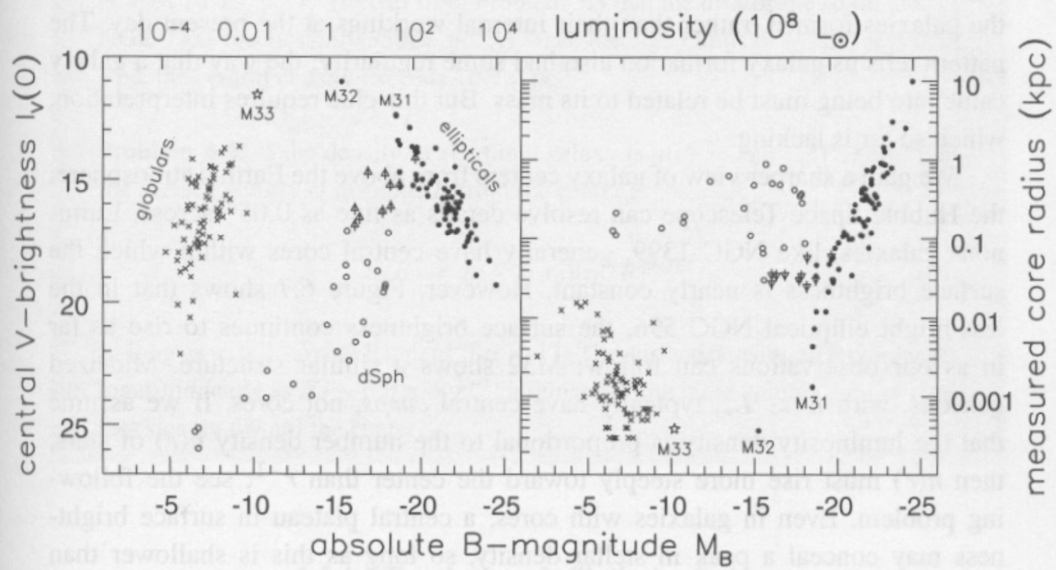


Figure 6.6 Central surface brightness $I_V(0)$ in mag arcsec^{-2} in the V band, and core radius r_c, measured from the ground, plotted against B-band luminosity M_B. Filled circles are elliptical galaxies and bulges of spirals (including the Andromeda galaxy M31); open circles are dwarf spheroidals; crosses are globular clusters; the star is the nucleus of Sc galaxy M33. Arrows show ellipticals in the Virgo cluster; here, seeing may cause us to measure too low a central brightness, and too large a core – J. Kormendy.

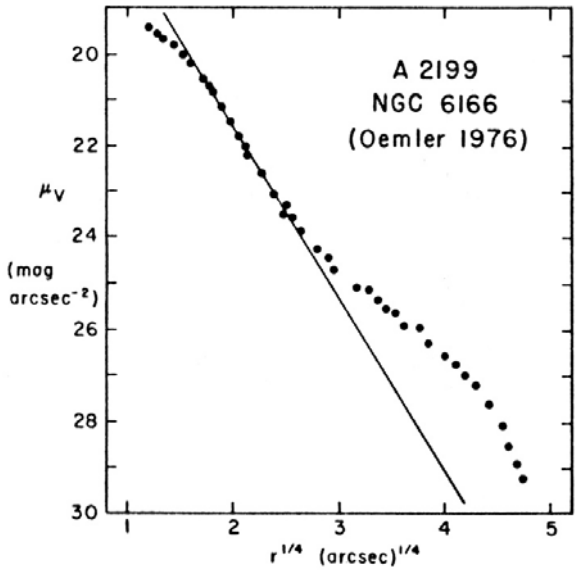
•This shows, that just like with stars, the properties of galaxies are not random: there is a certain degree of coherence.

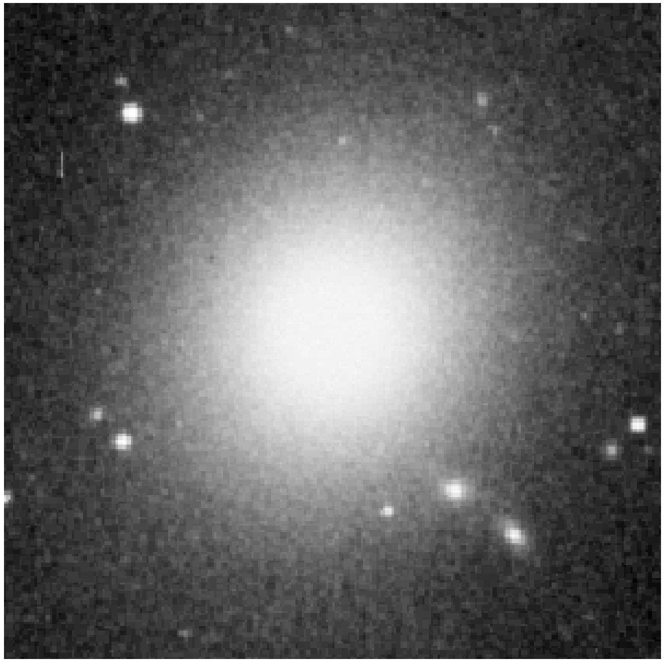
The outskirts of elliptical galaxies

- Surface brightness profile of giant ellipticals often shows an excess of light in the outer parts (compared to de Vaucouleurs profile).

- Such galaxies are known as cD Galaxies.
    - Usually at the center of clusters of galaxies

- Excess indicates the presence of an extended halo.
    - The cD halos could belong to the cluster rather than to the galaxy.





M87, the central cD galaxy in the Virgo cluster of galaxies.

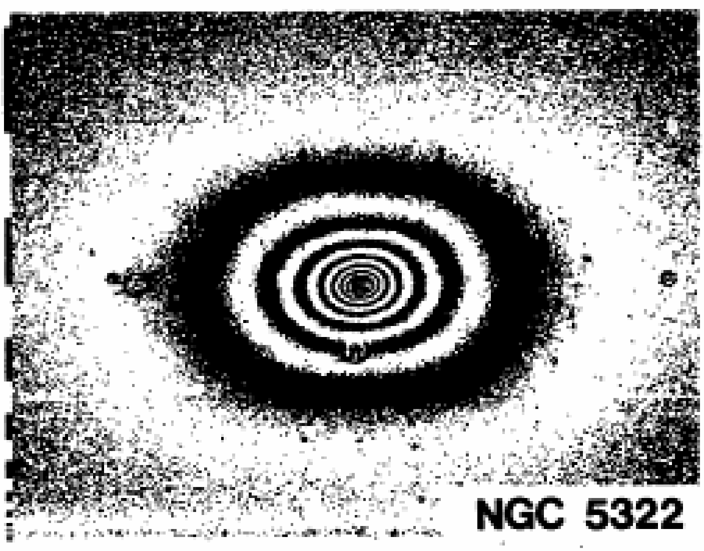
Isophotes

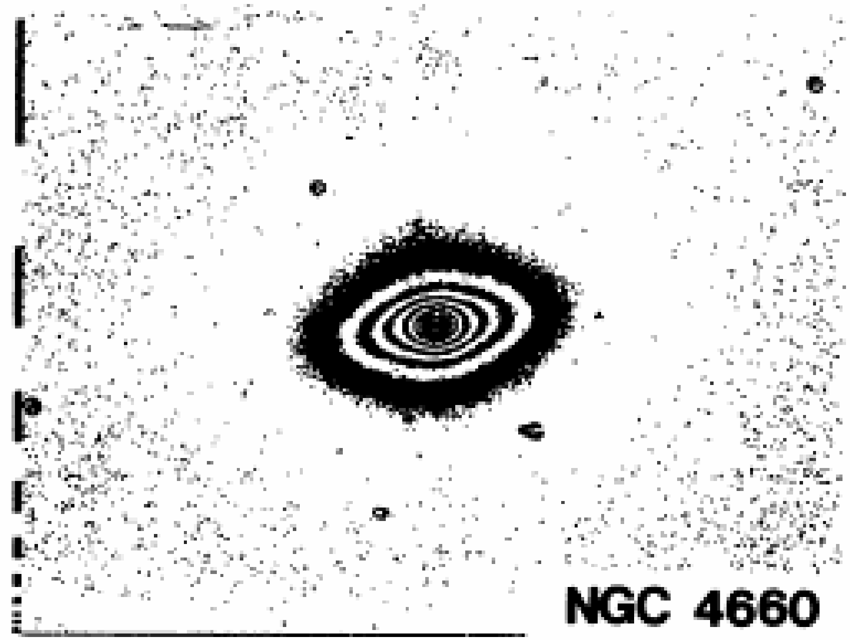
- Isophotes are contours of constant surface brightness.

- The ratio of semi-major to semi-minor axis measures how far the isophote deviates from a circle $e = 1 - b/a$.

    - Used to classify E galaxies by type En, where $n = 10\,(1-b/a)$

    - Note that this classification depends strongly on viewing angle

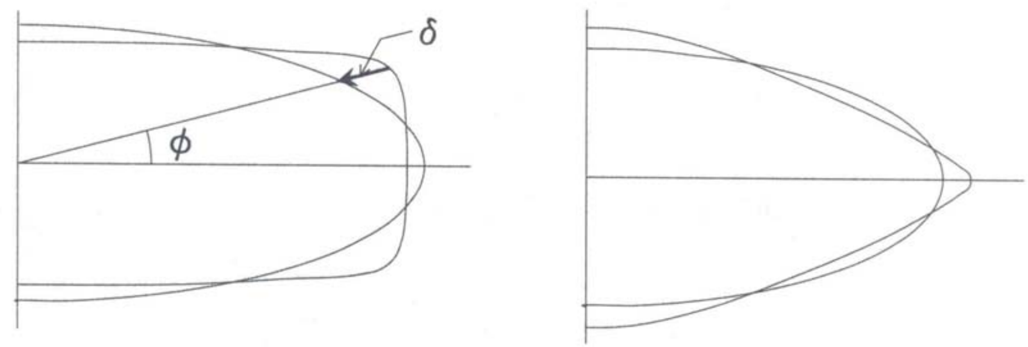




Deviations from ellipses

Isophotes are not perfect ellipses:

- Excess of light on the major axis: disky
- Excess on the "corners" of the ellipse: boxy.



The diskiness/boxiness of an isophote is measured by the difference between the real isophote and the best-fit ellipse:

$$\delta(\phi) = \langle\delta\rangle + \Sigma\, a_n \cos n\phi + \Sigma\, b_n \sin n\phi$$

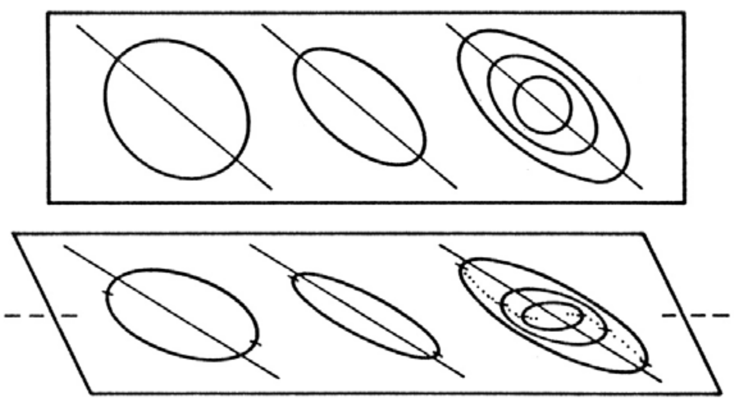
- If isophotes have 4-fold symmetry: terms with $n < 4$ and all b_n should be small. The value of a_4 tells us the shape:
  - $a_4 > 0$: disky E
  - $a_4 < 0$: boxy E

Isophote twisting

If intrinsic shape of a galaxy is triaxial, the orientation of the projected ellipses depends on:

- the inclination of the body
- the body's true axis ratio.

See figure below:

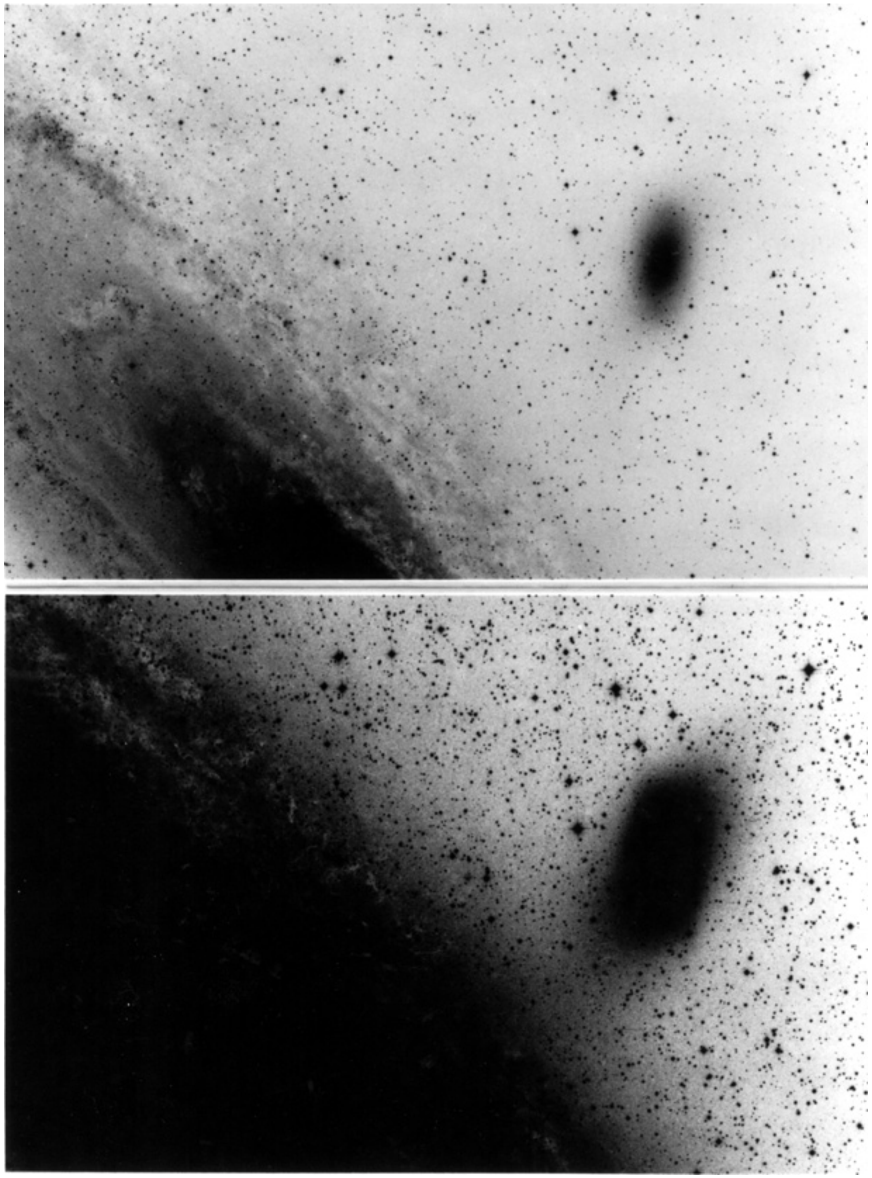


Since the ellipticity changes with radius, even if the major axis of all the ellipses have the same orientation, they appear as if they were rotated in the projected image.

This is called isophote twisting.

It is not possible, from an observation of a twisted set of isophotes to conclude whether there is a real twist, or whether the object is triaxial.

Twisted isophotes in a satellite galaxy of Andromeda (M31).



The shallower exposure shows the brightest part of the galaxy.

The deeper exposure shows the weaker more extended emission.

A twist between both images of the same galaxy are apparent (the orientation in the sky is the same in both figures).

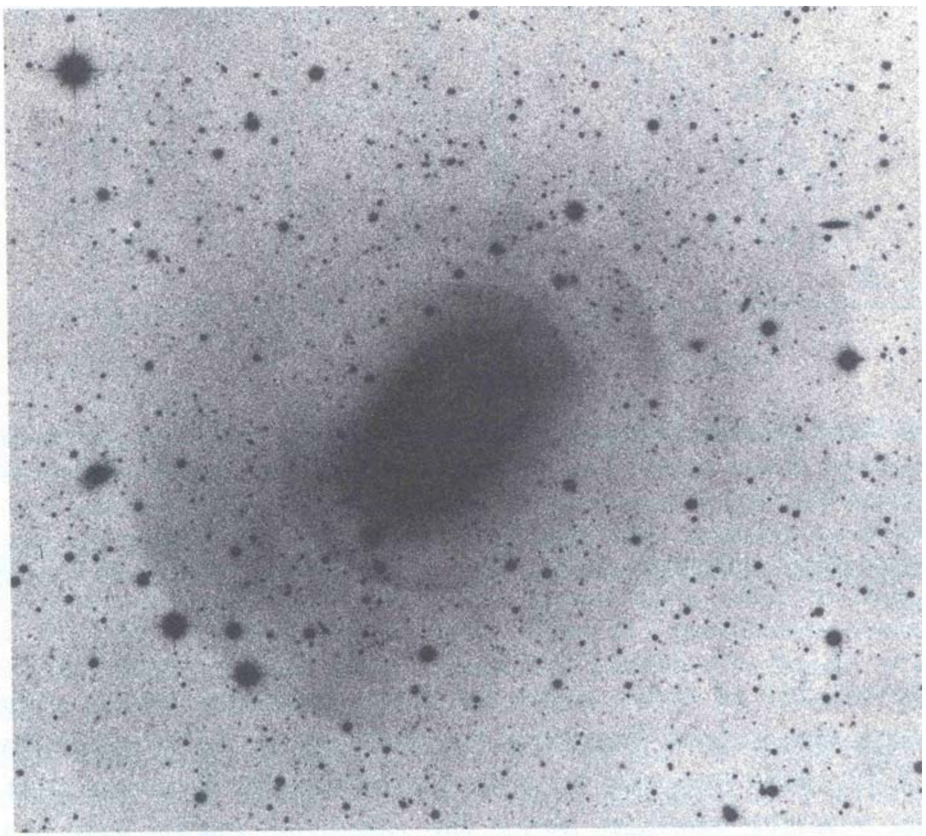
- Boxy galaxies are
    - more luminous in general,
    - probably triaxial
    - more likely to show isophote twists

- Disky E are
    - midsize,
    - more often oblate,
    - and faster rotators.

Some suggest that disky E can be considered an intermediate class between the big boxy ones and the S0s.

Fine structure

About 10 to 20% of the elliptical galaxies contain sharp steps in their luminosity profiles.



These features are known as ripples and shells.

Ripples and shells have also been detected in S0 and Sa galaxies.

It is not clear whether this is present in later-type galaxies because it is difficult to detect in those cases…

They are probably the result of the accretion/merger of a small galaxy on a very elongated (radial) orbit (Quinn 1984).

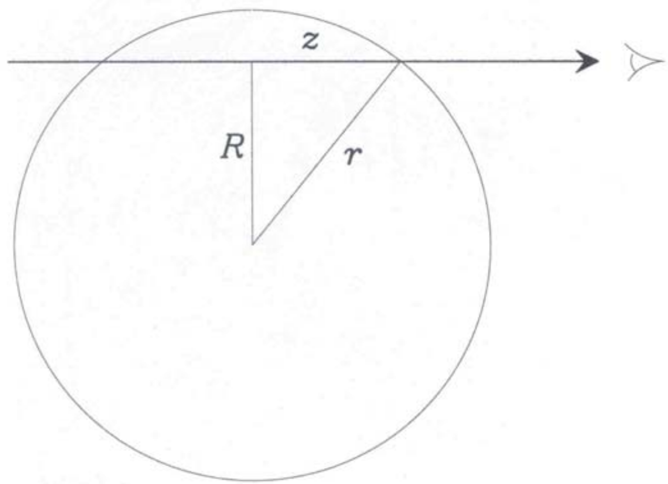
De-projection of galaxy profiles

Is it possible from the observed surface brightness profile I(**R**) to derive the true 3-D distribution of light j(**r**) of the galaxy?

If I(**R**) is circularly symmetric, it is possible that *j*(**r**) is spherically symmetric.

In this case

$$I(R) = \int_{-\infty}^{\infty} dz\, j(r) = 2\int_R^{\infty} \frac{j(r)\, r\, dr}{\sqrt{r^2 - R^2}}$$



This is an Abel integral equation for j as a function of I.

De-projection: Abel integral

Its solution is:

$$j(r) = -\frac{1}{\pi}\int_r^\infty \frac{dI}{dR}\frac{dR}{\sqrt{R^2 - r^2}}$$

A simple pair that can be used to represent the profile of elliptical galaxies, is:

$$I(R) = \frac{I_0}{1 + (R/r_0)^2} \qquad \longleftrightarrow \qquad j(r) = \frac{j_0}{[1 + (r/r_0)^2]^{3/2}}$$

This surface brightness profile is known as the Hubble law. Notice that for R >> r_0: $I \sim R^{-2}$, and $j \sim r^{-3}$.

De-projection: Non-spherical case

- If the isophotes are not circularly symmetric, then the galaxy cannot be spherically symmetric, but it can still be axisymmetric.

- In general the line of sight will be inclined at an angle with respect to the equatorial plane of an axisymmetric galaxy.

- In that case, there are infinite de-projected profiles that match an observation.

    - E.g. when observed from the pole, both a spherical galaxy as well as any oblate or prolate ellipsoid will produce the same projected distribution

Shapes of elliptical galaxies

What can we learn from the distribution of observed apparent ellipticities about the true (intrinsic) distribution of axis ratios?

In the most general case, the (luminosity) density $\rho(\mathbf{x})$ can be expressed as $\rho(m^2)$, where:

$$m^2 = \frac{x^2}{\alpha^2} + \frac{y^2}{\gamma^2} + \frac{z^2}{\beta^2}$$

The contours of constant density are ellipsoids of m^2 = constant.

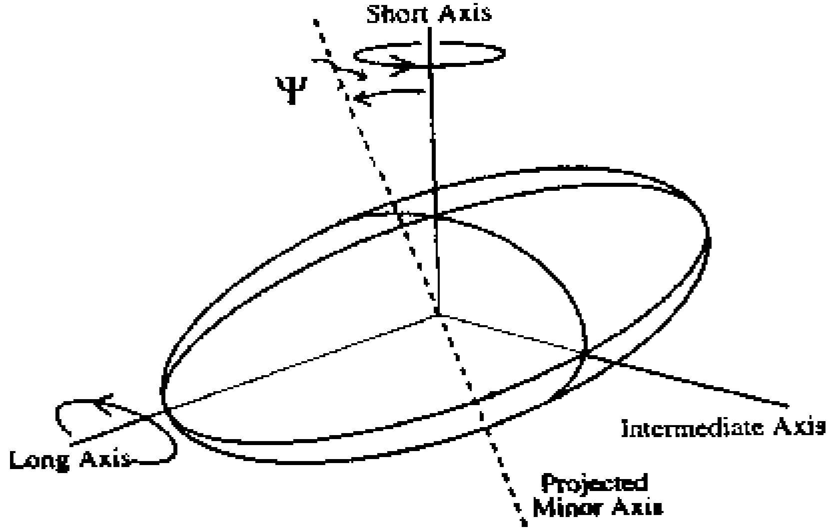
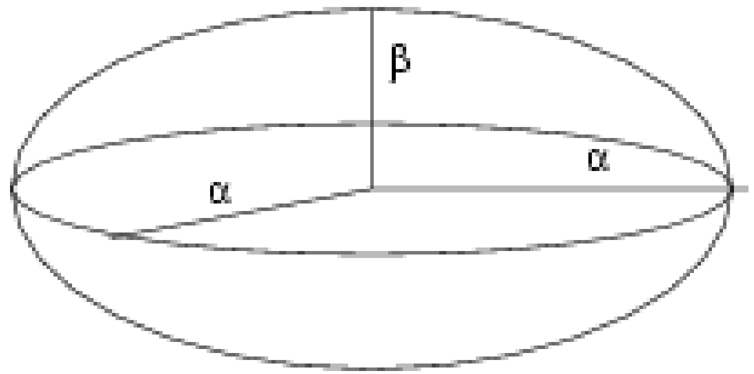


Figure 2.2: *The projection of a Prolate-Triaxial model*

$\alpha \neq \gamma \neq \beta$: triaxial
$\alpha = \gamma < \beta$: prolate (cigar-shaped)
$\alpha = \gamma > \beta$: oblate (rugby-ball)

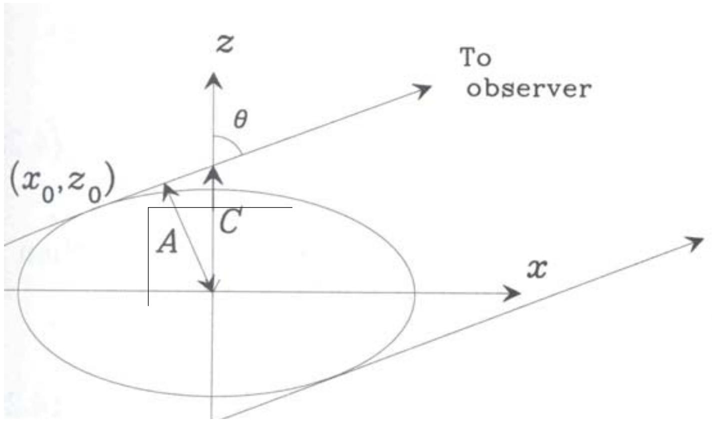
Assume that elliptical galaxies are oblate spheroids: $q = \beta/\alpha$



- An observer looking down the z-axis: E0 galaxy
- When viewed at an angle: system will be elliptical, with axis ratio $q_0 = b/a$.

How is q_0 related to α and β ?

Apparent and intrinsic shapes



The observed axis ratio is $q_0 = A/(m\ \alpha)$.

The line of sight intersects the ellipsoid at (x_0, z_0) at the constant-density surface m.

Galaxy is observed at inclination angle θ

The segment $A = C \sin\theta$, while $C = z_0 + (-x_0)/\tan\theta$; and $\tan\theta = dx/dz$.

Differentiating $d(m^2 = x^2/\alpha^2 + z^2/\beta^2) = 0$, we find $\tan\theta = -z_0/x_0\ \alpha^2/\beta^2$.

Replacing, $C = m^2\beta^2 / z_0$. Finally $q_0 = m\,\beta^2 / (\alpha\, z_0) \sin\theta$, or

$$q_0^2 = \cos^2\theta + (\beta/\alpha)^2 \sin^2\theta = \cos^2\theta + q^2 \sin^2\theta$$

The apparent axis ratio q_0 is always larger than the true axis ratio q: a galaxy never appears more flattened than it is.

Ellipticity distribution

Find the distribution of apparent ellipticities $f(q_0)$ produced by a distribution of oblate (or prolate) ellipsoids with axis ratios $q = \beta/\alpha$: $N(q)$

If the ellipsoids are randomly oriented wrt line-of-sight with angle θ, then the number of galaxies with true axis ratio in the interval $(q, q + dq)$, inclined an angle θ is $N(q)\, dq \sin\theta\, d\theta$

If there are $f(q_0)dq_0$ galaxies with (observed) axis ratios in $(q_0, q_0 + dq_0)$

$$f(q_0)dq_0 = \int dq\, N(q) \sin\theta\, d\theta$$

But $q_0 = q_0\,(q, \sin\theta)$,

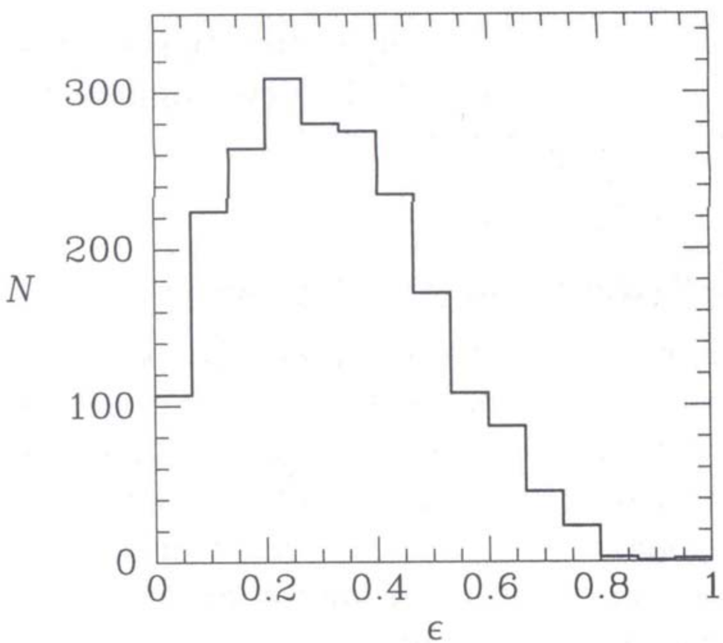
we can express $\sin\theta\, d\theta = q_0\, dq_0 / [(q_0^2 - q^2)^{1/2} (1 - q^2)^{1/2}]$

Distribution of ellipticities

Then

$$f(q_0) = q_0 \int_0^{q_0} \frac{N(q)\,dq}{\sqrt{q_0^2 - q^2}\sqrt{1-q^2}}$$

This is an integral equation for N(q), which can in principle be solved from the observed distribution of ellipticities.



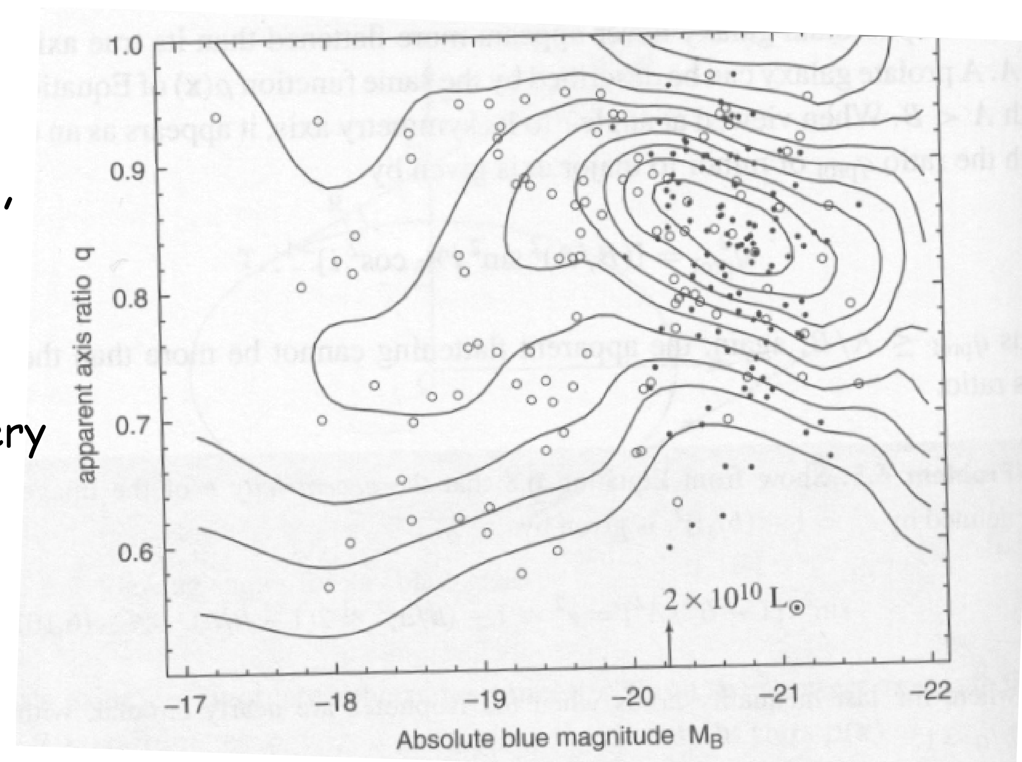
Observed distribution of apparent ellipticities ($\varepsilon = 1 - q_0$)

The sharp fall-off at $\varepsilon = 0$ ($q = 1$) is inconsistent with a distribution in which all galaxies are axisymmetric ellipsoids

Some amount of triaxiality is required…

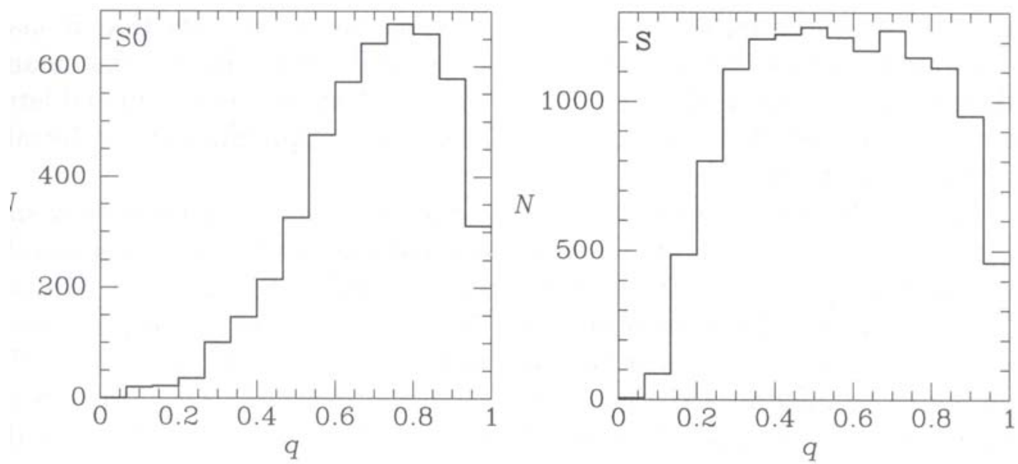
More on shapes

- The apparent shapes of small E are more elongated than large E
- On average, midsized ellipticals (M > -20), have q_0 ~ 0.75. If they are oblate, this would correspond to 0.55 < q < 0.7
- Very luminous E, with M < -20, have on average q_0 ~ 0.85.
- Very few are spherical on the sky -> very likely that most are actually triaxial.



Shapes of disk galaxies

Distribution of apparent axis ratios of a sample of ~5000 S0 galaxies (left) and ~13000 spiral galaxies (right)



•S0 galaxies the distribution of q_0 rises and has a sharp peak at $q_0 \sim 0.7$

•The distribution of spirals rises fast, but remains more or less constant above $q_0 \sim 0.3$.

Shapes of disk galaxies

Assume that spiral galaxies are axisymmetric oblate bodies. We can then use the equation derived:

$$\mathrm{F}(q_0) = q_0 \int_0^{q_0} \frac{N(q)dq}{\sqrt{q_0^2 - q^2}\sqrt{1-q^2}}$$

Note that if N(q) peaks at some value $q_a \ll 1$, then the distribution of apparent ellipticities q_0 is approximately independent of q_0 for $q_0 \gg q$

e.g. assume $N(q) = \delta(q - q_a)$.

Such a distribution fits well the observations.

This provides quantitative support to the impression that spiral galaxies are intrinsically quite thin.

The kinematics of stars in E galaxies

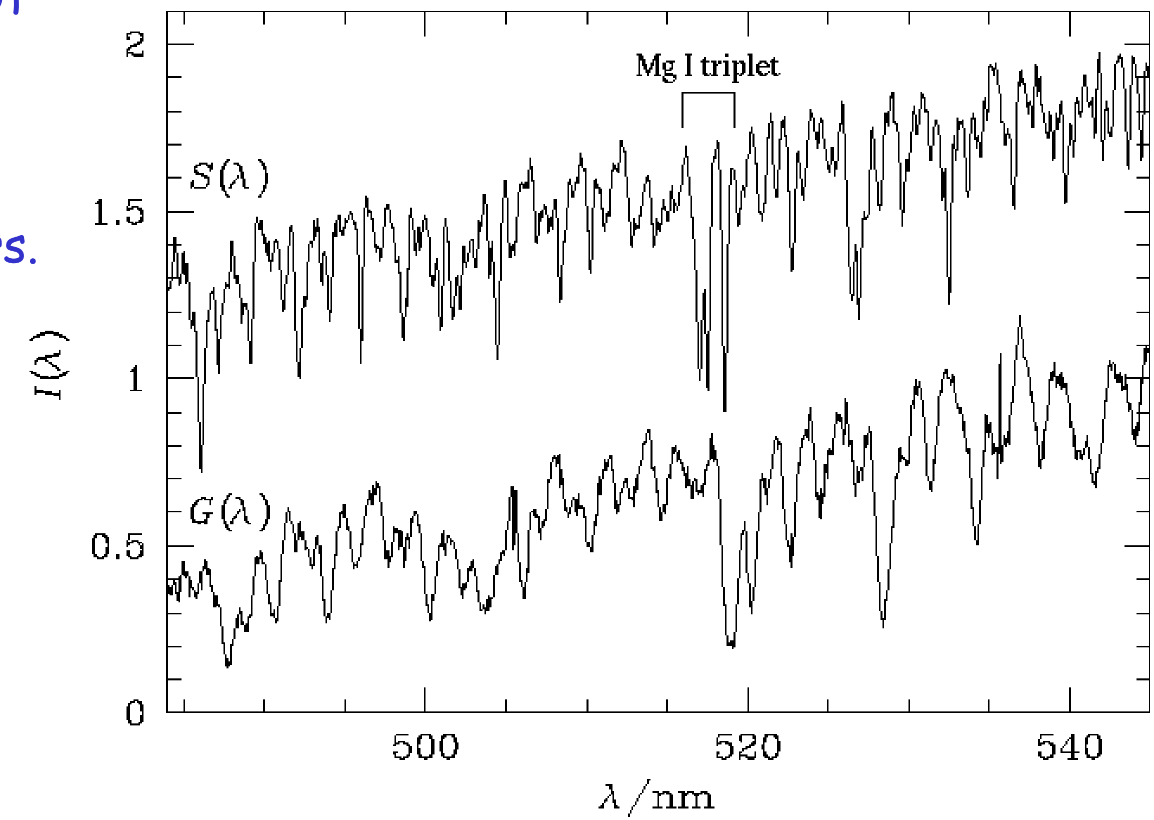
- Stars in E galaxies do not follow ordered motions
- Most of the kinetic energy is in random motions

- To measure the velocities of stars

- Use absorption lines: the result of the light from all stars.

- Each star emits a spectrum which is Doppler shifted in wavelength according to its motion.

- The resulting lines are wider.



Spectra and motions of stars in E

- Let the flux from a star be $F(\lambda)\, d\lambda$.
  - If the star moves away from us with a velocity $v_z \ll c$, the light we receive at λ, was emitted at $\lambda\,(1 - v_z/c)$.

- The total galaxy's flux on an image: $F_{gal}(x,y,\lambda)$
  - integral over all stars along the line of sight (z-direction).
  - The number density of stars at (x,y,z) with velocities in $(v_z, v_z + dv_z)$ is $f(\mathbf{r}, v_z)dv_z$

$$F_{gal}(x,y,\lambda) = \int_{-\infty}^{\infty} d\mathrm{v}_z\ \mathrm{F}(\lambda[1-\mathrm{v}_z/c]) \int_{-\infty}^{\infty} dz\ \mathrm{f}(\mathrm{r},\mathrm{v}_z)$$

- If the distribution function $f(\mathbf{r}, \mathbf{v})$ is known, and all stars were the same, we could derive the spectrum of the galaxy.

- In practice, one makes a guess for the $f(\mathbf{r}, v_z)$, which depends on a few parameters, and fixes those in order to reproduce the observed spectrum.

Spectra and motions of stars in E

A common choice is the *Gaussian*:

$$\int_{-\infty}^{\infty} f(r, v_z)\, dz \propto \exp[-(v_z - V_r)^2 / 2\sigma^2]$$

where $\sigma(x,y)$ is the velocity dispersion and $V_r(x,y)$ is the mean radial velocity

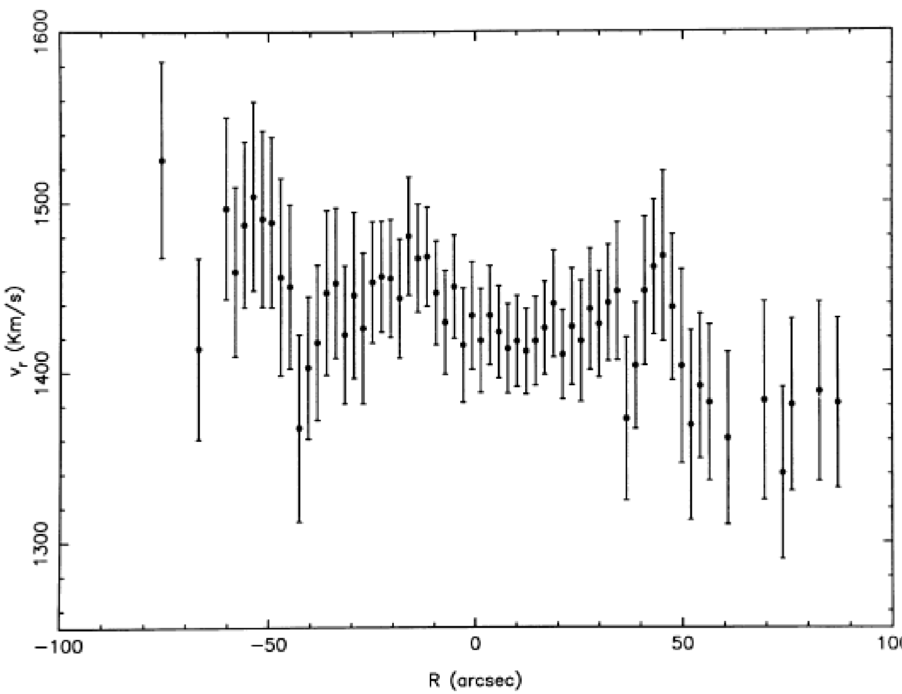


FIG. 2.—The rotation curve of NGC 1399

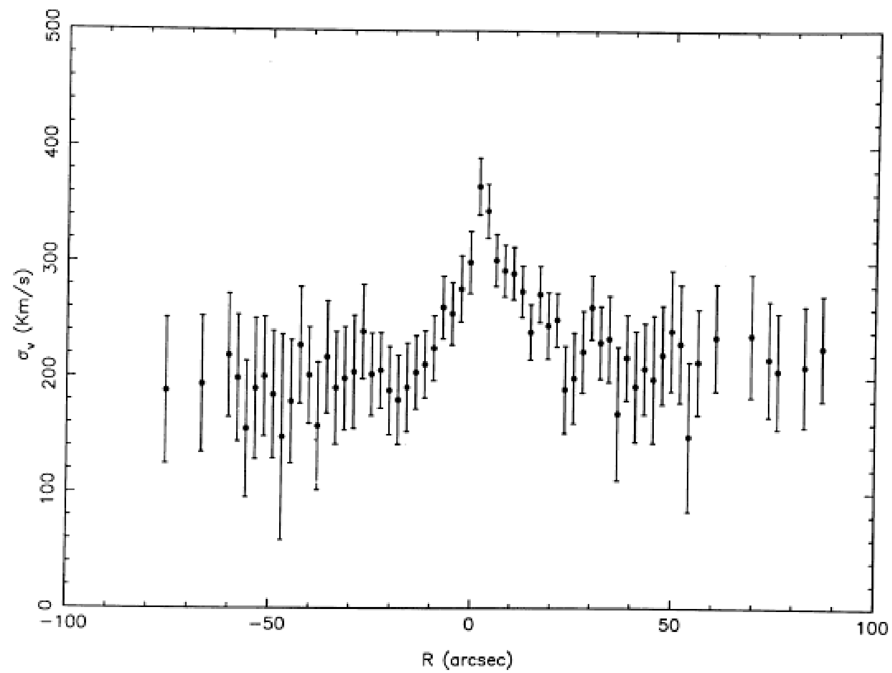
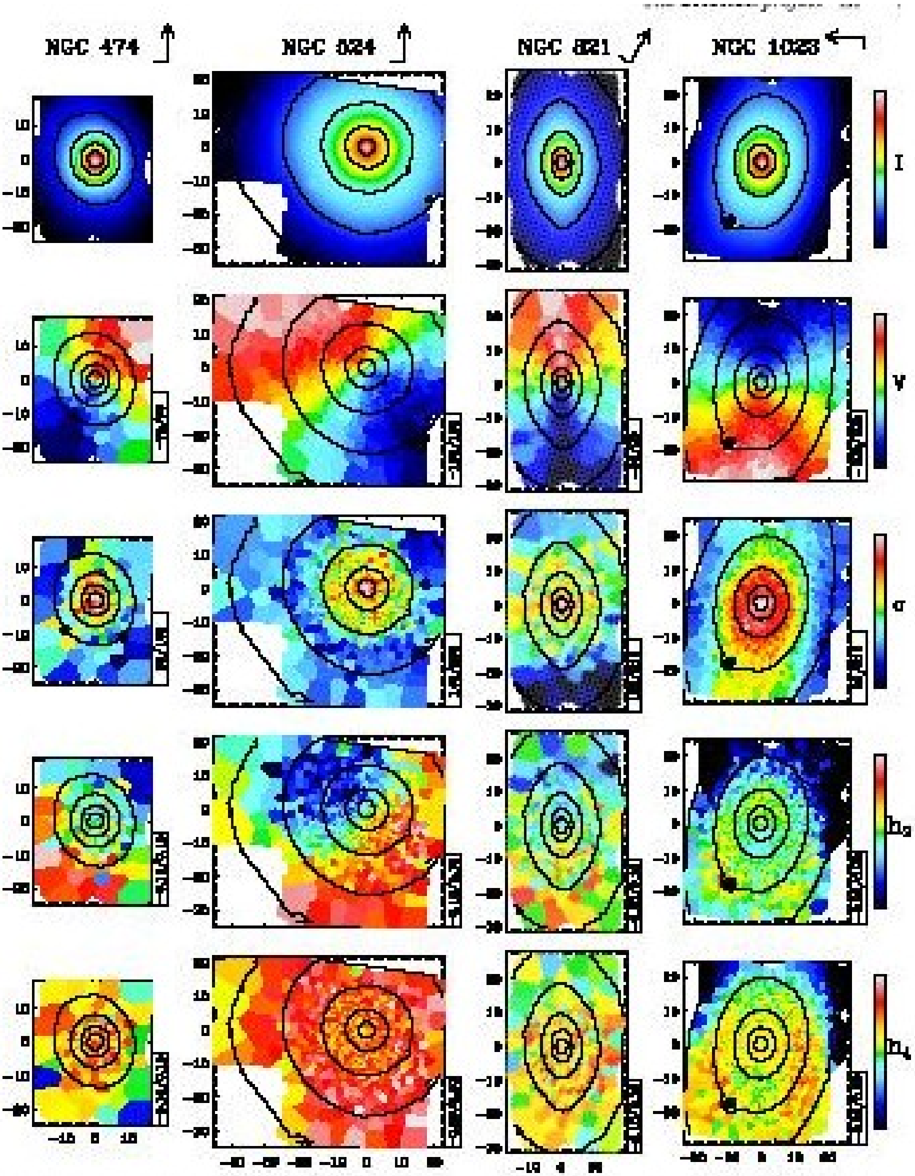


FIG. 3.—The velocity dispersion profile of NGC 1399. East is to the left, and west is to the right.

2D-maps of the kinematics



Integral field spectroscopy: each pixel in image has a corresponding spectrum.

This is an example from the SAURON team for 4 elliptical galaxies.

-variations of the velocity field
-higher moments across the galaxy
-departures from axial symmetry

E galaxies are far less regular than originally believed:

- counter-rotating cores,
- misalignments,
- minor axis rotation…

Do Elliptical galaxies rotate?

- Bright ellipticals (boxy)
  - Rotate very slowly (in comparison to their random motions: $v/\sigma \ll 1$)
  - Their velocity ellipsoids must be anisotropic (to support their shapes)

- Midsized ellipticals (disky)
  - fast rotators

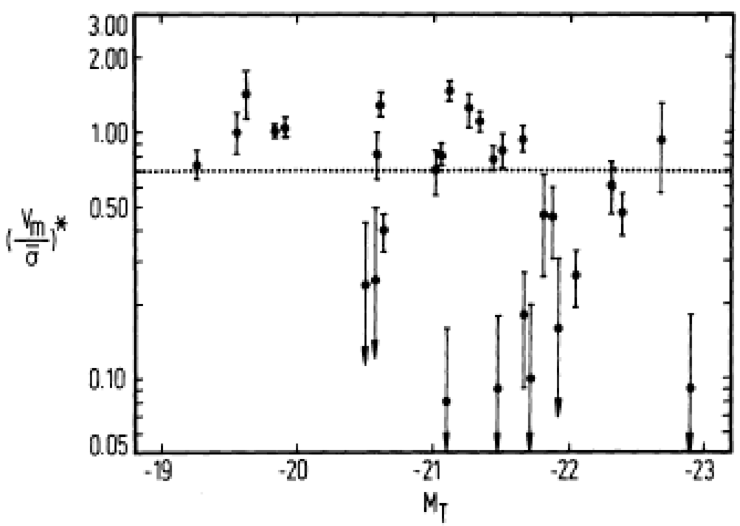


Fig.2: The anisotropies $(v_m/\bar{\sigma})^*$ against the absolute magitudes M_T of elliptical galaxies. Galaxies with $(v_m/\bar{\sigma})^* \lesssim 0.7$ most certainly have anisotropic velocity dispersions.

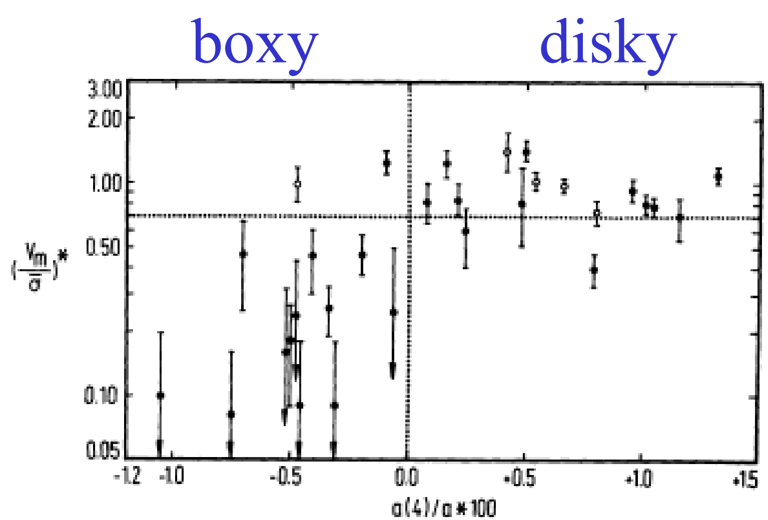


Fig.3: The anisotropies $(v_m/\bar{\sigma})^*$ in elliptical galaxies against their isophote parameters a(4)/a. Positive a(4)/a indicate peaked isophotes most probably caused by weak disk components, negative a(4)/a describe box-shaped isophotes. Weak ellipticals ($M_T > -20.5$) are denoted by open circles, luminous ellipticals ($M_T < -20.5$) by filled circles .

Scaling relations for E galaxies

E galaxies follow the Faber-Jackson relation

$$L_V \approx 2\times10^{10} L_{sun} \left(\frac{\sigma}{200\,\mathrm{km/s}} \right)^4$$

- More luminous galaxies have larger velocity dispersions
  - The range is: 50 km/s for dE up to 500 km/s for brightest E.

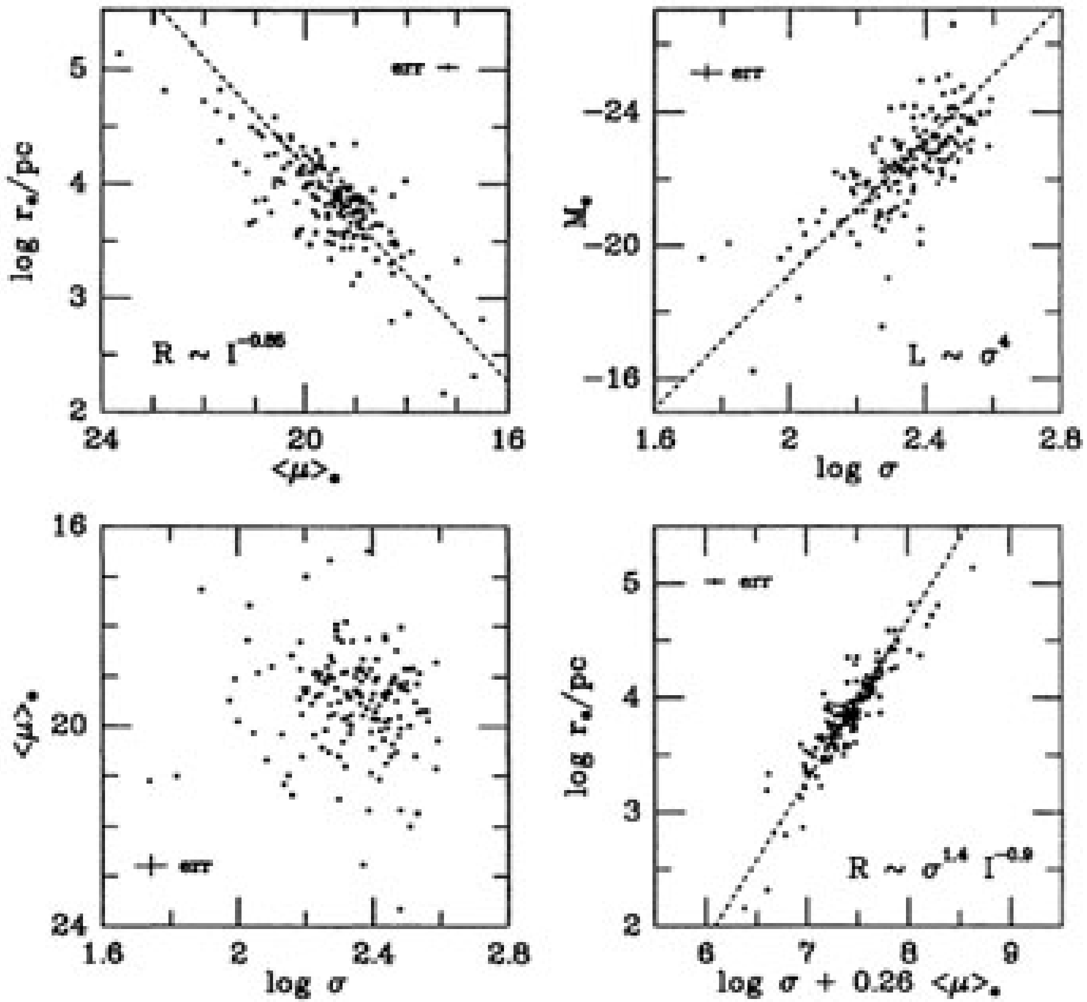
Elliptical galaxies lie close to a plane known as the fundamental plane. It is defined by:

- velocity dispersion σ
- effective radius R_e
- surface brightness I_e

Approximately

$$R_e \propto \sigma^{1.24} I_e^{-0.82}$$

The fundamental plane



Scaling relations for E galaxies

They are all projections of the fundamental plane relation (last panel)

They express some fundamental property of E galaxies.

They can be understood partially from the virial theorem

Kormendy & Djorgovski 1989

Stellar populations in E galaxies

- The stellar populations characteristics of E galaxies are derived from the integrated colours and spectra.
  - It is not possible to see individual stars in galaxies beyond 10-20 Mpc.

- The spectrum of an E galaxy resembles that of a K giant star.

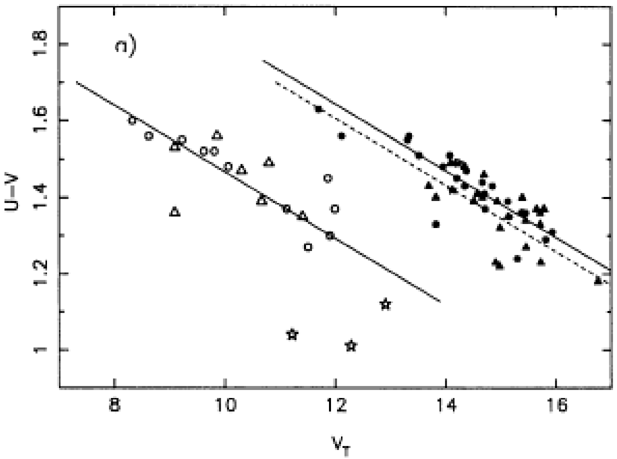
- E galaxies appear generally red:

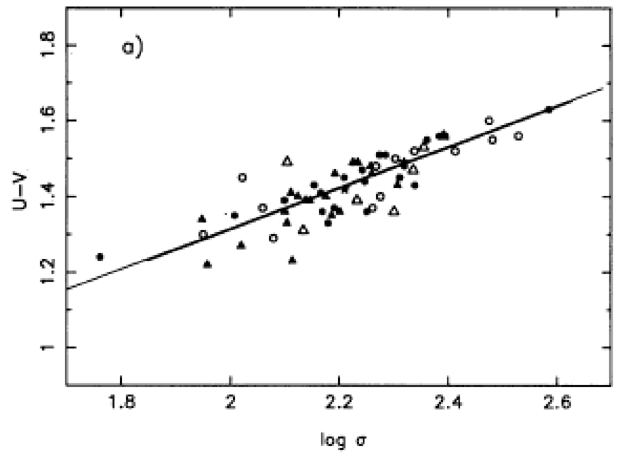
  - very few stars made in the last 1-2 Gyr (recall that after 1 Gyr, only stars with masses $< 2\ M_\odot$ are still on the main sequence)
  - Most of the light is emitted while stars are on the giant branch
  - The stars in the centres of E are very metal-rich, similar to the Sun

- This would seem to suggest that E galaxies are very old

CMDs for E galaxies

There is a relation between the colour, and total luminosity for E galaxies.





Sandage 1972

These plots show, for galaxies in the Coma and Virgo clusters, that

- Brighter galaxies are redder
- Fainter systems are bluer

This could be explained if small E galaxies were younger and/or more metal-poor
But metallicity can mimic age effects …. not possible to conclude what explanation is correct (need resolved deep colour-magnitude diagrams)

- We understand why stars occupy certain regions of the color-magnitude diagram:
    - the luminosity and temperature are controlled by the star's mass
    - the nuclear processes occurring inside the stars.

- Explaining the correlations in the properties of E galaxies is harder:

    - they reflect the conditions under which the galaxies formed, rather than their internal workings at the present day.

    - The well-defined patterns tell us that galaxy formation also had some regularity, and that the process must somehow be related to its mass.

Gas in E galaxies

- E galaxies contain very little cold gas:

  - Roughly 5 - 10 % of normal ellipticals contain HI or molecular gas
  - The big elliptical galaxies have less than 10^8 - 10^9 $M_\odot$ (compare to a large Sc which has 10^{10} $M_\odot$).

  There are some exceptions, but are usually peculiar (i.e. dust lanes, recent merger, etc).

- The average elliptical contains very large amounts of hot ionized gas.

  - emits in X-rays (T ~ 10^7 K)
  - located in a halo of ~ 30 kpc radius
  - roughly 10 - 20% of M_* is in this component

Boxy ellipticals tend to have more luminous X-ray halos and are also radio-loud (i.e. they emit in the radio wavebands).

Dark matter in E

- Cold gas in circular motion:
  - $M/L \sim 10 - 20\ M_\odot/L_\odot$ (for $r < 50$ kpc) >> expected for stellar pop

- Hot gas in hydrostatic equilibrium:
  - $dp/dr = -\rho\, GM(<r)/r^2$ and for perfect gas: $P = \rho\, k\, T/m$
  - $M(<r) = k/m\ r^2/(G\, \rho)\ d(-\rho\ T)/dr$

  (X-ray observations: give temperature and density structure)

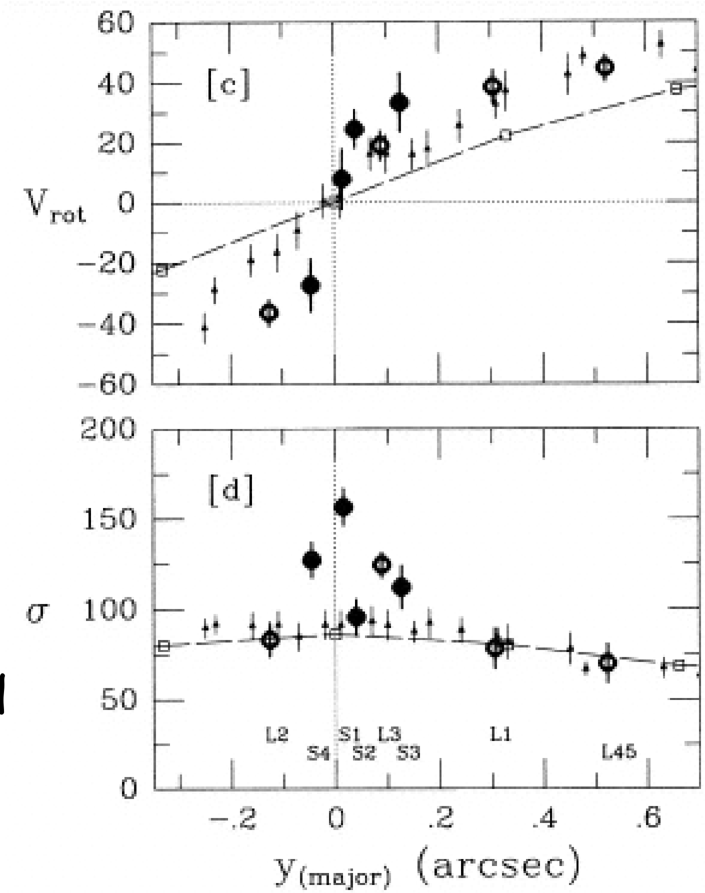
  - $M/L \sim 100\ M_\odot/L_\odot$ (for $r \sim 100$ kpc)

Black holes in the centres of E

- Some E galaxies have rising velocity dispersions closer to the centres.

- To keep such fast moving stars: very high densities are required
  - beyond what can be accounted for by the stars themselves.

- For example, for M32, a dwarf E satellite of M31, 2 million solar masses needed within the central 1 parsec!!



- The preferred explanation: super-massive black holes (SMBH).

- Some believe that every galaxy has a SMBH

The mass of the SMBH correlates with the total luminosity (or velocity dispersion) of the E galaxy.

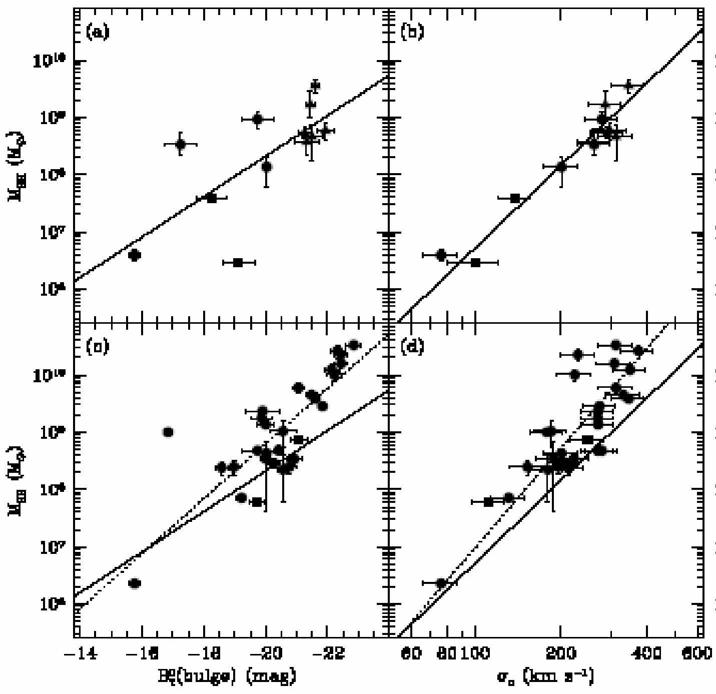


FIG. 1.— (a): BH mass versus absolute blue luminosity of the host elliptical galaxy or bulge for our most reliable Sample A. The solid line is the best linear fit (Table 2). Circles and triangles represent mass measurements from stellar and dust/gas disk kinematics respectively. The squares are the Milky Way ($M_\bullet$ determined from stellar proper motions) and NGC 4258 ($M_\bullet$ based on water maser kinematics), the only two spiral galaxies in the sample. (b) Again for Sample A, BH mass versus the central velocity dispersion of the host elliptical galaxy or bulge, corrected for the effect of varying aperture size as described in §2. Symbols are as in panel (a). (c): Same as panel (a) but for Sample B. Circles are elliptical galaxies, squares are spiral galaxies. The solid line is the same least-squares fit shown in panel (a); the dashed line is the fit to Sample B. All BH mass estimates in this sample are based on stellar kinematics. (d): Same as panel (b) but for Sample B. Symbols are as in panel (c).

$$M_{bh} \propto \sigma^{\alpha}$$

$$\alpha \sim 4.8$$

Magorrian et al 1998
Ferrarese & Merritt 2000
Gebhardt et al 2000