Determining the cosmological parameters Ω_m and σ_8 from peculiar velocity and density-contrast data

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Abstract

In the linear regime and at the far observer limit there is a simple relation between the two point correlation functions of the Gaussian smoothed density-contrast and peculiar velocity. This relation asserts that the derivative of velocity correlation function with respect to the smoothing scale is equal to the density contrast correlation function up to some constants. This relation has been shown to work beyond the linear regime to a certain extent. In this research project we explore, with numerical simulations, down to which (quasilinear) scales this equation holds. We find that this relationship holds down to a Gaussian smoothing scale of 5 h^{-1}Mpc with 16% error.

The second part of this Klein Onderzoek was to determine whether we could find an estimate for σ_8 from the peculiar velocity data using the same relation between the correlation functions. For this part we used data from the PSCz galaxy redshift catalogue and the SEcat galaxy peculiar velocity catalogue. Here we adopted the WMAP5 Ω_m value (0.27) and assumed that the velcity biasing is negligible. We estimate the value of σ_8 using a theoretical estimate for the density-variance, determined from the standard ΛCDM Model. σ_8 is obtained by fixing all cosmological parameters to the values measured by WMAP5. Using a chi-squared estimate, we find that $\sigma_8 = 0.73 \pm 0.12$, a result that is consistent with that obtained from WMAP.

1 Introduction

1.1 Cosmological Background

The model that most accurately describes our Universe as we see it today is the so-called ΛCDM model. It assumes a cosmological constant, the presence of cold dark matter and a scale-invariant spectrum of primordial density fluctuations. These pertubations are thought to originate from quantum fluctuations at the very early universe during the so-called inflationary epoch.
The most accepted explanation for the formation of structure is the gravitational instability theory, also known as Jeans instability. Jeans showed that minor density and velocity fluctuations with respect to a static, homogenous and isotropic background, could evolve as a function of time. The density ρ, full velocity $\mathbf{V}$ and the mean gravitational potential φ can be seen as a collisionless gas. Their evolution will be according to the fluid equations: the Euler equation, the continuity equation and the Poisson equation.

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \bigtriangledown)\mathbf{V} + \frac{1}{\rho} \bigtriangledown p + \bigtriangledown \varphi = 0 \tag{1}$$

$$\frac{\delta \rho}{\partial t} + \bigtriangledown \cdot (\rho \mathbf{V}) = 0 \tag{2}$$

$$\bigtriangledown^2 \varphi - 4\pi G \rho = 0 \tag{3}$$

It is useful to write these equations in comoving coordinates and express the cosmological fields in terms of the density-contrast and peculiar velocity. The density-contrast is defined as:

$$\delta(\mathbf{r}) = \frac{\rho(\mathbf{r}) - \overline{\rho}}{\overline{\rho}} \tag{4}$$

where $\rho(\mathbf{r})$ is the density at position $\mathbf{r}$ and $\overline{\rho}$ is the mean background density of the universe. The peculiar velocity is defined as:

$$\mathbf{v} = \mathbf{V} - H\mathbf{r} \tag{5}$$

where $\mathbf{V}$ is the physical velocity and $H\mathbf{r}$ is the velocity of the expansion of the universe. The gravitational potential pertubations in comoving coordinates are φ.
The fluid equations in comoving coordinates are:

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{a}(\mathbf{v} \cdot \bigtriangledown)\mathbf{v} + \frac{\dot{a}}{a}\mathbf{v} + \frac{1}{a} \bigtriangledown \varphi = 0 \tag{6}$$

$$\frac{\partial \delta}{\partial t} + \frac{1}{a} \bigtriangledown (1 + \delta)\mathbf{v} = 0 \tag{7}$$

$$\bigtriangledown^2 \varphi = 4\pi G a^2 \; \overline{\rho} \; \delta \tag{8}$$

Notice that in (eq. 6), $\dot{a}/a$ is defined as the Hubble parameter.

In the case of small density and velocity pertubations, the the nonlinear coupling terms in the continuity and Euler equation can be neglected. The set of fluid equations can be linearezed yielding:

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{\dot{a}}{a}\mathbf{v} = -\frac{1}{a} \bigtriangledown \varphi \tag{9}$$

$$\frac{\partial\delta}{\partial t} + \frac{1}{a} \bigtriangledown \cdot \mathbf{v} = 0 \tag{10}$$

$$\bigtriangledown^2 \varphi = 4\pi G a^2 \ \overline{\rho}\delta \tag{11}$$

Here a is the cosmological expansion factor and $\overline{\rho}$ is the cosmological background density.

1.2 The Cosmological density field

In order to solve the linearized fluid equations for pertubations in the density field, the time derivative of the linearezed continuity equation (eq. 10) is subsituted in the divergence of the linearezed Euler equation (eq. 9). The result is combined with the Poisson equation (eq. 11) to yield:

$$\frac{\partial^2\delta}{\partial t^2} \ + \ 2\frac{\dot{a}}{a}\frac{\partial\delta}{\partial t} = 4\pi G\overline{\rho}\delta \tag{12}$$

The result is a second-order partial differential equation in time alone. The solution can be split in a time component and a spatial component:

$$\delta(\mathbf{r}, t) = \delta_1(\mathbf{r})D_1(t) \ + \ \delta_2(\mathbf{r})D_2(t) \tag{13}$$

The first part of the solution reflects the growing mode, while the second part reflects the decaying mode. In this research project, the decaying mode will be neglected. The subscript is dropped and the solution for the density-contrast field will be:

$$\delta(\mathbf{r}, t) = \delta(\mathbf{r})D(t) \tag{14}$$

It is convient to think of a pertubation as a superposition of plane waves. This has the advantage that waves evolve independently while the pertubations are still small. The density pertubations can then be written as:

$$\delta(\mathbf{r}_1) = \sum_{\mathbf{k}} \delta_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}_1} \tag{15}$$

The goal is to find a linear relation between the two-point correlation functions of the density-contrast and the peculiar velocity. The two-point correlation function is defined as the probability for finding a typical neighbour at a distance r. The expectation value of finding δ_1 at $\mathbf{r_1}$ and δ_2 at $\mathbf{r_2}$ is defined as:

$$< \delta_1(\mathbf{r_1}) \ \delta_2(\mathbf{r_2}) >= \int\int \delta_{\mathbf{k_1}} e^{i\mathbf{k_1}\cdot\mathbf{r_1}} \delta_{\mathbf{k_2}} e^{-i\mathbf{k_2}\cdot\mathbf{r_2}} d^3\mathbf{k_1} d^3\mathbf{k_2} \tag{16}$$

The Wiener-Khinchin-theorem is used, which states that the power spectrum is the Fourier transform of the expectation function of the density-contrast:

$$< \delta_{\mathbf{k_1}} \ \delta_{\mathbf{k_2}} >= P_k \delta^3(\mathbf{k_1} - \mathbf{k_2}) \tag{17}$$

In this formula P_k is the Fourier transform of the power spectrum.This formula has a solution when $\mathbf{k} = \mathbf{k_1} = \mathbf{k_2}$. The distance between the two points is defined as $\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$. Because of isotropic arguments, we can say:

$$< \delta_1(\mathbf{r_1}) \ \delta_2(\mathbf{r_2}) >= \frac{1}{(2\pi)^3} \int P(k) e^{ik\cdot r} k^2 dk \ sin\theta d\theta \ d\phi \tag{18}$$

The result of the angular integral is equal to $4\pi\frac{sin(kr)}{kr}$. In this, the spherical zero-order Bessel function j_0 is recognised. The last step is to replace the expectation value with the desired two point correlation function.

$$\xi_d(r) = \frac{1}{2\pi^2}\int P(k)j_0(kr)k^2dk \tag{19}$$

1.3 The Cosmological velocity field

With the solution for our density-contrast field (eq. 14), solutions for the gravitational and velocity pertubation field can be derived. The linear relation between the two-point correlation function of the density-contrast field and the peculiar velocity field will be found by both coupling them to the gravitational potential. For the density, this is done in the Poisson equation (eq. 11). In the linear regime it can be shown that the peculiar velocity is drawn from a potential which can be coupled to the gravitational potential. (see Appendix A.1.1)
The density-contrast and the peculiar velocity are now both coupled to the gravitational potential pertubations, which means they can now be coupled to each other. The total derivation of this formula will be found in the Appendix (A.1.2). The final result is:

$$\bigtriangledown \cdot \mathbf{v} = -a \cdot \frac{\partial}{\partial t}\delta(\mathbf{r}, t) \tag{20}$$

We found for the solution of the density-contrast δ earlier (eq. 14). This gives:

$$\bigtriangledown \cdot \mathbf{v} = -\frac{1}{D} \cdot \frac{\partial D(t)}{\partial t}\delta(\mathbf{r}) \tag{21}$$

f is defined as the dimensionless linear velocity growth factor. The derivation of this formula is shown in the Appendix (A.1.3). The equation will become:

$$\bigtriangledown \cdot \mathbf{v} = -f(\Omega_m)H_0 \cdot \delta(\mathbf{r}) \tag{22}$$

The first approximation of the linear growth factor was made in 1980 by Peebles. He found:

$$f \approx \Omega_m^{0.6} \tag{23}$$

This was worked out furhter by Lahav et al. in 1991. They made an estimate for f in an universe with matter and a cosmological constant of [7]:

$$f(\Omega_m, \Omega_\Lambda) \approx \Omega_m^{0.6} + \frac{\Omega_\Lambda}{70}(1 + \frac{\Omega_m}{2}) \tag{24}$$

This clearly shows that the growth rate f mainly depends on the matter density Ω_m. The estimate of Peebles is therefore adopted in this study.

$$\bigtriangledown \cdot \mathbf{v} = -\Omega_m^{0.6}H_0 \cdot \delta(\mathbf{r}) \tag{25}$$

The divergence of the peculiar velocity is proportional to the density, which means that also it also is in Fourier space:

$$\bigtriangledown \cdot \mathbf{v}_k \propto \delta_k \tag{26}$$

and we find that:

$$\bigtriangledown \cdot \mathbf{v} = H_0 \bigtriangledown \int v_k e^{-i\mathbf{k}\cdot\mathbf{r}} d^3\mathbf{k} \tag{27}$$

This means that the peculiar velocity will be:

$$\mathbf{v}(\mathbf{r}) = \frac{-i\Omega_m^{0.6} H_0}{(2\pi)^3} \int \frac{\mathbf{k}}{k^2} \delta_k e^{-i\mathbf{k}\cdot\mathbf{r}} d^3k \tag{28}$$

We define the parameter β to be the ratio between the dimensionless linear growth factor $f(\Omega_m)$ (for which Peebles approximation is adopted (eq. 23)) and the bias parameter b. This latter is adopted to be equal to unity. β is substituted in the above equation:

$$\mathbf{v}(\mathbf{r}) = \frac{-i\beta H_0}{(2\pi)^3} \int \frac{\mathbf{k}}{k^2} \delta_k e^{-i\mathbf{k}\cdot\mathbf{r}} d^3k \propto - \bigtriangledown \varphi \tag{29}$$

The general form of the two-point correlation function is [8]:

$$\xi_{ij}(\mathbf{r}) = \langle v_i(\mathbf{x})\ v_j(\mathbf{x}+\mathbf{r})\rangle = \xi_\perp(r)\delta_{ij} + [\xi_{||}(r) - \xi_\perp(r)]r_i r_j \tag{30}$$

where $r = |\mathbf{r}|$, $r = \mathbf{r}/r$ and $\xi_\perp$ and $\xi_{||}$ are the transverse and the radial correlation functions. Since in the linear regime the peculiar velocity field is a potential field (see Appendix A.1.1 eq. 62), the two correlations functions are not independent from eachother [5]:

$$\xi_{||}(r) = \frac{d}{dr}[r\xi_\perp(r)] \tag{31}$$

The peculiar velocity field is adopted to be a Gaussian random field. Under that assumption the total correlation function $\xi_v(\mathbf{r}) = \langle v(\mathbf{x}) \cdot v(\mathbf{x}+\mathbf{r})$ is equal to [5]:

$$\xi_{\mathbf{v}}(r) = \xi_{||}(r) + 2\xi_\perp(r) \tag{32}$$

The two point correlation function of the peculiar velocity can be defined in terms of the density power spectrum.

$$\xi_{\mathbf{v}}(\mathbf{r}) = \frac{\beta^2 H_0^2}{(2\pi)^3} \int P_k j_0(kr) d^3\mathbf{k} \tag{33}$$

where j_0 is the zero-order spherical Bessel function and P_k is the Fourier transform of the power spectrum. The above defined two point correlation function is defined for the total peculiar velocity. Since our simulation data consists of the line-of-sight peculiar velocity, we find for the line-of-sight two point correlation function :

$$v_{l.o.s.} = \mathbf{v} \cdot \mathbf{r}_{l.o.s.} \tag{34}$$

So now we get:

$$\xi_v^{l.o.s.}(\mathbf{r}) = \frac{\beta^2 H_0^2}{(2\pi)^3} \times \int \frac{(\mathbf{r}_{l.o.s.} \cdot \mathbf{k})^2}{k^4} P_k j_0(kr) d^3\mathbf{k} \tag{35}$$

In the above equation P_k is the Fourier transform of the mass-density power spectrum, $\mathbf{r}$ is the vector seperating the two points, j_0 is the zeroth-order Bessel function and $r = |\mathbf{r}|$. One of the main assumptions in this research project is that we are working at the far observer limit. This means that $\mathbf{r}_{l.o.s.}$ and $\mathbf{k}$

are parallel. Another assumption of an isotropic field is adopted earlier, which results in symmetry between the line-of-sight and the two orthogonal directions. As a consequence of this, the total two point correlation function consists of three times the line-of-sight two point correlation function.

$$\xi_v^{l.o.s.}(r) = \frac{\beta^2 H_0^2}{3(2\pi)^3} \times \int \frac{P_k}{k^2} j_0(kr) d^3\mathbf{k} \tag{36}$$

In this Klein Onderzoek the fields are smoothed by a Gaussian filter. The pertubations on scales smaller then the smoothing radius will dissapear. The Fourier transform of the Gaussian smoothing kernel $e^{-\frac{k^2 R_s^2}{2}}$ will therefor be added to the equation. The derivative of the line-of-sight velocity two point correlation function with respect to the smoothing radius R_s will become:

$$\frac{d\xi_v^{l.o.s.}(r, R_S)}{dR_s} = -\frac{2}{3}\beta^2 H_0^2 R_s \int P_k e^{-k^2 R_s^2} j_0(kr) \frac{d^3\mathbf{k}}{(2\pi)^3} \tag{37}$$

In the above equation, the two point correlation for the density-contrast is recognized, which is given bij (eq. 19). The linear relation between the two-point correlation functions of the density-contrast and peculiar velocity fields can now be introduced:

$$\frac{d\xi_v^{l.o.s}(r, R_s)}{dR_s} = -\frac{2}{3}\beta^2 H_0^2 R_s \xi_d(r, R_s) \tag{38}$$

The two point correlation function in the limiting case of $r = 0$ is defined to be equal to the variance of the measured quantity. If we use this definition in above formula, we find the following linear relation between the variances of the peculiar velocity σ_v^2 and the density-contrast σ_d^2:

$$\frac{d\sigma_v^2(R_s)}{dR_s} = -\frac{2}{3}\beta^2 H_0^2 R_s \sigma_\delta^2(R_s) \tag{39}$$

This is the equation which will be used in this Klein Onderzoek. [15]

1.4 Going nonlinear

A number of assumptions were adopted by finding the relation between the variance of the density-contrast and the peculiar velocity (eq. 39). The main assumption is that the density-contrast is small $|\delta| \ll 1$ and the evolution can be described by the set of linearezed fluid equations. When the density pertubations are of the order of unity, the interaction between the different scales can no longer be neglected. The quasi-linear regime is entered. The Fourier transforms of the fluctuations in the density-contrast, peculiar velocity and gravitational potential will be represented by the set of non-linearezed fluid equations:

$$\frac{d\mathbf{v}(\mathbf{k})}{dt} + \frac{\dot{a}}{a}\mathbf{v}(\mathbf{k}) - \frac{1}{a}\int \frac{d\mathbf{k}'}{(2\pi)^3}[i\mathbf{v}(\mathbf{k}') \cdot (\mathbf{k} - \mathbf{k}')]\mathbf{v}(\mathbf{k} - \mathbf{k}') = \frac{1}{a} i\mathbf{k} \cdot \varphi(\mathbf{k}) \tag{40}$$

$$\frac{d\delta(\mathbf{k})}{dt} - \frac{1}{a} i\mathbf{k} \cdot \mathbf{v}(\mathbf{k}) - \frac{1}{a}\int \frac{d\mathbf{k}'}{(2\pi)^3} i\delta\mathbf{k}' \cdot \mathbf{v}(\mathbf{k} - \mathbf{k}') = 0 \tag{41}$$

$$\frac{\varphi(\mathbf{k})}{a^2} = -4\pi G\rho_u \frac{\delta(\mathbf{k})}{k^2} \tag{42}$$

These equations above show that in the general non-linear case, the continuity equation and the Euler equation contain mode-coupling terms. The evolution the density-modes **k** will no longer evolve independently from each other. [12]

The breakdown point of the linear regime is different for the density-contrast field and the peculiar velocity field. From the relation between the two (eq. 22), the peculiar velocity can be written as:

$$v(\mathbf{r}) = \frac{f(\Omega_m)}{4\pi} \int \delta(\mathbf{r}) \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|} d^3\mathbf{r} \tag{43}$$

This relation shows that in contrast to the density, the peculiar velocity is not a local property. The evolution of the peculiar velocity depends on different density-scales, which can be either linear or nonlinear. The scale of entering the quasi-linear regime therefor differs for these two observables.

1.5 σ_8 as a power spectrum normalization

The simplest inflationary ΛCDM model predicts a so-called scale invariant primordial power spectrum for the density, given by $P(k) \propto k^n$. There is a slight preference for the value of $n = 1$, which was predicted by Peebles and Yu (1970), Harrison (1970) and Zel'dovich (1972).
The power spectrum at the moment of the radiation-matter equality is the one observed today. In the radiation-dominated era, pertubations on scales smaller then the horizon can not grow. In the matter-dominated era this is possible, but the pertubations evolve with different rates. The combination of the two effects cause a bending in the orginally powerlaw spectrum on the scale of the horizon at the radiation-matter-equality. The index of the power spectrum will decrease by four at this bend. This is quantified in terms of the so-called transfer function, whose square is the quantity by which the primordial power spectrun is multiplied to obtain the final power spectrum. For linear pertubations it is defined as the the ratio of the Fourier components at an early time t_i and the the Fourier components at a later time t_f.

$$T(k, t_f) \equiv \frac{b(t_i)}{b(t_f)} \frac{F(k, t_f)}{F(k, t_i)} \tag{44}$$

An estimate for the transfer function for an universe dominated by dark matter over baryonic matter was made by Bardeen et al. [1], drawn from the work of Bond and Salazy (1983), Bond and Efstathiou (1984), Efstathiou and Bond (1985) and Bardeen (1985):

$$T(k) = \frac{ln(1 + 2.34q)}{2.34q} [1 + 3.89q + (1.61q)^2 + (5.46q)^3 + (6.71q)^4]^{-1/4} \tag{45}$$

where $q \equiv k/h\Gamma \ Mpc^{-1}$. The Γ function was defined by Bardeen et al. to be equal $\Gamma = \Omega_m h$. This was generalized by Sugiyama in 1995 [11] to:

$$\Gamma = \Omega_m h e^{-\Omega_b(1+\sqrt{2h}/\Omega_m)} \tag{46}$$

The baryonic density is small in the ΛCDM model. It influences the power spectrum by changing the the steepness of the slope after the turnaround point.

The main influence on the power spectrum comes from the cosmological parameters Ω_m, the matter density, and h, which is coupled to the Hubble parameter. The height of the peak of the power spectrum is determined by the product of $\Omega_m h$. Γ is inverse proportional to the horizon scale at radiation-matter equality, which scales as $1/\Omega_m h^2$, and thus sets the scale at which the bending takes place.

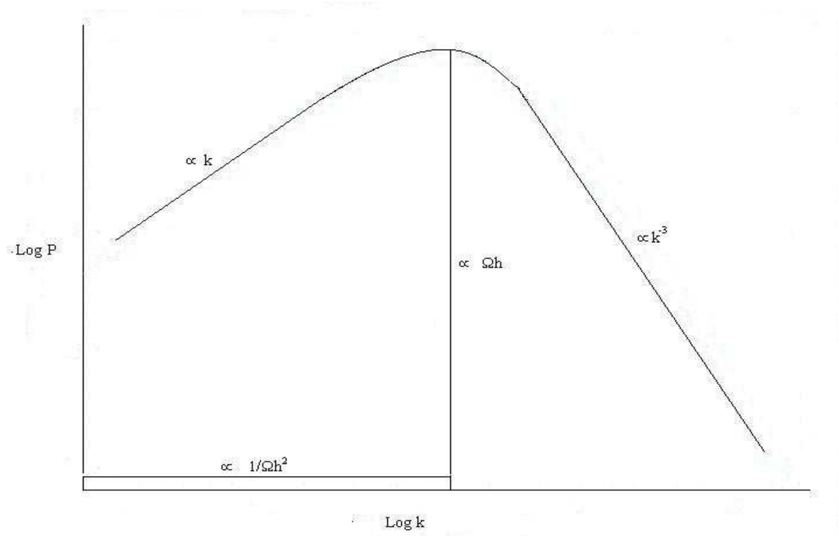

Figure 1: A simplified sketch of the powerspectrum

Part of this Klein Onderzoek is aimed at finding an estimate of the cosmological parameter σ_8 from peculiar verlocity data only. σ_8 is defined as the r.m.s. density variation when smoothed with a tophat-filter of radius of 8h^{-1} Mpc. [9] The definition of σ_8 in formula-form is given by:

$$\sigma_8^2 = \frac{1}{2\pi^2} \int W_s^2 k^2 P(k) dk \tag{47}$$

where W_s is tophat filter function in Fourier space:

$$W_s = \frac{3j_1(kR_8)}{kR_8} \tag{48}$$

where j_1 is the first-order spherical Bessel function. The parameter σ_8 is mainly sensitive to the power spectrum in a certain range of k-values. For large k, the filter function will become negligible and the integral will go to zero. For small k, the factor k^2 in combination with the power spectrum factor k^{-3} will make sure that the integral is negligible. In other words, σ_8 is mostly determined by the power spectrum within the approximate range $0.1 \leq k \leq 2$. Since σ_8 is only sensitive to a certain range of k, any difference in the values of the Hubble uncertaintenty, the baryonic matter density and the total matter density will influence the found estimate.

2 The purpose of this research

The first goal of this Klein Onderzoek is to determine to which scale the linear relation between the density-contrast and the peculiar velocity (eq. 39) is applicable. This simple relation between the two point correlation functions is found to work in the linear regime. Using eq. 39, the cosmological parameter $\beta = \Omega_m^{0.6}/b$ is estimated for different smoothing scales from simulation data. The results that were found are compared with the simulation parameters.

Also in this Klein Onderzoek, an estimate for the cosmological parameter σ_8 is found from peculiar velocity data only. Using the same linear relation (eq. 39) and adopting the WMAP5 value for Ω_m, the variance of the density-contrast is estimated using the SEcat galaxy peculiar velocity catalogue (see section 3 for definition) and a Gaussian smoothing filter ($\sigma_{d,G}$). A theoretical estimate for the variance of the density-contrast is made by using the transfer function determined by Bardeen et al.[1] and Sugiyama [11] ($\sigma_{d,theory}$). This is done for different smoothing scales. By multiplying $\sigma_{d,theory}$ with the ratio of a theoretical value for $\sigma_{8,TH}$ and an estimated value σ_8, the chi-squared test was performed.

$$\chi^2 = \frac{\sigma^2_{d,G} - \sigma^2_{d,theory}}{\sigma^2_{error}} \tag{49}$$

Using only peculiar velocity data when estimating σ_8 at a redshift close to zero is a favourable method, because the data is a direct probe of the underlying mass distribution and close to linearity.

3 The data: SEcat catalogue

In the second part of this research project the cosmological parameter σ_8 is determined out of peculiar velocity only. For this purpose, the SEcat galaxy peculiar velocity catalogue is used.
The SEcat catalogue is actually a combination of two homogeneous peculiar velocity catalogues: the SFI, a catalogue of peculiar velocities of spiral galaxies and the ENEAR catalogue, which is a catalogue of peculiar velocities of elliptical galaxies.([16] and references) The first catalogue has around 1300 objects and the second one around 2000, which are grouped in approximately 750 independent objects. [3] In the catalogue the radial velocity and distance are found for each object, corrected for the Malmquist bias, along with the velocity errors that mount up to ~ 19 percent of the galaxy distance. The SEcat catalogue has a range of 70 h^{-1} Mpc. The big advantage of this catalogue is that, because it consists of early-type and late-type galaxies, it samples both the high density regions and the low density regions. This minimizes the possible biases that might have affected the analyses based on a single population of objects. Moreover, the number of galaxies and the large sky coverage guarantees an dense and uniform sampling of the peculiar velocities.[15]

This is not the first time that this catalogue is used to estimate the parameter β. This is done earlier, with an estimate of $\beta = 0.51 \pm 0.06$. [16] Although a different method is used in this research project to get to an estimate, the expectation is that the results are consistent with this earlier estimate.

To find an accurate estimate of the variance of the peculiar velocity, the galaxies that are more then 7000 km/s away are excluded. Also the galaxies that are closer then 3000 km/s to us are excluded, because they do not fullfill the far-observer limit.

4 Method

4.1 Different methods

Many scientists have used the peculiar velocities of galaxies and their redshift-space positions to calculate an estimate of $\beta = \Omega_m^{0.6}/b$ under the assumption of linear theory and linear biasing. This was mostly done using two different methods: velocity-velocity comparison and density-density comparison.

4.1.1 Density-density comparison

In density-density analyses a three dimensional velocity field and a density field are derived from observations of the radial velocity and compared to the galaxy density field observed at redshift surveys. The estimates done with the density-density comparison are in general pretty large, close to the order of unity. One of the examples of this method is the comparison of the mass density field reconstructed with the POTENT method [2] from the MARK III catalogue of peculiar velocities [13] with the galaxy density field obtained from the IRAS PSCz 1.2 Jy redshift catalogue [10]. The result of this research was a rather large estimate of β, namely 0.89 ± 0.12 at a smoothing scale of 12 Mpc.[10] Calculations done earlier using the Velmond method [4] with data from the same catalogi led to a considerable lower estimate of β (~ 0.5). Another example is the in 2002 propsed UMV method [16]. This method was used to reconstruct the density-contrast and peculiar velocity field of the SEcat galaxy peculiar velocity catalogue and led by comparison with models obtained from the PSCz galaxy density-contrast survey to a value of $\beta = 0.57^{+0.11}_{-0.13}$. [16]

4.1.2 Velocity-velocity comparison

Here the observed galaxy distribution provides a mass density distribution from which the peculiar velocities can be calculated. These will then be compared to the observed ones. This type of comparison has been applied to almost all the peculiar velocities catalogues that exist and the results are systematically lower then those of the density-density comparison: around the value of $\beta = 0.4 - 0.6$.[14]. A velocity-velocity comparison is made using the UMV method with data from the SEcat galaxy peculiar velocity catalogue and the PSCz galaxy density-contrast catalogue to yield an estimate of $\beta = 0.51 \pm 0.06$. [16]

Velocity-velocity comparisons are generally regarded as more reliable as they require manipulation of the more homogeneous redshift catalogue data. This in contradiction to the density-density comparisons, which involve manipulation of the more noisier velocity data. [16]

4.2 Our method

In this research, a model-independent and linear relation is derived between the density-contrast and line-of-sight peculiar velocity variances assuming Gaussian smoothing:

$$\frac{d\sigma_v^2(R_s)}{dR_s} = -\frac{2}{3}\frac{f(\Omega_m)^2}{b^2} H_0^2 R_s \sigma_d^2(R_s) \tag{50}$$

The numerical calculation of the left hand side of the above equation is pretty straightforward. The derivative is obtained by infinite differencing:

$$\frac{d\sigma_v^2(R_s)}{dR_s} \approx \frac{\sigma_v^2(R_s + \triangle R_s) - \sigma_v^2(R_s)}{\triangle R_s} \tag{51}$$

One advantage of this proposed method is that the contribution of the measurement noise is low. Because of the smoothing involved, the variance is not greatly influenced by large random noise and because the comparison is done within the same region of space, the cosmic variance contamination contribution to the error analysis is avoided. The error itself can be calculated as follows: the noised associated with particle i is assumed to be $\epsilon(\mathbf{r}_i)$. Since smoothing is applied, the noise will get smoothed as well:

$$\epsilon^s(\mathbf{r}_i) = \sum_j \epsilon(\mathbf{r}_j) W_{R_s}(\mathbf{r}_i - \mathbf{r}_j) \tag{52}$$

Because the derived relation uses the two-point correlation functions, the two-point correlation function of the noise is calculated. The expectation value yields:

$$\langle \epsilon^s(\mathbf{r}_i)\ \epsilon^s(\mathbf{r}_l) \rangle = \sum_j \epsilon^2(\mathbf{r}_j) W_{R_s}(\mathbf{r}_i - \mathbf{r}_j) W_{R_s}(\mathbf{r}_l - \mathbf{r}_j) \tag{53}$$

where in the last equation the errors are assumed to be statistically uncorrelated. The derived relation (eq.39) is based on the variances of the parameters, so $\mathbf{r}_i = \mathbf{r}_l$ and the sum over all the data points is required. The result will then be the variance:

$$\sigma_N^2(R_s) = \frac{1}{N} \sum_{i,l} \langle \epsilon^2(\mathbf{r}_j) \rangle W_{R_s}^2(\mathbf{r}_i - \mathbf{r}_j) \tag{54}$$

So the noise variance that will add to the left hand side of the equation will be:

$$\frac{d\sigma_N^2(R_s)}{dR_s} \approx \frac{\sigma_N^2(R_s + \triangle R_s) - \sigma_N^2(R_s)}{\triangle R_s} \tag{55}$$

The values of the parameters that are used in the program, are based in the parameters in the ΛCDM Model. For the Hubble constant H and the matter density contribution Ω_m the values of respectively $100h$ km Mpc^{-1} s^{-1} and 0.27 are found.

5 First test: determining β on small scales

5.1 Earlier results

One of the biggest observations from which β can be determined is the WMAP-data. The results of the WMAP5 have only been recently presented. [6] The main goal of the mission was to estimate the parameters of the ΛCDM Model out of the observations of the Cosmic Microwave Background. Before any observations were done bounds were set on the parameters, taken from earlier observations. The WMAP data is also combined with the distance measurements from the Type Ia Supernovae (SNIa) and the Baryonic Accoustic Oscillations (BAO) in the distribution of galaxies. The results of the estimate for Ω_m with the combinations of different data-sets, are given in the table below. [6]

Parameter	Range	WMAP2008	WMAP2008+ BAO +SNIa
$\Omega_m h^2$	0.1308-0.1363	0.1326±0.0063	0.1369±0.0037

Table 1: The estimates of Ω_m, calculated with data taken from WMAP2008, combined with other data sets.

The approximation of Peebles (eq. 23), adopted in this research, is discussed in the introductory chapter. A value of h equal to 0.70 is adopted. From the two datasets above, the mean of the total matter density is $\langle \Omega_m \rangle = 0.2600$. When linear theory and linear biasing are assumed, the estimate for β from WMAP5 data can be calculated:

$$\beta = \frac{\Omega_m^{0.6}}{b} = 0.446 \tag{56}$$

Another estimation of β was calculated through the analysis of the SEcat catalogue using the UMV-method, with the constructed fields compared to those of the PSCz galaxy density contrast catalogue. This led to β-values from the comparison of the density and the velocity field yielding $\beta = 0.57^{+0.11}_{-0.13}$ and $\beta = 0.51 \pm 0.06$. [16] These are the most consistent results known, but other results have been published. (see section 4)

5.2 Testing the method

5.2.1 Parameters of our simulation

The first test done in this Klein Onderzoek, is to see whether β can be estimated through linear theory on small smoothing scales. The data used in this simulation is from a mock catalogue given by Geraint Harker. The length of the simulation box is 153.6h^{-1} Mpc and the simulation was build on a grid of 256^3 points. The matter density in this simulation has a value that is equal to the value in the ΛCDM Model: $\Omega_m \approx 0.275$. This yields a theoretical simulation estimate of $\beta = 0.456$, using Peebles (eq. 23). The value of the redshift is equal to zero. The simulation data provides us with four parameters: the peculiar velocity in the x-direction, in the y-direction, in the z-direction and the density-contrast.

5.2.2 Computer program

Our program, written in IDL (see appendix A.2.1), reads in the three velocities and the density given by the simulation data. These are smoothed with a Gaussian filter. The smoothing scales are focused on the smaller scales, since this is the area explored in this research project. For the velocity the same procedure is done: the variance of the velocity is calculated at a certain smoothing radius. But since differentiation is needed, the variance of the velocity at $R_s + \triangle R_s$ is also determined. $\triangle R_s$ is set at 0.01. Now all the parameters of the linear equation between the variances of the density-contrast and the peculiar velocity are know:

$$\frac{d\sigma_v^2(R_s)}{dR_s} = -\frac{2}{3}\beta^2 H_0^2 R_s \sigma_d^2(R_s) \tag{57}$$

$$\frac{\sigma_v^2(R_s) - \sigma_v^2(R_s + \triangle R_s)}{\triangle R_s} = -\frac{2}{3}\beta^2 H_0^2 R_s \sigma_d^2(R_s) \tag{58}$$

This simulation is done at a range of smoothing radii and for the different directions of velocities, trying to find a good estimate for β.

5.3 Results & discussion

The simulation yields the following results, found in figure 2 and table 2:

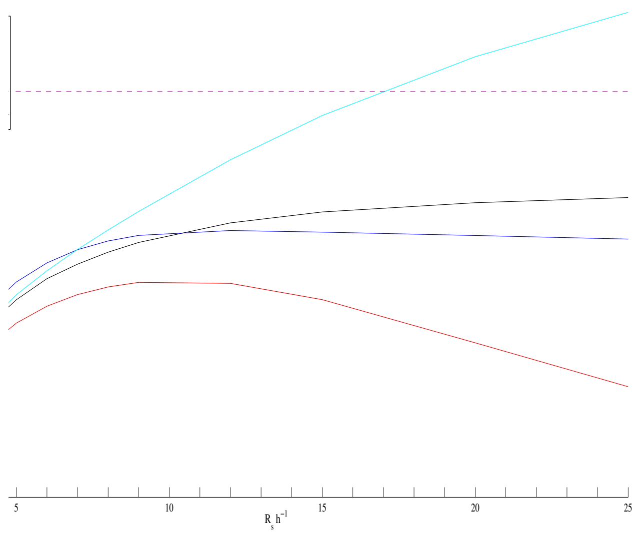

Figure 2: The results of our simulation. In this graph are the estimates of β plotted against the smoothing radius for the different velocities. The purple dashed line is the theoretical estimate for β according to Peebles. This simulation the parameters are: Ω_m=0.27, theoretical simulation estimate = 0.456 and gridsize=256^3.

The simulation was done with the value of Ω_m equal to 0.27 and linear theory and linear biasing are still assumed. This will give us a theoretical simulation value of β, of:

$$\beta = \frac{\Omega_m^{0.6}}{b} = 0.456 \tag{59}$$

In these results, a few prominent features can been seen. First of all, it is noticed that the value at which the velocities flatten out, is below the theoretical

Smoothing scale R_s	v_x	v_y	v_z	v
2	0.282	0.269	0.276	0.276
3	0.321	0.305	0.314	0.314
4	0.345	0.326	0.337	0.336
5	0.359	0.338	0.353	0.350
6	0.369	0.347	0.365	0.361
7	0.376	0.353	0.376	0.368
8	0.380	0.357	0.386	0.374
9	0.383	0.359	0.395	0.379
12	0.385	0.359	0.421	0.389
15	0.385	0.350	0.444	0.395
20	0.383	0.328	0.474	0.400
25	0.381	0.306	0.496	0.402

Table 2: Different estimates for β for different velocities.This simulation the parameters are: Ω_m=0.27, theoretical simulation estimate = 0.456 and gridsize=256^3.

simulation estimate. The likely reason for this is the way the velocity is normalized in the simulation. The specific simulation we use has an Ω_m= 0.1 at z= 0. To obtain the desired matter over-density, the simulation is analyzed at the redshift z_1 at which Ω_m= 0.27, which is not at z= 0. In order to get the correct simulation properties had it been with Ω_m=0/27 at z=0, the density is re-normalized to obtain σ_8=0.8 at redshift z_1. This operation is straightforward for the density. Unfortunatly however, this is not the case for the velocity, since this has an integral form and therefor depends on many scales. The simulation that we have, has been supposedly normalized for the velocities but we suspect that this is not done in a satisfactory way.

If the above is noted, the results can be discussed. It can been seen in the graph that the estimates of β of the peculiar velocity as a whole and the peculiar velocity in the x-direction nicely flatten out around 0.387. This is assumed to be 'the true value'. Down to smaller scales, we see that up till 5 h^{-1} Mpc the estimate for β is within 11 % error of the 'true value' Even at 4 h^{-1} Mpc the estimate is within 15%! This is better than expected.

On large scales, the graph does not flatten out, except for the velocity in the x-direction and the real peculiar velocity. Although the linear regime holds on large smoothing scales, the curve is not expected to flatten out completely. This is because the number of independent cells decreases as a function of smoothing scale,i.e. cosmic variance, so a very accurate estimate can not be obtained. But even with these remarks, the curves which represent the estimates at the velocity in the y- and z-direction are not even in the neighbourhood of the 'true value', the value of β where the other two velocity components flatten out. It was suspected that there was something in the structure influencing the peculiar velocity in these directions.

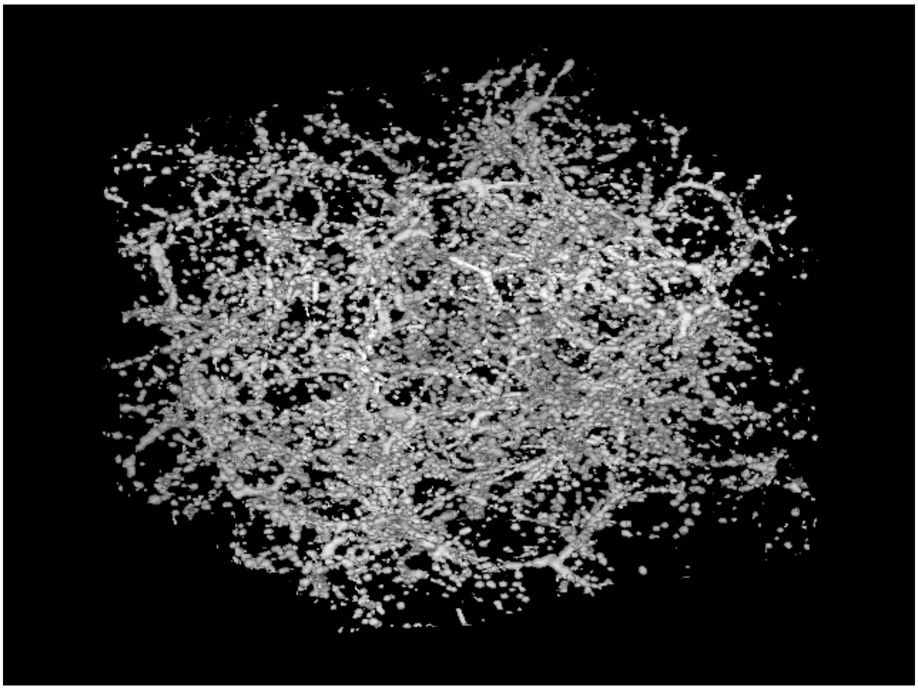

Figure 3: The density at a density-contrast of 5 is plotted in 3D. The density was smoothed first, with a smoothing scale of 2 Mpc.

When plotting the smoothed density-contrast in 3D, see figure 3, it was noticed that there were quite some filaments and walls in the simulation. As you can see in the graph, in the upper right corner, there are some large over-densities. As we look more closely to graph, it is noted that the bigger structures in the Cosmic Web are propagating more or less along the x-axis. This means that the structure will not have a big influence on the peculiar velocity in the x-direction. On the other hand, the influence of this big structure on the peculiar velocities in the other two directions is quite big. Because of the gravitational pull of the filaments and walls, the peculiar velocities in the y- and z-directions will be changed. This will lead to different estimates of β.

It was noted in the first part of the research that the theoretical simulation estimate was higher than anticipated because of an error in the scaling of the simulation data. Instead of the desired matter density value Ω_m of 0.27 at z=0, the simulation had an approximate value of $\Omega_m \approx 0.1$ at this particular redshift. This gives a theoretical simulation estimate of 0.273. At the redshift z=0, the peculiar velocities had to be re-scaled as well. This will influence our estimate of β. The results of this re-scaled data set are found in graph 4 and table 3.

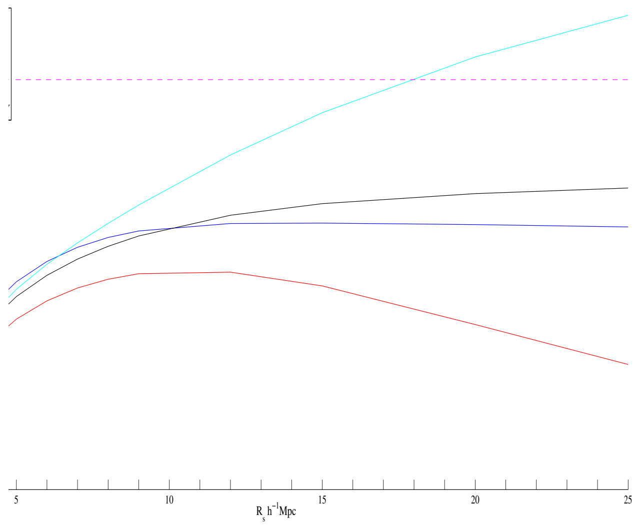

Figure 4: The results of our simulation. In this graph are the estimates of β plotted against the smoothing radius for the different velocities. The purple dashed line is the theoretical estimate for β according to Peebles. In this simulation the parameters are: $\Omega_m \approx$0.1, theoretical simulation estimate=0.273 and N=256^3.

Smoothing scale R_s	v_x	v_y	v_z	v
2	0.159	0.151	0.156	0.155
3	0.183	0.173	0.180	0.179
4	0.198	0.187	0.195	0.193
5	0.207	0.195	0.205	0.203
6	0.214	0.201	0.213	0.210
7	0.219	0.205	0.220	0.215
8	0.222	0.208	0.226	0.219
9	0.224	0.210	0.232	0.222
12	0.226	0.210	0.249	0.229
15	0.226	0.206	0.262	0.233
20	0.226	0.194	0.280	0.236
25	0.225	0.181	0.294	0.238

Table 3: Different estimates for β for different velocities. In this simulation the parameters are: $\Omega_m \approx$0.1, theoretical simulation estimate=0.273 and N=256^3.

The results of the estimates of β of this new simulation are also consistent with the results obtained from the earlier result. Again, we see that the value of flattening for the whole and the x-direction peculiar velocity is not near the theoretical simulation estimate. This is probably because of incapability of the re-scaling of the peculiar velocities. The estimate of β at which the curves flatten, is assumed to be the 'true value'. On a smoothing scale of 5 h^{-1} Mpc, the estimate of β is within 14% of this so-called 'true value'. On a smoothing scale of 4 h^{-1} this is within 19 %. The peculiar velocities in the y- and z-direction still show their deviation from the 'true value'.

As can been seen in table 3 and graph 4, the simulation data still not flattens out at the theoretical simulation estimate. This could be the case at low values for Ω_m, because the matter distribution is highly clustered. This means that there are quite some gridpoints in our simulation with a density-contrast of 0. The way to solve this, is to re-grid our simulation and this time use less gridpoints. So our simulation-data is re-gridded to a size of 64^3. The results of this simulation are found in graph 5 and table 4.

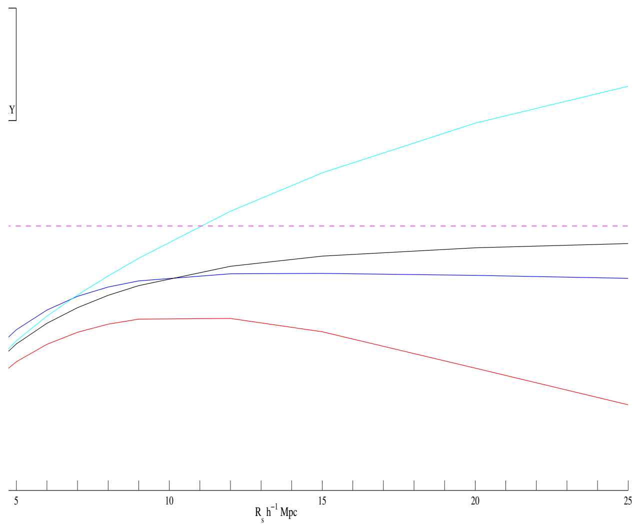

Figure 5: The results of our simulation. In this graph are the estimates of β plotted against the smoothing radius for the different velocities. The purple dashed line is the theoretical estimate for β according to Peebles. In this simulation the parameters are: $\Omega_m \approx 0.27$, theoretical simulation estimate=0.488 and N=64^3.

As can been seen in the graph 5, the flattening of the curves of the whole and x-direction peculair velocity, is now closer to the theoretical simulation estimate. The fact that these two values do not completely coïncide, is probably because of the cosmic variance: the patches choosen are not entirely identical. The curves of the peculiar velocity in the y- and z-direction still show the large deviation, as expected. On a smoothing scale of 5 h^{-1} Mpc the estimate of β is within 13 % of the flattening value of the whole and x-direction peculiar velocities. For the smoothing radius of 4 h^{-1} Mpc this is 19%. All these values are again consistent with the earlier results.

The most prominent feature of all the above graphs, is that the value at which the velocities flatten out is (sometimes) quite a bit below the theoretical simula-

Smoothing scale R_s	v_x	v_y	v_z	v
2	0.318	0.305	0.309	0.311
3	0.363	0.346	0.353	0.354
4	0.394	0.373	0.384	0.384
5	0.414	0.391	0.406	0.404
6	0.428	0.404	0.424	0.419
7	0.438	0.412	0.439	0.430
8	0.445	0.418	0.452	0.439
9	0.449	0.422	0.465	0.446
12	0.454	0.422	0.499	0.459
15	0.454	0.413	0.526	0.467
20	0.453	0.387	0.561	0.473
25	0.451	0.361	0.587	0.476

Table 4: Different estimates for β for different velocities. In this simulation the parameters are: Ω_m ≈0.27, theoretical simulation estimate=0.488 and N=64^3.

tion estimate made using the Peebles approximation. Although the simulation data sets used are based on the same set, they are fundamently different. It is therefore a surprise that the curves of the whole and the x-direction velocity do not flatten out at the theoretical simulation estimate. It is possible that there is an error in the written computer program. To test this, another data file was given as input as the program. This simulation data was provided by E. Romano-Diaz and also used to test the computer program in [15]. The data set delivered uses the parameters which correspond to an Einstein de Sitter universe. This beholds that $\Omega_m = 1$. The number of the grid points is equal to 128 and the box size is 100 h^{-1} Mpc. According to the estimate of Peebles for β (eq. 23), this means that β is equal to 1. The results of the simulation with the new data for an Einstein de Sitter universe are found below, in graph 6 and table 5.

Smoothing scale R_s	v_x	v_y	v_z	v
2	0.514	0.537	0.515	0.522
3	0.650	0.684	0.650	0.662
4	0.735	0.780	0.732	0.749
5	0.788	0.843	0.778	0.804
6	0.822	0.890	0.802	0.839
7	0.845	0.927	0.814	0.863
8	0.862	0.959	0.818	0.882
9	0.876	0.987	0.820	0.897
12	0.912	1.051	0.816	0.932
15	0.938	1.081	0.813	0.950
20	0.933	1.054	0.806	0.937
25	0.892	0.994	0.792	0.896

Table 5: Different estimates for β for different velocities. In this simulation the parameters are: Ω_m=1, theoretical simulation estimate=1 and N=128^3.

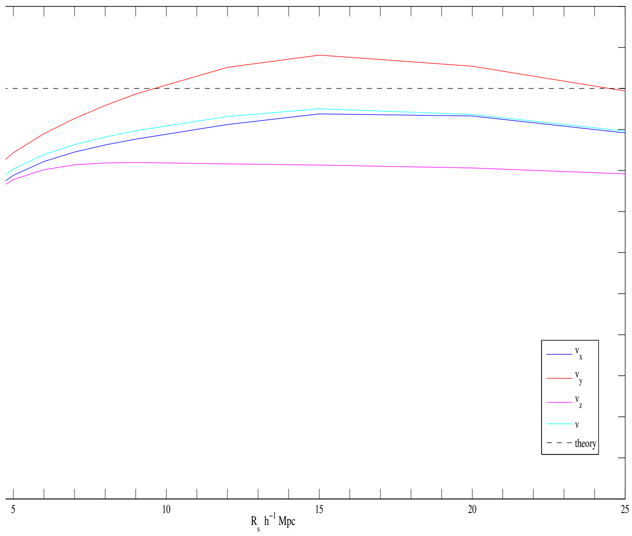

Figure 6: The results of our simulation. In this graph are the estimates of β plotted against the smoothing radius for the different velocities. The red dashed line is the theoretical estimate for β according to Peebles. In this simulation the parameters are: Ω_m=1, theoretical simulation estimate=1 and N=128^3.

The first thing that is noted, is that the graph again not completely flattens out at the theoretical estimate. Cosmic variance and the decrease of the number of independent cells are again the most likely reason for this. When comparing the last two simulations, the last set of results are closer to the theoretical simulation estimate then the set before that. It is also been seen that this last set of data does not have this large deviation in the y- and z-direction, so that also these velocity-curves nicely flatten out as was expected. The mean value around which flattening takes place is $\langle\beta\rangle = 0.930$. From the graph above, the mean estimate at a scale of 5 h^{-1} Mpc is 0.800. This is 20% of the theoretical estimate made using Peebles' assumption and 16% of the mean flattening value. This is not bad, considering that our simulation estimate at the smoothing scale of 12, where the linear regime is considered to be valid, is a few percent off of the theoretical estimate.

6 Second test: σ_8 from peculiar velocity data

6.1 Earlier results

The second part of our Klein Onderzoek constisted of estimating a value for σ_8 from peculiar velocity data only. This is done before from the data taken on the WMAP mission. Again, there are two datasets, built up the same way as in the last section. [6] The estimates are put in the table below.

Parameter	Range	WMAP2008	WMAP2008+ BAO +SN
σ_8	0.787 - 0.811	0.796±0.036	0.817±0.026

Table 6: The estimates of σ_8, calculated with data taken from WMAP5, combined with other datasets.

The mean estimate of σ_8 from the above two data sets is $\langle\sigma_8\rangle = 0.81$.

6.2 Testing our method

To find an estimate for the cosmological parameter σ_8, the chi squared test was used:

$$\chi^2 = \sum_{i=1}^{N} \frac{(y_i - f_i)^2}{\sigma_i^2} \tag{60}$$

In the above formula, y_i are the measured values, f_i are the theoretical determined values and σ_i is the error in the standard deviation of the measured values. For y_i, the results of a computerprogram are used, which estimates the density variance from the SEcat galaxy peculiar velocity catalogue according to the linear equation derived and used earlier (eq. 39). This is done for a number of smoothing radii. Since the catalogue consists of a large number of points that are not all independent of each other, the number of degrees of freedom is approximated by the number of independent cells N. The number of independent cells is roughly equal to:

$$N = \frac{(70h^{-1})^3}{\sqrt{(2\pi)^3} R_s^3} \tag{61}$$

where R_s is the smoothing radius. This number is calculated out of the standard integral of the Gaussian distribution without its normalisation factor. (see Appendix A.2.2)
A theoretical estimate of the density variance is found by approximating the transfer function for cold dark matter (Bardeen et al. [1]) with the Sugiyama estimation. [11] In the chi-squared test, the difference of the theoretical approximation and the measured value is divided by the error in the measured value. The sum over these steps is called χ^2. The point where χ^2 has its minimum, is the best approximation for the parameter σ_8.

6.2.1 Parameters

The parameters used are all in the transfer function of cold dark matter [1]. As has been shown earlier in this report (see section Introduction), this depends on the cosmological parameters Ω_m, Ω_b and h. The values choosen for these parameters are the ones that are determined by the ΛCDM model and are respectively 0.27, $0.019/h^2$ and 0.7. There were we are differing the value for Ω_m, we will point out what the used values are.

6.2.2 Computer program

The computer program that is used to find an estimate for the measured data calculated the variance of the density under the assumption that Ω_m has a value of 0.27. Using the equation derived earlier, the estimates for the density-variance on different smoothing radii are found. Also the number of independent cells are calculated.
The theoretical values used in the chi-squared distribution are the results of another computer program, with the estimated power spectrum equal to that of the Cold Dark Matter power spectrum and with the ΛCDM model values for the cosmological parameters. (see appendix A.2.2)

6.3 Results & discussion

The different results are plotted in a different graphs below:

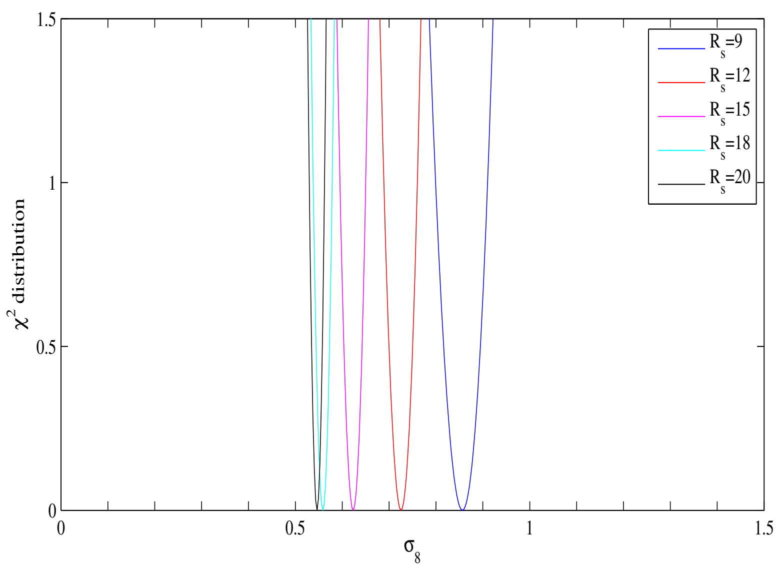

Figure 7: Estimates of σ_8 out of the peculiar velocity data. In the graph is the χ^2-distribution plotted versus the estimate of σ_8. This was done for different smoothing scales.

The estimates at the smoothing scales of $R_s = 9$ and $R_s = 12$ are the two that have within error bars, the estimate of the WMAP5. This was to be expected because these are the smoothing radii that can be best represented by the linear regime. As can been seen from the table, the error in the estimate of σ_8

Smoothing scale R_s	Estimate for σ_8	Number of independent cells
9	0.84 ± 0.09	87
12	0.73 ± 0.12	37
15	0.62 ± 0.14	19
18	0.56 ± 0.17	11
20	0.55 ± 0.19	8

Table 7: Estimates of σ_8 out of peculiar velocity data.

increases. The reason for this is that the error in proportional to the square root of the number of independent cells, which is proportional to the inverse of the smoothing radius cubed. Because the number of independent cells decreases with increasing smoothing radius, the estimate of σ_8 gets less reliable.

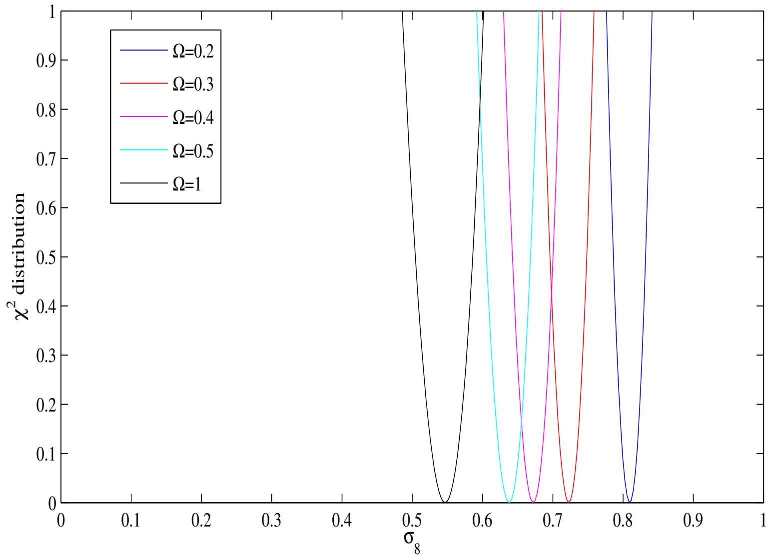

Figure 8: Estimates for σ_8 done on a smoothing scale of $R_s = 12$ while fixing the value of Ω_m.

The value for the total matter density $\Omega_m = 0.2$ and $\Omega_m = 0.3$ are again within error bars of the the value that was found using the WMAP data. This is also a good indication that the ΛCDM model, the model that is used to describe the universe as seen today, is pretty accurate. This model has adopted a value of $\Omega_m = 0.27$.

The χ^2 probability function is also plotted. From this function the likeliness that a certain value for σ_8 is found. The probability function is a pretty complicated function, but in this case can be represented by only the exponential part. Because this distribution is a Gaussian function, it can be normalised to get an estimate for the error in this function. As can been seen in the table, the errors are particulary low. (within 7% of the measured value of σ_8)

Matter density Ω_m	estimate for σ_8
0.2	0.81 $\pm$ 0.14
0.3	0.73 $\pm$ 0.12
0.4	0.67 $\pm$ 0.11
0.5	0.64 $\pm$ 0.11
1	0.54 $\pm$ 0.09

Table 8: The estimates for σ_8 on smoothing scale R_s=12 with different values for the matter density Ω_m.

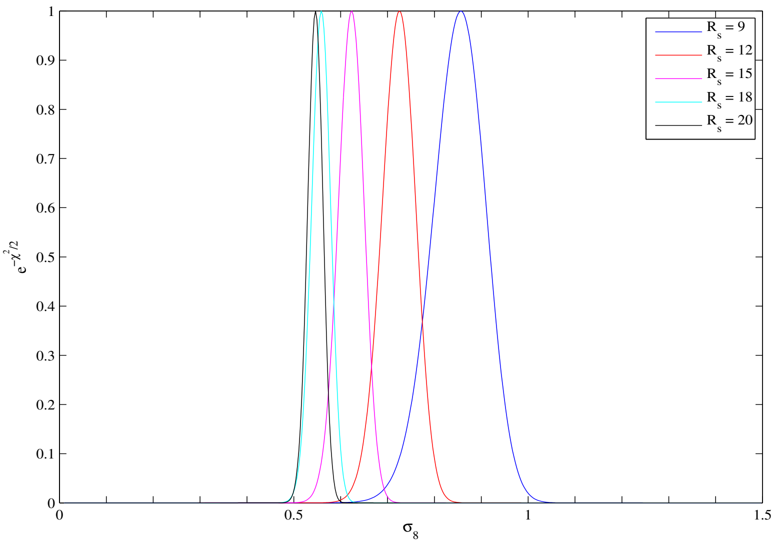

Figure 9: Plotting the χ^2 probability distribution function while fixing the smoothing scale.

7 Summary & Discussion

Looking at the estimates for β calculated from simulation data of the peculiar velocity and the density-contrast by using the linear relation between the correlation functions of the peculiar velocities and the densities (eq: 39), two regimes can be discussed.

The purpose of the first part of this Klein Onderzoek was to see whether the linear relation was applicable to smaller smoothing scales. The results for the various simulation sets show that the estimate of β down to a scale of 5 h^{-1} Mpc are in all our simulations within 16 % of the flattening value of the peculiar velocities. This was better than expected on forehand. The reason why the linear relation works so well on these small smoothing scales could be a function of our choosen filter. If instead of a Gaussian filter, a top-hat filter was used with the same volume the range of the smoothing scales extended more towards the linear regime. On larger scales a small deviation was expected at these scales because of the decrease of the number of independent cells as a function

Smoothing radius R_s	mean μ	error σ
9	0.84	0.06
12	0.73	0.04
15	0.62	0.03
18	0.56	0.02
20	0.55	0.02

Table 9: The specifications of the probability distribution of the chi-squared

of smoothing radius. In a few of the used data sets, a large deviation was found in the peculiar velocity curves in the y- and z-direction. This is probably because of a large over-density in the Cosmic Web. The other peculiar velocity curves flatten out nicely, as expected. Because of the difficulty of re-scaling the peculiar velocity, not all data sets flatten out at the theoretical estimate made using Peebles' approximation. (eq. 23)

The second part of this Klein Onderzoek was to determine an estimate of σ_8 from peculiar velocity data only. The data used orginates from the SEcat galaxy peculiar velocity catalogue. Using the derived linear relation for the two-point correlation function of the peculiar velocity and the density-contrast (eq. 39), a calculated variance for the density-contrast is retrieved. A theoretical estimate was made by integrating the transfer function for a cold dark matter spectrum (eq. 45) using Gaussian smoothing, multiplied by the ratio of the theoretical σ_8 and the estimated σ_8. To compare the two estimates of the density-contrast variance, the chi squared test was used. First an estimate of σ_8 was calculated for different smoothing scales under the assumption that the matter density was equal to $\Omega_m = 0.27$. The estimates for σ_8 on the smooothing scales of $R_s = 9$ and $R_s = 12$ conlude within their errorbars the estimate from the WMAP5 data [6]. The next step is to see what the best estimate is for the matter density Ω_m at the smoothing scale $R_s = 12$. By adopting the earlier found value for σ_8 to be the right one, the resulats showed that the best estimate of the matter density is roughly $\Omega_m = 0.3$. Again, this is in agreement with the WMAP5 data [6] and good confirmation for the ΛCDM model.

There is also another way to find confirmation for the ΛCMD model. When the results from the last calculations in the second part of this Klein Onderzoek are plotted in the combination $\sigma_8\Omega_m^{1.2}$, the following is found:

Matter density Ω_m	estimate for $\sigma_8\Omega_m^{1.2}$
0.2	0.12
0.3	0.0.17
0.4	0.23
0.5	0.27
1	0.55

Table 10: The estimates for σ_8 on smoothing scale R_s=12 with different values for the matter density Ω_m.

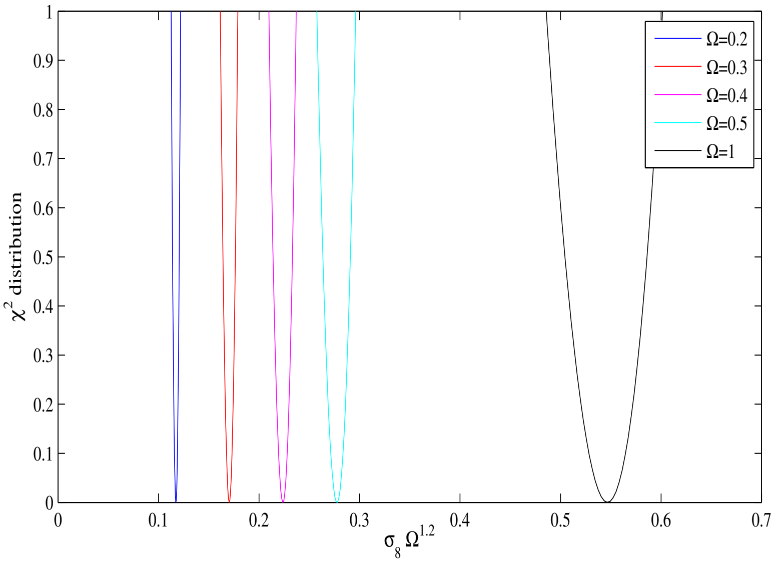

Figure 10: Estimates for σ_8 done on a smoothing scale of $R_s = 12$ while fixing the value of Ω_m. This time the parameter $\sigma_8\Omega_m^{1.2}$ is plotted.

This combination of parameters is plotted because it shows whether the cosmological parameter σ_8 is a good indicator for the matter density. The expected value is simply the multiplication of the σ_8 found earlier in this Klein Onderzoek with the ΛCDM model value for the matter density Ω_m. This gives us an expected value of 0.152. By varing the matter density, the value of $\sigma_8\Omega_m^{1.2}$ differs significantly. Plotting this combination of parameters is therefore a good probe for the ΛCDM model.

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A Appendix

A.1 Derivations

A.1.1 Peculiar velocity as potential flow in the linear regime

A velocity $\mathbf{v}$ can always be decomposed into a potential flow component $\mathbf{v}$ and a rotational flow component $\mathbf{v}_{rot}$, which creates vorticity. The first one can be written as a gradient of a scalar potential, while the the second one can been seen as the curl of some vector potential.

$$\mathbf{v} = \bigtriangledown\psi \tag{62}$$

$$\mathbf{v}_{rot} = \bigtriangledown \times \mathbf{B_v} \tag{63}$$

If this distinction in velocity is entered in the continuity equation for pertubations, the rotational flow component of the velocity will fall out of the equation: $\bigtriangledown \cdot \mathbf{v}_{rot} = \bigtriangledown \cdot (\bigtriangledown \times \mathbf{B_v}) = 0$. This means that only the potential flow component of the velocity is coupled to the density pertubations.

$$\bigtriangledown \cdot \mathbf{v} = -a\frac{\partial \delta}{\partial t} \tag{64}$$

$$\bigtriangledown \cdot \mathbf{v}_{rot} = 0 \tag{65}$$

If the above equation is adopted in the linearezed Poisson equation, the potential flow component of the velocity can be coupled to the gravitational potential.

$$\bigtriangledown \cdot \mathbf{v} = -a \bigtriangledown \cdot \frac{\partial}{\partial t} \frac{\bigtriangledown \varphi}{4\pi G \rho_u a^2} \tag{66}$$

The evolution of the both velocity-components can now be described by the Euler equation:

$$\frac{\partial a\mathbf{v}}{\partial t} = - \bigtriangledown \varphi \tag{67}$$

$$\frac{\partial a\mathbf{v}_{rot}}{\partial t} = 0 \tag{68}$$

From (eq. 68) it is seen that the rotational component of the velocity evolves like a^{-1}. Assuming that there were primordial vorticity primordial flows, it is shown that they have decayed to zero while the universe expanded. There is no system that generates vorticity flows, so the assumption that there is no vorticity in the linear regime can be adopted. This means that the peculiar velocity is equal to only the gradient of the gravitational potential pertubation. This is the first step of the relation between the velocity, density and the gravitational potential.

A.1.2 Dimensionless linear velocity growth factor

$$\frac{1}{D}\frac{\partial D}{\partial t} = \frac{1}{a}\frac{a}{D}\frac{dD}{da}\frac{da}{dt} \tag{69}$$

$$= H(t)\frac{a}{D}\frac{dD}{da} \tag{70}$$

$$= H(t)\frac{dlnD}{dlna} \tag{71}$$

$$= H(t)f \tag{72}$$

With f is the dimensionless linear velocity growth factor, which depends on the curvature in our universe (Ω_0) and the cosmological constant Λ.

A.1.3 Relation between peculiar velocity and density contrast

$$\bigtriangledown \cdot \mathbf{v}_{||} = -a \bigtriangledown \cdot \frac{\partial}{\partial t}\left(\frac{\bigtriangledown \phi}{4\pi G\overline{\rho}a^2}\right) \tag{73}$$

$$\bigtriangledown \cdot \mathbf{v}_{||} = a \bigtriangledown \cdot \frac{\partial}{\partial t}\frac{\mathbf{g}}{4\pi G\overline{\rho}a} \tag{74}$$

$$\bigtriangledown \cdot \mathbf{v}_{||} = a \cdot \frac{\partial}{\partial t}\frac{\bigtriangledown \mathbf{g}}{4\pi G\overline{\rho}a} \tag{75}$$

$$\bigtriangledown \cdot \mathbf{v}_{||} = a \cdot \frac{\partial}{\partial t}\frac{-4\pi G\rho_u \delta a}{4\pi G\overline{\rho}a} \tag{76}$$

$$\bigtriangledown \cdot \mathbf{v}_{||} = -a \cdot \frac{\partial}{\partial t}\delta(\mathbf{r}, t) \tag{77}$$

A.1.4 Number of independent cells

The number of independent cells is equal to:

$$N = \frac{V}{V_{cell}} \tag{78}$$

The total volume of a cell can be found according to:

$$V_{cell} = \int_0^R f(r)r^2 dr d\Omega \tag{79}$$

where f(r) is the density distribution function. The density-contrast is a Gaussian distribution:

$$V_{cell} = 4\pi \int_0^R e^{\frac{-r^2}{2R_s^2}} r^2 dr \tag{80}$$

This is a standard intgral and its result is equal to:

$$V_{cell} = \sqrt{(2\pi)}^3 R_s^3 \tag{81}$$

This gives a number of indepent cells equal to:

$$N = \frac{(70h^{-1})^3}{\sqrt{(2\pi)}^3 R_s^3} \tag{82}$$

This results in, sorted by smoothing radius:

Smoothing radius R_s	Number of independent cells N
9	84
12	36
15	19
18	11
20	8

A.2 Computer programs

A.2.1 Simulating β

```
pro readPSCz,vx,vy,vz,density,ngrid
; reading in the peculiar velocities and the density-contrast
boxsize=0.0D
omegabox=0.0D
lambdabox=0.0D
hubblebox=0.0D
ngrid=0L

OPENR,Unit,'dohgrid.dat', /F77\_UNFORMATTED,/GET\_LUN

READU,Unit,boxsize,omegabox,lambdabox,hubblebox,ngrid

density=fltarr(ngrid,ngrid,ngrid)
vx=fltarr(ngrid,ngrid,ngrid)
vy=fltarr(ngrid,ngrid,ngrid)
vz=fltarr(ngrid,ngrid,ngrid)

READU,Unit,density
READU,Unit,vx
READU,Unit,vy
READU,Unit,vz

close,unit
return
end
;

pro calculation
; Writing a program to smooth the density and velocity. Grid of N x N x N
readPSCz,vx,vy,vz,density,ngrid
; defining the different smoothing scales
r=[2,3,4,5,6,7,8,9,12,15,20,25]

;  simulation grid scale
N=256l
rc=154./float(N)
n2=N/2
denFT=FFT(density,-1,/double)
```

```
vxFT=FFT(vx,-1,/double)

; Define k-space grid
ii=indgen(N)
zk=fltarr(N)
zk(0:n2)=(ii(0:n2)/double(N))*2.*!pi/rc
zk(n2+1:N-1) = -((N-ii(n2+1:N-1))/double(N))*2.*!pi/rc
xk=zk \& yk=zk
dis=fltarr(N,N,N)
for i=0,N-1 do for j=0,N-1 do dis(i,j,*)=sqrt(xk(i)$^2$+yk(j)$^2$+zk(*)$^2$)

for q = 0,11  do beggin
    rs=r[q]

; Fast Fourier Transform of the Gaussian
    gfft=exp(-(dis*rs)^2/2.)
; Inverse of the Multiplied Fast Fourier Transform
    dens=FFT((gfft*denFT),1,/double)
    vxs=FFT((gfft*vxFT),1,/double)

; The velocity is read in row for row
    dummy=fltarr(N^3)
    dummy(*)=float(vxs)

; Calculation of the smooth velocity variance
    sigmv_s=stddev(dummy,/double)

; Calculation of the smooth verlocity variance at r+dr for the differation
    dr=0.01
    rs1=rs+dr
    gfft= exp(-(dis*rs1)^2/2.)
    vxs = FFT((vxFT*gfft),1,/double)
    dummy(*) = float(vxs)
    sigmv_s1 = stddev(dummy,/double)

; Variance of the density
    dummy(*)=float(dens)
    sigm_d=stddev(dummy,/double)

; Declaration of constants
; H is the Hubble constant
    H = 100
    beta=SQRT(1.5*((sigmv_s^2-sigmv_s1^2)/dr)*(1./(rs*sigm_d^2*H^2)))
    print, rs ,beta
endfor
end
```

A.2.2 Theoretically finding σ_d

```
;unnormalized Power spectrum routine
```

```
function p,k
rs=0.1
xn=1
h=0.71
omegab = 0.019/h^2
omegam = 0.27
; defining the LambdaCDM transferfunction
q = k/(omegam*h*exp(-omegab-sqrt(h/0.5)*(omegab/omegam)))
t = alog(1.+2.34*q)/(2.34*q)
t = t*(1.+3.89*q+(16.1*q)^2+(5.46*q)^3+(6.71*q)^4)^(-.25)
p = (k^(xn))*t^2*exp(-(k*rs)^2)
return,p
end

;=====================================================
;sigma 8 Normalization function
function pnorm,k
pi = 3.1415926540
k8 = k*8.
; definition of sigma_8^2 multiplied by the transferfunction of
; the LambdaCDM model
pnorm = k^2*p(k)*(3.*bessel1(k8)/k8)^2/(2.*!pi^2)
return,pnorm
end

;=====================================================
;a Spherical Bessel function of order 1
function bessel1,x
eps = 1.e-6
cosx=cos(x)
sinx=sin(x)
if x  le eps then begin
bessel1=0.333333333330*x*(1.0-0.10*x*x*(1.0-x*x/28.0))
endif else begin\\
bessel1=(sinx/x-cosx)/x
 endelse
return,bessel1
end

;======================================================
function normfactor,sgm8
; determining of the theoretical sigma_8
factor = qsimp('pnorm',1.e-4,1.e1,/DOUBLE)
cnorm=((sgm8)^2/factor)
return,cnorm
end

;==========================================================
function PGSmooth,k
Rs=12.
```

```
; applying Gaussian smoothing
return, k^2*p(k)*exp(-k^2*Rs^2)/(2.*!pi^2)
end

;===========================================================
function GausVar,cnorm
; determining of the theoretical estimate of sigma_d multiplied by
; the ratio of the Sigma_8's
sigm_d=cnorm*qsimp('PGSmooth',1.e-4,1.e1,/DOUBLE)
return,sigm_d
end

pro PSroutine,sgm8arr,chi2arr
f= 0.1327987
error=3.7127365E-03
n=200
sgm8min=0.01
sgm8max=1.5
sgm8arr=findgen(n)*(sgm8max-sgm8min)/float(n)+sgm8min
chi2arr=fltarr(n)
for i=0,n-1 do begin
sgm8=sgm8arr(i)
cnorm=normfactor(sgm8)
sigm\_d=GausVar(cnorm)
chi2arr(i)=((f^2-sqrt(sigm\_d)^2)^2)/error^2
endfor
end
```