# UNIVERSITY OF GRONINGEN

# BACHELOR THESIS

ASTRONOMY & PHYSICS

# Gamma-Ray Bursts as Source of High Energy Cosmic Neutrinos

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#### Abstract

In this thesis we investigate gamma-ray bursts, and in particular how high energy cosmic neutrinos can be produced and what the timing is between  $\gamma$ -rays and these high energy neutrinos. In the first part of this thesis basic concepts and observational constraints of GRBs are discussed. We find that the production of 10 TeV neutrinos is possible and should potentially be detectable at earth using neutrino telescopes like IceCube. We predict no significant The time difference between the detection of 10 GeV neutrinos and the  $\gamma$ -rays.

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# 1 Introduction

Gamma-ray bursts are short intense bursts of photons with energies of 100 keV to 1 MeV. When Gamma-ray bursts (GRBs) were discovered in the end of the 1960, it was unclear what causes GRBs and numerous theories were proposed, most involving events in our galaxy like bursts on galactic neutron stars. When the BATSE detector was launched in spring of 1991, BATSE observed an uniform distribution. This meant that GRBs should have an extra-galactic origin. This idea is confirmed by BeppoSAX, that measured the redshift of multiple GRBs and confirmed that they have an extra-galactic origin and have cosmological distances[1]. The fact that GRBs have cosmological distances makes them the most luminous objects in the universe. GRBs are able to release an energy of  $10^{51} - 10^{54}$  ergs in a few seconds or less. Because of these high luminosity, GRBs are rare events in our universe. At the time that BATSE was observing GRBs, it observed an average of one burst per day. This implies that, using a very simple model, that a GRB happens once every million year in a galaxy[1].

Multiple models were proposed to explain GRBs in the past of which many have been disproven by BATSE (models that say that GRBs are produced in our galaxy or because they do not produce short intense bursts)[1]. Nowadays we have a good idea what can produce GRBs and right now there are two popular models that together can explain the observed GRBs, this will be discussed in section 2. After this in section 3, a look is taken at observational constraints on GRBs like the distribution on the sky, typical spectra of GRBs, timing structure, observed energy of GRBs and the redshift distribution of GRBs.

After the observational part we introduce the basics of shock physics in section 4 plane shock waves and spherical blast waves are explained. After this we will study ultrarelativistic blast waves known as fireballs, and their deceleration phase in section 5.

In section 6 we look at the energetics of GRBs and how much energy is converted to radiation and how much is converted to low energy neutrinos produced due to annihilation of electrons and positrons. Subsequently a look is taken at neutrino production due to internal collisions which are promissing candidates for high energy cosmic neutrinos (section 7). Next a look is taken at the time difference between the detection of  $\gamma$ -rays and neutrinos, which is highly interesting for neutrino telescopes like Icecube (see section 9).

Still today gamma-ray bursts and neutrinos are not completely understood, because of this the combining neutrinos and gamma-ray burst is quite challenging. Besides this, combining the two, brings together two extremes, neutrinos which are by far the lightest particles of the standard models and gamma-ray burst which are by far the most energetic events in the univsers. This combination makes high energy neutrino production by gammaray bursts one of the most challenging and beautiful topics in astronomy and astrophysics right now.

# 2 GRB models

In this section we look at various models that can produce a GRB, but first we look at general accepted ingredients of GRB models.

# 2.1 Ingredients of GRB models

Gamma-ray Burst are rare violent phenomena, with variabilities on time scales of less than a second. These GRBs are the result of the forming or merging of compact objects releasing energies of the order of  $10^{51}$  erg to  $10^{54}$  erg, which is as much energy as the rest mass of our own sun or a neutron star  $E \approx M_{\odot}c^2$ . This results in the formation of a relativistic shock wave with a Lorentz factor  $\gamma > 100$ . The existence of the relativistic shock wave is observationally confirmed and results in the fact that the photons of the GRB are blueshifted in the observer's frame. Besides this due to relativistic beaming only a fraction  $1/\gamma$  of the source is observed. In most cases, the bursts are not spherically symmetric, but they have jets. This means that observed GRBs are pointing towards us. During the evolution of a GRB the jet consists of multiple shock waves which propagate all with a slightly different speed resulting in internal collisions in the jet which dissipate energy. Besides this, also energy is dissipated after the initial explosion when the jets of the GRB are slowed down and collide with the interstellar medium, this is called the afterglow. In almost all models of the evolution of GRBs, the initial formation of the relativistic shock wave is not important for the evolution of the GRB, which means that determining the origin of GRBs is challenging work[2, 1].

# 2.2 The 2 types of GRBs

Observationally it is very clear that there are two distinct types of GRBs, hard short bursts and long hard bursts. Of these two, the detection of the short bursts is more difficult but recent progress in observations have result in good detections of short burst and long bursts. The main differences between long and short burst are the time of the burst and the hardness. The time of the burst is expressed in a quantity called t90, which is the time it takes for the detectors to receive 90% of the flux of the GRB. Hardness is an observed property which basically is the ratio of an high energy band divided by a lower energy band[1, 3]. The fact that there are two distinguished classes of GRBs was already known before 2000, when two distinct types of GRBs where clearly visible from the BATSE data as can be seen in figure  $1[1]^1$ .

#### 2.3 Binary neutron star mergers

Binary neutron star mergers or neutron star-black hole mergers are among the compact object mergers the best candidates for GRBs. These mergers happen because binary orbits

 $<sup>^1\</sup>mathrm{For}$  a more extended explaination on t90 and hardness see appendix 1



Figure 1: Plot of the hardness against t90 using the BATSE data

of compact objects spiral into each other due to gravitational radiation, when the two neutron stars or neutron star and black hole merge, large amounts of energy escape in neutrinos and gravitational radiation. Besides this also a small fraction of the energy escapes as photons resulting in an observable GRB[1]. The standard model of the merger of compact objects which result in a GRB is that during the merger a rotating black hole is formed with an accretion disk, which result in jets in the direction of the rotation axis[3].

Nowadays the observations of neutron stars in our own galaxy using radio and X-ray suggest that neutron star mergers happen at a rate of around  $\approx 10^{-6}$  event per year per galaxy. This rate is quite close to the rate of short GRBs in galaxies. The rate of neutron star-black hole mergers does also have a comparable rate of around  $\approx 10^{-6}$  event per year per galaxy, based on detections in radio and X-ray. This coincidence means that neutron star-neutron star mergers and neutron star-black hole mergers are good candidates for GRBs and together can explain the amount of observed short GRBs quite good. Besides this other compact mergers could also produce GRBs[1, 4].

#### 2.4 Long duration bursts

It seems that the long duration gamma-ray bursts are becoming better understood[5, 4]. These long duration bursts are probably caused by collapsers. These are collapsing low metallicity Wolf-Rayet stars that have completed all stages of nucleosynthesis. Wolf-Rayes stars are stars that have a mass in the range of 20  $M_{\odot} - 100 M_{\odot}$  and are rapidly rotating. When these stars complete all stages of nucleosynthesis the core will collapse to a neutron star, but for these stars the pressure of neutron degeneracy is not strong enough to support the star so these stars will further collapse to a black hole. When the surrounding material of the star will be sucked in the black hole much energy is created resulting in a blast of

 $\gamma$ -rays, along the rotating axis of the black hole with a typical width of 3°[5, 4]. Wolf-Rayet stars are massive stars with short lifetimes, this means that they trace star forming regions.

# **3** Observational constraints of GRBs

## 3.1 Distribution of GRBs on the sky

Nowadays it is known that GRBs are distributed uniformly over the whole sky, because they are at cosmological distances. There is a correlation between GRBs and Abell clusters, which does not mean that GRBs are associated with Abell clusters but that GRBs trace the large-scale structure of the universe the same way as Abell clusters do[1]. Besides this, long GRBs are associated with star formation, this means that long GRBs are formed in galaxies with a high star formation rate and that the burst distribution in galaxies follows the light distribution of the host galaxy[2].

Because long GRBs follow the star formation rate this means that GRBs have probably something to do with stellar deaths like supernovae and hypernovae. The first observational evidence of this was found in 1998, in this year GRB 980425 took place after which SN 98bw was discovered within the error box of the position of the GRB. These supernova and GRB were strange. In the GRB there was no high energy spectrum and furthermore the supernova was exceptionalkly bright compared to other supernova, having an energy 10 times higher then usual supernovae. Besides this SN 98bw also had components expanding with subrelativistic speeds of around  $v \sim 0.3c[2, 4]$ .

On the other hand short GRBs are found more in early type galaxies. This means that short GRBs correspond to older stars. Furthermore short GRBs are also found to have older progenitors than type Ia. Some short GRBs are also found to be in in older regions of late type galaxies, also implying that short GRBs originate from old stars[3].

#### 3.2 Typical spectra

The spectrums of GRBs is non-thermal and differs strongly between different bursts, despite this there is a fit for the spectra of GRBs, called the Band spectrum. It basically is a spectrum that consists of 2 power laws which join smoothly at the break energy  $(\tilde{\alpha} - \tilde{\beta})E_0[2, 6]$ , with the property that the derivatives are continuous[6]. The Band spectrum is given by [2, 6]

$$N(\nu) = N_0 \begin{cases} \left(h\nu\right)^{\tilde{\alpha}} \exp\left(-\frac{h\nu}{E_0}\right), & \text{for } h\nu < (\tilde{\alpha} - \tilde{\beta})E_0; \\ \left[\left(\tilde{\alpha} - \tilde{\beta}\right)E_0\right]^{\left(\tilde{\alpha} - \tilde{\beta}\right)} \left(h\nu\right)^{\tilde{\beta}} \exp\left(\tilde{\beta} - \tilde{\alpha}\right), & \text{for } h\nu > (\tilde{\alpha} - \tilde{\beta})E_0. \end{cases}$$
(1)

The Band spectrum has the property that it can describe a broad range of spectra. Like single power laws  $(E_0 = \infty)$ , energy exponentials  $(\tilde{\alpha} = -1, \tilde{\beta} = -\infty)$  and photon exponential  $(\tilde{\alpha} = 0, \tilde{\beta} = -\infty)$ . The Band spectrum is thus able to describe the broad range of spectra observed in GRBs and even other processes however, it does not provide any relation to the underlying physical processes of the GRB itself[6].

The Band spectrum is also able to describe a subgroup of bursts called NHE burst (no high energy burst), in these bursts there is an absence of photons in the high energy part of the spectra. These burst have the property that they have no hard component and therefore have a very negative value of  $\tilde{\beta}[2]$ .

#### 3.3 Time spectra

For GRBs there seems to be no standard time spectrum, but rather there is an enormous diversity in the different possible time spectra. Figure 2 shows 4 examples of time spectra from the BATSE Catalogue. As can be seen there is a huge diversity between these 4, but what these GRBs all have in common is the existence of spikes with typical times  $\delta T$  and a chaotic behaviour of the time spectrum. Because of these short time fluctuations an upper limit can be set on the radius of the source. Often the variable time scale is  $\delta T \approx 10$  ms. This results that the object which produce GRBs have a size of  $E_i < c\delta T \approx 3000 \text{km}[1]$ . Furthermore because GRBs have in general a time spectra which consists of short peaks and is chaotic, explaining this behaviour is quite challenging and can be solved using the fireball model, which will be explained in section 5.



Figure 2: Total number of counts versus time for several bursts from the BATSE Catalogue. It can be clearly seen in this plot that there is a huge diversity in the different temporal structure observed in GRBs[1].

#### 3.4 Observed energy

The  $\gamma$ -ray detectors that search for GRBs and other  $\gamma$ -ray phenomena detect the flux of the photons at the different  $\gamma$ -ray energies. When a GRB is seen they immediately look at its position to accurately measure the flux. Using the observed flux it is possible to calculate the complete isotropic energy of the burst. The isotropic energy is the energy of the burst if it is assumed that the burst is completely spherical symmetric, this is called  $E_{iso}$  and is given by [1]

$$E_{iso} = 4\pi D^2 F = 10^{50} \text{ergs} \left(\frac{D}{3000 \text{ Mpc}}\right)^2 \left(\frac{F}{10^{-7} \text{ergs/cm}^2}\right).$$
 (2)

In reality some or all GRBs are beamed and as a result the real energy is given by

$$E = \frac{\Omega}{4\pi} E_{iso}.$$
 (3)

Where  $\Omega$  is the beaming angle[1].

#### 3.4.1 Observed intensity

From the detection of GRBs we know that they have a quite high  $\gamma$ -ray intensity of the order of  $10^{-7}$ erg cm<sup>-3</sup>. If this  $\gamma$ -ray intensity is used to calculate the source of the explosion it results in an totally unrealistic amount of  $\gamma$ -rays in a very small volume. From the observational fact that GRBs are detected, we know that GRBs are transparent to  $\gamma$ -rays. But if we assume that the explosion of the GRB is non-relativistic we would predict no  $\gamma$ -rays at Earth, because of the high density of  $\gamma$ -rays that immidiately make the explosion opaque for  $\gamma$ -rays (not transparent)[1].

This means that in the case of a nonrelativistic explosion we would not expect any  $\gamma$ -rays at earth. This problem can be solved by the fireball model if we use Lorentz factors  $\gamma \sim 1000$  which implies that the emitting plasma can be transparent for  $\gamma$ -rays and still can observe a large number of energetic  $\gamma$ -rays at earth[1]. The fireball model will be explained in section 5.

# 3.5 Redshift distribution

Gamma-ray bursts are cosmological events and have a distribution which is shown in figure 3[1, 7, 8]. This distribution indicates that GRBs are cosmological with an average redshift of 2.2.



Figure 3: Redshift distribution of GRBs[7, 8].

On average short GRBs are closer than long GRBs[3], besides this there is also evidence that long GRBs have an higher rate in the early universe than at low redshifts in the early universe the environments of heavy stars was metal-poor[8].

# 4 Shock waves

In this section we study non-relativistic shock waves, first we look at boundary conditions on the shock fronts after which we discuss the spherical blast waves. In this part we assume that the reader is familiar with basic hydrodynamics, if this is not the case we recommend the reader to read appendix 2, which explains the basics.

## 4.1 Shock waves in a simple fluid

To begin our discussion of shock waves we will first consider the case of a planar shock wave in a polytropic fluid  $(P = P(\rho))$ . Without lose of any generality for a planar shock wave, it can be chosen that the direction of the shock is in the x-direction, and the plane of the shock is in a fixed place in the y-z plane. Also it will be assumed that the velocity vector lies in the x-z plane and that derivatives with respect to the y and z axis are zero for all properties  $(\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0)$ . To get a better illustration of what the shock wave does look like, see figure 4[9].



Figure 4: The geometry of a thin planar shock wave, where the shock wave is traveling to the right. In this figure the quantities labeled with an 1 are pre-shock and those labeled with a 2 are post-shock[9].

The fluid equations of hydrodynamics can be rewritten in terms of equations 4, 5, 6 and  $7^2$ ,

<sup>&</sup>lt;sup>2</sup>For a detailed derivation see appendix 13.4

where h is the enthalpy per unit mass and  $\varepsilon$  is the internal energy per unit mass [9].

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \left( \rho u_x \right) = 0, \tag{4}$$

$$\frac{\partial(\rho u_x)}{\partial t} + \frac{\partial}{\partial x} \left(\rho u_x^2 + P\right) = 0, \tag{5}$$

$$\frac{\partial(\rho u_z)}{\partial t} + \frac{\partial}{\partial x} \left(\rho u_x u_z\right) = 0, \tag{6}$$

$$\frac{\partial}{\partial t} \left( \rho \left( \frac{u^2}{2} + \varepsilon \right) \right) + \frac{\partial}{\partial x} \left( \rho u_x \left( \frac{u^2}{2} + h \right) \right) = 0.$$
(7)

These 4 equations which are the 4 conservation equations for a plane wave in a polytropic fluid, in this equation it is used that  $u^2 = u_x^2 + u_z^2$ . Further it can be seen that all the 4 conservation equations have the same general form given by equation 8, where Q is some quantity and F is the flux of that quantity[9, 10].

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} = 0. \tag{8}$$

Using the assumption of an infinitely small shock equation 8 implies that the flux-density of this quantity is conserved before and after the shock<sup>3</sup>. This means that four conserved fluxes can be obtained and thus 4 equations. These equations are called the Rankine-Hugoniot jump conditions and are given by equations 9 till 12[9, 10].

$$\rho_1 u_{x,1} = \rho_2 u_{x,2},\tag{9}$$

$$\left(\rho u_x^2 + P\right)_1 = \left(\rho u_x^2 + P\right)_2,\tag{10}$$

$$\rho_1 u_{x,1} u_{z,1} = \rho_2 u_{x,2} u_{z,2}, \tag{11}$$

$$\left(\rho u_x \left(\frac{u^2}{2} + h\right)\right)_1 = \left(\rho u_x \left(\frac{u^2}{2} + h\right)\right)_2.$$
(12)

These jump conditions give the conditions that a planar shock wave should satisfy. But what would these equations imply for the conditions in front and behind the shock wave? In this part we discuss how these equation can be rewritten and what this means. Using these equations it is convenient to define the constant  $J = \rho_i u_{x,i}$ . Using this, the above equations

 $<sup>^3\</sup>mathrm{For}$  a detailed explanation why this is the case see appendix 13.5

can simply be rewritten to the following equations [9, 10].

$$\rho_1 u_{x,1} = \rho_2 u_{x,2} \equiv J,\tag{13}$$

$$\rho_1 u_{x,1}^2 + P_1 = \rho_2 u_{x,2}^2 + P_2, \tag{14}$$

$$u_{z,1} = u_{z,2},$$
 (15)

$$\frac{u_{x,1}^2}{2} + h_1 = \frac{u_{x,2}^2}{2} + h_2.$$
(16)

Now equation 14 and 16 can be rewritten to obtain what is called the Rayleigh line and the Rankine-Hugoniot shock adiabat which are given by equation 17 and 18, where a new quantity called,  $\mathcal{V}$  is defined, which is the specific volume  $(\mathcal{V} = \frac{1}{a})[9, 11]$ .

$$J^2 = \frac{P_2 - P_1}{\mathcal{V}_1 - \mathcal{V}_2},\tag{17}$$

$$\frac{\gamma}{\gamma - 1} \left( P_2 \mathcal{V}_2 - P_1 \mathcal{V}_1 \right) = \frac{1}{2} \left( \mathcal{V}_2 + \mathcal{V}_1 \right) \left( P_2 - P_1 \right). \tag{18}$$

Using the Rayleigh line, the velocity difference between the velocity before and after the shock can be calculated. Using the fact that  $u_1 - u_2 = J(V_1 - V_2)$  this results in[11],

$$\Delta V = V_1 - V_2 = \sqrt{(P_2 - P_1)(V_1 - V_2)}.$$
(19)

For this problem it is useful to know what happens to the density of the fluids before and after the shock wave. We define the compression ratio, which basically is the ratio between the density after and before the shock (see equation 20). Using this and equation 18 it can be shown that this reduces to equation 21[9].

$$r = \frac{\rho_2}{\rho_1} = \frac{\mathcal{V}_1}{\mathcal{V}_2} = \frac{v_1}{v_2},$$
(20)

$$r = \frac{\frac{\gamma+1}{\gamma-1}P_2 + P_1}{\frac{\gamma+1}{\gamma-1}P_1 + P_2}.$$
(21)

For a GRB and other shock waves it is interesting to look at what happens if there is a strong shock wave, which means  $P_2 \gg P_1$ . In this situation the compression ratio can be rewritten to equation 22[9, 10].

$$r = \frac{\gamma + 1}{\gamma - 1}.\tag{22}$$

In the case of an ideal gas the compression ratio is 4, because  $\gamma = \frac{5}{3}$ , on the other hand if the gas is ultrarelativistic the compression ratio becomes 7 ( $\gamma = \frac{4}{3}$ ). For a shock wave the Rankine Hugoniot Jump conditions can be rewritten in terms of the mach number instead of the velocity[10, 11].

$$\frac{\rho_2}{\rho_1} = \frac{u_{x,1}}{u_{x,2}} = \frac{(\gamma+1)\mathcal{M}_1^2}{(\gamma-1)\mathcal{M}_1^2 + 2},\tag{23}$$

$$\frac{P_2}{P_1} = \frac{2\gamma \mathcal{M}_1^2 - (\gamma - 1)}{\gamma + 1},$$
(24)

$$\frac{T_2}{T_1} = \frac{(2\gamma \mathcal{M}_1^2 - (\gamma - 1))((\gamma - 1)\mathcal{M}_1^2 + 2)}{(\gamma + 1)^2 \mathcal{M}_1},$$
(25)

$$\mathcal{M}_{2}^{2} = \frac{2 + (\gamma - 1)\mathcal{M}_{1}^{2}}{2\gamma\mathcal{M}_{1}^{2} - (\gamma - 1)}.$$
(26)

In the case of the strong shock limit which is of main interest in the case of astrophysical explosions these equations reduce to

$$\rho_2 = \left(\frac{\gamma+1}{\gamma-1}\right)\rho_1,\tag{27}$$

$$\Delta u = u_{x,2} - u_{x,1} = \left(\frac{2}{\gamma+1}\right) u_{shock},\tag{28}$$

$$P_2 = \left(\frac{2}{\gamma+1}\right)\rho_1 u_{shock}^2,\tag{29}$$

$$T_2 = \frac{\gamma - 1}{\gamma + 1} \frac{P_2}{P_1} T_1.$$
(30)

# 4.2 Spherical blast wave in a simple fluid

In explosions plane shock waves are just approximations far from the start of the shock wave. In this section we look at the most interesting time of the explosion, right after a lot of energy is realized and the shock wave starts to expand. This can be described as a blast wave, which is a wave which is formed after a lot of energy is realized in a small area. In this section spherical symmetry is assumed to understand a blast wave, this results in equation 31 for the continuity equation, equation 32 for the Euler equation, equation 33 for the energy equation and equation 34 for the adiabaticity of the fluid elements<sup>4</sup>[10].

<sup>&</sup>lt;sup>4</sup>These equations can easily be derived if one takes into account that  $\nabla \cdot \mathbf{F} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 F_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta F_{\theta}) + \frac{1}{r \sin \theta} \frac{\partial F_{\phi}}{\partial \phi}$ , which reduce to  $\nabla \cdot \mathbf{F} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 F_r)$  in the spherical symmetric case.

$$\left(\frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r}\right) \rho = -\frac{\rho}{r^2} \frac{\partial}{\partial r} (r^2 u_r), \qquad (31)$$

$$\left(\frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r}\right) u_r = -\frac{1}{\rho} \frac{\partial P}{\partial r},\tag{32}$$

$$\frac{\partial}{\partial t} \left( \rho \left( \frac{1}{2} u_r^2 + \varepsilon \right) \right) = -\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \left( \frac{1}{2} \rho u_r^2 + h \right) u_r \right), \tag{33}$$

$$\left(\frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r}\right) \ln\left(p\rho^{-\gamma}\right) = 0.$$
(34)

The spherical blast wave problem is a self similarity problem, which means that all functions of the problem can be expressed as a dimensionless function times initial conditions of the problem which gives them the desired dimension. This means that we can rewrite the density, velocity and pressure to the following expressions[10].

$$\rho = \rho_0 \breve{\rho}(\xi), \tag{35}$$

$$u = \dot{R}\breve{u}(\xi),\tag{36}$$

$$P = \rho_0 \dot{R}^2 \check{P}(\xi). \tag{37}$$

Because of this the partial differential equation can be rewritten into coupled ordinary differential equations, this means that the differential equations reduce to the following dimensionless differential equations [10]

$$(\breve{u} - \xi)\frac{\mathrm{d}\breve{\rho}}{\mathrm{d}\xi} = -\frac{\breve{\rho}}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\breve{u}\right),\tag{38}$$

$$(\breve{u} - \xi)\frac{d\breve{u}}{d\xi} - \frac{3}{2}\breve{u} = -\frac{1}{\breve{\rho}}\frac{d\breve{P}}{d\xi},\tag{39}$$

$$(\breve{u} - \xi) \left( \frac{1}{\breve{P}} \frac{\mathrm{d}\breve{P}}{\mathrm{d}\xi} - \gamma \frac{1}{\breve{\rho}} \frac{\mathrm{d}\breve{\rho}}{\mathrm{d}\xi} \right) - 3 = 0.$$
(40)

These ordinary differential equations can be solved using the boundary conditions on the shock front. This means that the shock conditions are valid at radius  $\xi = 1$ , where we define the dimensionless variable  $\xi = \frac{r}{R(t)}$ , and where the strong shock condition is valid at  $\xi = 1$ , resulting in the following boundary conditions at  $\xi = 1[10]$ .

$$\breve{\rho} = \frac{\gamma + 1}{\gamma - 1},\tag{41}$$

$$\breve{v} = \frac{2}{\gamma + 1},\tag{42}$$

$$\breve{P} = \frac{2}{\gamma + 1}.\tag{43}$$

Furthermore the total energy is given by equation 44, if for the moment, the energy of the external medium is ignored[10].



$$E = \int_{0}^{R} \left(\frac{1}{2}\rho u^{2} + \frac{P}{\gamma - 1}\right) 4\pi r^{2} \mathrm{d}r$$
(44)

Figure 5: The evolution of the scaled velocity (red), scaled mass density (blue) and the scaled pressure (green) as function of the dimensionless variable  $\xi$  for a ultrarelativistic gas[12].

Using the above differential equations and the boundary conditions, the spherical blast wave problem can be solved numerically or analytically. In this case we use the results from Cococubed to get an idea of what the solution will look like[12]. The Cococubed simulation uses an initial density profile of the form of  $\rho = \rho_0 r^{-\omega}$  and besides this the only other parameter which needs to be known for the problem is the adiabatic index  $\gamma$ . In the case of an adiabatic index of  $\gamma = \frac{4}{3} \approx 1.3$ , the solutions of the density, pressure and velocity will look like the curves shown in figure 5. In the case that the gas is ideal, the solutions will change a little bit and will look like figure 6[12]. As can be seen in these figures the solutions look similar with little differences for the different situations.



Figure 6: The evolution of the scaled velocity (red), scaled mass density (blue) and the scaled pressure (green) as function of the dimensionless variable  $\xi$  for a ideal gas[12].

#### 4.2.1 Dimensional analysis on the problem

In the case of a spherical blast wave the solution might depend on the two dimensional parameters of the problem which are E and  $\rho_0$ . To make a length-scale of these two parameters a time dependence needs to be added. This results in equation 45, where  $\alpha$  is a constant

which depends on the initial conditions [10]. In general the initial value for the constant is determined to be around 1 for an ultrarelativistic as well as for an ideal gas [10, 13].

$$R = \alpha \left(\frac{Et^2}{\rho_0}\right)^{\frac{1}{5}} \tag{45}$$

# 5 Fireball

"A fireball is a large concentration of energy (radiation) in a small region of space in which there are relatively few baryons" according to Piran[1]. The unavoidable outcome of a fireball is a relativistic particle flow that eventually will be converted into radiation. There are two interesting kinds of fireballs: One that is described by pure radiation resulting in a photonlepton fireball and one with baryons where the baryons have a large influence on its further evolution[1]. An important property of a fireball is that is has a high ratio of energy density to rest mass resulting in ultra relativistic velocities[14].

Initially the fireball has a high opacity due to free electron-positron pairs which means that the radiation in the fireball cannot escape. This results in an adiabatic expansion of the fireball which cools down the fireball till the temperature drops below the pair production temperature resulting in a transparent fireball. In some cases these fireballs also contain baryonic matter from the start explosion or from the surroundings of the explosion. In the case of baryons in the fireball the opacity will be higher due to the electrons from the baryonic matter resulting in a later escape of the radiation and the conversion from radiation energy into the energy of the bulk motion of the baryons[14].

# 5.1 Different regimes

In the evolution of fireballs two important transitions take place. The first transition that takes place is the transition from the optical thick adiabatic expanding phase to the optical thin phase where the photons and electrons are decoupled and where the  $\gamma$ -rays will escape. An other important transition in the evolution of a fireball is the change from radiation-dominated to matter-dominated. In this context radiation is dominant when  $\eta > 1$  (where  $\eta$  is given by equation 46) and matter-dominated when  $\eta < 1$ . The total result of a fireball depends on these two important transitions. When the transition from optical thick to optical thin happens earlier then the transition from radiation-dominated to matter-dominated most of the energy will be still in the radiation resulting in huge amounts of  $\gamma$ -rays. If on the other hand the fireball becomes matter-dominated before it becomes optical thin, most of the energy of the fireball will be converted to high energy cosmic particles rather then  $\gamma$ -rays, resulting in fewer photons[14].

$$\eta \equiv \frac{E}{mc^2} \tag{46}$$

How and when these transitions take place depends on the initial ratio of radiation energy to mass,  $\eta_i$ . Because of this we can separate 4 different kinds of fireballs with different outcomes. These 4 different regimes can be separated by means of the initial ratio of radiation energy to mass,  $\eta_{pair}$  and  $\eta_b$  where these are defined by equation 47 and 48.  $E_i$  is the initial energy,  $R_i$  the initial radius,  $\sigma_T$  the Thompson cross section and a the radiation constant[14].

$$\eta_{pair} = \sqrt{\frac{3\sigma_T^2 E_i a T_p^4}{4\pi m_p^2 c^4 R_i}} \approx 3 \cdot 10^{10} E_{52}^{1/2} R_{i7}^{-1/2},\tag{47}$$

$$\eta_b = \left(\frac{3\sigma_T E_i}{8\pi m_p c^2 R_i^2}\right)^{\frac{1}{3}} \approx 10^5 E_{52}^{1/3} R_{i7}^{-2/3}.$$
(48)

(i)  $\eta_i > \eta_{pair}$ . In this type of fireball the effect of baryons can be ignored and therefore the evolution is that of a pure photon-lepton fireball( $\tau_b \ll 1$ ). When at the end of the evolution the temperature drops below  $T_p$  (pair production temperature) and  $\tau_p$  becomes 1, the fireball is radiation-dominated and most of the energy escapes as radiation[14].

(ii)  $\eta_{pair} > \eta_i > \eta_b$ . In this type of fireball the opacity of the baryons becomes significant resulting in a temperature that drops much further below  $T_p$  before the fireball becomes transparent. When the fireball eventually becomes transparent, most energy still escapes as radiation[14].

(iii)  $\eta_b > \eta_i > 1$ . In this case the fireball becomes matter-dominated before it becomes optical thin, this has as a result that most of the energy is converted to the bulk kinetic energy of the baryons, this case is considered the most promising case for GRBs[14].

(iv)  $\eta_i < 1$ . In this regime the fireball behaves Newtonian. Because of the low  $\eta_i$  almost all the energy is rest energy resulting in an expansion which will never become near relativistic and can be described by a classical blast wave[14].

#### 5.2 Relativistic scaling laws

Lets consider a spherical symmetric blast wave. In this case the relativistic conservation equations for baryon number and energy momentum reduce  $to^5$ 

$$\frac{\partial}{\partial t}(n\gamma) + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2nu) = 0, \qquad (49)$$

$$\frac{\partial}{\partial t} \left( e^{\frac{3}{4}} \gamma \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 e^{\frac{3}{4}} u \right) = 0, \tag{50}$$

$$\frac{\partial}{\partial t}\left(\left(n+\frac{4}{3}e\right)\gamma u\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(n+\frac{4}{3}e\right)u^2\right) = -\frac{1}{3}\frac{\partial e}{\partial r}.$$
(51)

To get a better feeling for these equations it is useful to transform the equations to an other

 $<sup>^{5}</sup>$ For the relativistic fluid equations see appendix 13.6

set of variables  $(r, t) \rightarrow (r, s \equiv t - r)$ . The equations then reduce to<sup>6</sup>[14]

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2nu) = -\frac{\partial}{\partial s}\left(\frac{n}{\gamma+u}\right),\tag{52}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2e^{\frac{3}{4}}u\right) = -\frac{\partial}{\partial s}\left(\frac{e^{\frac{3}{4}}}{\gamma+u}\right),\tag{53}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(n+\frac{4}{3}e\right)u^2\right) = \frac{\partial}{\partial s}\left(\left(n+\frac{4}{3}e\right)\frac{u}{\gamma+u}\right) + \frac{1}{3}\left(\frac{\partial e}{\partial s} - \frac{\partial e}{\partial r}\right).$$
(54)

In the situation of a fireball it is interesting to look at what happens if  $\gamma \gg 1$ . In this case the the right hand side of equations 52, 53 and 54 become very small compared to the left hand side. So as a first approximation the right hand side can be set to zero, resulting in the following conditions[14].

$$r^2 n\gamma = \text{constant},$$
 (55)

$$r^2 e^{\frac{3}{4}} \gamma = \text{constant},$$
 (56)

$$r^2\left(n+\frac{4}{3}e\right)\gamma^2 = \text{constant.}$$
 (57)

With these constants it is possible to rewrite equations 52-54 to scaling laws in the different regimes. In the case of radiation dominated where  $e \gg n$  this reduces to the equations below, where  $T_{obs} \propto \gamma e^{\frac{1}{4}}[14]$ ,

$$\gamma \propto r, \quad n \propto r^{-3}, \quad e \propto r^{-4}, \quad T_{obs} \propto \text{constant.}$$
 (58)

This can be done for a matter-dominated fireball resulting in the following equations [14]

$$\gamma \propto \text{constant}, \quad n \propto r^{-2}, \quad e \propto r^{-\frac{8}{3}}, \quad T_{obs} \propto r^{-\frac{2}{3}}.$$
 (59)

In general the radiation dominated phase ends when all the internal energy is converted into kinetic energy of baryons, this happens at the typical radius  $R_L = \eta R_0$ , after this radius the Lorentz factor  $\gamma$  becomes constant[13].

In the case of a fireball it is even possible to write equations 55-57 such that they are valid in both regimes. For this we define a quantity D, which is given by [14]

$$\frac{1}{D} \equiv \frac{\gamma_0}{\gamma} + \frac{3\gamma_0\rho_0}{4e_0\gamma} - \frac{3\rho_0}{4e_0}.$$
(60)

In this case the scaling relations can be rewritten to

$$r = \frac{r_0 \gamma_0^{1/2} D^{3/2}}{\gamma^{1/2}}, \quad \rho = \frac{\rho_0}{D^3}, \quad e = \frac{e_0}{D^4}.$$
 (61)

<sup>&</sup>lt;sup>6</sup>For a detailed derivations see appendix 13.8

#### 5.3 Evolution of fireballs

Using these relations we can make a plot of the different quantities as function of radius, this results in figure 7 and figure 8[14]. In the first part of the evolution of a fireball, in the radiation dominated phase, the fireball can be approximated by a pulse of energy with a frozen radial profile. This approximation helds quite well for the first part of the evolution. But fastly starts to break down at the biggest and smallest radii. When eventually the fireball enters the matter dominated phase which is at a radius of  $R_L = \eta R_0^7$  (red line). In this phase the frozen pulse pulse approximation is no longer valid and multiple shells start to catch up on other shells, which eventually will results in internal collisions which start to take place at a radius of  $R_s = \eta^2 R_0$  (violet line) till the phase at which the fireball slows down to its Newtonian phase [1, 13].

To explain the evolution of the fireball we explain the evolution of the multiple physical parameters one at a time. Initially our fireball starts at a radius which is in figure 7 and 8 given by 10<sup>6</sup> cm. At this initially radius a lot of energy is concentrated in a small volume and because of this, electron-positron pairs are created and the fireball starts to expand. The start of a fireball has a lot in common with our own big bang as will be explained in the following parts. In the first part of the evolution, the fireball is radiation dominated. This means that the Lorentz factor of the fireball starts to increase linearly until it reaches a radius of  $R_L = \eta R_0$  (red line), at which point the radiation dominated phase ends and the matter dominated phase starts. In this phase the Lorentz factor stays constant till the fireball starts to deaccelerate due to the interstellar medium around the fireball. This means that the matter dominated phase ends after the photon sphere, which is given as the green line in figure 7[1, 13, 14]. The photon sphere indicates after which radius photons can escape, this means that photons produced before the photon sphere will be reabsorbed by the shock waves.



Figure 7: Evolution of the Lorentz factor ( $\gamma$ ) and the energy density as function of the radius Besides the evolution of the Lorentz factor, in figure 7 also the evolution of the energy <sup>7</sup>In this equation  $R_0$  is the typical radius found from the time spectra, which is defined as  $R_0 = c\delta t$ 

density as function of radius is shown. As can be seen at the beginning of the fireball a huge amount of energy is present. This energy density starts to decrease while the fireball starts to expand. In the first part of the evolution the expansion is radiation dominated and this means that the energy density decreases as  $\propto r^{-4}$ , which is the same as the radiation dominated phase of the Big Bang. After this the radiation dominated phase ends and the matter dominated phase is reached. In this phase the energy density starts to decrease less slow and decreases as  $\propto r^{-8/3}$  which is very close to the matter dominated phase in our big bang, where the energy density decreases as  $\propto r^{-3}$ . This means that the energy density of a fireball scales more or less the same way as that of our Big Bang. But besides this it needs to be noticed that the evolution of the big bang is simpler than the evolution of a fireball. This is the case because the big bang is homogeneous and isotropic while the fireball model that describes GRBs is anisotropic which means that the problem becomes mathematical more complicated[1, 13, 14].

Also the evolution of the density and the energy over mass ratio is quite well understood and shown in figure 8. It can be seen in the case of density we have two regimes. The first regime is the radiation dominated phase in which the density scales as  $\propto r^{-3}$ . When the matter dominated regimes is reached the density starts to scale differently and will decreases as  $\propto r^{-2}[1, 13, 14]$ .



Figure 8: Evolution of the density and the energy over mass ratio  $(\eta)$  as function of the radius

Furthermore the energy over mass ratio  $(\eta)$  is also shown in figure 8. As can be seen  $\eta$  initially is high but will decrease in the first part as  $\propto r^{-1}$ , when almost all energy is used to obtain the high Lorentz factor,  $\eta$  will drop below 1 and the matter dominated phase is completely reached, this means that  $\eta$  immediately starts to scale differently and will decrease further as  $\propto r^{-2/3}[1, 13, 14]$ .

The matter dominated phase approximates the evolution of the fireball well but starts to break down shortly after the photo sphere is reached. The photo sphere can be calculated using equation 62, and has a typical distance of around  $10^{13}$  cm. In figure 7 and 8 the photo sphere is shown as the green line at a radius of around  $0.6 \cdot 10^{13}$  cm.

$$R_{photon} = \sqrt{\frac{\sigma_T E}{4\pi m_p c^2 \eta}}.$$
(62)

#### 5.4 Deceleration phase

After the photosphere is reached the evolution of the fireball starts to enter the deceleration phase. In this phase of the fireball there are two intrinsic length scales that influence the further evolution of the spherical shock wave. The first length scale is the width  $\Delta$  of the relativistic shell which is of the order of  $\Delta \sim r/\gamma$ . the second length scale is the Sedov length which is given by  $l = (E/n_1m_pc^2)^{1/3} \approx 10^{18} \text{ cm}[15]$ . Using both these length scales the further evolution of a GRB can be described. During this stage of the evolution the shock starts to interact with the interstellar medium, this can be described by two shocks, a forward and a reverse shock. In this case there are 3 important length radii, that describe what happens. The first is the radius at which the reverse shock becomes relativistic and starts to reduce the Lorentz factor of the shock wave. The second radius  $R_{\Delta}$  is the radius at which the reverse shock has crossed the shell and the third radius  $R_{\gamma}$  is the radius at which the total mass of the ISM is  $M/\eta[13]$ .

Initial fireball size	$R_0 = c\delta t[14]$
matter dominated	$R_L = R_0 \eta [1, 13]$
Internal collisions	$R_s = R_0 \eta^2 [13]$
Photo sphere	$R_{photon} = \sqrt{\frac{\sigma_T E}{4\pi m_p c^2 \eta}} [1]$
External shocks radius (RRS case)	$R_{\Delta} = l^{3/4} \Delta^{1/4} [1, 13]$
External shocks radius (NRS case)	$R_{\gamma} = l/\eta^{2/3}[13]$
Relativistic reverse shock radius	$l^{3/2}\Delta^{-1/2}\eta^{-2}[13]$
Sedov length	$l = (E/n_1 m_p c^2)^{1/3} [15]$

Table 1: Table of important radii of GRBs

Depending on the conditions there are two different cases. The first case is called the Newtonian reverse shock case and for this case we have  $R_s < R_{\Delta} < R_{\gamma} < R_N$ . The spreading of different shock waves is important and results in the release of a lot of energy. Also this case experiences a reverse shock wave that is just mildly relativistic compared with the forward propagating ultrarelativistic shock wave[15, 13].

In the second case, which is called the relativistic reverse shock case, we have that  $R_N < R_{\gamma} < R_{\Delta} < R_s$ . In this case there is reverse shock wave that quickly becomes relativistic and due to this there is no spreading. This means that internal collisions are unimportant and a frozen radial profile exists[15].

For the production of neutrinos the Newtonian reverse shock case is of most interest because of the existence of internal collisions before the photo sphere. After  $r > R_{\gamma}$  in the Newtonian reverse shock case or  $r > R_{\Delta}$  in the relativistic reverse shock case, the shock wave enters the regime of the relativistic Blandford-McKee self-similar deceleration phase that has a decreasing Lorentz factor that scales as  $\gamma \propto (E/\rho)^{1/2} R^{-3/2}$ , the Blandford-McKee self-similar solution starts to breaks down when the shock has a volume of  $l^3$  and enteres the nonrelativistic self-similar Sedov-Taylor solution that describes the further evolution of the shock wave of the GRB[13].

In figure 9 and 10 a plot of the evolution for the Newtonian reverse shock and the relativistic reverse shock are shown[13].



Figure 9: Evolution of the Lorentz factor  $\gamma$  of a Newtonian reverse shock wave with a low final Lorentz factor. The thick solid line represents the average value, the thin solid line represents the value just behind the forward shock, the dotted line represents the maximal value and the dash-dotted line, the analytical estimate[13].



Figure 10: Evolution of the Lorentz factor  $\gamma$  of a relativistic reverse shock wave, the thick solid line represents the average value, the thin solid line represents the value just behind the forward shock, the dotted line represents the maximal value and the dash-dotted line, the analytical estimate[13].

# 6 Energetics of gamma-ray bursts

In this section we discuss how efficiently the energy of the GRB is converted to radiation, whereafter we discuss how much energy can be converted as pre-GRB neutrinos due to pair annihilation. The rest of the energy of the explosion will be converted to higher energy neutrinos and cosmic rays.

## 6.1 Radiation

GRBs are famous for their high energy output in  $\gamma$ -rays in the  $10 - 10^3$  keV energy band. The short time variability in many GRBs is believed to arise due to mainly internal shocks in which multiple ejecta from the collision with different velocities collide producing short time variability in the  $\gamma$ -ray spectrum. Typical energies of GRBs, assuming isotropic explosions, is of around  $10^{53}$  ergs. But this is not the complete story, by far not all energy of a GRB is converted to radiation. Just a fraction of the total kinetic energy of a GRB is converted to thermal energy. This thermal energy is shared between protons, neutrons, electrons and the magnetic field. Of this thermal energy around one-third goes to the electrons which is the only thermal energy which is able to radiate away as  $\gamma$ -rays[16].



Figure 11: Efficiency for the conversion of the initial energy in a GRB in the energy band  $10 - 10^3$  keV, via internal shocks for different duration times of the GRB. The continuous curve corresponds to a maximum Lorentz factor of 200 and the dotted curve a maximum Lorentz factor of 500, both with a minimum Lorentz factor of 5. The dashed curve has a minimum Lorentz factor of 50 and a maximum of 200[16].

As can be seen in figure 11, the efficiency of producing radiation is around 1%-2% for long

duration GRBs while for short duration GRBs the efficiency of producing  $\gamma$ -rays is much lower then 1% in all cases. In this analysis the main contributor is Bremsstrahlung from the electrons and positrons. This means that the rest of the energy is going somewhere else, the most likely way of these high energy loss is due to neutrino production in the fireball and cosmic rays like high energy protons[16].

# 6.2 Energy loss due to neutrino production from $e^- - e^+$ annihilation

In the case of energy loss by  $e^- - e^+$  annihilation, the energy loss rate is given by [16]

$$\frac{\mathrm{d}E_n}{\mathrm{d}t} = -2n_e c\sigma_e \epsilon_e (4\pi r^2 r_0 n_e) \tag{63}$$

In this equation  $E_n = E/N$  (E, total energy and N, number of shells),  $n_e$  is the number density of electrons,  $\epsilon_e$  is the mean thermal energy of the electrons,  $\sigma_e$  is the cross section for  $e^- - e^+$  annihilation to produce neutrinos of all the flavors, which is given by  $\sigma_e = 2 \times 10^{-44} (\epsilon_e/1 MeV)^2 \text{cm}^2$ , further  $E_n \approx 12\pi r^2 r_0 n_e \epsilon_e \gamma$ ,  $n_e = 2.34 \times 10^{34} T_{10}^3 \text{ cm}^{-3}$  ( $T_{10} = T/10$ Mev), and  $\epsilon_e = 3.15kT$ . If these equations are combined this results in [16]<sup>8</sup>

$$\frac{\mathrm{d}\ln E}{\mathrm{d}t} = -\frac{9.5 \times 10^3}{\gamma} T_{10}^5 \tag{64}$$

This equation can be integrated to find the total energy loss due to electron-positron annihilation to neutrinos and is given by [16]

$$\ln\left[\frac{E(2t_0)}{E(t_0)}\right] = -1.9 \times 10^3 t_0 \left(\frac{T_0}{10 \text{Mev}}\right)^5$$
(65)

In this equation  $t_0$  is the largest value of  $r_0/c$  and the time when the shells become optical thin to electron neutrinos. Shells become optical thin to electron neutrinos when their  $T_0 \leq 10.2 \text{MeV}[16]$ .

These pre-GRB neutrinos will escape from the GRB with an energy between 10 and 30 MeV. These neutrinos would not be detectable at Earth using todays neutrino telescopes.

<sup>&</sup>lt;sup>8</sup>For derivation of this equation see appendix

# 7 Neutrinos

Neutrinos are subatomic particles from the standard model which have no charge have a low mass and are only participating in the weak interaction and gravitational force. Therefore the neutrino is by far the strangest particle from the standard model. Today neutrinos can be detected with enormous detectors like Icecube and Super-Kamiokande. Because of this a new branch of science has appeared. The science that connects the production of neutrinos with other phenomenons in the universe. Till now it is known that supernovae produce neutrinos, by a the famous detection of Super-Kamiokande and other neutrino observatories, they detected neutrinos a pulse of neutrinos just before the light of supernova 1987A reached the earth[17].

# 7.1 Neutrino production

There are multiple ways to produce neutrinos. In this thesis we mainly focus on one method to produce high energy neutrinos but first give a short summary of the multiple ways to produce high energy neutrinos. One way to produce neutrinos is by means of internal collisions, in this proces multiple shells are produced that move with Lorentz factors that are slightly different from each other. Because of this faster shells start to catch up with slower moving shells causing collisions between shells, producing high energy neutrinos. This method of internal collisions is the main focus of this thesis. Besides the internal collisions neutrinos can also be produced due to the reverse shock of the shock wave. Furthermore the collision of the shock waves with the interstellar medium will result in the production of neutrinos. This method will probably produce less energetic neutrinos than those generated by internal shocks because when the shock wave collides with the interstellar medium the speed of the shock wave is already much lower than initially. The last method is due to jets drilling through the envelope of the progenitor producing collisions that produce neutrinos[3].

In addition there is production of less energetic neutrinos during a GRB. During the formation of a GRB probably a lot of energy escapes as low energy neutrinos due to inverse beta decay. Also during and after the explosion the neutrons in the explosion will decay to protons and low energy beta decay neutrinos. These low energy neutrinos are not of our interest, this means that we mainly focus on the most promising candidate, internal collisions, to produce high energy cosmic neutrinos[1].

# 7.2 Pp and pn-interaction

pp and pn collisions are different ways to produce high energy neutrinos but they are based on the same mechanism. What these two interactions have in common is that by this interaction a proton has an interaction with a neutron or an other proton. As a consequence of this high energy collisions mesons are produced. After this collision the mesons decay to lighter elementary particles. In these collisions mainly two types of mesons are produced, the pions and kaons. After the production the kaons and pions decay to produce  $\gamma$ -rays and neutrinos[18]. In our analysis we will assume that there are mainly pions formed and therefore ignore heavier mesons like kaons which potentially can be produced in shell collisions.

#### 7.2.1 Looking at the available energy

In GRBs multiple shock waves are formed, these are formed shortly after each other resulting in multiple shock waves that catch up on eachother. This means that between these different shock waves there is a difference between the Lorentz factor. This means that during the collisions of protons with photons, neutrons or other protons, there is extra available energy. In general the distance traveled by different shells is given by

$$d_1 = c\beta_1 t_1 = c\left(1 - \frac{1}{2\gamma_1^2}\right)t_1,$$
(66)

$$d_2 = c\beta_2 t_2 = c\left(1 - \frac{1}{2\gamma_2^2}\right)t_2.$$
(67)

Using this, the typical difference between two moving shells can be calculated and the difference in Lorentz factor can be calculated in the case that the shells are at the same place,

$$0 = d_1 - d_2 = (t_1 - t_2) + \frac{t_2}{2\gamma_2^2} - \frac{t_1}{2\gamma_1^2},$$
(68)

$$= \Delta t - \frac{\bar{t}}{2} \left( \frac{1}{\gamma_1^2} - \frac{1}{\gamma_2^2} \right).$$
 (69)

This can be rewritten to

$$\Delta t = \frac{\bar{t}}{2} \left( \frac{1}{\gamma_1^2} - \frac{1}{\gamma_2^2} \right),\tag{70}$$

$$=\frac{t}{2}\left(\frac{1}{(\gamma-\Delta\gamma)^2}-\frac{1}{(\gamma+\Delta\gamma)^2}\right),\tag{71}$$

$$=\frac{t}{2\gamma^2}\left(\frac{1}{(1-\frac{\Delta\gamma}{\gamma})^2}-\frac{1}{(1+\frac{\Delta\gamma}{\gamma})^2}\right),\tag{72}$$

$$=\frac{\bar{t}}{2\gamma^2}\left(1+\frac{2\Delta\gamma}{\gamma}-1+\frac{2\Delta\gamma}{\gamma}\right),\tag{73}$$

$$=\frac{2t\Delta\gamma}{\gamma^3},\tag{74}$$

$$\to \Delta \gamma = \frac{\Delta t \gamma^3}{2\bar{t}}.\tag{75}$$

Using this result it is possible to calculate typical differences between Lorentz factors. In general typical Lorentz factors are around  $\gamma = 1000$  and average times are of the order of  $10^{13}/c \approx \frac{10^3}{3}$ s, further typical time differences in GRBs are of the order of  $\Delta t = 10^{-4}$ . this means that realistic differences of Lorentz factors can be around  $\Delta \gamma \approx \frac{3 \cdot 10^{-4} 10^9}{2 \cdot 10^3} \approx 100$ . This

high value means that typical shells can move towards each other in the center of mass with typical Lorentz factors of  $\gamma \approx 100$ , which means that during a pp or pn collision there is an available energy of  $E = 2\gamma m_p c^2$ , which is around 200 GeV.

## 7.3 Pion multiplicity

As discussed in the previous subsection, during the collisions there is 200 GeV available energy to produce particles. So what does this mean for the amount of produced pions and there energy? To calculate this we look at the pion multiplicity of pp collisions at a different energies. As shown in figure 12, it can be seen that at 10 GeV in a pp collision around 6 pions are produced. If this is linearly extrapolated, this means that in totally 20 pions can be produced for an available energy of 200 GeV. Which also means that every pion on average will get an energy of 10 GeV.



Figure 12: Pion multiplicity per participating nucleon[19]

#### 7.3.1 Pion decay or pion interaction

Once it is known how many pions can be produced in internal collisions we look at the production of neutrinos which are produced during the decay of pions. Our first assumption is that during the fireball, the density is such that equation 76 is valid. In equation 76, l is the mean free path of the particles and  $\tau$  is the lifetime of the particles[20].

$$\gamma = \frac{E}{mc^2} < \frac{l}{c\tau}.$$
(76)

If this relation is obeyed it means that the pions will not interact with the shock waves and will decay, using this condition an upperlimit for the density can be calculated. Because we know that the mean free path is given by  $l = (\sigma n)^{-1}$  and the cross section can be written as  $\sigma = \frac{\langle m \rangle}{\lambda}$ . We can rewrite equation 76 to

$$\rho < \frac{\lambda}{c\tau\gamma}.\tag{77}$$

Using this equation it can be calculated that in general, if the density is below  $1.5 \cdot 10^{-4}$  g cm<sup>-3</sup> or  $1.5 \cdot 10^{-3}$  g cm<sup>-3</sup> the density is such that the pions will decay instead of have an interaction with the shock waves. For this reasonable values for  $\gamma$  were assumed of between 100 and 1000, and a  $\lambda = 120$  g cm<sup>-2</sup>. As can be seen in previous graphs of the density, these values are reached easily infront of the photon sphere. This means that this production mechanism of neutrino production is a realistic candidate. It needs to be added that the graphs 7 and 8 have a relative low  $\eta$  which means that the density of this fireball is kind of an upperlimit for the density, this means that there are also fireballs with a much lower density.

#### 7.3.2 Decay processes

When the previous mentioned conditions are satisfied, pions will decay and produce neutrinos via the following decay processes for the 3 different pions[21],

$$\pi^0 \to 2\gamma,\tag{78}$$

$$\pi^+ \to \mu^+ + \nu_\mu, \tag{79}$$

$$\pi^- \to \mu^- + \bar{\nu}_\mu. \tag{80}$$

As can be seen, the neutral pion decays to 2 photons and the charged particles produce a neutrino and decay to a muon. After this the muon will also decay, producing 2 extra neutrinos in the following reactions[21],

$$\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e, \tag{81}$$

$$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e. \tag{82}$$

As can be seen, on average every pion produces two neutrinos, this means that during the collision on average two neutrinos are produced and in every three collisions two photons are produced. The neutrinos will escape due to the long mean free path they have, but the produced  $\gamma$ -rays by  $\pi^0$ -decay will be reabsorbed by the shock wave and only escape when it reaches the photon sphere. Using all this information, it is also possible to estimate the energy of the neutrinos, which will be 1/4 of the energy of the pion. This means that the energy of the neutrinos are on average 2.5 GeV.

#### 7.4 Lorentz boost for neutrinos

During the collision of shells of baryons in a GRB pions are formed. Pions are unstable and decay to a muon and muon-neutrino. During this decay the direction of the muon-neutrino or electron-neutrino is arbitrary in the frame that moves with the center of mass. To calculate the observed energy of a arbitrary neutrino at a large distance, the energy in the frame that moves with a Lorentz factor ( $\gamma \sim 100 - 1000$ ) need to be transformed back to the observer frame which is not moving. To do this a Lorentz boost tensor is used to calculate the energy in the observers frame.

The first part of calculating the energy of the neutrino in the observed frame, is calculating the direction of the neutrino in the moving frame. To do this a random point on a sphere need to be generated. It is known that the solid angle of a sphere is given by equation 83, which can be rewritten to equation 84.

$$d\Omega = \sin\theta \, d\phi \, d\theta, \tag{83}$$

$$= \mathrm{d}\phi\,\mathrm{d}(\cos\theta). \tag{84}$$

This means that it is possible to generate random points on a sphere by calculating the angles  $\phi$  and  $\theta$  in the following way, in which R represents a random number between 0 and 1.

$$\theta = \arccos(2R - 1),\tag{85}$$

$$\phi = 2\pi \cdot R. \tag{86}$$

This procedure was used to generate 1000 random points on a sphere. This resulted in the following 3d plot, it can be seen in this plot that the distribution has no preferred direction.

Because we are interested in the situation shown in figure 14, the direction of the angle  $\phi$  does not matter at all. This means that in the simulation, only random angles  $\theta$  are generated. After all the random angles  $\theta$  are generated, the goal is to calculate the energy in the observers frame. This can be done using a Lorentz boost  $(p^{\mu\prime} = \Lambda^{\mu}{}_{\nu}p^{\nu})$ . Using this and the approximation for neutrinos that  $E^2 - p^2 = m^2 \approx 0 \rightarrow E^2 = p^2$ , the four-momentum can be rewritten to.

$$\begin{bmatrix} E\\ p_x\\ p_y\\ p_z \end{bmatrix} = \begin{bmatrix} E\\ E\cos\theta\\ E\sin\theta\cos\phi\\ E\sin\theta\cos\phi \end{bmatrix} = E \begin{bmatrix} 1\\ \cos\theta\\ \sin\theta\cos\phi\\ \sin\theta\cos\phi \end{bmatrix} , \qquad (87)$$

Without loss of any generality it can be said that the frame is moving in the x-direction. This means that the observed energy can be calculated using a Lorentz boost tensor



Figure 13: Plot of 1000 random points on a sphere  $\theta$   $\gamma = 1000$ 

Figure 14: Situation of the moving shell and the random angle  $\theta$ 

$$p^{\mu\prime} = E \begin{bmatrix} \gamma & \beta\gamma & 0 & 0\\ \beta\gamma & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1\\ \cos\theta\\ \sin\theta\cos\phi\\ \sin\theta\cos\phi \end{bmatrix} = E \begin{bmatrix} \gamma(1+\beta\cos\theta)\\ \gamma(\beta+\cos\theta)\\ \sin\theta\cos\phi\\ \sin\theta\cos\phi\\ \sin\theta\cos\phi \end{bmatrix} ,$$
(88)

In this equation  $\beta$  is given by  $\frac{\sqrt{\gamma^2-1}}{\gamma} \approx 1 - \frac{1}{2\gamma^2}$ . Using these equations a program was written that calculated the observed energy in the observed frame (for source code see appendix 13.12.3). In the case of a shock wave with a Lorentz factor of  $\gamma = 1000$  this results in the energy spectrum for 1000 neutrinos shown in figure 15.

In general it is also possible to find the energy spectrum by analytical means. Because of the randomness on a sphere,  $\cos \theta$  varies between 1 and -1 with equal probability. This directly means that for any energy between  $E_0(\frac{1}{2\gamma})$  and  $2\gamma E_0(1-\frac{1}{(2\gamma)^2})$ , there is an equal probability of finding a neutrino in this energy range, from this it can be seen that the maximum energy of an neutrino is  $2\gamma E_0$ .


Figure 15: Energy spectrum for a GRB with a Lorentz factor of  $\gamma = 1000$  and 30.000 neutrinos

# 7.5 Realistic spectrum

Because in practice the energy spectrum of neutrinos in the comoving frame is not equal to a delta function but often has the shape of a gaussian function with a tail. These kind of distributions look like the beta function and some  $\chi^2$ -distributions. Because of this similarity the standard Python function are used, these function are numpy.random.beta and numpy.random.chisquare. These function generate random numbers on the domain of these probability functions. This resulted in the following realistic energy spectrum of neutrinos produced by internal collisions in a GRB.



Figure 16: Energy spectrum for a GRB with a Lorentz factor of  $\gamma = 1000$ . Monte Carlo simulation of 300.000 neutrinos plotting only neutrinos above 1 GeV

As can be seen in figure 16, a lot of high energy neutrinos can be produced of the order of

 $10^{3.5}$  GeV. These neutrinos can potentially be detected by neutrino telescopes.

# 8 Icecube

The IceCube Neutrino Observatory is the first detector of its kind, specially designed to observe the most violent events in space. These observations are done from deep inside the South Pole ice. IceCube is a cubic-kilometer neutrino detector made of Antartic Ice. It has a hexagonal shape and extends to a depth of around 2500 meters. IceCube itself consists of two parts, a surface array, called Icetop and a deeper denser inner detector called deepcore. Deepcore consists of 86 vertical strings consisting of 60 modules with photomultiplier tubes (DOMs), these 86 strings are spread equally over a hexagonal grid[22].



Figure 17: Schematic image of Icecube[22]

# 8.1 Detection

Today IceCube has a detection threshold of around 10 GeV. The detection of neutrinos is done using the fact that when neutrinos collide with the ice they produce electrons, muons or tauons. This depends on the initial flavor of the neutrinos<sup>9</sup>. When these electrons, neutrinos or tauons are produced, they are detected using the produced Cherenkov radiation of the charged leptons. This Cherenkov radiation is detected using the DOMs in the deepcore of IceCube which measure the different arrival times of the Cherenkov light. Using this a difference can be seen between electrons, muons and tauons. Using this it can be determined what flavor the detected neutrinos has[22].

<sup>&</sup>lt;sup>9</sup>for detailed reactions see appendix 13.11



Figure 18: Example of an interaction of a neutrino with the ice[22]

# 8.2 Astrophysical neutrinos above 1 TeV

Between 2010 and 2012, IceCube has detected numerous events with energies of more than 1 TeV. Using these events IceCube is able to set constraints on the energy spectrum of the astrophysical neutrino flux. This was done using the fact that at IceCube they detect a flux which consists of two components, an atmospheric component and a part that consists of high energy neutrinos from astrophysical origin. Because today it is unknown what exactly produces the high energy neutrinos and the fact that there is a limited number of detected neutrinos events, icecube can only test very simple models.[23]

For the models of the high energy neutrinos, it is assumed that they are distributed isotropically on the sky, follow a power-law energy distribution and arrive at the Earth in equal amounts of  $\nu_e$ ,  $\nu_{\mu}$  and  $\nu_{\tau}$  due to neutrinos oscillations. This all means that the astrophysical neutrino flux can be approximated by equation 89. In which  $\Phi_0$  is the neutrinoantineutrino flux for each flavor at the energy  $E_0 = 10^5$  GeV,  $\Phi_0$  is given in the units  $\text{GeV}^{-1} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1} [23]^{10}$ .

 $<sup>^{10}\</sup>mathrm{In}$  which the units of  $\mathrm{sr}^{-1}$  stands for the steradians of the detector and not the steradians of neutrinos sources

$$\Phi_{\rm astro} = \Phi_0 \left(\frac{E}{E_0}\right)^{-\gamma} \tag{89}$$

Between 2010 and 2012 IceCube had detected 388 events, of which 106 had more than 10 TeV of energy and 9 had even an energy above 100 TeV. This resulted in the following constraint of the neutrino flux[23]

$$\Phi_{\nu} = 2.06^{+0.4}_{-0.3} \times 10^{-18} \left(\frac{E_{\nu}}{10^5 \text{ GeV}}\right)^{-2.46 \pm 0.12} \text{ GeV}^{-1} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}, \quad 25 \text{ TeV} < E_{\nu} < 1.4 \text{ PeV}$$
(90)

This analysis did not take into account any correlation with astrophysical events[23]. After this 1 year later a new analysis was done, resulting in a more accurate constraint on the astrophysical energy flux of neutrinos which is given by  $10^{-8}$  GeV cm<sup>-2</sup> s<sup>-1</sup> sr<sup>-1</sup>[24].

#### 8.2.1 Neutrinos as unique probe

In many astrophysical events, we expect that pions are produced in the interaction of cosmicrays with radiation or gas or in the most violent events itself in the universe. These produced neutrinos can travel long distances, during which they barely interact and reach their final destiny undisturbed compared to high energy photons which are often absorbed by gas clouds or interact with the CMB photons (GZK effect), and charged particles which are deflected by magnetic fields. This makes neutrinos an unique probe for studying violent events in the universe[24].

### 8.3 GRB limits Icecube

To explain the detected astrophysical neutrino flux detected by IceCube, multiple sources for high energy neutrinos were proposed. One of the most promosing candidates to explain the astrophysical neutrino flux are GRBs. The models of GRBs have proposed certain neutrino fluxes, but today these fluxes seems to be too high. IceCube has found an upperlimit on the neutrino flux from GRBs which is  $2.06^{+0.4}_{-0.3} \times 10^{-8}$  GeV cm<sup>-2</sup> s<sup>-1</sup> sr<sup>-1</sup>. Using the available data IceCube was able to set a limit on the neutrino energy flux which resulted in figure 19. As can be seen in this plot, the model of Ahlers et al can be ruled out, because it expects a too high neutrino flux[25].

At IceCube one is presently investigating if this non-observance of neutrinos is due to inadequate modelling of the flux or due to a time delay between the emission of neutrinos and gamma rays from a GRB.



Figure 19: Exclusion plot of energy flux and neutrino break energy[25]

# 9 Time difference of the detection of $\gamma$ -rays and neutrinos

Till today IceCube has not detected any prompt neutrinos of GRBs, because of this we are wondering why this is the case and one of the reasons why we do not detect any neutrinos could be because there is some kind of time delay between the arrival time of photons and neutrinos at Earth. In this analysis we only look at the time difference between the detection of  $\gamma$ -rays and the neutrinos produces in internal collisions.

### 9.1 Time difference due to the neutrinosphere and photosphere

During the first part of the fireball the neutrinos escape ealier than the photons, because the neutrinosphere is almost present at the radius at which the neutrinos are produced, which is smaller than the photosphere. This means that neutrinos built up a time difference between photons because they escape earlier than the photons. Produced photons are only able to escape after they have passed the photonsphere which is a few orders of magnitude bigger than the neutrinosphere.

In the most general case the internal collisions take place between distances of  $10^{11}$  cm –  $10^{14}$  cm. At the time of the first collisions the photons are not able to escape yet, this means that neutrinos are detected before the detection of the  $\gamma$ -rays. The photons escape at a typical distance of  $(\sigma_T E/4\pi m_p c^2 \eta)^{1/2} \approx 10^{13}$  cm[1].

Using these typical distances it is possible to estimate the time difference between the detection of the neutrinos and photons. Assuming that neutrinos already travel with the speed of light and the photons will escape at the photonsphere and will be produced by thermal photons in the shock wave, we obtain

$$\Delta t = \frac{s}{c} \left(\frac{1}{\beta} - 1\right) = \frac{s}{c} \left(\frac{\gamma}{\sqrt{\gamma^2 - 1}} - 1\right) = \frac{s}{2c\gamma^2}.$$
(91)

Using the typical distance of  $s \approx (10^{13} - 10^{12})$  cm =  $.9 \cdot 10^{13}$  cm and typical Lorentz factors of  $\Gamma \sim 100 - 1000$  the expected time difference is between  $\Delta t = 15 \ \mu s$  and  $\Delta t = 1.5$  ms. This is not a significant time difference between the detection of neutrinos and  $\gamma$ -rays.

## 9.2 Time difference due to the stellar wind of the stars

Possible candidates of long GRBs are heavy stars of around  $20M_{\odot} - 100M_{\odot}$ , these stars produce during their lifetime stellar winds, these stellar winds form a hydrogen cloud around the star. If during a GRB the photons and neutrinos escape, the neutrinos will travel unhindered to the earth. The light will travel a bit slower then the speed of light because it has a refraction index due to the intersellar medium it travels through. In general the time delay of the light and the neutrinos is given by

$$dt = \frac{dr}{c}(n-1).$$
(92)

To calculate this, the refraction index need to be calculated for  $\gamma$ -rays. In general the refraction index is given by [26]

$$n = 1 + \delta(E) + i\beta(E) \approx 1 + \delta(E).$$
(93)

In this equation  $\beta$  represents the influence of absorption and this will be ignored, because only the photons which are not absorbed are interesting for the problem. In general  $\delta$  is given by

$$\delta = \frac{r_e n}{2\pi} \left(\frac{hc}{E}\right)^2 f_1. \tag{94}$$

In this equation  $r_e$  is the classical electron radius, n the number density of atoms, E the energy of the photons and  $f_1$  a constant which is approximately equal to Z.

To calculate the time difference one more quantity need to be determined, which is the number density of the matter around the star, this is given by

$$\mu m_H n = \rho = \frac{\mathrm{d}M}{\mathrm{d}V} = \frac{\mathrm{d}M}{4\pi r^2 \mathrm{d}r} = \frac{\frac{\mathrm{d}M}{\mathrm{d}t}}{4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t}} = \frac{\dot{M}(r)}{4\pi r^2 v(r)}.$$
(95)

This results in,

$$dt = \frac{dr}{c}\delta = \frac{dr}{c}\frac{r_e n}{2\pi} \left(\frac{hc}{E}\right)^2 f_1,$$
(96)

$$= \frac{\mathrm{d}r}{c} \frac{r_e}{2\pi} \left(\frac{hc}{E}\right)^2 \frac{Z}{\mu m_H} \frac{\dot{M}(r)}{4\pi r^2 v(r)}.$$
(97)

This can be integrated to give,

$$\Delta t = \frac{r_e}{8\pi^2} \frac{h^2 c}{m_H E^2} \int_{r_0}^{r_{final}} \frac{Z(r)\dot{M}(r)\mathrm{d}r}{\mu(r)v(r)r^2}.$$
(98)

In most cases it can be assumed that the stellar wind has a constant velocity and mass loss and consists of pure hydrogen, this means that the above equation can be rewritten to,

$$\Delta t = \frac{r_e}{8\pi^2} \frac{h^2 c \dot{M}}{m_H E^2 v} \int_{r_0}^{r_{final}} \frac{\mathrm{d}r}{r^2},$$
(99)

$$= \frac{r_e}{8\pi^2} \frac{h^2 c \dot{M}}{m_H E^2 v} \left( \frac{1}{r_0} - \frac{1}{r_{final}} \right).$$
(100)

In all most all cases of stellar winds  $r_{final} \gg r_0$ , this means that the time delay is given by,

$$\Delta t = \frac{r_e}{8\pi^2} \frac{h^2 cM}{m_H E^2 v r_0}.$$
 (101)

For typical values the equation can be rewritten to,

$$0.7 \left(\frac{\dot{M}}{10^{-4} \,\mathrm{M_{\odot} \, yr^{-1}}}\right) \left(\frac{10 \,\mathrm{keV}}{E}\right)^2 \left(\frac{100 \,\mathrm{km \, s^{-1}}}{v}\right) \left(\frac{10^{13} \,\mathrm{cm}}{r_0}\right) \,\mathrm{fs.}$$
(102)

This result basically means that the time difference between the detection of neutrinos and the detections of photons can be ignored.

#### 9.2.1 Comparison with optical light

So the  $\gamma$ -rays have a short time delay compared to the neutrinos. So how does this change if the photons are optical photons? In this case we know that the refractive index is given by,

$$n \approx 1 + e\rho. \tag{103}$$

In which  $\rho$  is the density of the medium and e is an emperical constant. In the case of the air on earth, these values are n = 1.000293 and  $\rho = 1.2754 \cdot 10^{-3}$ . Using these values e can be found to be  $e = 2.297 \cdot 10^{-1}$  cm<sup>3</sup> g<sup>-1</sup>. This means that,

$$\Delta t = \frac{\mathrm{d}r}{c} e\rho. \tag{104}$$

Which immediately reduces to,

$$\Delta t = \frac{eM}{4\pi cvr_0},\tag{105}$$

$$\Delta t = 3.84 \cdot 10^2 \left( \frac{\dot{M}}{10^{-4} \,\mathrm{M_{\odot} \, yr^{-1}}} \right) \left( \frac{100 \,\mathrm{km \, s^{-1}}}{v} \right) \left( \frac{10^{13} \,\mathrm{cm}}{r_0} \right) \,\mathrm{s.}$$
(106)

This difference is significant, but in the case of GRBs there is no optical light from the GRB but from the afterglow, so this does not matter for the time difference between the neutrinos and  $\gamma$ -rays.

### 9.3 Time difference due to mass of neutrinos

Today it is not yet known what the precise mass of neutrinos are. Nowadays there are upper limits for the neutrinos mass found by looking at decay spectra, this resulted in an upper limit for the electron neutrino of 2 eV, this low mass can potentially result in a measurable time difference between the detection of neutrinos and photons of a GRB at cosmological distances. The time difference between the photons and neutrinos is given by

$$dt = \frac{ds}{c} \left(\frac{1}{\beta} - 1\right),\tag{107}$$

$$=\frac{\mathrm{d}s}{c}\Big(\frac{\gamma}{\sqrt{\gamma^2-1}}-1\Big).\tag{108}$$

In the case that the Lorentz factor is  $\gamma \gg 1$ , these relations reduce to

$$dt = \frac{\gamma ds}{c} \left( \frac{1}{\sqrt{\gamma^2 - 1}} - \frac{1}{\gamma} \right), \tag{109}$$

$$\approx \frac{\gamma \mathrm{d}s}{c} \left(\frac{1}{\gamma} + \frac{1}{2\gamma^3} - \frac{1}{\gamma}\right) = \frac{\mathrm{d}s}{2\gamma^2 c}.$$
(110)

This equation can be calculated easily if the gamma factor does not depend on the redshift, if this is the case. The equation reduces to

$$\Delta t = \frac{\operatorname{ltt}(z)}{2\gamma^2}.$$
(111)

In this equation ltt(z) is the light travel time. The light travel time as function of redshift is given by figure 20[27].



Figure 20: lighttraveltime as function of redshift[27].

This means that the time difference between the detection of neutrinos and photons can be rewriten in terms of LLT(z), where LLT(z) is the light travel time in Gyr and the energy and mass of the neutrino  $(\gamma = \frac{E}{m_{\nu}c^2})$ :

$$\Delta t = \frac{\pi \cdot 10^{16} \cdot \text{LLT}(z)}{2\frac{E^2}{m_{\nu}^2 c^4}} = 5\pi \cdot 10^{-2} \text{LLT}(z) \left(\frac{m_{\nu}}{1 \text{ eV}}\right)^2 \left(\frac{10 \text{ TeV}}{E}\right)^2.$$
 (112)



Figure 21: Time difference as function of redshift, for a neutrino with E = 10 TeV and E = 10 GeV.

This means that the time difference between the detection of photons and neutrinos is very small and can in general be ignored. To make the estimates more accurate of the time difference, lets assume that neutrinos undergo redshift the same way as photons undergo redshift and thus  $E \propto (z+1)^{-1}$ , this means that the energy spectrum changes and the time difference should increase less rapidly with redshift. This means that  $\gamma = \frac{E}{m_{\nu}}$  can be rewritten to  $\gamma = \frac{E_0}{m_{\nu}}(1+z)$ . In general this means that the time difference is given by

$$\Delta t = \int_0^z \frac{\mathrm{d}(\mathrm{llt}(z))}{2\gamma^2},\tag{113}$$

$$= \frac{m_{\nu}^2}{2E_0^2} \ 1 \ \text{Gyr} \ \int_0^z \frac{\mathrm{d}(\mathrm{llt}(z))}{(1+z)^2},\tag{114}$$

$$= 5\pi \cdot 10^{-2} \left(\frac{m_{\nu}}{1 \text{ eV}}\right)^2 \left(\frac{10 \text{ TeV}}{E}\right)^2 \int_0^z \frac{\mathrm{d}(\mathrm{LLT}(z))}{(1+z)^2}.$$
 (115)

This results in a time difference which is a factor 3 smaller as shown in figure 22 This all basically means that the time difference between the detection of neutrinos and photons is negligible.



Figure 22: Time difference as function of redshift, for a neutrino with E = 10 TeV and E = 10 GeV if it arrives at the earth. This plot also takes into account that the energy of neutrinos decreases due to redshift.

# 10 Summary

In this thesis we have discussed the different progenitor models of GRBs of which the main candidates are neutron star-neutron star mergers for short GRBs and collapsers for long GRBs. Also the observational constraints of GRBs and basic shock physics were discussed, to eventually the physics of relativistic shock waves like fireballs. After this we looked at the energetics of GRBs, like the photon production efficiency which seems to be quite low of around 1%, after this also a short look was taken at low energy neutrinos formed due to the annihilation of electron-positron pairs. As last we looked at neutrino production in GRBs due to internal collisions and came to the conclusion that 10 TeV neutrinos can be produced in ordinary GRBs. Besides this we looked at the time difference between the detection of neutrinos and photons taking into account earlier escape of neutrinos, refractive indices of clouds around the GRB and a finite mass for neutrinos. This resulted in time differences of the order of millisecond neutrinos, which is not a significant time difference between the detection of neutrinos and photons.

# 11 Discussion

Still there are some open questions that need to be answered. The absolute estimate of the amount of neutrinos at Earth is a real challenge, and could not be estimated properly within the scope of this work, This means that it is not possible to look if this GRB model of the fireball produces a reasonable amount of neutrinos which are not above the limits set by IceCube. Furthermore in this thesis we only looked at the prompt neutrinos production and pre-GRB neutrinos. Besides this also neutrinos can be produced in external collisions, which could have a time delay compared to the  $\gamma$ -rays detected at Earth. Furthermore it still remains a mystery what can cause the observed  $\gamma$ -rays produced at Earth. These open questions could be answered in future research.

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# 13 Appendix

### 13.1 T90 and Hardness

GRBs are often classified by 2 important quantities, t90 and hardness. t90 is the time interval in which the  $\gamma$ -ray detectors receive 90% of the flux of the total burst. This means that in the case of a spectra as shown in figure 23, t90 can be calculated by first subtracting the  $\gamma$ -ray background shown in blue after which the 90% of the area in this graph is selected and the coresponding time it takes to receive this flux. How to precisely calculate t90 goes behond the scope of this thesis, and so we will not discuss that. Furthermore we also have the property hardness. When in 1991 the BATSE was launched with as main goal to specially target GRBs it had 4 channels. Hardness is defined on the received flux of channel 2 and 3 of BATSE, This means that Hardness =  $I_3/I_2$  and in the case of figure 23 this means that hardness is the blue area divided by the red area[1].



Figure 23: The left plot is a plot of a typical time spectra, blue is the  $\gamma$ -ray background and red is 90% of the received flux. On the right is a plot of an energy spectra, in which red is the channel 2 and blue channel 3[1].

### 13.2 Basic astrophysical hydrodynamics

Before looking at shocks it is useful to first look at the basics of astrophysical hydrodynamics. Hydrodynamics is the physics of fluids and knows a long history which goes back till the old Greeks with Archimedes and Archimedes principle which says that the buoyancy is equal to the weight of the displaced fluid. After this discovery of Archimedes there was no further development of hydrodynamics until the seventeenth and eighteenth century. Since this time with Pascal, Newton and Euler in the seventeenth and eighteenth century and other like Lord Kelvin and Helmholtz in the nineteenth century the field of hydrodynamics came to its full form as it is now tought in undergraduate courses on hydrodynamics[28].

Hydrodynamics is a field of physics which has a strong mathematical basis. This basis is founded on what are called conserved quantities, These conserved quantities are result in conservation equations which can be derived from the fundamental Boltzmann equation. In hydrodynamics there are 5 important conservation laws. The first equation is the continuity equation which says that mass is conserved and is given by equation 116, where  $\rho$  is the density of the fluid and  $\vec{\mathbf{u}}$  is the bulk velocity[29].

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{\mathbf{u}}) = 0.$$
(116)

Besides the conservation of mass, it is also assumed that momentum  $(\vec{p} = m\vec{v})$  is also conserved. Using the Boltzmann equation this results in what is called the Euler equation which is given by equation 117. Besides these equations it is usefull to define something which is called the Lagrangian derivative which is defined by equation 118 which can be used to rewrite the Euler equation in vector form to equation 119[29].

$$\frac{\partial \vec{\mathbf{u}}}{\partial t} + (\vec{\mathbf{u}} \cdot \vec{\nabla}) \cdot \vec{\mathbf{u}} = -\frac{\vec{\nabla}P}{\rho} + \vec{\mathbf{f}},\tag{117}$$

$$\left(\frac{\partial}{\partial t} + \left(\vec{\mathbf{u}} \cdot \vec{\nabla}\right)\right) f = \frac{Df}{Dt},\tag{118}$$

$$\frac{D\vec{\mathbf{u}}}{Dt} = -\frac{\vec{\nabla}P}{\rho} + \vec{\mathbf{f}}.$$
(119)

Furthermore in hydrodynamics it is assumed that there is conservation of energy, this can also be derived from the Boltzmann equation and results in equation 120, this one will be written in Einstein summation form because of the complexity of the equation. In equation 120,  $\rho\varepsilon$  is the specific internal energy,  $\pi_{ik}$  the viscous stress tensor and  $F_k$  the conducting heat flux [29].

$$\frac{\partial}{\partial t} \left( \frac{\rho}{2} u_k u_k + \rho \varepsilon \right) + \frac{\partial}{\partial x_k} \left( \frac{\rho}{2} u_i u_i u_k + u_i (P \delta_{ik} - \pi_{ik}) + \rho \varepsilon u_k + F_k \right) = -\rho u_k \partial_k \Phi.$$
(120)

In the case that a fluid is completely adiabatic we obtain an extra conservation equation in hydrodynamics for the entropy, which is given by [10]

$$\frac{Ds}{Dt} = 0. \tag{121}$$

#### 13.2.1 Polytropic fluids

Polytropic fluids are fluids for which the pressure is only dependent on the local density, and so the pressure of these fluids is independent of the temperature and other parameters. In general a polytropic fluid does satisfy equation 122 where K is a constant and n is the polytropic index[29].

$$P = K\rho^n. \tag{122}$$

One important application of polytropic fluids is that under adiabatic circumstances all fluids behave like polytropes because  $P\rho^{-\gamma} = \text{constant}$  under these circumstances, in the case of an adiabatic fluid the polytropic index is given by the adiabatic index ( $\gamma = \frac{C_P}{C_V}$ ). In the case of an adiabatic gas like specific enthalpy and the specific internal energy become easier to calculate and are related by equations 123 and 124, which mean that the enthalpy and the specific internal energy are related to the pressure over density ( $P/\rho$ ) by nothing more then a constant which is a function of the adiabatic index[29].

$$h = \frac{\gamma P}{(\gamma - 1)\rho},\tag{123}$$

$$e = \frac{P}{(\gamma - 1)\rho}.$$
(124)

Besides this the enthalpy of an adiabatic fluid can also be calculated relatively easy, using equation 125. This equation can be derived from the first law of thermodynamics<sup>11</sup>[30].

$$s = s_0 + \frac{k_B \ln K}{\mu m_H (\gamma - 1)}.$$
 (125)

#### 13.2.2 The speed of sound

To understand shock waves first a short introduction to sound waves and the speed of sound is given. The sound speed is the speed at which small amplitude waves travel through a medium, we can separate 2 different forms of the speed of sound. The first sound speed is the adiabatic speed of sound which is the speed of sound for adiabatic fluid, given by

<sup>&</sup>lt;sup>11</sup>For a detailed derivation of this equation see appendix 13.3

equation 126. The other speed of sound is one at constant temperature and is called the isothermal speed of sound and is given by equation 127[9].

$$c_s^2 = \left(\frac{\partial P}{\partial \rho}\right)_S,\tag{126}$$

$$c_{s,T}^2 = \left(\frac{\partial P}{\partial \rho}\right)_T.$$
(127)

Of these 2 sound speeds, the adiabatic sound speed will be mainly used in calculations involving shock waves, because shock waves are adiabatic. The adiabatic sound speed can be rewritten to a different form using the fact that for any adiabatic fluid  $P\rho^{-\gamma} = \text{constant}$ , which results in equation 128[9].

$$c_s^2 = \frac{\gamma P}{\rho}.\tag{128}$$

Besides these we will also define what is called the Mach number, this is a number which indicates the velocity of the flow in units of the speed of sound and this is given by equation 129[9].

$$M_i = \frac{v_i}{c_s} = \begin{cases} < 1 : \text{ subsonic,} \\ = 1 : \text{ sonic,} \\ > 1 : \text{ supersonic.} \end{cases}$$
(129)

## 13.3 Entropy of a polytropic fluid

In this section the specific entropy for a polytropic fluid will be derived. In the case of a polytropic fluid it is known that equations 123 and 124 are valid. Besides these it is also known that  $\rho V = M$ , U = Me and S = Ms. Using these known equations and the first law of thermodynamics it can be shown that the entropy of a polytropic fluid is given by equation 125[30].

dU = TdS - PdV (First law of thermodynamics)  $dS = \frac{dU}{T} + \frac{P}{T}dV$  (rewriting)  $Mds = \frac{M}{T}de + \frac{\rho k_B}{\mu m_H}dV$  (using known equation)

 $\frac{\mu m_H}{k_B} ds = \frac{\mu m_H}{kT} de + \rho d\left(\frac{M}{\rho}\right)$ (rewriting)  $= \frac{\rho}{T} - \rho \frac{d\rho}{T}$ (rewriting)

$$P = \frac{\rho^2}{\rho^2} \qquad (rewriting)$$

$$= \frac{\mathrm{d}e}{(\gamma - 1)e} - \frac{\mathrm{d}\rho}{\rho} \qquad (rewriting)$$

$$= \frac{1}{\gamma - 1} d\ln e - d\ln \rho$$
 (rewriting)

$$e = \frac{K\rho^{\gamma - 1}}{\gamma - 1} \tag{known}$$

$$d\ln e = \frac{\gamma - 1}{K\rho^{\gamma - 1}} \left( \frac{\mathrm{d}k\rho^{\gamma - 1}}{\gamma - 1} + \frac{\mathrm{d}\rho\rho^{\gamma - 2}(\gamma - 1)}{\gamma - 1} \right) \qquad (\text{calculate } \mathrm{d}\ln e)$$
$$= \frac{\mathrm{d}k}{M} + (\gamma - 1)\frac{\mathrm{d}\rho}{M}$$

$$k^{-1} (\gamma - 1) \rho$$

$$= d \ln k + (\gamma - 1) d \ln \rho$$

$$\frac{\mu m_H}{k_B} ds = \frac{1}{\gamma - 1} d \ln K$$
(substitute in above equation)
$$s = s_0 + \frac{k_B \ln K}{\mu m_h (\gamma - 1)}$$
(integrate)

# 13.4 Conservative equations for planar shocks

In this part the conservative equations for a planar shock will be derivated in more detail. First we start with deriving equation 4, followed by 5, 6 and as last by the derivation of equation 7.

## 13.4.1 Rewriting the continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{\mathbf{u}}) = 0 \qquad (\text{Writing out divergence})$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \left( \rho u_x \right) + \frac{\partial}{\partial y} \left( \rho u_y \right) + \frac{\partial}{\partial z} \left( \rho u_z \right) = 0 \quad \text{(Derivatives with respect to y and z are zero)} \\ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \left( \rho u_x \right) = 0 \quad (4)$$

## 13.4.2 Rewriting the Euler equation x-component

In this derivation it is explicit assumed that there is no external force or this external force is constant.

$$\frac{\partial u_x}{\partial t} + u_x \frac{\partial}{\partial x} u_x = -\frac{1}{\rho} \frac{\partial P}{\partial x} \qquad \text{(Euler equation)}$$

$$\rho \frac{\partial u_x}{\partial t} + \rho u_x \frac{\partial}{\partial x} u_x + \frac{\partial P}{\partial x} = 0$$
 (Rewriting)

$$\rho \frac{\partial u_x}{\partial t} + u_x \left( \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_x)}{\partial x} \right) + \rho u_x \frac{\partial}{\partial x} u_x + \frac{\partial P}{\partial x} = 0$$
 (Adding zero)

$$\rho \frac{\partial u_x}{\partial t} + u_x \left(\frac{\partial \rho}{\partial t}\right) + u_x \left(\frac{\partial (\rho u_x)}{\partial x}\right) + \rho u_x \frac{\partial }{\partial x} u_x + \frac{\partial P}{\partial x} = 0$$
 (Rewriting)

$$\frac{\partial(\rho u_x)}{\partial t} + \frac{\partial}{\partial x} \left(\rho u_x^2 + P\right) = 0 \tag{5}$$

## 13.4.3 Rewriting the Euler equation z-component

$$\frac{\partial u_z}{\partial t} + u_x \frac{\partial}{\partial x} u_z = -\frac{1}{\rho} \frac{\partial P}{\partial z}$$
 (Euler equation)

$$\rho \frac{\partial u_z}{\partial t} + \rho u_x \frac{\partial}{\partial x} u_z = 0 \qquad (\text{Rewriting})$$

$$\rho \frac{\partial u_z}{\partial t} + u_z \left( \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_x)}{\partial x} \right) + \rho u_x \frac{\partial}{\partial x} u_z = 0$$
 (Adding zero)

$$\frac{\partial(\rho u_z)}{\partial t} + \frac{\partial}{\partial x} \Big(\rho u_x u_z\Big) = 0 \tag{6}$$

#### 13.4.4 Rewriting the Energy equation

Assuming agian that the external forces can be assumed constant or not present, no viscousity and no heat conducting .

$$\frac{\partial}{\partial t} \left( \frac{\rho}{2} u_k u_k + \rho \varepsilon \right) + \frac{\partial}{\partial x_k} \left( \frac{\rho}{2} u_i u_i u_k + u_i (P \delta_{ik} - \pi_{ik}) + \rho \varepsilon u_k + F_k \right) = -\rho u_k \frac{\partial \Phi}{\partial x_k}$$
(Energy equation)  

$$\frac{\partial}{\partial t} \left( \frac{\rho}{2} u_k u_k + \rho \varepsilon \right) + \frac{\partial}{\partial x_k} \left( \frac{\rho}{2} u_i u_i u_k + u_i P \delta_{ik} + \rho \varepsilon u_k \right) = 0$$
(Using assumptions)  

$$\frac{\partial}{\partial t} \left( \rho \left( \frac{u^2}{2} + \varepsilon \right) \right) + \frac{\partial}{\partial x} \left( \rho u_x \left( \frac{u^2}{2} + \varepsilon + \frac{P}{\rho} \right) \right) = 0$$
(Rewriting)  

$$\frac{\partial}{\partial t} \left( \rho \left( \frac{u^2}{2} + \varepsilon \right) \right) + \frac{\partial}{\partial x} \left( \rho u_x \left( \frac{u^2}{2} + h \right) \right) = 0$$
(7)

### 13.5 Conservation in an infinitly small shock

In shock physics a lot of the equations look like equation 8, these equations with Q a quantity and F the flux of this quantity imply that the flux is conserved. Why that will be shown in this appendix by integrating and rewriting equation 8. In this analysis it is assumed that the shock has a thickness  $l_s$  and that the center of the shock is at x = 0.

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} = 0$$
(8)
$$\int_{-\frac{l_s}{2}}^{\frac{l_s}{2}} dx \frac{\partial F}{\partial x} = -\int_{-\frac{l_s}{2}}^{\frac{l_s}{2}} dx \frac{\partial Q}{\partial t}$$
(Rewriting and integrating)
$$F_2 - F_1 = -\int_{-\frac{l_s}{2}}^{\frac{l_s}{2}} dx \frac{\partial Q}{\partial t}$$
(calculating left side)
$$\Delta F = \frac{l_s}{2} \left( \frac{\partial Q_2}{\partial t} + \frac{\partial Q_1}{\partial t} \right)$$
(estimating right side integral)
$$\Delta F = 0$$
(take the limit of  $l_s \to 0$ )

This means that in an infinitly thin shock the fluxes are conserved [9, 10].

### 13.6 Relativistic hydrodynamics

In this thesis the focus is mainly on gamma-ray burst, these burst realize an energy which is of the order of  $E \sim 10^{53}$  ergs in just a few seconds. Because of these high energies it is save to assume that these burst do not behave non-relativistic but rather ultrarelativistic, such that typical speeds of the explosion are of the order of  $v \sim c$ , in this situation the fluid equations need to be adjust to the relativistic case, which results in the relativistic Euler equation which is given by equation 130, where  $T^{\mu\nu}$  is the energy momentum tensor given by equation 131. In these 2 equation  $u^{\mu}$  is the four-velocity and  $g^{\mu\nu}$  is the metric<sup>12</sup>[31, 32].

$$T^{\mu\nu}_{;\nu} = 0,$$
 (130)

$$T^{\mu\nu} = (\rho + P)u^{\mu}u^{\nu} - Pg^{\mu\nu}.$$
(131)

If for the moment the gravity is ignored and it is assumed that space time behaves special relativistic. Then the above equations reduce to the less complicated equation 132 and 133[31].

$$\partial_{\nu}T^{\mu\nu} = 0, \tag{132}$$

$$T^{\mu\nu} = (\rho + P)u^{\mu}u^{\nu} - P\eta^{\mu\nu}.$$
(133)

# **13.7** Basic equations and conventions in relativity

In general relativity one defines things like covariant derivatives, these derivatives are encountered a lot in relativistic situations like relativistic hydrodynamics. The covariant derivatives are defined by equation 134, where the  $\Gamma$ s are what is called Christoffel symbols these symbols are defined by equation 135 and 136. In the case of special relativity one can easily see that the Christoffel symbols become zero and so in special relativity the covariant derivative is equal to the derivative (equation 137)[33].

$$T^{\mu\nu}_{;\nu} \equiv \partial_{\nu}T^{\mu\nu} + \Gamma^{\mu}_{\nu\rho}T^{\rho\nu} + \Gamma^{\nu}_{\nu\rho}T^{\mu\rho}, \qquad (134)$$

$$\Gamma_{\mu\nu\rho} = \frac{1}{2} \Big( \partial_{\rho} g_{\mu\nu} + \partial_{\nu} g_{\mu\rho} - \partial_{\mu} g_{\nu\rho} \Big), \tag{135}$$

$$\Gamma^{\mu}_{\nu\rho} = g^{\mu\sigma}\Gamma_{\sigma\nu\rho},\tag{136}$$

$$T^{\mu\nu}_{;\nu} = \partial_{\nu} T^{\mu\nu}. \tag{137}$$

# 13.8 Change of variables in partial differential equations

In the general case that one wants to change the variables of a partial differential equation (PDE), from say variables  $(a, b) \rightarrow (c, d)$  one needs to do calculate the old derivatives in the new variables. So in this general case that reduces to

 $<sup>^{12}</sup>$ For more information about the covariant derivative see appendix 13.7

$$\frac{\partial}{\partial a}\Big|_{b} = \frac{\partial c}{\partial a}\Big|_{b}\frac{\partial}{\partial c}\Big|_{d} + \frac{\partial d}{\partial a}\Big|_{b}\frac{\partial}{\partial d}\Big|_{c}$$
(138)

$$\frac{\partial}{\partial b}\Big|_{a} = \frac{\partial c}{\partial b}\Big|_{a}\frac{\partial}{\partial c}\Big|_{d} + \frac{\partial d}{\partial b}\Big|_{a}\frac{\partial}{\partial d}\Big|_{c}$$
(139)

These rules can be used. for any general transform of variables.

## 13.8.1 Rewriting the relativistic fluid equations

In this case a transformation from (r, t) to  $(r, s \equiv t - s)$  is made. Using this it is obtained that the old derivatives become in terms of the new derivatives

$$\frac{\partial}{\partial t}\Big|_{r} = \frac{\partial s}{\partial t}\Big|_{r}\frac{\partial}{\partial s}\Big|_{r} + \frac{\partial r}{\partial t}\Big|_{r}\frac{\partial}{\partial r}\Big|_{s} = \frac{\partial}{\partial s}\Big|_{r}$$
(140)

$$\frac{\partial}{\partial r}\Big|_{t} = \frac{\partial s}{\partial r}\Big|_{t}\frac{\partial}{\partial s}\Big|_{r} + \frac{\partial r}{\partial r}\Big|_{t}\frac{\partial}{\partial r}\Big|_{s} = -\frac{\partial}{\partial s}\Big|_{r} + \frac{\partial}{\partial r}\Big|_{s}$$
(141)

## Rewriting equation 49 to equation 52

$$\frac{\partial}{\partial t}(n\gamma) = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2nu) \tag{49}$$

$$\frac{\partial}{\partial s}(n\gamma) = \frac{1}{r^2}\frac{\partial}{\partial s}(r^2nu) - \frac{1}{r^2}\frac{\partial}{\partial r}(r^2nu) \qquad \text{(using equation 140 and 141)}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n u) = \frac{1}{r^2} \frac{\partial}{\partial s} (r^2 n u) - \frac{\partial}{\partial s} (n \gamma) \qquad (rewriting)$$

$$= \frac{\partial}{\partial s} (n u - n \gamma)$$

$$= -\frac{\partial}{\partial s} (n (\gamma - u)) \qquad (reverse taylor expansion)$$

$$= -\frac{\partial}{\partial s} \left(\frac{n}{\gamma + u}\right) \qquad (52)$$

Rewriting equation 50 to equation 53

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 e^{\frac{3}{4}} u \right) = -\frac{\partial}{\partial t} \left( e^{\frac{3}{4}} \gamma \right)$$

$$= -\frac{\partial}{\partial s} \left( e^{\frac{3}{4}} \gamma \right) + \frac{1}{r^2} \frac{\partial}{\partial s} \left( r^2 e^{\frac{3}{4}} u \right)$$
(using equation 140 and 141)
$$= -\frac{\partial}{\partial s} \left( e^{\frac{3}{4}} (\gamma - u) \right)$$

$$= -\frac{\partial}{\partial s} \left( \frac{e^{\frac{3}{4}}}{\gamma + u} \right)$$
(50)
$$(51)$$

Rewriting equation 51 to equation 54

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(n+\frac{4}{3}e\right)u^2\right) = -\frac{\partial}{\partial t}\left(\left(n+\frac{4}{3}e\right)\gamma u\right) - \frac{1}{3}\frac{\partial e}{\partial r}$$

$$= -\frac{\partial}{\partial s}\left(\left(n+\frac{4}{3}e\right)\gamma u\right) + \frac{\partial}{\partial s}\left(\left(n+\frac{4}{3}e\right)u^2\right) + \frac{1}{3}\left(\frac{\partial e}{\partial s} - \frac{\partial e}{\partial r}\right)$$

$$= -\frac{\partial}{\partial s}\left(\left(n+\frac{4}{3}e\right)u(\gamma-u)\right) + \frac{1}{3}\left(\frac{\partial e}{\partial s} - \frac{\partial e}{\partial r}\right)$$
(51)
$$= -\frac{\partial}{\partial s}\left(\left(n+\frac{4}{3}e\right)u(\gamma-u)\right) + \frac{1}{3}\left(\frac{\partial e}{\partial s} - \frac{\partial e}{\partial r}\right)$$
(54)

# 13.9 Pressure in a fireball

 $P = -\left(\frac{\partial U}{\partial V}\right) \qquad (\text{definition of pressure})$  $= -\left(\frac{\partial (Ve)}{\partial V}\right)$  $= -\left(e + V\frac{\partial e}{\partial V}\right) \qquad (\text{Rewriting})$  $= -\left(e + \frac{4\pi r^3}{3}\frac{\partial e}{4\pi r^2\partial r}\right)$  $= -\left(e + \frac{r}{3}\frac{\partial e}{\partial r}\right)$  $= -\left(\frac{e_0}{D^4} + \frac{r}{3}\frac{\partial e}{\partial \gamma}\frac{\partial \gamma}{\partial r}\right)$  $= -\left(\frac{e_0}{D^4} + \frac{r}{3}\frac{\partial e}{\partial D^{-1}}\frac{\partial D^{-1}}{\partial \gamma}\frac{\partial \gamma}{\partial r}\right)$  $= -\left(\frac{e_0}{D^4} + \frac{r}{3}4e_0D^{-3}\frac{\partial D^{-1}}{\partial \gamma}\frac{\partial \gamma}{\partial r}\right)$  $= -\frac{e_0}{D^3}\left(\frac{1}{D} + \frac{4r}{3}e_0D^{-3}\left(-\frac{\gamma_0}{\gamma^2} - \frac{3\gamma_0\rho_0}{4e_0\gamma^2}\right)\frac{\partial \gamma}{\partial r}\right)$  $= \frac{e_0}{D^3}\left(\frac{4r}{3}e_0D^{-3}\left(\frac{\gamma_0}{\gamma^2} + \frac{3\gamma_0\rho_0}{4e_0\gamma^2}\right)\frac{\partial \Gamma}{\partial r} - \frac{1}{D}\right)$ 

To calculate this we need to calculate  $\frac{\partial \gamma}{\partial r}$ , which is given by

$$r^{2}\gamma = r_{0}^{2}\gamma_{0}D^{3} \qquad (\text{known})$$

$$0 = r_{0}^{2}\gamma_{0}D^{3} - r^{2}\gamma \qquad (\text{rewriting})$$

$$= \frac{\partial}{\partial r} \left( r_{0}^{2}\gamma_{0}D^{3} - r^{2}\gamma \right) \qquad (\text{Implicit differentiation})$$

$$= r_{0}^{2}\gamma_{0}\frac{\partial D^{3}}{\partial r} - 2r\gamma - r^{2}\frac{\partial\gamma}{\partial r}$$

$$= 3D^{2}r_{0}^{2}\gamma_{0}\frac{\partial D}{\partial \gamma}\frac{\partial\gamma}{\partial r} - 2r\gamma - r^{2}\frac{\partial\gamma}{\partial r}$$

$$2r\gamma = \frac{\partial\gamma}{\partial r} \left( 3r_{0}^{2}\gamma_{0}D^{2}\frac{\partial D}{\partial \gamma} - r^{2} \right)$$

$$\frac{\partial\gamma}{\partial r} = \frac{2r\gamma}{3r_{0}^{2}\gamma_{0}D^{2}\frac{\partial D}{\partial \gamma} - r^{2}}$$

to calculate this part we need to calculate  $\frac{\partial D}{\partial \gamma}$ , which results in

$$\frac{\partial D}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left( \frac{1}{\frac{\gamma_0}{\gamma} + \frac{3\gamma_0\rho_0}{4e_0\gamma} - \frac{3\rho_0}{4e_0}} \right) \tag{142}$$

$$=\frac{\gamma_0\left(1+\frac{3\rho_0}{4e_0}\right)}{\left(\gamma_0\left(1+\frac{3\rho_0}{4e_0}\right)-\frac{3\rho_0}{4e_0}\gamma\right)^2}\tag{143}$$

# 13.10 Rewriting equation 63 to equation 64

$$\frac{dE_n}{dt} = -2n_e c\sigma_e \epsilon_e (4\pi r^2 r_0 n_e)$$
(63)
$$\frac{1}{E_n} \frac{dE_n}{dt} = -\frac{2n_e c\sigma_e \epsilon_e (4\pi r^2 r_0 n_e)}{E_n}$$
(Dividing by  $E_n$ )
$$\frac{1}{E} \frac{dE}{dt} = -\frac{2n_e c\sigma_e \epsilon_e (4\pi r^2 r_0 n_e)}{12\pi r^2 r_0 n_e \epsilon_e \gamma}$$
(Rewriting)
$$\frac{d\ln E}{dt} = -\frac{2}{3} \frac{\sigma_e cn_e}{\gamma}$$

$$= -\frac{2}{3} \frac{2 \times 10^{-44} (\epsilon_e / 1 \text{ MeV})^2 \text{ cm}^2 \cdot 3 \cdot 10^{10} \text{ cm} \text{ s}^{-1} \cdot 2.34 \times 10^{34} T_{10}^3 \text{ cm}^{-3}}{\gamma}$$

$$= -\frac{\frac{2}{3} \cdot 2 \cdot 3 \cdot 2.34}{\gamma} \left( \frac{\epsilon_e}{1 \text{ MeV}} \right)^2 T_{10}^3$$

$$= -\frac{9.36}{\gamma} \left( \frac{31.5 \text{ MeV } T_{10}}{1 \text{ MeV}} \right)^2 T_{10}^3$$

$$= -\frac{9.29 \times 10^3}{\gamma} T_{10}^5$$
(64)

# 13.11 Neutrino reactions for detection

Neutrinos are detected at IceCube by means of that they produce secondary particles, these are electron, muons or tauons. These particles move with speeds faster than the speed of light in the ice, with as consequence that they emit Cherenkov radiation. The six main reactions that produce these secondary particles are the following reactions:

$$\nu_e + n \rightarrow e^- + p$$
  

$$\bar{\nu}_e + p \rightarrow e^+ + n$$
  

$$\nu_\mu + n \rightarrow \mu^- + p$$
  

$$\bar{\nu}_\mu + p \rightarrow \mu^+ + n$$
  

$$\nu_\tau + n \rightarrow \tau^- + p$$
  

$$\bar{\nu}_\tau + p \rightarrow \tau^+ + n$$

## 13.12 Program codes

#### 13.12.1 Evolution of fireball code

```
Listing 1: Code that calculates the evolution of a fireball
```

```
\#!/usr/bin/env python
_{2} # made by Folkert Nobels
3 from __future__ import division
  import numpy as np
4
  from matplotlib.pyplot import figure, show, savefig, ylim, xlim, close, xlabel
      , ylabel, grid, title, legend, axvline
6 import scipy.optimize as sco
_{7} mH = 1.67e-24 # in grams
  rad_{-a} = 7.5646e - 15
8
  kB = 1.380658e - 16
9
  sigmaT = 6.6524e - 25
10
11
12 import scipy.constants as scc
13 import matplotlib
14 import matplotlib.pyplot as plt
  matplotlib.rcParams.update({ 'font.size': 22})
15
16
  c = s c c . c
17
  pi = scc.pi
18
19
  # calculate the arbitrary constant D
20
  def valueD (gamma, gamma0, rho0, e0):
21
       over = gamma0/gamma + (3*gamma0*rho0)/(4*e0*gamma) - (3*rho0)/(4*e0)
22
       return 1/over
23
24
25 # function to calculate radius
  def radius (r0, gamma, gamma0, rho0, e0):
26
       return r0*gamma0**.5 * valueD(gamma, gamma0, rho0, e0)**1.5 /gamma**.5
27
28
  # function to calculate density
29
  def rho(gamma, gamma0, rho0, e0):
30
       return rho0/valueD(gamma, gamma0, rho0, e0)**3
  # function to calculate energy density
33
  def eee (gamma, gamma0, rho0, e0):
34
       return e0/valueD(gamma, gamma0, rho0, e0)**4
35
36
  # function to calculate the baryonic loading
37
  def bloading (gamma, gamma0, rho0, e0):
38
       return e0/(rho0 * valueD(gamma, gamma0, rho0, e0))
39
40
41 # function to calculate the pressure
42 def pressure (gamma, gamma0, rho0, e0, r0):
       return (e0/valueD(gamma, gamma0, rho0, e0)**3) * (4*radius(r0, gamma,
43
     gamma0, rho0, e0) * gamma0/gamma**2 *(1 + 3 * rho0 / (4 * e0)) *
```

```
der_radius (gamma, gamma0, rho0, e0, r0) - 1/valueD (gamma, gamma0, rho0, e0)
       )
44
_{45} # function to calculate the derivative of gamma with respect to r
46 def der_radius (gamma, gamma0, rho0, e0, r0):
        return (2*radius(r0, gamma, gamma0, rho0, e0) * gamma)/(3 *r0**2 *gamma0
47
       *valueD(gamma, gamma0, rho0, e0) **2 * der_D(gamma, gamma0, rho0, e0, r0) -
       radius(r0, gamma, gamma0, rho0, e0)**2)
48
  \# function to calculate the derivative of D with respect tot gamma
49
  def der_D(gamma, gamma0, rho0, e0, r0):
50
        b = 3 * rho0 / (4 * e0)
        a = gamma0 * (1 + b)
        return a / (a - b* gamma)**2
54
  def pressure_zero (T, rho, P):
        return rad_a * T**4 / 3 + 2 * rho* kB* T / mH - P
56
57
  eta = 10
58
  gamma0=90
  eee0 = 1e10
  rho0 = eee0/eta
61
62
63 r00=1e7
_{64} #gamma_arr = np. linspace (1,1400,1000)
_{65} | #rad = radius (r00, gamma_arr, gamma0, rho0, eee0)
66 \mid \log \text{gamma}_{arr} = \text{np.linspace}(0, \text{np.log}(1288), 1000)
  gamma_arr=np.exp(loggamma_arr)
67
  gamma_arr = np.append(gamma_arr,gamma_arr[-1])
68
  rad = radius(r00, gamma_arr, gamma0, rho0, eee0)
70
  gamma_arr = np.append(gamma_arr,gamma_arr[-1])
71
r_{2} rad = np. append (rad, 10 * * 13)
  print rad
73
74
75
  \#raddd = np.linspace(1e6, 1e11, 100)
76
_{77} #gamma = []
78 #for i in raddd:
79 #
         print i
  #
         gamma.append(sco.bisect(radiusfind,gamma0,1e4,args=(gamma0,rho0,eee0,r00)
80
      , i ) ) )
81
  # calculation of all variables
82
|rho_arr| = rho(gamma_arr, gamma0, rho0, eee0)
  eee_arr = eee(gamma_arr, gamma0, rho0, eee0)
84
<sup>85</sup> bloading_arr = bloading(gamma_arr, gamma0, rho0, eee0)
  press_arr = pressure (gamma_arr, gamma0, rho0, eee0, r00)
86
87
|\#\text{eee\_arr} = \text{np.append}(\text{eee\_arr}, \text{eee\_arr}[-1])
|\text{rho}_{arr} = \text{np.append}(\text{rho}_{arr}, \text{rho}_{arr}[-1])
```

```
bloading_arr = np.append(bloading_arr, bloading_arr[-1])
90
   press_arr = np.append(press_arr, press_arr[-1])
91
92
93
   T = (3 * press_arr / rad_a) **.25
94
   T2 = mH * press_arr / (rho_arr * kB)
95
   , , ,
96
  T = []
97
   for i in xrange(0, len(press_arr)):
98
        T.append( sco.newton(pressure_zero, 1e6, args=(rho_arr[i], press_arr[i]) )
99
        )
100
   print T
101
   , , ,
102
   E_{tot} = 1e49
103
   Re = (sigmaT * E_tot / (4 * pi * 1.6 e - 3 * eta)) * * .5
104
   print Re
106
   fig = figure()
108
   frame = fig.add_subplot(211)
110
   frame.set_xscale("log", nonposx='clip')
111
   frame.set_yscale("log", nonposy='clip')
   print len(rad), len(gamma_arr)
113
   frame.plot(rad,gamma_arr,lw=3)
114
   plt.axvline(eta*r00, c='r', lw=4)
   plt.axvline(Re, c='g', lw=4)
116
   #plt.axvline(eta*eta*r00, color='#BD0BD9', lw=4)
117
  xlim(10 * *6, 10 * *13)
118
119 frame.axes.xaxis.set_ticklabels([])
   title ('The Lorentz factor and energy density', y=1.14)
120
  #xlabel('radius')
121
   grid (True)
122
   ylabel('$\Gamma$')
123
   plt.gcf().subplots_adjust(bottom=0.15, left=0.2, top=0.85)
124
125
126
   , , ,
127
   rho_arr = rho(gamma_arr, gamma0, rho0, eee0)
   eee\_arr = eee(gamma\_arr, gamma0, rho0, eee0)
129
   bloading_arr = bloading(gamma_arr, gamma0, rho0, eee0)
130
   press_arr = pressure(gamma_arr, gamma0, rho0, eee0, r00)
   , , ,
   frame=fig.add_subplot(212)
134
   frame.set_xscale("log", nonposx='clip')
135
  frame.set_yscale("log"
                            , nonposy='clip')
136
  frame.yaxis.set_ticks([10**14, 10**10, 10**6, 10**2, 10**-2, 10**-6])
137
  frame.plot (rad [.9e13>rad], eee\_arr [.9e13>rad], lw=3)
138
139 \text{ xlim} (10 * *6, 10 * *13)
```

```
140 plt.axvline(eta*r00, c='r', lw=4)
   plt.axvline(Re, c='g', lw=4)
141
   #plt.axvline(eta*eta*r00, color='#BD0BD9', lw=4)
142
<sup>143</sup> #title ('Energy density as function of r')
   xlabel('Radius (cm)')
144
   grid (True)
145
   ylabel('Energy density')
146
   plt.gcf().subplots_adjust(bottom=0.15, left=0.15, top=0.85)
147
148
149
   , , ,
150
   frame=fig.add_subplot(111)
151
   frame.set_xscale("log", nonposx='clip')
152
   frame.set_yscale("log", nonposy='clip')
153
154
   frame.plot(rad, press_arr)
155
156
157
   title ('Pressure as function of r')
   xlabel('radius')
159
   grid (True)
160
   ylabel('Pressure')
161
162
   frame=fig.add_subplot(236)
163
   frame.set_xscale("log", nonposx='clip')
frame.set_yscale("log", nonposy='clip')
164
165
166
   frame.plot(rad,T)
167
   #frame.plot(rad,T2)
168
169
   title ('Temperature as function of r')
170
   xlabel('radius')
171
   grid (True)
172
   ylabel('Temperature')
173
   , , ,
174
175
   show()
176
```

#### 13.12.2 Band function code

```
#!/usr/bin/env python
  from __future__ import division
2
3
  import scipy.constants as scc
4
  import numpy as np
5
  from matplotlib.pyplot import figure, show, savefig, ylim, xlim, close, xlabel
6
      , ylabel, grid, title, legend, axvline
  import sys
7
9
10 # import units
11 h = scc.h*10**7 \# h in erg s
  kevtoerg = 1.6021773e-9
12
13
14
  # Band spectrum function
15
  def bandtot(nu,a,b,E0,N0):
16
        nu1 = nu[h*nu <=(a-b)*E0]
17
        nu2 = nu[h*nu>(a-b)*E0]
18
        N1 = N0 * (h*nu1)**a * np.exp(- h*nu1/E0)
19
        N2 = N0* ((a-b)*E0)**(a-b) *(h*nu2)**b * np.exp(b-a)
20
        return np. concatenate ((N1,N2),0)
21
22
23
24
25
_{26}|a = float(sys.argv[1])
_{27} b = float (sys.argv [2])
_{28} E0kev = float (sys.argv[3])
_{29} N0 = float (sys.argv [4])
  print 2*a
30
         '# Calculate Band Spectrum \#'
  print
31
  print '# a = ', a
32
  print '\# b =', b
33
  print '# E0 =', E0kev, 'keV'
34
  print '# N0 =',N0
35
36
37
  Ei = 0.1 * kevtoerg
38
  Ef = 1e5 * kevtoerg
39
40
  E0 = E0 \text{kev} * \text{kevtoerg}
41
42
  nu = np.linspace(Ei/h, Ef/h, 10000)
43
44
  bandnu = bandtot(nu, a, b, E0, 1)
45
46
_{47} fig = figure()
```

```
Listing 2: Band function code
```

```
_{48} frame = fig.add_subplot(121)
  frame.set_xscale("log", nonposx='clip')
49
  frame.set_yscale ("log", nonposy='clip')
50
  frame.plot(h*nu/kevtoerg,bandnu)
54
  title ('Band Spectrum')
  xlabel('h \setminus u / 1 \text{ keV}')
  grid (True)
56
57 ylabel ('N(\backslash nu) ')
_{58} frame = fig.add_subplot(122)
59 frame.set_xscale("log", nonposx='clip')
  frame.set_yscale("log", nonposy='clip')
  frame.plot(h*nu/kevtoerg,nu**2*bandnu)
61
  title ('Band Spectrum \lambda \in F_{-} \in 
62
63
  xlabel('h \setminus u / 1 \text{ keV}')
64
  grid (True)
65
  ylabel('\\\nu F_\\nu $')
66
67
68
  savefig('picture.png')
69
70
  show()
```

#### 13.12.3 Neutrino energy spectrum code



```
_{1} #!/usr/bin/env python
_{2} # made by Folkert Nobels
_{3} # Physicist and Astronomer
4
  from __future__ import division
5
6 import numpy as np
7 import numpy.random as rand
s from matplotlib.pyplot import figure, show, xlim, ylim, contour, clabel, title
  import sys
  from mpl_toolkits.mplot3d import Axes3D
10
12
  \# function that calculates the observed energy
13
  def obs_energy(gamma, E, cos):
14
       return E*(gamma + (gamma**2 - 1)**.5 * cos)
15
16
  def obs_xmomentum (gamma, E, cos):
17
       return E*( (gamma**2 - 1)**.5 + gamma* cos
18
                                                         )
19
_{20}|E0 = float(sys.argv[1])*1e6
21 Gamma = float (sys.argv [2])
_{22} point = float (sys.argv[3])
```

```
23 print 'Start program'
_{24} print 'E0 = ', E0/1e6, 'MeV'
  print 'gamma =', Gamma
25
  print 'Number of points =', point/1e6, 'million'
26
27
 \# calculate random angles
28
29
  d\cos = rand.uniform(1, -1, point)
  phi = 2*np.pi*rand.uniform(1,0,point)
30
31
_{32} # calculate observed energy
a_{33} energy = obs_energy (Gamma, E0, dcos)
  xmomentum = obs_xmomentum (Gamma, E0, dcos)
34
35
  \# calculate theta and sin theta
36
  theta = np. \arccos(dcos)
37
  dsin = np.sin(theta)
38
39
  ymomentum = E0*np.sin(theta)
40
41
  angle = np.tan(ymomentum/xmomentum)
42
43
  # plotting part
44
_{45} fig = figure()
_{46} frame = fig.add_subplot(1,1,1)
47 frame.hist(energy/1e9, bins=100, color='b', alpha=0.8,normed=1)
  frame.set_title('Distribution of energy')
48
  frame.set_xlabel('energy (in GeV)')
49
  frame.set_ylabel('Propability')
50
  frame = fig.add_subplot(1, 2, 2)
52
  frame.hist(np.log10(energy), bins=100, color='b', alpha=0.8,normed=1)
53
54 frame.set_title('Distribution of energy')
  frame.set_xlabel('log energy (in log eV)')
55
  frame.set_ylabel('Propability')
56
57
58
  show()
59
60
_{61} fig = figure()
_{62} frame = fig.add_subplot(1,1,1)
63 frame.plot(np.cos(phi)*np.sin(angle),np.sin(phi)*np.sin(angle),'.')
64 frame.set_title('Angles due to relativistic beaming')
65 frame.set_xlabel('Projected y-axis')
  frame.set_ylabel('Projected z-axis')
66
  xlim(-.05,.05)
67
  ylim(-.05,.05)
68
69
  show()
70
71
72 # 3d plot
73
```

```
_{74} fig = figure()
<sup>75</sup> frame = fig.add_subplot(111, projection='3d')
76 frame.scatter(dcos, dsin*np.cos(phi), dsin*np.sin(phi))
77 frame.set_xlabel('x-axis')
78 frame.set_ylabel('y-axis')
  frame.set_zlabel('z-axis')
79
80
  show()
81
82
83
_{84} fig = figure()
s frame = fig.add_subplot(1, 1, 1)
86 frame.plot(theta, energy/1e6, '.')
87 frame.set_title('Energy as function of the angle')
set_xlabel('$\\theta$ (in radians)')
<sup>89</sup> frame.set_ylabel('Energy (in MeV)')
90 xlim (0, np. pi)
  show()
91
```

Listing 4: Neutrino energy spectrum module

```
1 #!/usr/bin/env python
2
3 from __future__ import division
4 import numpy as np
5
6 def obs_energy(gamma,E,cos):
7 return E*(gamma + (gamma**2 - 1)**.5 * cos)
```
## 13.12.4 Realistic neutrino energy spectrum code

Listing 5: Realistic neutrino energy spectrum code

```
#!/usr/bin/env python
  from __future__ import division
2
3
4 import numpy as np
  from matplotlib.pyplot import figure, show, xlim, ylim, contour, clabel, title
5
  import numpy.random as rand
6
7
  from test_mod import obs_energy
9
10 import matplotlib.pyplot as plt
11 import matplotlib
  matplotlib.rcParams.update({ 'font.size': 22})
12
  number = 1e5
14
  Gamma=1e3
15
16
 \# two body decay
17
18 # working very well
19 d\cos 2 = rand.uniform(-1,1,number/2)
  d\cos 2prime = np.\cos(np.\arccos(d\cos 2) + np.pi)
20
  d\cos 2 = np. concatenate((d\cos 2, d\cos 2prime))
21
22
  #np.transpose(np.vstack((dcos2,dcos2prime))).reshape(number,1)
23
24
25
26
27
  # four body decay
28
29 \mid d\cos 1 = rand.uniform(-1,1,(number,4))
  d\cos 3 = d\cos 1 [:,3]
30
  d\cos 4 = d\cos 1 [:, 0:3]
32
33
_{34} # random energies 4
  energy rand = rand.uniform (0, 1, (number, 4))
35
  summ = np.sum(energyrand, axis=1)
36
37
summ = np.transpose(np.vstack((summ,summ,summ))))
  energyrand = energyrand/summ * 1e9
39
40
  energyrand2 = 2.5e9*5*rand.beta(2,8,(len(dcos4),3))
41
42 print energyrand2
  print energyrand2.max()
43
44
_{45} # energy neutrinos
  energy = obs_energy (Gamma, energyrand2, dcos4)
46
47
48
```

```
49
50 # energy photons
  energyp = obs_energy (Gamma, 5e8, dcos2)
51
_{52} energy p_2 = obs_energy (Gamma, 2.5 e8, dcos3)
53
  # energy photons better
54
  numbs = (2.5 \, \text{e}8 + 5 \, \text{e}8)/5 \, \text{e}5
56
  d\cos 5 = rand.uniform(-1,1,numbs*number)
57
  energyp3 = obs_energy (Gamma, 5e5, dcos5)
58
59
  energyp = np.concatenate((energyp, energyp2))
60
  #energyp[energyp>1e10]
61
62
  ratio = len(energyp3 | energyp3 > 2.5e4])/len(energy[energy > 1e10])
63
  \#energyp = np.vstack((energyp, energyp2)).reshape(2*number, 1)
64
65
66
  energy_neu = energy_reshape(3 * len(energy), 1)
67
  energy_neu = energy_neu [energy_neu >1e10]
68
  print energy_neu.max()
70
71
_{72} fig = figure()
_{73} frame = fig.add_subplot(111)
  frame.hist(np.log10(energy_neu/1e9), log=True, bins=50, color='b', alpha=0.8,
74
      normed=True)
_{75} frame.set_title('Distribution of energy (neutrinos)', y=1.08)
  frame.set_xlabel('Energy (in log GeV)')
76
  #frame.set_xscale("log", nonposx='clip')
77
_{78} #xlim (0,1500)
<sup>79</sup> plt.gcf().subplots_adjust(bottom=0.15, left=0.2, top=0.85)
  frame.set_ylabel('log(Propability)')
80
  , , ,
81
_{82} frame = fig.add_subplot(222)
s3 frame.hist(energyp/1e9, bins=100, color='b', alpha=0.8)
84 frame.set_title('Distribution of energy (photons)')
<sup>85</sup> frame.set_xlabel('energy (in log GeV)')
86 #frame.set_xscale("log", nonposx='clip')
87 frame.set_ylabel('Propability')
  \#xlim(0, 1500)
88
89
_{90} frame = fig.add_subplot(223)
91 frame.hist(energyp3/1e6, bins=100, color='b', alpha=0.8)
  frame.set_title('Distribution of energy (photons)')
92
93 frame.set_xlabel('energy (in log MeV)')
  frame.set_ylabel('Propability')
94
  , , ,
95
  show()
96
97
_{98} # calculation of amount of neutrinos expected to detect by icecube
```

```
99 # 1 sec measure

100 counts = 5000

101

102 neutrinocount = counts / ratio

103

104 neutrinoice = neutrinocount * 1e6

105

106 print neutrinoice
```

## 13.12.5 Pre-GRB neutrino energy loss code

Listing 6: Pre-GRB neutrino energy loss code

```
\#!/usr/bin/env python
  from __future__ import division
2
3
  import numpy as np
4
  from matplotlib.pyplot import figure, show, savefig, ylim, xlim, close, xlabel
6
      , ylabel, grid, title, legend
  def pairannilation (r0,T0):
8
       \# return fraction energy loss due to production of neutrinos by pair
9
      annilation
       \# where T0 is given in MeV and t0 in seconds
10
        t0 = r0 / (2.99792458 * 10 * * 10)
        if T0>10.2:
12
             \ln = 0
13
        else:
14
             \ln = -1.9 * 10 * 3 * t0 * (T0 / 10) * 5
        return 1 - np.exp(ln)
16
  def decayparticle (r0,T0,td,Enu,md):
18
       \# return fraction energy loss due to decay of particles that produce
19
      neutrinos
        t0 = r0 / (2.99792458 * 10 * * 10)
20
        \ln = -t0 / td * Enu / (8*T0) * (md / T0) **.5 * np.exp(-md/T0)
21
       return 1 - np.exp(ln)
22
23
_{24} #print pairannilation (10**7,10)
  \#print decayparticle (10**7,15,2.2*10**-6,70,105.66)
25
_{26} #print decay particle (10**7, 15, 2.55*10**-8, 29, 139.6)
27
_{28} T = np.linspace (0.1, 10.2, 1000)
_{29}|T2 = np.linspace(0.1, 15, 1000)
_{30} #pair = pairannilation (10**7,T)
31 pair =[]
32 for i in T2:
        pair.append(pairannilation(10**7,i))
33
34
```

```
_{35} decay1 = decayparticle (10**7, T2, 2.2*10**-6, 70, 105.66)
_{36} decay2 = decayparticle (10**7, T2, 2.55*10**-8, 29, 139.6)
  together = decay1 + decay2 + pair
37
38
_{39} fig = figure()
_{40} frame = fig.add_subplot(111)
41 #frame.set_xscale("log", nonposx='clip ')
42 #frame.set_yscale("log", nonposy='clip ')
43 frame.plot (T2, pair, label='e^{-} = e^{+} annihilation')
44 frame.plot(T2, decay1, label='muon decay')
45 frame.plot(T2, decay2, label='pion decay')
46 frame.plot(T2, together, label='total energy loss')
_{47} ylim (0,1)
_{48} xlim (0,15)
49 title ('Energy loss due to neutrino production')
50 xlabel('\ (\ T \ T \ T \ MeV \ S')
51 legend()
52 grid (True)
53 ylabel('relative energy loss')
54 savefig('picture.png')
  show()
56
57
58 print 1.9891*10**33 *1.4* (2.99792458*10**10)**2
```

## 13.12.6 Time difference due to finite mass neutrino code

Listing 7: Time difference due to finite mass neutrino code

```
#!/usr/bin/env python
  from __future__ import division
2
3
4 import numpy as np
  from matplotlib.pyplot import figure, show, legend, grid, savefig
5
  from scipy.interpolate import interp1d
6
7
  import matplotlib.pyplot as plt
8
  import matplotlib
9
  matplotlib.rcParams.update({ 'font.size': 22})
11
12
  redshift = [0, .1, .2, .3, .4, .5, .6, .7, .8, .9, .1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.65, .1, .65]
13
      1.8, 2, 2.2, 2.5, 3, 3.5, 4, 5, 6, 7, 8, 9, 10
14 # light travel time in Gyr
15 lighttraveltime = [0, 1.31, 2.451, 3.448, 4.323, ]
      5.093, 5.773, 6.375, 6.911, 7.389, 7.817, 8.202, 8.549,
      8.862,9.146,9.405,9.751,10.055,10.404,10.703,
      11.075, 11.549, 11.898, 12.163, 12.534, 12.779, 12.950, 13.075, 13.169, 13.243
16
  f = interp1d(redshift, lighttraveltime, kind='cubic')
  newz = np. linspace(0, 10, 1e6)
18
  valuesf = f(newz)
19
20
  deltaT = []
21
22
  for i in xrange (0, \text{len}(\text{newz}) - 1):
23
       if i == 0:
24
             deltaT.append( ( valuesf[i+1] - valuesf[i] )*(newz[i]+1)**(-2) )
25
        else:
26
             deltaT.append( (valuesf[i+1] - valuesf[i])*(newz[i]+1)**(-2) +
27
      deltaT[i-1])
28
  deltaT.append(deltaT[-1])
29
30
  deltaT = np.array(deltaT)
31
  deltaT = 5*np.pi*1e-2 * deltaT *1e6
32
33
34
35 # plot interpolated accurate result
_{36} fig = figure()
_{37} frame = fig.add_subplot(1,1,1)
38
39 frame.set_title('Light travel time')
40 frame.plot(newz, f(newz))
41 frame.set_xlabel('Redshift')
42 frame.set_ylabel('Light travel time (Gyr)')
43 grid()
```

```
44 show()
45
46
  deltat = f(newz) * 5 * np. pi * 1e - 2 * 1e6
47
48
  fig = figure()
49
  frame = fig.add_subplot(1,1,1)
50
51
52 frame.set_title('Time difference')
frame.plot(newz, 4* deltat, label='m_{\lambda} = 2 eV')
_{54} frame.plot(newz, deltat, label='m_{(\ueble u)} = 1 \text{ eV'})
_{55} frame.plot(newz, deltat/4, label='m_{\langle nu_e \rangle} = 1/2 \text{ eV}')
56 frame.set_xlabel('Redshift')
  frame.set_ylabel('time difference (in ns)')
57
58 #legend()
  grid()
59
  show()
60
61
  fig = figure()
62
  frame = fig.add_subplot(1, 1, 1)
63
64
  frame.set_title('Time difference', y=1.08)
65
_{66} frame.plot(newz, 4*deltaT/1e6,label='m_{\chi} \approx 2 eV',lw=3)
67 frame.plot(newz, deltaT/1e6, label='m_{\langle nu_e \rangle} = 1 \text{ eV}', lw=3)
68 frame.plot(newz, deltaT/4/1e6, label='m_{\{\setminus nu_e\}} = 1/2 \text{ eV}', lw=3)
  frame.set_xlabel('Redshift')
69
70 frame.set_ylabel('time difference (in ms)')
_{71} plt.gcf().subplots_adjust(bottom=0.15, left=0.2, top=0.85)
72 #legend()
73 grid()
r4 savefig('timedifference4.pdf')
75
76
  show()
```