## Review of stellar evolution and color-magnitude diagrams

- The evolution of stars can be used to study the properties of galaxies
- Very characteristic features pinpoint at the age (chemistry) of the stars
- Knowledge mostly based on star clusters in the MW



## Which is a globular and which an open cluster? How do you know?




## Important timescales

- The main sequence lifetime is the amount of time a star is supported by hydrogen to helium thermonuclear conversion.
- This lifetime scales with mass of a star

$$
\tau^{\mathrm{ms}}=10 \mathrm{~A}\left(\mathrm{M} / \mathrm{M}_{\odot}\right)^{1-\mathrm{b}} \mathrm{Gyr}
$$

where $\mathrm{b}>2$, and A is of order unity

- A solar-type star spends roughly 10 Gyr on the main sequence
- A very massive star quickly drifts away from the main sequence. For a $10 \mathrm{M}_{\odot}$ star, $\tau^{\mathrm{ms}} \sim 0.06 \mathrm{Gyr}$.
- Let $E^{m s}$ be the total energy released by a star while on the main sequence. If $L$ is its luminosity, then $\mathrm{E}^{\mathrm{ms}}=\mathrm{L} \tau^{\mathrm{ms}}$ where $\tau^{\mathrm{ms}}$ is the main sequence lifetime.
- The energy produced in the conversion of a mass of hydrogen $d \mathrm{M}$ is $d \mathrm{E}=0.0067$ $d \mathrm{M} \mathrm{c}^{2}$
- Thus, if a mass $\alpha$ of H (of the total mass of the star M ) is burned on the main sequence, the energy released is $\mathrm{E}^{\mathrm{ms}}=0.0067 \propto \mathrm{M} \mathrm{c}^{2}$
- Therefore the main sequence lifetime is $\tau^{\mathrm{ms}}=0.0067 \alpha \mathrm{M} \mathrm{c}^{2} / \mathrm{L}$
- If roughly $10 \%$ of the mass is consumed on the main sequence, then $\alpha=0.1$. Using the appropriate units, this implies that $\tau_{\mathrm{MS}}=10 \mathrm{M} / \mathrm{M}_{\odot}\left(\mathrm{L} / \mathrm{L}_{\odot}\right)^{-1}[\mathrm{Gyr}]$
- For stars of solar metallicity, the relation between luminosity and mass is not linear: $\quad \mathrm{L} / \mathrm{L}_{\odot}=\mathrm{A}\left(\mathrm{M} / \mathrm{M}_{\odot}\right)^{\mathrm{b}} \quad$ where $\mathrm{b} \sim 4$


## Examples of globular clusters



Similar sequences

The exact location of the features in the HR diagram depends

- age
- metallicity,
- a quantity related to He or other element abundances.


## Resolved CMDs in galaxies

Nearby disk stars: wide range of ages (how do we know?)

Due to continuous (and ongoing) star formation


## The CMD of the nearby Carina (dwarf spheroidal galaxy)

## It results from "the superposition of the tracks followed by several globular clusters of different ages"

Several isochrones are needed to explain the stellar content of this galaxy


Figure 4.9 Left, color-magnitude diagram for the Carina dwarf spheroidal galaxy. Right, superposed isochrones give the locus of metal-poor stars $\left(Z=Z_{\odot} / 50\right)$ at ages of 3 Gyr (solid), 7 Gyr (dotted), and 15 Gyr (dashed); we see young red clump stars close to $B-R, \mathrm{~m}_{R}=(1,20)$, and old stars on the horizontal branch. Carina's distance modulus is taken as $(m-M)_{0}=20.09$; dust reddening is assumed to dim stars by 0.108 magnitudes in $B$ and 0.067 magnitudes in $R$ - T. Smecker-Hane; A. Cole, Padova stellar tracks.

## Stars and galaxies

- A galaxy's luminosity, colors, and spectra are the result of the superposition of the light emitted by its stars.
- The integrated spectral energy distribution* (total energy output) of a galaxy gives us an indication of the stellar types that are present
- a galaxy with many young stars, will be blue
- an elliptical galaxy is generally red, has the color of a K giant

Comparison of the optical spectrum


- Some differences: spectrum is shifted and lines are broader for the E galaxy. Why?
- Chemical elements observed reflect the chemical composition of the stars

Formally $\quad \mathrm{L} \sim \Sigma \mathrm{L}_{\mathrm{i}, \mathrm{star}} \mathrm{W}_{\mathrm{i}, \text { star }}$
The luminosity is the superposition of the luminosities emitted by all stars in a galaxy, where the sum is over types of stars.

The weights are given by the number of stars of a given age and type
If all the stars had the same age, and there were equal numbers of stars of each luminosity, then the weight would be the same for all: $\mathrm{L} \sim \sum_{\mathrm{i}} \mathrm{N}_{\text {star }} \mathrm{L}_{\mathrm{i}, \mathrm{star}}$

To understand the fluxes, colors and spectra of galaxies:
-sum up library of stellar spectral energy distributions
-know the luminosity function of stars (how many stars of each luminosity-> initial mass function and star formation history)

This is known as Synthesis of Stellar Populations

## Stellar populations synthesis studies

- The goal is to relate the observed galaxy's colors and spectral features to the underlying stellar composition
- Simplest case:
- Single stellar population (SSP): all stars have the same age and metallicity
- E.g. a globular cluster.
- More generally:
- sum of SSPs coupled to a star formation history
- Find answers to:
- What was the star formation history of the system?
- When did most of the stars form?
- When was the last burst of star formation?
- Could a significant amount of mass be in non-stellar form?
- Dark-matter content


## The initial mass function

- The initial mass function $\xi(\mathrm{M})$ specifies the distribution in mass in a freshly formed group of stars. The number of stars in ( $\mathrm{M}, \mathrm{M}+\mathrm{dM}$ ) is

$$
\mathrm{dN}=\xi(\mathrm{M}) \mathrm{dM}
$$

- Knowledge of the IMF is important for galaxy evolution
- The luminosity, color evolution of a population depends on number of stars of a given type
- Not known if the IMF is universal
- everywhere the same (independent of environment)
- always the same (was it different in the past?)
- Measuring the IMF is difficult
- requires an unevolved stellar population (open clusters) or corrections for stellar evolution for older populations ...
- knowledge of the mass of each star (binaries)


## Examples

- The simplest case is the Salpeter mass function. It is a power-law function of the mass:

$$
\xi(\mathrm{M}) \propto \mathrm{M}^{-2.35}
$$

- It diverges at the low-mass end; need to introduce a lower mass cut-off.
- The Scalo initial mass function is a broken power-law

$$
\begin{aligned}
& \xi(M) \propto M^{-2.45}, \text { for } M>10 M_{\odot} \\
& \xi(M) \propto M^{-3.27}, \text { for } M_{\odot}<M<10 M_{\odot} \\
& \xi(M) \propto M^{-1.83}, \text { for } M<1 M_{\odot}
\end{aligned}
$$

## Stellar populations: analytic approach

(Based on Tinsley [ ApJ 173, L93 (1972) and ApJ 186, 35 (1973)])

- For a single stellar population, would like to know:
- total luminosity evolution
- color evolution
- which stars dominate
- Luminosity evolution: $\mathrm{L}=\Sigma \mathrm{L}_{\mathrm{i}}=\mathrm{L}_{\mathrm{MS}}+\mathrm{L}_{\mathrm{GB}}$

1. Main sequence:
$\mathrm{L}_{\mathrm{MS}}=\int \mathrm{L} \mathrm{dN}=\int \mathrm{L}(\mathrm{M}) \xi(\mathrm{M}) \mathrm{dM} \propto \int \mathrm{M}^{\mathrm{b}} \mathrm{M}^{-\alpha} \mathrm{dM}$

$$
\mathrm{L}_{\mathrm{MS}} \propto \mathrm{M}_{\mathrm{TO}}{ }^{\mathrm{b}-\alpha+1}
$$

- Since $b-\alpha+1>0$, the luminosity contribution by the MS is larger when $M_{T O}$ is larger, i.e. for younger ages
- Since the $\mathrm{M}_{\mathrm{TO}}$ decreases with time, $\mathrm{L}_{\mathrm{MS}}$ will also decrease in time

2. Giant branch

$$
\begin{aligned}
& \mathrm{L}_{\mathrm{GB}}=\mathrm{N}_{\mathrm{GB}}<\mathrm{L}>\quad \text { where } \quad \mathrm{N}_{\mathrm{GB}}=\xi\left(\mathrm{M}_{\mathrm{TO}}\right) \Delta \mathrm{M} \\
& \text { Here } \Delta \mathrm{M} \sim \mathrm{dM} / \mathrm{dt}_{\mathrm{GB}} \propto \mathrm{M}_{\mathrm{TO}}{ }^{\mathrm{b}}
\end{aligned}
$$

Therefore

$$
\mathrm{L}_{\mathrm{GB}} \propto \mathrm{M}_{\mathrm{TO}}{ }^{\mathrm{b}-\alpha}
$$

The luminosity of the GB is larger for young ages, i.e. it also decreases with time

- The total luminosity of a single stellar population is time-dependent.
- The population becomes fainter with time
- The relative contribution of stars on the GB and on the MS is
$\mathrm{L}_{\mathrm{MS}} / \mathrm{L}_{\mathrm{GB}} \propto \mathrm{M}_{\mathrm{TO}}$

Since $\mathrm{M}_{\mathrm{TO}}$ decreases in time, the contribution of main sequence compared to giant branch stars becomes less important with time. For $\mathrm{M}_{\text {TO }} \sim 1 \mathrm{M}_{\odot}$ ( 5 Gyr and beyond) the light is dominated by stars on the GB

- The evolution in color of the population can be computed, e.g. from $(U-B)=-2.5 \log _{10} L_{U} / L_{B}+(U-B)$ 。

The color of a single stellar population does not evolve strongly after roughly 1.5 Gyr.

## Stellar populations: numerical results

- Analytic models are a bit limited because of the simplifications made.
- Numerical models to follow evolution of SSP
- These models proceed as follows:
- set initial abundance of H , He and other elements
- initial mass function
- evolve the population forward in time: solve the stellar structure equations for each star
- calculate the luminosities and colors for all the stars in the population
- compute the total luminosity of the population, as well as the colors in different bands


## Results: photometry

Evolution of (B-V), (V-K) and mass-to-light ratio (solar metallicity and Salpeter IMF)

- Note that the population becomes redder with time, initially very rapidly but rather slowly after roughly 5 Gyr.
- It is very hard to distinguish 11 Gyr from 14 Gyr population, unless the colors have been measured with very high accuracy
- recall isochrones closely packed



## More results: spectra



Figure 6.19 Spectra for a 'galaxy' that makes its stars in a $10^{8} \mathrm{yr}$ burst, all plotted to the same vertical scale. Emission lines of ionized gas are strong 10 Myr after the burst ends; after 100 Myr , the galaxy has faded and reddened, and deep hydrogen lines of A stars are prominent. Beyond 1 Gyr , the light dims and becomes slightly redder, but changes are much slower - B. Poggianti.

