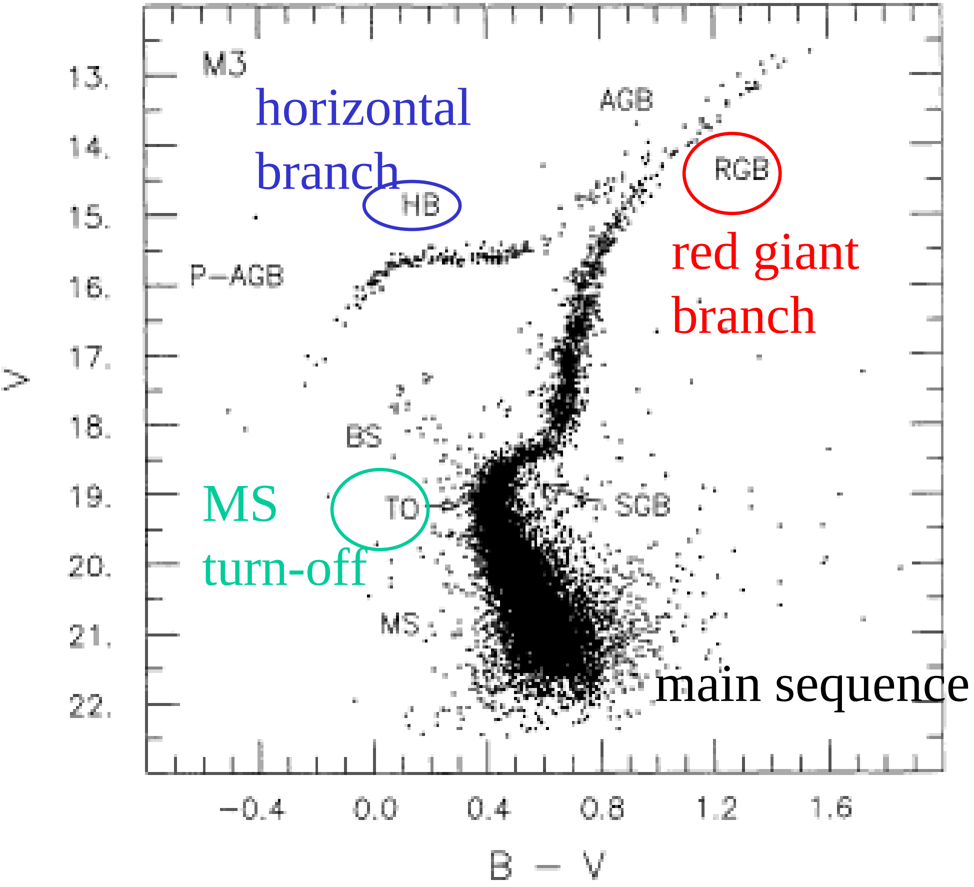
Review of stellar evolution and color-magnitude diagrams

- The evolution of stars can be used to study the properties of galaxies

- Very characteristic features pinpoint at the age (chemistry) of the stars

- Knowledge mostly based on star clusters in the MW



horizontal branch

red giant branch

MS turn-off

main sequence

Important timescales

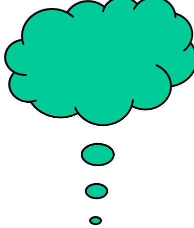
- The **main sequence lifetime** is the amount of time a star is supported by hydrogen to helium thermonuclear conversion.

- **This lifetime scales with mass of a star**

$$\boxed{\tau^{ms} = 10\ A\ (M/M_{\odot})^{1-b}\ \mathrm{Gyr}}$$

where $b > 2$, and A is of order unity

- A solar-type star spends roughly 10 Gyr on the main sequence
- A very massive star quickly drifts away from the main sequence. For a 10 $M_{\odot}$ star, $\tau^{ms} \sim 0.06$ Gyr .



The above expression follows from the following reasoning

- Let E^{ms} be the total energy released by a star while on the main sequence. If L is its luminosity, then $E^{ms} = L \ \tau^{ms}$
  where τ^{ms} is the *main sequence lifetime.*

- The energy produced in the conversion of a mass of hydrogen dM is $dE = 0.0067 \ dM \ c^2$

- Thus, if a mass α of H (of the total mass of the star M) is burned on the main sequence, the energy released is $E^{ms} = 0.0067 \ \alpha \ M \ c^2$

- Therefore the main sequence lifetime is $\tau^{ms} = 0.0067 \ \alpha \ M \ c^2/L$

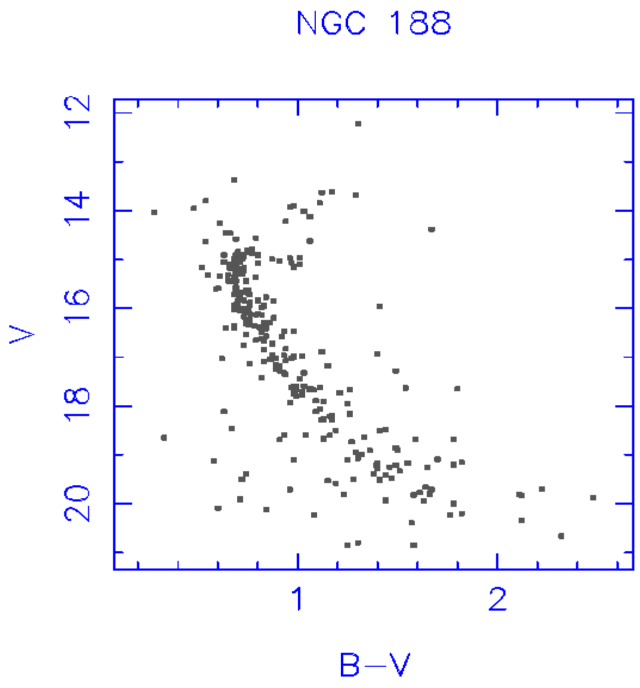
- If roughly 10% of the mass is consumed on the main sequence, then $\alpha = 0.1$. Using the appropriate units, this implies that $\tau_{MS} = 10 \ M/M_\odot \ (L/L_\odot)^{-1} [\mathrm{Gyr}]$

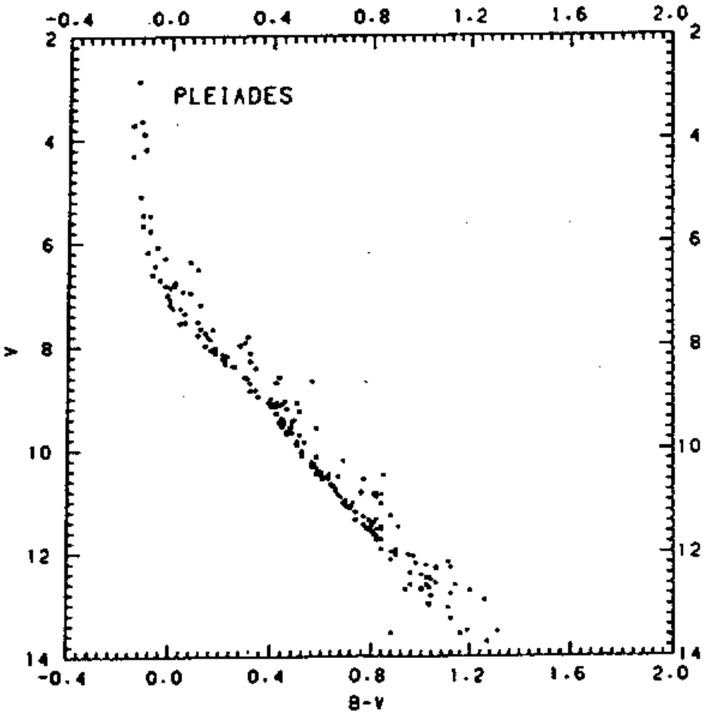
- For stars of solar metallicity, the relation between luminosity and mass is not linear: $L/L_\odot = A \ (M/M_\odot)^b$ where b~4

Open clusters

Clusters are crucial for understanding stellar evolution:

- –all stars formed simultaneously (same age)
- –all have same composition (generally close to solar)
- –all at same distance

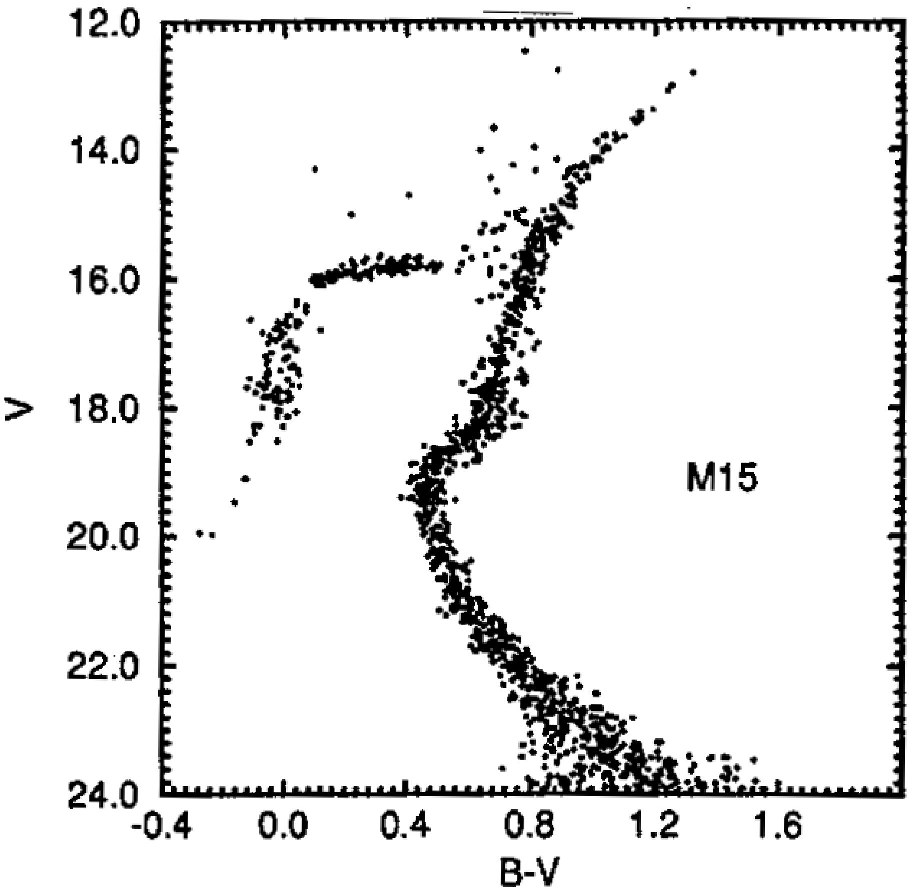


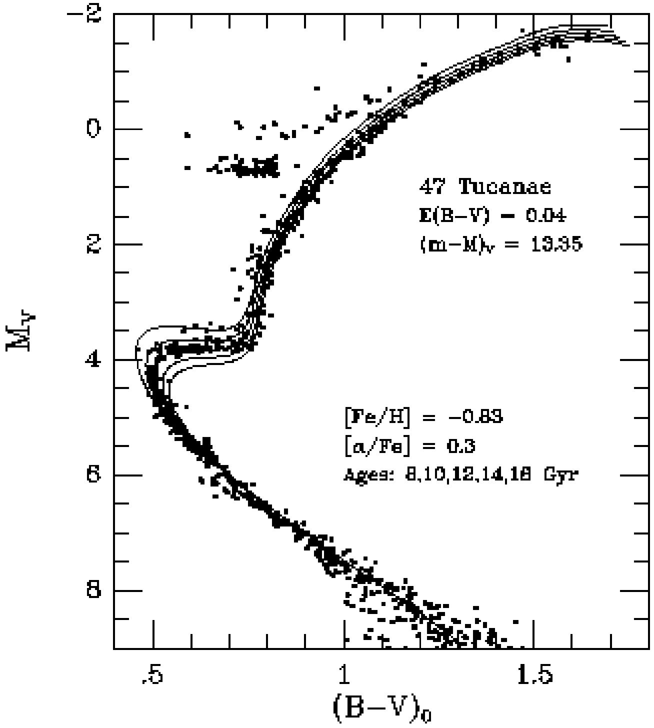


Globular clusters

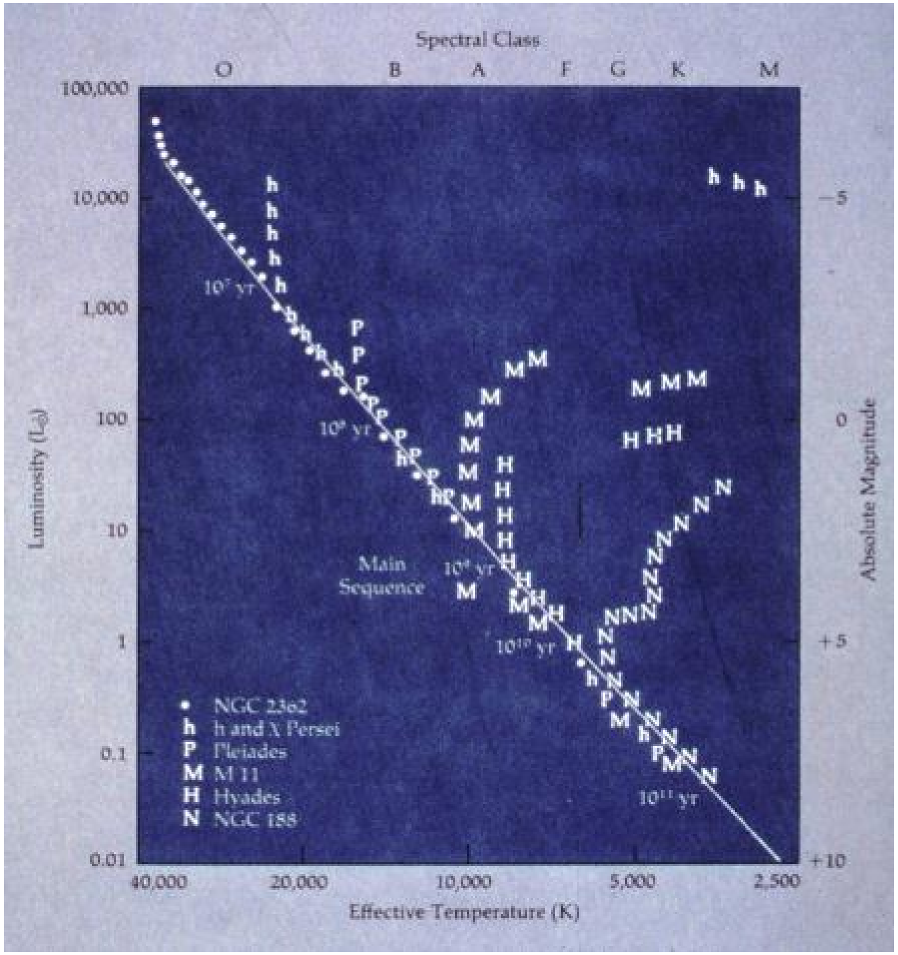
These also important for stellar evolution.

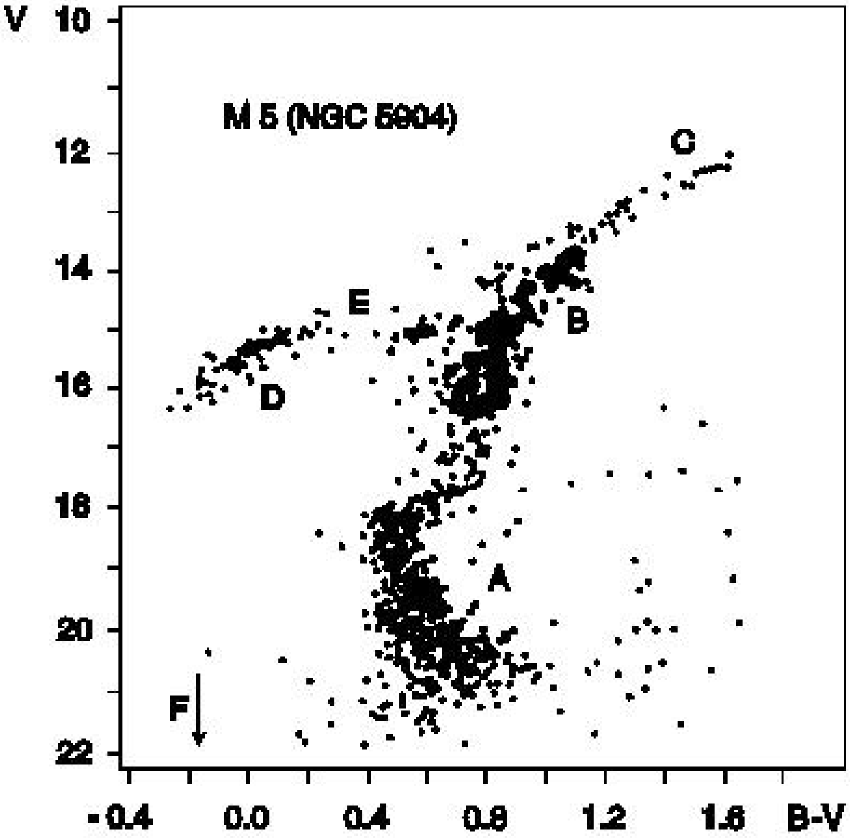
Groups of stars of the same age, but typically older and more metal-poor



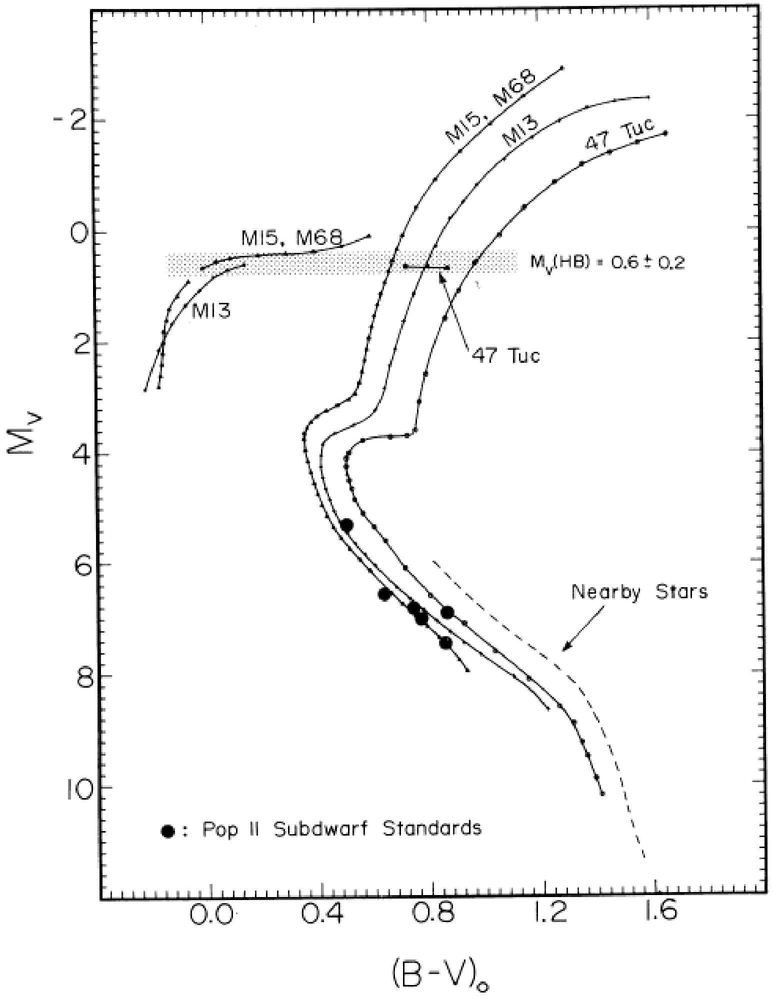


Which is a globular and which an open cluster? How do you know?





Examples of globular clusters



Similar sequences

The exact location of the features in the HR diagram depends

- age
- metallicity,
- a quantity related to He or other element abundances.

Distances to globular clusters

- Its stars are (essentially) located all at the same distance.

- The distance can be determined using **main sequence fitting.**
  - Use the MS defined by subdwarf stars* for which the trigonometric parallaxes are known (the absolute magnitudes of these stars are known).
  - Shift this main sequence vertically, until it overlaps with that of the globular cluster.
  - The shift in magnitude the distance D to the cluster, since $m = M - 5 + 5 \log (D)$.

* we shall see later that most globular clusters are metal-poor, and hence the term "subdwarfs" to refer to the MS dwarfs

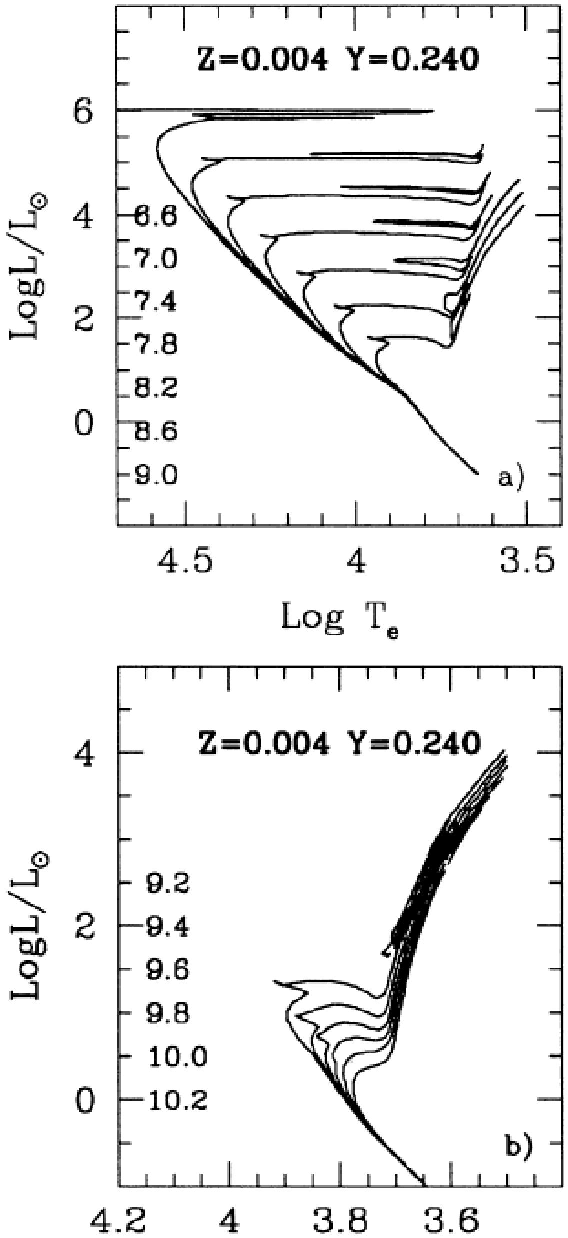
The effect of age in CMDs

- Isochrones:
  - theoretical distribution of group of stars with the same age
  - example: from 4 Myr to 10.6 Gyr

    (the labels give the log(age)).

- "Old" isochrones are much closer together than the "younger" isochrones

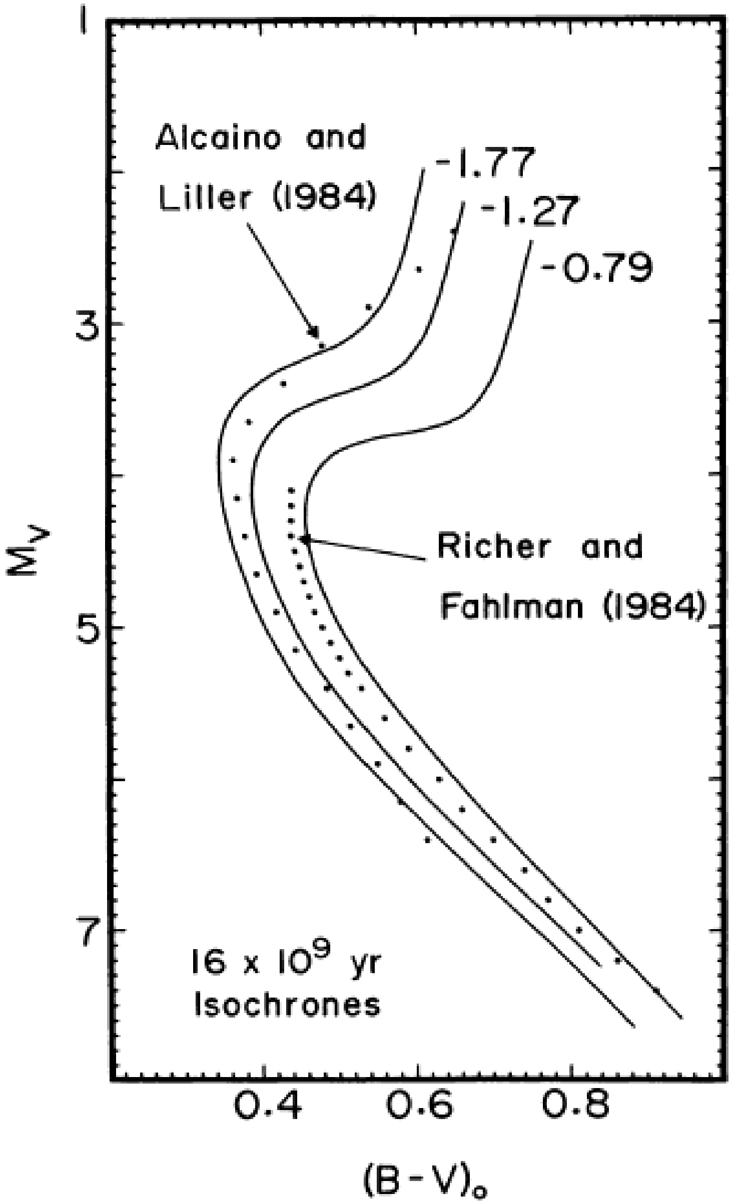
  -> Easy to measure the age of a young population but difficult for an old pop



The effect of metallicity

- Isochrones for stars of the same age but with different metallicities.
- The MS and the GB become brighter and bluer the lower the metallicity.
- The location of certain features allow determination of the metallicity.
  - For example, the absolute magnitude of the horizontal branch

    $M_v(HB) = 0.17\,[Fe/H] + 0.82$



The effect of metallicity in stars

- Stars of lower metallicity are brighter and hotter (at fixed mass):
  - This is due to the lack of heavy elements, which lowers the opacity, making more easy for the photons to escape.
  - The luminosity (and temperature) are higher.

- At fixed luminosity, a change in metallicity of -1.7 dex shifts the ZAMS to the blue by 0.06 Teff, or by $(V\text{-}K) - (V\text{-}K)_0 = -0.32$.

- At fixed color, such a ZAMS is shifted down by approx. 1 magnitude. This is why the low metallicity ZAMS dwarfs are called subdwarfs.

Ages of globular clusters from turn-off location

- Sharp turn-off shows that stars in globular clusters have the same age

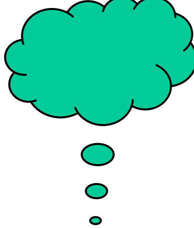
- Location of TO (turn-off) changes as cluster gets older

$\longrightarrow$ the absolute magnitude of the turnoff point $M_v(TO)$ is direct measure of the cluster age.

- From theoretical modelling:

$$M_v(TO) = 2.70 \log(t/Gyr) + 0.30\,[Fe/H] + 1.41$$

  - To derive age t:
    - the apparent magnitude of the main sequence TO
    - the distance to the cluster
    - the metallicity from spectra



Caveats ("uncertainties")

- Because the isochrones become very vertical at the turnoff, it is difficult to accurately measure the location of the TO in the CMD.

- Moreover, errors in the distance determination will also affect the age determination:

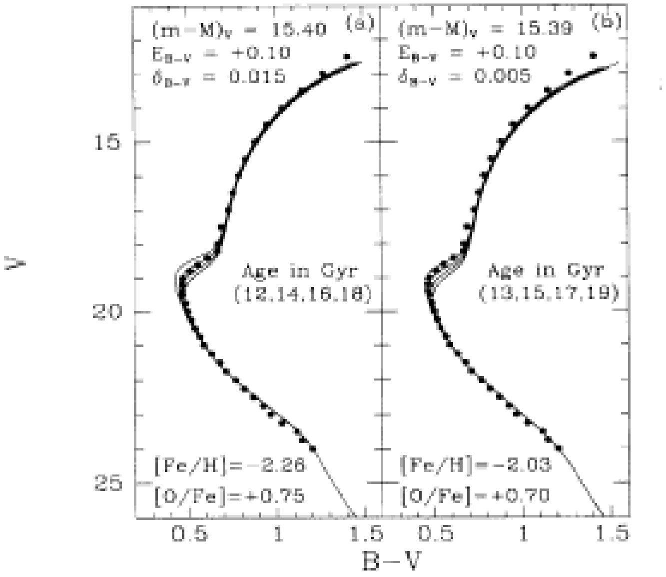
  A 10% distance error implies a 0.2 mag error in the distance modulus (m - M) which translates into a 0.2 mag error in the absolute magnitude of the TO.

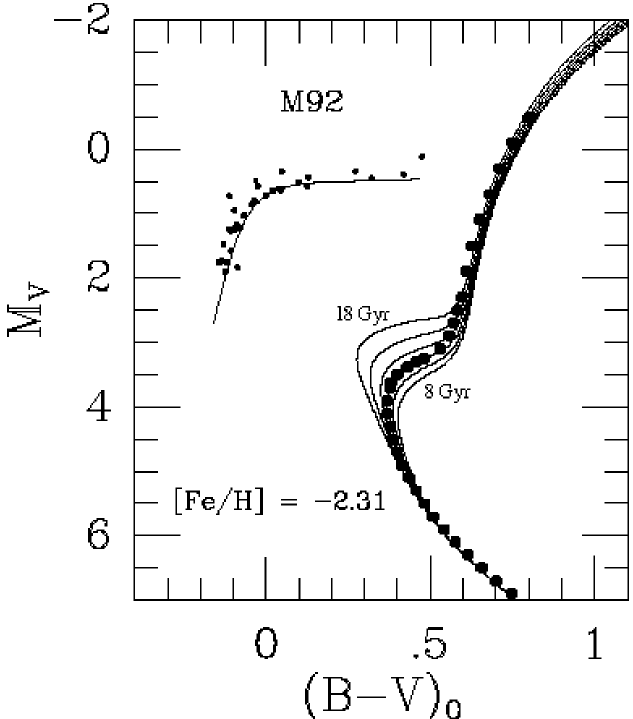
- This in turn produces a 20% error in the age (demonstrate this using the above Eq.!)

Ages from isochrone fitting

- Use **all features** in the CMD to find the best model that fits the data

- Distance is a free parameter
- Sometimes the metallicity and other element abundances are varied to obtain a good fit





Ages from ΔV method

- Age estimator that depends on two features of the CMD diagram to avoid uncertainties introduced by unknown distance.

- The quantity

$$\boxed{\Delta V = M_V(\mathrm{TO}) - M_V(\mathrm{HB})}$$

  measures the difference in magnitude between the turnoff point and the location of the horizontal branch.

- Good estimator:
  - It is independent of distance.
  - The brightness of the horizontal branch does not depend strongly on age
    - Duration of the HB phase is very short, of the order of 0.1 Gyr. So stars ought to have similar ages to be located on the HB at the same moment in time.

- The brightness of the TO depends strongly on age.

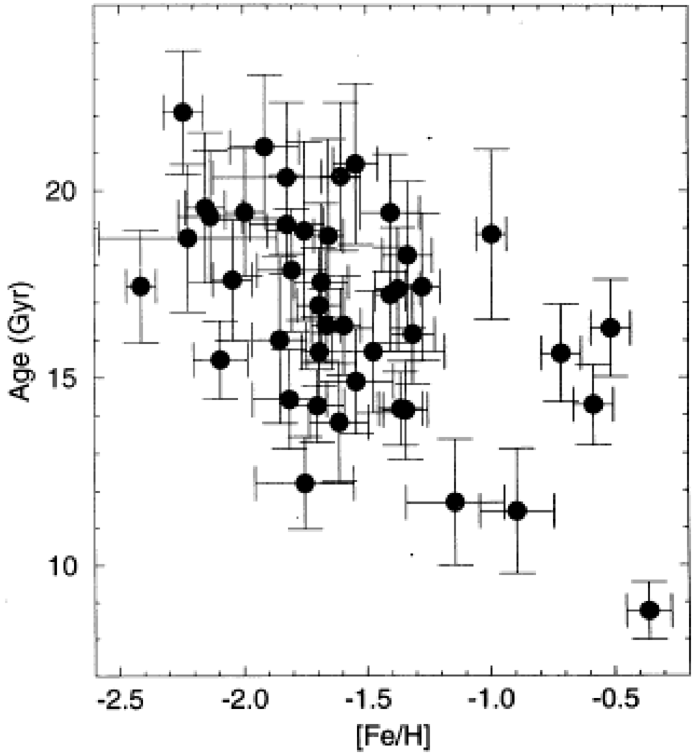
- Using previous equations, it is easy to show that

$$\boxed{\Delta V = 2.7 \log(t/\mathrm{Gyr}) + 0.13\,[\mathrm{Fe/H}] + 0.59}$$

Results

- Globular clusters are generally very old.

- Epoch of globular cluster formation finished a long time ago for our Galaxy.



- More robust to measure relative ages than absolute ages (because of modelling uncertainties).

- Therefore, better to say that there is a trend for younger clusters to be more metal-rich.

The initial mass function

- The initial mass function $\xi(M)$ specifies the distribution in mass in a freshly formed group of stars. The number of stars in $(M, M+dM)$ is

$$\boxed{dN = \xi(M)\, dM}$$

- Knowledge of the IMF is important for galaxy evolution
  - The luminosity, color evolution of a population depends on number of stars of a given type
- Not known if the IMF is universal
  - everywhere the same (independent of environment)
  - always the same (was it different in the past?)

- Measuring the IMF is difficult
  - requires an unevolved stellar population (open clusters) or corrections for stellar evolution for older populations …
  - knowledge of the mass of each star (binaries)

The luminosity function

- The luminosity function $\Phi(M_\lambda)$ measures the number of stars in a given absolute magnitude range $(M_\lambda, M_\lambda+dM_\lambda)$.
  - Sometimes the luminosity function is given per unit volume (i.e. number of stars per absolute magnitude interval and per pc^3)

- The luminosity function depends on time:
  - stellar evolution

    The initial LF (when all stars are on the MS) is denoted as $\Phi_0(M_\lambda)$.

    If none of the stars has evolved away from the MS, then $\Phi(M_\lambda) = \Phi_0(M_\lambda)$

  - ongoing star formation

    The number of stars with a given luminosity changes because new stars are formed.

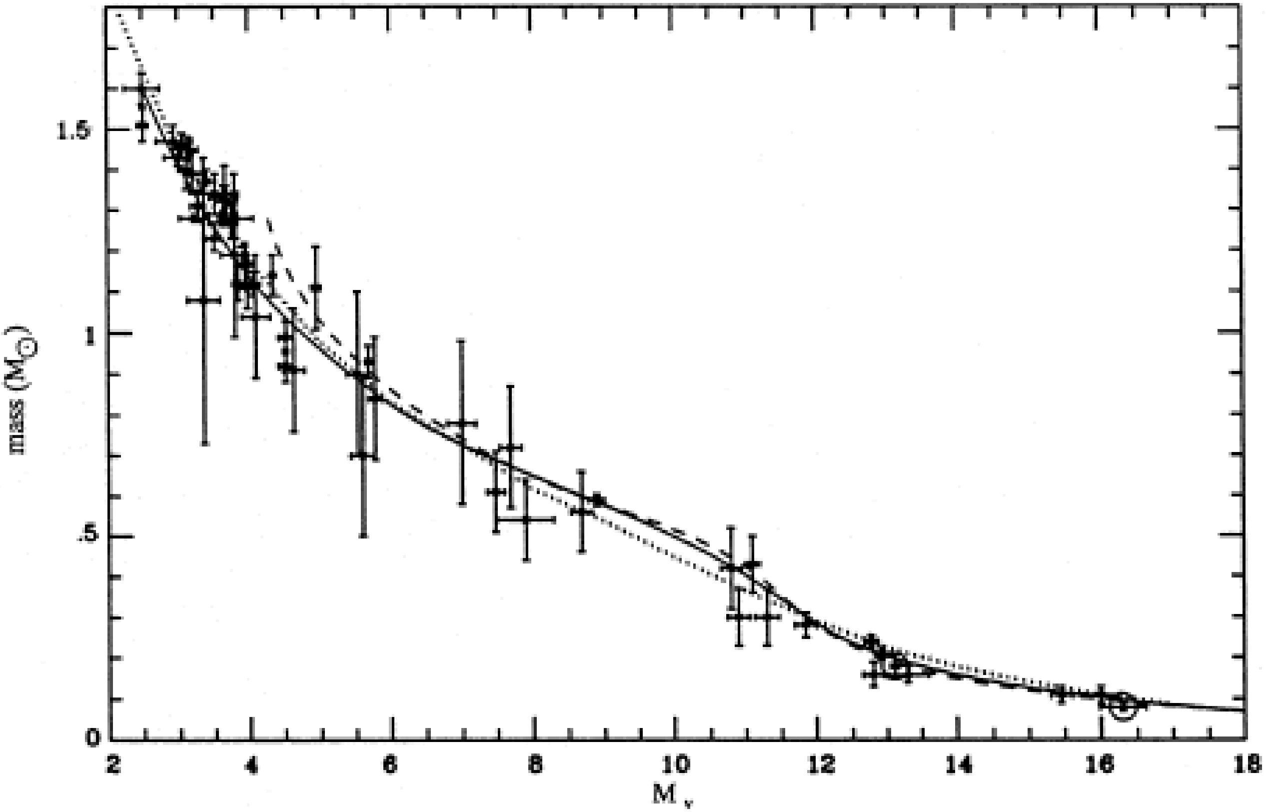
The IMF and the LF

The IMF and the luminosity function are related for a population of stars born at the same time:

$$\boxed{\xi(M)\, dM = \Phi_0(M_\lambda)\, dM_\lambda}$$

Mass and luminosity are not independent.

Their relation is often determined empirically using main sequence binary stars.



Examples

- The simplest case is the Salpeter mass function. It is a power-law function of the mass:

  $\xi(M) \propto M^{-2.35}$

  - It diverges at the low-mass end; need to introduce a lower mass cut-off.

- The Scalo initial mass function is a broken power-law

  $\xi(M) \propto M^{-2.45}$, for $M > 10\ M_\odot$
  $\xi(M) \propto M^{-3.27}$, for $M_\odot < M < 10\ M_\odot$
  $\xi(M) \propto M^{-1.83}$, for $M < 1\ M_\odot$

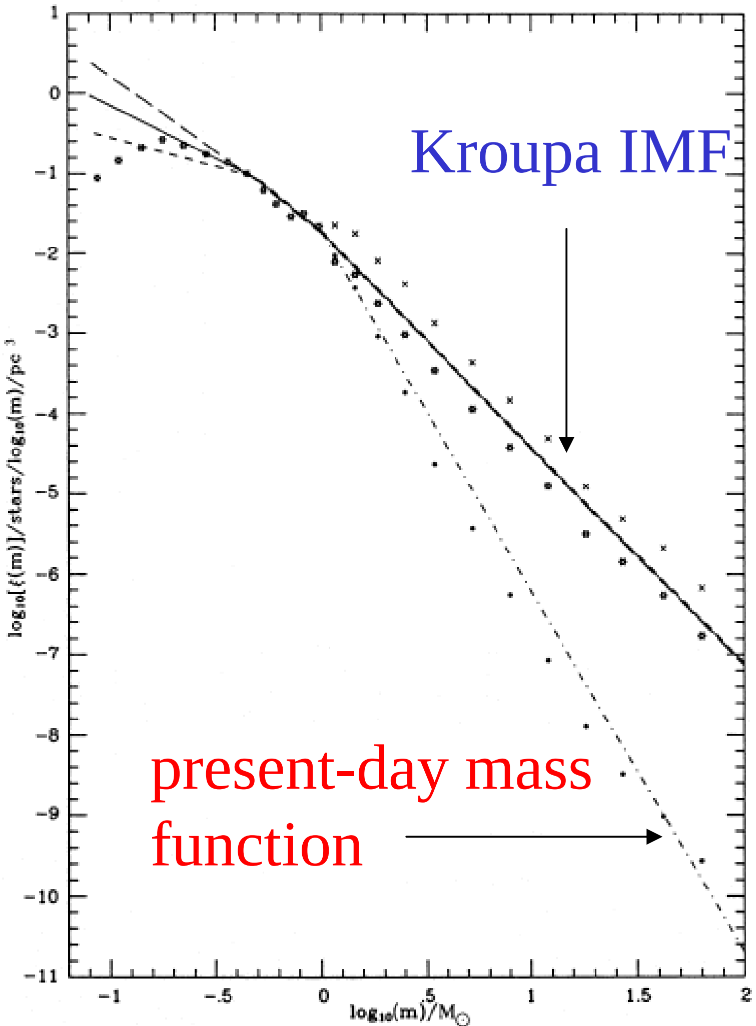
- The Kroupa initial mass function, derived for the solar neighbourhood

$$\xi(M) \propto M^{-4.5}, \text{ for } M > 10\,M_\odot$$

$$\xi(M) \propto M^{-2.2}, \text{ for } 0.5\,M_\odot < M < 10\,M_\odot$$

$$\xi(M) \propto M^{-1.2}, \text{ for } M < 0.5\,M_\odot$$

The solid line represents the Kroupa initial mass function, while the dot-dashed curve is the present-day mass function. The two coincide for masses below ~ 1 $M_\odot$



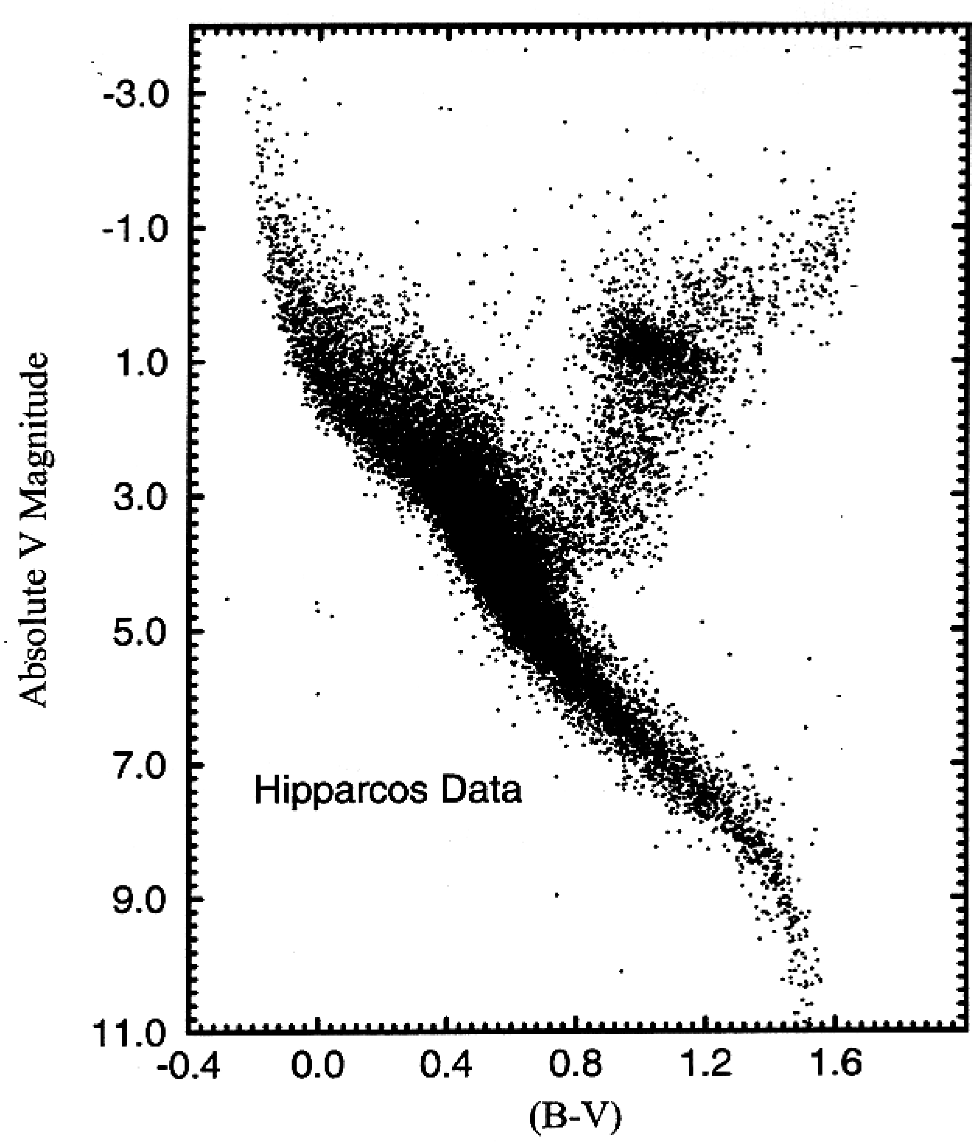
Kroupa IMF

present-day mass function

Resolved CMDs in galaxies

Nearby disk stars: wide range of ages (how do we know?)

Due to continuous (and ongoing) star formation



The CMD of the nearby Carina (dwarf spheroidal galaxy)

It results from "the superposition of the tracks followed by several globular clusters of different ages"

Several isochrones are needed to explain the stellar content of this galaxy

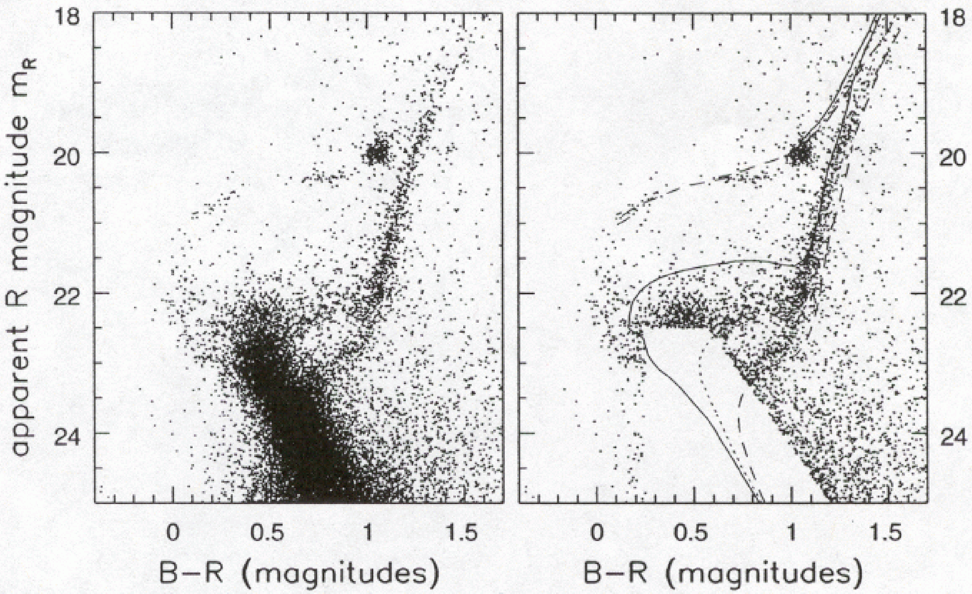


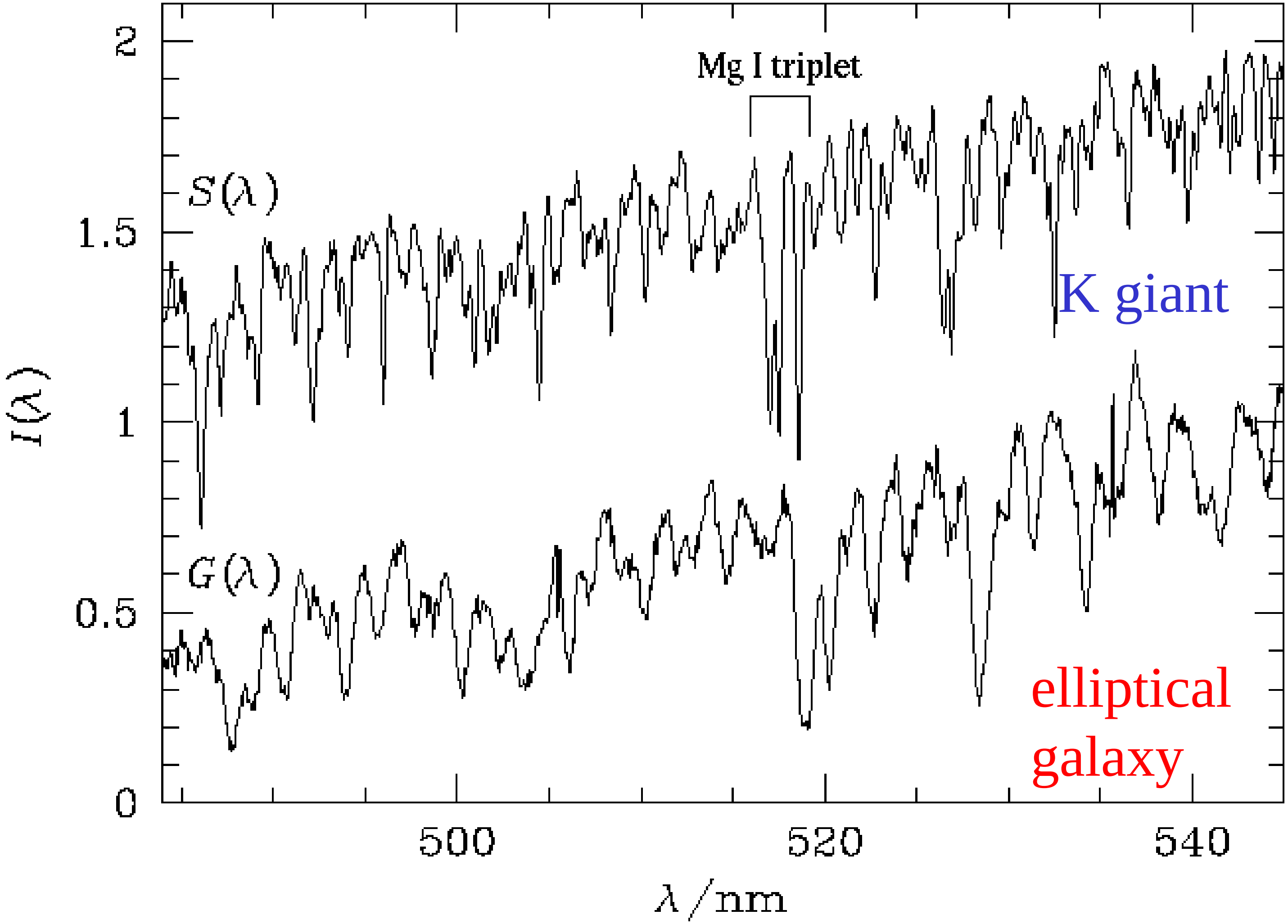
Figure 4.9 Left, color-magnitude diagram for the Carina dwarf spheroidal galaxy. Right, superposed isochrones give the locus of metal-poor stars ($Z = Z_\odot/50$) at ages of 3 Gyr (solid), 7 Gyr (dotted), and 15 Gyr (dashed); we see young red clump stars close to $B - R$, $m_R = (1, 20)$, and old stars on the horizontal branch. Carina's distance modulus is taken as $(m - M)_0 = 20.09$; dust reddening is assumed to dim stars by 0.108 magnitudes in B and 0.067 magnitudes in R – T. Smecker-Hane; A. Cole, Padova stellar tracks.

Stars and galaxies

- A galaxy's luminosity, colors, and spectra are the result of the superposition of the light emitted by its stars.

- The integrated spectral energy distribution* (total energy output) of a galaxy gives us an indication of the stellar types that are present
  - a galaxy with many young stars, will be blue
  - an elliptical galaxy is generally red, has the color of a K giant

Comparison of the optical spectrum



- Some differences: spectrum is shifted and lines are broader for the E galaxy. Why?
- Chemical elements observed reflect the chemical composition of the stars

Formally $L \sim \Sigma_i^{N_{star}} L_{i,star}\, w_{i,star}$

The luminosity is the superposition of the luminosities emitted by all stars in a galaxy, where the sum is over types of stars.

The weights are given by the number of stars of a given age and type (i.e. luminosity function).

If all the stars had the same age, and there were equal numbers of stars of each luminosity, then the weight would be the same for all: $L \sim \Sigma_i^{N_{star}} L_{i,star}$

To understand the fluxes, colors and spectra of galaxies:

- sum up library of stellar spectral energy distributions
- know the mass or luminosity function of stars

This is known as Synthesis of Stellar Populations

Stellar populations

- A stellar population is a group of stars with similar properties
    - abundance patterns (although not necessarily same metallicity)
    - kinematics

- Single stellar population (SSP): all stars have the same age and metallicity.
    - The simplest example is a globular cluster.

- Population I and population II are frequently used
    - Pop. I: the stars in disk (and more generally the recently formed component of star-forming galaxies)
    - Pop.II: stars in the halo; it is the metal-poor component of galaxies
    - Pop.III: stars of zero-metallicity (the first stars)

Stellar populations synthesis studies

- The goal is to relate the observed galaxy's colors and spectral features to the underlying stellar composition

- Find answers to:
  - What was the star formation history of the system?
  - When did most of the stars form?
  - When was the last burst of star formation?
  - Could a significant amount of mass be in non-stellar form?
  - Dark-matter content

Stellar populations: analytic approach

(Based on Tinsley [ApJ 173, L93 (1972) and ApJ 186, 35 (1973)])

- For a single stellar population, would like to know:
  - total luminosity evolution
  - color evolution
  - which stars dominate

- Luminosity evolution: $L = \Sigma L_i = L_{MS} + L_{GB}$

1. Main sequence:

$$L_{MS} = \int L\, dN = \int L(M)\, \xi(M)\, dM \propto \int M^b M^{-\alpha}\, dM$$

$$\boxed{L_{MS} \propto M_{TO}^{\,b - \alpha + 1}}$$

- Since $b - \alpha + 1 > 0$, the luminosity contribution by the MS is larger when M_{TO} is larger, i.e. for younger ages
- Since the M_{TO} decreases with time, L_{MS} will also decrease in time

2. Giant branch

$L_{GB} = N_{GB} <L>$ where $N_{GB} = \xi(M_{TO})\, \Delta M$

Here $\Delta M \sim dM/dt\; t_{GB} \propto M_{TO}^{\gamma+1}$

Therefore $\boxed{L_{GB} \propto M_{TO}^{\gamma\ -\ \mathrm{alpha}\ +\ 1}}$

The luminosity of the GB is larger for young ages, i.e. it also decreases with time

- The total luminosity of a single stellar population is time-dependent.
- The population becomes fainter with time

- The relative contribution of stars on the GB and on the MS is

  $L_{MS}/L_{GB} \propto M_{TO}^{\,b - \gamma}$

  Since $(b - \gamma) \sim 1$ -->

  For 5 Gyr and beyond ($M_{TO} \sim 1\ M_\odot$) the light is dominated by stars on the GB

- The evolution in color of the population can be computed , e.g. from

  $(U - B) = -2.5 \log_{10} L_U/L_B + (U - B)_\odot$

  The color of a single stellar population does not evolve strongly after roughly 1.5 Gyr.

Stellar populations: numerical results

- Analytic models are a bit limited because of the simplifications made.

- Numerical models to follow evolution of SSP

- These models proceed as follows:

  - set initial abundance of H, He and other elements
  - initial mass function

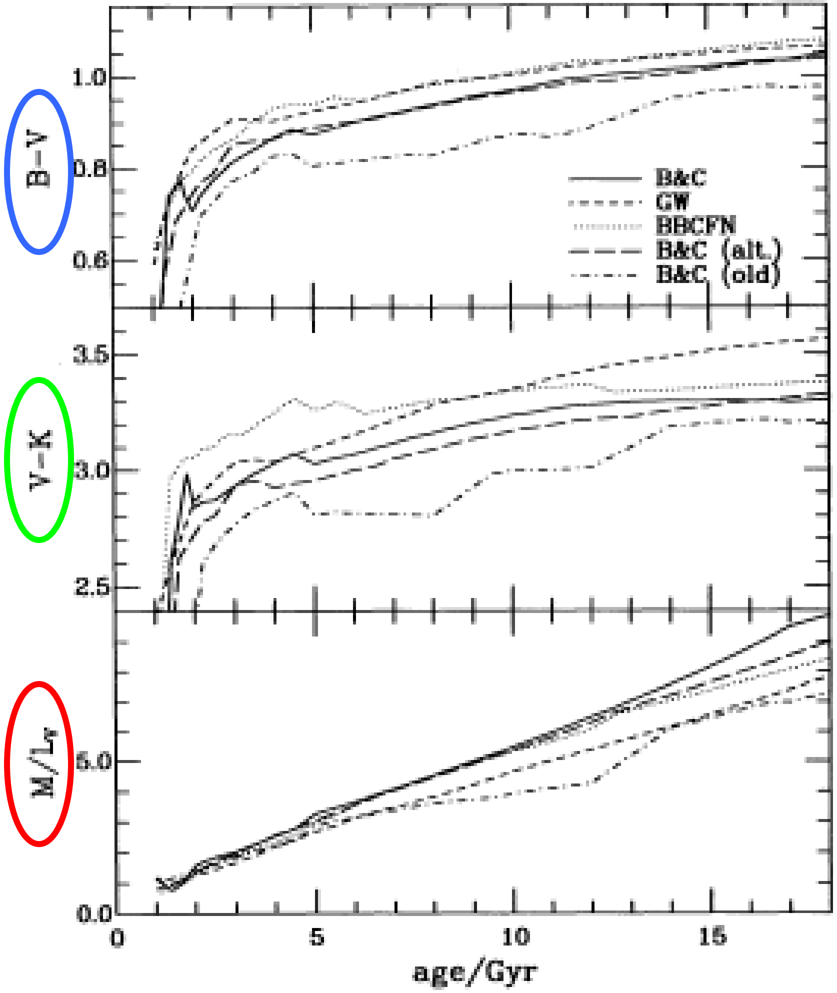
  - evolve the population forward in time: solve the stellar structure equations for each star
  - calculate the luminosities and colors for all the stars in the population

  - compute the total luminosity of the population, as well as the colors in different bands

Results: photometry

Evolution of (B-V), (V-K) and mass-to-light ratio

(solar metallicity and Salpeter IMF)



- Note that the population becomes redder with time, initially very rapidly but rather slowly after roughly 5 Gyr.

- It is very hard to distinguish 11 Gyr from 14 Gyr population, unless the colors have been measured with very high accuracy
  - recall isochrones closely packed

More results: spectra

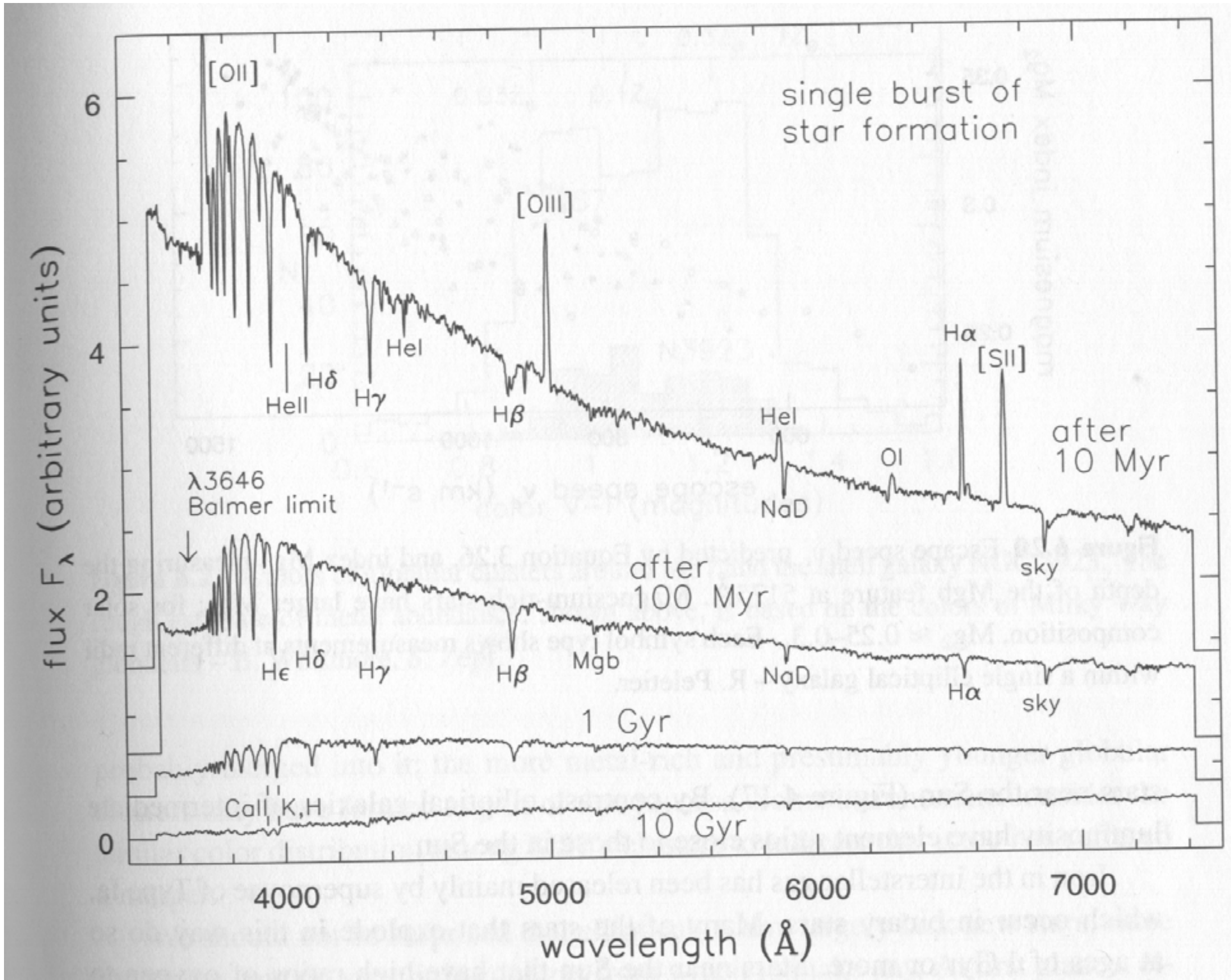


Figure 6.19 Spectra for a 'galaxy' that makes its stars in a 10^8 yr burst, all plotted to the same vertical scale. Emission lines of ionized gas are strong 10 Myr after the burst ends; after 100 Myr, the galaxy has faded and reddened, and deep hydrogen lines of A stars are prominent. Beyond 1 Gyr, the light dims and becomes slightly redder, but changes are much slower – B. Poggianti.