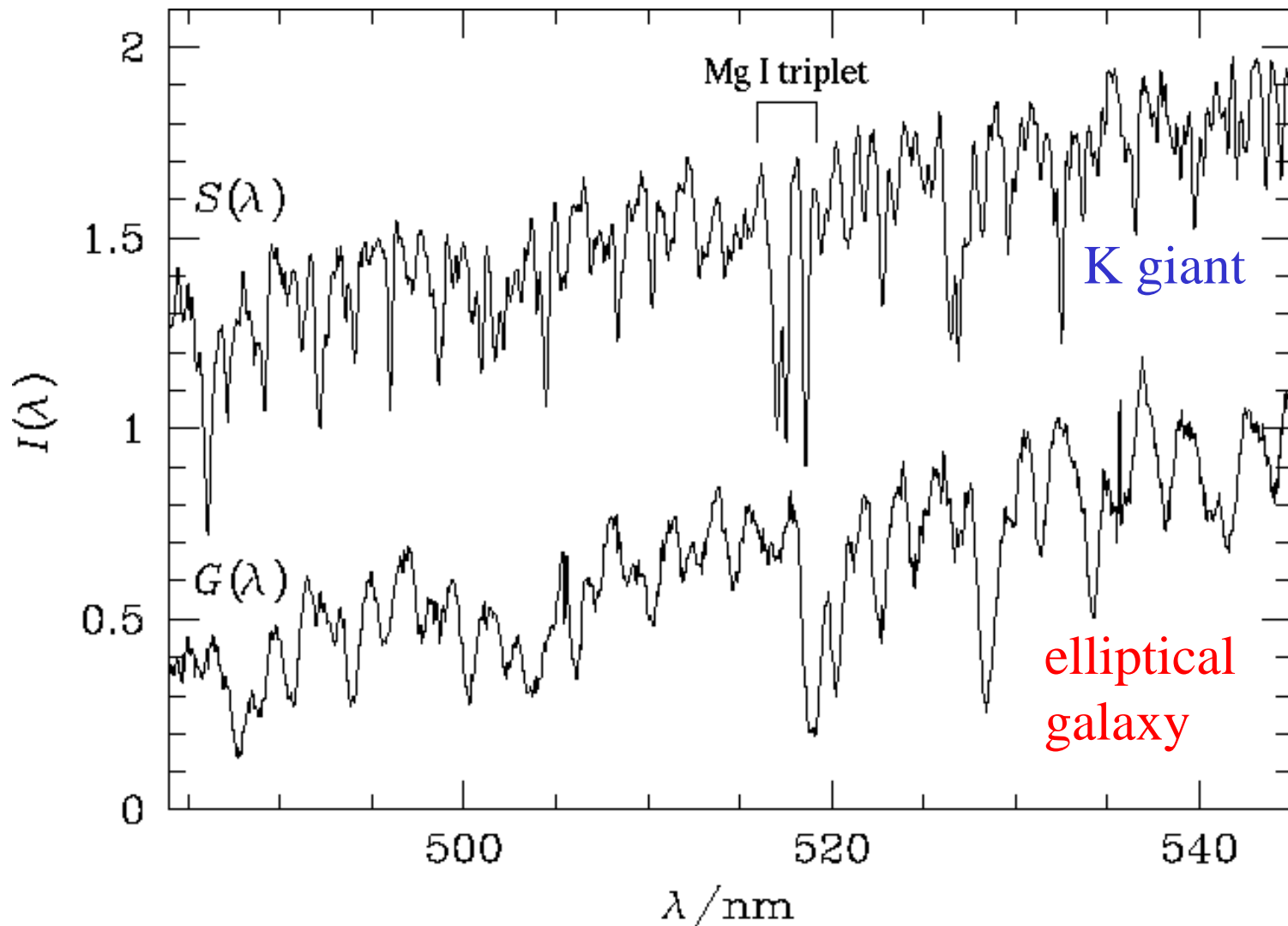


# Stellar populations

- Galaxies are made up of stars
- Hence, the light from a galaxy and its characteristics (colors) depends on the properties of its stars:
  - If there is are large numbers of young stars, the galaxy will appear very blue
  - The spectral energy distribution of early-type galaxies resembles that of a star of spectral type G5.
  - For very late-type galaxies, this can range from A5 to G5.
- The integrated spectral energy distribution\* of a galaxy gives us an indication of the stellar types that can be found in that galaxy.
  - The spectral energy distribution is given by the total flux as function of wavelength or frequency. In other words, it measures the energy output (or luminosity) in different wavelengths.



- Optical spectra K giant and a typical elliptical galaxy. Note the close resemblance.
- Some differences: spectrum is shifted and lines are broader for the E galaxy. Why?

- Since galaxies are composed of stars, then it is possible to understand their colors, spectra, and spectral energy distributions (or luminosities in different wavebands) as the result of the superposition of the light emitted by its stars.
- The chemical elements observed reflect the chemical composition of the photospheres of these stars (weighted in some way).
  - If the stars in the galaxy had the same age, and there were equal numbers of stars of each luminosity, then the weight would be the same for all of them, e.g.

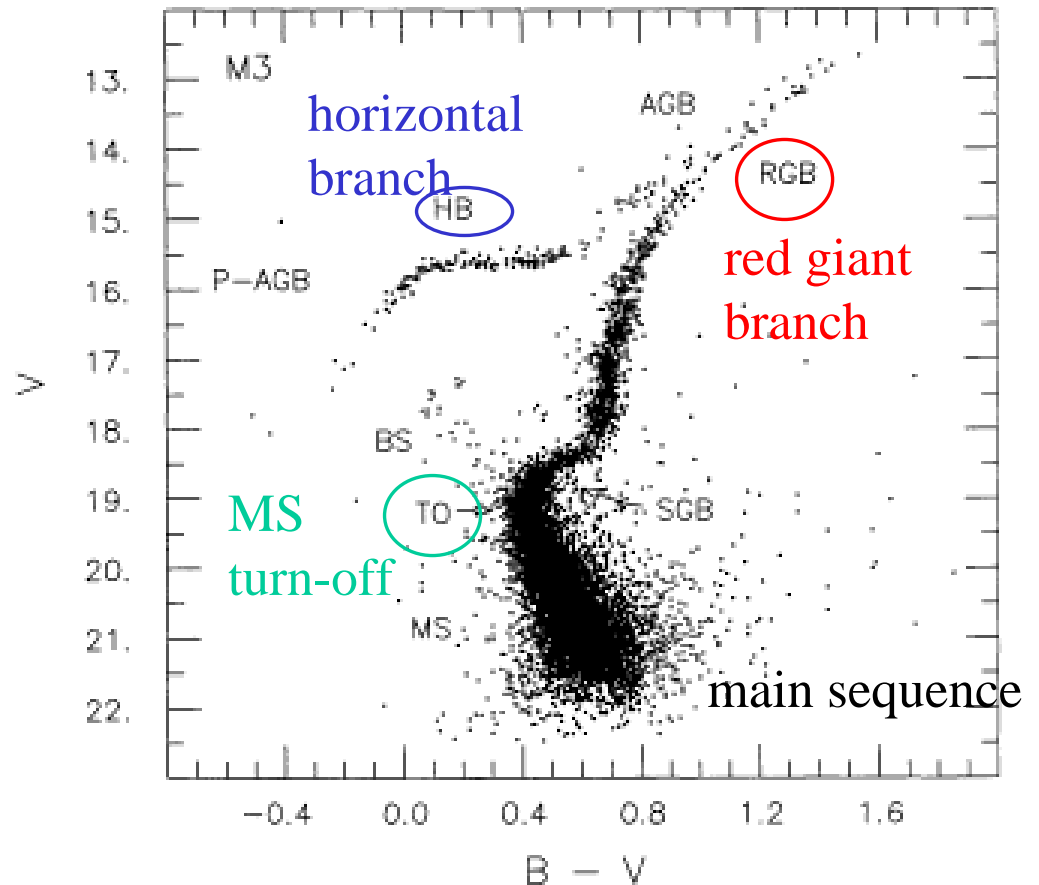
$$L \sim \sum_i^{N_{stars}} L_{i,star} w_{i,star} \longrightarrow L \sim \sum_{i=1}^{N_{stars}} L_{i,star}$$

- Therefore to reproduce the fluxes, colors and spectra of galaxies, all that is necessary is to add up a library of stellar spectral energy distributions with some understanding of the mass or luminosity function of stars in a galaxy\*

\* which essentially define the weights  $w_i$

# Review of stellar evolution and color-magnitude diagrams

- Stars evolve, and this evolution can be exploited to study the properties of galaxies
- Our understanding of their evolution comes from the study of star clusters in the MW

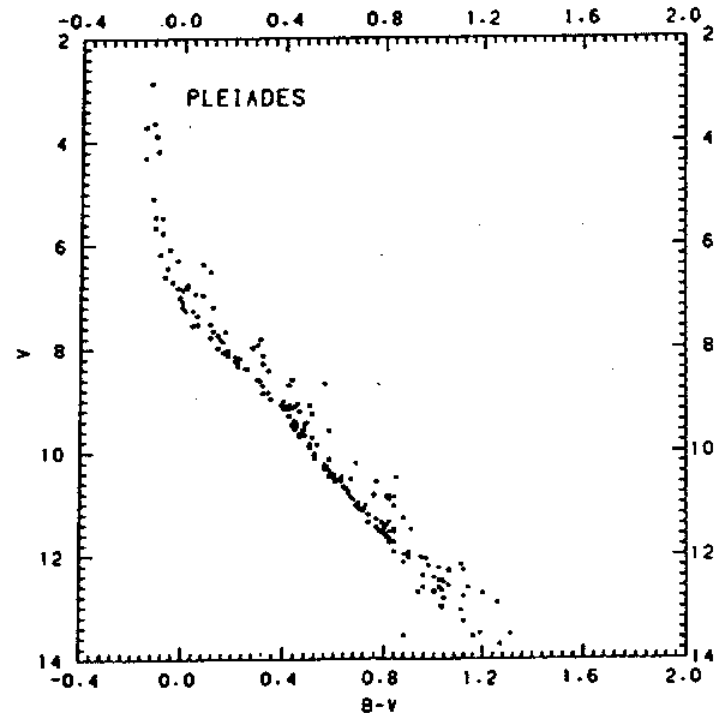
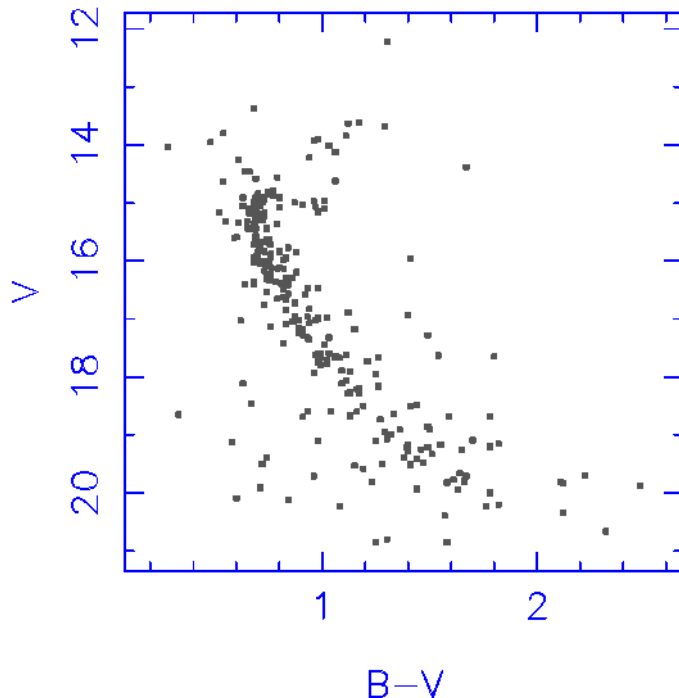


# Open clusters

Clusters are crucial for understanding stellar evolution, since

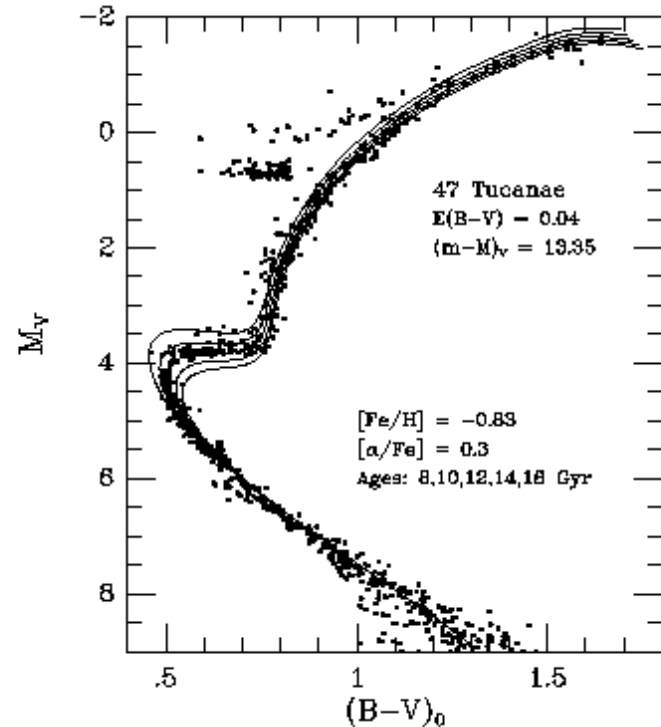
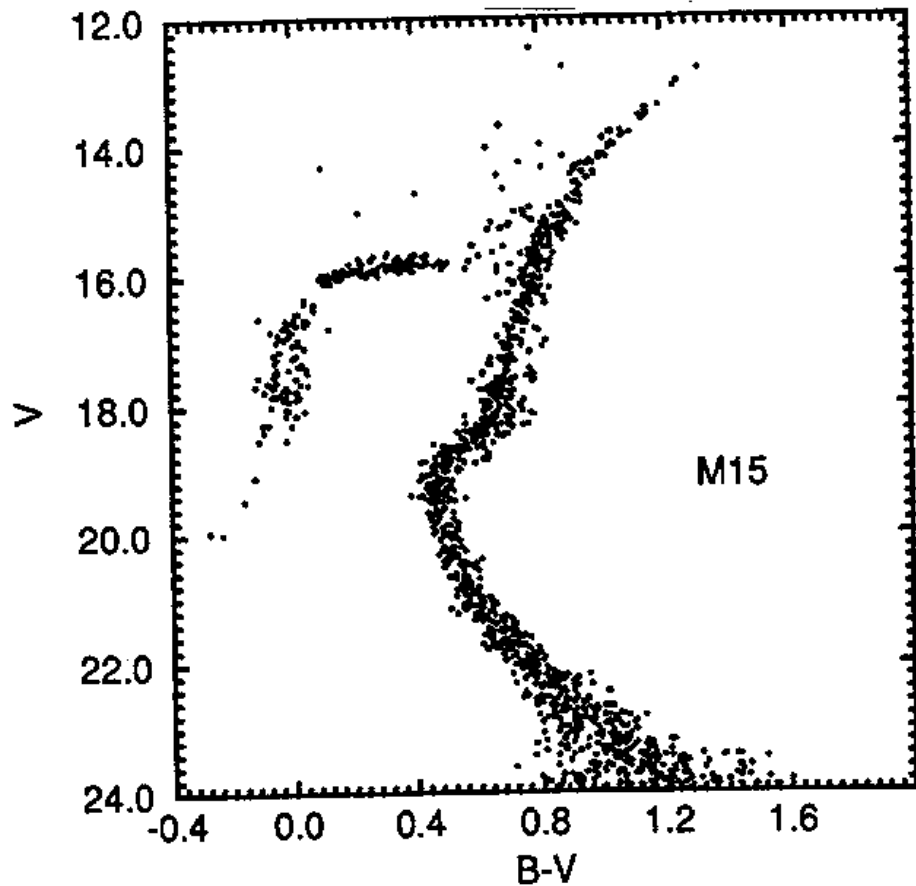
- all stars formed simultaneously (**same age**)
- all have **same composition** (generally close to solar)
- all at **same distance**

NGC 188



# Globular clusters

Globular clusters are also important to understand stellar evolution. They too constitute groups of stars of the same age, but they are typically older (how could you tell?) and more metal-poor



# Some important timescales

- The **main sequence lifetime** is the amount of time a star is supported by hydrogen to helium thermonuclear conversion.
- **How does this lifetime scale with mass and luminosity of a star?**
  - Let  $E^{\text{ms}}$  be the total energy released by a star while on the main sequence. If  $L$  is its luminosity, then  $E^{\text{ms}} = L \tau^{\text{ms}}$  where  $\tau^{\text{ms}}$  is the *main sequence lifetime*.
  - The energy produced in the conversion of a mass of hydrogen  $dM$  is  $dE = 0.0067 dM c^2$
  - Thus, if a mass  $\alpha$  of H (of the total mass of the star  $M$ ) is burned on the main sequence, the energy released is  $E^{\text{ms}} = 0.0067 \alpha M c^2$
  - Therefore the main sequence lifetime is  $\tau^{\text{ms}} = 0.0067 \alpha M c^2 / L$

- If we assume that roughly 10% of the mass is consumed on the main sequence, then  $\alpha = 0.1$ . Using the appropriate units, this implies that

$$\tau^{ms} = 10 \frac{M}{M_{\odot}} \frac{L_{\odot}}{L} \quad \text{Gyr}$$

- For stars of solar metallicity, the relation between luminosity and mass is not linear:  $L/L_{\odot} = A (M/M_{\odot})^b$

$M > 20 M_{\odot}$	$A = 81;$	$b = 2.14$
$2 M_{\odot} < M < 20 M_{\odot}$	$A = 1.78;$	$b = 3.5$
$M < 2 M_{\odot}$	$A = 0.75;$	$b = 4.8$

- Therefore

$$\tau^{ms} = 10 A (M/M_{\odot})^{1-b} \text{ Gyr}$$

- A solar-type star spends roughly 10 Gyr on the main sequence
- A very massive star quickly drifts away from the main sequence. For a  $10 M_{\odot}$  star,  $\tau^{ms} \sim 0.06$  Gyr .

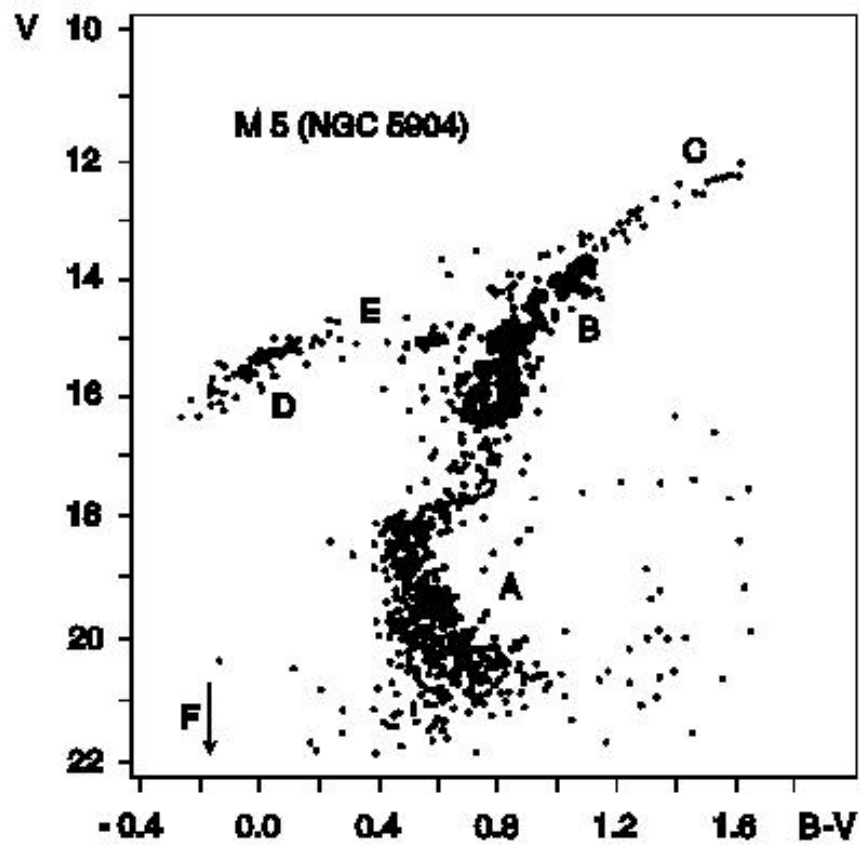
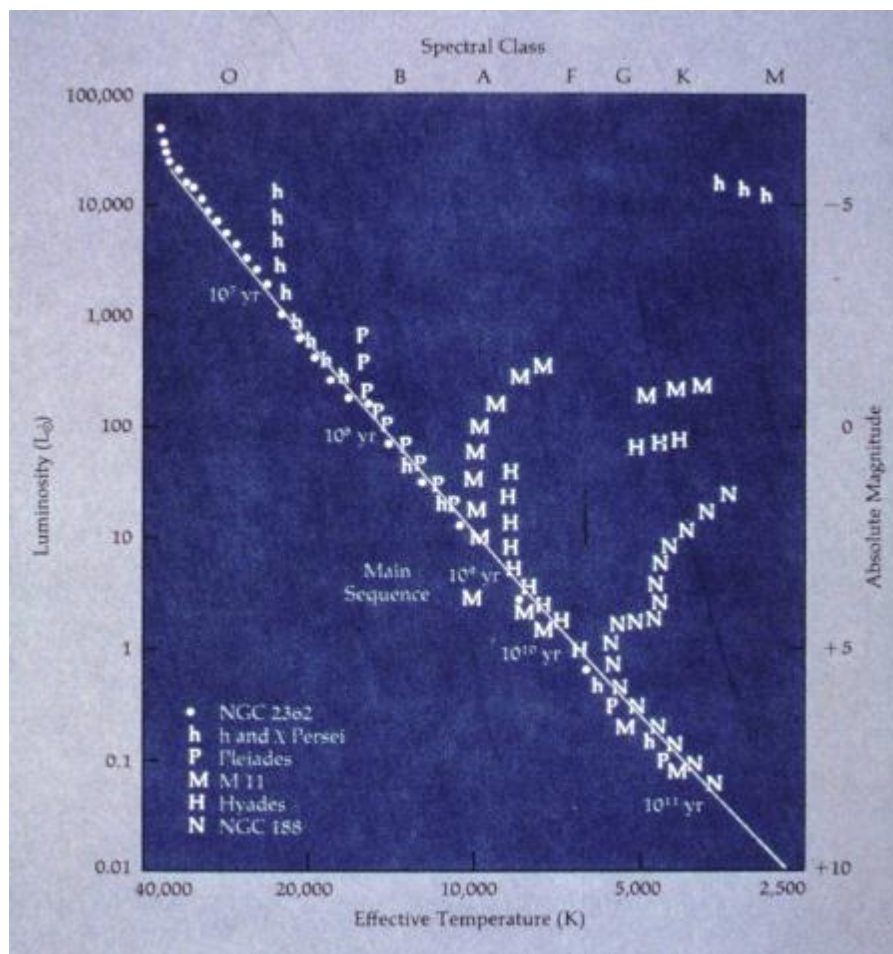


# Globular clusters CMDs in more detail

- For nearby globular clusters (in Milky Way and satellites) it is possible to observe stars individually, and measure their colors and apparent magnitude, i.e. construct a CMD.
- Because the stars are distributed in a small region of space, we may assume that they are located all at the same distance. This distance can be determined using **main sequence fitting**.

The idea is to use the MS defined by subdwarf stars\* for which the distances have been measured accurately using trigonometric parallaxes (**the absolute magnitudes of these stars are known**). Then, we can shift this main sequence vertically, until it overlaps with the main sequence of the globular cluster. **The shift in magnitude then gives us the distance D to the cluster, since  $m = M - 5 + 5 \log(D)$ .**

\* we shall see later that most globular clusters are metal-poor, and hence the term “subdwarfs” to refer to the MS dwarfs

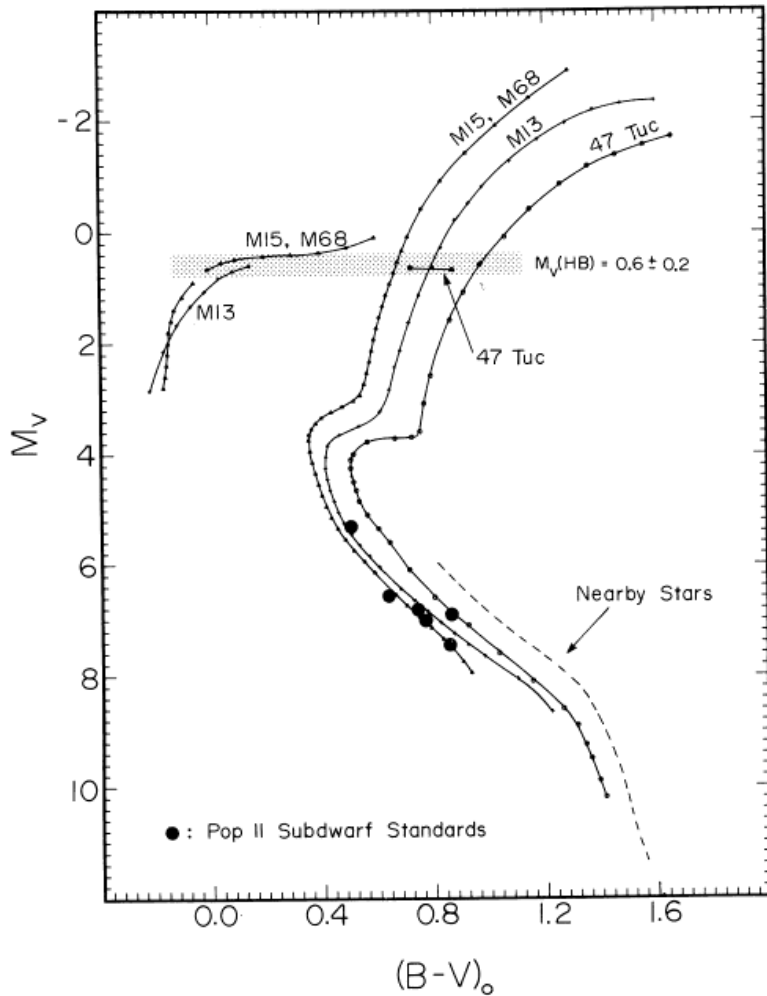


# More on GC CMDs

Stars located at the turnoff are evolving out of the main sequence just now. The more massive stars, have already evolved away, while those below will still spend sometime on the main sequence. This implies that *the age of the cluster is given by the age of the stars located at the main sequence turn-off.*

The fact that there are no stars evolving off the main sequence below the turnoff point shows that the cluster does not contain stars older than the MS lifetime of the turnoff stars. The sharpness of the various features in the observed CMDs can be used to constrain how long star formation lasted in a globular cluster. This timescale is smaller than 2% of the current age of the cluster.

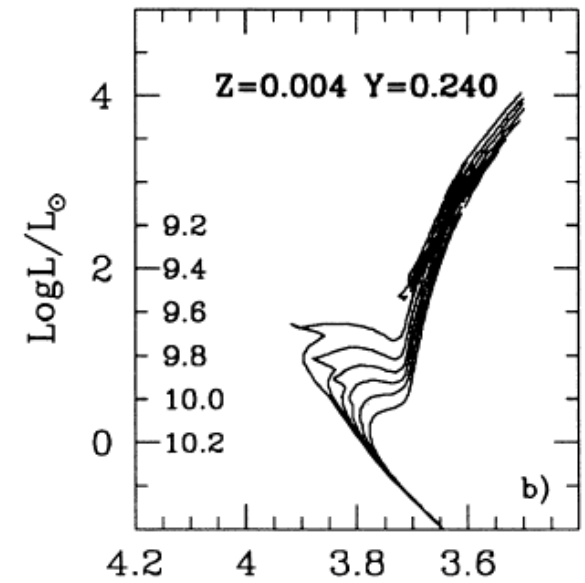
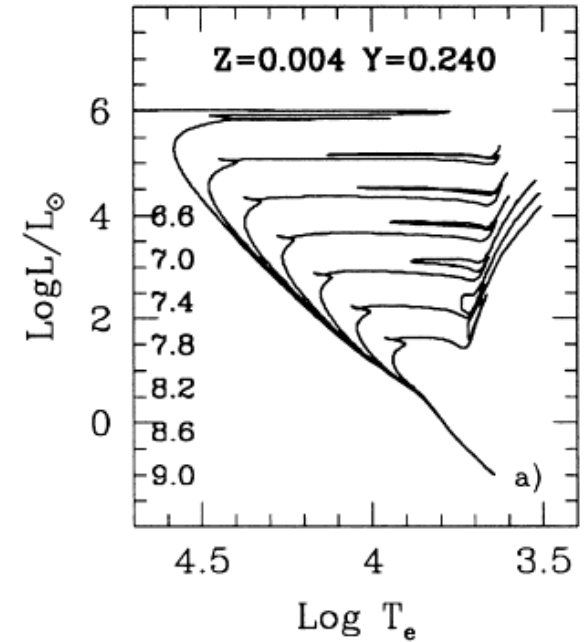
# Comparisons



- This plot shows the location of the principal sequences of 3 globular clusters : M15 (metal-poor), M13 (intermediate metallicity) and 47 Tuc (fairly metal-rich).
- Although the stars follow similar sequences, the **exact location** of these sequences depends both **on age and metallicity**, and possibly a 3rd quantity related to He or other element abundances.

# The effect of age in CMDs

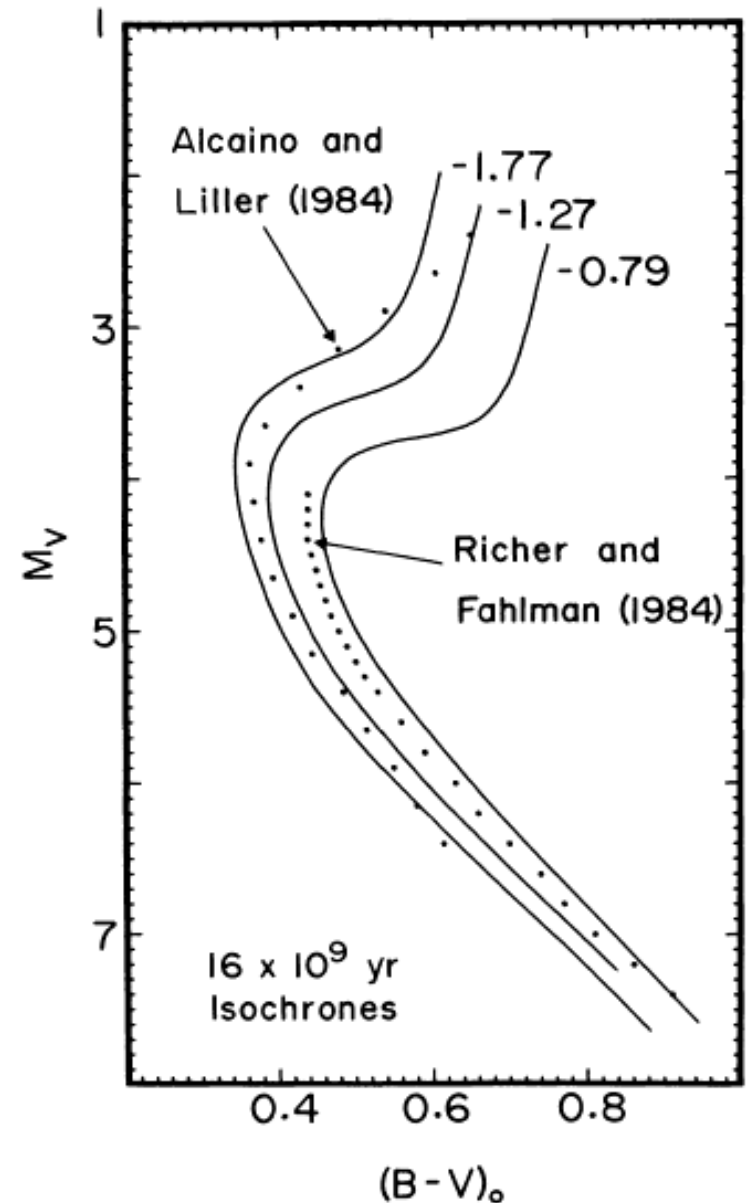
- These plots show theoretical curves (**isochrones**) of the CMD of a group of stars (formed at the same time) at different times (**ages**): from 4 Myr to 10.6 Gyr (the labels give the logarithm of the age).
- Notice how the SGB becomes narrower the older the stars are. This feature is very characteristic of globular clusters.
- Note as well how much closer together the "old" isochrones in comparison to the "younger" isochrones: It is relatively easy to measure the age of a young population of stars to high accuracy, but difficult for an old population.



# The effect of metallicity

This plot shows theoretical curves (**isochrones**) for coeval group of stars with a fixed age (16 Gyr in this case), but with **different metallicities**.

Notice how the MS and the GB become brighter and bluer the lower the metallicity.



# The effect of metallicity in stars

- Stars of lower metallicity are brighter and hotter: This is due to the lack of heavy elements in their atmospheres, which implies that more UV photons can escape and the star becomes bluer (i.e. hotter).
- At fixed luminosity, a change in metallicity of -1.7 dex shifts the ZAMS to the blue by 0.06 Teff, or by  $(V-K) - (V-K)_0 = -0.32$ .
- At fixed color, such a ZAMS is shifted down by approx. 1 magnitude. This is why the low metallicity ZAMS dwarfs are called subdwarfs.
- The location of certain features in the CMD can allow one to determine the metallicity of a population. For example, the absolute magnitude of the horizontal branch
  - $M_V(\text{HB}) = 0.17 [\text{Fe}/\text{H}] + 0.82$

# The ages of globular clusters from the turn-off location

- The presence of a sharp turnoff shows that the stars in globular clusters formed in all at the same time. Since the location of the TO changes as the cluster gets older, the absolute magnitude of the turnoff point  $M_v(\text{TO})$  provides a direct measure of the cluster age.
- From theoretical modelling, it has been found that

$$M_v(\text{TO}) = 2.70 \log (t/\text{Gyr}) + 0.30 [\text{Fe}/\text{H}] + 1.41$$

- Therefore, to derive  $t$ , we need:
  - the apparent magnitude of the main sequence TO
  - the distance to the cluster
  - the metallicity from spectra



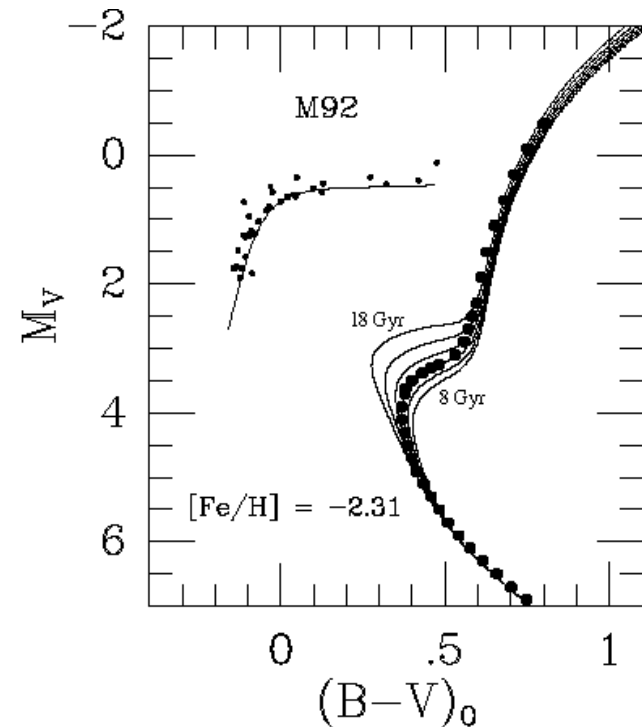
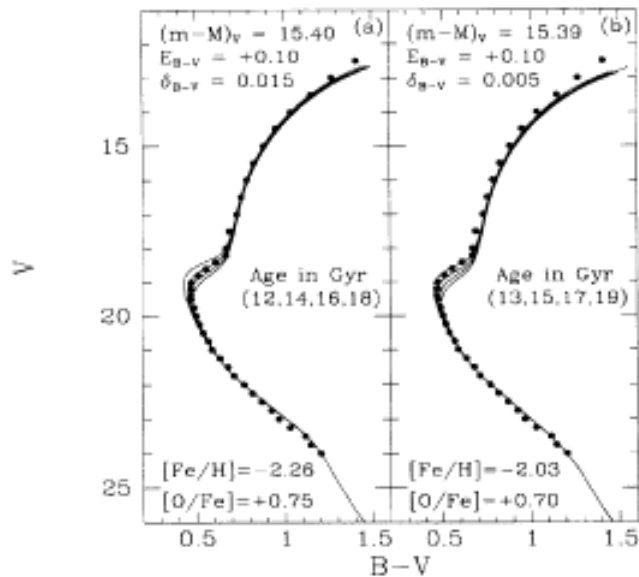
- Because the isochrones become very vertical at the turnoff, it is difficult to accurately measure the location of the TO in the CMD.
- Moreover, errors in the distance determination will also affect the age determination:

A 10% distance error implies a 0.2 mag error in the distance modulus ( $m - M$ ) which translates into a 0.2 mag error in the absolute magnitude of the TO.

- This in turn produces a 20% error in the age (demonstrate this using the above Eq.!).

# Ages from isochrone fitting

- Use the different features in the CMD to find the best model that fits the data
- This method leaves the distance also as a free parameter to be determined by the model that best fits the observed data.
- Sometimes it is also necessary to vary the metallicity and other element abundances to obtain a good fit to the data (as shown in the left panels).



# Ages from $\Delta V$ method

- One of the biggest problems in the previous methods is that they rely on the distance to the cluster (derived either through main sequence fitting; the method itself or using standard candles such as RR Lyrae stars). It is possible to avoid this problem by using an estimator which depends on two features of the CMD diagram.
- The quantity  $\Delta V = M_V(\text{TO}) - M_V(\text{HB})$  measures the difference in magnitude between the turnoff point and the location of the horizontal branch. It is therefore independent of distance.
- The brightness of the horizontal branch depends little on the mass of the stars which populate it, and therefore on the age of those stars. This is because the duration of the HB phase is very short, of the order of 0.1 Gyr. This implies that stars ought to have similar ages to be located on the HB at the same moment in time.

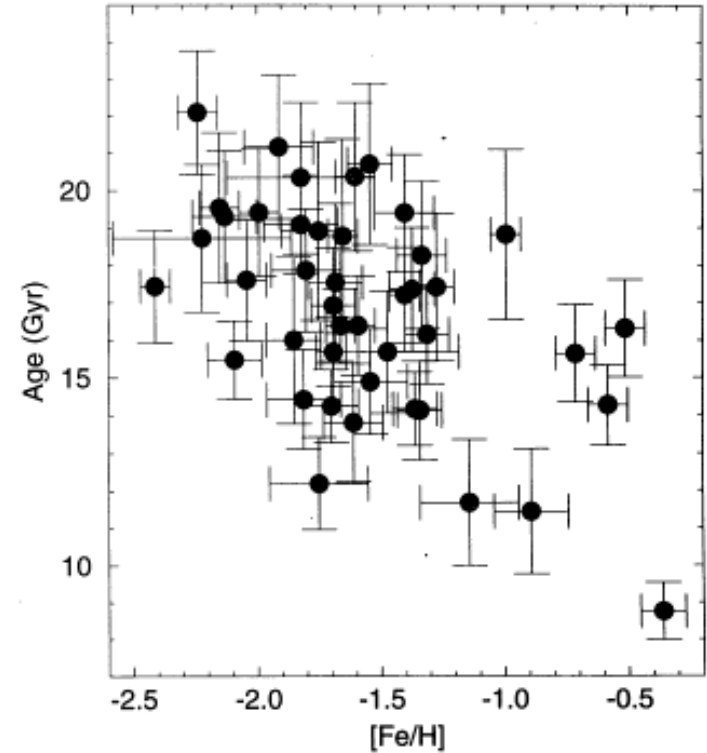
- On the other hand, the brightness of the TO depends strongly on age.
- Therefore the quantity  $\Delta V$  should provide a good measure of the age of the cluster.
- Using previous equations, it is easy to show that

$$\Delta V = 2.7 \log(t/\text{Gyr}) + 0.13 [\text{Fe}/\text{H}] + 0.59$$

# The ages

- Globular clusters are generally very old.

In this plot, some of the ages appear greater than the age of the Universe ( $13.7 \pm 0.2$  Gyr according to latest cosmological measurements). This is no longer the case: models have been improved and distance measurements have systematically changed since the Hipparcos satellite calibrated the subdwarfs distances better (making the globular clusters more distant, and hence its stars intrinsically brighter, or younger) .



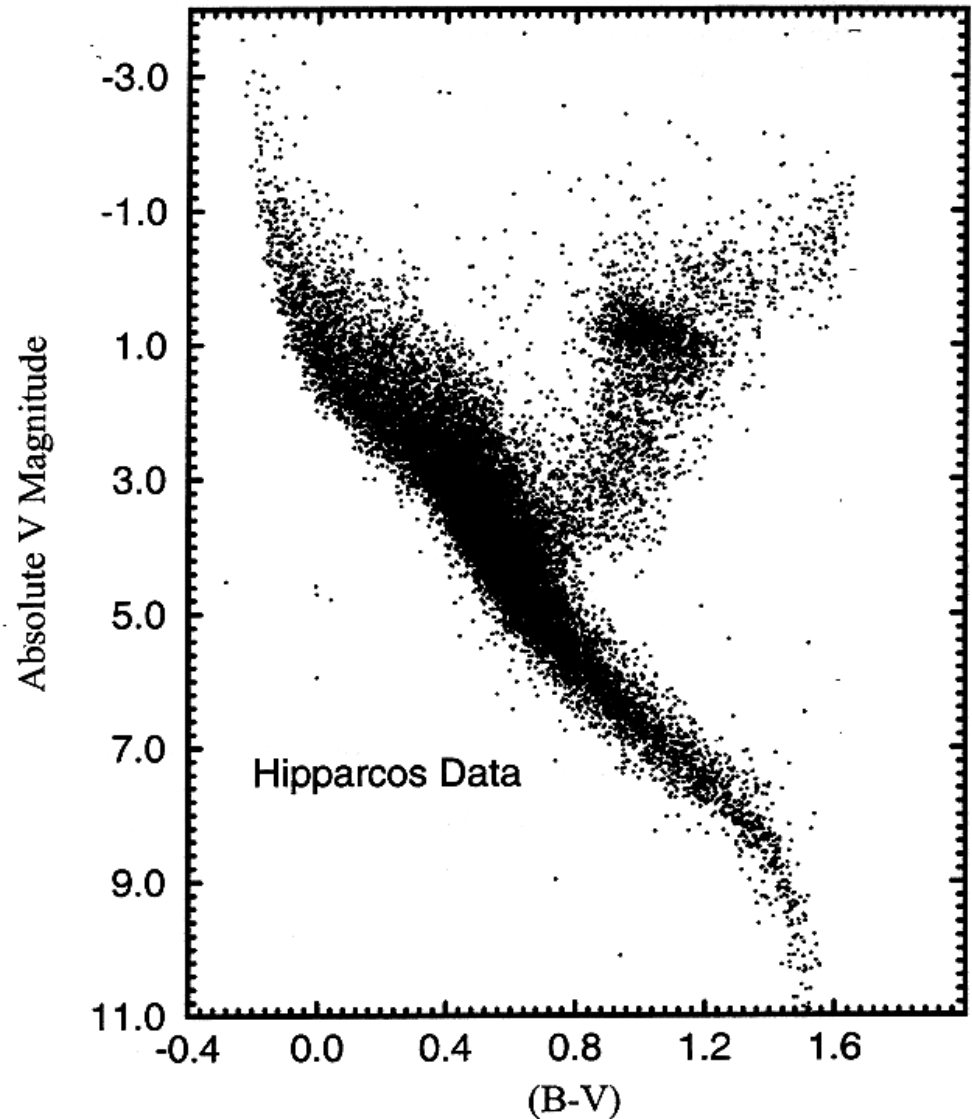
- It is more robust to measure relative ages than absolute ages. Therefore, in the above plot we may actually derive that there is a trend for younger clusters to be more metal-rich.

- We can also say that the epoch of globular cluster formation finished a long time ago for our Galaxy.

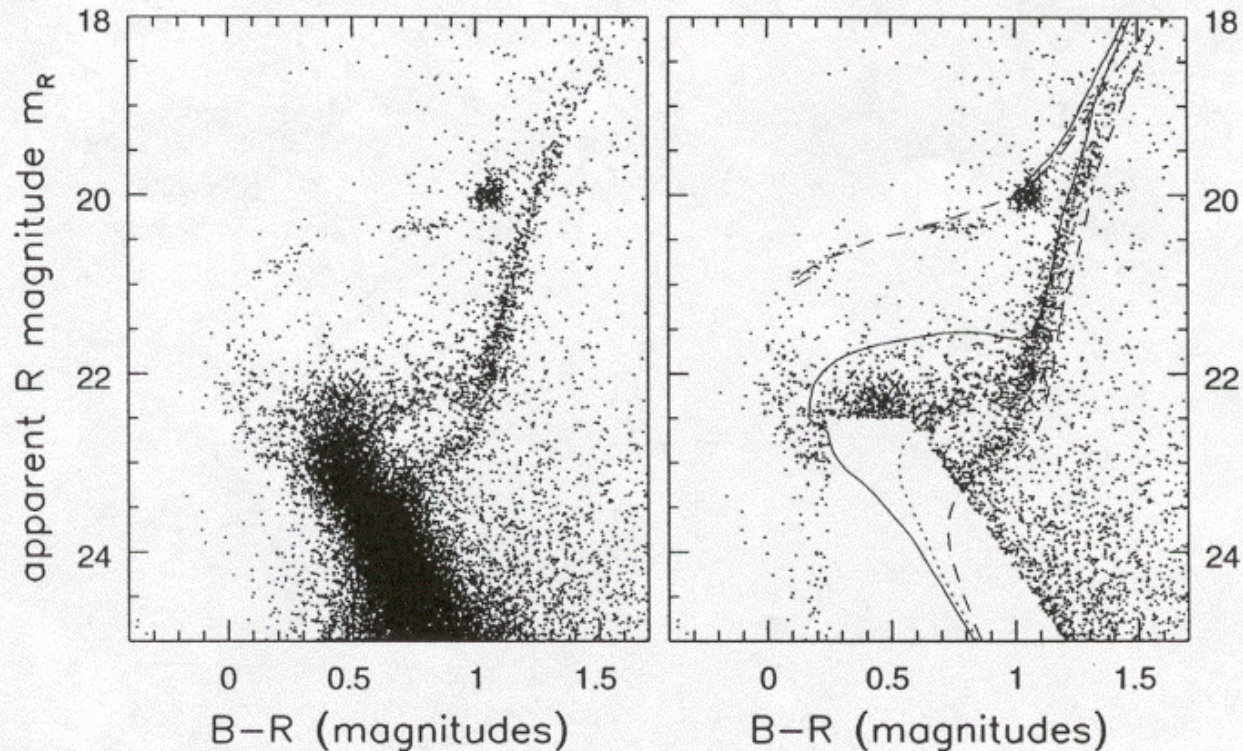
# Resolved CMDs in galaxies

Nearby disk stars:  
not just one  
age, but wide  
range!

Continuous (and  
ongoing) star  
formation



The CMD of a dwarf spheroidal (satellite of the Milky Way). Notice that it can be thought of as the superposition of several globular clusters of different ages, as indicated by the loci on the right-hand side panel.



**Figure 4.9** Left, color-magnitude diagram for the Carina dwarf spheroidal galaxy. Right, superposed isochrones give the locus of metal-poor stars ( $Z = Z_{\odot}/50$ ) at ages of 3 Gyr (solid), 7 Gyr (dotted), and 15 Gyr (dashed); we see young red clump stars close to  $B - R, m_R = (1, 20)$ , and old stars on the horizontal branch. Carina's distance modulus is taken as  $(m - M)_0 = 20.09$ ; dust reddening is assumed to dim stars by 0.108 magnitudes in  $B$  and 0.067 magnitudes in  $R - T$ . Smecker-Hane; A. Cole, Padova stellar tracks.

# Stellar populations

- Basic concept: a stellar population is a group of stars with similar properties:
  - abundance patterns (although not necessarily same metallicity)
  - kinematics
- The simplest example of a stellar population is one in which all stars formed at the same time (**common age**). Such a group of stars is commonly known as a **single stellar population**. The simplest example is a globular cluster.
- For the Milky Way, the terms population I and population II are frequently used
  - Pop. I: the stars in the disk (and more generally the recently formed component of star-forming galaxies)
  - Pop.II: stars in the halo; it is the metal-poor component of galaxies



# Stellar populations

- For most galaxies, the light we observe results from the contribution of thousands to millions of stars. What does this light (colors, spectrum) tell about the nature of the stars in the galaxy?

Even though it is not possible to construct the CMD diagram from a single (integrated) spectrum, it is still possible to constrain the mean metallicity and age of a galaxy.

The simplest case is when all stars in a system formed at the same time (this is the case for globular clusters). It is possible to regard a galaxy with any history of star formation as the superposition of single stellar populations of different ages and/or metallicities.

# Stellar populations synthesis studies

- The goal is to relate the observed colors, absorption line and emission line features to the underlying stellar composition
- It attempts to answer questions such as
  - what fraction of the stars come from each unresolved class?
  - What was the star formation history of the system?
  - When was the last time a given galaxy had a burst of star formation?
  - could a significant amount of mass be in non-stellar form?  
Examples would be white dwarfs, black holes, neutron stars, clouds of molecular hydrogen..... In combination with dynamical studies, can help to quantify the amount of dark-matter present in a galaxy.

# The initial mass function

- The initial mass function  $\xi(M)$  specifies the distribution in mass of stars in a freshly formed stellar population:

$$dN = N_0 \xi(M) dM$$

where  $dN$  is the number of stars with mass in the interval  $(M, M+dM)$ .  $N_0$  is a normalization factor, which depends on how many stars were initially formed and on the normalization of  $\xi(M)$  itself.

- Since in any star formation episode, most of the stars have low mass (but not necessarily contribute a substantial fraction of the total mass of the population), the IMF is normalized as  $\int dM M \xi(M) = M_{\odot}$

With this normalization  $N_0$  is the total number of solar masses formed in the burst of star formation.

The IMF can only be determined observationally for a relatively unevolved stellar population. Its determination requires counting the number of stars of a given mass. This implies knowing the mass of each star. Each of these aspects has its own difficulties.

For example, one important question is to determine if the IMF is universal

- everywhere the same (independent of environment)
- always the same (was it different in the past?)

To address this issue

- need to **measure the IMF in a variety of environments** (the Solar neighbourhood, open clusters, globular clusters, other regions of the Milky Way, other nearby galaxies, ...). In the large majority of cases, some **fraction of the stars will have evolved away from the main sequence, and hence have changed their mass substantially**. Will need to correct for this effect.

- it is very difficult to measure the mass of a star. The only way is to use binary stars.

# The luminosity function

- The luminosity function  $\Phi(M_\lambda)$  measures the number of (main sequence) stars in a given absolute magnitude range  $(M_\lambda, M_\lambda + dM_\lambda)$ .
  - It is possible to measure this quantity observationally.
- The fact that stars evolve will make the luminosity function depend on time. If we are interested in the determining the luminosity function at the time of formation of the stars  $\Phi_0(M_\lambda)$ , this evolution needs to be taken into account.
  - If the population under study is coeval, then  $\Phi(M_\lambda) = \Phi_0(M_\lambda)$
  - If instead, the star formation rate is constant (like in the Solar Neighborhood), then the initial luminosity function  $\Phi_0(M_\lambda)$  is related to today's  $\Phi(M_\lambda)$  as

$$\Phi_0(M_\lambda) = \Phi(M_\lambda) \times t/T_{\text{ms}}(M) \text{ for } t > T_{\text{ms}}(M), \text{ note that } M = f(M_\lambda) \text{ and}$$
$$\Phi_0(M_\lambda) = \Phi(M_\lambda) \text{ otherwise}$$

- Sometimes the luminosity function is given per unit volume (i.e. number of stars per absolute magnitude interval and per  $\text{pc}^3$ )

# The IMF and the LF

- The IMF and the luminosity function are related, since they both give the number of stars in a given mass or absolute magnitude range (and these quantities are related)

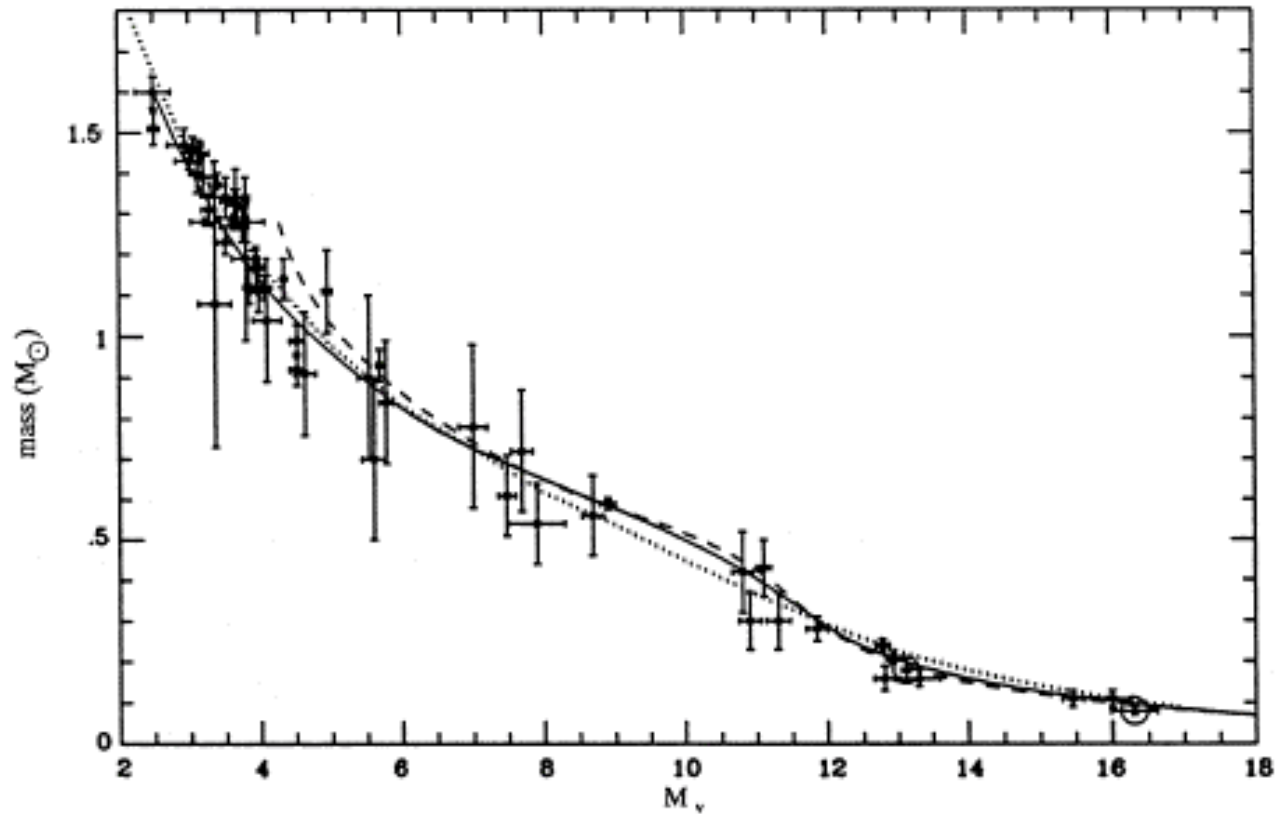
$$\xi(M) dM = \Phi_0(M_\lambda) dM_\lambda$$

or

$$\xi(M) = \Phi_0(M_\lambda) dM_\lambda / dM$$

- The relation between absolute magnitude and mass of a star is not easy to compute theoretically (it requires good models of stellar interiors -evolution- as well as stellar atmospheres -where the light originates).
- It is often determined empirically using binary stars

This plot shows the masses of main sequence binary stars with a given absolute magnitude.



# Examples of IMF

- The simplest case of an initial mass function is that which is a power-law function of the mass:

$$\xi(M) \propto M^{-2.35}$$

- This is the **Salpeter initial mass function**.
- Note that it diverges at the low-mass end. This can be circumvented by introducing a lower mass cut-off.

- The **Scalo initial mass function** is

$$\xi(M) \propto M^{-2.45}, \text{ for } M > 10 M_{\odot}$$

$$\xi(M) \propto M^{-3.27}, \text{ for } M_{\odot} < M < 10 M_{\odot}$$

$$\xi(M) \propto M^{-1.83}, \text{ for } M < 1 M_{\odot}$$



- Another example of a broken power-law is given by the **Kroupa initial mass function**, derived for the solar neighbourhood. In this case

$$\xi(M) \propto M^{-4.5}, \text{ for } M > 10 M_{\odot}$$

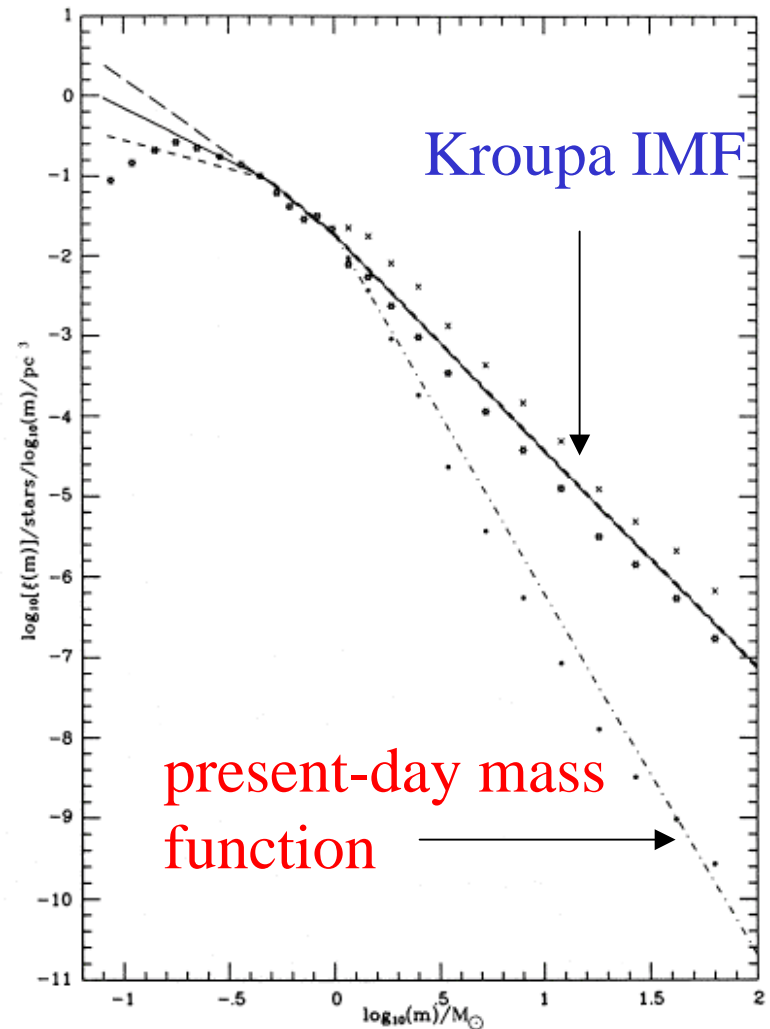
$$\xi(M) \propto M^{-2.2}, \text{ for } 0.5 M_{\odot} < M < 10 M_{\odot}$$

$$\xi(M) \propto M^{-1.2}, \text{ for } M < 0.5 M_{\odot}$$

The **solid line** represents the **Kroupa initial mass function**, while the **dot-dashed curve** is the **present-day mass function**. The two coincide for masses below  $\sim 1 M_{\odot}$ .

Solid dots correspond to present-day mass function measured by Scalo (who corrects for stellar evolution).

The different symbols (crosses and asterisks) are for different assumed ages for the stellar disk (recall that one needs to take into account for the stars that have evolved out of the MS)



# Stellar populations: analytic results

- The luminosity of a single stellar population varies with time.
- Stars more massive than  $1.25 M_{\odot}$  live less than 5 Gyr. Thus any single stellar population older than 5 Gyr will contain stars of mass  $< 1.25 M_{\odot}$ . Such stars are the most luminous while they are on the RGB, HB and AGB phase.
- **Therefore, most of the light from an old(ish) single stellar population comes from evolved stars.**
- The color of a single stellar population does not evolve strongly after roughly 1.5 Gyr (after stars with mass  $> 2 M_{\odot}$  have died).

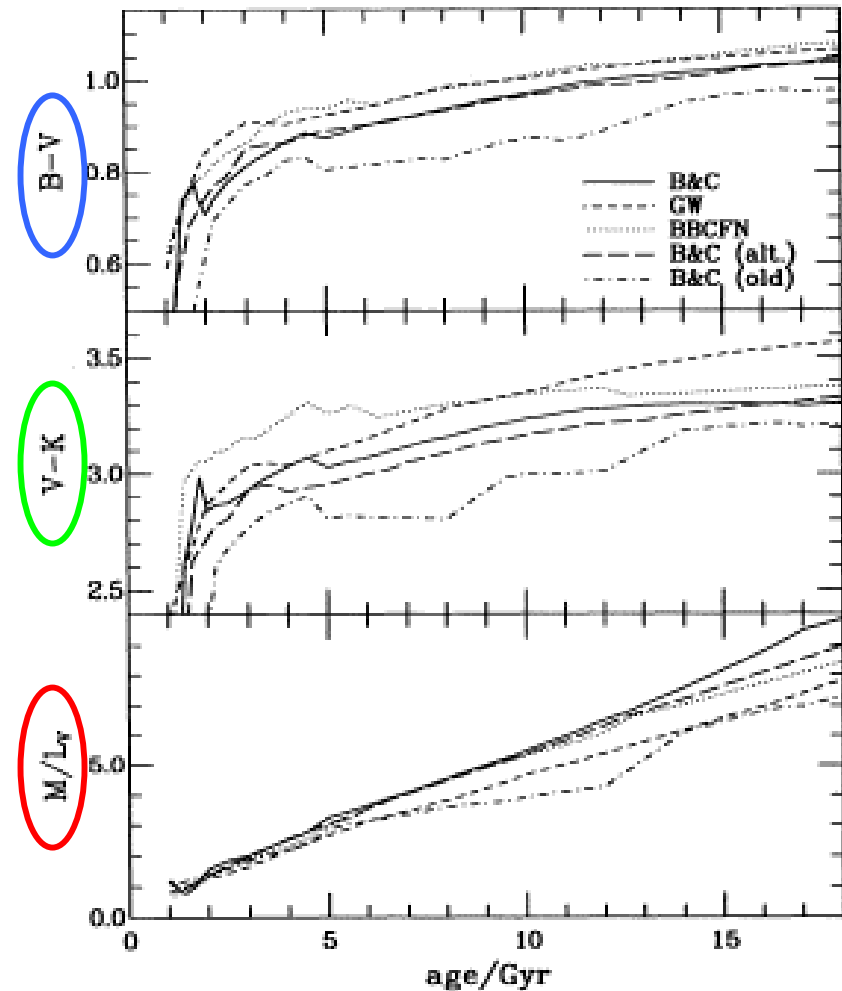
See Tinsley [ ApJ 173, L93 (1972) and ApJ 186, 35 (1973)] and the lecture notes

# Stellar populations: numerical results

- The analytic models presented are useful to gain insight into the properties of single stellar populations, but they are limited because of the simplifications made.
- It is possible to derive the evolution of a single stellar populations by turning to numerical models.
- These models proceed as follows:
  - assume an initial abundance of H, He and other elements
  - assume an initial mass function
  - evolve the population forward in time: solve the stellar structure equations for each star
  - calculate the luminosities and colors for all the stars in the population
  - compute the total luminosity of the population, as well as the colors in different bands

# Results

- Focus on the solid curves. They show the evolution of (B-V), (V-K) and **mass-to-light ratio** as a function of time, for a group of stars with solar metallicity and for a Salpeter IMF.
- Note that the population becomes redder with time, initially very rapidly but rather slowly after roughly 5 Gyr.
- In particular, it is very hard to distinguish a 11 Gyr from a 14 Gyr population, unless the colors have been measured with very high accuracy.



- Single stellar populations are the building blocks of galaxies
- They can be combined with a star formation rate (i.e. that gives the number of stars formed per unit time) to “construct” a model for the stellar populations of a galaxy
- SSPs are therefore very important for understanding the properties of galaxies and how these have evolved in time