## A hierarchy of voids

Master Thesis

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### Abstract

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Most of the volume in the universe that we observe nowadays is occupied by cosmic voids. The formation and evolution of these voids can potentially provide important clues on dark energy models and other cosmological parameters. We discuss, on the basis of existing literature, how the emergence of the filamentary cosmic web may be described from the perspective of voids. Within this study we investigate the hierarchical evolution of the void population.

The identification of voids in galaxy redshift surveys and N-body simulations is not straightforward. This is mainly because the criteria for void identification are not well defined. The result of this is a range of different void finding algorithms being used. In our study we use an extended version of the Watershed Void Finder by Platen et al. [84]: the Multiscale Watershed Void Finder. For the original Watershed Void Finder, we find that the resulting void populations depend strongly on the user defined filter radius of the density field. The Multiscale Watershed Void Finder does not use a single filter radius. Instead it uses a set of progressively smaller filter radii. For each filter radius we only take into account the watershed basins that meet a certain predefined density criterion. We combine the voids that we identify for the different filter radii into a new segmentation. This allows us to probe the hierarchy of voids over a range of scales.

By using the Multiscale Watershed Void Finder we try to preserve the multiscale character of the cosmic web. In this way we can quantify the concept of void persistence and demonstrate the existence of an evolving population of voids.

After a general introduction into the subject of large-scale structure and the cosmic web in chapter 1, we will discuss the formation and evolution of voids in chapter 2. The concept of numerical simulations, the Delaunay Triangulation Field Estimator and the main simulation used for this research are discussed in chapter 3. In chapter 4 we introduce the Multiscale Watershed Void Finder and in chapter 5 we present our results with respect to the evolution of void abundances, void sizes and void shapes. The concept of void persistence is discussed in chapter 6. In chapter 7 we demonstrate that the hierarchical nature of the void population is quite well described by the adhesion model.

We have applied the Multiscale Watershed Void Finder to different dark energy models and compared the resulting void populations. The obtained differences between these populations turn however out to be strongly related to differences in  $\sigma_8$ . Our findings are presented in chapter 8.

### Contents

#### Abstract

1	Cosmology, large-scale structure and the cosmic web								
	1.1	The standard cosmological model: $\Lambda CDM$	1						
	1.2	Large-Scale Structure formation	4						
		1.2.1 The initial density field	4						
		1.2.2 Linear perturbation theory	5						
		1.2.3 Hierarchical clustering	8						
		1.2.3.1 The Press-Schechter formalism	9						
		1.2.4 The Excursion Set Formalism	11						
	1.3	The Cosmic Web	13						
		1.3.1 Zel'dovich approximation	13						
		1.3.2 Adhesion approximation	15						
<b>2</b>	$Th\epsilon$	Cosmic Web from the void perspective	17						
	2.1	Voids	17						
		2.1.1 Observations and N-body simulations	19						
		2.1.2 Characteristics	22						
		2.1.3 Dark energy and void shapes	23						
	2.2	P. From isolated voids to the sociology of voids							
	2.3	3 Extending the Excursion Set Approach							
		2.3.1 Void spectrum: SvdW analysis	28						
		2.3.2 Void spectrum: corrections	31						
		2.3.3 Void spectrum update: $V dn$ model	31						
	2.4	Tidal influences on void evolution	32						
	2.5	Voidpatch and void hierarchy	37						
3	Sim	ulating the Universe	39						
	3.1	N-body simulations with GADGET-2	40						
		3.1.1 Gravitational softening	40						
		3.1.2 The TreePM method	41						
	3.2	From a system of particles to a grid of densities: DTFE	42						
	3.3	Simulations parameters	44						
	3.4	Example run: density slices	44						

i

4	The Multiscale Watershed Void Finder       4         4.1 Void finders       4	<b>17</b> 47
	4.2 The Multiscale Watershed Void Finder	48 49
	4.2.1.1 Different filter radius, different void population 5	51
	4.2.2 Shell crossing criterion	53
	4.2.3 Multiscale filtering procedure	53
	4.2.4 The resolution of the simulation	50
<b>5</b>	Results: void abundances, sizes, shapes and velocities 5	58
	5.1 Evolution of void abundance	58
	5.2 Void sizes $\ldots$ $\ldots$ $\ldots$ 5.2 Void sizes $\ldots$ $\ldots$ 5.2 Void sizes $\ldots$ $\ldots$ 5.2 Sizes Sizes $\ldots$ 5.2 Sizes Size	59
	$5.3$ Void shapes $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$	61
	5.4 Outflow: the velocity divergence $\ldots \ldots $	64
6	Void persistence and merger trees	<b>38</b>
	$6.1  \text{Merger trees} \dots \dots$	69
	6.2 The persistence diagram	74
	6.3 Void persistence: statistics	76
	6.4 Disentangling mergers from squeezed voids: velocities and Lagrangian space 7	79
7	Void hierarchy and the adhesion model	33
•	7.1 Adhesion models	83
	7.2 Void identification with the MWVF	84
	7.3 Merger trees and void persistence	84
	7.4 Conclusions $\ldots$	89
0	Void biomonology for different Don's Energy models	פר
0	8.1 Simulations	9 <b>0</b> 02
	8.2 Evolution of void abundance	95 95
	8.2.1 The evolution of $\sigma_{\rm s}$	96
	8.3 Void shapes	97
9	Summary and discussion 10	00
A	knowledgements 10	)3
$\mathbf{A}$	The Watershed Transform 10	)4
	A.1 Greyscale image and topographic distance	04
	A.2 Segmentation algorithm	05
р	The principle of persistence	)e
В	The principle of persistence 10	JU
$\mathbf{C}$	Dark energy models 10	)8
	C.1 Models	08

	C.1.1	Dark energy in $\Lambda$ CDM	108
	C.1.2	Dynamical dark energy models: quintessence	109
C.2	Param	neterization	109

#### l Chapter

# Cosmology, large-scale structure and the cosmic web

#### 1.1 The standard cosmological model: $\Lambda CDM$

Modern cosmology is based on the cosmological principle, stating that at the largest scales the universe is isotropic and homogeneous. The most important piece of evidence for the cosmological principle is that of the almost isotropic Cosmic Microwave Background (CMB) radiation.

With Edwin Hubble, we now know that we live in an expanding universe. With the exception of galaxies in the Local Group, all galaxies in the universe are receding from us with a velocity that is proportional to their distance, described quantitatively by the Hubble law:

$$\vec{v} = H(t) \cdot \vec{r} \tag{1.1}$$

in which H(t) is the Hubble constant describing the expansion rate of the universe. Normally the Hubble constant is parametrized by h, such that H = 100h km s<sup>-1</sup> Mpc<sup>-1</sup>. The latest value for the present-day value of the Hubble constant  $H_0$  is given in table 1.1.

The notion of an expanding universe naturally leads to the introduction of a coordinate system that moves along with this expansion: comoving coordinates  $\vec{x}$ . These comoving coordinates are related to physical coordinates through:

$$\vec{r} = a(t) \cdot \vec{x} \tag{1.2}$$

in which a(t) is the scale factor of the universe. If there were no peculiar velocities, galaxies would remain fixed in the comoving coordinate system. Combining equations 1.1 and 1.2, one can easily derive that:

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} \tag{1.3}$$

Within modern physics, gravitation is described by Einsteins theory of general relativity. According to general relativity, all particles move along straight lines in spacetime, spacetime itself being a curved four-dimensional geometry. The local curvature of spacetime is related to the local energy density of matter and radiation, as described by the Einstein Field Equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}$$
(1.4)

where  $R_{\mu\nu}$  is the Ricci curvature tensor,  $g_{\mu\nu}$  the metric tensor,  $\Lambda$  the cosmological constant and  $T_{\mu\nu}$  the stress-energy tensor.

The Einstein Field Equations turn out to have an exact solution for a universe that fulfills the cosmological principle<sup>1</sup>. This solution was independently found by Friedmann-Lemaître-Robertson-Walker (FLRW), and is for this reason named the FLRW-metric:

$$ds^{2} = c^{2}dt^{2} - a(t)^{2} \left[ dx^{2} + S_{k}(x)^{2} d\Omega^{2} \right]$$
(1.5)

with

$$S_k(x) = \begin{cases} R_0 \sin(x/R_0) & (k=+1) \\ x & (k=0) \\ R_0 \sinh(x/R_0) & (k=-1) \end{cases}$$
(1.6)

corresponding to the three geometries allowed by the cosmological principle, respectively positively curved space (k = +1), flat space (k = 0) and negatively curved space (k = -1). For non-flat geometries, the curvature is described by the radius of curvature  $R_0$ .

On the largest scales, the universe can be described as a fluid with density  $\rho$  and pressure P. Within the FLRW-metric, the evolution of the scale factor a(t) for such a fluid (i.e. the universe) is given by the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2} \tag{1.7}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3P}{c^2}\right) + \frac{\Lambda c^2}{3} \tag{1.8}$$

The second equation is also known as the acceleration equation and is derived from the first Friedmann equation. In the equations  $\rho$  represents the energy density of the universe, k the curvature (+1, 0 or -1) and  $\Lambda$  the cosmological constant. This cosmological constant can be added as an extra term in the Einstein Field Equations. Originally, it was introduced by Einstein to create a static universe. Describing the cosmological constant also as some kind of a fluid with associated energy density  $\rho_{\Lambda}$ , we may define  $\rho_{\Lambda} \equiv \Lambda c^2/(8\pi G)$ . In this way, the Friedmann equations can be rewritten as:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \tag{1.9}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3P}{c^2}\right) \tag{1.10}$$

<sup>&</sup>lt;sup>1</sup>Notice that such a universe can be expanding or contracting

with  $\rho = \sum \rho_i$  the sum over all species that contribute to the energy density, including  $\rho_{\Lambda}$ .

Solving the first Friedmann equation requires one to know how the energy density of the universe evolves with time. This time dependence  $\rho(t)$  may be derived from the first law of thermodynamics, and is captured in the fluid equation:

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{P}{c^2}\right) = 0 \tag{1.11}$$

The relation between the pressure P and energy density  $\rho$  is described by the equation of state. In general, we may write the equation of state as

$$P_i = \omega_i \rho c^2 \tag{1.12}$$

with  $\omega_i$  some constant that depends on the species being considered.

From the first Friedmann equation one may derive the critical density  $\rho_c$  for our universe to be flat (i.e. k = 0). Recalling the definiton of H given by equation 1.3, the critical density is given by:

$$\rho_c = \frac{3H^2}{8\pi G} \tag{1.13}$$

Now we are able to define the cosmological density parameter  $\Omega$ , which expresses the density in terms of the critical density:

$$\Omega \equiv \frac{\rho}{\rho_c} = \frac{8\pi G\rho}{3H^2} \tag{1.14}$$

		WMAP	<u>Planck</u>
Hubble constant	$H_0$	$70.4^{+1.3}_{-1.4} \text{ km s}^{-1} \text{ Mpc}^{-1}$	$67.4^{+1.4}_{-1.4} \text{ km s}^{-1} \text{ Mpc}^{-1}$
density dark energy	$\Omega_{\Lambda,0}$	$0.728\substack{+0.015\\-0.016}$	$0.686\pm0.020$
density dark matter	$\Omega_{\mathrm{dm},0}$	$0.227\pm0.014$	0.268
density baryonic matter	$\Omega_{\mathrm{b},0}$	$0.0456 \pm 0.0016$	0.049
density radiation	$\Omega_{ m r,0}$	$\sim 10^{-5}$	$\sim 10^{-5}$

TABLE 1.1: Estimates of parameters in the ACDM model from both the WMAP (Jarosik et al. [51]) and Planck satellite (Planck Collaboration [82]).

The energy content of the universe is made up by different species: radiation, matter and dark energy. For each of these species we can define a separate density parameter, respectively  $\Omega_{\rm r}$ ,  $\Omega_{\rm m}$  and  $\Omega_{\Lambda}$ . Moreover, the contribution from matter may be divided into contributions from baryonic matter and dark matter:  $\Omega_{\rm m} = \Omega_{\rm dm} + \Omega_{\rm b}$ . Table 1.1 gives an overview of the estimates for the Hubble parameter and present-day density parameters resulting from both the WMAP (Jarosik et al. [51]) and Planck satellite (Planck Collaboration [82]). At the time we have started this study, the WMAP data represented the most up-to-date values. While writing this thesis, the first results of the Planck satellite have been published. These estimates are also included in table 1.1. The values in table 1.1 form the basic parametrization of modern cosmology: one in which the evolution of the universe is dominated by both the cosmological constant and cold dark matter. This universe is known as the concordance or  $\Lambda$ CDM model. It will be the reference point of this study. Another important type of universe that is often used to asses cosmological results, is a flat universe that consists of matter only. Such a universe is also known as an Einstein-de Sitter universe and will appear a few times in this thesis.

#### **1.2 Large-Scale Structure formation**

In the previous section we have assumed a homogeneous and isotropic universe: the basic assumption for the framework of modern cosmology. Empirically, one can however rather easily establish that this assumption is not correct. The universe around us does in fact reveal a wealth of structures like human beings, cities, planets, and stars. Hence the cosmological principle does at least not hold on the relatively small scales that we live in.

Within the standard  $\Lambda$ CDM-cosmology, the very early universe after the Big Bang is thought to consist of an almost isotropic and homogeneous density field. Within this density field, small fluctuations of order  $10^{-5}$  are present (confirmed by fluctuations in the Cosmic Microwave Background). These fluctuations are thought to be the seeds for the variety of structures that are observed in the universe nowadays.

It turns out that on the largest scales ( $\gtrsim 100$  Mpc) the present-day universe may still be considered as a homogeneous and isotropic medium. The largest structure that can be observed in the universe is that of the cosmic web: a structure of sheets and filaments of galaxies separated by voids, the nodes of these sheets and filaments appearing as superclusters. The galaxies that reveal this structure are only the lighthouses that mark the underlying distribution of dark matter. A theoretical description of how these structures grow out of the initially smooth density field is provided by gravitational clustering theory, which is the topic of discussion in this section.

#### 1.2.1 The initial density field

The initial density field can be prescribed by a field  $\delta(\vec{x})$ , that describes the perturbation of the density at certain comoving coordinates  $\vec{x}$  with respect to the background density  $\rho_u$ :

$$\delta(\vec{x}) = \frac{\rho(\vec{x})}{\rho_u} - 1 \tag{1.15}$$

From this equation it is easily seen that  $\delta(\vec{x}) = 0$  implies that the density at  $\vec{x}$  is equal to the background density  $\rho_u$ . Since negative densities do not exist, one will always find  $\delta(\vec{x}) \geq -1$ .

Most of the inflation models<sup>2</sup> predict that the initial field of density perturbations is Gaussian in nature. Even if the density field  $\delta(\vec{x})$  is a superposition of independent stochastic variables that are non-Gaussian in nature, the Central Limit Theorem predicts that this could still result in an initial density field that is Gaussian in nature. Although

<sup>&</sup>lt;sup>2</sup>The inflation model is invoked in physical cosmology to solve both the horizon and flatness problem that arise in the standard cosmological model. Basically it assumes that the very early universe has gone through a phase of exponential expansion, in which the quantum fluctuations that were present in the pre inflation universe were blown up by a factor of  $\sim e^{60}$  and form the seeds for structure formation in the universe.

a certain level of non-Gaussianities is allowed by most inflation models (in fact, the level of possible non-Gaussianity could help to better constrain the correct inflation model), we will in this thesis assume a Gaussian initial density field  $\delta(\vec{x})$ . This will also be our reference point in discussing the basics of large scale structure formation.

The statistical properties of a Gaussian initial density field may be completely described by its second order moments. Given the existence of a galaxy at position  $\vec{x_1}$ , the function  $\xi(r_{12})$  represents for the discrete case the excess probability of finding a galaxy at position  $\vec{x_2}$  at distance  $r_{12}$  from  $\vec{x_1}$ , i.e. the two-point correlation function. The continuous equivalent, the autocorrelation function, is defined as:

$$\xi(r_{12}) = \langle \delta(\vec{x_1})\delta(\vec{x_2}) \rangle \tag{1.16}$$

such that the probability of finding a pair of galaxies in the volumes  $dV_1$  and  $dV_2$  is given by  $dP = \rho_u^2 (1 + \xi(r_{12})) dV_1 dV_2$ .

In general, setting up a random field is much more convenient in Fourier space. The value at a particular position  $\vec{x}$  in the field  $f(\vec{x})$  can be considered as a superposition of many different wave modes:  $f(\vec{x}) = \Sigma \hat{f}_k e^{-i\vec{k}\cdot\vec{x}}$ . In Fourier space the counterpart of the autocorrelation function, which defines the statistical properties of our field in real space, is the power spectrum P(k). The power spectrum defines the amount of power for the different wave modes (characterized by their wavenumber k),

$$(2\pi)^3 \delta_D(\vec{k} - \vec{k'}) P(k) \equiv \langle \delta_k \delta_{k'} \rangle \tag{1.17}$$

Within cosmology the power spectrum is a quantity of utmost importance: for a Gaussian primordial density field, it completely specifies the statistical properties of the field. Since there is no reason to assume that there exists a preferred length scale within the universe, the primordial power spectrum is often modelled by a featureless power law:  $P(k) = A \cdot k^n$ . Within such a power law spectrum, it is the index *n* that governs the balance between the power at small and the power at large scales. A special case of such a power law spectrum is the scale invariant Harrison-Zel'dovich spectrum, predicted by inflation, with n = 1: spectra with n > 1 have more power on the smaller scales whereas spectra with n < 1 have more power on the larger scales. The most common spectrum is  $\Lambda$ CDM, which combines a powerlaw primordial power spectrum with a post-horizon modulation determined by CDM.

#### 1.2.2 Linear perturbation theory

As long as the density perturbations are small, that is to say  $\delta(\vec{x}, t) \ll 1$ , structure growth may be reasonably well described by linear perturbation theory.

Besides the density perturbation  $\delta(\vec{x}, t)$ , we have to introduce three other perturbation quantities at this point. These are:

- The peculiar velocity  $\vec{v} = \vec{u}(\vec{r},t) \vec{v}_H(\vec{r},t) = a(t)\dot{\vec{x}}$
- The potential perturbation  $\phi(\vec{x},t) = \Phi(\vec{r},t) \frac{1}{2}a\ddot{a}x^2$
- The peculiar gravity  $\vec{g}_{\rm pec} = \vec{g} \vec{g}_0 = -\frac{\vec{\nabla}\phi}{a}$

On a length scale L, that is typical for describing the formation and evolution of the cosmic web, the cosmos may be treated as a fluid (that is, the mean distance of interaction  $\lambda$  for fluid particles is much shorter than this typical length scale L). There are basically three equations that govern the evolution of a (cosmic) fluid. Expressed in physical coordinates  $\vec{r}$ , these are:

The continuity equation 
$$\frac{\partial \rho}{\partial t} + \vec{\nabla}_r \cdot \rho \vec{u} = 0 \qquad (1.18)$$

The Euler equation 
$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla}_r)\vec{u} = -\frac{1}{2}\vec{\nabla}_r P - \vec{\nabla}_r \Phi \qquad (1.19)$$

The Poisson equation 
$$\nabla_r^2 \Phi = 4\pi G(\rho + \frac{3P}{c^2})$$
 (1.20)

To deal with these equations we have to make some simplifications. First of all, the epoch of structure formation that we are interested in is dominated by matter and dark energy, allowing us to neglect the contribution of radiation to the energy density of the universe (currently estimated to be only  $\Omega_{\rm r,0} \sim 10^{-5}$ ). As a consequence of its negative pressure, there are no fluctuations in the energy density of dark energy and hence one will always find  $\delta_{\Lambda} = 0$ . Altogether, this implies that in describing the formation of structure in the universe we will only be dealing with the matter component of the fluid equations. The matter component consists out of both dark matter and baryonic matter. For baryonic matter, the pressure is in general neglible compared to the density, i.e.  $P_b \ll \rho_b/c^2$ . Dark matter does by definition only interact gravitationally and is for this reason a pressureless medium. Therefore the second simplification to be made is that pressure forces may be neglected, allowing us to drop the pressure terms in both the Euler and Poisson equation (eqs. 1.19 & 1.20).

Now that we are left with a much more idealized, but justifiable form of the fluid equations, we can transform these equations in three ways. First of all we will make a transformation from physical coordinates to comoving coordinates. Second of all we will express the equations in terms of the perturbations quantities discussed in the beginning of this section. And eventually we will linearize the equations (that is, neglect all higher-order terms). The mathematical derivation that is involved will not be discussed here (the interested reader is referred to e.g. van de Weygaert [110]). After having performed these transformations, we end up with the following linearized fluid equations expressed in comoving coordinates and perturbation quantities:

Linearized continuity equation	$\frac{\partial \delta}{\partial t} + \frac{1}{a} \vec{\nabla} \cdot \vec{v} = 0$	(1.21)
Linearized Euler equation	$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} = -\frac{1}{a}\vec{\nabla}\phi$	(1.22)
Linearized Poisson equation	$\nabla^2 \phi = 4\pi G a^2 \rho_u \delta$	(1.23)

It turns out that these linearized fluid equations can be solved analytically for a matter dominated universe. In order to do so, one has to take the divergence of the linearized Euler equation (eq. 1.22), and replace in this equation the linearized continuity equation (eq. 1.21) and the linearized Poisson equation (eq. 1.23). After having performed these replacements, one ends up with a second order partial differential equation containing only the perturbation quantity  $\delta$ :

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = \frac{3}{2}\Omega_0 H_0^2 \frac{1}{a^3}\delta \tag{1.24}$$

where we have used that for a matter dominated universe:  $\Omega_0 = 8\pi G \rho_0 / 3H_0^2$  and  $\rho_u(a) = \rho_0 a^{-3}$ .

The general solution to this equation consists of the sum of a growing mode solution  $D_1(t)$  and a decaying mode solution  $D_2(t)$ :

$$\delta(\vec{x}, t) = D_1(t)\Delta_1(\vec{x}) + D_2(t)\Delta_2(\vec{x})$$
(1.25)

In the context of structure formation the decaying mode solution soon becomes insignificant, so that normally we only need to consider the growing mode solution. One can show that for an Einstein-de Sitter universe, the growing mode solution scales with time as:

$$D_1(t) \propto t^{2/3}$$
 (1.26)

Furthermore, from equation 1.25 one can see that while in the regime where linear perturbation theory is valid, a density perturbation at spatial coordinates  $\vec{x}$  at time  $t_2$  is related to the density perturbation at time  $t_1$  through:

$$\delta(\vec{x}, t_2) = \frac{D(t_2)}{D(t_1)} \delta(\vec{x}, t_1)$$
(1.27)

The formal solution for the growing mode of equation 1.24, as is for example derived in Lahav and Yasushi [58], may in terms of redshift z be written as:

$$D(z) = \frac{5\Omega_{m,0}H_0^2}{2}H(z)\int_z^\infty \frac{1+z'}{H^3(z')}dz'$$
(1.28)

In general, this equation requires numerical evaluation, although for pure matter dominated universes ( $\Omega_{\Lambda} = 0$ ) it can be solved analytically. It turns out that for a universe with  $\Omega_{\Lambda} \neq 0$ , one can approximate the solution to the growing mode equation by the following fitting formula (see e.g. Lahav and Yasushi [58]):

$$D(z) = \frac{5\Omega(z)}{2(1+z)} \frac{1}{\Omega^{4/7}(z) - \Omega_{\Lambda}(z) + [1 + \Omega_m(z)/2][1 + \Omega_{\Lambda}(z)/70]}$$
(1.29)

Defining the dimensionless linear velocity growth factor as

$$f \equiv \frac{d\ln D}{d\ln a} = \frac{1}{HD} \frac{dD}{dt}$$
(1.30)

one can show that the peculiar velocity field  $\vec{v}$  evolves linearly proportional to the peculiar gravity field  $\vec{g}$ :

$$\vec{v} = \frac{Hf}{4\pi G\rho_u}\vec{g} \tag{1.31}$$

For a matter-dominated universe, this dimensionless linear velocity growth factor may be approximated by (Peebles [80]):

$$f(\Omega_m) \approx \Omega_m^{\gamma} \tag{1.32}$$

Originally Peebles [80] used  $\gamma = 0.6$  for a pure matter universe. Linder [64] showed that for a  $\Lambda$ CDM universe it is better to use  $\gamma = 0.55 + 0.05[1 + \omega(z = 1)]$ . From the last equation one can conclude that f is only weakly dependent on  $\Omega_{\Lambda}$  (through the equation of state parameter  $\omega(z = 1)$ ), confirming the assumption made earlier about neglecting the dark energy component in the evolution of perturbation quantities.

By now, we have arrived at one of the most important results of linear perturbation theory: in the linear regime, the induced peculiar velocity field  $\vec{v}$  is linearly proportional to the peculiar gravity field  $\vec{g}$ , that is generated by the density perturbation field  $\delta(\vec{x}, t)$ .

In summary, the starting point of classical structure formation is an assumed Gaussian primordial density field  $\delta_{\rm p}(\vec{x})$ . The tiny perturbations that are present in this initial density field induce perturbations into the gravitational background field, creating a peculiar gravity field  $\vec{g}$ . As a consequence of this peculiar gravity field, a peculiar velocity field  $\vec{v}$  is created: matter starts to move from lower density regions to higher density regions, a process which may be referred to as structure formation. It turns out that the first phases of structure formation are reasonably well described by linear perturbation theory, which has been the subject of discussion in this section.

Once the initial density perturbations have grown sufficiently large  $(\delta(\vec{x}, t) \approx 1)$  it is no longer sufficient to use linear perturbation theory in order to describe the evolution of growing structures in the universe. We are now in a phase that is referred to as nonlinear clustering. In order to describe the evolution of density perturbations one has to use the full nonlinear fluid equations. Characteristic of the nonlinear regime is a phenomena called mode coupling: a transfer of power between the different (Fourier) scales in the matter distribution on which structure formation takes place. This in contrast to linear structure formation where all the scales of structure formation evolve independently of each other.

#### 1.2.3 Hierarchical clustering

Since baryonic matter comprises only a relatively small fraction of the total mass in the universe, it is generally assumed to follow closely the distribution of dark matter. In other words, galaxies may be seen as the beacons of the cosmic web marking the underlying distribution of dark matter. The galaxies in our universe are thought to have formed in dark matter haloes, which are the high density structures within the field of matter density perturbations  $\delta(\vec{x})$ . Describing galaxy formation by looking at the formation of dark matter haloes in the density field, has the advantage that dark matter only interacts via the always attractive force of gravity. This allows one to initially neglect the complex fluid-mechanical and radiative behaviour of a baryonic gas. Classically, the formation of dark matter haloes in the universe is described by the Press-Schechter formalism.

#### 1.2.3.1 The Press-Schechter formalism

The simplest way to describe the formation of an object out of the primordial density field  $\delta_{\rm p}(\vec{x})$ , is to consider an isolated spherical overdensity consisting of a pressureless and collisionless medium. The equation of motion for a mass shell of such an overdensity is solely governed by the interior mass of the shell.

The time evolution of the mass shell may be described by three different phases: expansion, collapse and virialization. Initially, the overdensity is moving along with the Hubble flow, although its expansion slows down due to the gravitational attraction of the interior mass. At turnaround, expansion comes to a halt and the overdensity decouples from the Hubble flow and starts to collapse. During the collapse, gravitational potential energy is converted into random motions. Eventually the collapse comes to a halt, and a virialized object has formed.

Gunn and Gott [42] have shown that for a spherical isolated overdensity in an Einsteinde Sitter universe, one can derive critical values for the overdensity  $\delta$  that correspond to the events of turnaround and collapse (formation of a virialized object). Extrapolated from the linear regime these values are

$$\delta_{L,ta} = \frac{3}{20} (6\pi)^{2/3} \approx 1.062$$

$$\delta_{L,c} = \frac{3}{20} (12\pi)^{2/3} \approx 1.686$$
(1.33)

whereas the corresponding nonlinear values are given by  $\delta_{\rm NL,ta} \approx 4.55$  and  $\delta_{\rm NL,c} \approx 177$ .

A Gaussian primordial density field, linear gravitational growth and the spherical collapse model can be combined into what is referred to as the classical Press-Schechter (PS) formalism (Press and Schechter [87]). The PS formalism is a simplified model for describing the time evolution of the abundance of dark matter haloes n(M, z) in an analytical way on the basis of the initial density field.

Within the spherical model, we have seen that the average overdensity of a clump of mass should exceed some critical value  $\delta_c$  for it to be associated with a gravitationally collapsed object. According to the PS approach, the location and properties of these bound objects can be derived by smoothing the initial linear density field with a filter function (often a Gaussian for analytic convenience). For a filter with characteristic length scale  $R_{\rm f}$ , the associated mass scale is  $M \sim \rho_0 R_{\rm f}^3$ .

Assuming a Gaussian initial density field, the probability that a fluctuation on the scale  $R_{\rm f}$  corresponds to an (average) density perturbation  $\delta$  is given by:

$$P(\delta) = \frac{1}{\sqrt{2\pi}\sigma(R_{\rm f})}e^{-\frac{\delta^2}{2\sigma^2(R_{\rm f})}}$$
(1.34)

In this equation  $\sigma^2(R_{\rm f})$  is the variance of the density field filtered on a scale  $R_{\rm f}$ . The probability that a given point lies in a region with  $\delta > \delta_{\rm c}$  may be obtained by integrating equation 1.34 from  $\delta_{\rm c}$  to infinity:

$$P(\delta > \delta_{\rm c}|R_{\rm f}) = \int_{\delta_{\rm c}}^{\infty} P(\delta) \mathrm{d}\delta = \frac{1}{2} \left[ 1 - \operatorname{erf}\left(\frac{\delta_{\rm c}}{\sqrt{2}\sigma(R_{\rm f})}\right) \right]$$
(1.35)

According to the PS approach, the probability of equation 1.35 is proportional to the probability that a given point has ever been part of a collapsed object on a scale larger than  $R_{\rm f}$ . The problem with this approach is, that even if  $\sigma(R_{\rm f}) \to \infty$ , the fraction of collapsed objects is only  $\frac{1}{2}$  since only overdense regions participate in spherical collapse. To overcome this problem, Press-Schechter multiplied equation 1.35 by an ad hoc factor of two<sup>3</sup>.

Defining  $\nu = \delta_c / \sigma(M)$ , incorporating the factor 2 and rewriting equation 1.35 in terms of mass, we end up with the following fraction of the universe being condensed into objects with mass > M:

$$F(>M) = 1 - \operatorname{erf}(\nu/\sqrt{2})$$
 (1.36)

This equation enables us to determine the comoving number density n(M)dM of objects within the mass range [M, M + dM].

$$n(M)dM = \frac{\rho_0}{M} \cdot \left| \frac{dF}{dM} \right| dM = \frac{\rho_0}{M^2} \left| \frac{d\ln\sigma}{d\ln M} \right| \sqrt{\frac{2}{\pi}} \nu e^{-\nu^2/2}$$
(1.37)

For the case of a power spectrum  $P(k) \propto k^n$ , the variance of the density field  $\sigma^2(M)$  filtered on a scale  $R_{\rm f}$  that corresponds to a mass scale M turns out to be

$$\sigma^2(M) = \left< \delta^2 \right> = A M^{-(3+n)/3} \tag{1.38}$$

One may now define the following characteristic mass scale:

$$M^* = \left(\frac{2A}{\delta_c^2}\right)^{3/(3+n)} \tag{1.39}$$

Rewriting equation 1.37 in terms of mass by using the characteristic mass scale  $M^*$  yields:

$$n(M)dM = \frac{1}{2\sqrt{\pi}} (1 + \frac{n}{3}) \frac{\rho_0}{M^2} \left(\frac{M}{M^*}\right)^{(3+n)/6} e^{-\left(\frac{M}{M^*}\right)^{(3+n)/3}}$$
(1.40)

Recalling that for an Einstein-de Sitter universe (equation 1.26), density perturbations grow as  $\delta \propto t^{2/3}$ , one can show that the characteristic mass scale evolves as:

$$M^* = M_0 \cdot \left(\frac{t}{t_0}\right)^{4/(3+n)}$$
(1.41)

As time progresses, the characteristic mass scale  $M^*$  increases, which translates into an increasing number density of high mass objects according to equation 1.40. Furthermore, for n > -3 (which is generally assumed to be true) the slope of the mass spectrum n(M)dM is negative. Altogether, this implies that structure formation is hierarchical: the smaller structures are the first to become nonlinear, continuously clustering together to form larger and larger objects.

<sup>&</sup>lt;sup>3</sup>The extended Press-Schechter formalism provides us with a physical justification of this factor  $\frac{1}{2}$ .



FIGURE 1.1: Figure from Brinchmann [22] illustrating excursion set formalism. In this figure S represents the rms filtered value of  $\delta$ , increasing from left to right along the x-axis. Since large S implies filtering on a smaller radius  $R_f$ , and  $M \propto R_f^3$ , the mass decreases from left to right.

#### 1.2.4 The Excursion Set Formalism

The extended Press-Schechter formalism (EPS) or excursion set formalism (Press and Schechter [87], Peacock and Heavens [79], Bond et al. [16], Sheth [100]) provides us with an interesting, physically intuitive way of describing the evolution of the density at a particular point  $\vec{x}$ . We define S(M) as the fluctuation of our density field filtered on a mass scale M, such that

$$S(M) = \sigma^2(M) \tag{1.42}$$

At S = 0, when the density field is filtered on the scale of the universe, we have  $\delta = 0$ by definition. Starting from this point, we are going to follow the trajectory of  $\delta$  for increasing S (i.e. a decreasing smoothing radius and corresponding mass scale). The basic assumption of the exursion set formalism is that for each  $\vec{x}$ , there exists a trajectory  $\delta(S = \sigma^2(M))$ . This trajectory describes the evolution of the overdensity as a function of the rms filtered value of the variance of the density field. Examples of such trajectories are shown in figure 1.1.

For a Gaussian initial density field, the Fourier components  $\hat{\delta}(\vec{k})$  are independent of each other. This implies that if we apply a sharp k-space filter to the density field and follow the evolution of  $\delta(\vec{x})$ , each step is independent of the previous step so that the trajectory  $\delta(S)$  at  $\vec{x}$  is essentially transformed into a random walk,

$$\delta(\vec{x}) = \int \frac{\mathrm{d}\vec{k}}{(2\pi)^3} \hat{W}_f(k) \hat{\delta}(\vec{k}) e^{-i\vec{k}\vec{x}}$$
(1.43)

Recalling that regions of space for which  $\delta > \delta_c$  may be associated with gravitationally collapsed objects, one can conclude that the first time a trajectory  $\delta(S)$  crosses the threshold  $\delta = \delta_c$  (first upcrossing) sets the maximum mass of an object of which the

region can be part. For example, trajectory B of figure 1.1 crosses the line  $\delta = \delta_c$  for the first time at  $S_3 = \sigma^2(M_3)$  so that it may be identified as being part of an object with mass  $M_3$ . However, it could be that a certain trajectory  $\delta(S)$  crosses the barrier  $\delta_c$ at  $S_M$  but has already crossed  $\delta_c$  for some  $S < S_M$ . In that case, the object identified at  $S_M$  is in reality part of a larger mass object and should not be taken into account. This is referred to as the cloud-in-cloud problem and should be accounted for in the halo mass spectrum.

Formally, to derive a mass spectrum on the basis of the excursion set formalism, one should determine the distribution of first upcrossings of the random trajectory  $\delta(S)$ . In other words, we want to know the probability that a random walk  $\delta(S)$  crosses  $\delta_c$  at a mass scale  $S_M$  but has not crossed  $\delta_c$  for any  $S < S_M$ . The probability distribution  $\Pi(\delta_L, S_M | \delta_c)$  of trajectories with a linearly extrapolated density value  $\delta_L$  at a smoothing scale  $S_M$  that have not crossed  $\delta_c$  for  $S < S_M$  is found by solving the Fokker-Planck equation (Bond et al. [16], Zentner [119]):

$$\frac{\partial \Pi}{\partial S} = \lim_{\Delta S \to 0} \left\{ \frac{\langle (\Delta \delta)^2 \rangle}{2\Delta S} \frac{\partial^2 \Pi}{\partial \delta^2} - \frac{\langle \delta \Delta \delta \rangle}{\Delta S} \frac{\partial \Pi}{\partial \delta} \right\}$$
(1.44)

If one chooses to use a sharp k-filter to smooth the density field, the change in density contrast  $\Delta \delta_L(S)$  in going from  $S \to S + \Delta S$  is independent of the previous value  $\delta_L(S)$  such that  $\langle \delta_L(S) \Delta \delta_L(S) \rangle = 0$  and we end up with

$$\frac{\partial \Pi}{\partial S} = \frac{1}{2} \frac{\partial^2 \Pi}{\partial \delta^2} \tag{1.45}$$

The probability that a trajectory  $\delta(S)$  has a density value  $\delta_L(S_M)$  at  $S_M$ , regardless whether it crossed the barrier  $\delta_c$  for  $S < S_M$  or not, is given by the Gaussian distribution

$$\Pi(\delta_L, S_M) = \frac{1}{\sqrt{2\pi S}} \exp\left(-\frac{\delta_L^2}{2S_M}\right)$$
(1.46)

Suppose that we have a trajectory that reaches the threshold  $\delta_c$  for some  $S < S_M$ . After reaching this threshold, it may either cross  $\delta_c$  or not cross  $\delta_c$ , the probabilities of both being equal since it is a random step process. For example, trajectories B and B' in figure 1.1 both reach  $\delta_c$  at  $S_3$  and should be identified with an object of mass  $M_3$ . Trajectory B crosses  $\delta_c$  whereas B' does not cross  $\delta_c$  and is merely a reflection of B around  $\delta_c$ . Notice that for each trajectory with  $\delta_L(S_M) > \delta_c$  there exists a reflected trajectory that ends up the same distance below  $\delta_c$ . The probability of first upcrossings may therefore be obtained by subtracting the reflected distribution from equation 1.46

$$\Pi(\delta_L, S|\delta_c) = \frac{1}{\sqrt{2\pi S_M}} \left\{ \exp\left(-\frac{\delta_L^2}{2S_M}\right) - \exp\left(-\frac{(\delta_L - 2\delta_c)^2}{2S_M}\right) \right\}$$
(1.47)

Integrating this distribution over all values of  $\delta_L$  gives the probability that the threshold has at least once been crossed. Hence, the corresponding probability that the considered location is part of an object with mass  $\leq M$  is given by

$$P(S_M|\delta_c) = 1 - \int d\delta_L \Pi(\delta_L, S_M|\delta_c) = 1 - \operatorname{erf}\left\{\frac{\delta_c}{\sqrt{2}\sigma(M, t)}\right\}$$
(1.48)

We have seen that the PS formalism shows us that structure formation is hierarchical. In addition, the excursion set formalism teaches us how to deal with the so-called 'cloud-incloud' problem. This knowledge turns out to be essential for gaining an understanding of the evolution of the underdense regions in the universe, which is slightly more difficult and will be discussed in the next chapter.

#### 1.3 The Cosmic Web

One of the most striking features of the matter distribution in the universe is the appearance of a weblike geometry: sheets and filaments of galaxies are separated by voids, the nodes of these sheets and filaments appearing as superclusters. This structure is known as the cosmic web and can be recognized in both the local distribution of galaxies and the distribution of particles in N-body simulations as is shown in figure 1.2.

The excursion set formalism provides us with a purely local description for the evolution of isolated overdensities. In reality, these overdensities are not isolated but embedded in the surrounding density field  $\delta(\vec{x}, t)$ . Moreover, the initial overdensities are in general not completely spherical. During gravitational collapse, these asymptries tend to be amplified, giving rise to the highly anisotropic structures that form the skeleton of the cosmic web. This anisotropic nature of gravitational collapse is best illustrated by the Zel'dovich approximation.

#### 1.3.1 Zel'dovich approximation

The Zel'dovich approximation (Zel'dovich [118]) is a first order Lagrangian approach that for the first time showed in an analytical way the anisotropic nature of the matter distribution in the universe. Within this approximation the initial displacement of particles is determined and it is assumed that particles continue to move in this direction. The Eulerian position  $\vec{x}(t)$  of a particle in the Zel'dovich approximation is given by:

$$\vec{x}(t) = \vec{q} + D(t)\vec{\Psi}(\vec{q})$$
 (1.49)

where  $\vec{q}$  describes the initial comoving coordinates of the particle (i.e. Lagrangian position) and  $\vec{\Psi}(\vec{q})$  is the time-independent (initial) displacement vector which is scaled by the linear growth factor D(t). Since particles are assumed to maintain their initial velocity, the displacement vector  $\vec{\Psi}(\vec{q})$  only depends on the initial potential field:  $\vec{\Psi}(\vec{q}) \propto \nabla \phi$ .

From mass conservation between the two different coordinate systems we have

$$\rho \cdot d^3 \vec{x} = \rho_0 \cdot d^3 \vec{q} \quad \rightarrow \quad 1 + \delta = \frac{1}{J} \quad \text{with} \quad J = \left| \frac{\partial \vec{x}}{\partial \vec{q}} \right| = |I + T_{ij}| \tag{1.50}$$



(a) Map of the galaxy distribution resulting from the final data release of the 2dF Galaxy Redshift Survey (2dFGRS). Within this spectroscopic survey spectra were obtained for 245,591 objects resulting in 221,414 reliable galaxy redshifts. Image from Colless et al. [28].



(b) Distribution of dark matter in the Millenium simulation. [Image courtesy: V. Springel and the Millenium simulation.]

FIGURE 1.2: The matter distribution is organized in a weblike geometry.

In the last relation J represents the Jacobian of the mapping and  $T_{ij} = \left| \frac{\partial \vec{\Psi}}{\vec{q}} \right|$  the deformation tensor.

Rewriting equation 1.50 in terms of the eigenvalues  $\lambda_i$  of the matrix  $T_{ij}$  gives us

$$1 + \delta = \frac{1}{[1 - D(t)\lambda_1][1 - D(t)\lambda_2][1 - D(t)\lambda_3]}$$
(1.51)

The absolute values of these eigenvalues may be identified as the three axes of the deformation ellipsoid. If one of the eigenvalues  $\lambda_i$  is positive, we have  $1 + \delta \rightarrow \infty$  as  $D(t) \rightarrow 1/\lambda_i$  and a positive eigenvalue will therefore eventually imply gravitational collapse along that particular axis. In that sense, we can identify four different morphological objects that together build up the cosmic web:

- clusters:  $\lambda_1 \approx \lambda_2 \approx \lambda_3 > 0$
- filaments:  $\lambda_1 \approx \lambda_2 > 0$
- walls:  $\lambda_1 > 0$
- voids:  $\lambda_1, \lambda_2, \lambda_3 < 0$

The Zel'dovich approximation normally takes longer to break down than the first order Eulerian approach of linear perturbation theory discussed previously. As long as sheets of matter do not cross, the Zel'dovich approximation has proven to be rather accurate, even in the mildly nonlinear regime. The reason for discussing it here, is that nearly every N-body code uses it to set up quasi-linear initial conditions. Also for this research project, the initial conditions generator that was used (N-genic) evolves the primordial density field  $\delta_p(\vec{x})$  to a later timestep by using the Zel'dovich approximation. Only after this step, the real N-body simulation is started (notice that the computational costs of applying Zel'dovich approximation are far less than carrying out the real N-body simulation).

#### 1.3.2 Adhesion approximation

Although the Zel'dovich approximation gives a rather accurate description for the first phases of structure formation, it will at some point break down. Describing structure formation with the Zel'dovich approximation implies that sheets of matter can move through each other without feeling their mutual gravitational attraction. Therefore as soon as these sheets start to cross each other, the Zel'dovich approximation starts to break down.

One considerable improvement of the Zel'dovich approximation is the adhesion approximation. This approximation introduces an artificial viscosity term  $\nu$ , leading to an equation which is well known. The artifical viscosity term takes into account the selfgravity of the forming structures as soon as matter starts to move through each other. This viscosity term causes the matter that moves through each other, to stick together. Hence this approximation is called the adhesion model.



FIGURE 1.3: The cosmic web in the adhesion approximation. [Image courtesy: Johan Hidding.]

If we define  $u \equiv (d\vec{x}/dD)$  as a new comoving velocity (with *D* the linear velocity growth factor), one can show that the Zel'dovich equation of motion becomes (Shandarin [99], Hidding [45]),

$$\frac{\partial \vec{u}}{\partial D} + (\vec{u} \cdot \nabla)\vec{u} = 0 \tag{1.52}$$

Adding the artificial viscosity term of the adhesion model to this equation, we obtain,

$$\frac{\partial \vec{u}}{\partial D} + (\vec{u} \cdot \nabla)\vec{u} = \nu \nabla^2 \vec{u}$$
(1.53)

This equation is known as Burgers' equation. The viscosity term prevents crossing of particle orbits and therefore preserves shocks after they form. This allows one to follow the emergence of the cosmic web to much later times than the Zel'dovich approximation. An example of the cosmic web in the adhesion model is shown in figure 1.3. A more detailed discussion of the adhesion model is provided by e.g. Shandarin [99] and Hidding [45].

## Chapter 2

## The Cosmic Web from the void perspective

In the previous chapter we have seen that the structure of the cosmic web consists of sheets, elongated filaments and clusters of galaxies that are separated by large empty voids. Classically, the formation, evolution and dynamics of the foamlike cosmic web is described by the hierarchical clustering of overdense objects. Although most of the mass in the universe can be found in these virialized structures, most of its volume is occupied by underdense regions. For this reason describing structure formation from the perspective of voids seems a natural choice. Within this approach, the universe is considered as a collection of expanding voids, with filaments and sheets appearing at the intersections of the void boundaries (Icke [50], Sheth and van de Weygaert [101], Aragon-Calvo and Szalay [2]).

In this chapter, we first discuss some basic characteristics of voids before we apply the excursion set formalism to voids in order to derive a void spectrum. Although the excursion set formalism provides us with an interesting analytical approach to describe void evolution, some of its assumptions are unrealistic. We will discuss these assumptions before taking a closer look on the interplay between void evolution and the large-scale tidal field in which they are embedded. We conclude this chapter by discussing the void-patch formalism, which tries to describe the hierarchical evolution of a void dominated universe. The existence of this void hierarchy is the main subject of this study and will be discussed and demonstrated in this thesis.

#### 2.1 Voids

As soon as the first density perturbations decouple from the Hubble flow, matter starts to contract in recognizable features. Underdense regions in between these features expand with a super-Hubble velocity. From the void perspective, the effective repulsive gravity resulting from this super-Hubble expansion forces matter to contract further into the features that form the boundaries of the voids (i.e. sheets, filaments and clusters). Theoretical models of void formation confirm that in a void-based description of structure formation, matter in between the void bubbles is organized into the filamentary network of the cosmic web (Icke [50], Regős and Geller [89], Sheth and van de Weygaert [101]).

As pointed out by van de Weygaert and Platen [113], the study of structure formation from the void perspective is interesting for a number of reasons. First of all, voids form a key element of the cosmic web: most of the universe's volume is occupied by voids. Voids also contain significant information on the underlying cosmological scenario and global cosmological parameters. These imprints may for example be found in the outflow velocities and corresponding redshift distortions (Dekel and Rees [33], Martel and Wasserman [66], Ryden and Melott [94]), but also in their intrinsic structure, shape and mutual alignent (Park and Lee [76], Platen et al. [85], Bos [18]). Before discussing the basic characteristics of voids, we will take a closer look at the voids found in both observations and N-body simulations.



FIGURE 2.1: Slice through the SDSS galaxy distribution. Each point represents a galaxy, the earth being at the centre. [*Image courtesy:* M. Blanton and the Sloan Digital Sky Survey.].

#### 2.1.1 Observations and N-body simulations

Already within the first galaxy surveys (Chincarini and Rood [25], Gregory and Thompson [41], Einasto et al. [37]) the existence of large regions that are almost completely devoid of any galaxies was established. One of the largest voids known to exist is the Boötes void, discovered by Kirshner et al. [55], [56]. The key position occupied by voids in the arrangement of matter in the structure of the cosmic web was already recognized with the first CfA redshift slice (de Lapparent et al. [30]). Later redshift surveys, especially the 2dFGRS and SDSS surveys (Colless et al. [28], The SDSS collaboration [107]) have confirmed this view. The prominence of void regions in the galaxy distribution is clearly visible in 2dFGRS and SDSS sky maps, an example of which is shown in figure 2.1.

Pan et al. [74] have created a catalog of voids in the nearby universe based on the void finding algorithm by Hoyle and Vogeley [49] applied to data release 7 (DR7) of the SDSS. The most up-to-date inventarisation is that of Sutter et al. [106], who have used the Watershed Transform (see chapter 4 and appendix A) to produce a public void catalog that is also based on SDSS DR 7. For producing this catalog they have used both the main galaxy catalog (out to z = 0.2) and the luminous red galaxy catalog by Kazin et al. [54] (z = 0.16 - 0.44). From these two catalogs, respectively four and two volume-limited samples have been constructed. Redshift bins and maximum K-corrected magnitudes  $M_r$  of these samples are given in table 2.1 (more details in Sutter et al. [106]). Voids in these samples are identified by using a modified version of the parameter-free void finder ZOBOV (Neyrinck [71]). The galaxy distribution and corresponding void distribution for the samples dim1 and dim2 are shown in figure 2.3 for a 25 deg opening angle. The size distribution of the void populations obtained for the different samples is presented in figure 2.2. This figure clearly shows that the obtained void population depends strongly on the galaxy sample used to trace the underlying matter distribution.



FIGURE 2.2: Distribution of void radii for the different samples in the public void catalog. It is clearly visible that the obtained void population depends strongly on the galaxy sample being used. Figure from Sutter et al. [106].

Sample name	$z_{\min}$	$z_{\min}$	$M_{r,\max}$
dim1	0.0	0.05	-18.9
$\dim 2$	0.05	0.1	-20.4
bright1	0.1	0.15	-21.35
bright2	0.15	0.20	-22.05
lrgdim	0.16	0.36	-21.2
lrgbright	0.36	0.44	-21.8

TABLE 2.1: Redshift bins and *K*-corrected maximum magnitudes of the volume-limited samples used by Sutter et al. [106] to produce their public void catalog.





FIGURE 2.3: Spatial distribution of voids in the public void catalog of Sutter et al. [106] for the samples dim1 and dim2. Galaxies and void centers within a 25 deg opening angle are rotated about the x-axis to lie in the xy plane. Void centres have different colors based on their size. Figure from Sutter et al. [106].

Supplementary to the galaxy distributions obtained by redshift surveys, the existence of void regions can be shown observationally by looking at the large scale peculiar velocity field. This peculiar velocity field is a manifestation of the effective gravitational push of the underlying matter distribution: matter is attracted to overdense regions and flows away from underdense regions. This manifests itself in the observed large-scale motions (Dekel [32], Branchini et al. [20]). Dekel and Rees [33] have used outflow velocities from voids to establish lower bounds on  $\Omega$ . For the galaxies in the Local Group, Tully et al. [108] decomposed their peculiar velocities into three components: away from the Local Void and in the direction of the Virgo Cluster and on a larger scale the Centaurus cluster. He demonstrated that indeed you can feel the gravitational push by the Local Void. On the basis of the peculiar velocity of an isolated galaxy in the Local Void and the standard cosmological paradigm they estimate the size of the Local Void to be at least 23 Mpc.



FIGURE 2.4: Matter distribution of the local universe reconstructed on the basis of the peculiar velocity field. Notice in particular the outflow patterns in the voids. Figure from Courtois et al. [29]

Since the peculiar velocity field is a manifestation of the effective gravitational push of the underlying matter distribution, the peculiar velocity field can be inverted to reconstruct the underlying density distribution (Bertschinger and Dekel [9], Romano-Díaz and van de Weygaert [92]). Because of the relatively high uncertainties in peculiar velocities at higher redshifts this reconstruction is currently restricted to the local universe. An interesting example of such a reconstruction made by Courtois et al. [29] is shown in figure 2.4.



FIGURE 2.5: Six timeframes (a = 0.05, 0.15, 0.35, 0.55, 0.75, 1.0) of the evolution of a deep void in a numerical simulation. This particular void was found in a  $\Lambda$ CDM scenario with an n = 0 power-law power spectrum. Image from Platen [83].

The view of a universe that is built by a collection of expanding void bubbles, the boundaries of which mark the outline of the cosmic web, is confirmed by various numerical studies (e.g. Martel and Wasserman [66], Regős and Geller [89], Dubinski et al. [34], van de Weygaert and van Kampen [115], Goldberg and Vogeley [39], Colberg et al. [26], Padilla et al. [72], Aragon-Calvo and Szalay [2]). Figure 2.5 shows a very good example of the evolution of a void in a numerical simulation by Platen [83] in a ACDM scenario.

#### 2.1.2 Characteristics

The underdensities of the initial density field  $\delta_{\rm p}(\vec{x})$  represent the seeds of the voids that will eventually form as the universe evolves. The net deficit of matter within voids induces a negative peculiar gravity that points outwards. As a consequence of this "repulsive gravity" matter moves out of the void region and the void expands with respect to the background universe. Usually, the density profile of a void increases from the centre to its boundaries. This implies that the outward gravitational acceleration is highest at the centre and gradually decreases outwards. Matter at the centre therefore moves out faster than surrounding material closer to the boundary of the void. Matter starts to accumulate in ridges, and eventually this process may lead to interior mass shells overtaking exterior shells: shellcrossing (only occurs if the initial density profile is sufficiently steep). One can show that the density profile of a spherical tophat void evolves into a so-called "bucket shape" (see figure 2.6): matter accumulates in steep ridges around the void whereas the interior region behaves as an expanding mini-low density-FRW universe (Palmer and Voglis [73], Sheth and van de Weygaert [101]).



FIGURE 2.6: Left: Evolution of a tophat void with an initial linearly extrapolated underdensity of  $\delta_L = -10.0$  and initial radius of  $R_0 = 5.0 \ h^{-1}$  Mpc. Subsequent timesteps show that the density profile of such a void evolves into a characteristic "bucket shape". Right: Evolution of a void with the same initial density deficit and radius, only this time with a more realistic density profile (angular averaged SCDM profile). Both profiles clearly show that voids expand, become more empty and form steep density ridges. Figure from van de Weygaert and Bond [112].

Bertschinger [7] showed that once voids have gone through the event of shellcrossing, expansion slows down and they enter a phase of self-similar expansion. Similar to the collapse of a spherical overdensity at  $\delta_{\rm c,L} \approx 1.69$ , one can identify the event of shell-crossing with the formation of a mature void (Blumenthal et al. [12], Sheth and van de Weygaert [101], Dubinski et al. [34]). For an isolated spherical tophat void in an EdS universe, shellcrossing occurs at a linearly extrapolated density value  $\delta_{\rm sc,L} = -2.81$ . At this point the void will have expanded by a factor 1.72 such that the density is only  $1.72^{-3} \approx 20\%$  of the original value, and the corresponding nonlinear density value of the void is  $\delta_{\rm sc} \approx -0.8$ .

#### 2.1.3 Dark energy and void shapes

Recent studies showed that void shapes may be used as a precision probe of the nature of dark energy (Park and Lee [76], Park and Lee [76], Lavaux and Wandelt [60], Biswas et al. [11], Bos et al. [19]).

The shape of a void may be characterized by fitting an ellipsoid to it. The characteristics of an ellipsoid are captured by its ellipticity  $\epsilon$ , oblateness p and prolateness q,

$$\epsilon = 1 - \frac{c}{a} \tag{2.1}$$

$$p = \frac{b}{a} \tag{2.2}$$

$$q = \frac{c}{b} \tag{2.3}$$

In section 2.4 we discuss how the large scale tidal field can influence the evolution of voids and their shapes. For the most extreme cases it may even lead to the complete collapse of a void. The large scale tidal field is in turn intimately related to the nature of dark energy in the universe. So indirectly, void shapes are a reflection of the nature of dark energy.

The tidal tensor  $T_{ij}$ , defined as the traceless component of the second derivative of the gravitational potential,

$$T_{ij} = \frac{\partial^2 \phi}{\partial x_i \partial x_j} - \frac{1}{3} \nabla^2 \phi \delta_{ij}, \qquad (2.4)$$

is related to the sphericity  $s = 1 - \epsilon$  and oblateness p of a void through its eigenvalues  $\lambda_1 > \lambda_2 > \lambda_3$ :

$$\lambda_1(p,s) = \frac{1 + (\delta_v - 2)s^2 + p^2}{p^2 + s^2 + 1}$$
(2.5)

$$\lambda_2(p,s) = \frac{1 + (\delta_v - 2)p^2 + s^2}{p^2 + s^2 + 1}$$
(2.6)

with  $\delta_v = \sum_{i=1}^3 \lambda_i$ . The probability density function f(s) for the sphericity s is given by (Park and Lee [76]):

$$f(s; z, R_L) = \int_s^1 \mathcal{P}[p, s|\delta = \delta_v; \sigma(z, R_L)] dp$$

$$= \int_s^1 dp \frac{3375\sqrt{2}}{\sqrt{10\pi}\sigma^5(z, R_L)} \exp\left(\frac{-5\delta_v^2 + 15\delta_v(\lambda_1 + \lambda_2)}{2\sigma^2(z, R_L)}\right)$$

$$\times \exp\left(-\frac{15(\lambda_1^2 + \lambda_1\lambda_2 + \lambda_2^2)}{2\sigma^2(z, R_L)}\right) (2\lambda_1 + \lambda_2 - \delta_v)$$

$$\times (\lambda_1 - \lambda_2)(\lambda_1 + 2\lambda_2 - \delta_v) \frac{4(\delta_v - 3)^2 ps}{(p^2 + s^2 + 1)^3}$$
(2.7)

In this distribution function,  $\sigma(z, R_L)$  is the linear rms value of the density field filtered on a scale  $R_L$  at redshift z, defined as

$$\sigma^{2}(z, R_{L}) \equiv D^{2}(z) \int_{0}^{\infty} \frac{k^{2} \mathrm{d}k}{2\pi^{2}} P(k) W^{2}(kR_{L}) \mathrm{d}\ln k$$
(2.8)

with D(z) the linear velocity growth factor,  $W(kR_L)$  the top-hat window function and P(k) the linear power spectrum.  $\sigma^2(z, R_L)$  is the key quantity that relates the cosmological parameters and the shape distribution (Lee and Park [61]).

Defining the mean ellipticity as

$$\langle \epsilon \rangle = \int \epsilon f(\epsilon, z) \mathrm{d}\epsilon$$
 (2.9)

equation 2.7 shows us that the mean ellipticity decreases with redshift z. The rate at which the mean ellipticity decreases depends on the cosmological parameters and may be used to descriminate different values of  $\omega_a$  for dark energy models (Lee and Park [61]). Using the voids identified with the WVF for a filter radius of 1.5 Mpc, Bos et al. [19] find both the predicted decrease of  $\epsilon_{\text{mean}}$  with redshift as well as a difference between the dark energy models considered. The differences for the dark energy models are however strongly related to the evolution of  $\sigma_8$ , which is different for the dark energy models considered.

#### 2.2 From isolated voids to the sociology of voids

The analytical prescription of void evolution from the previous section is restricted to isolated voids. However, in reality voids are not isolated entities. As voids expand they will eventually encounter other features of the Megaparsec universe such as filaments, walls and other voids. This gives rise to a void sociology in which the population of voids evolves hierarchically over time as they interact with their large-scale environment. The two main processes dictating the evolution of voids in their environment are the merging of smaller voids into larger voids and the collapse of smaller voids embedded in larger overdensities. But not only is the evolution of voids determined by the direct encounters with the surrounding elements of the cosmic web. The large-scale tidal field resulting from the mass distribution in which they are embedded can potentially have a significant influence on their dynamics and evolution and may be the origin of shape deformations.

Given that a merger between two voids actually implies the gradual disappearance of material in between these voids, one would expect that a large void is characterized by a rich substructure related to its formation process. Rieder et al. [90] have used the very high resolution  $\Lambda$ CDM simulation CosmoGrid to study the formation of dark matter haloes within the tenuous void infrastructure. With this research they aim to gain a better understanding of the relation between the formation of galaxies (in particular void galaxies) and the large-scale structure in which they are embedded. Figure 2.7 shows one of the void systems they identified. Although this dark matter halo lies within a void, the high resolution allows one to see the rich substructure by which it is surrounded.

A better analytical description of voids evolving in a large-scale environment is provided by the excursion set formalism for voids.

#### 2.3 Extending the Excursion Set Approach

The excursion set approach discussed in the previous chapter for dark matter haloes can be extended to voids (Sheth and van de Weygaert [101]). We can characterize the formation of a mature void by the event of shellcrossing at a linearly extrapolated density value of  $\delta_{sc,L} = -2.81$ . In extending the excursion set approach to void formation we may therefore replace the barrier of collapse at  $\delta_{c,L} = 1.69$  by the barrier of shellcrossing at  $\delta_{sc,L} = -2.81$ .

Similar to the cloud-in-cloud problem, when we apply the excursion set formalism to underdense regions we encounter the void-in-void problem: small voids that are actually part of voids on a larger mass scale S(M) should not be taken into account. Such voids



FIGURE 2.7: One of the void systems identified by Rieder et al. Plotted are the projected density of the void system CVG-G and its large-scale environment. First three frames show respectively the XZ plane, YZ plane and XY plane in a slice of  $7 \times 7$   $h^{-1}$  Mpc and 1  $h^{-1}$  Mpc thickness. The last frams shows a  $1 \times 1$   $h^{-1}$  Mpc zoom-in on the void system. Image from Rieder et al. [90].



FIGURE 2.8: Illustration of cloud-in-cloud, cloud-in-void, void-in-void and void-incloud process. Figure from Sheth and van de Weygaert [101].

may have existed at earlier times, but at the time of analysis they have merged into a larger structure and do not exist anymore. Although these voids do pierce the barrier  $\delta_{sc,L}$  for a certain mass scale S(M), they have already pierced this barrier at a higher mass scale S(M'). For this reason, they are part of the higher mass object. These voids should be eliminated from the list of void candidates.

We have implicitly assumed that a collapsing halo can perfectly exist within in an expanding void. For voids, there is an extra complication because voids that are part of a collapsing structure on a higher mass scale S(M) cannot exist. Such voids will eventually be squeezed out of existence and should not be taken into account if one wants to describe the void spectrum. This is referred to as the void-in-cloud problem. Figure 2.8 gives an overview of all the four processes.

According to the model of Sheth and van de Weygaert [101], extending the excursion set approach to voids implies that besides the barrier of shellcrossing at  $\delta_{sc,L} = -2.81$ , we



FIGURE 2.9: Illustration of the two-barrier excursion set approach for voids. For a particular point in space, the red and green trajectories represent the overdensities  $\delta$  of the filtered density field at these locations. For increasing S(M) the trajectories correspond to a random walk if one applies a sharp k-space filter. The only real void in this illustration is V1. Void V2 of the red trajectory has merged with the larger void V1 and for the green trajectory V2 was part of a collapsing halo H1 at a larger mass scale.

have to incorporate a second barrier in the formalism: the barrier of collapse, corresponding to  $\delta_{c,L} = 1.69$ . This second barrier takes into account the void-in-cloud problem. The analytical description of the void mass spectrum  $n_v(M)$  now essentially boils down to the analysis of the fraction of Brownian random walks that undergo shellcrossing at a mass scale S(M) and did neither cross  $\delta_{sc}$  nor  $\delta_c$  at a mass scale S'(M') < S(M) (i.e. taking into account both the void-in-void and the void-in-cloud problem). See figure 2.9 for an illustration of such random walks.

#### 2.3.1 Void spectrum: SvdW analysis

On the basis of the two-barrier excursion set formalism, Sheth and van de Weygaert [101] (SvdW hereafter) derived an analytical approximation for both the mass and radius distribution of the void population. We define  $\mathcal{F}(S, \delta_v, \delta_c)$  as the fraction of random walks that cross  $\delta_v$  at S but have neither  $\delta_v$  nor  $\delta_c$  crossed for any s < S. SvdW have shown that this distribution is given by

$$S\mathcal{F}(S,\delta_{\rm v},\delta_{\rm c}) = \sum_{j=0}^{\infty} \frac{j^2 \pi^2 \mathcal{D}^2}{\delta_v^2 / S} \frac{\sin(j\pi\mathcal{D})}{j\pi} \exp\left(-\frac{j^2 \pi^2 \mathcal{D}^2}{2\delta_v^2 / S}\right)$$
(2.10)

in which

$$\mathcal{D} \equiv \frac{|\delta_{\rm v}|}{\delta_{\rm c} + |\delta_{\rm v}|} \tag{2.11}$$

represents the void-and-cloud parameter that takes care of the impact of halo evolution on the void population. According to SvdW, the sum in equation 2.10 can be approximated by

$$\nu f(\nu) \approx \sqrt{\frac{2\nu}{\pi}} \cdot \exp\left(-\frac{\nu}{2}\right) \cdot \exp\left(-\frac{|\delta_{\rm v}|}{\delta_{\rm c}}\frac{\mathcal{D}^2}{4\nu} - 2\frac{\mathcal{D}^4}{\nu^2}\right) \tag{2.12}$$

where

$$\nu \equiv \frac{\delta_v^2}{S} \equiv \frac{\delta_v^2}{\sigma^2(M)} \tag{2.13}$$

and

$$\nu f(\nu) \frac{\mathrm{d}\nu}{\nu} = S\mathcal{F}(S, \delta_{\mathrm{v}}, \delta_{\mathrm{c}}) \frac{\mathrm{d}S}{S}$$
(2.14)

The void mass spectrum  $n_{\rm v}(M)dM$  is related to  $S\mathcal{F}(S, \delta_{\rm v}, \delta_{\rm c})$  through

$$n_{\rm v}(M) {\rm d}M = \frac{\rho_u}{M^2} \cdot S\mathcal{F}(S, \delta_{\rm v}, \delta_{\rm c}) \cdot \frac{{\rm d}\ln\sigma(M)}{{\rm d}\ln M}$$
(2.15)

Combining this with the approximation in equation 2.12, the mass spectrum can be written as

$$n_{\rm v}(M){\rm d}M \approx \sqrt{\frac{2\nu}{\pi}} \cdot \frac{\rho_u}{M^2} \cdot \exp\left(-\frac{\nu}{2}\right) \cdot \exp\left(-\frac{|\delta_{\rm v}|}{\delta_{\rm c}}\frac{\mathcal{D}^2}{4\nu} - 2\frac{\mathcal{D}^4}{\nu^2}\right) \cdot \left|\frac{{\rm d}\ln\sigma(M)}{{\rm d}\ln M}\right|$$
(2.16)

This double-exponential function is characterized by a cut-off at both small and large values of  $\nu$ , related to small voids being squeezed by larger overdensities and large voids corresponding to high- $\sigma$ -fluctuations in the initial density field. Equation 2.16 gives us some feeling for the importance of the void-and-cloud parameter  $\mathcal{D}$ . If  $\mathcal{D} \to 0$ , the second exponential approaches unity and hence we are left with a distribution that corresponds to a single barrier at  $\delta_{v}$ . As  $\mathcal{D}$  increases (i.e.  $\delta_{v}$  and  $\delta_{c}$  come closer together), the probability of a small void being squeezed by a larger overdensity will also increase.

The simplest way to convert this mass spectrum into a distribution of void sizes is by setting the comoving volume V to

$$V = \frac{M}{\rho_u} \times 1.7^3 \tag{2.17}$$

in which the value of 1.7 corresponds to the expansion factor of the radius of a spherical tophat void at the time of shellcrossing. The distribution of void radii  $n_{\rm v}(r)$  resulting from this model is shown in figure 2.10.

The fraction of mass residing in voids is obtained by integrating equation 2.16 and is given by:

$$\int dS \mathcal{F}(S, \delta_{v}, \delta_{c}) = 1 - \mathcal{D} = \frac{\delta_{c}}{\delta_{c} + |\delta_{v}|}$$
(2.18)

For  $\delta_c = 1.69$  and  $\delta_v = -2.81$  we see that  $\sim 37\%$  of the mass in the universe resides in voids. Equation 2.17 suggest that the volume fraction of voids is  $\sim 1.7^3$  times higher.



FIGURE 2.10: Distribution of void radii on the basis of the two-barrier excursion set approach in an EdS model with  $P(k) \propto k^{-1.5}$ . Top left panel shows fraction of mass in voids with radius r. Bottom left panel shows number density of voids as a function of radius. Top right panel shows cumulative void volume fraction. In these three figures  $\delta_v = -2.81$ , dashed and solid curves correspond to respectively  $\delta_c = 1.06$  and  $\delta_c = 1.69$  whereas the dotted curve shows the overabundance of small voids as a result of completely ignoring the void-in-cloud process. Bottom right panel shows evolution of cumulative volume fraction of voids for  $\delta_c = 1.69(1 + z)$  with z = 1, 0.5, 0 from left to right. Figure from Sheth and van de Weygaert [101].

The fact that this fraction exceeds unity suggests that voids do indeed occupy most of the universe volume, but it also gives us a hint that some of the assumptions made by the excursion set formalism are clearly unrealistic.

#### 2.3.2 Void spectrum: corrections

We have seen that the excursion set approach provides us with an elegant way of giving an analytical prediction for the mass spectrum of both haloes and voids. The reader should realize, that this description is only statistical in nature. This makes it in some sense unrealistic. Two points in the density field that lie close together will probably pierce the barriers  $\delta_c$  or  $\delta_v$  for different S and will therefore be identified as being part of two different objects. In reality however, the likelihood that these two points do actually belong to the same object is significant.

The SvdW excursion set model for voids uses the initial Lagrangian density field  $\delta_{\rm p}(\vec{x})$  to identify the void population at different times by filtering this field with a sharp k-space filter. The subsequent assumption is that the trajectory of linear overdensity versus filter scale one obtains for a particular point in space corresponds to a sequence of uncorrelated steps. However, as argued by Paranjape et al. [75], for a given position in space the overdensity one obtains for a given smoothing scale is in fact correlated to the overdensities on all other smoothing scales.

The excursion set approach for voids takes into account the void-in-cloud problem by introducing a two-barrier-system. The voids which are surrounded by an overdensity on a larger scale that has collapsed by the time of evaluation, are eliminated from the list of possible void candidates. This procedure does not take into account the population of voids that are sufficiently underdense and surrounded by an overdensity that has not completely collapsed. These overdensities may not have completely collapsed but have nevertheless influenced the evolution of these voids by squeezing it partially and hence modifying their radii and shapes. Paranjape et al. [75] propose in their paper to modify the original SvdW approach by considering the mapping from Lagrangian space to Eulerian space. In their model, the fixed barrier  $\delta_v = -2.81$  is replaced by a time dependent barrier  $\delta_v(t)$  that gives the value of the linearly extrapolated density contrast in a Lagrangian region containing mass M which evolves into the Eulerian volume V at time t.

#### 2.3.3 Void spectrum update: Vdn model

The original SvdW model assumes that as voids expand, their number densities are conserved. As a consequence of this assumption, the volume fraction of voids that they obtain exceeds unity. In reality, voids will however encounter other voids and merge to form larger voids. Jennings et al. [52] propose to fix the unphysical behaviour introduced by the assumption of isolated spherical voids by not conserving their number density, but their volume fraction. This model is referred to as the Vdn model.

Jennings et al. [52] also for the first time compared the excursion set model as well as the Vdn model directly with the results of N-body simulations. The results of these simulations are shown in figure 2.11. These turn out to be consistent with the Vdnmodel but not with the original SvdW model. Although the shape of the SvdW model


FIGURE 2.11: Void abundances obtained in simulations by Jennings et al. [52]. The simulations assumed a  $\Lambda$ CDM cosmology and were carried out for boxes of 64, 128, 256 and  $500h^{-1}$  Mpc. All of these boxes containted 256<sup>3</sup> particles, except the last one which containted 400<sup>3</sup> particles. Also shown are the predicted void abundances by the SvdW model and the Vdn model. The predicted range corresponds to the interval  $\delta_c = [1.06 - 1.69]$ . Figure from Jennings et al. [52].

abundance function is correct, it overpredicts the number of voids as a consequence of conserving their number density.

## 2.4 Tidal influences on void evolution

In describing the universe by looking at the void component of the cosmic web, the picture that has emerged so far is one of a universe that is filled with expanding void bubbles that constitute most of its volume. Although the excursion set approach presents an interesting way of deriving analytical expressions for the void spectrum, there seems to be an intricate interplay between the evolution of voids and their surroundings. A more realistic description of structure formation from the void perspective clearly requires a non-local treatment of voids that are embedded in a large-scale tidal field resulting

from the surrounding mass distribution. This external tidal field can potentially have a significant influence on their evolution.

Isolated voids that were initially aspherical are expected to become more spherical as time progresses (Icke [50]). For non-isolated voids embedded in a large-scale tidal field this is not necessarily a valid assumption. Platen et al. [85] find a prolate population of voids which is in agreement with Park and Lee [76] and Shandarin et al. [98] who state that in realistic cosmological circumstances voids tend to be non-spherical. The homogeneous ellipsoidal model (White and Silk [117], Eisenstein and Loeb [38], Bond and Meyer [13]) therefore provides us with a better description of the evolution of underdense regions than the spherical model does. Also the ellipsoidal model is a key element for the voidpatch formalism, which seeks to describe the hierarchical evolution of a voiddominated universe.

Although voids are highly nonlinear entities, their minimum density contrast  $\delta$  cannot exceed -1. This has led to the expectation that the influence of external structures on their evolution is substantially larger than for haloes. Van de Weygaert (manuscript, 1996, private communication) has investigated the evolution of underdense homogeneous ellipsoids under the influence of an artificially imposed external tidal field. In this section, the results of this analysis will shortly be summarized.

As a starting point in the analysis, underdense regions are approximated by the homogeneous ellipsoid model. The equations of motion for such an isolated underdense ellipsoid in an expanding background were already derived by Icke [50], with the conclusion that as voids evolve they tend to become more spherical. It turns out that sphericity for such an ellipsoid is only achieved at late times as the void has expanded by many scale factors. In reality, such a void will most likely have run into other large-scale structures well before sphericity is achieved. Van de Weygaert therefore considered an underdense ellipsoid being acted upon by an external tidal field caused by matter fluctuations outside the void itself:

$$E_{mn} \equiv \frac{\partial^2 \phi_{\text{ext}}}{\partial r_m r_n} - \frac{1}{3} (\nabla^2 \phi_{\text{ext}}) \delta_{mn}$$
(2.19)

According to van de Weygaert, the total gravitational potential  $\phi(\vec{r})$  in an underdense ellipsoid is given by:

$$\phi(\vec{r}) = \frac{2}{3}\pi G\rho_u \sum_m r_m^2 + \frac{1}{2}\sum_{m,n} \phi_{mn}^{\text{ell}} r_m r_n + \frac{1}{2}\sum_{m,n} E_{mn} r_m r_n$$
(2.20)

This expression contains three components,

- (i) Contribution from homogeneous background  $\phi_{\rm b}(\vec{r}) = \frac{2}{3}\pi G\rho_u(r_1^2 + r_2^2 + r_3^2)$
- (ii) Contribution from underdense ellipsoid  $\phi_{\text{ell}} = \frac{1}{2} \sum_{m,n} \phi_{mn}^{\text{ell}} r_m r_n$
- (iii) Contribution from external field  $\phi_{\text{ext}} = \frac{1}{2} \sum_{m,n} E_{mn} r_m r_n$

For the second term, Lyttleton [65] showed that if the principal axes of the ellipsoid coincide with the axes of the coordinate system, the expression for the potential of an ellipsoid with effective density  $(\rho_e - \rho_u)$  becomes

$$\phi_{\rm ell}(\vec{r}) = \pi G(\rho_e - \rho_u) \sum_m \alpha_m r_m^2 \tag{2.21}$$

where the coefficients  $\alpha_m$  dependent on the principal axes  $c_m$  of the ellipsoid

$$\alpha_m = c_1 c_2 c_3 \int_0^\infty (c_m^2 + \lambda)^{-1} \prod_{n=1}^3 \frac{1}{\sqrt{c_n^2 + \lambda}} d\lambda$$
 (2.22)

In other words, this correponds to

$$\phi_{mn}^{\text{ell}} = 2\pi G(\rho_e - \rho_u) \alpha_m \delta_{mn} \tag{2.23}$$

As an immediate consequence of the assumption of a homogeneous ellipsoid, the total potential in equation 2.20 is a quadratic function of coordinates. The acceleration at each location in the ellipsoid is:

$$\frac{d^2 r_m}{dt^2} = -\vec{\nabla}\phi = -\frac{4\pi}{3}G\rho_u r_m(t) - \sum_n \phi_{mn}^{\text{ell}} r_n(t) - \sum_n E_{mn} r_n(t)$$
(2.24)

The linear dependence of the acceleration on  $\vec{r}$  implies that we can relate the location of a certain mass element  $\vec{r}(t)$  to its initial location  $\vec{r}_i(t_i)$  through

$$r_m(t) = \sum_k R_{mk}(t) \cdot r_{ki} \tag{2.25}$$

Initially  $R_{mn}$  is a diagonal matrix such that  $R_{mn}(t_i) = R_m(t_i)\delta_{mn}$ . Equation 2.25 can be inserted into the expression for the acceleration (eq. 2.24) to yield:

$$\frac{\mathrm{d}^2 R_{mk}}{\mathrm{d}t^2} = -\frac{4\pi}{3} G R_{mk} - \sum_n \phi_{mn}^{\mathrm{ell}} R_{nk} - \sum_n E_{mn} R_{nk}$$
(2.26)

Since we are only interested in how the external tidal field can influence the (shape) evolution of voids, we can make the simplifying assumption that the external tidal field tensor is alligned with the mass tensor. This means that  $E_{mn}$  is a diagonal matrix, such that

$$\sum_{n} E_{mn} R_{nk} = E_{mm} R_{mk} \tag{2.27}$$

Combining this assumption with the expression for  $\phi_{mn}^{\text{ell}}$  from equation 2.23, the evolution of the matrix elements  $R_{mk}$  is described by

$$\frac{\mathrm{d}^2 R_{mk}}{\mathrm{d}t^2} = -2\pi G \left[ \alpha_m \rho_e + \left(\frac{2}{3} - \alpha_m\right) \rho_u \right] R_{mk} - E_{mm} R_{mk}$$
(2.28)

Knowing that  $R_{mk}$  is initially a diagonal matrix, equation 2.23 shows us that it will remain so, implying that we can write

$$\frac{\mathrm{d}^2 R_m}{\mathrm{d}t^2} = -2\pi G \left[ \alpha_m \rho_e + \left(\frac{2}{3} - \alpha_m\right) \rho_u \right] R_m - E_{mm} R_m \tag{2.29}$$

The functions  $R_1(t)$ ,  $R_2(t)$  and  $R_3(t)$  that we obtain in this way can be interpreted as the scale factors of the principal axes of our homogeneous ellipsoid:  $c_m(t) = R_m(t) \cdot c_{m,i}$ .

We can rewrite equation 2.29 by:

0	D (3 T 2)	( ( <b>2</b> 0)	0 0	
Case	$E_{11}/(\frac{3}{4}H_0^2)$	$E_{22}/(\frac{3}{4}H_0^2)$	$E_{33}/(\frac{3}{4}H_0^2)$	Collapse
(a)	0.0	0.0	0.0	Ν
(b)	-0.1	-0.1	0.2	Ν
(c)	-0.25	-0.25	0.5	Ν
(d)	-0.5	-0.5	1.0	Ν
(e)	-1.0	-1.0	2.0	Ν
(f)	-1.75	-1.75	3.5	Υ
(g)	-2.0	-2.0	4.0	Υ

TABLE 2.2: Void configurations used for solving equation 2.33. Results are shown in figure 2.12.

• Noticing that mass conservation implies  $\rho_e(t) = \rho_{e,i}/(R_1R_2R_3)$ . Defining the initial density contrast of the ellipsoid as  $\delta_i$ , we have

$$1 + \delta_i = \frac{\rho_{e,i}}{\rho_{u,i}} \tag{2.30}$$

and

$$\frac{\rho_e(t)}{\rho_{u,i}} = \frac{1+\delta_i}{R_1 R_2 R_3}$$
(2.31)

• Introducing the dimensionless time variable  $\tau$ :

$$\tau \equiv t\sqrt{2\pi G\rho_{u,i}} = t\sqrt{\frac{3}{4}\Omega H^2 \left(\frac{a}{a_i}\right)^3}$$
(2.32)

• Assuming that the tidal tensor evolves linearly, such that  $E_{mm}/(\Omega H^2) \propto D(t)$ 

Applying these modification we end up with:

$$\frac{\mathrm{d}^2 R_m}{\mathrm{d}\tau^2} = -\left[\frac{\alpha_m (1+\delta_i)}{R_1 R_2 R_3} + \left(\frac{2}{3} - \alpha_m\right) \left(\frac{a_i}{a}\right)^3 + \frac{4E_{mm,0}}{3\Omega_0 H_0^2} D(t) \left(\frac{a_i}{a}\right)^3\right] R_m \tag{2.33}$$

Van de Weygaert has evaluated this equation for various values of the initial density deficit  $\delta_i$  and diagonal components of the traceless tidal tensor  $E_{mm,0}$ . Each of the calculations was based on an initially spherical void in an EdS  $\Omega = 1$  universe in which  $H_0 = 50$  km s<sup>-1</sup> Mpc<sup>-1</sup>. For the initial velocity associated with the void,  $v_m(t_i) = (dR_m/dt)r_{m,i} = v_{\text{Hubble},m}(t_i) + v_{\text{pec},m}(t_i)$ , the growing mode solution of linear perturbation theory was adopted (Peebles [80]),

$$v_{\text{pec},m}(t_i) = \frac{2f(\Omega_i)}{3H_i\Omega_i}g_{\text{pec},m}(t_i) = -\frac{1}{2}H_if(\Omega_i) \left[\alpha_{m,i}\delta_i + \frac{4E_{mm,0}}{3\Omega_0H_0^2}D_i\right]r_{m,i}$$
(2.34)

with  $\alpha_{m,i}$  the shape factor of the ellipsoid at time  $t_i$ .

Table 2.2 lists the configurations for which equation 2.33 was evaluated. Corresponding results are shown in figure 2.12 and show us that an external tidal field may eventually induce the collapse of a void.



FIGURE 2.12: The evolution of comoving size  $c'_m = c_m/a(t)$  of the axes (topleft), axis ratios  $c_2/c_1$  and  $c_3/c_1$  (topright), the ratio  $\rho_e/\rho_u$  (bottom left) and the velocity  $v_m$  along the axes of the ellipsoid in units of  $v_H$  (bottom right) for the seven initial void configurations summarized in table 2.2. Initially at  $a_i = 0.01$  all underdensities are spherical with  $\delta_i = -0.02$ . All underdensities evolve under the influence of an axisymmtric external tidal field with  $E_{mm,0}/(\frac{3}{4}H_0^2) = (-E, -E, 2E)$ , the amplitude Eincreasing for the different configurations. Legend: (a) solid; (b) dot; (c) short dash; (d) long dash; (e) dot-short dash; (f) dot-long dash; (g) short dash-long dash. Figure from Van de Weygaert.

#### 2.5 Voidpatch and void hierarchy

The excursion set approach provided us with an analytical description for the evolution of voids embedded in their large-scale environment. As mentioned before, the excursion set formalism is a local description in the sense that it fails to take into account the spatial and geometrical setting in which the void is located. This description is therefore only statistical in nature. A non-local description of the hierarchical buildup of voids taking into account the large-scale tidal field in which they are embedded, the main subject being investigated within this research, is addressed by the voidpatch formalism.





The original peak-patch formalism by Bond and Meyer [13], [14], [15] tries to reconstruct the evolution of the cosmic web skeleton on the basis of the information that is stored in the initially density field  $\delta_{\rm p}(\vec{x})$ . The first step of the formalism consists of determining the maxima of the primordial field  $\delta_{\rm p}(\vec{x})$ . These maxima are considered to be the seeds for the clusters (peak patches) that will form later on, and form the nodes of the filamentary cosmic web. Peak patches in the density field are identified by filtering  $\delta_{\rm p}(\vec{x})$  on progressively smaller scales. At all cosmic times the scale of the peak  $R_{\rm pk}$  is the largest scale at which it according to the homogeneous ellipsoidal model collapsed along all three dimensions. Thereafter an exclusion algorithm is applied to make sure that there are no overlapping patches. The subsequent evolution of each of the peak patches is dictated by the combination of a smooth linear background field  $F_{\rm b}(\vec{x})$  that is responsible for the evolution of the cluster as a whole and a nonlinear small-scale field  $F_{\rm f}(\vec{x})$  that is responsible for the substructure within the clusters itself,

$$\delta_{\rm pk}(\vec{x}) = \delta_{\rm b} + \delta_{\rm f} \tag{2.35}$$

Assuming that the dynamics of peak patches as a whole is mainly determined by the linear large-scale background field, one may for example apply the Zel'dovich approximation to displace the patches from their Lagrangian initial positions to their presentday Eulerian positions.

The tidal shear field in between two clusters that are relatively close to each other almost always gives rise to the contraction of matter into anisotropic filaments. This leads to the characteristic cluster-filament-cluster pattern which is the main building block of the cosmic web. In that sense, the locations of the peak patches can indeed be used to reconstruct the outline of the cosmic web on the basis of the initial density field, as was shown by Bond et al. [17]. Figure 2.13 shows an illustration of the three main steps involved with the peak patch formalism.

In analogy to the peak-patch formalism one can identify the minima of the initial density field  $\delta_{\rm p}(\vec{x})$  and follow their subsequent evolution (Platen et al., unpublished). Knowing that most of the universe's volume is nowadays occupied by voids, this should lead to a cellular pattern of voids outlining the structure of the cosmic web in between them.

As the void population evolves, the material between two different voids slowly dilutes such that two voids might end up as one larger void: a void merger. Small voids that are part of a larger overdense structure will disappear as the surrounding structure collapses. A void region can also be squeezed as a result of the expansion of surrounding voids, such that this region eventually ends ups as an overdense structure that marks the boundaries between the voids by which it is compressed. The combination of these processes gives rise to a hierarchy of voids: initially there are only small voids but mergers between voids and the collapse of small voids cause an increasing characteristic size of the void population during the evolution of the universe.

# Chapter 3

# Simulating the Universe

In 1941 Holmberg [48] simulated encounters between stellar systems by using light bulbs. A stellar system was represented by 37 light bulbs, each of which represented a simulation 'particle'. The intensity of light was assumed to be proportional to the gravitational potential. This experiment can be considered as the analog precursor of N-body simulations. The first digital N-body simulation has been carried out by von Hoerner [116].

Numerical simulations have played an important role in establishing the ACDM model as the leading theoretical paradigm for cosmological structure formation. Modern cosmological research and numerical simulations are therefore inseparable. In fact, N-body simulations are the only tool for the cosmologist to study the full range of structure formation, from the initially linear stages to the highly nonlinear stages. Amongst others, N-body simulations have been used to study galaxy clustering (Aarseth et al. [1]) and large-scale structure formation (Efstathiou et al. [36]). Techniques with respect to particle-based computer simulations are reviewed by Hockney and Eastwood [47], while a more modern review of the algorithms being used within N-body simulations is provided by Dehnen and Read [31]. A review of the role of N-body simulations within the context of structure formation is given by Bertschinger [8].

Since structure formation is dictated by the laws of gravity, cosmological simulations that study this process only need to take care of gravitational forces without considering the complex hydrodynamical and radiative behaviour of a baryonic gas. In essence this is rather simple: a particular particle moves under the influence of the mutual gravity of all the other particles, implying that at each timestep there are 6N ordinary differential equations that have to be integrated. However, as the number of particles increases, the number of particle-particle interactions that have to be taken into account grows as  $N^2(+N)$ . Since the number of particles that build up our surrounding universe is almost infinitely large, direct N-body simulations are of cause not feasible. The phase space density in the simulations being considered is therefore sampled by a finite number N of tracer particles. The reader should always realize that these "particles" do not refer to real physical particles and are merely used as an approximate discretization of space. An exception to this discretization is formed by Hahn et al. [44], who follow the evolution of a continuous dark matter manifold in phase space. Describing the universe with computer simulations as well as the need for efficient algorithms require a number of approximations and simplifications with respect to real space. Using cosmological simulations for scientific research therefore inevitably requires a good understanding of the simulation methods being used as well as its limitations.

The work done in this thesis is mostly based on N-body simulations with the cosmological simulation code GADGET-2 (Springel [103]), an improved version of the original simulation code GADGET-1 (Springel et al. [104]). For the subsequent processing of the resulting data to a density grid and eventually to a network of voids we use respectively the Delaunay Triangulation Field Estimator (Schaap and van de Weygaert [96], Cautun and van de Weygaert [24]) and the the Watershed Void Finder (Platen et al. [84]). The basic principles of the GADGET-2 and the Delaunay Triangulation Field Estimator are discussed in the upcoming sections. We finish this section with an overview of the parameters we have used for our simulations

#### **3.1** N-body simulations with GADGET-2

The cosmological simulation code GADGET-2, developed by Springel [103], is a Tree-SPH code that has been optimized for parallelization. Simulations with GADGET-2 may contain both collionless dark matter particles and ideal gas particles. The hydrodynamical behaviour of the ideal gas is followed by means of smoothed particle hydrodynamics (SPH) whereas gravitational forces are normally determined by using a tree method. For cosmological simulations GADGET-2 offers an additional TreePM algorithm, where only short range gravitational forces are determined by the tree method and long range forces by a particle-mesh (PM) method in Fourier space. Since our research is mainly based on cosmological simulations with GADGET-2, we will discuss relevant features of the simulation code hereafter.

Initial conditions for our simulations were generated by using the initial condition generator N-genic for GADGET-2. Before starting the real simulation, the initially Gaussian random field is evolved in time by using the previously discussed Zel'dovich approximation (to a redshift where the ZA is still valid) (Klypin and Shandarin [57]).

#### 3.1.1 Gravitational softening

Looking at Newtons law of gravity

$$F_G = G \frac{m_1 m_2}{r^2}$$
(3.1)

for the gravitational force between two point masses, one can easily see that the gravitational force may become very large as two particles come close to each other. Knowing that the (massive) 'particles' in our simulations do in fact not represent real particles but serve as a tracer of the underlying phase space density, close encounters between such particles may lead to unrealistic behaviour. The mass of a simulation particle is therefore usually spread out over a certain region of space by using a normalized gravitational softening kernel of comoving scale  $\epsilon$  to represent the density distribution  $\tilde{\delta}(\vec{x})$ . Within GADGET-2 the density distribution of particles is modelled by using the spline kernel such that  $\tilde{\delta}(|\vec{x}|) = W(|\vec{x}|, 2.8\epsilon)$  where

$$W(r,h) = \frac{8}{\pi h^3} \begin{cases} 1 - 6\left(\frac{r}{h}\right)^2 + 6\left(\frac{r}{h}\right)^3, & 0 \le \frac{r}{h} \le \frac{1}{2} \\ 2\left(1 - \frac{r}{h}\right)^3, & \frac{1}{2} < \frac{r}{h} \le 1 \\ 0, & \frac{r}{h} > 0 \end{cases}$$
(3.2)

The choice of gravitational softening length  $\epsilon$  should be high enough to prevent unrealistic large forces for close encounters between particles. But it should also be low enough to minimize the influence on the dynamical evolution of the global system. The problem of choosing an optimal softening length is not solved yet and therefore still a subject of discussion.

#### 3.1.2 The TreePM method

Short-range forces in GADGET-2 are determined by using an octagonal tree algorithm (Barnes and Hut [4]). In this method the particle distribution is used to subdivide space into a Barnes-Hut oct-tree. Starting at the complete simulation box (the trunk), space is continuously subdivided into eight daughter nodes with a side length that is half of the original cube until each cube contains at most one particle (the leaves). This procedure is illustrated in figure 3.1.



FIGURE 3.1: Illustration (in two dimensions) of the construction of a BH tree by subdividing space hierarchically into smaller cells until each cell contains only one particle. Figure from Springel et al. [105].

Gravitational forces are determined by "walking" the tree from the trunk to the leaves. Depending on the distance to the particles being considered and the required accuracy, the force between this particle and the center of mass of all particles in a particular branch is used as an approximation to the gravitational force. If this approximation does not comply with the required accuracy the tree is opened further to obtain better accuracy but to the costs of more CPU power. In general this will imply that particles that are close to the one being considered are directly taken into account whereas particles at increasing distances are grouped into ever-larger subcells. Tree methods reduce the number of calculations from  $\sim N^2$  to  $\sim N \log N$  with respect to direct N-body simulations.

Traditional particle-mesh (PM) methods convert the distribution of particles to a grid of densities. Normally this is done by splitting the particle masses, by means of an interpolation scheme, between the grid cells surrounding the particle. From this density grid the corresponding potential field  $\phi(\vec{x})$  may be obtained by using a Fast Fourier Transform to solve the Poission equation in Fourier space. The solution is thereafter transformed back to real space. Using the same interpolation scheme as was used to distribute the mass of a particle between the surrounding cells, the potential field at the position of a certain particle may be determined from the values of the potential field at the surrounding cells. In this way particles do not interact directly but only through the mean potential field  $\phi(\vec{x})$  that can be used to determine the gravitational force on the particle. Although PM codes are fast they have the disadvantage of becoming inaccurate at scales approaching the distance between mesh cells. GADGET-2 therefore uses a treePM method.

Within the treePM method, short range forces are determined by using the tree method discussed previously whereas long range forces are determined by the PM method. Using exponential factors to suppress the long range and short range potential below and above some scale  $r_s$ , the Fourier potential field  $\phi_k$  can be written as

$$\phi_k = \phi_k^{\text{long}} + \phi_k^{\text{short}} \tag{3.3}$$

$$\phi_k^{\text{long}} = \phi_k \exp(-k^2 r_s^2) \tag{3.4}$$

$$\phi_k^{\text{short}} = \phi_k (1 - \exp(-k^2 r_s^2)) \tag{3.5}$$

The exponential factor suppresses the short-range force to  $\sim 1\%$  of its Newtonian value for  $r \approx 4.5r_s$ . To limit computational costs, in practice  $\phi^{\text{short}}$  is not determined for radii larger than a certain pre-defined radius.

Altogether the treePM algorithm is a bit slower than traditional PM method, but using the tree method for short-range forces leads to a significant improvement of the accuracy of the simulation.

### 3.2 From a system of particles to a grid of densities: DTFE

Once a GADGET-2 simulation has finished, the typical result is a number of snapshots containing the positions and velocities of the particles at certain predefined timesteps  $t_i$ . Before we can use the Watershed Transform to identify the voids, we have to reconstruct the density field (i.e. rectangular grid of densities). For this, we use the Delaunay Triangulation Field Estimator (DTFE) by Schaap and van de Weygaert [96] (implementation by Cautun and van de Weygaert [24]).

At the basis of the DTFE lies the assumption that the distribution of simulation particles constitutes an unbiased representation of the underlying density field. Using DTFE for reconstructing the density field has the advantage that it preserves both the multiscale character and the local geometry of the point distribution. Important characteristics of DTFE are that it does not depend on user defined parameters and that it is volume weighted. The latter is important in the sense that it allows for a fair comparison with analytical predictions which are also volume weighted.

The DTFE density estimate is based on the Delaunay triangulation of the particle distribution  $\mathcal{P}$ . In three dimensions this corresponds to the set of tetahedra between points  $p \in \mathcal{P}$ , such that none of their circumscribed spheres contains any of the points p.



FIGURE 3.2: Voronoi (left) and Delaunay tesselation (right) of a two dimensional point distribution. Shaded regions show the Voronoi cell (left) and contiguous Voronoi cell (right) of the red point. Figure from Schaap [95]

From this triangulation one may construct its dual, the Voronoi tesselation, as well as the contiguous Voronoi cells of the points p, which is formed by the union of all Delaunay tetahedra of which the point p is one of the vertices. A visual representation of the three triangulations in two dimensions, is given in figure 3.2.

Within DTFE the density estimate at the locations p of the particle distribution, is taken to be inversely proportional to the normalized volume  $V(\mathcal{W}_p)$  of the contiguous Voronoi cell

$$\hat{\rho}(\vec{x}_p) = (1+D)\frac{m}{V(\mathcal{W}_p)} \tag{3.6}$$

where the particles have equal mass m and there are D particles defining the contiguous Voronoi cell. One can show that the use of contiguous Voronoi cells, ensures mass conservation in the simulation box (Schaap and van de Weygaert [96], Schaap [95], van de Weygaert and Schaap [114]).

Knowing the density estimates at the vertices of a Delaunay tetahedron m, one may find the density at a location  $\vec{x}$  inside such a tetahedron by means of multidimensional linear interpolation

$$\rho(\vec{x}) = \hat{\rho}(\vec{x}_p) + \widehat{\nabla\rho}|_m \cdot (\vec{x} - \vec{x}_p) \tag{3.7}$$

in which  $\nabla \rho|_m$  is the estimated constant density gradient within the tetahedron m and  $\vec{x}_p$  is one of the vertices of the tetahedron. Once we know the Delaunay tetahedron of which a particular gridpoint is part, this procedure allows a complete reconstruction of a density grid within the simulated volume.

In a similar way, DTFE can also be used to reconstruct velocities. By reconstructing velocities in this way, one directly obtains velocity divergences, velocity shear components and velocity vorticities. The interested reader can find more details on the DTFE algorithm and velocity reconstructions in Schaap [95], van de Weygaert and Schaap [114], Bernardeau and van de Weygaert [6] and Romano-Díaz and van de Weygaert [92].

#### **3.3** Simulations parameters

The main simulation on which this study is based consists of  $384^3$  particles sampled on a regular cubic grid of  $768^3$  gridpoints. Unless stated otherwise, the results presented hereafter are based on this simulation. Relevant parameters for this simulation are listed in table 3.1. These parameters are based on the Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) temperature and polarization observations (Jarosik et al. [51]).

Number of particles	$384^{3}$
Boxsize	$200 {\rm ~Mpc}$
Starting redshift	49
Energy density matter $\Omega_{m,0}$	0.272
Energy density dark energy $\Omega_{\Lambda,0}$	0.728
Hubble constant $H$	$70.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$

TABLE 3.1: Simulation parameters for the main simulation of this study.

#### 3.4 Example run: density slices

During the simulation run, we have created 40 snapshots at regular time intervals (from a = 0.025 to a = 1.0). For each of these snapshots we have used the DTFE to obtain estimates for the density on a 768<sup>3</sup> regular grid. Figure 3.3 shows a number of DTFE density slices resulting from our simulation.

Starting with the upper left slice at a = 0.025, we see a density field that is still very close to the homogeneous and isotropic initial density field. The upper right slice already shows the outline of a very tenuous filamentary network that is forming as a consequence of the tiny perturbations that were present in the primordial density field. Within the middle left slice we can see how the first clusters of galaxies with density contrasts  $\delta \sim 10^3$  start to form. These clusters form the nodes of the filaments that define the spine of the cosmic web. As time progresses into the last three slices, more and more matter starts to accumulate into clusters and filaments of galaxies, giving rise to an increasingly sharp contrast to the more and more empty voids that lie in between.

Although underdense regions can clearly be recognized within slices 2 and 3, the density contrast with the other large-scale features is still not too high. This is a sign that the voids in these slices have not had the time to form the steep density profile by which they are characterized. Hence these voids are not mature voids yet. Without quantifying, we can see that within the last three slices the density contrast between voids and their surrounding elements really starts to become sharp. One could therefore expect that it is around this time that the first mature voids start to form. This question is closely related to the exact definition of a void (which is currently not available) and the corresponding void finder algorithm to be used. These issues will be discussed in the next chapter.

Notice in particular the hierarchical evolution of the void population. Small voids are the first to become sufficiently underdense to be considered a mature void. Mergers between



FIGURE 3.3: Slice through our simulation volume for successive values of the cosmological expansion factor a (a = 0.025, 0.20, 0.40, 0.60, 0.80 and 1.0). Density estimates are obtained by DTFE.

# Chapter 4

# The Multiscale Watershed Void Finder

# 4.1 Void finders

One of the important problems for the study of voids is the fact that there is not an exact definition of a void available. This has led to the development of a broad range of void finders, all incorporating their own definition of a void into the corresponding algorithms. As a consequence, comparing results between studies that have used different void finders is not necessarily a straightforward thing to do.

Within the different classes of void finders, we may distinguish void finders that are based on the distribution of galaxies and haloes (Brunino et al. [23], Göttlober et al. [40], Hoyle and Vogeley [49]) and void finders based on the distribution of dark matter. The latter class of void finders is of particular interest for this study, since the simulations that we use produce a distribution of dark matter particles. The intention is to identify voids in this distribution of dark matter particles. Void finders within the dark matter distribution can, amongst others, be based on:

- Identification of spherical regions around local density minima (Brunino et al. [23]).
- Irregularly shaped underdense region formed by merging spherical regions around local density minima that have a mean density lying below a certain threshold (Colberg et al. [26]).
- The tidal field tensor  $T_{ij}$  resulting from an expansion series of the equation of motion of a test particle within the smoothed density field (Hahn et al. [43]). Regions of space where  $T_{ij}$  has no positive eigenvalues are identified with voids.
- Identification of connected underdense grid cells that have a density below a certain threshold or lie within a certain isodensity surface (Plionus and Basilakos [86], Shandarin et al. [98])
- Segmentation of a density grid. Two related void finders in this context that result in quite similar voids are ZOBOV (Neyrinck [71]) and the Watershed Void Finder (Platen et al. [84]). ZOBOV identifies the minima of the density grid resulting



FIGURE 4.1: Applying different void finders to the same underdense region can lead to different void identifications. Figure from Colberg et al. [27].

from the Voronoi Tesselation Field Estimator. Particles are thereafter partitioned into zones around these minima, resulting in minimum zones that are joined by a flooding procedure. The Watershed Void Finder applies the Watershed Transform to a density grid resulting from the DTFE. We use an extended Multiscale version of the original WVF that is discussed extensively in this chapter.

Colberg et al. [27] have compared the identification of voids by a number of void finders. Figure 4.1 clearly illustrates that different void finders may lead to different voids.

Within this study we have used an extended multiscale filtering version of the Watershed Void Finder by Platen et al. [84]. This void finder will be discussed hereafter.

## 4.2 The Multiscale Watershed Void Finder

For the identification of underdense regions in our simulations we have used an extended multiscale filtering version of the Watershed Void Finder (WVF) by Platen et al. [84]. This extended Multiscale Watershed Void Finder (MWVF) allows us to study the hierarchy of voids. After discussing the original WVF and the consequences of the choice of filter radius required by the WVF, we introduce the modifications we have applied to the original version. These modifications consist of

- (i) The introduction of a density criterion for a watershed basin to be identified as a void
- (ii) The use of a multiscale filtering procedure instead of choosing one single filter radius

#### 4.2.1 The original Watershed Void Finder

The WVF uses the Watershed Transform (for more details see appendix A) to construct a segmentation of the density field. The basic idea of a watershed segmentation is best illustrated by the analogy with a geographical landscape containing hills and valleys. Within such a landscape, water will always tend to flow in the direction of a local height minimum. To create a watershed segmentation of such a landscape, one has to identify the regions for which water will flow to the same local basin and partition the landscape accordingly. The Watershed Transform (WST) uses the principle of flooding to create such a segmentation. At each local minimum a water source is placed, such that the water level rises equally throughout the landscape. As the water level rises, more and more of the landscape will become flooded. At some point different basins meet up, the ridges between them defining the boundaries of a segment. Eventually this leads to a complete watershed segmentation of the landscape. This process is illustrated in figure 4.2.



FIGURE 4.2: Illustration of the principle of the WST. Starting from the local minima the landscape of the leftmost frame is continuously flooded by water (dotted plane). As soon as basins meet up a 'dam' is created, eventually leading to a network of 'dams' that forms the complete watershed segmentation of the landscape (right frame). Figure from Platen et al. [84].

The WVF is based on the same principles as discussed for the geographical landscape, only now the landscape is replaced by the cosmological density field. We may identify voids as the basins of the density field, whereas walls and filaments form the ridges that separate them. The rectangular density grid that results from applying the DTFE to the particle distribution of a particular snapshot (section 3.2) serves as the input for the WVF to construct a watershed segmentation of that particular snapshot. Basic steps of the WVF algorithm are summarized hereafter.

- (i) *Filtering*. The DTFE density field is first smoothed by using a Gaussian kernel and discretized into a finite set of density contour levels.
- (ii) *Find local minima*. The local minima of the smoothed density field are those pixels that are solely surrounded by pixels with a higher density value.
- (iii) Flooding and segmentation. Starting at the locations of the identified minima, density contours levels are continuously increased (flooding). Pixels with a density value lower than the flooding threshold are added to the basin of a particular minimum. As soon as a pixel is reached by two different basins, this pixel is identified



FIGURE 4.3: DTFE density slice through the simulation volume at a = 1.0 (*left*) and the corresponding watershed segmentation for a filter radius of 1.0 Mpc (*right*). Also shown is a zoom in for a region containing a number of clusters.

as belonging to the boundary between the corresponding basins (segmentation). See figure 4.2 for an illustration of this step.

(iv) *Hierarchy correction*. Segmentation boundaries that have density values below a certain critical density threshold  $\delta_t$  are removed such that only physically significant voids are preserved.

Except for the filter radius, that the WVF uses for filtering the original DTFE density field, the WVF is free of user defined parameters for the void identification process. Moreover, the WVF does not make any a priori assumptions about the geometry of a void, making it an appropriate tool for the study of void hierarchy and evolution.

Figure 4.3 shows the last slice of the density slices of figure 3.3 and its corresponding watershed segmentation for a filter radius of 1.0 Mpc.

#### 4.2.1.1 Different filter radius, different void population

The WVF is not entirely free of user defined parameters. Before the actual WST is applied, the DTFE density field is filtered with a Gaussian filter that has a user defined filter radius. As the density field is filtered on progressively larger scales, more and more of the internal substructure of voids is smoothed out by the filtering procedure. Figure 4.4 shows us how an increasing filter radius leads to less and less detected substructure. Looking at figure 4.4, one may already conclude that the choice of filter radius has direct consequences for the characteristic population of watershed voids that one will detect.



FIGURE 4.4: Same slice through the watershed segmentation of our simulation volume for filter radii of 1.0, 2.0 and 4.0 Mpc. At the background the corresponding DTFE density field is plotted. For a filter radius of 4.0 Mpc, we can see that there is still a lot of substructure within the voids that is not detected by the WVF at that radius.

We define the volume of a void V as

$$V = \frac{\# \text{ of void gridpoints}}{\text{total } \# \text{ of gridpoints}} \times V_{\text{simulation}} = \frac{\# \text{ of void gridpoints}}{768^3} \times 200^3 \text{ Mpc}^3 \qquad (4.1)$$

with  $V_{\text{simulation}}$  the volume of the simulation box. This enables us to define the (equivalent) radius of a void as  $R = \left(\frac{3}{4\pi}V\right)^{1/3}$ .

Analyzing the average void radius and corresponding standard deviation as a function of cosmological expansion factor a for different filter radii, one can clearly see that the choice of filter radius directly affects the obtained void population. The results of this analysis are shown in figure 4.5.



(a) Number of voids as a function of cosmological expansion factor a for filter radii of 0.75, 1.5 and 3.0 Mpc.



(b) Average void radius of the void population (*thick lines*) and corresponding standard deviation (*shaded regions*) as a function of cosmological expansion factor a for filter radii of 0.75, 1.5 and 3.0 Mpc.

FIGURE 4.5: The choice of filter radius has direct consequences for the population of voids one obtains.

#### 4.2.2 Shell crossing criterion

Applying the WVF to a simulation volume leads to a complete segmentation of this volume into watershed basins. Not all of these watershed basins do necessarily correspond to real voids. This brings us back to the question what a real void actually is. We have seen that in analogy to the virialization of dark matter haloes, the formation of a void is characterized by the epoch of shell crossing at  $\delta_{sc} = -0.8$ . In that sense, the voids that we detect should at least have undergone some evolution before we identify them as a real void. As a measure for the evolutionary stage of a void, we may define the 'emptiness' of a watershed basin as the percentage of its gridpoints that have a density below  $\delta_{sc} = -0.8$ . Figure 4.6 clearly shows that as time progresses, the emptiness of the watershed basins increases.



FIGURE 4.6: As the universe evolves voids grow more empty. From left to right the same slice through the simulation is shown for redshifts z = 1.5, 0.43, 0. The colourcoding of the voids corresponds to the percentage of their gridpoints that have a density  $\delta < -0.8$  (righthand colorbar).

To distinguish the real voids in the collection of watershed basins, we can set a requirement for the emptiness of a watershed basin. The requirement we have used in our analysis is a minimum emptiness of 50%. Although this might seem an arbitrary choice, we end up with a final void volume fraction of  $\sim$ 70% which does not seem to be unreasonable. If we had chosen to adopt a criterion of 40% or 60%, we would have ended up with a final void volume fraction of respectively  $\sim$ 95% and  $\sim$ 30% In the analysis that follows, only the watershed basins that fulfill the 50% emptiness criterion are considered to be voids unless stated otherwise.

#### 4.2.3 Multiscale filtering procedure

We have seen that applying the WVF to our DTFE density field leads to a complete watershed segmentation of the density field. Evidently, the obtained watershed segmentation depends strongly on filter radius with larger radii probing the void population higher in the hierarchy. To probe this hierarchy of voids we chose not to use a single filter radius but rather a set of progressively smaller filter radii. Within this study we use the set of filter radii  $R_{\rm f} = 4.0, 3.0, 2.0, 1.5, 1.0, 0.75, 0.50$ and 0.25 Mpc. This is the range over which we probe the void population. We take  $R_{\rm f} = 4.0$  Mpc as the largest filter radius, because for larger filter radii the number of voids with an emptiness above 50% becomes too small (e.g. for  $R_{\rm f} = 8.0$  Mpc we find only 3 voids). The smallest filter radius of  $R_{\rm f} = 0.25$  Mpc is similar to the intergrid distance. Combining the voids identified for progressively smaller filter radii with the emptiness criterion of 50%, we can summarize our void finding algorithm by the flow diagram depicted in figure 4.7.



FIGURE 4.7: Flow diagram of the Multiscale Watershed Void Finder algorithm.

The procedure of the Multiscale Watershed Void Finder is illustrated by figure 4.8. Considering one particular slice through the simulation volume, we see how voids detected at progressively smaller filter radii are added to the new segmentation. Voids detected for the different filter radii are only added if their emptiness is more than 50% and not more than 20% of their gridpoints are registered as belonging to a void detected at a larger filter radius.



FIGURE 4.8: Illustration of the Multiscale Watershed Void Finder prodedure. Starting with the voids identified at a filter radius of 4 Mpc that have an emptiness above 50%, voids meeting the emptiness criterion at progressively smaller filter radii are added to the void population. Voids identified at smaller filter radii are only included into the population if not more than 20% of their gridpoints have been registered as belonging to voids detected at larger filter radii. Colors of the voids correspond to voids detected for a filter radius of 4 Mpc (*red*), 3 Mpc (*yellow*), 2 Mpc (*green*), 1.5 Mpc (*blue*), 1 Mpc (*area*), 0.75 Mpc (*number*), 0.50 Mpc (*area*), and 0.25 Mpc (*area*).

#### 4.2.4 The resolution of the simulation

In our discussion with respect to the emptiness criterion we made the choice to adopt a criterion of 50%. This is to some extent an arbitrary choice. In order to have a multiscale filtering procedure, we have to choose which watershed basins we want to take into account and which basins we do not want to take into account. If we would not adopt such a criterion, already the basins identified for the largest filter radius would lead to a complete segmentation of the density field. We have chosen to adopt an emptiness criterion. The value of this criterion we want to choose in such a way that the void population of the final snapshot occupies a reasonable fraction of the simulation volume. We have analysed the volume occupation by the obtained void populations for emptiness criteria of 40%, 50% and 60%. The evolution of volume occupation for these criteria is plotted in figure 4.9. It turns out that at a = 1.0, for an emptiness criterion of 50%, 68.3% of the simulation volume is occupied by the voids we identify. If we had used an emptiness criterion of 40% or 60%, this number would have been respectively 94.8% or 30.1%. Although we realize the choice is arbitrarily, a final volume occupation of  $\sim 70\%$  does not seem to be unreasonable.



FIGURE 4.9: Percentage of the simulation volume occupied by voids as a function of cosmological expansion factor a. Void volume fractions are shown for an adopted emptiness criterion of 40, 50 and 60%.

It turns out that the the emptinesses of the voids that we identify depends strongly on the resolution of the simulation that we consider. In chapter 8 we use simulations that only contain  $256^3$  particles. If we still adopt an emptiness criterion of 50% for these simulations, the population of voids that we obtain for the ACDM scenario occupies only  $\sim 4.5\%$  of the simulation volume. Although we do not exactly know what the volume occupation of our voids should be, this number seems to be unreasonable. Moreover, the number of voids we would get by holding on to the same criterion is too small to obtain any significant results. In order to investigate the volume occupation by the voids we identify in the low resolution simulations, we identify the voids for emptiness criteria of 20, 30, 40 and 50% in one of the low resolution  $\Lambda$ CDM simulations we use in chapter 8. The corresponding results are shown in figure 4.10. From these results we conclude that adopting an emptiness criterion of 20% results in a final volume occupation of  $\sim 70\%$ . This is similar to the volume occupation we obtain for the high resolution simulation.



FIGURE 4.10: The evolution of volume occupation, for emptiness criteria of 20, 30, 40 and 50%, for the voids we identify in one of the low resolution (256<sup>3</sup> particles)  $\Lambda$ CDM simulations that we use in chapter 8. For an emptiness criterion of 20%, we obtain a final volume occupation of ~70%, which is similar to what we obtain for the high resolution simulation.

The lower the resolution of a simulation, the more massive the 'particles' in the simulation and the less particles available for void identification. From this, we expect that in a lower resolution simulation, less substructure is resolved. This leads to the identification of larger voids in the simulation volume. These voids will in general have a lower emptiness than the voids identified in a high resolution simulation and this may to some extent explain the dependence of the void emptinesses on the resolution of the simulation. Compare this with the results of Sutter et al. [106], who find different void population in their public void catalog depending on the galaxy sample they use to identify the voids.

# Chapter 5

# Results: void abundances, sizes, shapes and velocities

In this chapter we will discuss basic characteristics of the void population we obtain by applying the Multiscale Watershed Void Finder to our simulation. Starting with the evolution of void abundance we will subsequently discuss the void size and void shape distributions.

# 5.1 Evolution of void abundance

Figure 5.1 shows the number of voids that we find as a function of cosmological expansion factor. Compare this with figure 4.5, where we can clearly see that for all filter radii the number of voids increases until almost up to a = 1.0. However, if one chooses to apply a multiscale filtering procedure, the number of voids reaches a maximum at  $a \sim 0.8$ . Before this maximum the number of voids increases because more and more of the watershed basins pass the critical emptiness threshold of 50%. Even though this process continues after  $a \sim 0.8$ , we observe that the total number of voids decreases again. There are four (not strictly independent) processes that can be held responsible for this decreasing number of voids:

- More voids at a larger filter radius are now below the critical emptiness threshold of 50%. As soon as such a large scale void is below this critical threshold, this may imply that several small scale voids are now part of the substructure of the large scale void and therefore not taken into account anymore.
- Some of the ridges separating two voids gradually disappear. At some point these voids can merge to form one larger void. This implies the original ridges to have become part of the rich substructure that characterizes the voids.
- Two expanding (larger) voids can squeeze a smaller void, that lies in between, out of existence.



FIGURE 5.1: The number of voids detected as a function of cosmological expansion factor a. The number of voids shows a peak around a = 0.8.

• A small void may become part of an overdensity collapsing at a larger mass scale (void-in-cloud).

### 5.2 Void sizes

Figure 5.2 shows the average void radius as a function of cosmological expansion factor a as well as the standard deviation of the distribution. As the universe grows older, the average radius increases. There are a number of processes responsible for this increase. First of all, more large scale voids reach the critical emptiness threshold of 50%. This process is related to smaller voids merging to form larger voids. Small scale voids may also disappear because they have been squeezed by larger voids or by a collapsing overdensity. Furthermore, we observe that the width of the radius distribution, characterized by the standard deviation, grows as time progresses.

Comparing the obtained distribution of void radii with the distribution obtained by Sutter et al. [106] (see figure 2.2), we observe that voids obtained in our simulation are much smaller than the ones that have been determined from their observational catalog. We suspect that one of the most important explanations for this difference is the (different) kind of tracer particles that have been used. The galaxies that lie at the basis of their void identification process are much sparser sampled than the dark



FIGURE 5.2: The average radius of the voids in our population as a function of cosmological expansion factor a (thick line) and the corresponding standard deviation (shaded region) obtained by assuming a Gaussian distribution. For the expansion factors a = 0.50, 0.75 and 1.00 we show the real distribution of void sizes together with the Gaussian fit. As time progresses, the distribution becomes more skewed towards the smaller voids.

matter 'particles' in our simulation. This probably implies that we detect a lot more substructure, effectively leading to a population of smaller voids. As one may conclude from figure 2.2, the choice of the galaxy sample used to trace the underlying dark matter distribution already has a strong influence on the obtained void population. Compare this with our discussion in section 4.2.4, where we conclude that the population of voids we obtain depends on the resolution of the simulation.

### 5.3 Void shapes

Characterizing the shapes of the voids that we find in our simulations can be done by fitting an ellipsoid to the volume elements that belong to a particular void. In order to do so, we can define the shape tensor  $S_{ij}$  as

$$S_{ij} = \Sigma_k (\delta_{ij} \vec{x}_k - x_{ki} x_{kj}) \tag{5.1}$$

where the sum k is over all N volume elements of the void and  $\vec{x}_k = \vec{r}_k - \vec{r}_c$  represents the position of element k with repect to the void centre  $\vec{r}_c$ . The shape tensor is similar to the moment of inertia tensor  $I_{ij}$ , but is weighted by volume instead of by mass and therefore represents a more natural reflection of the void shape because the mass in voids tends to concentrate towards the boundaries.

Defining  $s_1 \ge s_2 \ge s_3$  as the eigenvalues of the shape tensor  $S_{ij}$ , the semi-axes of the ellipsoid may be inferred from an ellipsoid with semi-axes  $a \ge b \ge c$ ,

$$a^{2} = \frac{5}{2N} \{s_{1} + s_{2} - s_{3}\}$$

$$b^{2} = \frac{5}{2N} \{s_{1} + s_{3} - s_{2}\}$$

$$c^{2} = \frac{5}{2N} \{s_{2} + s_{3} - s_{1}\}$$
(5.2)

Hence, we can characterize the shape of a void by determining its shape tensor  $S_{ij}$  as well as the corresponding eigenvalues  $s_1, s_2, s_3$ , and subsequently fit an ellipsoid to the void that fulfills the relations of equation 5.2. The ellipsoid characteristics are captured by the ellipticity  $\epsilon$ , the oblateness p and the prolateness q,

$$\epsilon = 1 - \frac{c}{a} \tag{5.3}$$

$$p = \frac{b}{a} \tag{5.4}$$

$$q = \frac{c}{b} \tag{5.5}$$

The volume of an ellipsoid is given by  $V = \frac{4\pi}{3}abc$ . The average ellipticity, oblateness and prolateness as a function of cosmological expansion factor *a* found in our simulation are shown in figure 5.3. Also shown in this figure are the median ellipticity, oblateness and prolateness as a function of the void radius for the final snapshot of the simulation. In addition we have also plotted a two-dimensional scatter diagram of  $\frac{b}{a}$  versus  $\frac{c}{a}$  for the voids detected in the final snapshot of the simulation which is shown in figure 5.4.



FIGURE 5.3: Average ellipticity  $\epsilon$  (a), oblateness p (b) and prolateness q (c) of the void distribution as a function of cosmological expansion factor (*thick lines*) together with the corresponding standard deviation of the distribution (*shaded regions*). The distributions appears to be approximately constant over time. Also shown are the binned distributions of radius versus median ellipticity (d), oblateness (e) and prolateness (f) for the final snapshot of the simulation at a = 1.0. Although the effect is small, the larger voids seem to be slightly more roundish than the smaller ones.



FIGURE 5.4: Two-dimensional plot of the ratios  $\frac{b}{a}$  and  $\frac{c}{a}$  for the semiaxes a > b > c of the ellipsoids that have been fitted to the voids detected in the final snapshot of our simulation. The void population looks quite spherical.

From figure 5.3 we observe no evolution of the average void shape with cosmological time. Within this analysis we have assumed that the shape of a void can be characterized by fitting an ellipsoid to it. This is only a crude approximation. Visual inspection of a number of voids learns us that they can have rather irregular shapes (an example of which is shown in figure 5.5). Supplementary to describing the shape of a void by fitting an ellipsoid, we have therefore also quantified the void shape by defining the dimensionless form factor F (see e.g. van de Weygaert [109]) as

$$F = 36\pi \frac{V^2}{A^3}$$
(5.6)

The form factor describes the relation between the volume of a void V and its surface area A. It is defined such that F = 1.0 for a sphere.

To determine the form factor F for the watershed voids in our simulation, we use for V the number of grid cells that correspond to a void. For each of the void's (cubic) grid cells we determine the number of sides that are adjacent to a cell that is not part of the void. The sum of all these sides, we take to be the area of the void. The average form factors for the voids in our simulation, as a function of cosmological expansion factor, are shown in figure 5.6. Also shown in this figure is the average form factor as a function of the void radius. No significant trend is observed with respect to the average form factor as a function of cosmic time. The larger voids seem to have a slightly smaller form factor



FIGURE 5.5: Example of a void with a number of irregularities on its surface.

than the smaller voids, implying that they have a relatively larger area with respect to their volume than the smaller voids. This could imply these voids are more elongated, which would contradict our previous finding that larger voids tend to be slightly more roundish than the smaller voids. We therefore suspect that the slight decrease in form factor with void radius has to do with irregularities on the surfaces of larger voids. These irregularities could lead to a relatively large increase of the area of a void and hence to a smaller form factor. An example of a void with such irregularities is shown in figure 5.5.

### 5.4 Outflow: the velocity divergence

The rate at which material flows away or into a particular region of space may be characterized by the divergence of the velocity field. We define  $\theta$  as the divergence of the velocity field, normalized with respect to the Hubble constant  $H_0$ ,

$$\theta \equiv \frac{\nabla \cdot \vec{v}}{H_0} = \frac{1}{H_0} \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$
(5.7)

The divergence of the velocity field is related to the density contrast  $\delta$  through the continuity equation (Peebles [80]). In the regime of linear perturbation theory this relation is given by

$$\theta = \frac{\nabla \cdot \vec{v}(\vec{x}, t)}{H} = -f(\Omega_m)a(t)\delta(\vec{x}, t)$$
(5.8)



FIGURE 5.6: Average form factor F(a) of the void distribution as a function of cosmological expansion factor (*thick line*) together with the corresponding standard deviation of the distribution (*shaded region*). This distributions appears to be approximately constant over time. Also shown are the binned distributions of radius versus average form factor (b) for the final snapshot of the simulation at a = 1.0. Although the effect is small, the larger voids seem to be have a slightly larger area with respect to their volume than the small voids have.

Since the minimum value of the density contrast  $\delta$  is -1, equation 5.8 defines an upperbound for the maximum value  $\theta_{\text{max}}$  that the velocity divergence can obtain. This upper bound corresponds to the maximum expansion rate of voids. Bernardeau [5] derived a second-order approximation to equation 5.8 for the nonlinear regime, which is given by

$$\theta = 1.5 f(\Omega_m) \{ 1 - [1 + \delta(\vec{x})]^{2/3} \}$$
(5.9)

Using  $\delta = -1$  as the minimum density contrast and recalling that  $f(\Omega_m) \approx \Omega_m^{0.55}$ , we obtain

$$\theta_{\max} = 1.5\Omega_m^{0.55} \tag{5.10}$$

Inserting  $\Omega_m = 0.272$ , which we used in our simulation, into equation 5.10, we expect to find a cutoff at  $\theta_{\text{max}} \approx 0.73$  in the probability distribution of  $\theta$ .

For all gridpoints in the final snapshot of our simulation we determine the velocity divergence, expressed via  $\theta$ , by using the DTFE. The corresponding probability densityu function of  $\theta$  that we find, is shown in figure 5.7. In this figure we can clearly distinguish the cutoff at  $\theta_{\text{max}} \approx 0.73$ . Notice that figure 5.7 is in fact a reflection of the density distribution  $\delta$ , which illustrates the intimate connection between the two quantities.



FIGURE 5.7: Probability distribution of the velocity divergence  $\theta$  for all void gridpoints in the last timestep of our simulation. One can clearly recognize the cutoff in this distribution around  $\theta \sim 0.7$ .

Knowing the velocity divergence for all gridpoints, we can for each void determine its average divergence. The probability density function of average void divergences is shown in figure 5.8. This distribution appears to be a sharper, more narrow version of the distribution shown in figure 5.7. Remember that a void is in general characterized by a velocity profile that is highest at the centre and decreases outwards. If we take the average of such a distribution, the values that one finds are biased towards the centre of the original profile and the distribution does as a matter of fact become more narrow.



FIGURE 5.8: Probability distribution of the average velocity divergences  $\theta$  of the voids we identify at a = 1.0.

For large voids we expect the velocity divergence to be positive. In other words, material will flow out of the larger voids. The reason for this is that large voids dominate their surroundings and the effective gravitational push of these objects ensures a positive (average) velocity divergence. Moreover, the probability that such a rare object is part of a collapsing region on an even larger mass scale is negligibly small. In figure 5.9 we plot the average divergence  $\theta$  of a void against its radius R. On the basis of this figure we conclude that the largest voids are indeed characterized by a positive velocity divergence, and hence outflow of material.



FIGURE 5.9: The average divergence of a void  $\theta$  plotted against its equivalent radius R. Red and green dots represent respectively a positive and negative velocity divergence. The largest voids have a positive velocity divergence.
# Chapter 6

## Void persistence and merger trees

As the universe evolves, small voids merge and form larger and larger voids. Small voids may also be squeezed in between other voids or a larger overdensity. Together these processes give rise to an evolving population of voids. Since voids form the main building blocks of the cosmic web, this also means an evolving cosmic web.

Within this chapter we will attempt to make a connection between the voids that are detected at different cosmic epochs. When we compare two subsequent timesteps, some of the void boundaries may have become so diluted that they are not detected anymore by the Watershed Void Finder. In our simulations this would represent a void merger (remember that in the last step of the watershed void finding algorithm a hierarchy correction is applied in which all segmentation boundaries with  $\delta_t < -0.8$  are removed).

By using the MWVF, it may be that in going from one to the other timestep a watershed basin at a larger filter radius becomes sufficiently underdense to be identified as a void. In that case several small scale features, that previously led to the identification of a number of subvoids in this region, now have been smoothed out of the density field. The disappearance of these small scale features may be a consequence of a void merger. But it may also be the consequence of a small void being squeezed by other voids or a surrounding overdensity. Although there are actually two different processes responsible for the evolving hierarchy of voids, it is impossible to distinguish these processes by only comparing the voids identified at the different timesteps.

As we try to relate voids identified at different cosmic epochs, we are actually trying to describe the more general concept of void persistence. Within this chapter we visualize the concept of void persistence by using merger trees and persistence diagrams. Thereafter we discuss a number of statistics that we obtain with respect to void persistence. The more general concept of persistence homology and persistence diagrams is described in appendix B. At the end of this chapter, we discuss the additional information that would be required to allow one to disentangle the different processes responsible for the evolving void hierarchy. We use velocity divergences to make an estimate of the relative importance of both processes.

#### 6.1 Merger trees

The concept of void persistence can be visualized by constructing merger trees. For dark matter haloes, merger trees are more common and often used to reconstruct their mass assembly history (e.g. Kauffmann and White [53], Somerville and Kolatt [102], Neistein and Dekel [69], Parkinson et al. [77]). A merger tree for voids allows one to visualize how several small scale voids continuously merge and eventually form one large void.

The starting point of the merger trees that we construct is a void detected at a = 1.0. We start by identifying all the gridpoints that belong to this void. From that point we move to the next snapshot of watershed voids (at an earlier time a = 0.95). We identify for each of the gridpoints that belong to our original void, the voids that are found at these same gridpoints in the new watershed file at a = 0.95. For each of the voids that is found in the same region as the original parent void, we determine the percentage of gridpoints that belong to the parent void. If this percentage is more than 50%, the void is considered to be a child of the parent void. With respect to the persistence of these children as the one that survives (i.e. the one that persists). The other children disappear in proceeding to the next time step. From this point, all the children of the parent void become parents themselves and the process is repeated for the "new" parent voids to create the next layer of the merger tree. The algorithm for constructing a merger tree is illustrated by the flow chart in figure 6.1

The merger trees we construct are based on the voids identified with the Multiscale Watershed Void Finder. Figure 6.2 shows the merger of two different voids in our simulation. The first one corresponds to void 103, the largest void of our simulation with an equivalent radius of 26.1 Mpc. The second tree corresponds to void 140, an average void with an equivalent radius of 11.2 Mpc.

We show the merger tree of void 21 in figure 6.3. The equivalent radius of this void is 16.4 Mpc. In order to gain a better understanding of how this merger tree represents the evolutionary path of the void, we show in figure 6.4 a three dimensional representation of the different voids that build up the parent void for a number of time steps. All of the merger trees are supplied with both a redshift and time axis, the latter being based on a  $\Lambda$ CDM cosmology. Within a merger tree, solid lines represent the voids that survive (persist) to the next time step. Dotted lines correspond to voids that disappear in going from one to the other time step.



FIGURE 6.1: Flow chart illustrating the algorithm for creating a merger tree.



(a) Mergertree of void 103



(b) Mergertree of void 140

FIGURE 6.2: Mergertrees of void 103, the largest void of our simulation with an equivalent radius of 26.1 Mpc, and void 140, an average void with equivalent radius 11.2 Mpc.



FIGURE 6.3: Mergertree of void 21 with an equivalent radius of 16.4 Mpc. Figure 6.4 shows for a number of layers the three dimensional representation of the child voids that build up the parent void.



FIGURE 6.4: Visualization of the three dimensional evolution of void 21 for a number of time steps. In order of appearance in the merger tree, purple, blue, black, red, orange, brown, yellow, gray, green and white colors correspond to the branches splitting of at the fourth layer.

#### 6.2 The persistence diagram

A merger tree is a very good tool to visualize the intricate substructure that eventually forms the parent void. One merger tree however only contains information with respect to one parent void. Although the information contained in a persistence diagram is not as rich as in a merger tree, it does allow us to visualize information from all the voids in the simulation.

Within our discussion of merger trees in the previous section we have already seen that if several voids merge to form one parent void, we identify the largest of these child voids as the one that survives. The other voids disappear when proceeding to the next time step. In that sense, we may identify for each of the voids in the simulation a 'life line<sup>1</sup>' that connects voids persisting through different cosmic epochs (notice that this 'life line' can also be just one point for a void that has neither a parent nor children). Information concerning the 'life lines' of all the voids can be stored in a persistence diagram. The persistence diagram, and the more general concept of persistence homology, has its origin in mathematical topology (Edelsbrunner and Harer [35]). The principle of persistence is explained in more detail in appendix B. Within this chapter we apply these ideas to the persistence of voids.

To construct a persistence diagram we start with the first (earliest) watershed file for which voids are identified. For these voids we register the cosmological expansion factor of that snapshot as their time of birth  $a_{\text{birth}}$ . Thereafter we determine for each of these voids if they have a parent in the subsequent watershed file. This is considered to be the case if more than 50% of the gridpoints of a void lie within the region of one and the same parent void. If one of the voids turns out not to have a parent, it will not 'persist' to the next time step. In that case we register its time of death  $a_{\text{death}}$  as the time of the corresponding snapshot file. All voids for which we registered both  $a_{\text{birth}}$  and  $a_{\text{death}}$  are represented by a point in the persistence diagram. In the persistence diagram we plot  $a_{\text{birth}}$  versus  $a_{\text{death}}$ . Notice that the concepts of birth and death do in this case truly correspond to the appearance and disappearance of voids in the cosmic web and are not only a mathematical concept.

From this point we proceed to the next snapshot. The time of birth  $a_{\text{birth}}$  for the parent voids is set equal to that of its child, if it has one. Otherwise the time of birth of the parent void is set equal to the time of the corresponding snapshot (i.e. the time it is detected for the first time). In this respect it is good to remember that if a parent void has more than one child, the largest of these children survives. The time of birth will in that case be equal to the time of birth of the largest child void.

The persistence diagram resulting from applying the above procedure to the subsequent snapshots of our simulation is plotted in figure 6.5. Within this persistence diagram we have plotted the logarithm of the number density of registered points. As a consequence of the limited amount of snapshots available,  $a_{\text{birth}}$  and  $a_{\text{death}}$  can only have certain discrete values corresponding to those of the snapshots (although in reality they could have all values in between). It is therefore preferable to plot the (interpolated) density of points instead of plotting the points directly.

<sup>&</sup>lt;sup>1</sup>These life lines correspond to the solid lines in a merger tree.



FIGURE 6.5: Persistence diagram

#### 6.3 Void persistence: statistics

Supplementary to the merger trees and the persistence diagram, we have derived a number of additional statistics that we can use to quantify the concept of void persistence. First of all, for each of the available snapshots in which voids were identified (starting at a = 0.3), we determine the average number of subvoids that the voids in this snapshot have with respect to the voids in the previous snapshot at  $a = a_{\text{original}} - 0.05$ . In other words, for each void we determine which gridpoints belong to this void. Then we proceed to the previous timestep, and identify the voids that are found at these gridpoints. The number of voids that we identify in this way and belong for 50% or more to the original void, is what we define as the number of subvoids. Notice that in this way voids can have zero subvoids. The higher the average number of subvoids, the more subvoids disappear in going from one to the other timestep. In that sense, the average number of subvoids may be used a measure for the evolution rate of voids.

The average number of subvoids obtained in this way is plotted as a function of cosmological expansion factor a in figure 6.6. From this figure, the evolution rate of voids, appears to increase until  $a \sim 0.95$  at which it has a maximum and starts to decrease.



FIGURE 6.6: Average number of subvoids as a function of cosmological expansion factor (solid line) together with the corresponding stander error of the distribution (shaded region). The number of subvoids at cosmological expansion factor a is determined with respect to the voids identified at a - 0.05.

Considering all the voids that have been detected in our simulation (integrated over the full range of expansion factors), we have also determined the (binned) average number of subvoids with respect to the previous snapshot (at  $a = a_{\text{original}} - 0.05$ ) as a function of the equivalent radius of the void. The same statistic has been derived by considering only the voids at z = 0 and determining the subvoids with respect to the snapshot at z = 1.0. Resulting diagrams are shown in respectively figure 6.7 and figure 6.8. From these figures we may clearly observe a general trend that a larger void contains more



FIGURE 6.7: Average number of subvoids (binned) integrated over the full range of expansion factors as a function of the equivalent void radius. If a void is identified at cosmological expansion factor a the number of subvoids is determined at a - 0.05. Also plotted are the standard errors of the data points in the distribution.

subvoids. Since small voids continuously merge to form larger and larger voids, this result is in agreement with our expectations.

Let us make the assumption that the population of subvoids has an average radius of  $\overline{R}_{sub}$ . Knowing that the volume of a void is related to its radius through  $V_{void} \sim R_{void}^3$ , we can approximate the number of expected subvoids for a void with radius  $R_{void}$  by

$$\#_{\rm subvoids} \sim \left(\frac{R_{\rm voids}}{\overline{R}_{\rm sub}}\right)^3 \sim R_{\rm void}^3 \tag{6.1}$$

From this relation we do indeed expect that a larger void is characterized by more subvoids. If we compare two subsequent timesteps, there is a significant probability that a large void is dominated by large subvoids. But if we compare two timesteps that are separarated by a larger timespan, the original population of small subvoids has the time to fully condense into the large voids that we observe nowadays. Therefore we expect equation 6.1 only to hold if we compare timesteps with a large timespan in between. We test our assumption, expressed via equation 6.1, by fitting a least squares third order polynomial to the datapoints in figure 6.8. On the basis of this fit we conclude that equation 6.1 is a reasonable approximation, if the time span between the snapshots to be compared is sufficiently high.



FIGURE 6.8: Average number of subvoids (binned) of the voids identified at z = 0 with respect to the voids identified at z = 1 as a function of the equivalent radius of the void. Standard errors of the data points in this distribution are also plotted. In addition we show is the fit  $\#_{\text{subvoids}} = 0.0012996 \cdot R_{\text{voids}}^3$  (blue). This fit shows that equation 6.1 is a reasonable approximation for the expected number of subvoids, as long as the time in between the snapshots that are compared is sufficiently high for the subvoids to condense into the parent voids.

#### 6.4 Disentangling mergers from squeezed voids: velocities and Lagrangian space

In establishing the existence of a void hierarchy we encounter situations like the one illustrated in figure 6.9. Within this figure we see that in going from one to the other timestep, the central red void disappears. But what has happened to this void? On the one hand it may have merged with the green void because the segmenting filament has become too diluted to be detected. On the other hand it may also have been squeezed by the surrounding voids. On the basis of figure 6.9 we are unable to tell what happened. To disentangle these processes, additional information is required.

One possible method to distinguish mergers from squeezed voids is looking at the velocity profile inside the void. This is illustrated by figures 6.10b and 6.10c. For a merger, the direction of the velocity profile will in general be away from the centre of the void due the expansion. A void that is being squeezed, is expected to have a velocity profile that is in general pointing towards the centre of the void. Notice however, that it is still possible that a void expands along one of its axes and yet gets squeezed along another axis.

Another approach may be found in looking at voids in Lagrangian space. Within this method, we determine as soon as a void is born which simulation particles lie within the void. We label these particles accordingly. At the moment the void disappears, we identify the locations of these same particles. If the void has been squeezed by other large-scale features, the particles will be found in a relatively small region of space. If on the other hand the void has merged with another void, the particles will be distributed in a way that still characterizes the original void infrastructure. The interior region of a void may be characterized by a Delaunay grid of the particles that we find inside the original void. If we create this Delaunay grid for the same particles at the moment a void disappears, this allows us to for example compare the original and final area/volume of the Delaunay triangulation of these particles. If this is approximately the same, we would have a merger whereas if the final Delaunay surface/volume is significantly smaller we would have a void that has been squeezed. This approach is illustrated by figures 6.10 a, d, e.

A third method to distinguish mergers from voids that are crushed, is provided by considering velocity divergences. From figure 6.10 we may conclude that for a merger, material will in general tend to flow out of the void. For a void that is being crushed on the other hand, material flows into to the void. The outflow of matter into or out of a void, may be characterized by the velocity divergence as discussed in section 5.4. Notice that this method is restricted to voids that are squeezed in volume but discards the fact that a void can still expand and get squeezed (e.g. a void can get squeezed along one axis but still expand along the other axes). To investigate the importance of these processes, one has to consider the velocity field more carefully by for example determining the shear of the velocity field. This is something we leave for future research to be done.

The voids that we find in the persistence diagram, represent those voids that disappear as a consequence of the evolutionary processes discussed in this section. For each of these voids we determine the average velocity divergence, characterized by  $\theta$ . The resulting distribution of  $\theta$  is shown in figure 6.11. Assuming that a positive divergence corresponds to a merger, and a negative divergence to a squeezed void, we conclude



FIGURE 6.9: In going from one to the other timestep in the above figure, the red void disappears and becomes part of the green void. By only comparing the Eulerian shape of the voids we cannot determine if the red void has disappeared because its internal substructure slowly diluted and it has merged with the green void or because it has been squeezed by the surrounding voids.



FIGURE 6.10: Two processes are responsible for the evolving hierarchy of voids: mergers between voids and crushing of small voids. Both processes have a characteristic velocity pattern, illustrated by b and c. Comparing the original distribution of the particles in a void (a), with the final distribution of these particles, the original void infrastructure is still visible for a merger (d) whereas the particles are found in a small region for a crushed void (e).

that 58% of the voids disappear because of mergers and 42% because they collapse as a consequence of surrounding overdensities or other voids. Because large voids will in general dominate their surroundings, we do not expect large voids to disappear as a consequence of collapse. Figure 6.12 shows the velocity divergences of the disappearing voids plotted against their radii, confirming our expectation that if large voids disappear, this is probably due to a void merger. Hence large voids that disappear should be characterized by a positive velocity divergence, which is what we observe in figure 6.12.



FIGURE 6.11: Distribution of average velocity divergences  $\theta$  for the voids disappearing as a consequence of hieararchical void evolution.



FIGURE 6.12: The average velocity divergence of the voids that disappear as a consequence of hierarchical void evolution plotted against their equivalent radius. Red and green dots represent respectively a positive and negative velocity divergence.

#### | Chapter

## Void hierarchy and the adhesion model

In the previous chapter we demonstrated the hierarchical evolution of the void population in our N-body simulation. One of the best models to describe hierarchical structure formation is the adhesion model. The adhesion model, discussed in section 1.3.2, is an improved version of the Zel'dovich approximation that uses an artificial viscosity  $\nu$  to take into account the selfgravity of the developing structures. The adhesion term models the hierarchical evolution of the developing structure quite well, without having to take care of the highly nonlinear structure formation on the smaller scales.

In this chapter we use the adhesion model to visualize and demonstrate the hierarchical buildup of voids. In order to do this, we compare two adhesion models characterized by power spectra with different levels of large scale power. We compare the two models quantitatively by using the tools developed in the previous chapter.

#### 7.1 Adhesion models

In chapter 1 we discussed how the statistical properties of the primordial density field in Fourier space, are captured by the power spectrum. The power spectrum P(k) defines the amount of power for the different wave modes and is defined as:

$$(2\pi)^3 \delta_D(\vec{k} - \vec{k'}) P(k) \equiv \langle \delta_k \delta_{k'} \rangle \tag{7.1}$$

Primordial power spectra are often modelled by a power law spectrum  $P(k) \propto k^n$ . A power spectrum with n = 1 is referred to as the scale invariant Harrison-Zel'dovich spectrum, predicted by inflation. Power spectra with n > 1 have more power on the smaller scales (remember  $k = \frac{2\pi}{\lambda}$ ), spectra with n < 1 have more power on the larger scales.

In this chapter we test the effect of the power spectrum on the hierarchical buildup of voids. In order to do this, we compare two simulations with a different power spectrum. The simulations that we use for this purpose are two-dimensional versions of powerlaw power spectra cosmologies, modelled on the basis of the adhesion formalism (see section (1.3.2)). These capture the large-scale weblike structure very well, at the cost of discarding the highly nonlinear structure on smaller scales. We choose to use the 2D version as it allows us to follow structure formation over a significantly larger dynamic range than would be possible in the equivalent 3D situation. We consider the power spectra

$$P(k) \propto k^1 (\mathrm{P-model})$$
 (7.2)

$$P(k) \propto k^{-1} (M - model)$$
 (7.3)

For each of these models we have 86 density snapshots, sampled on a  $1024 \times 1024$  density grid. The simulation data for this analysis has kindly been provided by Johan Hidding (Hidding et al. [46]). Figure 7.1 shows the final snapshots of the simulations we use. In addition, this figure also shows a more detailed zoom in of a smaller region of the slice. For the M-model, we show in figure 7.2 the time evolution of this zoom in. Comparing the two adhesion models, it is clearly visible that for the P-model there is much more small scale power than there is for the M-model. It also produces voids that appear to be much deeper than in the M-model.

From figure 7.2 we can clearly observe the hierarchical evolution of the void population. Starting with the upper left slice, we see a population of mostly small voids. Already the outline of a higher void hierarchy may be recognized by looking at the thicker filaments. As time progresses, more and more of the smaller voids disappear. Their surrounding filaments continuously merge to form larger and larger voids and become part of larger filaments or end up as part of the substructure of a void higher in the hierarchy.

#### 7.2 Void identification with the MWVF

We have extended the MWVF to identify voids in two-dimensional simulations. The viscosity term of the adhesion model gives rise to a very sharp outline of the emerging cosmic web. As a consequence, voids in the adhesion model are very deep and have a corresponding high emptiness. In order to identify the void populations in the adhesion model with the MWVF, we adopted an emptiness criterion of 90%. This results in respectively  $\sim 82\%$  and  $\sim 85\%$  of the last simulation slice being occupied by voids for the P- and M-models. Figure 7.3 shows 6 subsequent timesteps for the P-model with the voids that we identify. The same timesteps are shown for the M-model in figure 7.4. The slices that we show in these figures correspond to the same region as the zoom in of figure 7.1.

#### 7.3 Merger trees and void persistence

To visualize the effect of the power spectrum on the hierarchy of voids, we use merger trees and persistence diagrams. The construction of both merger trees and persistence diagrams, is discussed in chapter 6.

For the P-model we have constructed the merger tree of the largest void that we find in the last snapshot of the simulation. This merger tree is shown in figure 7.5. To visualize how this merger tree evolves, we also show a number of snapshots of the subvoids that



(a) Final snapshot of P-model  $(P(k) \propto k^1)$ 



(b) Final snapshot of M-model  $(P(k) \propto k^{-1})$ 





FIGURE 7.2: Six subsequent timesteps of the M-model  $(P(k) \propto k^{-1})$ , clearly visualizing the hierarchical buildup of structure in the adhesion model.



FIGURE 7.3: Six subsequent timesteps of the P-model  $(P(k) \propto k^1)$ . The dark (flooded) regions represent the voids identified with the MWVF.



FIGURE 7.4: Six subsequent timesteps of the M-model  $(P(k) \propto k^{-1})$ . The dark (flooded) regions represent the voids identified with the MWVF.

will eventually form the parent void at t = 10. Notice that the units of time have no physical meaning and are only used to indicate subsequent snapshots.

In constructing a merger tree for the M-model, we have identified a void that is similar in size to the one for which we created a merger tree in the P-model. This merger tree and the corresponding snapshots are shown in figure 7.6. In comparing the two merger trees, we observe that the evolution of the void in the M-model is already from the beginning dominated by a large void (the one that forms the "life line" of the parent void). This void subsequently merges a few times with voids that are much smaller. The void that we follow for the P-model on the other hand, evolves out of several small scale voids that merge together to form the parent void. Even for the snapshot preceding the final one, the subvoids are still quite similar in size. This is clearly a manifestation of the excess power on the smaller scales.

Using all 86 snapshots, we construct a persistence diagram for both the P- and the Mmodel. These diagrams are shown in figure 7.7. Comparing the persistence diagrams, voids in the P-model appear to have longer lifetimes. As soon as voids start to mature, evolution in the M-model is probably soon dominated by the larger voids that we expect to be more abundant than in the P-model. The majority of these voids will probably survive till the end of the simulation and therefore never form a point in the persistence diagram. In the P-model we have on the other hand a lot of small voids. Not all of these voids can survive till the end of the simulation, but they are more likely to survive at least a number of timesteps. This might explain the broader distribution we find in the persistence diagram of the P-model with respect to the M-model. Also shown in this figure is the ratio of the number of points in the persistence diagram of the P-model and the persistence diagram of the M-model. The fact that this number is almost everywhere larger than one, tells us that the number of points in the persistence diagram of the Pmodel is much larger than the number of points for the M-model (these numbers are respectively 21581 and 12459). On the basis of the persistence diagrams and merger trees we have created, we conclude that within the P-model, the hierarchical evolution of voids is much stronger. This is exactly what one would expect for a power spectrum with a higher spectral index.

#### 7.4 Conclusions

The adhesion models are optimally suited for following the hierarchical evolution of the filaments, walls, voids and there connections to the cosmic web. In this chapter we have applied our tools to a set of cosmological adhesion models to show that they really sense the underlying hierarchical nature of the cosmic matter distribution.



FIGURE 7.5: Mergertree for largest void in the P-model  $(P(k) \propto k^1)$ .



(b) Subsequent snapshots of the void region.

FIGURE 7.6: Mergertree of a void in the M-model  $(P(k) \propto k^{-1})$ . This void is similar in size to the one used for the P-model.



FIGURE 7.7: Persistence diagrams for both the P-model (a) and the M-model (b). Also shown is the ratio of the number of points we find in the persistence diagram of the P-model and the M-model (c).

## Chapter 8

# Void hierarchy for different Dark Energy models

As an application of the Multiscale Watershed Void Finder, we compare the void populations for three different models of dark energy. The dark energy models we consider are referred to as respectively LCDM, RP and SUGRA. We discuss these models in appendix C. A more extensive discussion of the models is provided by Bos [18] and Bos et al. [19].

#### 8.1 Simulations

Our analysis with respect to the different dark energy models is based on simulations by Bos et al. [19]. For these simulations, the N-body code P-GADGET-3 has been used. This is an extended version of GADGET-2 with the ability to specify the mode of dark energy evolution.

We compare the void populations resulting from 8 different, low-resolution simulation runs. For each simulation run, with the same initial conditions, we follow the subsequent evolution of the system for the three dark energy models discussed in appendix C. Altogether, we consider 24 simulation runs: 8 runs for each of the three dark energy models. The specific dark energy model parameters are given in table 8.1. For an explanation of these parameters, the reader is referred to appendix C. The general cosmological parameters of the simulation runs are summarized in table 8.2. These parameters are the same for the three models.

model	$\alpha$	$\omega_0$	$\omega_a$
LCDM	-	-1.0	0
$\operatorname{RP}$	0.635	-0.9	0.0564
SUGRA	0.635	-0.9	0.452

TABLE 8.1: Specific parameters of the different dark energy models used in the simulations by Bos et al. [19]. For an explanation of the parameters, see appendix C.

Number of particles	$256^{3}$	
Boxsize	$300 {\rm ~Mpc}$	
Starting redshift	60	
Energy density matter $\Omega_{m,0}$	0.268	
Energy density dark energy $\Omega_{\Lambda,0}$	0.732	
Hubble constant $H$	$70.4 \rm \ km \ s^{-1} \ Mpc^{-1}$	

TABLE 8.2: General cosmological parameters of the simulations runs by Bos et al. [19] that we use to study void populations for different dark energy models.

For each simulation run we have snapshots at z = 0, 0.1, 0.25, 0.51, 1.0 and 2.04. As we explain in chapter 4, the low resolution of the simulations we use in this chapter implies that we have to adjust our emptiness criterion for the void identification process. If we still adopt a criterion of 50%, we end up with a void population in the LCDM simulations that occupies only  $\sim 4\%$  of the simulation volume. This seems to be rather unrealistic. On the basis of the test run that we describe in chapter 4, we adopt an emptiness criterion of 20% for the low resolution simulations in this chapter. This results in an average volume occupation of  $\sim 71\%$  for the LCDM simulations,  $\sim 62\%$  for the RP simulations and  $\sim 41\%$  for the SUGRA simulations. The evolution of the volume occupation by the voids identified with the MWVF is shown in figure 8.1.



FIGURE 8.1: The evolution of the volume occupation by the voids we identify for the three different dark energy models. For each timestep we show the average value of the 8 simulations runs we consider for each of the models. Also shown is the corresponding standard deviation of this distribution.

#### 8.2 Evolution of void abundance

We use the MWVF to analyse the evolution of void abundance for the three dark energy models LCDM, RP and SUGRA. Figure 8.2 shows the results of this analysis. Within this figure, we plot both the absolute number of voids that we obtain for the three populations as well as the number of voids normalized with respect to the number of voids identified in the LCDM model.



(a) Average number of voids as a function of a. This average has been determined from 8 different simulations runs. The errorbars correspond to the standard deviations of these distributions.



(b) The evolution of the average number of voids normalized with respect to the average number of voids found for the LCDM model.

FIGURE 8.2: The evolution of void abundance for the three dark energy models considered.

From figure 8.2, we observe that voids start to form first in the LCDM model. Especially the evolution of void abundance for the RP model, seems to be a shifted version of the LCDM simulation. As if void formation in the RP model is a bit delayed with respect to the LCDM model. In reality, the differences between the models are small, but apparently the LCDM voids reach the critical emptiness criterion of 20% a bit earlier. For both the LCDM and RP models we find a similar void abundance pattern as the one we encountered in chapter 5. At some point, the watershed basins start to reach the critical emptiness of 20%: the number of voids begins to increase. Mergers between voids and crushing of small voids give rise to an evolving void population. The number of voids therefore does at some point reach a maximum after which it start to decline again. Although at a = 1.0, the number of voids in the SUGRA models still increases, one can imagine that this distribution will also reach a maximum at some point. Hence we expect that this distribution is eventually similar to the others in that it will also reach a maximum.

#### 8.2.1 The evolution of $\sigma_8$

The amount of clustering is normally characterized by the normalization parameter  $\sigma_8$ . This parameter probes the power spectrum at a scale of  $8h^{-1}$  Mpc. As is explained in Bos et al. [19], the simulations for the different dark energy models that we use here, are normalized at the CMB by using the relation

$$\sigma_{8,DE} = \sigma_8 \frac{D_{\Lambda CDM}(z_{CMB})}{D_{DE}(z_{CMB})} \tag{8.1}$$

with the assumption that  $z_{CMB} = 1089$  and D is the linear velocity growth factor. The latter is dependent on the dark energy model through H. Bos et al. [19] find that this rescaling relation leads to differences in the amount of clustering, which will evidently influence structure formation in the different dark energy models. Bos et al. [19] argues that the obtained differences between the dark energy models they obtain, are in fact related to the different evolution of  $\sigma_8$  in these models. In other words, if we would compare the results of e.g. the LCDM and SUGRA model for the same value of  $\sigma_8$ , the obtained differences are expected to disappear. This implies that the different expansion histories of the dark energy models are expressed via the evolution of  $\sigma_8$ .

In order to test the relation between the evolution of  $\sigma_8$  and the results we obtain, we estimate the values of  $\sigma_8$  for each of the snapshots in our simulation. In figure 8.3 we plot the evolution of void abundance as a function of the average value of  $\sigma_8$  for the corresponding snapshots. From this figure, one may conclude that indeed the evolution of void abundance is related to the different evolution of  $\sigma_8$  for the dark energy models. All three dark energy models now seem to be part of one and the same void evolution abundance curve.



FIGURE 8.3: The evolution of void abundance appears to be expressed via  $\sigma_8$  with  $\sigma_8$ . Plotting the estimates for the average value of  $\sigma_8$  versus the number of voids for the snapshots of the three dark energy models, the data points seem to be part of the same void evolution abundance curve.

#### 8.3 Void shapes

In section 2.1.3, we have seen that void shapes have the potential to be used as a precision probe of dark energy. Here we test if we can find any differences between the void shapes of the dark energy models resulting from the void populations identified with the MWVF.

We determine the void shapes in the same way as described in section 5.3. The evolution of the mean ellipticity as a function of a is plotted in figure 8.4. It turns out that the mean ellipticities do not vary significantly between the different models, neither do we observe a significant variation of  $\epsilon_{\text{mean}}$  with a. Bos et al. [19] find a slow increase of  $\epsilon_{\text{mean}}$  with a, but the effect is only small. Although the large standard deviations of our distributions would allow such an increase, it is striking that the mean values of  $\epsilon$  do not show any sign of evolution.

To make sure the shape parameters we determine are correct, we have checked our data for consistency with that of Bos et al. [19]. In order to do this, we determine the ellipticities for all watershed basin detected at a filter radius of  $R_{\rm f} = 1.5$  Mpc for both the LCDM and SUGRA dark energy model. This allows for a comparison with the results by Bos et al. [19]. The evolution of the mean ellipticities we obtain in this way are plotted in figure 8.5. We conclude that the evolution that is observed is consistent with the one reported by Bos et al. [19].

The fact that the mean ellipticities we find for the voids identified with the MWVF do not show any sign of evolution is surprising. Remember that also for the high resolution simulation,  $\epsilon_{\text{mean}}$  is found to be almost constant over cosmic time (figure 5.3). At this



FIGURE 8.4: The evolution of the mean void ellipticity  $\epsilon_{\text{mean}}$  for the three dark energy models. We do not observe differences in  $\epsilon_{\text{mean}}$  between the different models. Neither do we observe any evolution with a.

moment it is not clear how this observation is exactly related to the void population we obtain with the MWVF.



FIGURE 8.5: We checked our data for consistency with Bos et al. [19] by considering the mean ellipticities of all the watershed basins detected for a filter radius of 1.5 Mpc. We consider both the LCDM and SUGRA dark energy model. The decrease with redshift and difference between the LCDM and SUGRA model is similar to the one reported by Bos et al. [19].

# Chapter 9

## Summary and discussion

Most of the volume in the universe is occupied by large empty voids that are practically devoid of any galaxies. Within this thesis we discuss how the emergence of the filamentary cosmic web may be described from the perspective of cosmic voids.

The identification of voids in galaxy redshift surveys and N-body simulations is not straightforward. This is mainly because the criteria for void identification are not well defined. This results in a range of different void finding algorithms being used. We discuss the important classes of void finding algorithms that are nowadays used in this thesis.

The broad range of void finding algorithms in use, automatically brings one to the question "what is the best void finding algorithm?". Although some void finders do arguably not result in natural voids, the answer to this question strongly depends on the subject that one tries to investigate. In this study we try to investigate the multiscale character of the void population. For this reason, the void finder we use should closely follow the real geometry of the cosmic web. In that sense, we argue that the Watershed Void Finder (WVF) by Platen et al. [84] is a good candidate because it is almost free of user defined parameters and does not make any a priori assumptions on the geometry of a void. One important user defined parameter of the WVF, is the filter radius. We show that the void population obtained by the WVF strongly depends on the filter radius defined by the user.

Moreover, the WVF completely segments a simulation volume into its watershed basins. Not all of these basins necessarily correspond to 'true' voids (e.g. because there is a lot of substructure in the basin that is smoothed out by the filtering procedure). With this in mind, we have developed an extension of the original WVF: the Multiscale Watershed Void Finder. This void finder was specifically developed to study the existence of a void hierarchy.

Within the MWVF algorithm, we choose not to define every watershed basin as a void. For each watershed basin, we determine the fraction of gridpoints that have a density below  $\delta = -0.8$  (shellcrossing for isolated spherical void). We call this fraction the emptiness (E) of a void. Only if the emptiness is higher than some critical value  $E_c$ , we identify the watershed basin as a 'true' void. The Multiscale Watershed Void Finder does not use a single filter radius. Instead it uses, as the name suggest, a set of progressively smaller filter radii. For each filter radius that we consider, we only take into account the basins that meet our emptiness criterion. We combine the voids that we identify for the different filter radii into a new segmentation. This allows us to probe the hierarchy of voids over a range of scales.

The introduction of an emptiness criterion is in a sense rather artificial, but allows us to apply a multiscale filtering procedure. Choosing one single emptiness criterion, we find that the volume occupation by the voids we detect with the Multiscale Watershed Void Finder depends strongly on the resolution of the simulation. The sparser sampling of simulation 'particles' for low resolution simulations, will in general result in larger voids that have a lower emptiness. The same effect may be recognized in the identification of voids in galaxy redshift surveys. In this thesis we show the distribution of void radii for the public void catalog by Sutter et al. [106] for different galaxy samples. This clearly shows us that the obtained radii distribution depends strongly on the sample of galaxies used to define voids.

We have chosen the emptiness criterion to be such, that a reasonable fraction of the simulation volume is occupied by the final void population we obtain. For the high resolution simulation (384<sup>3</sup> particles), we find for emptiness criteria of 40%, 50% and 60% a final volume occupation of respectively ~95%, ~70% and ~30%. Current literature suggests that most (at least more than half) of the volume of the universe is occupied by voids. We expect that a volume occupation of ~70% is not unreasonable, although we have no numbers to support this assumption. On the basis of this assumption we choose to adopt an emptiness criterion of 50% for our high resolution simulation. For the low resolution simulations we use later on, this same criterion results in a final volume occupation of ~4%. We have therefore adjusted the emptiness criterion to 20% for the low resolution simulation. This results in a volume occupation of ~70%, similar to that obtained in the high resolution simulation.

The use of a multiscale filtering procedure, forces one to introduce some kind of a selection criterion. Otherwise, one already obtains for the largest filter radius a complete watershed segmentation of the density field. We select voids on the basis of their emptiness. The tuning of our emptiness criterion, to obtain a reasonable volume occupation is rather artificial. For future research it would be good to consider other selection criteria, that are more closely related to the physics of void formation.

We analyze the characteristics of the void population obtained with the MWVF and their corresponding time evolution. One of the most striking features of this analysis is the characteristic evolution of void abundance. As soon as the first voids start to mature and reach the critical emptiness, the number of voids begins to increase. But voids may also disappear because they merge with other void or collapse under the influence of other large scale features. The MWVF allows us to probe this hierarchical void evolution. The resulting void abundance evolution curve shows us that at some point the number of voids reaches a maximum and starts to decline again. This shows us that at some point the rate at which voids disappear because of hierarchical evolution processes, becomes larger than the rate at which voids reach the critical emptiness threshold. In the analysis of the size distribution of the voids we identify, we observe that the average radius of the void population increases with cosmic time. This reinforces our conclusion that voids evolve hierarchically.

To characterize the shapes of the voids in our population, we determine their ellipticities, oblatenesses and prolatenesses. There seems to be a slight tendency for larger voids to more roundish (based on their ellipticities), but the effect is only small. This contradicts the smaller form factors we find for larger voids. We suspect that the latter might be explained by irregularities on the surfaces of larger voids, but we have no statistical evidence to support this. With respect to the time evolution, we find to our surprise that the shape parameters we use to describe the voids are almost constant with time.

We applied the MWVF to a number of low resolution simulation runs (by Bos et al. [19]) with the same initial conditions but different models to describe the evolution of dark energy. The shape of the void abundance evolution curve is similar to the one obtained for the high resolution simulation. The different dark energy models seem to be shifted with respect to each other. Knowing that the evolution of  $\sigma_8$  is different for the three dark energy models, we have for each of the snapshots in these simulations determined estimates for  $\sigma_8$ . If we plot the void abundance as a function of  $\sigma_8$ , the reported differences between the models disappear. The differences in void abundance are therefore expected to be related to the different evolution histories of  $\sigma_8$  that characterize the dark energy models.

There have been numerous claims that void shapes may be used as a precision probe for the nature of dark energy. The mean ellipticity of voids is expected to decrease with redshift. Bos et al. [19] find both a decrease of  $\epsilon_{\text{mean}}$  with redshift and differences in the distributions of  $\epsilon$  for the dark energy models considers. The void populations that we obtain by applying the MWVF is once again surprisingly constant with time. At this moment it is not clear why the voids that we identify appear to have such a constant mean ellipticity. For the time being this remains an open question and future research has to show how this observation is related to the physics behind the void formation process and the void identification process that is being used.

With this study we aimed to demonstrate the hierarchical evolution of the void population. Using merger trees and persistence diagrams, we translated the hierarchical buildup of voids into the more general concept of void persistence. We discuss the processes responsible for the hierarchy of voids that we observe as well as possible suggestions on how these processes might be distinguished from each other. We make the assumption that mergers between voids are characterized by a positive velocity divergence whereas voids that are being squeezed are characterized by a negative velocity divergence. On the basis of this assumption we conclude that 58% of the voids disappear as a consequence of mergers and 42% of these voids disappear because they have been squeezed. This is however a oversimplified model. A more careful analysis of the velocity field in voids or a Lagrangian analysis of the particles inside voids will in future research have to show how these processes are exactly related to each other.

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## The Watershed Transform

Appendix A provides an overview of the Watershed Transform that lies at the basis of the original and Multiscale Watershed Void Finder. This appendix is based on appendices 2A and 2B from Platen [83].

The Watershed Transform has its origin in the field of image analysis and mathematical morphology. Mathematical morphology was originally introduced by the mathematicians Georges Matheron and Jean Serra in 1964 and provides techniques for extracting image components. More details can be found in for example Matheron [67], Serra [97] and Meyer and Beucher [68]. Applications of mathematical morphology are amongst others to be found in material science, medical imaging and pattern recognition. In our case, the Watershed Transform is used to identify voids in a cosmological density field. In that context, the general concept of an "image" in mathematical morphology can be replaced by "cosmological density field" for the purposes we are interested in.

### A.1 Greyscale image and topographic distance

A cosmological density field  $f(\vec{x})$  may be mapped onto an image  $\mathcal{F}$ ,

$$f(\vec{x}) \to \mathcal{F}(\vec{x})$$
 (A.1)

The image  $\mathcal{F}$  is a function in *n*-dimensional lattice space (usually n = 2 or n = 3)  $\mathcal{Z}^n$ ,

$$\mathcal{F}: \mathcal{Z}^n \to \mathcal{Z} \tag{A.2}$$

In practice the pixels of the image lattice can only attain a certain number of discrete values, allowing us the define a greyscale image as an image that can only have N different discrete values

$$\mathcal{F} = \{\mathcal{F}_i, i = 1, ..., N\}$$
(A.3)

For an image we can define the topographic distance  $\mathcal{T}(x, y)$  between two points in  $\mathcal{Z}^n$  defined with respect to the image map  $\mathcal{F}$ . In the limit of a continuous map  $\mathcal{F}$ ,  $\mathcal{T}(x, y)$ 

is defined as the minimum length through the "image landscape",

$$\mathcal{T}(x,y) \equiv \inf \int_{\Gamma} |\nabla \mathcal{F}(\gamma(s))| \mathrm{d}s$$
 (A.4)

That is,  $\mathcal{T}(x, y)$  represents the minimum image path length  $\int |\nabla \mathcal{F}(\gamma(s))| [s]$  along the path  $\gamma(s)$  in the set of all possible paths  $\Gamma$ . More intuitively,  $\mathcal{T}(x, y)$  represents the path of steepest descent, the track a droplet of water would follow as it flows down a mountaion surface.

### A.2 Segmentation algorithm

The Watershed Transform tries to segment an image into its watershed basins. A watershed basin is defined as the collection of points which are closer in topographic distance to the defining minimum than to any other minumum. There are numerous algorithms available to create the watershed segmentation of an image. The WVF is based on the watershed transform algorithm by Beucher and Meyer [10]. The steps involved in this algorithm are:

- <u>Initialization</u> Each of the N greyscale levels of the image is allocated a queue. All of the pixels of the image are initialized by adding them to the queue corresponding to their greyscale level.
- <u>Finding minima</u> A minimum is a plateau at an altitude *h* from which it is impossible to reach a point of lower height without getting higher first (i.e. a droplet of water found at a minimum will stay there). Each minimum plateau is identified and tagged uniquely. Pixels representing minima are inserted into their corresponding minima queues.
- <u>Flooding</u> All pixels in the greyscale level queues are processed for increasing greyscale levels. If a pixel is surrounded by on or more processed neighbours belonging to the same minima queue, it is inserted in the corresponding minima queue. If it is surrounded by processed neighbours belonging to different minima queues, the pixel obtains a boundary tag. If none its neighbours is processed, the pixel is undetermined. As soon as all pixels in a greyscale queue are processed, the algorithm proceeds to the next greyscale level until all levels have been processed.

Before the above algorithm can be applied to the DTFE density field, this density field (that can have any value) has to be transformed into a greyscale image consisting of a finite set of density contour levels.

# Appendix B

## The principle of persistence

Persistent homology is a principle originating from mathematical topology. It is beyod the scope of this thesis to discuss the complete mathematical machinery that is involved with persistent homology. The interested reader can find more information in e.g. Edelsbrunner and Harer [35].

Perhaps the basic concept of persistence is best illustrated by the use of figure B.1. Every point in the two dimensional plane that we see on the left hand side of this figure is characterized by a scalar value  $f : \mathbb{X} \to \mathbb{R}$  where f is a continuous function and  $\mathbb{X}$  is a connected topological space (i.e. the two dimensional plane). The contours drawn in this plane represent equal values of f, lighter colours corresponding to larger values.



FIGURE B.1: A continuous function on a two dimensional plane (*left*). Comparing with a geographical landscape, the contours can be interpreted as equi-height contours increasing for lighter colours. By looking at the family of connected subspaces for increasing contour levels one can construct a merger tree (*right*). Figure from Edels-brunner and Harer [35].

Setting b as a threshold for our function f, we can define the family of nested subspaces  $X_a \subseteq X_b$  as the family of subspaces  $X_a$  that are topologically connected and have  $f(X_a) = a \leq b$ . Evaluating the family of subspaces for an increasing threshold 4b, the family evolves. Two subspaces can become topologically connected and merge into



FIGURE B.2: Illustration of how to construct a simple persistence diagram. As the y-threshold increases, new features (subspaces) are detected. The time a feature is detected corresponds to the time of birth in the persistence diagram (*right*). As soon as two features become connected (i.e. merging), the oldest feature survives whereas the younger feature dissapears. The latter leads to the creation of a new point in the persistence diagram.

one subspace. Eventually the threshold becomes high enough for  $X_a$  to be completely connected throughout the two dimensional plane and we are left with a single feature. The process of continuously increasing the threshold and looking at the obtained family of subspaces allows one to create a merger tree as is shown on the right hand side of figure B.1. At the junction of two merging paths in such a tree, the older one survives whereas the younger one is ended (the elder rule).

Considering the example above, we can describe persistence as the continued existence or occurence of certain (topological) features. As soon as a feature is detected for the first time, we can define this event as the 'birth' of the feature. If on the other hand at some point a feature dissapears because it merges with another (older) feature, we define this as the 'death' of the feature. Plotting the level (time) of birth along the y-axis versus the level (time) of death along the x-axis we obtain what is called a persistence diagram. The idea of a persistence diagram is illustrated by figure B.2.

The basic idea of visualizing persistence by using merger trees and persistence diagrams discussed in this appendix is applied in chapter 5 to visualize void persistence.

# Appendix C

## Dark energy models

In chapter 1 we have seen that the relation between pressure and density of a cosmic constituent j is captured by the equation of state, which in general can be written as:

$$P_j = \omega_j \rho_j c^2 \tag{C.1}$$

Inserting the equation of state in the acceleration equation (eq. 1.10), we end up with:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + 3\omega\rho\right) \tag{C.2}$$

Observations of SNe type Ia by Riess et al. [91] and Perlmutter et al. [81] have shown that the universe is nowadays in a stage of accelerated expansion. The only term in the Einstein field equations that can in fact be connected to this accelerated expansion is the cosmological constant  $\Lambda$ . The source of this term (on the right-hand side of the field equations) is known as "dark energy". Looking at the acceleration equation, one can easily derive that for a universe that is in a stage of accelerated expansion, one should have  $\omega_{\rm DE} < -1/3$ .

### C.1 Models

#### C.1.1 Dark energy in $\Lambda$ CDM

Within the standard  $\Lambda$ CDM cosmological scenario, dark energy is modelled by a cosmological constant with  $\omega_{\Lambda} = -1$ . The corresponding Friedmann equation for such a universe can be written as

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_{\rm m,0}}{a^3} + \frac{\Omega_{\rm r,0}}{a^4} + \Omega_{\Lambda,0} \tag{C.3}$$

#### C.1.2 Dynamical dark energy models: quintessence

Dynamical dark energy models that model the evolution of  $\omega_{\Lambda}(a)$  over time by a noninteracting scalar field  $\phi$  in a potential  $V(\phi)$  are called quintessence models. For these models we have

$$\omega_{\phi} = \frac{P_{\phi}}{\rho_{\phi}} = \frac{\frac{1}{2}\phi^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$
(C.4)

and the corresponding Friedmann equation is given by (Ratra and Peebles [88]):

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_{\rm m,0}}{a^3} + \frac{\Omega_{\rm r,0}}{a^4} + \Omega_{\Lambda,0} \cdot \exp\left(-3\int_{a_0}^a \frac{1+\omega_\phi(a')}{a'}da'\right) \tag{C.5}$$

According to quantum field theory, the equation of motion for a scalar field  $\phi$  is given by the Klein-Gorden equation:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0 \tag{C.6}$$

The properties of the dark energy model are determined by the potential  $V(\phi)$ . By making an appropriate choice for  $V(\phi)$ , quintessence models can be used to solve both the fine-tuning and coincidence problem. The two quintessence models that we have considered use an inverse power law potential (Ratra and Peebles [88]) and a generalized inverse power law potential (Brax and Martin [21]), the latter extending the former by including corrections from supergravity. These models are referred to as respectively RP and SUGRA and the corresponding potentials are given by

$$V_{\rm RP}(\phi) = \frac{\Lambda^{4+\alpha}}{\phi^{\alpha}} \tag{C.7}$$

$$V_{\rm SUGRA}(\phi) = \frac{\Lambda^{4+\alpha}}{\phi^{\alpha}} \exp(4\pi G\phi^2)$$
(C.8)

where  $\alpha \geq 0$  and  $\Lambda$  are free parameters.

The evolution of the equation of state parameter  $\omega(a)$  as a function of cosmic time for the three dark energy models is plotted in figure C.1.

### C.2 Parameterization

Describing different dark energy models (of which we discussed only 3), includes a number of different, unrelated parameters. This makes it difficult to compare the different models. Linder [63] showed that the dark energy (quintessence) models discussed here, may be fitted by the following formula:

$$\omega(a) = \omega_0 + \omega_a (1 - a) \tag{C.9}$$

The time dependence of the dark energy models in the simulations by Bos [18] that we use, is parameterized by this relation. The values of  $\omega_a$  for the three models have been determined by a  $\chi^2$  fit to the relations plotted in figure C.1.



FIGURE C.1: Evolution of the the equation of state parameter  $\omega$  as a function of cosmological expansion factor *a* for the three dark energy models considered. Figure from Bos [18].

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