Exercises for the course on Physics of Galaxies.I

- Stellar populations and photometric evolution

  1. Integrate the Salpeter mass function between a lower mass limit M_l and upper limit $M_u \gg M_l$ to find (a) the number of stars formed in the population; (b) their total mass; (c) their total luminosity. Show that the number and mass of stars depends mainly on the mass M_l of the smallest stars, while the luminosity depends on M_u, the mass of the largest stars. Taking $M_l = 0.3 M_\odot$ and $M_u \gg 5 M_\odot$, what fraction of all stars have $M > 5 M_\odot$? What fraction have $M > 1 M_\odot$?
  2. Use the (in-class) derived evolution of the luminosity of a single stellar population $L(t)$ to show that single stellar populations become fainter in time

- Elliptical galaxies

  1. If the luminosity density of stars in a galaxy is $j(r) = j_0 (r_0/r)^\alpha$, show that the surface brightness at distance R from the center is

     $$I(R) = I_0 (r_0/R)^{\alpha - 1} \qquad (1)$$

     as long as $\alpha > 1$. What happens if $\alpha < 1$? Compute the total luminosity of the system.
  2. The virial theorem relates the internal potential energy W and the kinetic energy K of a system in equilibrium through: $2K + W = 0$. Assuming that both the velocity dispersion σ and the mass-to-light ratio M/L are constant throughout a galaxy, and that no dark matter is present, use the virial theorem to show that
     - Since the potential energy $W \propto -GM^2/R_e$, where M is the total mass of the galaxy and R_e its effective radius, and the kinetic energy $K \sim M\sigma^2/2$, so the mass of the galaxy should be $M \propto \sigma^2 R_e$.
     - If the surface brightness $I(R)$ of all elliptical galaxies could be described by Sersic's law $I(R) = I_e \exp[-b(R/R_e)^{1/n} - 1]$ with the same value of n, explain why their total luminosity L should follow $L \propto I_e R_e^2$.
     - If all elliptical galaxies had the same mass-to-light ratio M/L and surface brightness at the effective radius I_e, the Faber-Jackson relation is expected.