# Investigating the origin of the Magorrian relation

On the evolution of super massive black holes and their host halos in the very early universe.

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#### Abstract

The relationship between galaxies and the supermassive black holes (SMBHs) found in their cores, plays a key role in the formation and evolution of both of these major constituents of the universe, as well as the evolution of the intergalactic medium. Studies of galaxy and SMBH co-formation and co-evolution are now among the central topics of research in cosmology. Yet the very origin, and the early growth phases of the SMBH are still not firmly established.

In this paper, we report on our investigations concerning the origin of the correlation between the mass of a galaxy and the mass of a SMBH, already found when the universe was only 1 Gyr old, known as the Magorrian relation (Magorrian et al. 1998). After examining the different physical processes and mechanisms that are involved with the growth of a galaxy and its SMBH, we run a series of simulations to emulate the evolution of the galactic system.

We find that it is possible to simulate the birth of a galaxy with an SMBH in its core within 1 Gyr, and even the first indications of a Magorrian relation. Furthermore, we examine the different physical parameters involved in the simulations to asses the relative importance of the different phases in the evolution of the developing galaxy and SMBH.

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## 1

## Introduction

Imagine a warm and beautiful summer night. Looking out to the stars, one sees a proper and quiescent night sky that would probably give anybody an impression of rest and order. One could even easily imagine that ancient cultures saw the light of the stars as the light of heaven, piercing through the outer globes of the celestial spheres. Giving a sneak peak to better times ahead. Could an image be more wrong?

The night sky may seem to be a calm and peaceful place from here, but in fact all we do see are the results of immense and powerful processes. These sparkling little stars of ours are in fact giant nuclear reactors, burning their fuels for billions of years. We see stars dying in enormous explosions. We see nurseries where stars come to live, blowing away all of their surroundings. We see turbulent streams of stars, violently seeking a way through the billions of stars that form our galaxy. And all of this revolves around one center; the supermassive black hole in the center of our galaxy.

By now the presence of supermassive black holes (hereafter SMBHs) at the centers of most galaxies appears to be firmly established. The masses of these SMBHs are in the range of  $10^6 - 10^9$  Solar Masses ( $M_{\odot}$ , Häring & Rix 2004) and various correlations have been observed between them and the properties of the galactic bulge hosting them.

The first of these correlations was between the mass of the SMBH  $M_{\bullet}$ and the mass of the galactic bulge  $M_{bulge}$ ; Magorrian et al. 1998 found that  $M_{\bullet} \sim 10^{-3} M_{bulge}$ . Laor (2001) pointed out the correlation between  $M_{\bullet}$ and the luminosity of the bulge  $L_{bulge}$ . More recently, a tighter relation was found between the mass of the SMBH and the bulge velocity dispersion  $\sigma_{bulge}$ , at some fiducial distance from the center, the  $M_{\bullet} - \sigma_{bulge}$  relation (Gebhardt et al. 2000; Merritt & Ferrarese 2001). An equally tight relation has also been detected between  $M_{\bullet}$  and the light profile of the galactic bulge, quantified by the Sersic index n (Graham et al. 2001). Subsequently, Marconi & Hunt (2003) showed that the correlation between the bulge luminosity and the SMBH mass becomes even tighter in the near infrared than



Figure 1.1: SMBH mass vs. bulge mass for 30 sampled galaxies. Two fits were made to the data showing. The solid line shows the fit with a slope of  $1.12 \pm 0.06$  found by (Häring & Rix 2004). The dashed line is a fit by Marconi & Hunt (2003) and shows a slope of  $1.06 \pm 0.09$ . Figure taken from Häring & Rix (2004).

in the optical.

Because these correlations extend well beyond the direct dynamical influence of the SMBH, the dominating idea is that there is a close link between the formation of SMBHs and the formation of their host galaxies. Most models put forward to account for the link between galaxy and SMBH formation, proceed along either or both of two routes to explain how SMBHs grow in mass. One way is through growth mainly by gas accretion within the host bulge. In this case a strong non-gravitational interaction between the growing SMBH and the bulge has to be invoked. The second route considers that the SMBH mass increases mainly by the merging of smaller precursors.

In order to investigate what are the driving mechanisms for achieving the found correlations, the evolution of both the galaxies and the black holes have to be examined.

Although most of the SMBHs we see in the galaxies in our local neighborhood - or even the one in our own Milky Way - are not associated with quasar activity today, presumably they represent the now-dormant counterparts to the quasar-powering engines known to exist at high redshifts (Fan et al. 2001). Under reasonable assumptions, the luminosity function of active galactic nuclei (AGN) can be explained by modeling mass accretion on to black holes that have the same mass range as observed in local spheroids. The standard expectation is that, once a primordial galaxy is populated with a so called 'seed black hole' at an early time, the black hole can grow by accreting the available mass. And it are these AGN that tell us that most of the SMBHs are already formed (or being formed) very early in our Universe. The highest redshift of a quasar discovered to date is  $z_{QSO} = 6.43$ , corresponding to QSO SDSS 1148+5251 (Fan et al. 2001). Accordingly, if SMBHs are the driving force in quasars, they first ones must

have been formed prior to  $z_{QSO}$ , or within  $t = 0.87 \ Gyr$  after the big bang in the concordance  $\Lambda$ CDM cosmological model. This requirement sets significant constraints on seed black hole formation and growth mechanisms in the early universe.

The barred spiral galaxy NGC 4258 (M 106), located at a distance of  $7.2 \pm 0.3$  Mpc (Herrnstein et al. 1999), presents a well-studied active nuclear region, classified as a Seyfert 1.9 (Ho et al. 1997). NGC 4258 is one of 22 nearby AGNs known to possess nuclear water masers (the microwave equivalent of lasers). The enormous surface brightness ( $\geq 10^{12}K$ ), relatively small sizes ( $\leq 10^{14}cm$ ) and narrow line-widths (a few  $kms^{-1}$ ) of these masers make them ideal probes of the structure and dynamics of the molecular gas in which they are embedded. Very-long-baseline interferometry (VLBI) observations of the NGC 4258 maser have provided the first direct image of the disc of material surrounding the SMBH inside an AGN (Greenhill et al. 1995); revealing a thin, differentially rotating warped disc in the nucleus, with a radiative efficiency of  $\epsilon \sim 0.2$  and which extends roughly between 0.14 and 0.28 pc in relation to the nucleus. The Keplerian rotation curves traced by the masers require a central binding mass  $M_{\bullet}$  of  $(3.9 \pm 0.1) \times 10^7 M_{\odot}$  (Herrnstein et al. 1999).

In this work, we analyze the different mechanisms for building a SMBH within a galactic bulge that satisfy the observations described above, and ask ourselves the question whether it is possible to make a SMBH with a mass of  $10^9 M_{\odot}$  as early as z = 6.43? And if so, are the found correlations - and with that in particular the Magorrian relation - a logical consequence of these precesses?

## The underlying physics

In 1798, the French mathematician and astronomer Pierre-Simon Laplace formulated the following theorem in his book 'Exposition du système du monde': "A luminous star, of the same density as the Earth, and whose diameter should be two hundred and fifty times larger than that of the Sun, would not, in consequence of its attraction, allow any of its rays to arrive at us; it is therefore possible that the largest luminous bodies in the universe may, through this cause, be invisible."

This is one of the first occurrences of the idea of a black hole. Laplace's theorem can be proven quite easily; in order to escape a gravitating object, like the Earth, a projectile must have a kinetic energy larger than the potential energy that is pulling it towards the surface. In other words, the velocity of the projectile has to be larger than the escape velocity. Since the escape velocity is proportional to the radius (when the density is kept the same), an object with the same density as the Earth has to have a radius 250 times the radius of the Sun, for the escape velocity to become as large as the speed of light. The corresponding mass of this "black star" would be  $\sim 10^8 M_{\odot}$ ; an immense object that, as we know now, could never exist. Still it is a beautiful idea, far ahead of its time.

By now we know, thanks to Einstein's theory of general relativity, that a black hole is a region of space-time in which gravity is so strong that nothing, not even light can escape it. Once a certain minimum of mass  $(3M_{\odot})$  is put into a small enough volume of space, it must eventually (after it has exhausted all of its fuel) collapse into a black hole. Where electrondegenerate or neutron-degenerate gas pressure is able to support a White Dwarf or a Neutron Star respectively, there is no physical mechanism that can stop the immense gravity when a black hole is formed. The volume of the contracting entity will decrease towards zero. Simultaneous, the density will grow towards infinity. In real physics neither of these phenomena can be exactly true. That is why the birth of this singularity (a singular point of zero volume and infinite density) marks the breakdown of the laws of physics, as we know them.

## 2.1. Basic black hole physics

#### 2.1.1 The Schwarzschild radius

For the description of a black hole, the same considerations are still used as Laplace did two centuries ago. For an object to escape, it has to have a speed larger than the escape speed of the gravitating object, so the sum of potential and kinetic energies has to be zero for an object that escapes to infinity and is in rest with respect to the object there:

$$(1/2)mv^2 - GmM/R = 0,$$

where v is the speed of the escaping projectile. Since the conservation of energy requires that this is the escape speed  $(V_{esc})$  at the moment of launch:

$$V_{esc} = \left(\frac{2GM}{R}\right)^{1/2}.$$

Because nothing can have a speed larger than the speed of light, the maximum escape speed is c. The expression for the escape speed then gives us an equation for the radius of a black hole:

$$R_{\bullet} = \frac{2GM_{\bullet}}{c^2},\tag{2.1}$$

where  $M_{\bullet}$  is the mass of the black hole. This radius is called the event horizon or the Schwarzschild radius and marks "the point of no return" for matter on its way into a black hole. In 1915 this expression was worked out by the German astrophysicist Karl Schwarzschild, shortly after Einstein published his general theory of relativity.

#### 2.1.2 Space-time around a black hole

To examine the strange structure of space-time around a black hole, let us do the following *Gedankenexperiment*. Imagine yourself in a spaceship, flying around a  $10M_{\odot}$  black hole on a perfect Keplerian orbit. To do your experiment you put on your space suit and jump, armed with a laser and a digital watch, towards the black hole. Sending laser pulses to the spaceship along the way.

As you are headed for the black hole, nothing strange happens. But as you get closer and closer, the strong gravity stretches you out from head to toe, and squeezes you together at the shoulders. The ultimate consequence of this will be that your feet will be pulled that much harder towards the black hole than your head, that you will be torn apart. But since this is a thought experiment, let us say that you will survive these gravitational tidal forces. At a certain point you will cross the event horizon, but nothing happens. There is no significant difference between the "inside" and the "outside" of a black hole. After this the journey will soon end, as you crash into the singularity and will (still) be destroyed.

For your colleagues in the spaceship this experiment will be less harmful, but it will take them a very long time to be witness to your experiences. And even then, they can only see part of it. As you fall towards the black hole, the laser pulses you send will be gravitationally red-shifted. The time between the laser pulses will also increase because of time-dilation effects described by general relativity. As you come closer to the event horizon the watches will get more and more out of synchronization. Your flight towards the black hole will seem to go slower and slower for the spaceship. Time slows down more and more. Even so much, that a laser pulse send towards the ship at the moment that you cross the Schwarzschild radius, will take an infinite amount of time to reach the spaceship; even though the laser pulse is moving with the speed of light.

#### 2.1.3 Rotating black holes

Karl Schwarzschild described with his equations a theoretical, stationary black hole. This model works well enough to calculate the distance to the event horizon, but lacks in describing the real state of a black hole.

When a large star collapses into a neutron star, angular momentum plays an important role. The original star rotates at a certain rate, perhaps once every twenty or thirty days. After the collapse, its angular momentum is transferred into the much smaller neutron star, which now spins around many times a second. In an equivalent way, a black hole must have angular momentum also. The first basic mathematical solution corresponding to rotating black holes was discovered by the New Zealand physicist Roy Kerr and is since then called the Kerr solution. Two years later, Ezra Newman found the axis-symmetric solution for Einstein's field equation for a black hole which is both rotating and electrically charged. This solution is known as the Kerr-Newman solution:

$$R_{\bullet} = \frac{G(M_{\bullet} + \sqrt{M_{\bullet}^2 - Q^2 - a^2})}{c^2},$$
(2.2)

where Q is the charge, and a is defined as the angular momentum per unit mass  $S M^{-1}$ .

The Kerr-Newman solution recognizes the same basic black hole properties found in the Schwarzschild solution. However, the rapid spin inherent to a Kerr black hole creates a considerably more complex and dynamic situation. First, the singularity is no longer a single point, but a warped area of space shaped like a ring. Second, the event horizon, marking the boundary between the black hole and the ordinary space beyond it, is moving in the same direction that the singularity is spinning. And as it moves, it drags part of the nearby region of space along with it - forcing the surrounding space to rotate in tornado-like manner.

While the frame-dragging only plays a significant role in regions close to the black hole's horizon, the spin of a black hole is also responsible an effect that is visible on far larger scales; due to the spin two narrow but powerful jets of gas are created that appear to be shooting out of some black holes. In reality these jets originate in the accretion disk. The mechanism responsible for the formation of these jets is still open to discrepancy. Some scientists believe the immense pressures produced by the rapidly rotating gases in the accretion disk create two vortexes, shooting two jets of hot gas outward at high speed in opposite directions. Another possible scenario involves a powerful magnetic field generated by the black hole's spin. Magnetic field lines anchored in the accretion disk and sticking out of it will be forced to spin along with the disks orbital motion. The electrical forces capture hot gas and plasma onto the spinning field lines. As these field lines spin, centrifugal forces should fling the plasma outward along them to form the two jets.

## 2.2. Seed black holes

All SMBHs start out small. With a small black hole that can grow to become the massive center of the large galaxies we see nowadays. Such a small starting black hole is called a seed black hole (SBH), with masses in the range of a few solar masses till even a couple of thousands. SBHs are formed with the death of a massive star, or because of the singular collapse of a large molecular cloud.

### 2.2.1 Forming seed black holes

#### Stellar collapse

According to recent semi-analytical simulations the very first stars in the Universe were likely created inside molecular clouds at redshifts well above  $z \sim 20$ . These clouds fragmented out of the first cores inside dark matter haloes. For common  $\Lambda$ CDM cosmologies these haloes are found to have masses of around  $M_{min} \sim 3 \times 10^5 h^{-1} M_{\odot}$  and have collapsed at redshift  $z \sim 24$ . In linear collapse theory this corresponds to a collapse from  $3\sigma$  density peaks in the initial matter density field. At these redshifts and in these regions the free-fall time is larger than the cooling time. Making it possible for the halo to cool on a timescale shorter than the gravitational infall time-scale. Since the clouds consist of mainly hydrogen and some deuterium at

these redshifts, the cooling is less efficient than at present times, so it proceeds much more slowly. This results in clouds with densities  $\rho \sim 10^5 cm^{-3}$ and temperatures  $T \sim 200K$ , where the Jeans mass  $(M_J \sim T^{3/2}/\rho^{1/2})$  is relatively high. The fragments that develop under these conditions at z > 20are thus much bigger than they would be at present, with similar conditions. Add the fact that the fragments can accrete large amounts of gas from the cloud without further fragmentation occurring, this eventually leads to the formation of a protostar.

The only physical phenomena that could halt the accretion process from the infalling layers is radiation pressure from the protostar. However, due to the lack of metals the infalling gas has too low an opacity for the radiation pressure to become significant. Also the effect of winds that could lead to large mass loss with population I stars, is negligible. As a result this will lead to very massive stars, some even with masses as large as  $10^3 M_{\odot}$ . These stars are known as population III stars.

There is still very little known about the initial mass function of these early stars. It is known that because of their large mass, most of them will end up as black holes with almost the same mass. The gravitational field of these massive stars is so strong that not even its own ejecta can escape. So we can assume that in each dark matter halo forming at  $z \sim 24$  with a mass larger than  $M_{min} \sim 3 \times 10^5 h^{-1} M_{\odot}$  a black hole will form with a mass of  $\sim 10^2 M_{\odot}$ , acting as the seed for the SMBHs that form the centers of galaxies seen in present day.

#### Singular cloud collapse

Another way of forming SBHs in the early universe is by way of singular cloud collapse. Spaans & Silk (2006) have examined the polytropic equation of state of an atomic hydrogen gas in primordial halos with baryonic masses of  $M_h \sim 10^7 - 10^9 M_{\odot}$ . For roughly isothermal collapse around  $10^4 K$  they find that the polytropic index stiffens to values well above unity. Under the assumptions of zero  $H_2$  abundance and very few metals ( $10^{-4}$  solar), fragmentation is likely to be inhibited for such an equation of state. With this they argue that - on purely thermodynamic grounds - a single black hole of  $\sim 10^4 - 10^7 M_{\odot}$  can form at the center of a halo for z = 10 - 20. Given that the free-fall time is less that the time needed for a resonantly scattered Ly $\alpha$  photon to escape the halo.

#### 2.2.2 Black hole mass density

Considering the initial matter density field, in which a SBH of  $M_{SBH} \approx 260 M_{\odot}$  is formed in every halo corresponding to a  $3\sigma$  density peak, Islam et al. (2003) calculate the global mass density contained in MBHs in the

following way:

$$\rho_{SBH} \leqslant \frac{0.0027\Omega_0 \rho_{crit} M_{SBH}}{10^5 M_{\odot} M p c^{-3}} \approx 2.9 \times 10^5 M_{\odot} M p c^{-3},$$

where the values  $\Omega_0 = 0.3$ ,  $\Omega_{\Lambda} = 0.7$  and h = 0.7 are used for a  $\Lambda \text{CDM}$  cosmology. Merritt & Ferrarese (2001) obtain an actual mass density for SMBHs of  $\rho_{SBH} \approx 5 \times 10^5 M_{\odot} M p c^{-3}$ .

A SMBH which has acquired its mass primarily by mergers of lower mass black holes, requires that the SBHs are  $\sim 500 M_{\odot}$  and all of their mass should end up in the resulting SMBH. This requirement can be met by the singular collapse theory of Spaans & Silk (2006), where they find  $M_{SBH} \sim 10^{-3} \times$ mass in the baryonic halo. However, for lower mass SBHs - formed in the collapse of a population III stars - growth through mergers cannot account for the observed mass density. Since these SBHs are expected to have masses  $< 500 M_{\odot}$ . This tells us that in this scenario, growth through accretion of gas must be very important in achieving the mass required for the present-day SMBH mass density.

## 2.3. Accretion

Observations of luminous quasars in the Sloan Digital Sky Survey (SDSS, Fan et al. 2001) give a very strong constraint on the high-redshift evolution of SMBHs. At redshifts as high as  $z \sim 6$  already SMBHs are found with luminosities corresponding to masses  $\approx 10^9 M_{\odot}$ . This means that when the Universe was approximately 1 billion years old, the small SBHs with masses  $\approx 10^2 M_{\odot}$  have already grown to masses on the same order as that of the giant dominating black hole in the center of our Milky Way.

#### 2.3.1 Eddington accretion

When our SBH is formed, it will start accreting its surroundings right away. Its strong gravitational field will pull everything on its path towards it. One of the most fundamental problems with the formation of SMBHs can already be seen here; the centrifugal barrier. The infalling object has to lose its angular momentum. Because individual stars have very few ways of angular momentum transfer, it is very hard for a black hole to accrete entire stars. If a star gets caught by a black hole, it will most likely be placed on an orbit around the black hole. From there it could be thrown into the black hole due to collision effects with encountering stars. But these effects are very rare, and are thus of minor importance for the process of accreting mass. Let alone be the sole mechanism for the growth of a SMBH.

So even though it is very difficult to accrete very massive objects like stars, it is very well possible to accrete gas. Gas will also be pulled onto an orbit around the black hole, and can dissipate its angular momentum more easily by turbulent viscosity inside the formed differentially rotating accretion disc. The effective kinematic viscosity of a turbulent process is usually written in the form  $\nu \sim d \cdot v$ , where d is the size and v is the turnover velocity of the largest eddies. For turbulence in an accretion disc we may surmise that the scale of the eddies is less that the scale height (or disc thickness) H, and that the turbulence is subsonic. Thus, the viscosity  $\nu$  in an accretion disc may be written as

$$\nu = \alpha c_s H = \alpha c_s^2 / \Omega,$$

where  $c_s$  is the speed of sound,  $\Omega$  is the angular velocity and  $\alpha$  is the alpha viscosity parameter (Shakura & Syunyaev 1973, Pringle 1981). When r is the radius of the disc, an accretion disc is a thin disc  $(H \ll r)$  where  $\alpha < 1$ . This gives the accretion disc a low hydrodynamic turbulent viscosity; a mechanism that can transfer angular momentum outwards, and thus makes it possible for the gas to contract into the black hole.

Even though the idea that turbulence accounts for the viscosity dates back almost to the conception of the accretion disc model. There is no adequate demonstration that discs are hydrodynamical turbulent. Simulations show that hydrodynamic turbulence does not develop spontaneous within the disc, and if the turbulence is provided by some means (e.g. convection), there is no net outward transport of angular momentum (Hawley & Balbus 1998).

Balbus & Hawley (1991) therefor proposed a magnetohydrodynamic (MHD) turbulent model that does give a decent description of a viscous accretion disc. The reason that MHD turbulence does succeed, is because the magnetic field fundamentally alters the the linear stabilizing properties of a differentially rotating flow. Weakly magnetized discs are dynamically unstable if the angular velocity decreases outward. This magnetorotational instability (MRI) model makes the presence of disc turbulence no more fundamentally mysterious than that of the convective turbulence which develops in the outer layers of low mass stars. In both cases a local linear stability criterion is violated.

In the meanwhile, the gas that is spiraling towards the black hole will gain kinetic energy and heats up, becomes ionized and emits electromagnetic radiation. This mechanism gives us an upper limit for the amount of matter that the black hole can accrete over a period of time: the Eddington accretion rate (Haiman 2004; Shapiro 2005). The Eddington luminosity is defined as

$$L_{edd} = \frac{4\pi GMcm_H}{\sigma_T},\tag{2.3}$$

in which G is Newton's gravitational constant, M is the mass of the attracting object, c is the speed of light,  $m_H$  is the mass of an hydrogen atom



Figure 2.1: Simulations of the growth of a black hole accreting at its Eddington limit  $(\eta = 1)$ , for different radiative efficiencies. On the vertical axis the dimensionless mass of the black hole  $M/M_{SBH}$  is plotted, where  $M_{SBH}$  is the mass of SBH. The simulations show that the radiative efficiency needs to be  $\epsilon \ll 1$ , for a SBH of ~  $10^2 M_{\odot}$  to grow to the observed SMBH masses of  $\approx 10^7 - 10^9 M_{\odot}$  (Häring & Rix 2004).

and  $\sigma_T$  is the Thompson cross section. Equation 2.3 describes the balance between the amount of radiation that the accretion disc produces, and the gravitational attraction of the black hole. So when the luminosity of an accretion disc is  $L_{edd}$ , it accretes on its maximum accretion rate. When the luminosity exceeds  $L_{edd}$  the infalling gas will be blown away by radiation pressure.

Since the mass of the accreting object is the only variable quantity in the equation above, it is very easy and convenient to rewrite it in terms of Solar masses:

$$L_{edd} = 1.25 \times 10^{38} \ erg \ s^{-1} (M/M_{\odot}).$$
 (2.4)

Now that we know the luminosity of the accretion disc, it is possible to derive the accretion rate with the help of Einstein's most famous equation  $E = Mc^2$  and the knowledge that the luminosity is a time derivative of energy:

$$\frac{dM}{dt} = \dot{M} = \frac{L_{edd}}{\epsilon c^2},\tag{2.5}$$

where  $\epsilon$  is the radiative efficiency. The radiative efficiency gives the fraction of  $Mc^2$  that is radiated away. In practise  $\epsilon$  will be on the order of 0.1, because most of the energy is lost as heat and entropy in dissipative processes inside the disc.

When we define  $\eta$  as the Eddington luminosity efficiency:

$$\eta = L/L_{edd},$$

we get an expression for the efficiency at which the black hole is accreting its surrounding matter. In most conventional models  $\eta < 1$ : the black hole is accreting with a sub-Eddington accretion rate. For  $\eta = 1$  the black hole accretes on its Eddington limit. Subsequently, there are models where  $\eta > 1$ where the accreted material is forced to withstand the radiation pressure. In this scenario the black hole is accreting with a super-Eddington accretion rate.

Combining equations 2.4 and 2.5 then gives us our expression for the Eddington accretion:

$$\dot{M}_{Edd} = \frac{\eta \alpha_{edd} M}{\epsilon M_{\odot} c^2},\tag{2.6}$$

where the factor  $\alpha_{edd}$  is defined as  $\alpha_{edd} \equiv 1.25 \times 10^{38} \ erg \ s^{-1}$ .

We can now also define the mass independent characteristic Eddington accretion timescale (or Salpeter timescale);

$$\tau_{edd} \equiv \frac{Mc^2}{L_{edd}} \approx 0.45 \ \epsilon \eta^{-1} \ Gyr. \tag{2.7}$$

Figure 2.1 shows simulations of the evolution of SBHs accreting at their Eddington limit, for different radiative efficiencies. It is possible to reach  $M_{\bullet} \sim 10^6 - 10^9$ , and thus equal to the observed quantities (Häring & Rix 2004), given that  $\epsilon \ll 1$  and the accretion has to be at the Eddington limit. Recent research done on NGC 4258 suggests a radiative efficiency of  $\epsilon \sim 0.2$ (Herrnstein et al. 2005), which strengthens our conclusion. However, the condition that accretion has to be at the Eddington limit for such a long period of time, with only the mass of the SBH to account for some range, describes a very strict picture. So even though Eddington accretion can give a SMBH a substantial part of its mass, it is to be questioned whether it is the single mechanism responsible for the growth of black holes.

#### 2.3.2 Bondi accretion

Another way of looking at the formation of a SMBH is from a more thermodynamical point of view. Consider a SBH in a very high density region. Since we assume that most SBHs are formed at redshifts z > 20 in rare  $3\sigma$ density peaks in the initial matter density field, this is not such a strange assumption. From there the gas could fall in via a radial manner.

This spherical inflow leads to a rapid growth of the SBH with highly super-Eddington accretion rates, and can therefor be used to explain the SMBHs at high redshifts. This form of accretion is called Bondi accretion (Bondi 1952; Volonteri & Rees 2005), and leads to accretion rates  $\dot{M}_{Bondi} \gg \dot{M}_{Edd}$ .

To evaluate this Bondi accretion we consider a steady state in which the velocity v is radial, and v, p and  $\rho$  are functions of r only. We assume the following polytropic equation of state:

$$\frac{p}{p_{\infty}} = \left(\frac{\rho}{\rho_{\infty}}\right)^{\gamma},\tag{2.8}$$

where  $p_{\infty}$  and  $\rho_{\infty}$  are evaluated at infinite r. This equation is valid for an adiabatic flow and gives a useful approximation for a radiating gas, provided that the polytropic index  $\gamma$  has a value of  $1 \leq \gamma \leq 5/3$ .

Mass conservation gives us an accretion rate:

$$\frac{dM}{dt} = \dot{M} = 4\pi r^2 \rho v. \tag{2.9}$$

Integrating equation 2.8 with Bernoulli's equation;

$$\frac{v^2}{2} + \int_{\rho_{\infty}}^{\rho} \frac{dP}{\rho} - \frac{GM}{r} = 0.$$

yields

$$\frac{v^2}{2} + \frac{c^2}{\gamma - 1} \left[ \left( \frac{\rho}{\rho_{\infty}} \right)^{\gamma - 1} - 1 \right] = \frac{GM}{r}, \qquad (2.10)$$

where  $c_s$  is the sound speed:

$$c_s^2 = \frac{\gamma p_\infty}{\rho_\infty}.\tag{2.11}$$

In order to obtain a single equation relating v and r (a relation now given by equations 2.9 and 2.10), we introduce the non-dimensional variables x, yand z, to replace r, v and p respectively:

$$r = xGM/c^{2},$$

$$v = yc_{s},$$

$$\rho = z\rho_{\infty}.$$
(2.12)

Equations 2.9 and 2.10 then take the form

$$x^2 yz = \lambda, \tag{2.13}$$

and

$$\frac{y^2}{2} + \frac{z^{(\gamma-1)} - 1}{\gamma - 1} = \frac{1}{x}.$$
(2.14)

Where  $\lambda$  is given by

$$\dot{M} = 4\pi\lambda (GM)^2 c^{-3}\rho_{\infty}.$$
(2.15)

After substituting the Mach number  $\mathfrak{M} = yz^{-(\gamma-1)/2}$  into equation 2.13, and multiplying this by  $(x^2/\lambda)^{2(\gamma-1)/(\gamma+1)}$ , it takes the form of

$$f(\mathfrak{M}) = \frac{1}{\lambda^{\alpha}} g(x). \tag{2.16}$$

Where

$$f(\mathfrak{M}) \equiv \frac{\mathfrak{M}^{2\alpha/(\gamma-1)}}{2} + \frac{1}{(\gamma-1)\mathfrak{M}^{\alpha}},$$
$$g(x) \equiv \frac{x^{2\alpha}}{\gamma-1} + \frac{1}{x^{1-2\alpha}}$$
(2.17)

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and

$$\alpha \equiv \frac{2(\gamma - 1)}{\gamma + 1}.$$
(2.18)

We now have an equation that relates the Mach number of infalling material to the distance of the surface of the accreting object. Because g(x) is the sum of one positive and one negative power of x, it must have a minimum. The corresponding minimum value of x is  $x_m = (5 - 3\gamma)/4$ . Since we are looking at the sound speed defined in equation 2.11,  $x_m$  can be substituted into equation 2.9. This results in the final expression for the Bondi accretion rate:

$$\dot{M}_{Bondi} = \frac{4\pi n_H m_H (x_m GM)^2}{c_s^3},$$
(2.19)

where  $\rho$  is written as  $n_H m_H$ .

The found expression for the Bondi accretion describes a modest accretion rate for a SBH with a mass  $M_{SBH}$  of  $\sim 10^{-8}M_{SBH}/year$  for  $t \ll \Gamma$ , where  $\Gamma \equiv (4\pi n_H m_H (x_m G)^2 M_{\odot}/c_s^3)^{-1}$ . However, when t approaches  $\Gamma$ , the accretion rate will display a strong asymptotic behavior, enabling the black hole to grow extremely rapid. With this behavior, the black hole could empty its mass reservoir in only a fraction of time. However, even though this is still a topic of much debate, the overall idea is that in reality this extreme accretion will only last for a very short period of time (Volonteri & Rees 2005). During this time the black hole is capable of accreting presumably several time its own mass, before feedback processes stop the accretion, and force the black hole to accrete with an Eddington accretion rate. The feedback processes stop the Bondi accretion when the radius of the black hole reaches the feedback radius, which is defined as:

$$r_f = \frac{r_{\bullet}}{r_{SBH}} = \frac{M_{\bullet}}{M_{SBH}},\tag{2.20}$$

where  $r_{\bullet}$  is the radius of the accreting SMBH and  $r_{SBH}$  is the radius of the original SBH.

There is still very little known about the value of  $r_f$  at which the Bondi accretion stops. According to Volonteri & Rees (2005) the feedback kicks in around  $r_f \sim 5-50$ . Furthermore, the sort of feedback is still under debate. Most likely this will be some sort of outflow which blows away most of the local radial gas cloud.

### 2.4. Mergers

The formation of a SMBH via accretion is only one of two complementary routes that can be followed in the study of SMBH growth; the other is that of a series of mergers with other black holes. In the currently favored cold dark matter cosmogonies (Spergel et al. 2007), present-day galaxies have been assembled via a series of mergers, from small-mass building blocks which form at early cosmic times. In this paradigm galaxies experience multiple mergers during their lifetime. If most galaxies host black holes in their center, and a local galaxy is made up by multiple mergers, then a black hole binary is a natural evolutionary stage. After each merger event, the central black hole already present in each galaxy would be dragged to the center of the newly formed galaxy via dynamical friction. If the two black holes get close ( $\sim 0.01 - 0.001$  pc) the black hole binary would coalesce via emission of gravitational radiation (Rees & Volonteri 2007).

The efficiency of dynamical friction decays as black holes get close and form a binary; the binary separation for  $M_{\bullet} \approx 10^5 - 10^8 M_{\odot}$  is then  $\sim 0.1 - 1$  pc. Emission of gravitational waves becomes efficient at binary separations about two orders of magnitude smaller. In gas-poor systems, the subsequent evolution of the binary, while gravitational radiation emission is negligible, may be largely determined by three-body interactions with background stars (Begelman et al. 1980). The binary captures stars that pass within a distance of order of the binary semi-major axis, and ejects them at much higher velocities (Quinlan 1996, Milosavljević & Merritt 2001, Sesana et al. 2006). Dark matter particles will be ejected by decaying binaries in the same way as the stars, i.e. by the gravitational slingshot. The hardening of the binary modifies the density profile, removing mass interior to the binary orbit, depleting the galaxy core of stars and dark matter, and slowing down further decay.

In gas-rich systems, however, the orbital evolution of the central SMBH is likely dominated by dynamical friction against the surrounding gaseous medium. The available simulations (Escala et al. 2004, Mayer et al. 2006, Dotti et al. 2006) show that the binary can shrink to about parsec or slightly subparsec scale by dynamical friction against the gas, depending on the gas thermodynamics. The interactions between a black hole binary and an accretion disc can also lead to very efficient transport of angular momentum, and drive the secondary black hole to the regime where emission of gravitational radiation forces the black holes to merge (Gould & Rix 2000, Armitage & Natarajan 2005).

To calculate the total cumulative effect of mergers on the eventual mass of the SMBH, this hierarchical build-up process is mainly studied by means of N-body simulations. The evolution of dark matter haloes can be modeled using merger trees based on the extended Press-Schechter formalism (Press & Schechter 1974, Lacey & Cole 1993). The fraction of mass f(M, t) in haloes with mass M, at time t, per interval dM is given by

$$\frac{df}{dM}(M,t) = \frac{\delta_c(t)}{(2\pi)^{1/2}\sigma^3(M)} \left| \frac{d\sigma^2(M)}{dM} \right| exp\left[ -\frac{\delta_c^2(t)}{2\sigma^2(M)} \right],$$

where  $\delta_c$  is the critical threshold value for the amplitude of the density fluctuations, and  $\sigma^2(M)$  the variance of the linear density field, when smoothed with a window function containing mass M. From this the the comoving number density of haloes of mass M, present at time t, per dM, is

$$\frac{dn}{dM}(M,t) = \left(\frac{2}{\pi}\right)^{1/2} \frac{\bar{\rho}}{M^2} \frac{\delta_c(t)}{\sigma(M)} \left| \frac{d \ln \sigma}{d \ln M} \right| exp\left[ -\frac{\delta_c^2(t)}{2\sigma^2(M)} \right], \quad (2.21)$$

where  $\bar{\rho}$  is the present mean mass density of the universe. With equation 2.21 it is possible to follow the merger tree through which a SMBH is formed.

Following the merger tree, it is possible to calculate a minimum multiplication factor  $\phi$ , that accounts for the total mass that is acquired through mergers. Consider a SBH that grows through both Eddington accretion and mergers. When we integrate equation 2.6 this gives

$$M_f = M_i \, exp\left(\frac{\eta \alpha_{edd}(t-t_i)}{\epsilon M_{\odot}c^2}\right),\tag{2.22}$$

where  $M_f$  is the final mass, equivalent to  $M_{\bullet}$ , and  $M_i$  is the initial mass, equivalent to  $M_{SBH}$ .

The right-hand side of equation 2.22 is independent of black hole mass. This fact makes it possible to disentangle and track separately the amplification of black hole mass by accretion from the amplification by discrete mergers. Let  $M_n(t)$  be the mass of the black hole at time t, following its *nth* merger with another black hole at time  $t_n$ , where  $t_n \leq t \leq t_{n+1}$ . Assume that the duration of a merger is much shorter than the time interval between mergers, and that the hole continues to accrete steadily throughout this interval.

Let  $\phi_n$  be the mass amplification of the hole following its *nth* merger with another black hole:  $\phi_n = M_n(t_n)/M_{n-1}(t_n) > 1$ . Then we may use equation 2.22 to calculate the total mass amplification from  $t_i$  to  $t_f$  according to

$$\frac{M_f}{M_i} = \frac{M_{\bullet}}{M_{SBH}} = \frac{M_N(t_f)}{M_0(t_i)} 
= \frac{M_0(t_1)}{M_0(t_i)} \frac{M_1(t_1)}{M_0(t_1)} \frac{M_1(t_2)}{M_1(t_1)} \cdots 
\cdots \frac{M_{N-1}(t_N)}{M_{N-1}(t_{N-1})} \frac{M_N(t_N)}{M_{N-1}(t_N)} \frac{M_N(t_f)}{M_N(t_N)} 
= exp[C(t_1 - t_i)] \times \phi_1 exp[C(t_2 - t_1)] \times \cdots 
\cdots \phi_{N-1} exp[C(t_N - t_{N-1})] \times \phi_N exp[C(t_f - t_N)] 
= \phi_1 \phi_2 \cdots \phi_N exp[c(t_f - t_i)],$$
(2.23)

where  $C \equiv \eta \alpha_{edd} / \epsilon M_{\odot} c^2$  and is essentially constant. Comparison of equations 2.22 and 2.23 reveals that the net mass amplification due to accretion



Figure 2.2: Models of the mass history of the SMBH in QSO SDSS 1148+5251, according to four different estimates gravitational of wave recoil effects, taken from Yoo & Miralda-Escudé (2004).The upper curve (thick solid line) shows the mass of the main halo progenitor.  $M_{SBH} = 10 M_{\odot}$ , and the recoil velocities are of order 1000, 1000, 400 and 50  $km \ s^{-1}$  for Fitchett, Kidder and Favata upper and lower models, respectively.

can be treated as a single multiplicative factor that is independent of the net amplification factor due to mergers:

$$\phi = \phi_1 \phi_2 \dots \phi_N. \tag{2.24}$$

Thus for the simple scenario envisioned here, a black hole can grow through Eddington accretion, even when this steady growth by accretion is interrupted by discrete, stochastic black hole mergers.

Note that it is possible that black hole mergers completely eject black holes from halo centers owing to gravitational wave recoil and thereby turn off accretion altogether (Hut & Rees 1992, Merritt et al. 2004, and Madau & Quataert 2004). Whenever the recoil velocity is larger than the escape velocity of the halo, the black hole is ejected form that halo. However, Yoo & Miralda-Escudé (2004) conclude from the most recent recoil calculations into simple models of dark halo mergers, that the kick velocities are not sufficiently large to impede black hole growth significantly. Haiman (2004) states that the kick velocities are large enough for relatively small black holes to be ejected from the halo, but that this could be left out of account when the black hole are formed relatively recent, with super-Eddington accretion times.

Figure 2.2 shows the mass history of the SMBH in the z = 6.43 quasar SDSS 1148+5251, taken from the models of Yoo & Miralda-Escudé (2004). The upper thick solid line shows the mass of the main halo progenitor in one realization of the merger tree, obtained by always choosing the branch of the most massive progenitor at every merger. The lower four lines show the mass of the black hole in this main halo progenitor, according to four different prescription for the gravitational wave recoil (or kick) velocity. The

first prescription is a quasi-Newtonian calculation of nonspinning black holes by Fitchett (1983). Secondly, Kidder (1995) added a post-Newtonian spinorbit correction to Fitchett's work that depends on the black hole spins. And finally, Favata et al. (2004) and Merritt et al. (2004) obtained a new estimate using black hole perturbation theory, and they provided an upper and a lower limit on the gravitational wave kick velocity. These limits are a compromise to the uncertainty due to the final plunging state that gives the dominant contribution. These two limits are shown as two separate estimates.

The mass of the SBHs is fixed at  $M_{SBH} = 10M_{\odot}$ , and they start growing as soon as they are formed, over an Eddington time of  $\tau_{edd} = 4 \times 10^7 yr$ (with  $\eta = 1$ , and  $\epsilon \sim 0.1$ ). At every merger, the mass of the two black holes is added when the recoil velocity is smaller than the escape velocity. This results in the sudden mass increase seen in the figure. Growth by gas accretion is presumed to continue immediately after mergers. whenever the recoil velocity exceeds the escape velocity, the black hole is removed and replaced by a new seed with  $M_{SBH} = 10M_{\odot}$ . Figure 2.2 shows that many black holes are ejected at high redshifts when they reside in low-mass haloes. However, when the mass of the haloes increases, fewer black holes are  $10^{-3}$  the mass of the progenitor halo.

Figure 2.3 shows a histogram of the final number of black holes that have merged into the final SMBH, as a function of their formation redshift (*upper panel*), and their contribution to the total mass of the final SMBH (*lower panel*). In other words, the number shown in the upper panel is



Figure 2.3: Composition of the final SMBH. Upper panel: Number of SBHs that merged into the final SMBH, as a function of their formation redshift. Lower panel: contribution to the final mass of the SMBH from the SBHs as a function of their formation redshift (Yoo & Miralda-Escudé 2004).

the amplification factor  $\phi$  form equation 2.24, and the lower panel shows the contribution of the total mass of all the SBHs to the final mass of the SMBH.

Figures 2.2 and 2.3 show that Yoo & Miralda-Escudé (2004) find an amplification factor of  $\phi \sim 10^4$  in the evolution of their SMBHs. Combining a continued mass growth of black holes from Eddington-limited ( $\eta = 1$ ) gas accretion at a radiative efficiency of  $\epsilon = 0.1$  with black hole mergers, they can easily account for the presence of SMBHs with  $M_{\bullet} \sim 10^9 M_{\odot}$ , at z = 6.43 (the redshift of quasar SDSS 1148+5251), starting from stellar mass SBHs of  $10M_{\odot}$ . With recoil velocities according to the Favata upper model, the simulations can even account for SMBHs with  $M_{\bullet} \sim 10^9$  at  $z \sim 10$  already. Typically, these black holes are formed by  $10^4$  black hole mergers, and subsequently grow by a factor of  $10^4$  in mass by gas accretion.

When a number of variations to the basic model are examined, Yoo & Miralda-Escudé (2004) show that increasing  $M_{SBH}$  from 10 to  $100M_{\odot}$  causes a very small increase of the redshift ( $\sim +1 z$ ) at which a mass of  $\sim 10^9 M_{\odot}$  can be achieved. Introducing the requirement that haloes will only merge with haloes of which their mutual mass ratio lies between 0.1 and 1, makes it difficult to reach a final SMBH of  $\sim 10^9 M_{\odot}$ . The Fitchett and Kiddler models reach a mass of only  $10^8 M_{\odot}$ . For the upper limit of the Favata model it is also very hard to reach the required  $4 \times 10^8 M_{\odot}$ , but for the lower limit to the recoil velocities the required mass can still be reached at z = 9.

Nevertheless, it is important to keep in mind that the amplification factor  $\phi$  can only be regarded as a crude estimate of the total influence of black hole mergers, since it can only be applied with constant Eddington accretion models. It is reasonably safe, however, to say that black hole mergers can account for an amplification factor of  $\phi \sim 10^3$ .

### 2.5. Star Formation

Now that we have a plausible representation for the growth of the massive black holes, it is time to look at the growth of the galactic bulges in which these SMBHs are located.

To model the star formation history (SFH) of a galaxy, we use the empirical Kennicutt-Schmidt law (Kennicutt 1998; Schmidt 1959) to describe the rate at which gas is converted into stars. In a simple self-gravitational halo, the large-scale star formation rate (SFR) is presumed to scale with the growth rate of perturbations in the gas reservoir. The Kennicutt-Schmidt law states that the surface density of star formation scales with the surface density of gas. From this it can easily be derived, that also the volume densities of star formation and gas scale with each other:

$$\rho_* \propto \rho_{gas}^{1.5},\tag{2.25}$$



Figure 2.4: Normalized density profiles for pseudo-isothermal spheres with different core radii  $r_0$ . The density profiles start out fairly flat, and proceeds to a slope of -2 when  $r >> r_0$ .

where  $\rho_*$  and  $\rho_{gas}$  are the volume densities of star formation and gas, respectively. Even though its origins are not yet fully understood and it is just beginning to be tested at high redshift, the Kennicutt-Schmidt law is widely used to interpret and describe star formation in galaxies (Erb 2007).

#### 2.5.1 The pseudo-isothermal sphere

As a model of the halo, we adopt a body of gas with a constant temperature and an equation of state corresponding to equation 2.8, known as an isothermal sphere; which has a density profile that decreases as  $\rho(r) \propto r^{-2}$ .

The general solution for the isothermal sphere is given by

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}$$

and this solution describes a model known as the *singular isothermal sphere* (Binney & Tremaine 1987).

Unfortunately, the singular isothermal sphere has infinite density at r = 0. To obtain a density profile that is well behaved at the origin, it is convenient to make use of the *pseudo-isothermal sphere*, which is given by:

$$\rho_{gas}(r) = \frac{\rho_0}{1 + (r/r_0)^2},\tag{2.26}$$

where  $\rho_0$  is the central density of the sphere,  $r_0$  is known as the core radius and r is the distance to the center of the halo. At  $r_0$  the density of the isothermal sphere falls to half of its central value, and it is called the core radius in analogy to the observational definition.

Figure 2.4 shows normalized density profiles for pseudo-isothermal spheres with different core radii  $r_0$ . The density profiles start out fairly flat, and proceed to a slope of -2, similar to the singular isothermal sphere, when  $r >> r_0$ .

In order to find the central density  $\rho_0$ , we integrate equation 2.26 over volume at  $t_0$ :

$$M_{gas} = \int_0^R 4\pi \rho_{gas} r^2 dr$$
$$= 4\pi \rho_0 \Big[ 1 - \tilde{r} \ tan^{-1}(\tilde{r}) \Big] Rr_0^2$$

where  $M_{gas}$  and R are the total mass and the total radius of the halo, respectively, and  $\tilde{r}$  is the dimensionless variable defined as

$$\tilde{r} \equiv \frac{R}{r_0}.$$
(2.27)

The central density can then be written as

$$\rho_0 = \frac{M_{gas}}{4\pi \left[1 - \tilde{r} \ tan^{-1}(\tilde{r})\right] R r_0^2}.$$
(2.28)

From equation 2.25 we know that  $\rho_* = K \rho_{gas}^{1.5}$ , where K is the Kennicutt-Schmidt constant. When we integrate this using equation 2.26 and the volume integral we get an expression for the SFR:

$$SFR = \int_0^R 4\pi \rho_* r^2 dr$$
  
=  $4\pi K \rho_0^{1.5} \Big[ log(\tilde{r} + \sqrt{1 + \tilde{r}}) - 1/2\sqrt{2} \Big] r_0^3.$  (2.29)

The factor K can be used to scale the SFR to empirical values, and is of order  $K \sim 10^2 - 10^3 \ cm^{3/2} \ kg^{-1/2} \ yr^{-1}$ , for starburst galaxies that form  $\sim 10 - 100 M_{\odot}$  per year.

#### 2.5.2 Evolved stars

During the growth of the SMBH, and the accompanying star formation phase, we have to consider the evolution of the massive stars in the forming galaxy. The very massive stars can have life times of only  $\sim 10^6 yr$ . When they have burned all of their fuel, and come to the end of their life cycle, they return a substantial part of their mass to the mass of the halo through supernova's and stellar winds.

To account for these evolved stars, we return a fraction  $\Upsilon$  of the mass in newly formed stars to the total mass of gas in the halo with each time step. This can be justified by the fact that the mass fraction of very massive stars,  $M \ge 10 M_{\odot}$ , in a typical IMF is very small;  $< 10^{-2}$  (Figure 2.5; Scalo 1986; Kroupa et al. 1993). So even though these stars are very massive and have a very short life time, the total amount of mass affected by this process is very small, since there are very few stars. At the same time, the major





fraction of stellar mass will be caught in stars that have a life time > 1 Gyr, and will thus have a life time larger than the time we focus on.

As a consequence, the fraction of massive stars will become smaller over time, as the IMF evolves towards the present-day mass function (PDMF; Figure 2.5). With that, the amount of mass that returns to the mass of gas in the halo will become smaller also; making the stellar feedback even less significant.

With the incorporation of this stellar feedback, we have a complete model that describes the evolution of the gas in our galactic halo, the star formation and the basic stellar evolution that takes place simultaneously with the growth of the SMBH.

## Results

With the mechanisms discussed in the previous chapter, we have the main ingredients to model the growth of a SMBH in a galactic bulge, and with that, asses the relative importance of these mechanisms. The evolution of a black hole is traced, along with its host halo, in a numerical simulation: a SBH is placed in a halo of gas, where it accretes mass to grow and evolve into a SMBH. At the same time, the gas in the reservoir is being turned into stars at a SFR as calculated in section 2.5.

## 3.1. The simulations

The SBH starts accreting in a radial manner with a Bondi rate, which is determined by the central density of the isothermal sphere that models the halo. During this accretion process, the SBH grows in mass and radius, forcing the accretion rate to increase very rapidly (see section 2.3.2). This makes it less likely that the super-Eddington accretion rate can be sustained.

As argumented in the previous chapter, feedback processes will thus stop the Bondi accretion when the radius of the black hole becomes a factor of 5-50 larger (Volonteri & Rees 2005). Even though this sort of feedback is still under debate, this will likely be some sort of outflow which blows away most of the local gas cloud. After the Bondi accretion phase, the black hole will go on accreting with an (sub-)Eddington accretion phase, until all the available gas of the halo is turned into stars or accreted by the SMBH.

The SFR at which gas is turned into stars, simultaneously with the accreting black hole, depends on the density of the gas - and thus the available mass  $(M_{\circ})$  - in the host halo. This SFR is calculated with every step, and both the mass of the formed stars and the mass of the black hole is then subtracted from the total mass of the gas in the reservoir. To account for evolved stars, we return a fraction  $\Upsilon$  of the total mass in stars to the gas in the halo, with each time step.



Figure 3.1: Time vs. mass. Simulation of the growth of the SMBH (solid line), the mass of the stellar population (dashed line) and the mass of the gas in the halo (dotted line) for out fiducial model:  $M_{\circ} = 10^{9} M_{\odot}$ ,  $R = 10^{2} kpc$ ,  $r_{0} = 10 \ kpc$ ,  $M_{SBH} =$  $2 \times 10^{2} M_{\odot}$ ,  $c_{s} = 10^{6} \ m/s$ ,  $\gamma = 1$ ,  $r_{f} = 10$ ,  $\epsilon = 0.15$ ,  $\eta = 1$ ,  $\Upsilon = 0.01$  and K = $10^{2} \ cm^{3/2} \ kg^{-1/2} \ yr^{-1}$ .

Figure 3.2: Redshift vs. mass. Simulation of the growth of the SMBH (solid line), the mass of the stellar population (dashed line) and the mass of the gas in the halo (dotted line) for our fiducial model, starting at redshift z = 24. The same parameters where used as with Figure 3.1

Within the simulations, we do not take into account the effects of mergers within our time frame. In section 2.4 we have shown that the total amount of mass acquired through mergers of our halo with other small (proto-)galaxies, or through mergers of our SMBH with individual black holes, can be represented by the multiplication factor  $\phi$ , which is typically of order  $10^3 - 10^4$ . Therefor, the end masses of the SMBH and bulge including the effect of mergers, can be simply found by multiplying the end masses of the simulations with a factor  $\phi$ .

#### 3.1.1 The fiducial model

Figures 3.1 and 3.2 show the results of the simulations. The solid line represents the mass of the black hole, the dashed line represents the mass of the stellar population, and the dotted line represents the mass of the gas in the halo. In Figure 3.1 time is plotted against the mass. Figure 3.2 shows redshift versus mass.

The Figures are made with the same set of parameters. This set forms our fiducial model and consists of the most likely parameters: mass of the halo  $M_{\circ} = 10^9 M_{\odot}$ , radius of the halo  $R = 10^2 \ kpc$ , core radius  $r_0 = 10 \ kpc$ ,  $M_{SBH} = 2 \times 10^2 M_{\odot}$ , sound speed  $c_s = 10^6 m/s$ , polytropic index  $\gamma = 1$ , feedback radius  $r_f = 10$ , radiative efficiency  $\epsilon = 0.15$ , Eddington luminosity efficiency  $\eta = 1$ , stellar feedback fraction  $\Upsilon = 0.01$  and the Kennicutt-Schmidt constant  $K = 10^2 \ cm^{3/2} \ kg^{-1/2} \ yr^{-1}$ .

The different stages of accretion by the developing SMBH can be seen in both Figures. The first epoch of Bondi accretion ends after approximately  $10^8 yr$  (Figure 3.1) or at  $z \approx 23$  (Figure 3.2), and is followed by the (sub-)Eddington accretion phase. This lasts until the SMBH and the stellar population (which is steadily growing in the meantime) have captured all of the available gas, after a few  $\tau_{edd}$  (~  $10^9 yr$ ) and at  $z \approx 4.5$ , in Figure 3.1 and 3.2, respectively.

One of the most important results of this study can be seen immediately: the model is able to produce a SMBH and a bulge of stars which are both massive enough, and that within a small enough time. A comparison of these results to the observations used for the Magorrian relation (Figure 1.1) shows that the SMBH is somewhat massive compared to the bulge, but the mass ratio of the SMBH and the bulge lies within the scatter of the plot.

### 3.1.2 Tweaking the fiducial parameters

In order to investigate the affect and importance of the different parameters assigned to the simulations, we have made simulations of the model in which we change the value of one of the parameters, and thus leave the values of the other parameters unchanged with respect to the fiducial values. Since not all parameters are independent, we only change the value of the feedback radius  $r_f$ , the core radius  $r_0$ , the radiative efficiency  $\epsilon$ , the mass of the SBH  $M_{SBH}$ , the mass of the halo  $M_{\circ}$  and the Kennicutt-Schmidt constant K. The results are plotted in subplots A through E of Figure 3.3.

In their paper, Di Matteo et al. (2005) stress the importance of a steady gas flow that feeds the emerging galaxy and its SMBH. This stream of gas can yield prolonged star formation at a steady rate, and could even fuel the SMBH in such a manner that it powers the quasar so strong that feedback processes, resulting from this, quench star formation and further black hole growth entirely. Subplot G of Figure 3.3 shows the effects of such a mass





Figure 3.3: Time vs. mass. Simulation of the growth of the SMBH (solid line), the mass of the stellar population (dashed line) and the mass of the gas in the halo (dotted line). The same fiducial parameters were used as in Figure 3.1, with the exceptions as described in the legends. Note: K in subplot F is in units of  $cm^{3/2} kg^{-1/2} yr^{-1}$ .

inflow in five scenario's. The amount of mass that flows into the system,  $M_i$ , is given in units of the halo mass  $M_{\circ}$ .

The results shown in Figure 3.3 clearly demonstrate the different impacts of the altered parameters. In subplot A, we change the feedback radius  $r_f$ . This is the radius at which the growing SBH is accreting so fast and extreme, that feedback processes stop the radial accretion, and force the black hole to go on via (sub-)Eddington accretion. Since the radius and the mass of a black hole are related through the Schwarzschild radius (equation 2.1), we know that  $R \propto M_{\bullet}$ . Therefore we also know that when the radius expands by a factor  $r_f$ , the mass increases by the same factor  $r_f$ ; as shown in subplot A.

Subplot B shows the influence of the density profile of the pseudoisothermal sphere on our model. With a decreasing core radius  $r_0$ , the density in the center of the isothermal sphere increases; forcing more gas onto the SBH. This will significantly advance the time in which the black hole has reached the feedback radius, and thus its (sub-)Eddington accretion phase. On the other hand, when the core radius increases, and the density profile becomes much flatter, more gas will be turned into stars, and the SBH growth will stagnate.

The largest amount of mass that a SMBH accretes, will be through (sub-)Eddington accretion. The efficiency of this process plays a major role in allowing the amount of matter that can be accreted, as shown in subplot C. Eventhough the influence of the radiative efficiency  $\epsilon$  is plotted, the Eddington luminosity efficiency  $\eta$  plays an equal role since  $\tau_{edd} \propto \epsilon \eta^{-1}$  (equation 2.6). The plot shows the immense effect that these parameters have on the growth of a SMBH and the mass ratio between the SMBH and the galactic bulge.

When the mass of the SBH is varied (subplot D), this has influence on the time the Bondi accretion phase lasts. Nevertheless, it has little effect on the end masses and mass ratio. The massive SBHs used for this simulation  $(M_{SBH} = 10^3 M_{\odot} \text{ and } M_{SBH} = 10^3 M_{\odot})$ , show that black holes formed through a singular cloud collapse (Spaans & Silk 2006) could still very well act as a SBH in our model.

In changing the mass of the halo out of which our galaxy forms, as is done in subplot E, we alter the end mass of our sellar population. Simultaneously, this changes the density of the gas reservoir, which reflects on the Bondi phase of the simulation. Still this has, like in subplot B, little influence on the end mass of the SMBH.

Subplot F shows the implications of the Kennicutt-Schmidt constant. Changing the rate at which gas is converted into stars affects the growth of the stellar population. With a large K of  $10^4 \ cm^{3/2} \ kg^{-1/2} \ yr^{-1}$ , the stars are formed so rapidly that there is no matter to feed the black hole and there is no SMBH growth. When K is small  $(K = 1 \ cm^{3/2} \ kg^{-1/2} \ yr^{-1})$ , the stellar population grows so slowly, that after a couple of Eddington times, the SMBH is more massive than the galacitc bulge. Eventhough these two extreme scenario's are physically not the most likely ones, the Kennicutt-Schmidt constant must have a strong influence on the resulting end masses and mass ratios.

In order to investigate the claims of Di Matteo et al. (2005), subplot G shows the effects of a steady gas inflow. Other parameters are the same as in our fiducial model. The simulations show that a very strong inflow  $(M_i \sim 10M_{\circ})$  affects the end masses. Furthermore, it is possible that the accreting SBMH grows for enough Eddington times, and thus will start accreting so extremely that a feedback process must kick in to halt this phase. However, more research has to be done to find an adequate description for the manner in which this feedback process affects the total scenario.

#### 3.1.3 The evolving Magorrian relation

Figure 3.4 shows the manner in which the ratio between the SMBH mass and the mass of the galactic bulge evolves during the first Gyr, for our fiducial model. Subplot A shows the time plotted against the mass ratio. Subplot B shows the mass of the bulge against the SMBH mass.

In the first  $\sim 10^8 yr$ , it is the galactic bulge that grows must faster than the black hole, and the ratio is too low, compared to the Magorrian relation (Häring & Rix 2004; Figure 1.1).

After these first  $10^8 yr$ , the (sub-)Eddington accretion lasts for several



Figure 3.4: Evolution of the Magorrian relation plotted as the time versus the mass ratio of the bulge mass with the SMBH mass  $(M_{\bullet}/M_{\star})$ , and as the mass of the SMBH  $(M_{\bullet})$  versus the bulge mass  $(M_{\star})$ , in subplots A and B respectively. The dotted line in both plots defines the Magorrian relation (Figure 1.1). Both plots show that the mass ratio is to low when the system is evolving less than  $\sim 2 \times 10^8 yr$ . After that, the SMBH becomes to massive (within a period of  $\sim 10^8 yr$ ), before the total amount of gas is accreted or put into stars and an equilibrium is reached.



Figure 3.5: SMBH mass vs. bulge mass for 32 of the 36 end masses from the simulations shown in Figures 3.1, 3.2 (both the fiducial model) and 3.3 (colored dots). The colored area indicates the same region as Figure 1.1 (Häring & Rix 2004).

Eddington times and is therefor responsible for a rapid increase in mass, driving the mass ratio to a value well above the Magorrian relation. Thereafter, the system has depleted all of the gas from the halo and turned it into stars or accreted it onto the SMBH. Eventhough the eventual mass ratio is larger than the factor  $10^{-3}$  predicted by the Magorrian relation, it lies within the scatter of the points of Figure 1.1.

The end masses of 32 out of the 36 simulations shown in Figures 3.1, 3.2 and 3.3 are plotted in Figure 3.5. The four points that were omitted are those where the SBH did not grow substantially; subplots B ( $r_0 = 80.0 \ kpc$ ), D ( $M_{SBH} = 1M_{\odot}$  and  $M_{SBH} = 10M_{\odot}$ ) and F ( $K = 10^4 \ cm^{3/2} \ kg^{-1/2} \ yr^{-1}$ ). The colored area of Figure 3.5 indicates the same region as Figure 1.1.

Note that the masses plotted in Figure 3.5 represent the masses of the evolving halo without any merging. For the masses of a system with mergers, both of the result masses have to be multiplied by the multiplication factor  $\phi$  (section 2.4), which is typically of order ~ 10<sup>3</sup>.

The two characteristic paths, that the mass ratio follows during its evolution in Figure 3.4, indicate an evolution in the Magorrian relation. Very young galaxies (with an age of < 1 Gyr, where stars are still being formed, and where the black hole is only accreting for ~ 1  $\tau_{edd}$ ) would have a mass ratio < 10<sup>-3</sup>. This could provide a method to falsify this model. However, the ambition to observe these stages of evolution would be a fairly bold one, since these processes occur at redshifts very far away from the present day observable universe ( $z \geq 10$ ).

## **3.2.** Discussion and Conclusions

The important result of these simulations is the fact that it shows that it is possible to create a SMBH in a galactic bulge within 1 Gyr (or before z = 6.43), and - even more important - that this SMBH is massive enough to fit into the Magorrian relation. This is in accordance with findings by Volonteri & Rees (2005), Yoo & Miralda-Escudé (2004) and Di Matteo et al. (2008).

One of the conclusions could even be, that the masses are too high; overshooting the mass range found by Häring & Rix (2004). However, as shown in Figure 3.3, it is very well possible that the SMBH becomes less massive by increasing the radiative efficiency  $\epsilon$  (subplot C), or reducing the Eddington luminosity efficiency  $\eta$ . This would result in a less massive SMBH, and thus a mass ratio closer towards the Magorrian relation.

In fact, this could very well be in agreement with recent findings. We use a radiative efficiency of  $\epsilon = 0.15$  (Herrnstein et al. 2005) as a fiducial parameter. This is an observed value. Secondly, we state that the Eddington luminosity efficiency  $\eta = 1$ . Pelupessy et al. (2007) find in their simulations that SBHs grow much to fast with such a low radiative efficiency, and claim that the accretion goes sub-Eddington; thus  $\eta < 1$ . Which could explain our too massive SMBH. However, the rate at which the black hole is accreting is an item which is still very much under debate. Even in the recent review paper by Djorgovski et al. (2008) the total efficiency of the (sub-)Eddington accretion phase (that is  $\epsilon \eta^{-1}$ ) is a controversial parameter. Apart from the massive SMBH, the total mass of the young galaxy and its central black hole is fairly high. When the amplification factor due to merging is incorporated, the mass of the stellar population is ~  $10^{12} M_{\odot}$ . This means that the system would end up in the right top corner of Figure 1.1. This is perfectly possible.

However, by simply decreasing the mass of the halo  $M_{\circ}$  and reducing the total radius R, it is no problem to produce a bulge of  $\sim 10^7 - 10^8 M_{\odot}$ with a SMBH of  $\sim 10^4 - 10^5 M_{\odot}$ , which would still satisfy the Magorrian relation.

With each time step in our model, we return a fraction  $\Upsilon$  of the total mass in newly formed stars to the total gas mass of the halo (Section 2.5.2). Because the largest part of the gas is turned into stars with intermediate masses ( $M \sim M_{\odot}$ , Figure 2.5), the majority of the gas will be captured in stars with life times much larger than the time span of our model. Therefore, we do not account for feedback processes caused by less massive stars. However, after several Gyr (at  $z \gtrsim 2$ ), when the intermediate mass stars come at the end of their life cycles, this could very well be a trigger for a new phase of stellar birth, as well as a new accretion phase for the SMBH. This could even be an explanation for the AGN peak found at  $z \sim 1 - 1.5$  (Madau 1998).

The point at which feedback processes stop the Bondi accretion phase of the black hole is another element of uncertainty. We use the ambiguous, but arguable definition of Volonteri & Rees (2005). Since this Bondi accretion phase resembles an adiabatic inflow model, it is an *advection dominated accretion flow*, or ADAF (Narayan & Yi 1994). Improved hydrodynamical simulations of SMBH evolution could shed a light on the different physical circumstances belonging to this ADAF, or its counterpart (containing also outflow), the *adiabatic inflow-outflow solutions*, known as ADIOS (Blandford & Begelman 1999). Nevertheless, the overall picture (a period of extreme inflow followed by an outflow) does not change dramatically. As we have seen in subplots A and B of Figure 3.3, the point at which the feedback processes stop the Bondi phase has no large impact on the eventual system - as long as there is a period of (sub-)Eddington accretion.

## 3.3. Future work

Even though the work described in this paper shows some very nice results. There is still a lot of work that can be done.

First of all, the model of the SMBH accretion could be improved in a number of ways. By including a numerical code that beholds up-to-date ADAF and ADIOS models (Blandford & Begelman 1999). This should give a better insight in the different physical processes involved, encompassing eg. gas cooling, magnetic fields, black hole spin and different feedback processes resulting from these processes, or resulting in it.

Second, the effect of mergers should be better examined. This would promote the model to a full size state-of-the-art N-body simulation, especially if it also includes the recent ADAF and ADIOS models. Still, the effect of mergers of different masses and proto-galaxies in different stages of evolution must have played a large role in the evolution of present day (local) galaxies.

Besides the Magorrian relation, the  $M_{\bullet} - \sigma_{bulge}$  (Gebhardt et al. 2000; Merritt & Ferrarese 2001) and other recent found relations (Section 1) should be investigated to provide a better understanding of the origin of galaxies and SMBHs in the very early universe. Or to provide a better understanding of the link between SMBHs and AGNs or starbursts.

Fortunately, formation and early evolution of galaxies and black holes are clearly among the most exciting subjects in cosmology today. The synergetic co-evolution and interplay between the galaxy and its SMBH touch upon numerous fundamental questions, and there are prospects for surprises and even detection of new astrophysical phenomena associated with the birth and growth of SMBHs (Djorgovski et al. 2008).

The subject is growing rapidly, providing both observational and theoretical challenges, and will undoubtedly be a fruitful playground for the forthcoming generations of large telescopes. Both in space and on ground, over the full range of wavelengths and especially for the promising field of gravitational wave astronomy.

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