SCALING LAWS
IN THE DISTRIBUTION OF GALAXIES
APPENDIX:

notes, comments and suggestions by R. van de Weygaert

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PAPER NUMBER: 5162003

In this appendix you find a listing of (often too detailed) comments, suggestions and thoughts which occurred to me while studying the manuscript. As usual an interesting text triggers many thoughts on related phenomena, concepts and studies. It may therefore be good to point out that by far most of the indicated items do not concern criticisms. They are usually mere suggestions for possible additions, alterations and reorganizations.

Often I am not sure whether they would indeed lead to an improvement of the current text, and I leave it up to the authors to judge whether any of these are indeed useful. Also, at many occasions the authors did touch upon the items mentioned as a remark. They were included nonetheless, as part of a suggestion to reorganize or relocate various sections. In view of the abundance of points adressed in the following, and limitations to the reach and extent of the intended review, the remarks will perhaps be of full use only for later work (book ?). Thus, they may of use partially so for the present review.

Most comments can be traced to five major considerations:

A. **Physics Review.** Reading the text as an interested reader from a different branch of physics. This results in emphasis on clarifying astrophysical/astronomical concepts, and at points seeking to take away possible confusion with similar concepts in other fields (e.g. the use of correlation functions in statistical physics and condensed matter physics).

B. **Review of Cosmology and Gravitational Clustering.** Main approach was that as a "review" of

- cosmological structure formation and the large scale structure of the Universe, with a specific focus on
- scaling properties of gravitational clustering.

This had repercussions in terms of:

- basic explanations of relevant concepts,
- a sufficiently broad coverage and representation of related and relevant issues (of the concept of clustering scaling),
- a sufficiently extensive and representative list of references (even though the list of suggested references is far from complete).

However, I realize that during the writing of this appendix I may have become too ambitious/interested, and given the extense of the subject and its many connections to related topics the list has grown far beyond original intention, and the authors are advise only to use those points which they find appropriate.

C. **Reorganization.** The text would benefit from some reorganization, i.e. reordering of a few major sections, and reshuffling and relocation of various minor parts (paragraphs, subsections) in the text. The latter involves sections that seem to be closely related, or here and there a few redundancies. The listed comments base themselves upon my recommendations.

D. **Scaling.** The concept of "scaling" has various meanings, dependent on the context. With respect to galaxy formation and galaxy structure it may refer to relations like the Tully-Fisher relation or fundamental plane relations. Also in the context of gravitational clustering one may interpret it in a few different ways. The review should offer explicit specification of the physical significance of the various scaling phenomena, and elucidate their (intrinsic) relationship and relation to cosmological observations.

E. **References.** Because websites of various survey programs and (CMB) experiments have become such essential tools and sources of information, I would find it highly recommendable to insert website references in a review such as this. In the following you'll find at occasions various suggestions.

F. **Illustrations.** The topic being far from trivial, I would recommend to insert a few more figures to clarify some of the material. In this report I added a great number of possibly useful illustrations.

Suggested Itinerary

Throughout this write-up of notes on the manuscript, I followed a suggestion for a re-ordering of the various chapters/sections in the manuscript.

This served two purposes. One was bringing together a few sections in different parts of the manuscript, sections that seemed to be more logically connected with each other than suggested by their original context. In addition, a major suggestion concerns the insertion of an extra chapter on gravitational instability theory, and nonlinear gravitational clustering phenomena. It followed my own investigation into the connection between gravitational clustering theory and the scaling phenomenon, triggered by the many comments in the review. The field being so rich, in my view it would be nice to have it included in the review. I would suggest this to follow immediately the introduction, incorporating various parts distributed through the manuscript. The subsequent follow-up of the observational reality and the clustering measures, and their interpretation within the context of the scaling issue would profit from the prior knowledge handed by such a section.

Nonetheless, I realize it has all evolved far beyond the original intention, driven by the fascination of the subject. Therefore I would (again) suggest the authors to only take out those parts they deem useful for their review. The notes offered here no doubt will come of use for other purposes too (preparation of a book).
In summary, the itinerary followed by myself is:

I. Introduction and Overview Cosmology

II. Gravitational Instability

including

[-] Nonlinear Clustering: Analytical Models

[-] Nonlinear Clustering: Correlation Function Evolution

[-] Nonlinear Clustering: Self-Similar Evolution

[-] Nonlinear Clustering: Hierarchical Structure Formation

[-] N-body simulations

III. Discovery of Cosmic Structure: Observational Reality

including:

[-] Cosmic Microwave Background

IV. Clustering Measures

V. Clustering Scaling and Biasing

VI. Fractal Characterization of Scaling

VII. Historic Hierarchical Models

VIII. Statistical Models

IX. Dynamical Models

X. Summary and Concluding Remarks

I. Introduction and Overview Cosmology

Recommendation to reorganize and expand the introduction section. This would help to provide physicists without astronomy background the astrophysical perspective and context of cosmology, structure formation, and the emergence of complex cosmic structures. For astrophysicists it should provide the rationale behind the search for scaling in the galaxy distribution.

Figure 1: The Coma Cluster of Galaxies. Photo: Omar Lopéz-Cruz & Ian Shelton

Ia. Introduction: Scaling and the Cosmos

1a) **Universe and Complex Structures** After a very short introduction (one paragraph) on the success of Big Bang cosmology, indicate that the Universe is abound with structures over a wide range of spatial and mass scales. Structures which should not have been there according to the ideal FRW cosmology. Their presence indicates the need for an extension of the physical principles underlying standard cosmology by a theory of structure formation (and as usual, an off-equilibrium system represents a key towards a much more profound understanding of the complete system, and in this context therefore to the early Universe). The observational reality indicates a cosmic matter distribution of substantial complexity, providing a key towards understanding the process of structure formation.

b) **Galaxies** Given the broad audience of physicists to whom this contribution is directed, an introduction on galaxies and on the galaxy distribution ... Following the idea that images are often worth a thousand words, and given the fact that a major fraction of the readership is not directly familiar with the astronomical background, I would welcome a complete page with an imagery of astronomical hierarchy: a galaxy ... its immediate surroundings (surrounding dwarfs ?) ... a cluster of galaxies (the Coma cluster image by Omar Lopez-Cruz provides a beautiful image) ... a supercluster, or part of (e.g. central concentration of clusters in the Shapley concentration), and finally the large scale galaxy distribution (perhaps the smashing image of the 2MASS infrared galaxy sky distribution, recently released). It may go along with a qualitative characterization of the galaxy distribution (see point 1). Later one could refer back to it (section D).

c) **Physics of Scaling** An explanation of why scaling relations are of interest in physics. This should basically be an extension of section C "Scaling Laws in physics": a (short) introduction on scaling laws in physics, indicating what such scaling laws may tell about the physical system at hand. Perhaps one or a few examples from other fields may be helpful references.

d) **Scaling in the Universe** Subsequently introducing them within the context of cosmology, the galaxy distribution and galaxies, given the general audience from all fields of physics, I would recommend a somewhat more specific motivation:

   - the theory of gravitational instability as basic paradigm behind the formation of cosmic structure from subgalactic scales upward ... (which should subsequently be expanded upon in the next section).
   - why we are interested in the galaxy distribution: cosmic fossil of the structure formation process
   - what visually and physically "*scaling*" in such an astrophysical context entails (examples of hierarchically embedded structures)
   - what we hope to learn from the underlying physical processes.

2. Already at the onset, in the introduction, the authors should more clearly specify how they confine and define the topic of this review. The review concentrates in particular on point distribution scalings in the galaxy distribution. This may indeed be understood as a manifestation of the gravitational clustering process. However, there are more astrophysical scaling relations which directly relate to this issue.

- There are various known galaxy scaling relations, such as the Tully-Fisher relation and fundamental plane relations. They are very likely deeply tied in with the galaxy formation process. Because they non-gravitational physics will likely be an important agent they do not relate to the subject of the review. Nonetheless, the review should clarify at the onset which subjects will be covered, and the rationale behind not including other fundamental relations.

- Scaling can also be recognized in various manifestations and stages of the gravitational clustering process. One example is that of the small-scale internal structure of collapsed halos, where the possible existence of *universal halo* scaling relations (the NFW relation: Navarro, Frenk & White 1996, 1997), indicates an intimate relationship with the nonlinear gravitational collapse process Also, the success of the Press-Schechter and excursion set formalism has taught us about the equally surprising self-similar nature of the condensation of the matter distribution in discrete collapsed clumps. In the early Universe, the primordial density perturbations on large scale appear to exhibit a largely scale-free distribution, which eventually appears to evolve into the asymptotic nonlinear extreme of *stable clustering* via a surprisingly systematic clustering evolution during the *transition* phase of cosmic structure development which uncovered by the *HKLM* formalism. In all these instances gravitational growth of structure apparently involves profound scaling behaviour. This aspect is extensively elaborated upon in the comments and suggestions concerning an additional chapter on the theory of gravitational clustering.

Ib. Statistical Mechanics and Self-Gravitating Systems

3. section IB 'Statistical mechanics':
   isn't another way of stating that the statistical mechanics of a self-gravitating system not being a totally nontrivial subject: gravity defines a system of negative specific heat.

4. section IB: 'two suggestions for ...'. Perhaps one should also mention shape statistics (Vishniac, later Babul & Starkman 1992), which tried to relate scaling to the shapes/geometry of structures. These, in a sense, are related yet again to Minkowski functionals. When limiting this section to mentioning VPFs and multifractal measures, it may be good to explain why this is so.

Ic. Cosmology Overview: contents

5. A general cosmological introduction should follow the *scene-setting* description mentioned above. Basically starting with what is now

a) "I. Physical Cosmology".
This includes part of first paragraphs of section I (before section A), most of section II, and also include III.E "The cosmological principle"

b) the Universe of galaxies (section III B.):

- "Cosmogony" and "Island Universes".

- "Island Universes": Wasn't it Kant who first suggested the concept of the nebulae being "island universes". When talking about a single galaxy (Herschel) it would be good to mention Kapteyn's Universe ... Section IIIB, footnote 2: Lindblad (1926), and Oort (1926, 1928): ie., it's fair to include Oort's 1926 result ...

c) Given the earlier global cosmology describe what galaxies are in a cosmological context, what their role is : ie. a short subsection putting later discussion on galaxy distribution in perspective, basically stating that the galaxies are used to map the structure of the Universe, as cosmic lampposts).
Section II B. hints at this, when mentioning gravitational lensing. It might be good to explicitly state that this potentially is the most promising technique to achieve this, but that as yet the galaxies still provide the most detailed view of the cosmic tapestry. A good recent review reference on dark matter/dark energy etc. is the Science special of June 20, 2003 ... (vol 300).

Id. Cosmology Overview: Antiquity

6a) III.E. "The Cosmological Principle". Given the historic emphasis in the early parts, perhaps good to point out that somehow it was Aristarchus who already formulated cosmic homogeneity, in that his cosmos had not special central location in the Universe. In Archimedes "Sand Reckoner" it is stated that Aristarchus thought the sun AND the stars were fixed, and that the latter were far removed. In other words, he equated sun and stars and implied a "non-preferred" location. Copernican principle should be "Aristarchean" principle, according to Harrison (1982). As for isotropy, this was almost literally formulated by Lucretius: "Whatever spot anyone may occupy, the universe stretches away from him just the same in all directions without limit" (Lucretius, De Rerum Natura, "The Nature of the Universe", 1st century BC).

b) III.A. "... none of them formulated ... mathematics concepts". I would find it appropriate to mention that in fact it WERE the ancients who made the unique step to suggest mathematics as the order behind the Universe. Pythagoras is definitely the first prominent scholar emphasizing the key role of maths (the spheres !!!), and explicitly Plato treats in his Timaeus the mathematical basis of the Universe (the mathematical world belonging to the ideal world of "forms", unlike the material world of the "becoming")! By the way, perhaps also appropriate to include a reference to Anaximander as first true cosmologist, whose idea of *Apeiron* is remarkably close to our concept of the *vacuum* as source of everything.

c) Possibly interesting and relevant references on ancient cosmology:

1. M. Wright, *Cosmology in Antiquity*, Routledge, 1995

2. C. Kahn, *Anaximander and the Origins of Greek Cosmology*, Hackett Pub., 1994

3. Plato, *Timaeus*,

- Loeb Classical Library: R. Bury, *Timaeus, Critias, Cleitophon, Menexenus, Epistles*, Loeb Lib. # 234, Harvard Univ. Press, 1960

- Penguin Classics: H.D. Lee (transl.), *Timaeus and Critias*, Penguin, 1972

4. F.M. Comford, *Plato's Cosmology; The Timaeus of Plato*, Hackett Pub., 1997 (reprint)

5. Lucretius, *De Rerum Natura*

- Loeb Classical Library: W. Rouse, M. Smith (transl.), *De Rerum Natura*, Loeb Lib. # 181, Harvard Univ. Press, 1975

- Penguin Classics: R.E. Latham (transl.), *On the Nature of the Universe*, Penguin, 1994

while a nice overview, summaries and references on cosmology in antiquity and on general cosmological views in world history may be found in:

6. E. Harrison, *Cosmology, The Science of the Universe*, Cambr. Univ. Press, 2nd ed., 2000

7. E. Harrison, *Masks of the Universe : Changing Ideas on the Nature of the Cosmos*, Cambr. Univ. Press, 2003

Ie. Cosmology Overview: Cosmological Principle

7. section III.E. "The Cosmological Principle":
   when mentioning the first demonstration of homogeneity is the scaling of the 2-pt function: my understanding is that Hubble's argument of number counts $N(m) \propto m^{0.6}$ was a rather robust argument, and still is, for homogeneity (on large scales). Perhaps, given the introductory nature of the cosmology section, it is worthwhile to list some of the major arguments for isotropy & homogeneity:

* Homogeneity:

- the spatial galaxy distribution in galaxy redshift surveys: 2dF and SDSS, possibly earlier also LCRS, do not reveal the reality of larger structures. Also the Broadhurst et al. pencil beam redshift survey did not find inhomogeneities larger than $150-200h^{-1}$Mpc.
- $N(m) \propto 10^{0.6m}$ (Hubble)
- scaling $w(\theta)$ with depth D
- convergence of the Local Group acceleration vector (the Cosmic Dipole)

* Isotropy (for sources at different depths of the Universe !!!!):

- CMB isotropy
- isotropy X-ray background
- isotropy galaxy sky distribution (on cosmologically relevant scales: e.g. radio source distribution)
- isotropy Gamma Ray Burst sources
- isotropy Hubble expansion (no evidence for anything in excess of a simple div v term).

VII. Historic Hierarchical Models

Principal suggestion is to replace this section to a later part.

8. Section D. “Hierarchical models”:
   SHIFT: shift the galaxy clustering models (IID) to a separate and later section, preceding either (following my preference):

a) the description of “modern” clustering models (section VII), specifically the “Statistical Models” (for which I suggest a separate chapter, see later).

b) in front of current section VII “Clustering Models”

c) directly following the observational descriptions of clustering, currently in section III and IV, “Discovery of Cosmic Structure”, the description of the galaxy distribution and redshift surveys (section B and C):

9. The historically interesting models may be a nice prelude to chapter on “Clustering Measures” as they discuss the clustering in quantitative terms. Also, it may be that it would better put in perspective the use of correlation functions as tool to search for scalings/hierarchies.

10. section D. “Hierarchical Models”:
    ‘The clustering together ...’: insert ‘ ... galaxies in successively ordered assemblies .... ’. A description/explantion of this, smaller compact objects grouped in ever larger assemblies, would be in place.
    Also, at the end of the introduction to this section it may be worthwhile to insert an extra line which places the subsequent subsections on hierarchical models in perspective.

II. Gravitational Instability

A major modification involves the insertion of an extra chapter on the theory of gravitational clustering and the emergence of structure in the Universe. For the perspective of the review and its focus on the process of gravitational clustering, a manifestation of the process of cosmic structure formation process, it would function as a reference point for the remainder of the review.

Also, this section is the natural location for an extensive treatment and discussion on the various scaling phenomena involved with gravitational clustering. This starts from the primordial initial conditions for structure formation, but becomes really compelling in the later nonlinear phases of the development of cosmic structure. The original manuscript contained a few scattered references to such scaling manifestations, but they did not command a systematic overview. Thus, this chapter seeks to gather the various relevant parts in conjunction with a presentation of additional and relevant key concepts.

In my opinion, processes such as the asymptotic nonlinearity extreme of *stable clustering*, the surprisingly systematic clustering evolution during the *transition* phase from the linear to the nonlinear phase of cosmic structure development which was uncovered by the *HKLM* formalism, the perhaps equally surprising profound self-similar nature of the condensation of the matter distribution in discrete collapsed clumps (Press-Schechter and excursion set formalism) and the recently discovered *universal* internal structure of the emerging matter clumps (the NFW profiles) are all manifestations of a profound and fascinating intrinsic *scaling* behaviour of the gravitational clustering process.

This review provides the natural platform to elucidate and discuss these aspects of gravitational clustering. Armed with this knowledge one may better appreciate the observational probes which provide information on the possible existence of such scaling in the real world, before finally proceeding with a systematic analysis of the repercussions of such scaling behaviour in the quest for the ultimate origin and source of the distribution of matter in our cosmos.

IIa. Eulerian Perturbation Theory: Fundamentals and Definitions

11. Section VII. "Clustering Models", section D.: Hydrodynamic Models for Clustering: The first part, including subsection 1 on "cosmological gas dynamics" would not only be an appropriate introduction to the theory of structure formation by gravitational instability, but as fundamental paradigm underlying the rest of the work forms the basis of the review. In my view, it would benefit the later discussions and the perspective in the paper if these sections were to be forwarded to this introductory part of the review. It helps in defining the (mathematical) language. It should be extended with a few elements:

- current subsection "1": the "Eulerian perturbation theory". After introducing basic concepts as δ, $\mathbf{v}$, $\mathbf{g}$, ϕ, continuity equation, Euler equation, the path is free to introduce the concepts that are so basic in assessing "scaling" in the matter distribution, the correlation function $\xi(r)$ and the power spectrum $P(k)$.

12. As this is mainly a theoretical underpinning of the later sections, the presentation here should introduce the autocorrelation function

$$\xi(r) \equiv \langle\delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r})\rangle \tag{1}$$

and its Fourier transform, the power spectrum $P(k)$,

$$P(k_1) \equiv (2\pi)^3 \, \delta_D(\mathbf{k}_1 - \mathbf{k}_2) \, \langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \rangle \tag{2}$$

For the following discussions, this would provide an important reference point.

- In section VI C, the review mentions the power spectrum. In that section on "clustering measures" the discussion should refer to $P(k0$ as a measurable characterization of clustering, to be estimated from observational data. However, in the present section it should be emphasized that $P(k)$ is *far more* than a measure of the matter distribution: it is the single most important physical function determining the cosmic matter distribution ($\equiv$ the *infrastructure of the Universe*), and as such represents a *fundamental* aspect of cosmological theory.

- The concept of a power-law power spectrum $P(k) = Ak^n$ may be used to introduce the concept of a scale-free (and self-similar) matter distribution, of relevance to "scaling". As remarked in VI C, even if the power spectrum is not a power law, over a smaller range one may define a *meaningful* power law slope $n(k)$,

- Later, when discussing the 2pt correlation function (section V), the authors should clarify the connection between (the discrete) *2pt correlation function* and the *autocorrelation function.* This connection is most instructively presented and clarified in the contribution by Bertschinger (1992, "New Insights in the Universe", eds. Martinez et al). See later discussion on correlation functions and clustering measures.

$$n(k) \equiv \frac{d \log P(k)}{d \log k} \tag{3}$$

which is in the order of $n \approx -1$ for galaxy cluster scales, $n \approx -1.5$ for galaxy scales, and has to be $n > -3$ to keep it a hierarchical scenario in which small scale fluctuations dominate over those at larger scales.

IIb. Lagrangian Perturbation Theory

13. Although the authors may consider to insert a short section on *basic* "Lagrangian perturbation theory" here, perhaps for consistency it is best to have this all combined in the final chapter on *Dynamical Models.* Perhaps better named *Lagrangian Dynamical Models*, it would combine section VII.C.2 *pancake and adhesion models*, VII.D. "Hydrodynamic models" (except for the first part up to VII.D.1. 'cosmological gas dynamics', which would be included in the section above) and VII.E. "Nonlinear dynamic models". I'll continue with some comments on Lagrangian models in the discussion on that chapter.

IIc. Nonlinear Clustering: Analytical Models

Missing largely in the review is a systematic theoretical treatment/extrapolation of the primordial linear perturbations towards a state of nonlinear clustering and the supposedly hierarchical nature of the buildup of structure in the most succesfull cosmological scenarios. Aspects of this are mentioned and covered in section VI C. on "Dynamical Models". However, while this follows a presentation of a set of "statistical models", it runs the risk of losing the connection with our basic understanding of structure formation. This is of such fundamental importance, also to the physics of scaling, that one should advice to reorganize this, and include substantial parts in this introductory section.
Because of the focus on "scaling" of clustering in the Universe, the nonlinear evolution is of fundamental importance to the rationale behind the review:

- *Scaling* would be the natural outcome of gravitational growth from power-law power spectrum initial conditions and ensuing hierarchical clustering.
- At the same time, it imposes constraints on the nature and reach of "scaling", the asymptotic digression towards homogeneity on Hubble scales does not accommodate any '*simple fractal models.*

14. It would be of benefit to the review if this connection/perception would be clarified. I therefore recommend a description of nonlinear gravitational clustering and the development of a clustering hierarchy in standard scenarios of structure formation (starting from Gaussian primordial conditions).

15. Distributed throughout the current text there are sections which all relate to the issue of structure formation, which perhaps are best in place by shifting them to a combined "structure formation theory" section, setting out the foundation of the review. Examples are the sections on the power spectrum and on stable clustering (VI C).

16. With respect to nonlinear gravitational clustering, four major topics may be identified, all with ramifications for *scaling* solutions:

    • *perturbation theory*, including *BBGKY hierarchy*, particularly oriented towards describing the early nonlinear stage.

    • asymptotic nonlinear clustering regime: the *stable clustering* regime.

    • the *transition* from linear to nonlinear clustering, for which a highly interesting "scaling" solution has been identified by Hamilton et al. (1991)

    • *hierarchical* clumping of matter into ever larger clumps, which has been succesfully described by the surprisingly universal Press-Schechter type formalisms (and excursion set approach), which may be understood as profound scaling solutions.

17. **Perturbation Theory**. The early onset of nonlinearity is described by *perturbation theory.* As yet this is mentioned in a systematic fashion in section VI C1, The **BBGKY hierarchy**. Although indeed in my opinion this is the appropriate place, it may be good to expend some words/sentences on perturbation theory at this stage. Also, it may be good to point out that although some interesting results (on various statistical parameters) may be obtained through this approach, its applicability as a model for clustering evolution is restricted. Prominence of the first higher order contributions are reached relatively fast after nonlinearity sets in, so that corresponding

cosmic phases can hardly be identified, while it becomes increasingly complex to compute higher order contributions. Its virtue relates more to identifying spatial scales at which one may find good estimators of cosmological parameters (e.g. the skewness of the density distribution provides a galaxy bias-independent measure of Ω).

- For the BBGKY hierarchy treatment, the reference to Davis & Peebles 1977 should absolutely be mentioned.
- Perhaps it is worthwhile to elaborate on BBGKY with one or two equations.

18. The text in the manuscript states that a primordial power law power spectrum will retain its power law character because of the scale free nature of gravity. This point deserves much more attention in the current review, being of *seminal* importance for understanding gravitational clustering (see the 4 points above). An aside:

    • This is strictly true only in an Einstein-de Sitter Universe. In an $\Omega_0 < 1$ Universe, there will be an imprint of the gradual change in global curvature: at expansion factor

$$a(t) \approx \left\{ \frac{1}{\Omega_0} - 1 \right\} \tag{4}$$

    the Universe becomes curvature-dominated, leading to a different expansion dynamics. Linear fluctuations will stop growing after this phase.

19. **Stable Clustering**. In a separate subsection the asymptotic limit of *stable clustering* model should be treated, presently found in section VI C. "Power Spectrum". I strongly urge the authors to replace this section largely, and move it to this earlier and in my opinion natural location.

    • The basic scenario and assumptions may be described more extensively: all matter *condensed* into virialized clumps whose internal structure does not change anymore. The *stable clustering* regime is the asymptotic limit in which the spatial distribution of the nonlinear clumps retains its current nature in *comoving space.*

    • Assuming a distribution of such nonlinear clumps with power-law profiles $\rho \propto r^{-\epsilon}$ (for a more physical study of this aspect see also later discussion on the possible existence of universal density profiles, following the work by Navarro, Frenk & White (1996)), scaling solutions may be readily found for this nonlinear clustering regime. The related section "scaling" in chapter 22 of Peebles, "Principles of Physical Cosmology" (pg. 545) provides a good background. Simple scaling considerations yields predictions for the density profiles of collapsed mass concentrations as a function of slope n of the primordial power spectrum $P(k)$. Important nonlinear regime scaling results concern the growth of the mass of characteristic nonlinear structures:

$$M_{nl} \propto t^{4/(3+n)}\,, \tag{5}$$

    and a power-law density profile

$$\rho(r) \propto r^{-\gamma}, \qquad \gamma = \frac{9+3n}{5+n}\,. \tag{6}$$

    • The power-law nature is of great significance in tying the *scaling* interest of the review to the physical theory of structure formation !!!!

    • On the basis of these considerations, Peebles (1974) derived the related scaling solution for the correlation function $\xi(r)$. This is elaborated upon in the comments on "clustering measures" (below).

20. As remarked slightly later (last paragraph), stable clustering can only be an asymptotically valid situation, assuming fully virialized and decoupled nonlinear clumps. It will arguably break down most prominently at active cluster outskirt regions and their supplying filamentary extensions, regions which play a key role in the evolution of the cosmic foam.

21. At this location, it would be appropriate to include the last paragraph of VI.C, "However, nowadays ... solutions exist", and define the concept of nonlinear wavenumber k_{NL}.

22. **Transition Regime** The evolution of the power spectrum, the correlation function, and in general the matter distribution in the nonlinear regime has been the subject of intense interest in the past decade. It has lead to fundamental analytical contributions uncovering profound "scaling" behaviour. In particular this concerns the HKLM procedure and follow-up studies (Hamilton et al. 1991),

23. As one of the major breakthroughs in insight of the clustering process, the HKLM procedure and scaling solutions deserve ample attention in the review. In addition to the seminal contribution by Hamilton et al. (1991), there have been various major follow-up studies, of which Jain, Mo & White (1995), Peacock & Dodds (1996), Padmanabhan (1995, 1996), Valageas, Lacey & Schaeffer (2000) are some of the most noteworthy ones. They have shown convincingly its succesfull description of N-body experiments of gravitational clustering, at first for power-law power spectrum scenarios, later also for CDM-type spectra.

24. In addition to the equation(s) of the scaling (mapping) procedure relating linear and nonlinear power spectrum and correlation function, it may be worthwhile to include a figure on the (scaling) evolution of the power spectrum, as it evolves from its linear to nonlinear stage. For a further elaboration on this point see (30) below. Notice that an important repercussion of this description is that the HLKM models keep power spectra with slopes always in excess of $n > -3$.

25. A major step in our understanding of the process of gravitational clustering is that the HKLM procedure has taught us that *three* distinctive regimes may be identified:
    - Linear clustering regime
    - Transition regime
    - Highly nonlinear clustering regime

IId. Nonlinear Clustering: Correlation Function Evolution

This section contains remarks on the evolution of correlation functions in the process of gravitational clustering. Although these would be in place in the later "clustering measure" section, I have a slight preference, be it with some reservations, to include it at this location.

CRUCIAL in the discussion on (power-law) scaling of correlation functions is the expected TIME EVOLUTION in the various clustering regimes of gravitational clustering. This aspect has been relegated to a rather hidden location in the manuscript (section VI.C), while I feel it would warrant at least a separate and considerably more expanded section. This may include analytical considerations, and possibly results from N-body simulations.

26. The first part of section VI.C treats the power spectrum definition, and its primordial origin. This may be better in the previously indicated introductory section (up to '.... $n \geq -3$ for galaxy scales.)

27. Most of the remainder of section VI.C may form the basis for a separate section/chapter on correlation function evolution, and refer back to the earlier basic section on gravitational perturbation theory.. Because the time evolution is closely related with the clustering regimes (linear, quasi-linear, nonlinear), this extra (sub)section should treat in more detail the relation between $\xi(r)$ and the matter distribution in these regimes. This touches at the core of the whole issue of "scaling", and thus certainly deserves more attention (see later). I find the treatment by J. Peacock in his book ("Cosmological Physics": 16.4 "nonlinear clustering evolution", pg. 509) an excellent basis for such a discussion. One may include both analytical arguments and arguments on the basis of N-body simulations.

28. Linear regime: for a power-law spectrum $P(k) \propto k^n$, the slope of $\xi(r)$ is $\gamma = 3 + n$. The amplitude of $\xi(r)$ increases as

$$\xi(r,t) \propto D(t)^2 = a(t)^2 g(\Omega) . \tag{7}$$

with $D(t)$ the global linear growth factor.

29. Referring back to the dicussion on non-linear clustering, it may be good to mention the $\xi(r)$ that would result from a cluster with power-law density profile with slope ϵ: a power-law function with slope $\gamma = 2\epsilon - 3$ (Peebles 1974). Also see McClelland & Silk (1977). For this configuration $\xi(r)$ is a reflection of the radial cluster density profile. In essence a scaling relation, it will be appropriate to mention it here.

30. In the nonlinear regime, one may return to the *stable clustering* regime introduced earlier in the discussion of gravitational clustering. Assuming a distribution of such nonlinear clumps with power-law profiles $\rho \propto r^{-\epsilon}$, the continuing decrease of the uniform background density with cosmic expansion ($\propto a^{-3}$), leads to a correlation function evolution (comoving):

$$\xi(r,t) \propto a(t)^{3-\gamma} , \tag{8}$$

which leads to a prediction of the nonlinear slope $\gamma (\equiv 3 + n_{NL})$ and the effective power spectrum index n_{NL}, by finding the point where the linear and nonlinear regime match (i.e. $\xi(r_0) = 1$),

$$\gamma = \frac{(3n+9)}{(n+5)} , \qquad n_{NL} = -\frac{6}{5+n} . \tag{9}$$

Subsequently, the evolution in between the above asymptotic linear and nonlinear regimes may be treated on behalf of:

31. results from N-body simulations. A notable example may be the results obtained from the Virgo consortium simulations (Jenkins et al. 1998). It may be worthwhile to include a figure of $\xi(r)$ evolution from this work.

32. in terms of insight very compelling *HKLM procedure* (Hamilton et al. 1991) description establishing, on what in essence are scaling arguments, an evolutionary link between the initial linear field clustering and the final nonlinear clustering. Not only did it provide a succesfull description of nonlinear clustering as confirmed by N-body experiments, it also identified a third regime of clustering evolution in addition to the asymptotic regimes of the initial *linear* regime and the nonlinear *stable clustering* regime, the *quasilinear transition* in which clustering steeply evolves from linear to nonlinear. During this *quasilinear* transition, the correlation function grows as

$$\xi(r,t) \propto D(t)^{(6-2\gamma)(1+\alpha)/3}, \tag{10}$$

with the transition parameter $3.5 \leq \alpha \leq 4.5$.

IIe. Nonlinear Clustering: Self-similar Evolution

Also absent from the review is a most telling and illustrative example of gravitational clustering and self-similar scaling: that of the evolution of a cosmological density field with emerging from a pure power-law power spectrum $P(k) \propto k^n$. For the appreciation of clustering "scaling" and its link with scale-free power-law power spectra $P(k)$ and the scale-free nature of gravity, the review would profit considerably from a treatment of such scenarios. In addition, it would offer the opportunity to include some highly illustrative figures of clustering in related N-body experiments.

33. There have been various studies based on N-body experiments of the full range, from the early linear regime until the ultimate nonlinear "stable clustering" regime, of clustering in scale-free scenarios. Amongst these, the most noteworthy are probably the study by Efstathiou, Frenk, White & Davis (1988), and subsequent systematic work by Jain & Bertschinger (1996: $n=-1$, 1998: $n=-2$) and the recent work by the Virgo consortium, Smith et al. (2003). Interesting is also the scale-free initial conditions gravitational clustering study with the inclusion of gas dynamical processes by Owen et al. (1998).

34. My recommendation is to include a figure akin to e.g. Figure 1 and Figure 2 of Smith et al., or a combination thereof. Of course, one may also produce simulations oneself. Scale-free initial conditions have the great virtue of showing that the matter configuration at later timesteps are scale-up versions of earlier timesteps, a perfect representation of perfect "scaling" (of what I name "structure scaling", see later).

35. As these studies do not only present the resulting clustering patterns, but also provide extensive analysis of the scaling of power spectrum, two-point correlation function, velocity-velocity correlation function, etc., the authors may also pay some attention to these aspects.

36. In its asymptotic limit, this directly relates to the renormalization group approach by Peebles (see later, IIi).

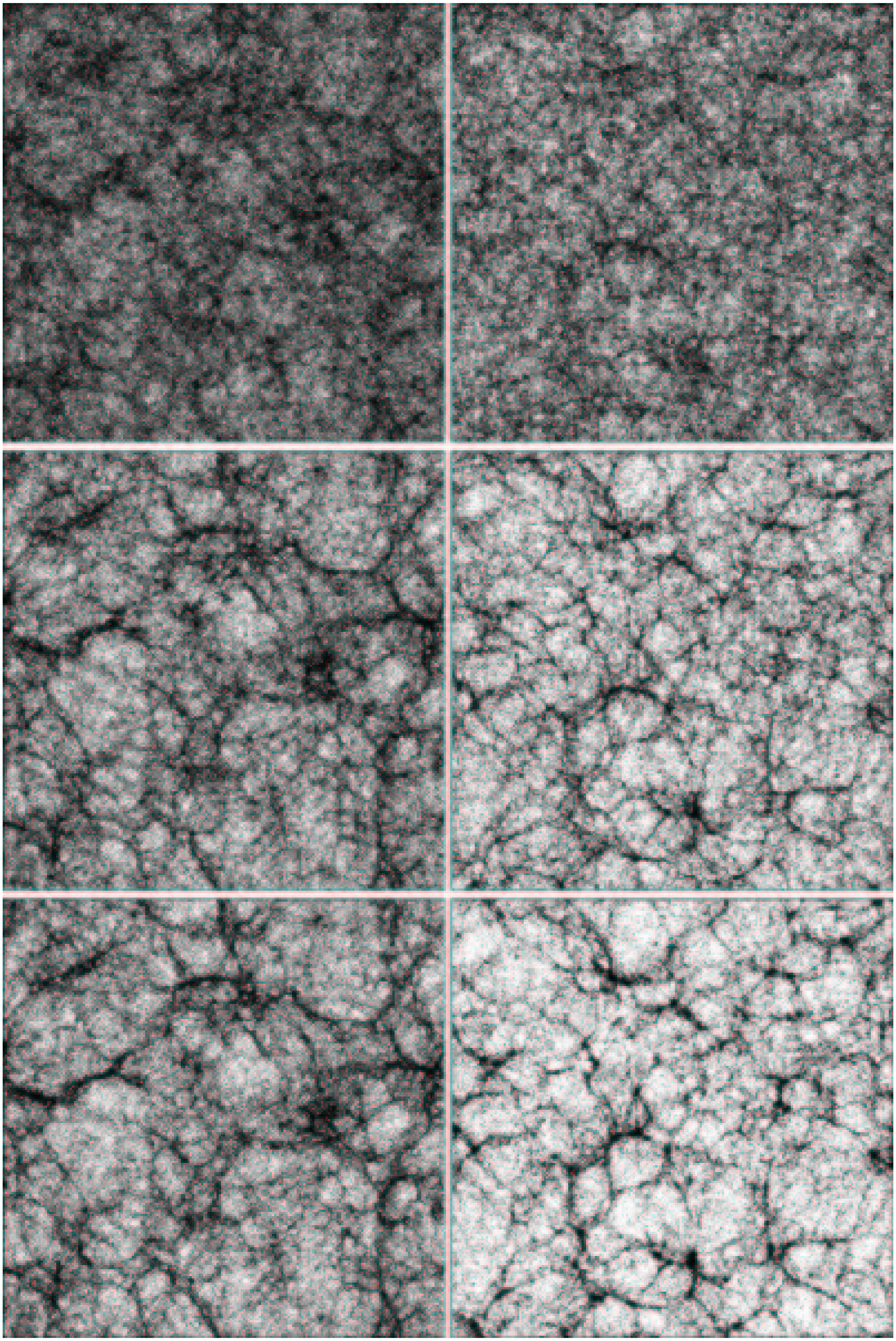


Figure 2: Gravitational clustering in scale-free scenarios. Left: $n = -2$ simulation, at $a = 0.2, 0.45, 0.55$. Right: $n = 1.5$ at $a = 0.25, 0.63, 1.0$. From Smith et al. 2003.

IIf. Nonlinear Clustering: Hierarchical Structure Formation

A major aspect of gravitational clustering is the formation of clumps of matter as the result of gravitational collapse. In a field of primordial random density fluctuations this may involve clumps over a broad range of spatial scales, and thus in mass. Under the condition that the power spectrum $P(k)$ obeys the condition that

$$\frac{d\log P(k)}{d\log k} > -3 \tag{11}$$

the implied collapse timescales as a function of mass scale are such that small-scale objects will form previous to being absorbed into larger encompassing matter fluctuations. This nonlinear *hierarchical* buildup of structure in the Universe has been analyzed extensively, and has lead to highly succesfull analytical descriptions.

Hierarchical Clump Evolution

These Press-Schechter type models involve a self-similar scaling of the population of collapsed matter halos/clumps. In my view it would be highly appropriate for this review to devote attention to this key aspect of gravitational clustering. These analytical descriptions are of such basic importance in our present-day understanding of the matter distribution in the Universe that in my view it belongs within the context of this introductory *gravitational instability* section.

37. The review would benefit from discussing in some detail the basic elements of the description of hierarchical clustering by Press-Schechter related models (Press & Schechter 1974). Formalized in the excursion set formalism (Bond et al. 1991) the discussion may also mention the *barrier excursion* concept for predicting the population of collapsed objects.

38. The discussion should also mention and shortly discuss the alternative models for the emergence of objects in a cosmic density field. Starting from the basic peaks model of BBKS (Bardeen et al., 1986), the most noteworthy models are:

    - the adaptive peaks model of Appel & Jones (1990). Recent work by Sheth & van de Weygaert (2003) proved it to provide a better approximation for the large mass (or deficit, for voids) tail of the object (void) distribution.

    - the attempt to phrase a dynamical model for the formation of collapsed objects in the cosmic density field taking into account the anisotropic collapse of peak regions: the *peak-patch* model by Bond & Myers (1996a,b,c). It did indeed prove to yield succesfull predictions of the evolution of the cluster population in the Universe.

    - The peak-patch formalism, based on the importance of tidal shear as shaping physical force behind the cosmic matter distribution, lead towards the study on the formation of the *cosmic web*, the word coined for the foamlike matter distribution observed in the galaxy distribution and in N-body simulations of cosmic structure formation (Bond, Kofman & Pogosyan 1996).

39. The literature on these models is substantial, and it may be good to e.g. explain that these models allow firm predictions of clump merger rates, the ancestry of current halos through *merger trees* (Kauffman & White 1993), of halo mass spectra seeking to explain those of clusters and galaxies (luminosity functions) and, of high interest to the review, the predictions it entails for correlation functions (see 38).

40. Recent work by Sheth & Van de Weygaert (2003) has shown that the excursion set approach can be succesfully invoked in describing an hierarchically evolving network of voids, offering a natural explanation for the cosmic foam as a *self-similarly* evolving packing of expanding voids. To this end the formalism needs to be extended into a *two-barrier problem.*

41. A tantalizing question with respect to the success of the Press-Schechter description is it being based on a simple spherical description of gravitational collapse. We know this is a gross oversimplification, far beyond validity for individual clumps. The Press-Schechter description basically involving a statistical average over collapsed objects, its succesful predictions seem to hint at a profound tendency of gravitational clustering. Nonetheless, recent modifications seeking to deal with the nonspherical nature of gravitational collapse have lead to a significant improvement in halo mass functions (Sheth & Tormen 1999).

42. These models forms the basis of the **halo model**, which is discussed in section VII C4, as an example of "Dynamical Models". In that section one can then shortly refer back to the models described here.

43. Interestingly, these models have also lead to predictions on the spatial distribution of matter, and specifically the 2pt correlation function, thus establishing a direct relation between "clustering scaling" and basic "theory of gravitational structure formation": e.g. Mo & White (1996) and Sheth (1998).

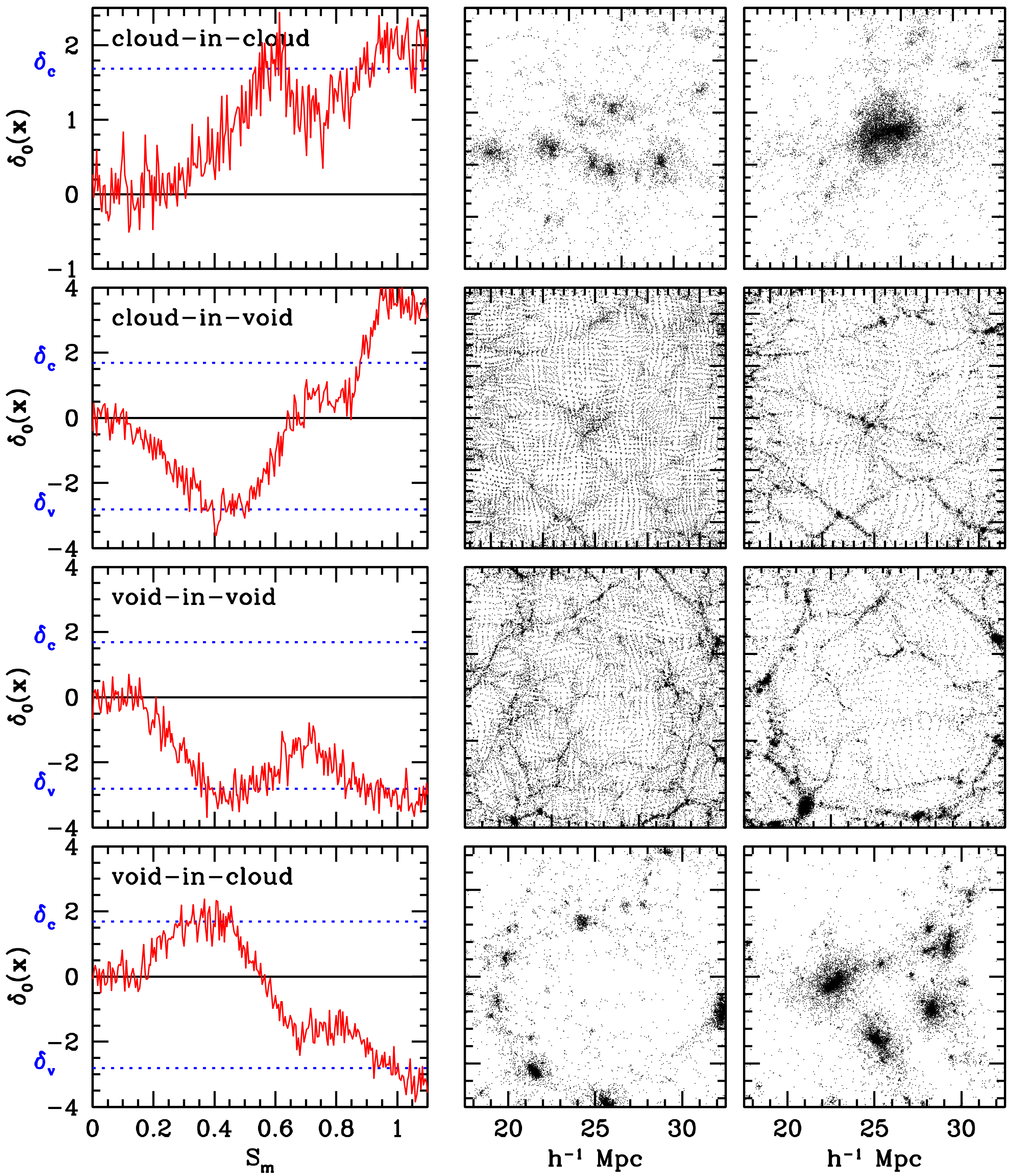
Void Hierarchy

Like the later remarks on the Voronoi model, these remarks involve personal interest (the latter in two meanings). Perhaps it would be a relevant remark concerning the "scaling" of the cosmic matter distribution. As yet the publication is in the finishing stage, and this material will therefore be "embargoed" until publication/acceptance.

44. In a very recent contribution, Sheth & Van de Weygaert (2003, shortly to be submitted) showed how the evolution of the foamlike web in the cosmic matter distribution may be seen as a self-similar evolution driven by the hierarchical evolution of the void population.

a) They find that at any cosmic epoch the voids have a size distribution which is well peaked about a characteristic void size which evolves self-similarly in time. This is in distinct contrast to the distribution of virialized halo masses which does not have a small-scale cut-off.

b) In this model, the fate of voids is ruled by two processes. The first process affects those voids which are embedded in larger underdense regions: the evolution is effectively one in which a larger void is made up by the mergers of smaller voids, and is analogous to how massive clusters form from the mergers of less massive progenitors. The second process is unique to voids, and occurs to voids which happen to be embedded within a larger scale overdensity: these voids get squeezed out of existence



cloud−in−cloud
cloud−in−void
void−in−void
void−in−cloud
δ_c
δ_v
$\delta_0(\mathbf{x})$
S_m
h^{-1} Mpc

Figure 3: Four Mode (Extended) Excursion Set Formalism: census of four hierarchical clustering modes. Each of the rows illustrates one of the four basic modes, from top to bottom: *cloud-in-cloud* process, cloud-in-void process, *void-in-void* process and *void-in-cloud* process. On each row, a mode is illustrated by means of three frames. On the left, the local *density fluctuation random walk*, plotting the local density perturbation $\delta_0(\mathbf{x})$ as a function of (mass) resolution scale S_m. In each graph, the dashed horizontal lines indicate the *collapse barrier* δ_c and the shell-crossing *void barrier* δ_v. A representative and visually appealing image of each process is provided by the particle distributions in the two frames on the righthand half of each row. They depict an appropriate example of an evolving particle distribution selected from an N-body simulation of cosmic structure formation, at two consecutive and representative timesteps. Whereas halos within voids may be observable (2nd row depicts a halo within a larger void), voids within collapsed halos are not (last row depict a small void which will be squeezed to small size as the surrounding halo collapses). It is this fact which makes the calculation of void sizes qualitatively different from that usually used to estimate the mass function of collapsed halos. From Sheth & Van de Weygaert 2003.

as the overdensity collapses around them. It is this second process which produces the cut-off at small scales.

c) In the excursion set formulation of cluster abundance and evolution, solution of the *cloud-in-cloud* problem, i.e., counting as clusters only those objects which are not embedded in larger clusters, requires study of random walks crossing *one-barrier*. For void evolution a similar formulation requires study of a *two-barrier* problem; one barrier is required to account for *voids-in-voids*, and the other for *voids-in-clouds*. Thus, in our model, the void size distribution is a function of two parameters, one of which reflects the dynamics of void formation, and the other the formation of collapsed objects.

IIg. Dark Matter Collapse: Universal Infrastructure ?

45. Missing largely in the review is a discussion on the intimate relationship between the distribution of matter on supra-galactic scales and that of the (dark) matter distribution on subgalactic scales, down to smaller than kpc ... Interestingly, the assumption of the "stable clustering" approximation is that of fully virialized clumps without internal structure. Perhaps one of the few truely fundamental contributions by N-body calculations has been the finding that there may be an intimate link, of a self-similar "scaling" nature, between the product of gravitational collapse over a wide range of spatial scales.

a) The topic of the review suggests that the enormous amount of work and discussion on the settling of dark matterin halos with apparent universal properties, on galaxy scales, perhaps down todwarf galaxy scales as small as 10^6-$10^7 M_\odot$ as well as on scales of galaxy clusters is highly relevant for the present discussion. These finding strongly argue for profound "scaling" physics.

b) Perhaps best-known in this context is the work on scaling of collapsed halo profiles (the work by Navarro, Frenk and White (1996, 1997), and the ensuing chain of publications). The existence of a universal (asymptotic) dark matter density profiles, for a slew of clustering scenarios, may have profound repercussions for our understanding of gravitational clustering. As this forms part of the rationale behind the present review, it should pay attention to this topic.

c) For the physical insight into these issues earlier work on the nonlinear collapse of density fluctuations is relevant: the collapse and the ensuing secondary infall of surrounding matter leads to a power-spectrum (slope) dependent halo density profile, akin to the n dependence of $\xi(r)$ for the stable clustering regime. The early work by Gunn & Gott (1972), Fillmore & Goldreich (1984) and Hoffmann & Shaham (1985) provide substantial insight in the related physics. Now for more complex power spectra (not pure power-law but CDM-type) more complex universal relations have been recovered which may indicate a profound link to the virialization processes involved with the formation of clumps. Personally, I find the suggestion that its origin should be found in the phase-space stratification of the infall process most suggestive (Taylor & Navarro)

IIh. Nonlinear Clustering: N-body Simulations and Experiments

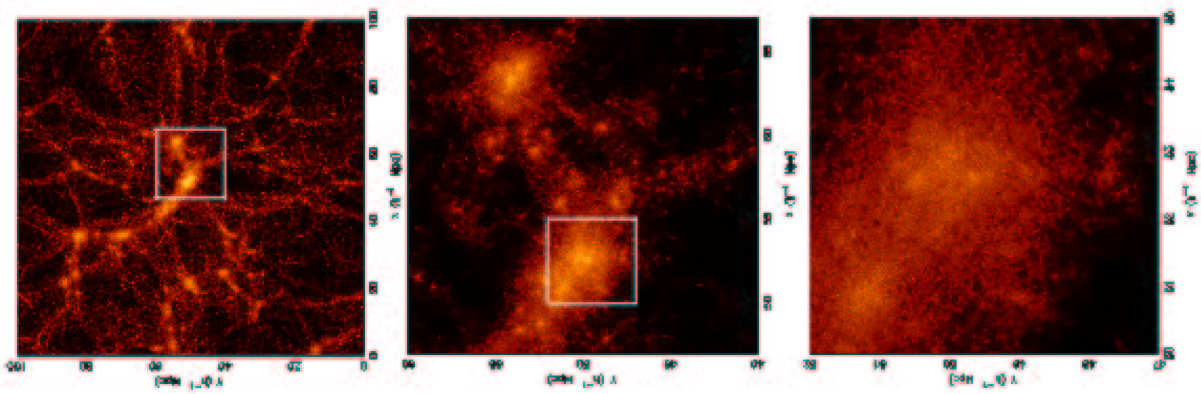


Figure 4: Three zoom-ins into a 128^3 SCDM N-body simulation, processed with the help of DTFE technique, showing the hierarchical nature of the resulting matter distribution. From: Schaap & Van de Weygaert 2000.

N-body "models" are treated in chapter VII "Clustering Models", However, in my view a more natural location would be in this chapter on the general background of gravitational clustering. Also, in view of the paramount role of cosmological simulations over the past 20 yeras, it may be good to upgrade the paragraph somewhat.

The suggested relocation stems from my opinion that N-body experiments should not be seen as models. They are tools for describing and investigating aspects of gravitational clustering within the current models and scenarios of cosmic structure formation and evolution, those aspects too complex to be assessed by means of analytical, statistical or simple numerical tools.

The advances in computing technology have been so vast, and the results of present-day N-body simulations so sophisticated and wide-ranging that the trade of N-body studies has established itself as a true new branch of scientific expertise. In addition to observational studies and pure theoretical/analytical work, the information produced by N-body experiments has become a necessary, substantial and complementary source for nearly all studies of cosmological structure formation.

On a more philosophical mode I find that the commonly used word of *N-body simulation* seems to indicate an intention of reproducing the Universe as complete as possible. A more modest word would be *N-body experiment*, which acknowledges the fact that computer simulations are always considerably restricted, often far more than acknowledge, in the amount of reality they contain, represent and reproduce. They are basically great and wonderful tools to probe particular complexities of reality, indicating which direction physical processes may work, but for always incapable of explaining it all !

Some issues the authors may elaborate upon:

46. Already at the start of the section, it would be nice to include a reference to the ARAA review by Bertschinger (1998), which stands as the sole best introduction into the subject of cosmological simulations

47. As for the historic background painted by the review in the first 4-5 paragraphs, I would like to add two key references.

a) The first N-body simulation, before the arrival of digital computers, was that by Holmberg (1941) of 2 colliding clusters. Using lightbulbs as particles, I find this experiment not only charming, but also testifying of genius and great inventiveness and ingenuity !

b) When mentioning that Aarseth et al. 1979 and Gott et al. 1979 (4th paragraph pg. 67) were the first papers using the modified code, the review should also mention the first cluster simulations by S. White (S.D.M. White 1976), still an outstanding reference.

48. The paragraph starting with "Is this enthusiasm justified" should not restrict its reservations to remarks on the lack of sufficiently reliable physical complexities such as gas dynamics, radiative transfer, etc. Perhaps best to make this into a separate subsection discussing systematically the practical virtues and shortcomings of N-body experiments. This not only concerns physical processes, but also technical details such as

   - limited mass resolution
   - limited spatial resolution
   - restricted dynamic range

a) For example, it is rarely appreciated that even in the most sophisticated codes the pretended density perturbation power spectra are only represented over a rather narrow range of scales. Thus, N-body experiments often do not really concern the cosmological scenario they pretend to represent, but a restricted version thereof. This issue of course ties in with the technical issue of discreteness mentioned in the text.

b) As for the analysis of the N-body simulations, I fully agree with the remark that studies usually restrict themselves to a few global (and standard) statistical analyses. May be stressed even more. I find it quite remarkable that while one of the main results of computer experiments has been the finding of a universal tendency of gravitational collapse to produce elongated or flattened anisotropic patterns, assembled into a foamlike structure pervading the whole Universe, this as yet has not lead to substantially more than qualitative remarks. While a few attempts to define proper statistical measures an exception to this, this lack of significant progress in

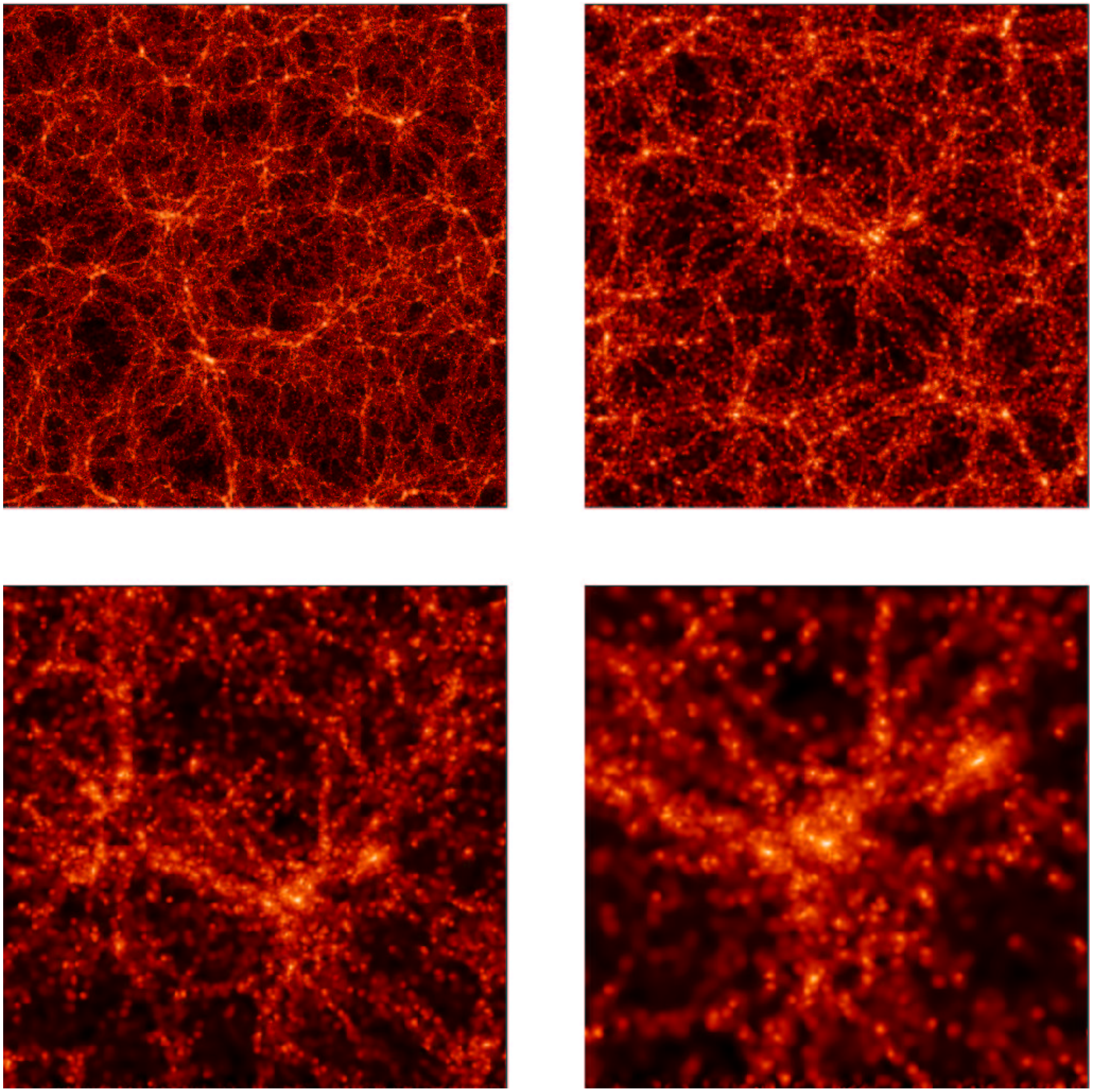


Figure 5: A mosaic of 4 blow-ups from a 256^3 particle simulation by the Virgo consortium, corresponding to a SCDM scenario mass distribution at $z = 0$. Courtesy: VIRGO/Colberg et al.

characterizing and quantifying the resulting complex matter distributions is also an illustration of an important but not always stated fact. N-body experiments are the computational equivalents of large observational datasets. True understanding can only be acquired through meticulous and incisive analysis, it is not the raw result of the simulations themselves (this is my political statement !).

49. Although the review here and there passes on remarks on the use of N-body experiments which not only include pure gravity but also more complex processes such as gasdynamical and star formation phenomena, it does not do real justice to the impressive efforts and advances in recent years. In my opinion it would be fair to elaborate on attempts to proceed along these lines:

- gas/fluid dynamics on the basis of SPH techniques.

- gas/fluid dynamics on the basis of fixed (Eulerian) grid methods: Cen (1992), and the related cosmo-hydrodynamical studies by Cen & Ostriker (e.g. Ostriker & Cen 1996).

- gas/fluid dynamics on the basis of moving grid methods (Gnedin 1995; Ue-Li Pen 1995; Xu 1997, for a general review see Mavripilis 1997, Ann. Rev. Fluid Mech. 29, 473)

- galaxy formation simulations on the basis of semi-analytical methods of heuristically encrypting crucial astrophysical processes (e.g. GIF simulations). These may be seen as pure "modelling", yet for consistency best kept in this section.

50a. To put the various advances in perspective, the authors may shortly discuss the various N-body techniques which allowed the progress in this field.

- Starting with the direct Particle-Particle codes, evolving into the state-of-the-art hardware GRAPE simulations ().

- Particle-Mesh codes to the P^3M codes (Efstathiou et al. 1985),

- Adaptive grid codes: AP^3M code by e.g. Couchman 1991, the adaptive grid code by Kravtsov, Klypin & Khoklov 1997 and the (many-level) adaptive mesh refinement AMR code by Bryan & Norman (Bryan & Norman 1997; Norman & Bryan 1998; e.g. Abel, Bryan & Norman 2000).

- Along a parallel line the treecodes (Barnes & Hut 1984), with their multipole force expansion technique.

- TREESPH. The treecode developed into ubiquitous tools, easier to accommodate gas dynamical processes: the TREESPH code by Katz & Hernquist (1989), combining gravity with Smooth Particle Hydrodynamics,

- culminating in the ubiquitious GADGET code of Springel (Springel, Yoshida, White 2001).

- Along yet another path developed the moving grid codes, combining the virtues of Lagrangian codes with Eulerian grid codes. Noteworthy are e.g. the moving mesh code by Gnedin (1995), the adaptive moving mesh code by Ue-li Pen (1995), the Delaunay adaptive mesh code by Xu (1997), and recently Trac & Ue-li Pen (2003).

- One may also discuss the codes extended with *radiative transfer* modules (e.g. Gnedin's and Abel's code). However, as this is not immediately related to the gravitational clustering issue, perhaps best point to stop the census.

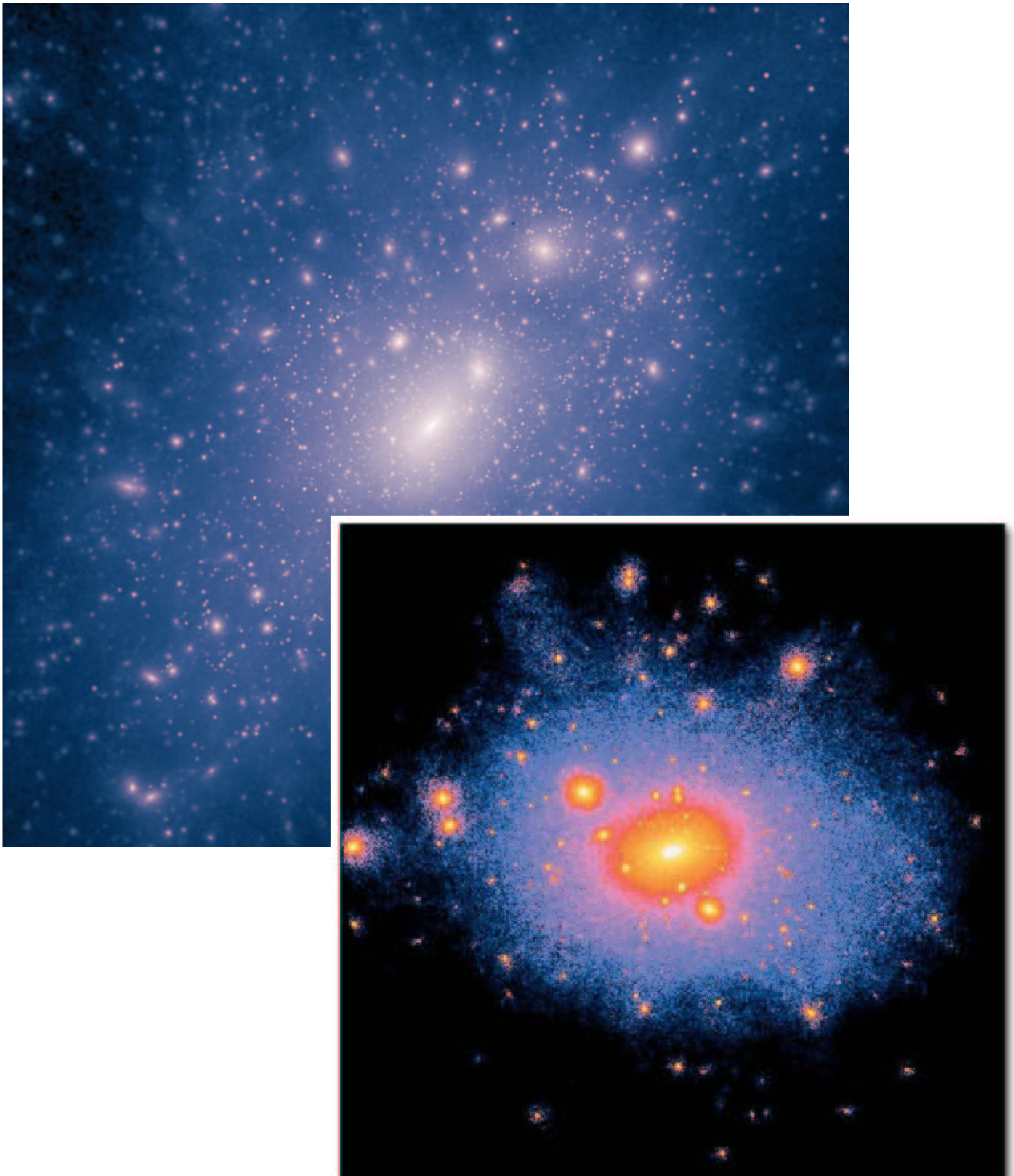


Figure 6: Similarity in the cosmic structure formation process: the formation of a cluster (from very high resolution simulation by V. Springel) and of a galaxy (T. Quinn), both in ΛCDM scenario.

b) The progress of the N-body simulations can also be appreciated from the gradual rise of number of particles (from a few 100, to 32^3 (e.g. Davis et al. 1985), with 128^3-256^3 becoming the current norm, and the state-of-the-art reaching up to 512^3-1024^3 (e.g. Hubble volume simulation).).

51. I would encourage also a more extensive "review" of influential N-body work, perhaps starting with the Efstathiou, White, Frenk & Davis (1985-1988) studies, which defined the first cosmological N-body references (in the sense that many cosmology interpretations referred to the results of these computer experiments), and finally leading up to the massive present-day simulations such as the Hubble Volume simulations (e.g. Jenkins et al. 2001)

52. Although the review contains an image of the Hubble Volume simulation of the Virgo consortium, I would recommend the use of one (or a few) other images. The Hubble Volume may show the state-of-the-art capacity of N-body simulation, for the subject of the review, scaling, it may be less illustrative. Thus it would be more interesting to include an illustration of true (intriguing) scaling in the Universe.

    • figure showing the similarities between galaxy-sized regions and evolution and those, later, on cluster-sized regions. Hence, I made an image of a truely awesome cluster simulation by V. Springel (Virgo consortium), together with a galaxy halo simulation (T. Quinn, Washington group). It provides a striking confirmation of the new knowledge provided by N-body work on the issue at hand (figure included in this report).

    • The Virgo website contains a nice image of 4 frames zooming in at different levels on one and the same simulation. For understanding of structure formation, at different spatial scales, this seems to be a more informative image (figure included).

    • The figure from the scale-free spectrum simulations by Smith et al. (2003, see section IIe) is another recommendable image of an N-body simulation pertaining to the issue of clustering scaling (figure included).

53. An interesting phenomenon is the rise of large (international) consortia for N-body simulations. Their website are important sources for scientific research, some even offering freely the resulting simulations. It may therefore be useful to include some of the most useful website references, e.g.

a) The first large coordinating effort was the GC3 consortium:

b) The University of Washington N-body shop:

   http://hpcc.astro.washington.edu/

c) The Virgo Consortium:

   http://www.mpa-garching.mpg.de/galform/virgo/index.shtml

   http://star-www.dur.ac.uk/

   including the Hubble Volume simulation:

   http://www.mpa-garching.mpg.de/galform/virgo/hubble/index.shtml

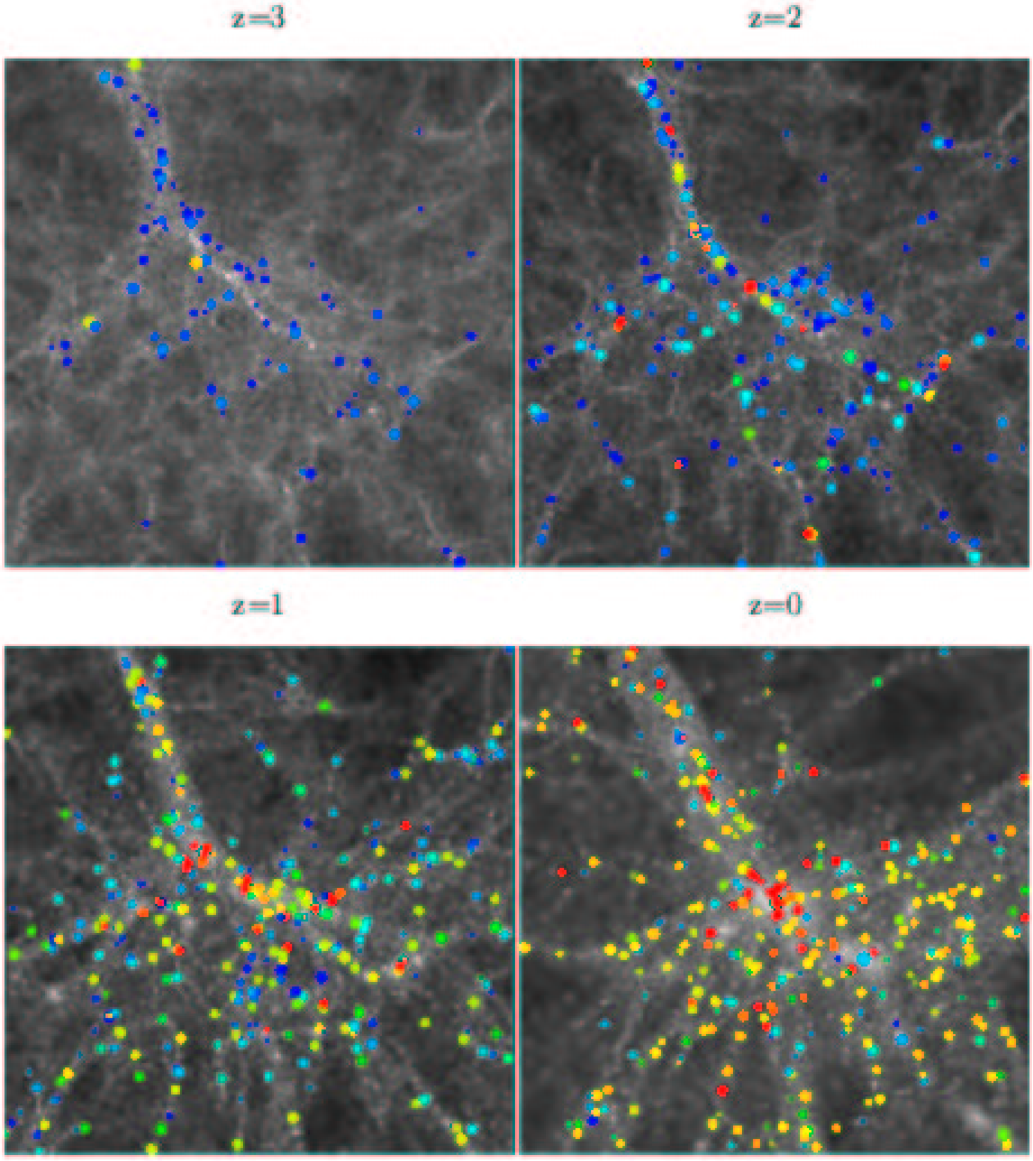


Figure 7: Evolving galaxy population in semi-analytical GIF simulation. Courtesy: Colberg & Diaferio/VIRGO/GIF.

"Semi-analytical" Modelling

54. N-body simulations have the disadvantage that they mainly concern the distribution of dark matter (or gas particles). For the predicted sites of the galaxies one needs to invoke a model description for the formation of galaxies. Because the review clearly tries to relate the statistical properties of the galaxy distribution, in particular the scaling, to the underlying process of gravitational growth of cosmic structure, I would recommend to extend the description on efforts towards this direction than the general remark on "... simple heuristic first steps".

a) The simple (simplistic) linear biasing prescriptions that were used in many studies since White et al. (1987).

b) The semi-analytical models of galaxy formation that have recently been combined with N-body simulations to yield "realistically" looking galaxy distributions in various cosmological structure formation models. As yet there are a few groups who have focussed on models along these lines:

• The GIF group (MPA, Munich). References: Kauffmann et al. (1999a, 1999b), Diaferio et al. 1999

http://www.mpa-garching.mpg.de/galform/gif/index.shtml

• The "Durham" group: e.g. Benson et al. 2001; Benson et al. 2003

• The "Santa Cruz" group: e.g. Somerville & Primack 1991

• The GALICS group (IAP, Paris): e.g. Hatton et al. 2003, Blaizot et al. 2003

http://galics.iap.fr/

IIi. Nonlinear Clustering: Renormalization Group

- In section VII.A.3, the *renormalization group* approach by Peebles (1980) is presented as a separate dynamical model. I would find it perhaps more appropriate to let it follow here, following the discussion on scale-free clustering. It seeking to bridge the difference between the limited dynamic range of N-body simulations and analytical descriptions (BBGKY hierarchy), the discussion on this aspect may be best placed at this location, fittingly closing the chapter on gravitational clustering and clustering scaling.

III. Discovery of Cosmic Structure: Observational Reality

55. Section IV, 'Discovering Cosmic Structure'. Before starting off with section A. 'Early catalog builders' I would find it good to have a general section explaining what the different sources of information on cosmic structure are. In this context it would be good to provide a short census of cosmic structures and objects which possibly contain information on the structure formation process (from globular up to supercluster). It would help at putting the ensuing sections in perspective.
    In other words, providing the perspective for the ensuing sections. Thus, possible identification:

    - galaxy distribution reflects the present-day matter distribution from vvmore than a hundred Megaparsec down to fine spatial scale

    - galaxy peculiar velocities probes dynamics of structure formation by tracing the implied matter migration streams ... due to lack of progress in dire state

    - gravitational lensing, weak and strong: perhaps most promising strategy for tracing the dark matter content in the Universe. Even though as yet outside of clusters the (large scale) cosmic shear has been measured "only" "statistically", it will not be long before true (spatial, even) maps of the matter distribution will be produced.

    - Absorption lines: in particular Ly a forest: Highly detailed view of gas distribution along a line of sight. Provides enormous amount of information on gas, yet needs understanding of gas dynamical processes (temperature, pressure and "equilibrium" of the gaseous medium) to interpret it within the context of the matter distribution.

    - cosmic microwave background: the primordial conditions. Having elaborated on these sources of structure in the Universe, it may be easy to note that as yet the galaxy distribution is still the only system around where one can probe structure deeply into the nonlinear regime where gravity has left its most prominent scaling signatures.

56. When adressing the issue on '(Discovering) Cosmic Structure' there is usually the issue of choosing between a focus on the 'observational technique'(redshift survey) or on the 'physical structures' themselves. The manuscript has apparently put the emphasis on the survey technique (redshift measurement).

    - Therefore there is less space and attention for genuine structures like 'clusters' and 'superclusters', and their role within the scheme of cosmic structure . Because one may argue that these mark a physically significant transition between a fully collapsed and virialized structure on the one hand (clusters) and a youthful emergent but not yet condensed structure (supercluster) on the other hand, this point should receive more attention (also so with respect to the intention of 'scaling studies' to identify underlying physics). With the enormous spread of observational potential and reach, in particular wrt. cluster research, the focus on redshift observations may rapidly lead to a somewhat biased view:

    - Suggestion, for purpose of transparency:

        - section on survey technology (the sky surveys and redshift surveys)

        - a separate and more compressed (!!!) section on the available redshift surveys, their technical specifications, and chronology.

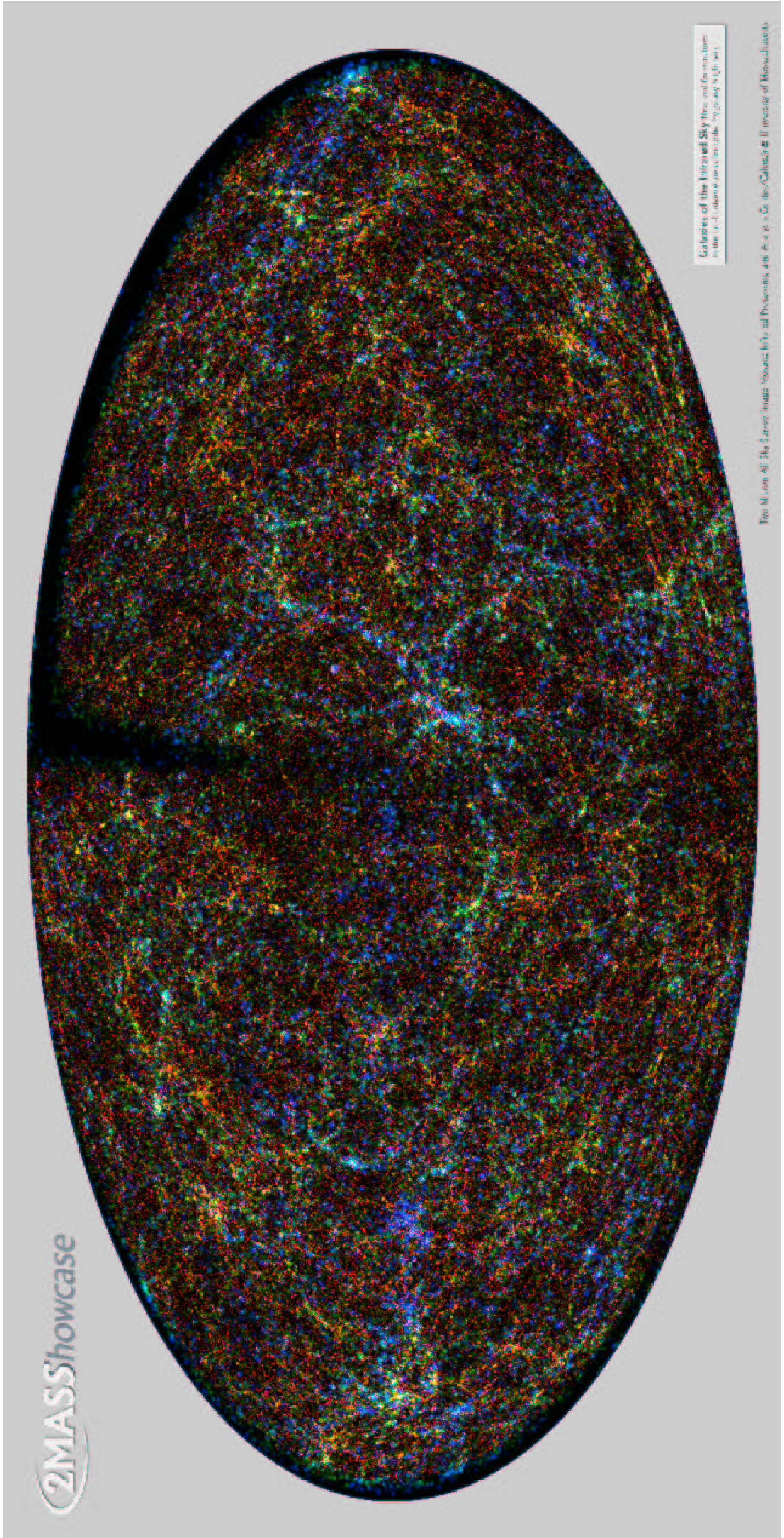

2MASSShowcase

Figure 8: Distribution of infrared galaxies on the sky, as seen by the 2MASS survey. Near and far structures in the local Universe are colour-coded by galaxy brightness. From: Image Credit: 2MASS/T. H. Jarrett, J. Carpenter, & R. Hurt

- a summarizing section focussing only on the identified and observed structures. Perhaps this may be combined with a separate discussion of the results of the 2dF and SDSS campaigns, as these will form the templates for much future work.

- For the presentation of 'discovering cosmic structure' it may be worthwhile to have more figures, to illustrate the diversity and nature of cosmic (infra)structure. Partially this may relate to a figure page at the beginning of the article, exposing the "hierarchy" of cosmic structure.

IIIa. Sky Distributions

57. Perhaps convert the 'Early catalog builders' section into a section on the 'galaxy sky distribution'. Perhaps good to have this in an entirely distinct section. Surely an illustration on the sky distribution should be included. Suggestions: the old Lick counts AND the newest 2MASS map (the colour one).

a) Also modern sky surveys like the APM and the recently released 2MASS sky distribution are relevant in this context. Ultimately, also the HDF is to be considered a sky distribution, in a very small angle of sky.

b) Also mentioned should be the superiority of sky surveys to have large numbers.

c) A major part of the SDSS is the 5-band sky survey. Mention estimated numbers of extragalactic objects. And mention the possibility to determine 'photometric redshifts' (including redshift range and error).

d) When discussing the Lick survey, it would be good to combine the part in IV.A ('The Lick Survey...') with subsection 1, followed by the paragraph 'But it were the Palomar ..." with subsection 2.

e) As in Fig. 5, it might be an interesting idea to have an illustration of the old Lick Survey count map (1 million galaxies), in cnojunction with e.g. the 2MASS sky distribution: both, even more than the APM map, show the rich texture of the galaxy distribution.

Deep Wide Angle Surveys

58. With the advent of gravitational lensing studies as a major probe for the cosmological mass distribution, a few large projects have started to probe the Universe through well-defined, uniform, deep wide-angle photometric surveys in patches of sky of one to few degrees. At the moment, the Canada-French-Hawaii-Telescope *Legacy Survey*, using the *Megaprime/Megacam* wide-angle camera on the CFHT, is probably the. In view of the impressive results concerning the discovery of cosmic shear (Van Waerbeke et al. 2000), the (weak) gravitational lensing by the large-scale inhomogeneous matter distribution, this project may hold the promise for one of the largest advances of our understanding of the Universe's infrastructure:

    http://www.cfht.hawaii.edu/Science/CFHLS/

Various other projects along a similar line are currently starting up, through the use of new telescope wide-field cameras (e.g. Omegacam on VST).

IIIb. Redshift Surveys

59. The section on redshift surveys should follow in this with the emphasis on the technical issues brought up in section B. What I missed was:

    • the remark that since the Universe contains complex and intricate structure, the projected sky distribution cannot reflect the full 3-D structure: the sky surveys are merely information-ridden projections of more complex geometries and patterns.

    • why redshift replaces the spatial depth I guess even part of the audience would not know why 'redshift' instead of 'distance' or 'depth'. Explaining that it provides a shortcut for distance in an Hubble expanding Universe would not be a luxury, and it would put also the section B2 on redshift distortions into perspective.

    • explain that redshift determinations are usually based on optical spectra. Perhaps specify the (atomic) lines that are most telling in this respect (given it concerns galaxy spectra). Yet, also mention the alternative of using the 21cm line at radio wavelengths (Giovanelli & Haynes, see later).

    • explain the various redshift survey volume strategies: slice, pencil-beam, full 3-D volume, complete sky (for local gravity studies), etc., and the rationale behind them: in essence, an application of 'stereology', the branch of stochastic geometry dealing with the measurement of structural information on the basis of sections.

    • in recent years, in particular with the advent of extensive and deep photometric surveys, the concept of 'photometric redshifts' has found widespread application. It may be worthwhile to mention and shortly discuss this technique.

a) In section B1 'Why do this':

b) suggestion to shift the part following 'Mapping the Universe ... unsatisfactory' in its entirety to the discussion on N-body simulations (VII.A.) It is more related to the rationale behind N-body simulations than to that of redshift surveys.

c) Also, swap subsection 2 with 3: the selection functions involved with redshift surveys are more important considerations than those of the distortions.

d) As for the selection effects, two additional ones are:

    • survey geometry. For example, the brick-geometry of the LCRS survey had to be taken into account very meticulously.

    • multi-fiber surveys often lead to a maximum number of surveys per observation. This often disturbs the high-density regions.

e) As for redshift distortions, first discuss the macroscopic effect of bulk motions (the latter part) and its distortive effect, followed by the more nonlinear signature of the finger-of-god effect.

f) indicate that the bulk motion distortion may provide an estimate of Ω, through its anisotropic effect on the 2pt correlation function (the latter may even incite an illustration: the result of the 2dF is still outstanding in this respect).

g) mention that the bulk flows are intimately coupled to the formation of structure as the *migration currents* involving the displacement of matter towards the emerging structures.

60. subsection 4 'corrections ...': 'There is, however, one universal correction':
It should be remarked that k-corrections are actually dependent upon galaxy type (difference between spirals and ellipticals), which is essential when seeking to compare spatial distributions between different classes of galaxies.

61. Compress the overview of the various redshift survey campaigns. In the manuscript the various redshift surveys, past, present and future are interwoven with a description of the structures revealed by the various surveys. It may be preferrable to treat the surveys and the revealed structures in distinct sections:
    - overview of available and coming redshift surveys
    - general census of revealed structures (the discovery of the cosmic foam, filaments, voids, great walls, etc.)
    - the work on specific clusters and superclusters may follow in a separate section, keeping in mind that the *cluster distribution* is another hierarchy level than the *galaxy distribution*.

Redshift Survey Overview

a) Section C: available redshift surveys. Some of the survey-specific details may not be necessary. I note a tendency to make remarks on the political issue of putting surveys in the public domain. It may be more apt to make one generic remark, with examples, in the 'redshift survey technique' section. For the further purpose of finding out about scaling it may be less necessary.

b) Combine the sections C1 and C2 on the CfA and SSRS, perhaps with a plot of the original slices ... they were the ones revealing the cosmic foam.

c) The ORS, QDOT and PSCz may be combined into a separate section, their goal being a complete coverage of the sky. Necessary for meaningful dynamical analysis: '...to get significant determinations of large-angle anisotropies in the local force field, dipole, quadrupole, ...' (remark would be included in paragraph 'The IRAS redshift catalogs ...'

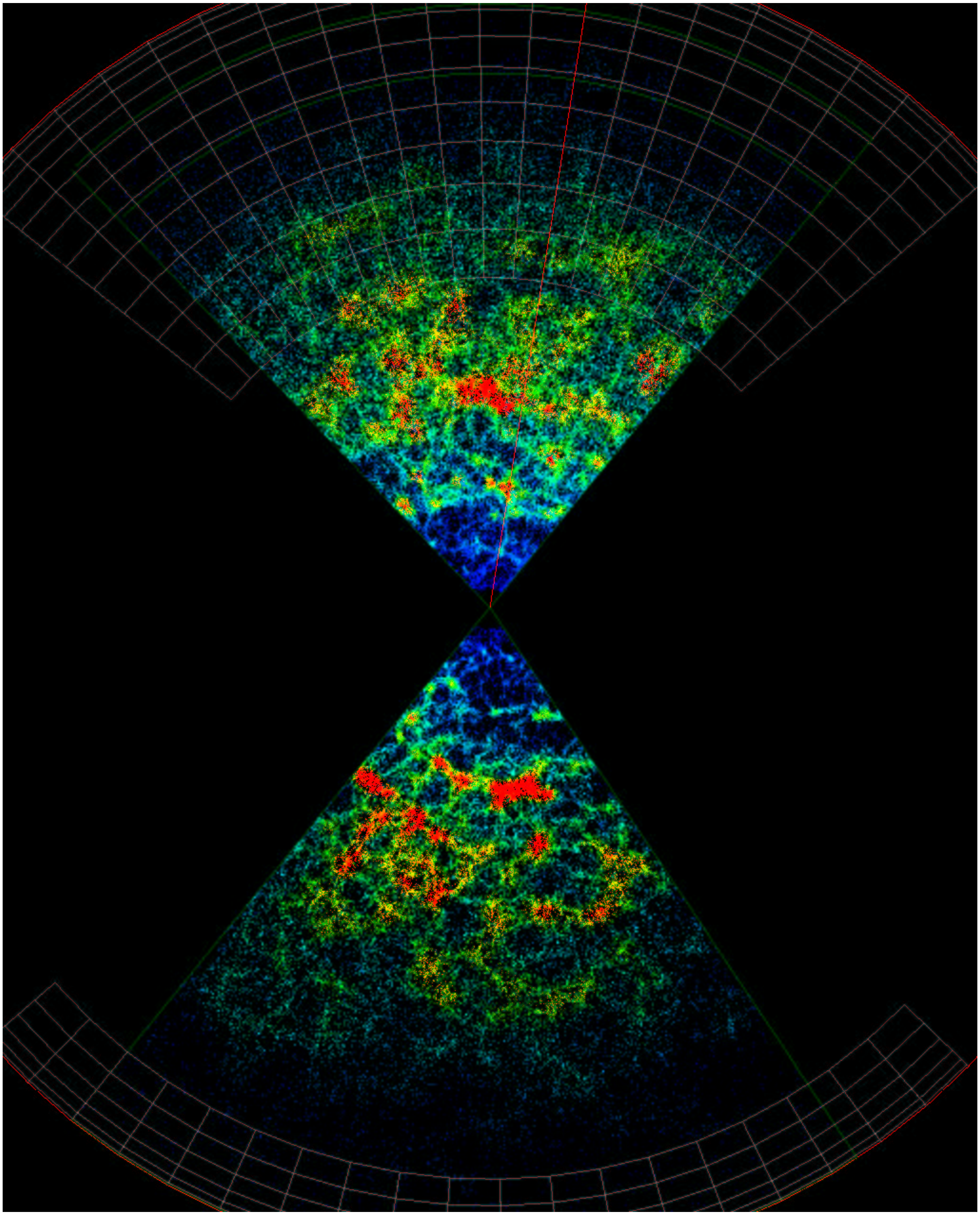
d) With 2MASS completed, a suggestive new sky-covering redshift survey would be based upon this catalog.

e) Inclusion of a figure with (smoothed) map of the PSCz survey with the derived peculiar velocity field (from e.g. Branchini) ? Would illustrate the use of such sky-covering maps for the study of local cosmic dynamics.

f) The last paragraph on the PSCz survey having been used for fractal studies should be relegated to specific sections on multifractal studies (section VIE).

g) Is it really necessary to treat the Centenary survey, the Stromlo-APM survey, and the Durham/UKST survey extensively ? Perhaps confining it to one/two paragraphs is sufficient.

h) The Las Campanas redshift survey may have sorted more information, certainly on the overall morphology of the structures. Perhaps the section should culminate in a discussion on the LCRS.



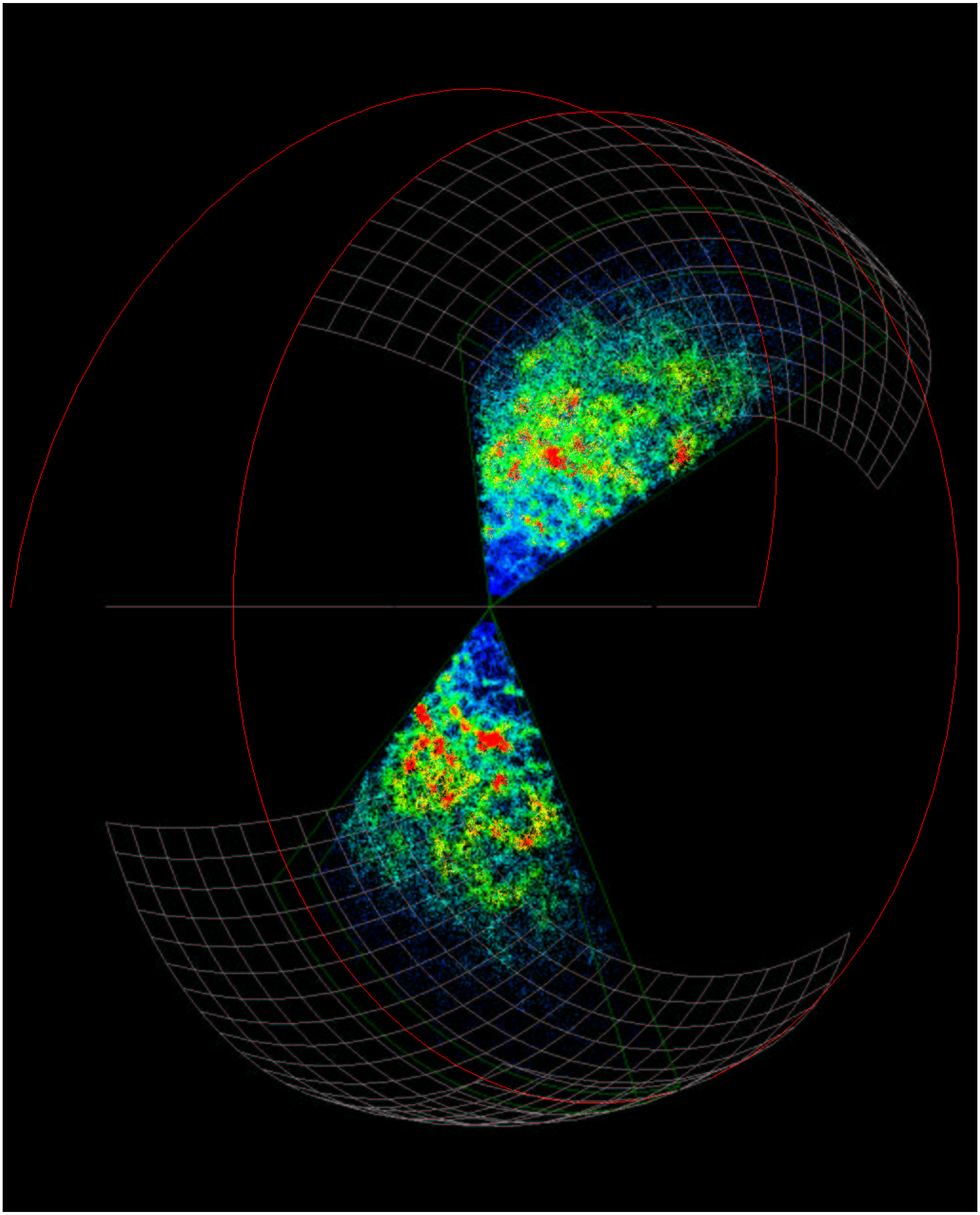


Figure 9: Top view of the 2dFGRS survey regions, with pixelized map of the galaxy distribution (courtesy: 2dFGRS consortium).

Figure 10: Side view of the 2dFGRS survey regions, with pixelized map of the galaxy distribution (courtesy: 2dFGRS consortium).

2dFGRS and SDSS Redshift Surveys

62. The 2dFGRS and SDSS redshift survey:
    Combined section, containing extensive descriptions of the characteristics and results of these surveys. They represent, for the coming years, the basis for our understanding of the spatial fabric of the cosmic galaxy distribution.

a) header suggestion, instead of "Surveys in Progress": something like the "New Maps of the Universe"

b) Given the interest in mentioning the policy concerning public accessibility of the survey data, it may be worthwhile to provide a reference to the 2dF website, as well as to them distributing CDs with the survey (the 100k june 2001 release).

b) Website references:
   - 2dFGRS:

     http://www.mso.anu.edu.au/2dFGRS/
   - SDSS:

     http://www.sdss.org/

c) in the meantime the final data release was on Jun 27, 2003, astro-ph/0306581 (with on page 2 both the sky coverage, and the "final" galaxy distribution).

d) a more extensive description of the technical aspects of the surveys. For the 2dF catalog it would be informative to have e.g. a description of the geometry of the survey volume (image ?).

e) provide a qualitative description of the structures observed in the 2dF map, their morphology, perhaps even with some "zoom-in". Also, given the interest in 2-pt correlation function, the sky-redshift 2-pt function published by the consortium may be a worthwhile product to show/discuss.

f) necessary to also inform about the quasar survey of 2dF, extending out much further.

g) There are impressive new images concerning the 2dF survey and its results around:

   http://magnum.anu.edu.au/~TDFgg/Public/Gallery/index.html

h) The description of the SDSS should contain a reference to the photometric survey (perhaps even more important), unprecedented uniform catalog in 5 colours !

i) Also mentioned should be the website of the SDSS survey (see above).

j) The colours enable the identification of very high redshift objects. In addition, there has been ample work on photometric redshifts (Connolly et al.).

j) It is only fair to provide references to the defining papers/work by Jim Gunn: the SDSS is his brainchild, both technically and observationally.

k) Akin to the 2dF the description of the morphology of the galaxy distribution, the discussion should be somewhat more extensive. The authors may contact A. Szalay, who has a beautiful "movie" showing the spatial galaxy distribution in very fine detail, showing in impressive detail the tenuous filamentary patterns pervading the local Universe ... (movie made by his son).

High Redshift Surveys

63. I am missing a reference to the pencil beam redshift surveys, starting with Broadhurst et al (1990), with in between improvements, and various others. In general, the authors may want to devote a section on these past high redshift survey efforts, including the current and ongoing ones (ESO/Sculptor by de Lapparent e.g).

b) Concerning the latter, the recent surge of activity for deep redshift surveys is highly relevant for the topic of the manuscript:

- The DEEP survey ($z \approx 1.3$) is the deepest uniform coverage redshift survey (Davis, Faber, Koo, Szalay etal.:

  http://deep.ucolick.org/

- GOODS, The Great Observatories Origins Deep Survey (Dickinson, Giavalisco et al), a multiwavelength deep survey combining the deepest surveys provided by HST, Chandra, XMM-Newton and SIRTF. Recently various papers released on astro-ph, also see website:

  www.stsci.edu/ftp/science/goods/

- the VIRMOS-VLT deep survey (le Fèvre et al.)

  http://www.astrsp-mrs.fr/virmos/vvds.htm

c) Particularly surprising is the recent work of the "Subaru Deep Field" (see Ouchi et al. 2004). This Lyα emission survey reveals a very surprising result on clustering at $z \approx 4$ (including 2-pt correlation functions):

  http://zone.mtk.nao.ac.jp/~kashik/sds.html

d) The manuscript is also advised to provide a reference, and short mentioning, of the work on Ly break galaxy studies (Steidel 1995). In particular the work by Giavalisco & Steidel (1998) paved the way towards a study of the clustering of these galaxies at high z, showing surprisingly strong levels of clumping.

e) Of course the major pitfall of the high redshift work is that the physical identity of Ly break galaxies is unclear, let alone that of the identity of Lyα emission regions (perhaps not even individual galaxies).

64. For the Lyα forest see below, point 35.

IIIc. Large Scale Structures: Census

65. Following the 'sky survey' and 'redshift survey' sections, it would be nice to include

a) a separate section on 'Structure Identification: Clusters and Superclusters', including both the parts on sky surveys (e.g. Abell) as well as redshift surveys and additional surveys.

b) a general outline of the structures that have been observed: clusters and superclusters

Clusters

66. It should be mentioned in the text that clusters are genuine physical objects, the most massive and most recently fully collapsed and virialized structures in the cosmic matter distribution.

67. Perhaps a similar section in the 'redshift' survey section (or an additional one after that) may treat the subject of galaxy clusters and their surveys as a combination of various surveys:

    • Optically, after the original work of Abell's and Zwicky's catalogues based on sky surveys

    • followed by large systematic redshift surveys: Stromlo-APM, ENACS (Katgert et al.), CNOC, 2dFGRS (de Propris et al.), REFLEX (Böhringer et al), Cluster Red-Sequence Survey (Gladders, Yee et al), the PDCS survey of distant clusters (Postman et al., ...).

    • X-ray based surveys (REFLEX): the most objective cluster survey is that of the X-ray REFLEX sample. In section E, it may be fair, given REFLEX owes most to H. Boehringer that references to REFLEX involve two of the describing papers with Boehringer as first author (only Guzzo 2002 is mentioned): Böhringer et al. 2001 (A&A 369, 826: sample definition) and Böhringer et al. 2002 (ApJ 566, 93: luminosity function). Also, it may be nice to have an image of an X-ray cluster (what about Fabian's recent Perseus cluster results ?). The figure shown in the review by Borgani & Guzzo (2001), comparing the REFLEX cluster sample with the LCRS galaxy distribution in the same region still occurs to be the best illustration of the relation between the two.

    • (weak & strong) lensing studies: the prospects of lensing studies of clusters (both weak and strong, the work by and following up on Kaiser & Squires, Ellis, Tyson, ... and for recent strong lensing:

    a worthwhile and highly illustrative image of a cluster, certainly recommendable to include, concerns the smashing set of cluster images, with lensing arcs, by Benitez, Broadhurst et al.: HST:

    http://hubblesite.org/newscenter/archive/2003/01/

    • systematic Sunyaev-Zel'dovich surveys: surveys soon coming on line: e.g Viper, VSZS, Romer et al., APEX, etc.

Superclusters

68. When addressing the identification of superclusters from sky surveys:

    • de Vaucouleurs was the one who identified the Local Supercluster (still the SG plane),

    • Refer to paper by Oort (1983, ARAA): it was not until after the review paper by Oort that the reality of superclusters got recognized.

    • For an astronomical lay public it may be good to explain the differences between clusters and superclusters. Superclusters are mere structures that started to contract or to detach from the cosmic expansion, but have not yet collapsed. As for identification, nearly all superclusters are identified via clusters, but they should be more.

- There is a nice supercluster website with beautiful maps of known superclusters:

  http://www.anzwers.org/free/universe/superc.html

69. While in section E the radio surveys are mentioned, I would find it appropriate to mention the 21cm line redshift survey of the Pisces-Perseus supercluster region by Giovanelli & Haynes in a supercluster section (also showing redshift surveys need not only be based upon optical spectral lines). It still yields the most telling example of a *filamentary* feature in the galaxy distribution. Also see earlier remarks on 21 cm line redshift surveys.

70. As I myself am getting charmed by the true monsters in our nearby Universe, the Shapley concentration and the Horologium supercluster, perhaps some mentioning of these would be warranted: the presence of these structures in the nearby stress our imagination and, perhaps even more important, those of our theories.

Voids

71. The concept of *voids* as one of the key aspects of the large scale matter distribution is largely missing (yet, see publication by Peebles, 2002). I feel it warrants specific attention. This includes the discovery of the Bootes void through a targeted redshift survey campaign (Kirshner et al. 1981, 1987), but also the attention void regions have received wrt. their sparse population of voids (e.g. the HI surveys by Szomoru et al. of the Bootes void).

IIId. The Global Universe

72. Section E: radio, X-ray and γ-ray sky. As may be clear from the above, I would prefer the X-ray cluster work to be contained in a cluster section following a general redshift survey section. Section E though should include a discussion on the X-ray background and its likely origin in unresolved high redshift AGNs (as the Chandra Deep Field seems to indicate).

b) Instead of the present section E title it may be good to limit this section to the topic of "clustering of radio galaxies and quasars" (in this respect the radio continuum work described may be combined with the X-ray background studies. I would split off the quasar distribution from section F and include it in E. Notice that both 2dF and SDSS have a lot to say on this subject from the optical point of view.

c) γ ray sky mentioned in the title, but not elaborated upon. Perhaps the gamma-ray burst may be mentioned and the fact that they define a near perfectly isotropic distribution on the sky.

d) In the context of wrapping up observational probes of structure in the Universe, future prospects also deserve mentioning. Earlier mention was given to the Sunyaev-Zel'dovich mapping of clusters. Additional enticing capabilities are HI 21 cm survey of the (re)ionization transition at the end of the Dark Ages. The LOFAR radiotelescope, close to being commissioned in either the Netherlands or Australia, and more future projects like SKA have the potential to provide detailed maps of the gas distribution at this decisive epoch, just before the Universe got populated with stars and galaxies (see various numerical/theoretical studies, e.g Madau, Meiksin & Rees 1997, Tozzi et al. 2000, Gnedin & Ostriker 1997, Ciardi & Madau 2003). Other interesting avenues are the search for the "lost" baryonic matter in the Universe, the warm and hot intergalactic medium supposedly confined to the filamentary structures in the Universe (Kaastra et al. 2003).

IIIe. Additional Cosmic Fossils

In addition to the cosmic galaxy distribution, the current manuscript restricts its discussion of probes of the cosmic matter distribution to merely one additional aspect, the cosmic microwave background as trace of the initial conditions out of which all structure in the Universe has arisen. However, in point (15) we have already indicated that there at least three other major probes of the fluctuation power spectrum, probing different spatial ranges: 1) cosmic peculiar velocity fields, 2) cosmic weak lensing by inhomogenous matter distribution and 3) absorption line systems probing the intergalactic gas distribution (Lyα forest).

73. Although it is notoriously difficult to sample the cosmic (peculiar) velocity field with any confidence beyond a depth of $\approx 100 - 150h^{-1}$Mpc, and the interpretation of the measurements are ridden by systematic effects, the estimates of the (velocity) power spectrum based on the available velocity data are interesting as they sample the scale at which the CDM power spectrum is supposedly turning over (references: Zaroubi et al. 1997, Freudling et al. 1999, Silberman et al 2001). In view of the discussion of the mass distribution asymptotically tending towards global homogeneity, particularly relevant wrt. fractal models, this seems also a relevant topic to address within the context of the current review.

74. In a review of the Megaparsec matter distribution the impressive advances of lensing studies in recent years, yielding the only entirely objective and unbiased probe of the matter distribution, should not remain unmentioned.

- Two ARAA reviews within the past few years may provide an impression of the promise of this field:

  Mellier, Y., 1999, ARAA 27, 127

  Refregier, A., 2003, ARAA 41, 645

- after early attempts by Tyson et al. (1990) it was the seminal work by Kaiser & Squires (1993), defining a powerful method to reconstruct the lensing mass distribution, which opened up the use of weak gravitational lensing towards probing the cosmic mass distribution.

- As yet, reconstructed "maps" were almost exclusively restricted to the locations with the highest concentrations of dark matter, clusters of galaxies. However, statistically the presence of (large-scale) *cosmic shear* was for the first time convincingly proven by Van Waerbeke et al. (2000). In the meantime the quality and quantity of measurements have become so good that a significant measurement of the lensing power spectrum is within reach, and present measurements have provided stringent constraints.

- The link between mass distribution and resulting lensing signal is most suggestively illustrated by (theoretical) maps such as those computed by Jain, Seljak & White (1997). Recently, techniques have been forwarded to reconstruct the 3-D spatial matter distribution from wide-angle weak lensing surveys (e.g. Hu & Keeton 2002). Surveys like the Legacy survey (see above, point 18) may be sufficiently good that they will yield the first spatial maps of massive and dense large scale matter features in between massive clusters, the surrounding filamentary and wall-like patterns.

75. **Lyα forest**: The section on the Lyα forest should be may be more substantial, it represents an additional entirely complementary probe of the (high redshift) matter distribution:

    - it is a unique probe of structure and clustering in the Universe.
    - and in the context of this review introduces the important concept of the intergalactic gas distribution out of which galaxies have formed.

    A discussion should involve some of the basic physics and of observational and statistical results. References to theoretical/numerical work showing this is gas in the cosmic web, work by Cen & Ostriker, Weinberg et al., on the fact that it is a probe of the thermal conditions of the gas in the early Universe (e.g. Schaye et al.). Perhaps an illustration of one quasar Ly alpha forest spectrum would be relevant.

IIIf. The Cosmic Microwave Background

Section G on the Cosmic Microwave Background needs a major rewrite after the release of the WMAP and the high resolution CBI results. Moreover, the review would benefit if this section would be rewritten, in order to embed it more strongly within the context of scaling and "hierarchical clustering". The introductory text in section G1 is so concise that an interested reader without background in the subject will probably fail to see the connection to the topic and focus of the review. A more extensive treatment of physics appears to be necessary to explain this.

Relevance for clustering scaling

76. The introductory text should point out why the microwave background structure represents a direct source of information on the initial conditions:

    - the CMB concerns photon temperature fluctuations, which besides a possibly intrinsic component in adiabatic structure formation models concerns the reaction of the radiation fluid to the primordial potential perturbations, the seeds for the formation of structure (and thus one should be careful in seeing this as a one-to-one map of the primordial mass distribution: "structure before our eyes").

a) even with the newest results of WMAP, balloonborne and groundbased experiments (Cosmic Background Imager in particular), the angular resolution is such that spatial scales significantly in excess of those of galaxies are resolved. The smallest fluctuations in WMAP are around supercluster scales, while the currently highest resolution obtained is that by CBI. The latter groundbased experiment reaches an angular resolution up to $l = 3500$ ($\theta \approx 6' - 15'$) and claim to have seen features with a mass of around $(5 - 80) \times 10^{14} \mathrm{M}_{odot}$, a genuine protocluster !!!! Instead of mentioning this in the last paragraph of G1, it should be clarified earlier, as it immediately relates to the impression that the CMB (already) shows embryonic galaxies.

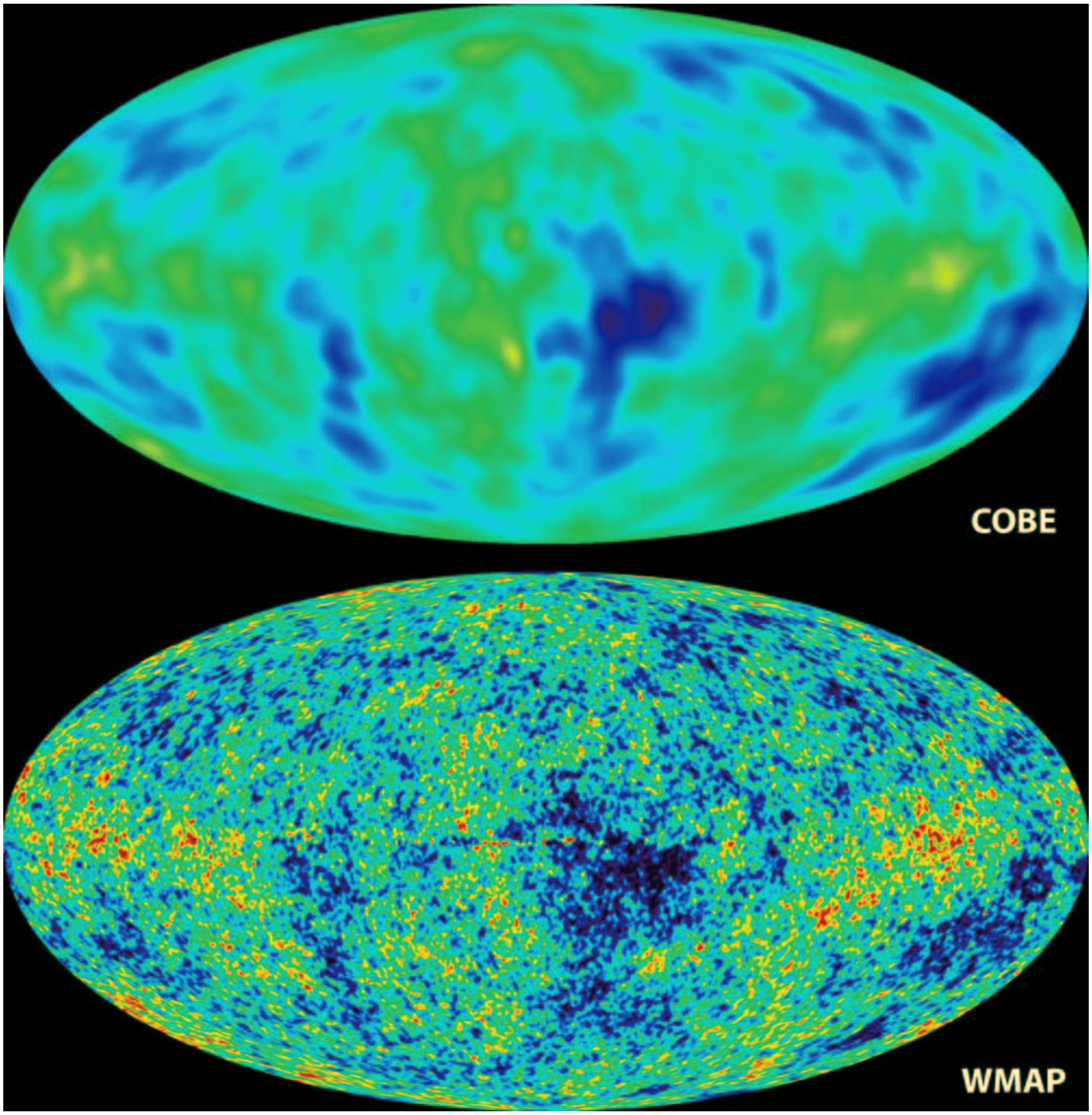


Figure 11: A comparison between the large-angle view of Sachs-Wolfe temperature fluctuations by COBE and the detailed view of mainly acoustic temperature fluctuaorkinm.tions seen by WMAP (courtesy NASA/GSFC).

b) direct CMB information on the topic of clustering "scaling" relates to the evidence of a primordial density field with a "hierarchical" density power spectrum $P(k)$, like the CDM-type spectra:

$$n_P \equiv \frac{d \log P(k)}{d \log k} > -3 \tag{12}$$

The text should emphasize therefore the attempts to measure the power spectrum of primordial fluctuations, and discuss the evidence that the spectra seem to be in concordance with power spectra leading to bottom-up clustering scenarios. The Cold Dark Matter scenarios are of course the most straightforward candidates, but a Warm Dark Matter spectrum may not be excluded.

c) When presenting section G3 on "initial conditions for galaxy formation" most of the discussion focusses on large scale perturbations, on scales larger than $\approx 150 - 200h^{-1}$Mpc. Galaxy formation pertains to substantially smaller scales, and the CMB section should therefore also pay attention to what happens on the smaller scales.

d) Because they are in excess of the typical matter-radiation scale the large scale are indeed in the realm of the primordial fluctuations spectrum. This is where the discussion on the Harrison-Zel'dovich spectrum, with a slope of $n = 1$ for the power-law power spectrum $P(k) \propto k^n$ refers to (first part section G3, pg. 37). As the matter-radiation scale is, for most viable cosmological models, very close to the (sound) horizon scale at recombination, this is the playground of pure Sachs-Wolfe (and early ISW) effects.

e) However, it is via WMAP and balloonborne experiments like Boomerang that a lot has been learnt about the smaller scales too. On these sub-horizon scales, as shortly hinted at in G3, the spectrum modifying physical processes kick in which finally mould the primordial Harrison-Zel'dovich power spectrum into CDM-type (or other) power spectra. For the galaxy distribution highly relevant is the evidence whether on these scales, approximately smaller than superclusters, still corresponds to a spectrum with hierarchical characteristics $n(k) < -3$. These kind of spectra are the most likely agents for the clustering scaling the review focusses on.

f) Therefore, more attention to the processes responsible for the measured CMB spectra is warranted, precisely on these smaller scales. Suggestion is to insert a subsection, after the introduction in subsection 1, shortly elaborating on the various processes involved with the measured CMB fluctuations. In particular to an non-expert audience this is quite necessary given the at first rather perplexing representation in terms of the undulating angular power spectrum $l(l+1)C_l$ (fig. 6).

g) Having elaborated on the various physical processes, one may then finish with the remark that proper evaluation of the full range of effects one needs to solve is complicated as one needs to evaluate the evolution of the photon (energy) distribution function while the photon mean free path is radically changing during recombination. Because a fluid approach does not suffice under such circumstances, one needs to solve the full Boltzmann equation for the evolving radiation-matter fluid to find the evolving photon distribution. The most efficient code accomplishing this (public domain) is CMBFAST.

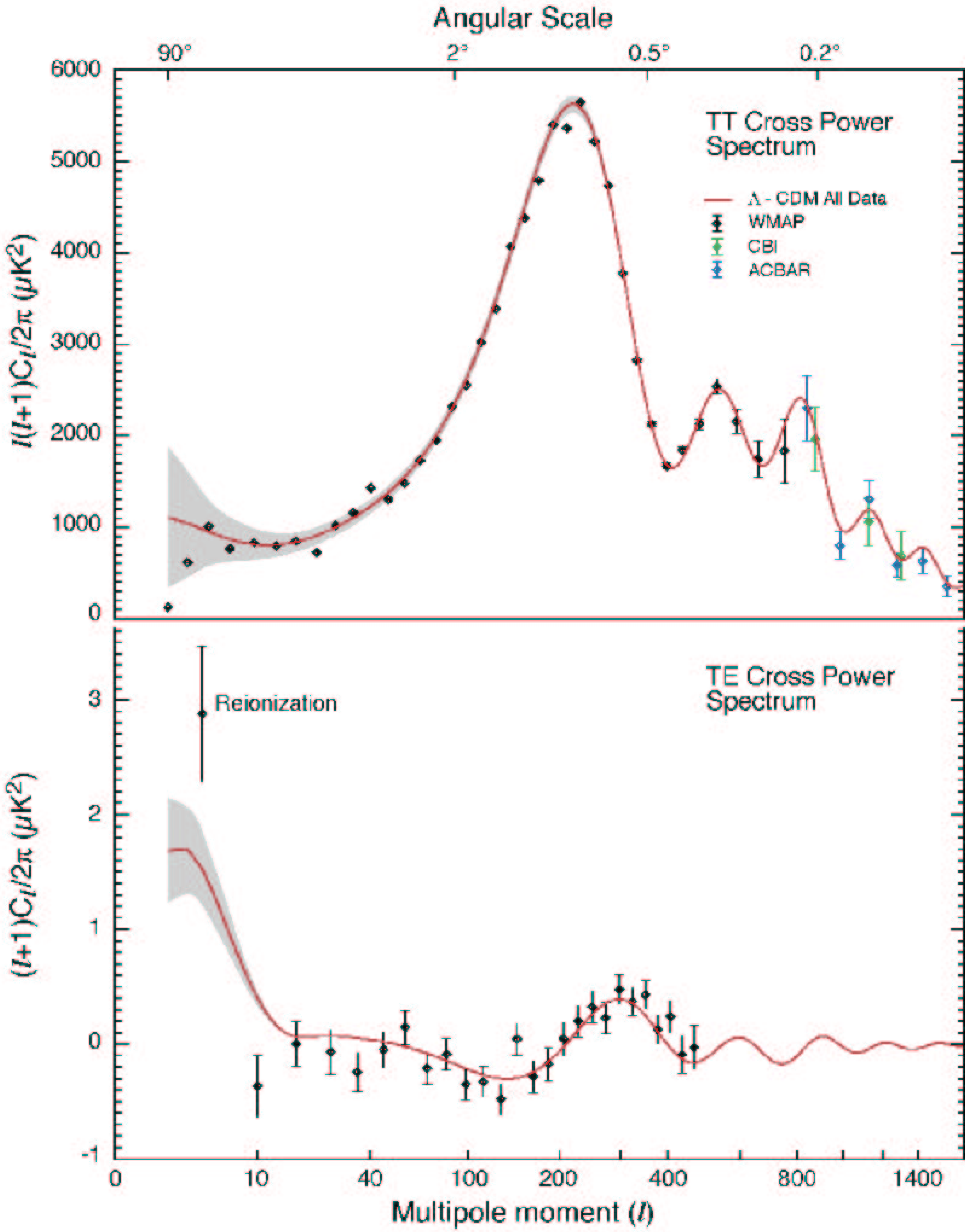

Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
TT Cross Power
Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
l(l+1)C_l/2π (μK²)
3
2
1
0
-1
Reionization
TE Cross Power
Spectrum
(l+1)C_l/2π (μK²)
0
10
40
100
200
400
800
1400
Multipole moment (l)

Figure 12: WMAP measured angular power spectrum of CMB temperature fluctuations (top) and temperature-polarization cross power spectrum (bottom).

Organization

77. It would be worthwhile to have an illustration with a genuine map of the CMB sky. I would suggest one of the figures available on the WMAP website, showing the map by COBE in conjunction with the much higher detail of the WMAP experiment of the full sky. See e.g. picture at top of my webpage:

    http://www.astro.rug.nl/~weygaert/iac2003.lecture.html

    For some drama, it may be worthwhile to zoom in upon a small region of the WMAP, showing details of the primordial density noise field (produce one's own "figure").

78. Together with the new maps, Figure 6 may be replaced by the newest estimated power spectrum, possibly a combined figure of the well sampled WMAP power spectrum with the far reaching CBI power spectrum. Also, one may contemplate to include the temperature-polarization cross-correlation function determined by WMAP.

79. Subsection G1 should be splitted in three subsections:
    - a genuine short introduction
    - physics
    - observations/data analysis

Introduction & Sachs-Wolfe effect

80. The introduction may correspond largely to 1st and 2nd paragraphs of section 1, involving in particular the **Sachs-Wolfe effect**.

a) It should be stated clear that the CMB fluctuations are to be divided in large scale and small scale fluctuations, defined by the angular scale of the horizon at recombination ($\theta \approx 1^\circ$). Above this scale the Sachs-Wolfe effect is dominant.

b) It may also elaborate on the language of the angular power spectrum. The CMB is a projection of spatial radiation temperature fluctuations, expressed in terms of spherical harmonics Y_l^m. Subsequently, remark that in terms of angular fluctuations, by means of spherical harmonics, a $P(k) \propto k^1$ Harrison-Zel'dovich spectrum generates Sachs-Wolfe effect temperature fluctuations which scale as $C_l \propto 1/l(l+1)$. This then clarifies immediately the normalization by $l(l+1)$ in the spectra (shown in e.g. Fig. 6).

CMB physics

81. When explaining about the physics behind the temperature perturbations, the review would benefit substantially from a more extended discussion of the effects occurring on scales relevant to the small-scale galaxy distribution (where scaling is important).

a) Instead of mentioning merely the Sachs-Wolfe effect and the Boltzmann code CMBFAST (preventing a blackbox), some of the physics behind the visible acoustic oscillations should be clarified in terms of the processes involved. I find the work

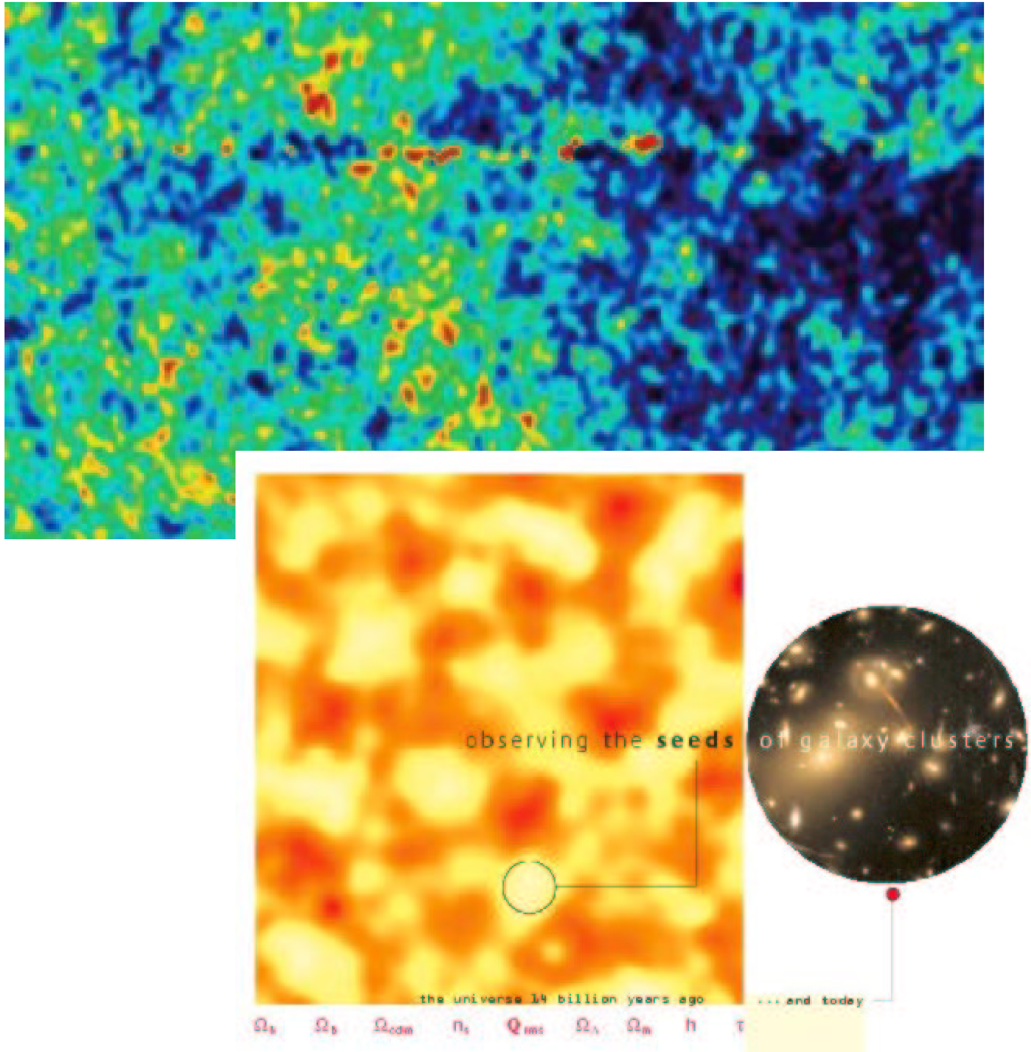


Figure 13: Structure in the Cosmic Microwave Background: a combination of a zoom-in onto a part of the WMAP sky map, in combination with a part of the high-resolution CBI map of the CMB. Indicated is the size of the region of a present-day galaxy cluster (image), which would emerge out of such a perturbation. Courtesy: NASA/GSFC (WMAP), CBI consortium.

by Wayne Hu amongst the most gratifying and impressive in explaining these. A reference to his website would certainly be apt:

http://background.uchicago.edu/∼whu/physics/physics.html,

as would be a reference to his thesis, one of the most marvellous Ph.D. theses I know of, see website:

http://background.uchicago.edu/∼whu/thesis/thesispage.html

other highly accessible references: Hu & Sugiyama (1995), the excellent short review Hu, Sugiyama & Silk (Nature, 1995), and the recent ARAA review Hu & Dodelson (2001).

b) Indicate or mention the various physical scales present in the CMB spectrum (see illustrations Hu, Sugiyama, Silk), list the (intrinsic) effects which play a role:

- **Intrinsic**: intrinsic (adiabatic) temperature fluctuations

- **Sachs-Wolfe effect**: the gravitational redshift + time dilatation photons suffer as they work their way through the gravitational potential fluctuations on the last scattering surface.

- **pressure waves/acoustic oscillations**: the pressure waves in the photon+baryon cosmic fluid, generated through the collapse and the reaction in terms of pressure resistance once a fluctuation enters the (sound) horizon. This small-scale effect is dominant on sub-(sound)horizon scales $< 1^\circ$, and conspicuously visible as the sequence of peaks first observed by Boomerang (first peak), now by WMAP up to the third peak, and even further by CBI (up to the Silk damping tail). It may not be too difficult to explain that this provides possibly the most objective measure of the Universe's geometry (total Ω) and induces a very characteristic scale in the C_l spectrum, the peaks and their locations. Also, one may elaborate shortly on the effects of the amount of baryonic matter (suppression second peak), the amount of dark matter (suppressing the amplitude of the radiation fluctuations), and other cosmic parameters.

- **Doppler shifts**: frequency shifts due to the motions of the primordial fluid. Minor wrt. intrinsic temperature fluctuations due to projection effects (while the name Doppler peaks for acoustic peaks is an historic accident).

- Notice: the acoustic oscillations of course also go along with velocity perturbations, which wrt. baryons express themselves in acoustic wiggles in the spatial density power spectrum $P(k)$, which may have been observed in the 2dF power spectrum.

- **Silk damping**: the washing out of fluctuations on very small (galaxy) scales due to photon diffusion, which will render it difficult to observe the primordial protogalaxies, and limiting direct observation to cluster scales and up.

- **Early Integrated Sachs-Wolfe effect**: the imprint of the early Matter-Radiation transition, accompanied by a change in cosmic expansion and thus evolution of comoving potential (which hardly evolves in the early matter epoch).

- **Late Integrated Sachs-Wolfe effect**: the imprint of the change of cosmic expansion at late transitions of cosmic equation of state and coupled expansion dynamics due to

  - curvature transition in a low-Omega Universe $\Omega_0 < 1$ at $a \approx (1/\Omega) - 1$

  - transition from Matter-dominated to Lambda-dominated Universe. Few recent extremely interesting publications on the latter, cross-correlating foreground galaxy samples with CMB (e.g. Scranton et al. 2003 for SDSS, Boughn & Crittenden 2003). Additional LISW sources are the impact of gravitational waves, nonlinear clustering (although better known as Rees-Sciama effect, unlike ISW not linear), cosmological artefacts, etc.

- the combination of the above leads to a characteristic cosmic parameter sensitivity in terms of four fundamental parameters. See Hu's page:

  http://background.uchicago.edu/~whu/araa/node15.html

82. As shortly hinted at towards the end of section G3, the detection of polarization in the CMB is among the most interesting and innovative developments of recent times. A whole new and complementary source of information on the primordial universe, in addition to the plain temperature perturbations, is opened up. Again, reference to the splendid website of Wayne Hu & Martin White, a CMB Polarization Primer:

    http://background.uchicago.edu/~whu/polar/webversion/polar.html

Additional references to the WMAP results, the DASI results shortly before that, the nice and concise comment by Hivon & Kamionkowski (2002, Science 298, 1349) and the review by Zaldarriaga (2003) may be invoked.
A short indication of the sources for CMB polarization may be gratifying (staying short from discussing the different E and B modes):

- intrinsic polarization as a consequence of Thomson scattering of the photons in the electron-photon plasma at recombination.
- tensor (gravitational wave) perturbations, the B-mode of polarization not yet detected.
- gravitational lensing of the CMB by foreground matter inhomogeneities
- the main reason for a lot of current excitement: the Thomson scattering by a reionized plasma at the end of the Dark Ages, and the estimate of an optical depth $\tau = 0.17$.

83. Would be fair to refer to the DASI discovery of the polarization,
preceding the WMAP release !

84. The current text does mention at a few locations *secondary* effects. Perhaps a slightly more systematic treatment in the Physics section may indeed be warranted. Important effects are the Sunyaev-Zel'dovich effect, the nonlinear clustering Rees-Sciama affect and of course the foreground polarization due to the reionization epoch. Like mentioned, the Sunyaev-Zel'dovich effect operates mainly on the small angular scales corresponding to the cores of galaxy clusters with sufficiently hot ICM. The others operate mostly on large angular scales. However, e.g. the primary effect of Silk damping will suppress detectable galaxy scale fossil signals !

Observations and Power Spectra

85. I recommend to include references to websites of various relevant CMB experiments:

- Satellite:
  - NASA Legacy Archive for the Microwave Bakcground:
    http://lambda.gsfc.nasa.gov/
  - COBE: vvvvvvvvv
    http://lambda.gsfc.nasa.gov/product/cobe/
  - WMAP:
    http://map.gsfc.nasa.gov/
    http://lambda.gsfc.nasa.gov/product/map/
  - Planck:
    http://astro.estec.esa.nl/SA-general/Projects/Planck/
- Balloonborne:
  - Boomerang:
    http://cmb.phys.cwru.edu/boomerang/

- Groundbased:
  - CBI:

    http://www.astro.caltech.edu/~tjp/CBI/

86. Having painted the physical background may have clarified why it is so relevant to measure the power spectrum C_l, the amplitude of the temperature perturbations as function of a scale. This involves tremendous technical challenges, it may perhaps be good to refer e.g to the website of M. Tegmark's, the CMB data analysis center:

    http://www.hep.upenn.edu/~max/cmb/pipeline.html

    discussing the various aspects involved in moulding a raw CMB temperature map into a optimally significant estimate of the CMB fluctuation spectrum C_l, currently up to a resolution as high as $l \approx 3500$ (WMAP probes up to $l \approx 1500$, CBI goes up to $l \approx 3500$, be it less densely sampled and with less significance).

87. Having elaborated earlier on the Sachs-Wolfe signature on large scales in the angular power spectrum C_l, for the Harrison-Zel'dovich spectrum leading to a horizontal line in the graph of $C_l l(l+1)$ versus l, it is straightforward to set up the argument of G3 on the estimate of the power-law index n from the observed CMB fluctuations.

88. Based on the earlier discussion on acoustic fluctuations, one may explain how the measured angular scale l of the first peak defines such a stringent constraint on the geometry of the Universe (flat).

89. Up to the third acoustic peaks has now been detected by WMAP (and balloon experiments). This informs us about the amount of baryons and the amount of nonbaryonic (CDM) dark matter.

IV. Clustering Measures

This part of the appendix concerns section V ("Measurements of Clustering") and section VI ("Further Clustering Measures") of the review. As various parts of these sections either overlap or are more related to each other than to parts in their respective sections, I would suggest some reorganization of the two sections. In addition, given the key role of these two sections within the review, and the extensive field of point clustering measures, the review should really elaborate more on various issues.

Instead of directly starting the section on "clustering measures" directly on the two-point correlation function, a more balanced account is perhaps to start with a general discussion and overview on clustering measures. The excellent book by Martinez & Saar, "Statistics of the Galaxy Distribution", will certainly provide some relevant material for point distributions.

90. First, the galaxy distribution is supposed to form a discrete representation of the underlying (dark) matter distribution. The statistical measures describing the point process of the galaxy distribution are supposedly related to the underlying continuous measures. In particular wrt. the correlation functions, the authors may issue statements about the relationships between the *continuous* and *discrete* measures.

The review puts heavy emphasis on the (2pt) correlation functions for analyzing the presence of scaling in their clustering. This evokes the question why the review chooses to do so, an issue which has not been adressed in the review. In fact, I would recommend a more general start, in which the review describes the various point distribution descriptors which may contain relevant information about intrinsic scaling properties. Some of the most straightforward and best known measures are in fact intimately related:

91a. the first order description of the cosmic density field, the 1-pt density distribution function (pdf of density).

b) counts-in-cells, closely related to the pdf of the density field, and the cumulants of these counts, closely related to the clustering/correlation descriptors of the point process.

c) the 2-pt correlation function $\xi(r)$, and the higher-order N-point correlation functions, quantifying a hierarchy of ordered clustering measures.

d) the power spectrum $P(k)$, and the range of higher order extensions (such as bispectrum), the Fourier transforms of N-point correlation functions.

92. As for the 1-pt probability density function, which should perhaps be the firs concept to mention in this section, it might be interesting to introduce the concept of a primordial Gaussian distribution gradually evolving into a increasingly skewed and asymmetric distribution, ultimately perhaps evolving into a lognormal distribution. Here i am also referring to the 2nd paragraph in VI.A. It should be no effort to state this is easy to comprehend as the negative δ values for the plain physical reason there one cannot be 'more empty than empty' are restricted to $\delta \geq -1$, while positive δ values can grow without limit. Interesting reference may e.g. be Kofman etal. 1994 on the evolution of the 1-pt distribution function evolving out of a Gaussian field.

93. In this respect, and with respect to its intimate link with the concept of fractal measures (section VI.E.4, VI.E.5, VI.E.6) it may be worthwhile to devote substantially more attention to the concept of *counts-in-cells*. It has been touched upon in section VI.E.2 and VI.E.3, but this basically only within immediate relation to the fractal description. More than any other measure closely related to the spatial density distribution in a density field, and defining the formal basis for the encryption of a multitude of spatial statistical measures, e.g. via cumulants, correlation functions, etc., it deserves an "independent" treatment. One may also refer to the large amount of work by Peebles, Gaztañaga, Colombi, Szapudi (and, also, Saslaw) and others: e.g. Peebles 1980, Gaztañaga (1992, 1994), Colombi et al. 1995, Szapudi, Meiksin & Nichol 1996, Szapudi 1997, 1998.

a) As for the attention devoted to scaling of the correlation functions, this aspect also warrants attention (as in VI.E).

Organization "Correlation Functions"/"Clustering Measures"

94. It may be helpful to reorganize the discussion in sections V and VI, so that various separate (sub)sections that logically belong together may be found in related parts, while the reader may feel the discussion to lead more poignantly to the relation with "clustering scaling".

+ It may be recommendable to shift the discussions on power-law clustering and the angular correlation function till shortly after the basic concepts of the spatial galaxy-galaxy correlation functions have been treated.

+ To prevent fragmentation of the (multi)fractal aspect of scaling, I would suggest to shift the references to correlation dimensions to a special chapter/section (including last part V.A.1, V.A.4, V.A.5 and VI.E.).

+ Special emphasis is placed on the "significance" of clustering scaling in the last section.

+ Although, a more optimal order may certainly be imaginable, one suggestion is the following (to which I will pertain in the coming pages):

  • **Two-point correlation function** (section B1):

  - Definitions and scaling

  - Relation (field) autocorrelation function and (discrete) 2-pt correlation function

  - Significance (physical) of the 2-pt correlation function.

  • **Higher-order correlation function** & Three-point correlation function (section VI.A and VI.B). Also the relation between continuous and discrete functions.

  • **Correlation Function Estimators**: two-point correlation function, possibly extended with estimators for higher order functions (section V.2).

  • **Power-law correlations**: power-law behaviour $\xi(r)$ (section V.A).

  • **Determinations (Measurements)** of the galaxy-galaxy 2-pt correlation function (part section V.B.1, section V.B.3)

  • **Angular two-point correlation function**:

  - $w(\theta)$,

- its relation to the spatial 2-pt correlation function $\xi(r)$ (Limber's equation), resulting power-law $w(\theta)$,

- the expected sample depth scaling in a Universe of large-scale homogeneity (large central part section V.B.1)

- Measured results for $w(\theta)$ and the finding of perfect depth scaling.

- **Redshift Distortions** Correlation Functions
- **Related Clustering Measures**: Power Spectrum
- **Additional Clustering Measures**

In the following, I will elaborate on the various aspects.

IVa. Two-point Correlation Function

95. Before entering the discussion on power-law clustering (section VII A) and observational results on the galaxy-galaxy correlation function (section VII B, subsection 3), it may be better to introduce in a formal fashion the concept(s) of two-point, three-point and general N-point correlation functions as descriptors/characterizations of the galaxy/matter distribution. Thus:

a) Shift section A till after the presentation of the basics.

b) Start with section B1 "definitions and scaling", on the spatial 2-pt correlation function.

96. Insert the arguments Bertschinger (1992, "New Insights in the Cosmos", eds. Martinez et al.) presented on the connection between the (continuous) autocorrelation function and the 2-pt correlation function. Notice that this involves the tacit assumption of the validity of the *ergodic theorem*, as the ensemble average of the autocorrelation function is replaced by a spatial average in the 2-pt function !!!!

97. Discuss in a more substantial fashion than normally encountered the (physical) significance of the 2-pt correlation function, i.e. clarify how it relates to the underlying mass distribution at various stages of cosmic evolution. Issues to be indicated:

    - first order measure of inhomogeneity/clumping, after the 1-pt density distribution function,

    - Fourier transform of the power spectrum, and as such a full characterization of the density field if this were Gaussian. This may be coupled to equation (30) in VI.A.

    - even for nonlinear circumstances, when it contains only a limited amount of information on the clustering process, it provides a direct link to the dynamics of the structure formation process through e.g. the *cosmic virial theorem* (in conjunction with the three-point correlation function).

    - (as mentioned later in VI.B and VI.E.3) for a particular class of nonlinear hierarchical models the whole range of N-point correlation functions are reduced to functionals of $\xi(r)$.

f) mention that its information content on the matter distribution is limited (as stated in 1st paragraph section VIA). A most insightful illustration was suggested by A. Szalay, and used by e.g Van de Weygaert (2002): Take a Voronoi foam toy model distribution, Fourier transform the density field, randomize the phases of the Fourier components, after which the inverse transform yields a Gaussian density field with

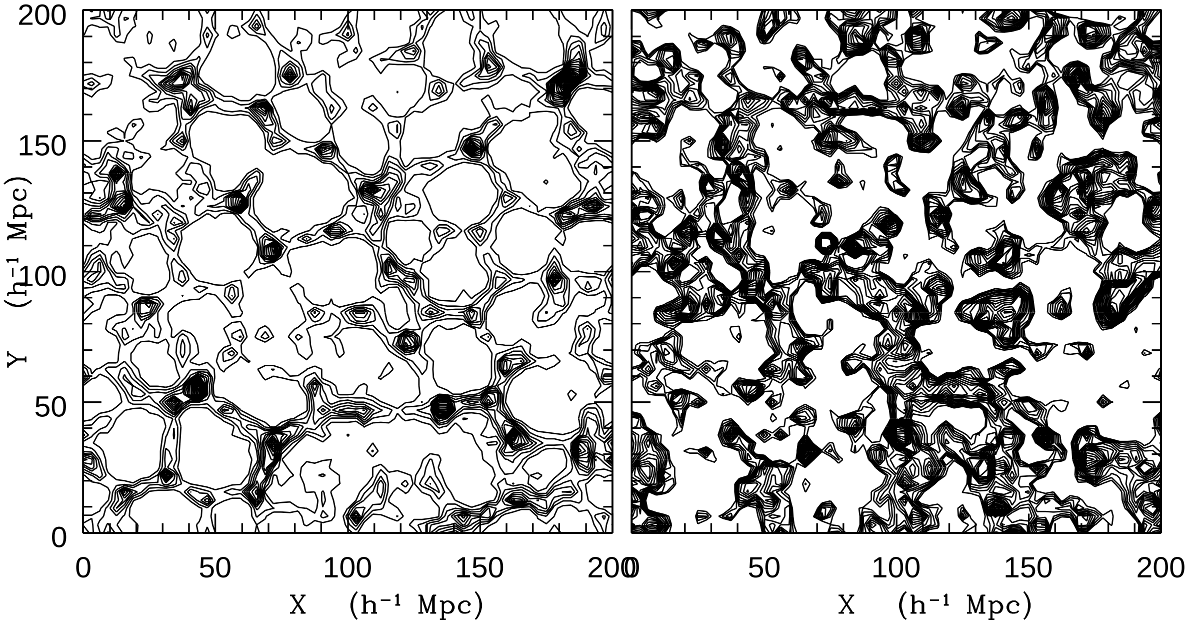


Figure 14: A density field with a characteristic cellular geometry (left), and its counterpart with the same power spectrum $P(k)$, yet scrambled phases. The contour levels of the righthand frame are chosen such that only positive δ levels are indicated. After a suggestion of Alex Szalay.

the same power spectrum $P(k)$ (and 2-pt function $\xi(r)$). The authors are welcome to use this illustration !

g) An issue of interest is whether one would see the imprint of nonlinear features in the galaxy distribution in the two-point correlation function. For example, Goldwirth, da Costa & van de Weygaert (1995) and Einasto et al. (1997).

IVb. Higher-order Correlation Functions

98. Section on Higher-order and Three-point correlation functions, as for $\xi(r)$ these all involve the basic "spatial definitions".

a) Perhaps join section VI.A and VI.B into one section

b) include definitions (by equation), and also include the concept of *general* and *reduced* correlation functions.

c) Also indicate the relation/difference between "continuous" field correlation function and the "discrete" N-point correlation function, and point out when the latter may be considered representative for the continuous function. Again, the reference above to Bertschinger's summerschool contribution (1992) may be very valuable.

d) Perhaps it would be good to include section VI.D., on the *bispectrum*, in this section of higher order correlation functions. As a separate subsection it would fit it very well with the context, while otherwise it would get a bit lost. I would find it fair to refer to the work by Scoccimaro (2000), providing a solid treatment of the bispectrum in theory and observation.

IVc. Correlation Function Estimators

99. Follow up with section VB.3, on the "Estimators". Most of this should naturally keep the focus on the 2-pt function, but additional remarks on estimators for higher order correlation functions might be relevant. A few remarks:

a) 3rd line, "For a discussion of them see ... ", include first and foremost Hamilton (1993) and Landy and Szalay (1993). Their discussion contains a large amount of information on the quality of other estimators.

b) when mentioning the kind of estimators favored by Pietronero (name in draft contains typo), one may also mention that in particular in small sample this operation tended to restrict determination of the 2-pt function to a few (central) points so that effectively one would obtain a "density profile" around these points and get completely dominated by cosmic variance effects (and completely discards the "ensemble average" aspect in the definition of the autocorrelation functions $\xi(r)$).

c) In addition to the mentioned estimators, the authors may mention an alternative estimator which

   - does not employ the galaxy counts in discrete distance bins (as indicated after eqn. 21),
   - uses a cumulative count (if I recall well, E. Saar set up an estimator following this technique).

- This technique proves to be more beneficial for probing the low-amplitude large-scale tail of $\xi(r)$, usually involving the regime where power-law behaviour no longer prevails and $\xi(r)$ is presented in a linear-linear plot. I myself used this to get significant estimates at distances beyond the correlation radius r_0, which has turned out to be rather succesfull (Van de Weygaert 2003a, 2003b, also 2002, and accompanying figures).

d) Recently, extremely efficient estimators of the two-point correlation function and higher order functions have been introduced (Szapudi 1997, Szapudi & Szalay 1998, Szapudi et al. 1998). These essentially enable highly efficient estimates for 2pt- and higher order correlations for catalogues containing millions of objects.

e) Perhaps also worthwhile to refer to the *sparse-sampling* strategy discussed by Kaiser (1986)

100. It is rather compelling to include a discussion on the expected errors of correlation function estimates and the various effects which influence these, such as sampling effects, discreteness effects, finite-volume and edge effects. In the meantime there is a substantial literature on the subject. Peebles (1973) and Kaiser (1986) were amongst the first suggesting substantial improvements to the simple *Poisson error bar*, while Ling, Frenk & Barrow (1986) suggested the *bootstrap and resampling* method for obtaining error estimates of $\hat{\xi}(r)$. Since these early studies work by e.g. Mo, Jing & Börner (1992) and more recent studies by e.g. Szapudi & Colombi (1996), Colombi, Szapudi & Szalay 1998 and Szapudi, Colombi & Bernardeau (1999) have been assessing the influence of the various errors.

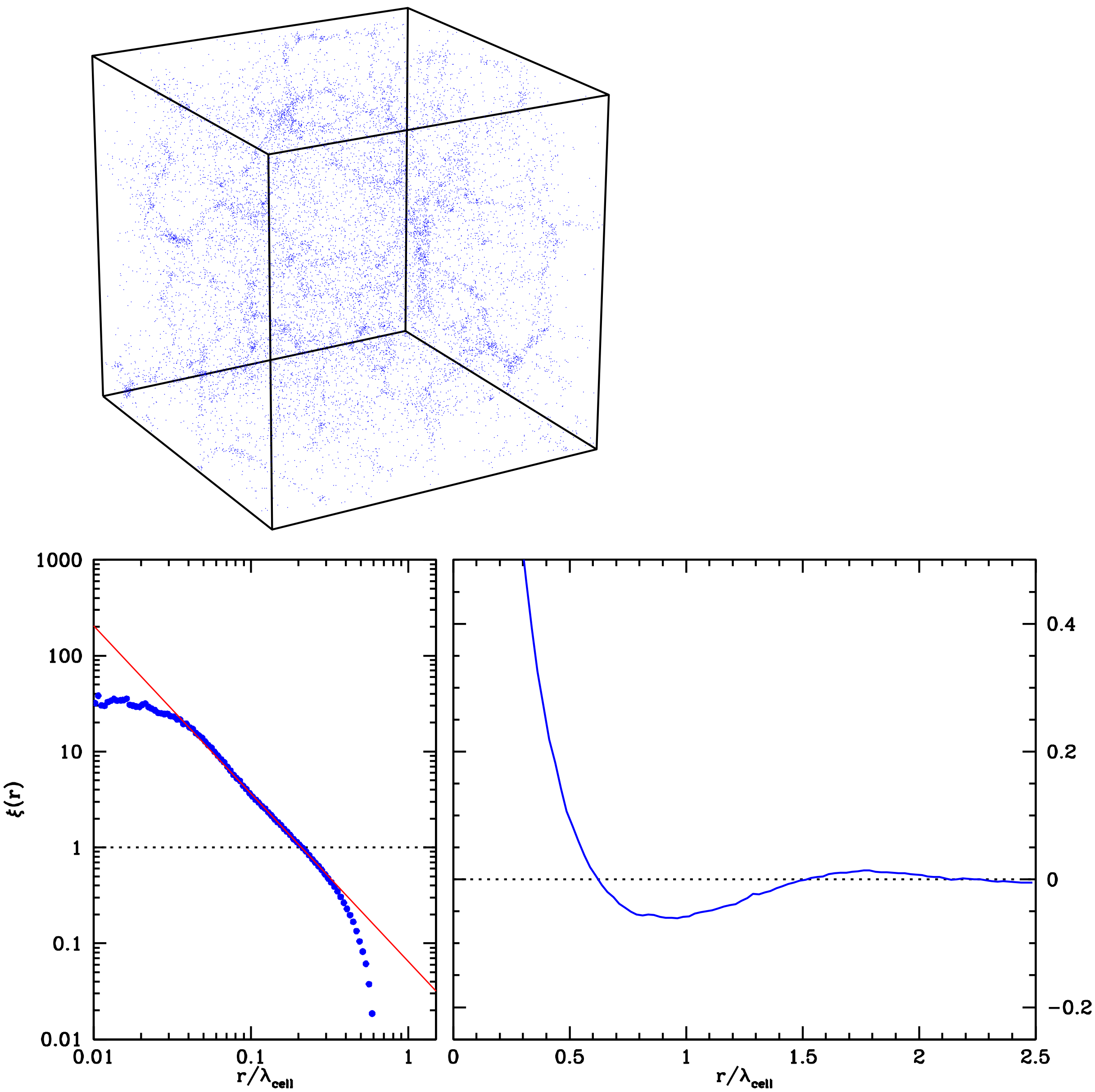


Figure 15: Two-point correlation function analysis of a selection of galaxies in a Voronoi kinematic model realization. Top frame: a spatial 3-D depiction of a full galaxy sample in a box of size $150h^{-1}$Mpc, at a stage corresponding to the present cosmic epoch $\sigma(8h^{-1}\mathrm{Mpc} \approx 1$. The cellular morphology with walls and filaments forms a marked pattern throughout the box, with sites of a few conspicuously dense cluster "nodes" standing out. Bottom left: a log-log plot of the $\xi(r)$, with distance r in units of the basic cellsize λ_{cell}. The power-law character of ξ up to $r \sim 0.5\lambda_c$ is evident. Bottom right: a lin-lin plot of ξ. The beautiful ringing behaviour out to scales $r \sim 2\lambda_{\mathrm{cell}}$ has been amply recovered. From: Van de Weygaert 2002, 2003b.

IVd. Power-law correlations

101. Discuss the power-law behaviour of the two-point correlation function $\xi(r)$. The first 3 paragraphs of VI.A should be the core of this section. However, given the FUNDAMENTAL importance of the power-law nature of $\xi(r)$ in this work, I would recommend to devote a few special lines and a one-line equation to this. As a suggestion I am quoting here from a recent text (Van de Weygaert 2002):

As for the real world, the most solid estimate of the spatial two-point correlation function of galaxies is inferred on the basis of the millions of objects in sky catalogues, through deprojection of the angular two-point correlation $\omega(\theta)$. On scales $\leq 5h^{-1}$Mpc the two-point correlation function is very well approximated by a power law, which implies a power-law spatial correlation function $\xi(r)$ (see e.g. Efstathiou 1996),

$$\xi_{gg}(r) \;=\; \left(\frac{r_o}{r}\right)^{\gamma}\,; \qquad\qquad \gamma \approx 1.8, \quad r_o \approx 5h^{-1}\mathrm{Mpc}\,. \tag{13}$$

Although direct estimates from 3-D redshift survey samples are complicated by discreteness noise, sampling and selection effects and redshift distortions, overall they tend to corroborate this power-law behaviour, also wrt. the parameter values (e.g. Davis & Peebles 1983).

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102. The definition of correlation functions in other branches of physics may differ in significant details, and e.g. the concept of *correlation length* may cause confusion to other physicists. Therefore I would suggest a pointed explanation of the definitions of *correlation length* and *coherence length* for a power-law correlation function. Possibly useful quote (Van de Weygaert 2002):

Conventional cosmological terminology expresses the amplitude of $\xi(r)$ in terms of the scale r_o,

$$\xi(r_o) \;=\; 1\,. \tag{14}$$

Conventionally denoted by the name "correlation length", we prefer the more correct name of "clustering length". Rather than a characteristic geometric scale, r_o is a measure for the "compactness" of the spatial clustering. A more significant scale within the context of the geometry of the spatial patterns in the density distribution is the scale at which

$$\xi(r_a) \;=\; 0\,. \tag{15}$$

As a genuine scale of coherence it is a highly informative measure for the morphology of nontrivial spatial structures, so that we reserve the name "correlation length" for this scale.

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103. While the power-law behaviour has been introduced, it should also be stated clearly that it is not clear to what extent this behaviour continues, although there are ample indications it breaks shortly after the correlation length r_0 (keeping to the cosmology definition). Here the discussion may pursue by pointing out it would be a natural consequence of reaching the "homogeneous"/"uniform" large scale matter distribution. Perhaps a generic model illustrating both regimes, a short-range power-law range and a large-scale tendency to homogeneity, is that of a galaxy distribution in a Voronoi foam (Van de Weygaert 2003a): which also illustrates the behaviour of $\xi(r)$ in the usual log-log plot, as well as in a lin-lin plot more apt for large spatial distances. Having depicted such behaviour, the result of Fig. 8 may be keenly understood.

IVe. Spatial Correlation Determinations and Measurements

104. Present the determinations/measurements of the spatial two-point correlation function $\xi(r)$, essentially section V.B.3., short of the paragraph on 'the angular correlation function for the SDSS".

IVf. Angular Correlation Function

105. After the section on the spatial correlation functions, and its small-scale power-law behaviour, the section on the "projected" angular two-point correlation function should follow. This would involve most of section 1B from 4th paragraph, pg. 40, starting from "A similar quantity ..." to the paragraph ending with '... 2dF and SDSS surveys':

a) 3rd line after eqn. 12, pg. 40: write result for angular correlation function in case of power-law correlation as a separate equation:

$$w(\theta) = A\,\theta^{1-\gamma}\,, \tag{16}$$

because the result is of central importance for the topic of the review.

b) Transfer the part of the text "Peebles (1993) ..", and rest of paragraph to a later part of this section. Mix it in within the paragraph 'The earliest catalogs available ...', as there are some redundancies.

c) Following the paragraph ending in equation $\mathcal{N} \propto D_*^3$, start with paragraph explaining the depth scaling relation, i.e. the paragraph starting with 'As the distance increases'. In my view the result may be explain physically as the consequence of 2 effects (one may even imagine a 'cartoon' figure to illustrate this):

- the angular extent of a given physical scale (and correlation length) is inversely proportional to the depth: $\theta_D \propto 1/D$

- the strength of the correlation diminishes due to the superposition of shells. Even when each of the shells contain pronounced clumpy features, the superposition of physically hardly correlated shells will lead to a increasingly random distribution on the sky and thus a dilution of the clustering signal on the sky. It may be evident that the effect is stronger as the depth D is larger: $w \propto 1/D$.

- A figure of appropriately scaled galaxy sky distributions (taken from an N-body simulation, or some 3-D toymodel) would perhaps provide a fitting illustration of the expected corrrelation function scaling.

106. present determinations $w(\theta)$, i.e.:

- part of paragraph starting with "Peebles (1993) has shown ..", mixed in with
- paragraph "The earliest ...".
- beautiful SDSS depth scaling relation recovered by Connolly (paragraph "Now we can do much better ..." and later paragraph, bottom pg. 44, also on SDSS result (to prevent redundancy).

IVh. Redshift Distortions Correlation Functions

107. In view of the later discussion on correlation function scaling, including my suggestion to include the discussion on the cluster correlation function as an example of "biasing" in that section, I would recommend to first discuss the redshift distortions of the 2pt correlation function (section V.D), and the information it may provide on the global cosmology.

108. Instead of the title "The pairwise velocity dispersion" one may perhaps use the more extensive "Redshift Distortions Correlation Function".

109. One may slightly change or specify the introduction around equation (25) and (26), which (26) concerns mainly the spherical model. Perhaps slightly more general velocity vector equations may also be good to introudce the factor $f(\Omega)$.

110. As for equation (27), it may be good to point out that the velocity power spectrum $\mathcal{P}_v(k) = \mathcal{P}(k)/k^2$ is different from the density power spectrum. Implicitly it is included in the eqn. (25), but at the same time not obvious. The immediate implication is a velocity field substantially more "quiet" than the density field.

111. The influence of cosmic flows on the correlation function is expressed in the breaking of its isotropy in redshift space. I would find it appropriate when this aspect, hinted at earlier, would here be mentioned in a somewhat more extended fashion. It should include the SEMINAL references to the work by a) Kaiser (1987) and b) Hamilton (1998). In addition it may include specific references to the work by Hamilton (1992, 1993) addressing the issue of how to infer Ω from such distortions, Zaroubi & Hoffman (1996) computing resulting distortion patterns in the linear theory of cosmic flows, and the work by Fisher et al. (1993) and Feldman, Kaiser & Peacock (1994) discussing the effects on the power spectrum estimate Hamilton (1992, 1993). Finally, it would be interesting to have included an illustration, for which I would recommend the redshift space correlation function $\xi(\sigma, \pi)$ of the 2dFGRS survey (Hawkins et al. 2002, astro-ph/0212375 see website).

112. Having treated the aspect of the large-scale cosmic migration flows, which have been studied extensively, it may be better to treat the small-scale velocity dispersion in a separate subsection. However, in the present manuscript, the discussion on σ_{12} looks rather lost within the larger context. The authors may sharpen the rationale of including this aspect.

113. In this section, one may also feel encouraged to address the issue of the velocity-velocity correlation function. Besides its definition and application to studies of gravitational clustering, the related concept of relative pair velocities is an important ingredient of studies of gravitational clustering in scale-free circumstances (see e.g. Efstathiou et al. 1988).

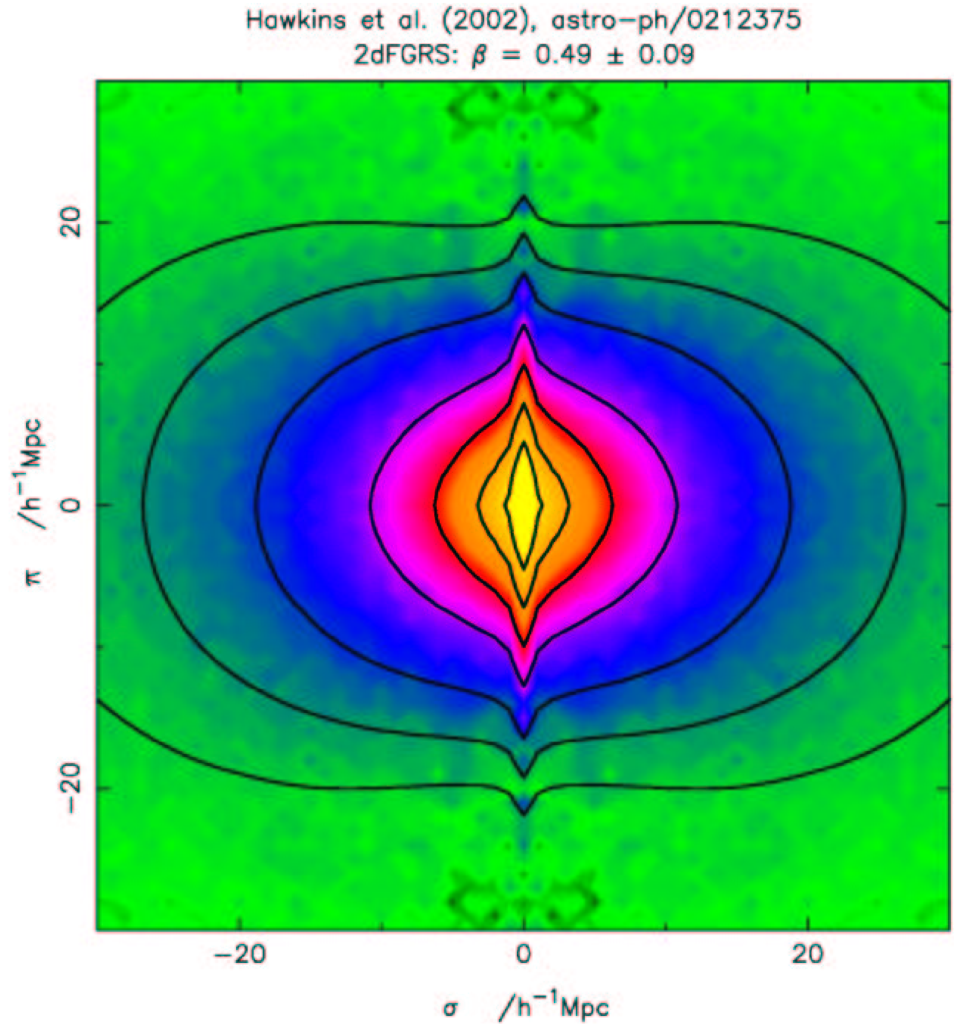


Figure 16: Redshift space two-point correlation function $\xi(\sigma, \pi)$ for the 2dFGRS survey. See: Hawkins et al. 2002

114. I am missing the concept of "cosmic virial equation", relating the velocity dispersion σ_{12} to the two-point and three-point correlation functions ξ and ζ. Wouldn't it be appropriate to include it as an equation in the text ? And, also to include references to the works by Peebles (1976, 1980) and Davis & Peebles (1977). An possibly interesting additional reference: Bartlett & Blanchard 1996.

115. A great number of studies addressed the issue of the pair velocity dispersion distribution for gravitational clustering models. This evidently being an important characteristic of the clustering process may deserve some interesting references for further study. Interesting recent ones are e.g. Sheth 1996, Juszkiewicz, Springel & Durrer 1999, Sheth et al. 2002, and one study specifically focussing on the issue of self-similarity and hierarchical clustering, Bharadwaj 1997.

IVi. Related Clustering Measures: Power Spectrum and Phase Correlations

Power Spectrum

Although the power spectrum is shortly mentioned, its close relationship to the correlation function $\xi(r)$ and thus to the issue of clustering scaling warrants more attention. Trying to determine the power spectrum $P(k)$ directly from the many sources of information on cosmic structure is one of the major activities of cosmological studies. While in principle containing the same information, for some aspects/regimes $P(k)$ is to be preferred to as clustering measure as $\xi(r)$, in particular so on spatial scales where $|\xi(r)| \ll 1$.

116. The discussion should perhaps seek to shift the balance slightly and mention/refer to at least some of the major references on direct measurements of $P(k)$. Provide some discussion of the various methods, particularly concentrating on the machinery that has been developed to extract significant estimates of $P(k)$ from galaxy redshift surveys (while major parts of the spectrum are covered by the e.g CMB measurements, weak lensing studies and the analysis of local cosmic flows).

117. Some suggestions for relevant references:

   - Feldman, Kaiser & Peacock , 1994

     Power-spectrum Analysis of Three-dimensional Redshift Surveys

   - Vogeley & Szalay

     Eigenmode Analysis of Galaxy Redshift Surveys. I. Theory and Methods

   - Tegmark, Taylor & Heavens, 1997

     Karhunen-Loève Eigenvalue Problems in Cosmology: How should we tackle large data sets

   - Hamilton, 1997

     Towards optimal measurement of power spectra - I. Minimum variance pair weighting and the Fisher matrix

   - Hamilton, 1997

     Towards optimal measurement of power spectra - II. A basis of positive, compact, statistically orthogonal kernels

   - Tegmark, Hamilton & Xu, 2002

     The power spectrum of galaxies in the 2dF 100k redshift survey

   - Dodelson et al., 2002

     The Three-dimensional Power Spectrum from Angular Clustering of Galaxies in Early Sloan Digital Sky Survey Data

   - Seljak, U., 1998

     Cosmography and Power Spectrum Estimation: a unified approach

Phase Correlations

118. Phase correlations are directly related to the nonlinear phase of structure evolution (starting from Gaussian Initial Conditions). Recently there have been various attempts to quantify this on the basis of spatial matter/galaxy/temperature distributions, it may be fitting to include appropriate references discussing either

the interpretation in terms of clustering, or the measurement and quantification of phases:

- Ryden & Gramman 1991; Scherrer, Melott & Shandarin 1991

- Jain & Bertschinger 1996, 1998 (also see earlier section on gravitational evolution scale-free power spectrum scenarios).

- Chiang & Coles 2000; Coles & Chiang 2000; Watts, Coles & Melott 2003; Watts & Coles 2003; Coles et al. 2003 (astro-ph/0310252).

IVj. Additional Clustering Measures

In addition, amongst the large number of additional suggested point clustering measures, there have been a few which also relate intrinsically to the "scaling issue", and/or aspects thereof. Except for the fractal descriptors (which I would prefer to concentrate in one central "chapter/section", some of these are certainly worthwhile to be mentioned:

119a. The *shape statistics* measure, see Babul & Starkman (1992) and Shan & Vishniac (1995), Davé et al. (1998).

b) Minkowski functionals (Mecke, Buchert & Wagner 1994, Kerscher 1996, and the large number of studies that followed up on this)

c) The SURFGEN technique developed by Sheth, Sahni, Shandarin & Sathyaprakash (2003), which seeks to define a cleanly defined method to describe the geometry and topology of the foamlike large scale structure patterns by defining a triangulated surface, which then can be analyzed by e.g. Minkowski functionals to characterize the surface.

d) Wavelets (see section on multifractal descriptors)

e) Void probability function (in some sense this belongs in the realm of number counts and is a "misnomer" in its suggestive reference to "voids").

V. Clustering Scaling & Biasing

Extended discussion on relations and meaning of clustering scaling and its manifestation in correlation function systematics (last part section V.B.1, section V.B.4, section V.B.5, part section V.C.3, and additional discussion). With clustering measures having been defined in the previous subsections, the last section of this Clustering Measure section may devote itself to the issue of Clustering Scaling and Biasing as revealed through these clustering measures. In particular this relates to the scaling of the two-point correlation function (section V.E.).

120. First, I would like to forward a suggestion for the discussion on scaling. This involves the introduction of three distinct (perhaps more) modes of scaling, implicitly present in studies on this issue.

    a) *structure scaling*

    b) *mass scaling.*

    c) *correlation scaling*

    In the following I will explain this distinction: *structure scaling* attempts to phrase what the power-law character of the correlation function implies in terms of structure growth. *Mass scaling* is what usually relates to *bias* of a population, and implies a "self-similar" clustering behaviour. Finally, *correlation scaling* is basically the statistical models forwarded by e.g Fry (1984a) and Balian & Schaeffer (1989a).

I leave it up to the authors to include this section in the general chapter/section on clustering measures, or to define a separate section on this issue.

Clustering Scaling Va: Scale-free Structure Evolution

121. Being a scale-free power-law function, whose functional behaviour remains mostly intact, the universal (function-preserving) amplitude evolution of $\xi(r)$ (certainly in the asymptotic linear and nonlinear regimes) means that the spatial distribution of galaxies within a particular volume will be the same as that in an proportionally larger volume at a later cosmic time: time evolution involves a *"self-similar mapping"* of the spatial point distribution, in concreto its statistical characteristics, from one scale to another. One would not be able to distinguish between a certain volume at one time and an appropriately larger volume at a later epoch:

122. In its purest and ultimate asymptotic extrapolation, this is what the *renormalization group* description of gravitational clustering seeks to describe (Peebles 1980, Couchman & Peebles 1980). Perhaps section VII.C.3 "Dynamical Models 3. Normalization Group" would be best placed within the "Gravitational Instability" chapter (see above), along with BBGKY hierarchy, halo model and others related to hierarchical clustering.

Clustering Scaling Vb: Population Dependence

123. In V.C. the review treats the *cluster-cluster* correlation function. The difference between the *galaxy-galaxy* clustering and the *cluster-cluster* clustering is what I rank as *mass scaling.* Because the cluster distribution appears to form an amplified reflection of the underlying galaxy distribution (which may be a similarly biased version of the underlying dark matter distribution). The *scaling* leads to a self-similarity

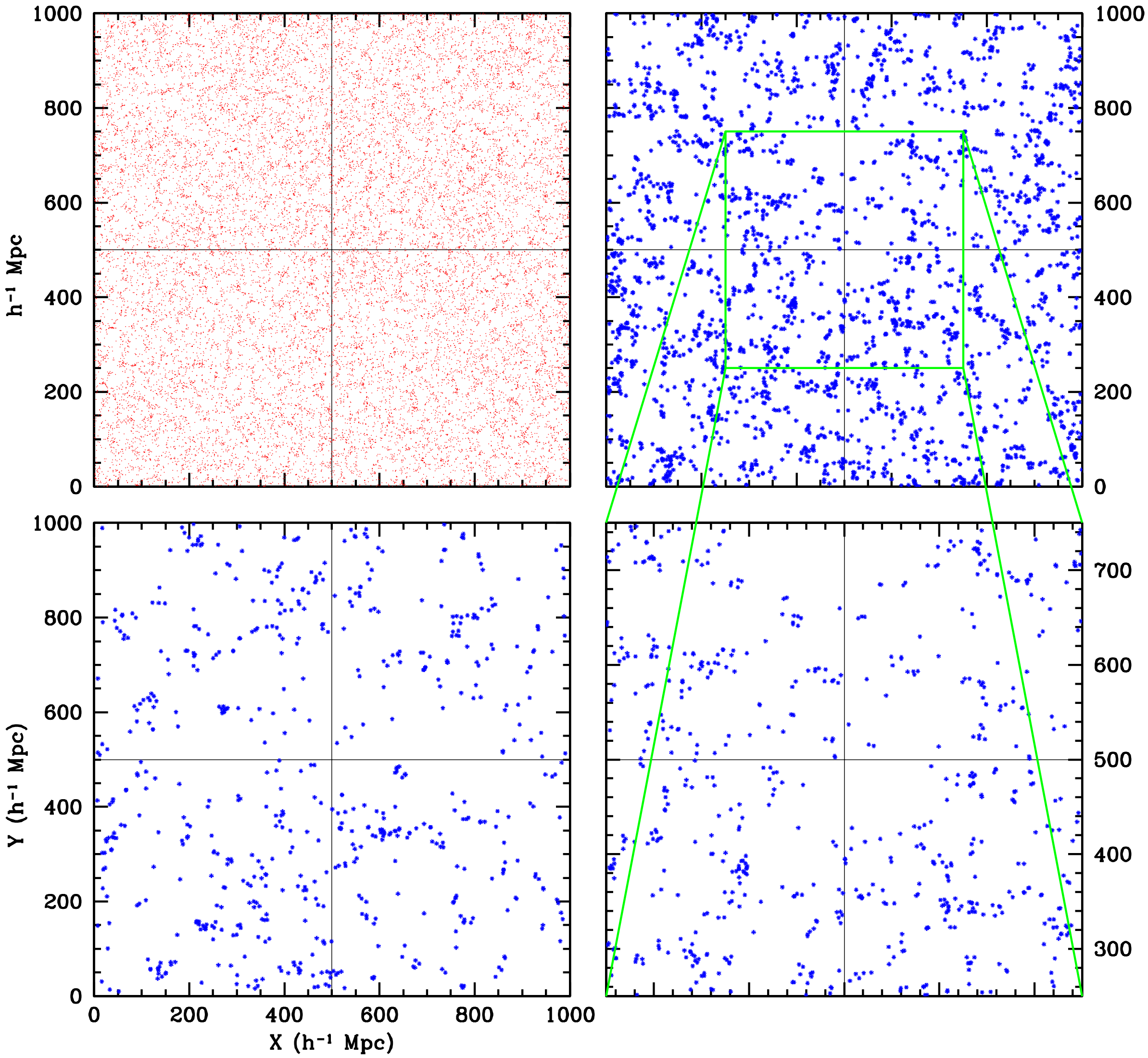


Figure 17: A depiction of the idea of 'self-similarity' in the vertex distribution. Out of a full sample of vertices (top left) in a central slice, (top right) the 20.0% richest vertices. Similarly, (bottom left) the 2.5% richest vertices. When lifting the central $1/8^{th}$ region out of the 20% vertex subsample in the (top righthand) frame and sizing it up to the same scale as the full box, we observe the similarity in point process between the resulting (bottom righthand) distribution and that of the 2.5% subsample (bottom lefthand). Self-similarity in pure form ! From Van de Weygaert 2002,2003b.

mapping, at the same cosmic epoch (!!!), between the distribution of the more clustered population in a proportionally larger volume and that of the distribution in a smaller volume containing the objects of the population with a less pronounced clustering.

I included a figure (from Van de Weygaert 2003b), showing how a small volume from a less clustered population (of 20% most massive "Voronoi vertices", topright, selected from the total vertex population, the red dots topleft) is compared (by zooming in into a central region, bottom right) with a population of more clustered objects (2.5% most massive vertices, bottom left). As both populations have a power-law $\xi(r)$ with the same slope $\gamma \approx 1.8$, differing only in amplitude, the bottom point distributions are each other equivalent in terms of statistical characteristics.

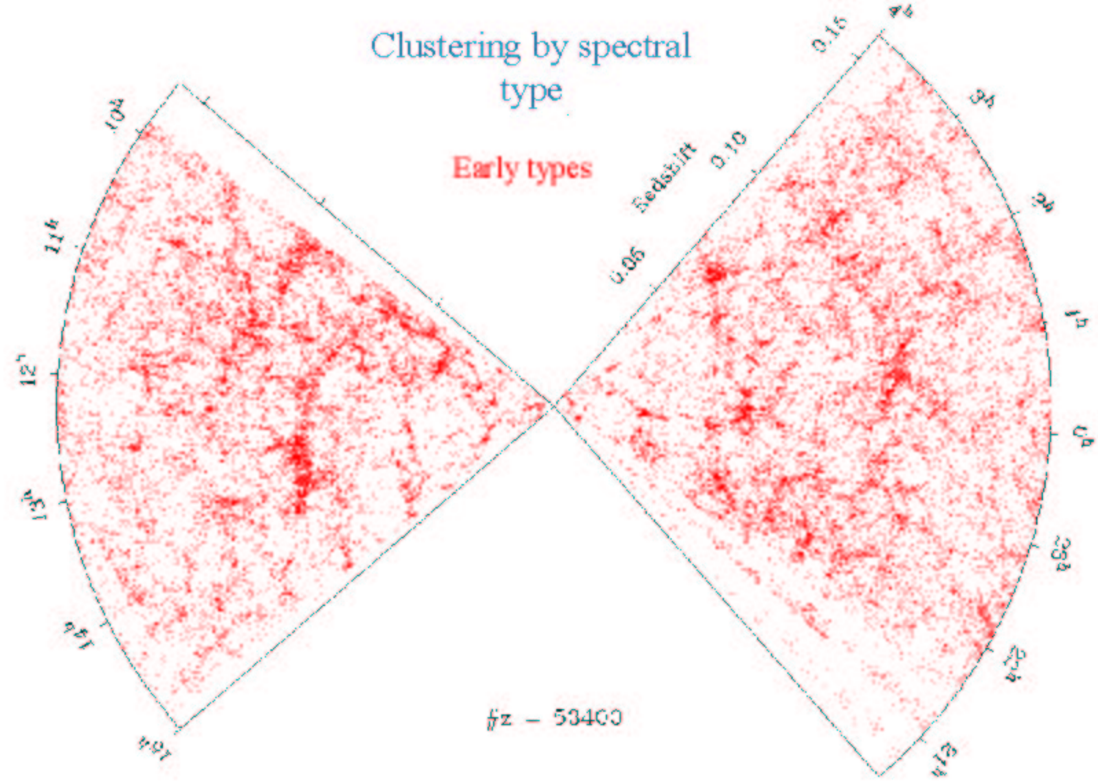
Galaxy Bias: Light versus Matter

The issue of light not tracing mass (section V.E) is in a sense intimately coupled with the issue of the contrast between cluster clustering and galaxy clustering (sections V.C). Both concern the issue of *biasing*. Although without clear preference, it might be most straightforward to first treat the issue of the light versus matter fluctuations, (currently section V.E.)

124. The first paragraph "It has long ..." should start a separate paragraph, including a sentence on that the difference between light and matter distribution concerns one of the most fundamental issues in our understanding of structure formation, the formation of galaxies. Following the remark in the 3rd paragraph on the galaxy formation process, and that our theoretical understanding of this issue is far from materializing. Include the last paragraph of V.E. in this introductory paragraph. Then continue with a second paragraph on the observational indications for such mass-light segregation. Separate these in two, one on *morphology segregation* and one on *luminosity segregation.*

 a) **Morphology Segregation**. A (very) recent analysis of high interest is the study of the 2dF survey assessing the clustering dependence on spectral type (Madgwick et al., 2003, astro-ph/0303668). This followed an earlier analysis of the luminosity function for the various galaxy spectral types (Madgwick et al., 2002). This relates directly and explicitly to the morphology-density relation (Dressler 1980), who established this not only for the dense environment of clusters, but also far out into the field. I myself find the Perseus-Pisces chain maps for various types of galaxies by Giovanelli, Haynes & Chincarini (1986) one of the most illustrative examples. Also the early 2dF survey 100k release included 2 different maps, one for early-type and one for late-type galaxies (by automatic classification, and uncorrected), which showed clearly the stark differences in spatial distribution.

 b) **Luminosity Segregation**. Two key references are Benoist et al. (1996), who found a significant dependence of clustering on luminosity for bright galaxies in the SSRS2 survey sample, while the most solid current result is the 2dF survey study by Norberg et al. (2002, figure included). See also website 2dF survey. They presented significant evidence for the dependence of the *correlation length* r_0 on luminosity of the galaxies. In same paragraph (not separate) include the galaxy luminosity dependence remark ("The recognition ..."). Would also be of benefit to include a (very) recent theoretical study on the dependence of galaxy luminosity functions on large scale environment (Mo et al., 2003, astro-ph/0310147).


Clustering by spectral type
Early types
0.05
0.10
0.15

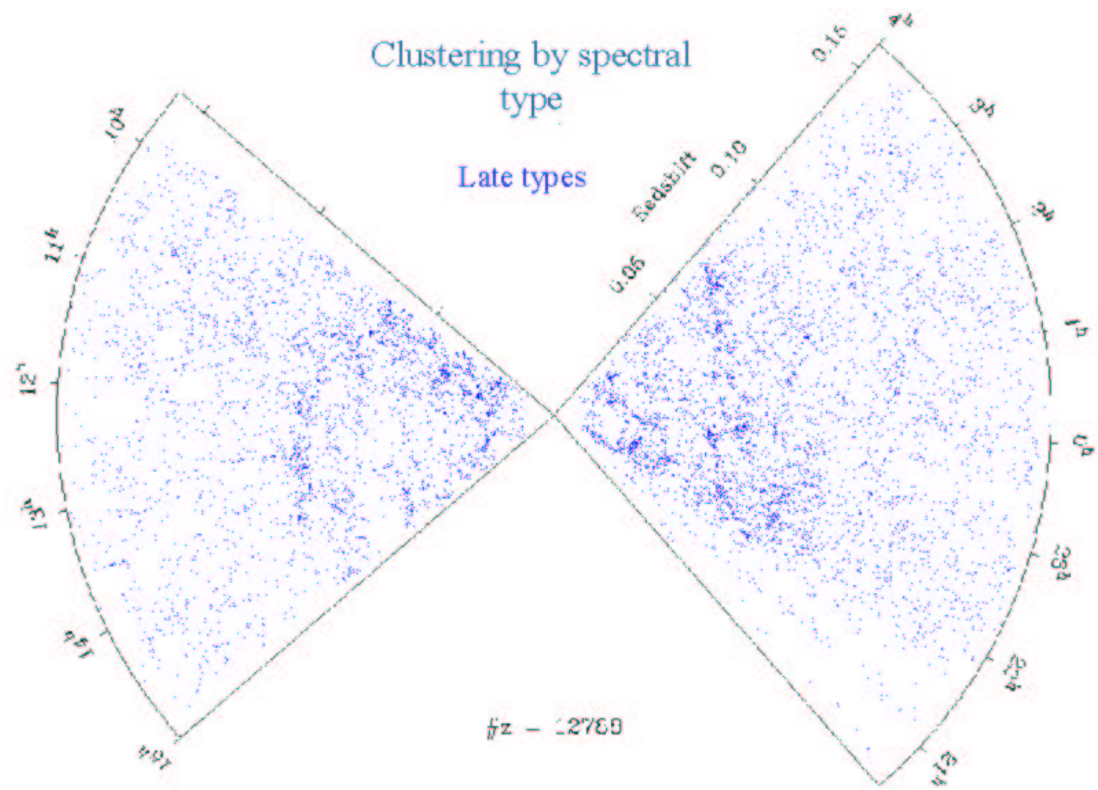

Clustering by spectral type
Late types
0.05
0.10
0.15

Figure 18: Morphological segregation in the 2dFGRS survey, on the basis of the 100k sample release. Top: distribution of early galaxies. Bottom: distribution late-type galaxies. From: 100k CD-ROM release. Courtesy: 2dF consortium.

125. Part of the first paragraph of V.CI discusses the issue of galaxy-galaxy clustering being a function of the particular selection criterion involved. It would therefore be more appropriate to include this aspect in the context of galaxy bias instead of in the context of cluster clustering.

126. The concept of galaxy biasing as formulated by Kaiser (1984), based on the identification of peaks in the primordial Gaussian density field, may be remarked upon as a simple ansatz. Basically it is an attempt to phrase the complexities of the galaxy formation process into an easy statistical formulation. The identification of a galaxy with a primordial peak involves the complications of choice for scale and amplitude of density fluctuations. Later sophisticated statistical models (e.g. the excursion set approach) tried to deal with this.

127. Besides the stochastic biasing model of Dekel & Lahav (1999), there have also been additional analytical studies of biasing in hierarchical scenarios of gravitational clustering (i.e. on a more fundamental level than the more heuristic *semi-analytical* models mentioned in the last paragraph of the N-body subsection, see above). The studies by Mo & White (1996) and Mo, Jing & White (1997) may be considered as fundamental contributions in this respect.

128. I find that section V.E.2 on 'mass and light fluctuations' would be better located in a section on 'counts-in-cells', also as the difference between mass and light fluctuations is hardly touched upon in the present text.

Cluster Clustering

Subsequently, the *mass scaling* section may turn to the clustering of clusters. A few remarks may be relevant, various concerning a possible reorganization of some sections.

129. I would urge the authors to start the discussion in section V.C. with explicitly mentioning (equation line) the estimate of the cluster-cluster correlation function, instead of incorporating it in the 'run' of the sentence. Indicate in particular also the estimated correlation length r_0 for clusters.

    a) Concretely, it implies replacing the paragraph from section V.C.3 starting with "The empirical ..." up to "... APM cluster samples".

130. Following this quantitative result, the now 2nd paragraph should follow stating that it was a surprise that a) clusters are clustered and b) they are more clustered than galaxies.

131. In the meantime, the best defined cluster sample around, certainly in terms of physical definition, is the REFLEX catalogue assembled by Hans Böhringer and collaborators. It would be appropriate to include the relevant references ($\xi_{cc}(r)$: Collins et al. 2000, $P(k)$: Schücker et al. 2001). Providing the cleanest cluster sample available, I repeat my remark that the figure shown in the review by Borgani & Guzzo (2001), comparing the REFLEX cluster sample with the LCRS galaxy distribution in the same region provides an ideal visualization of the relation and differences between the two.

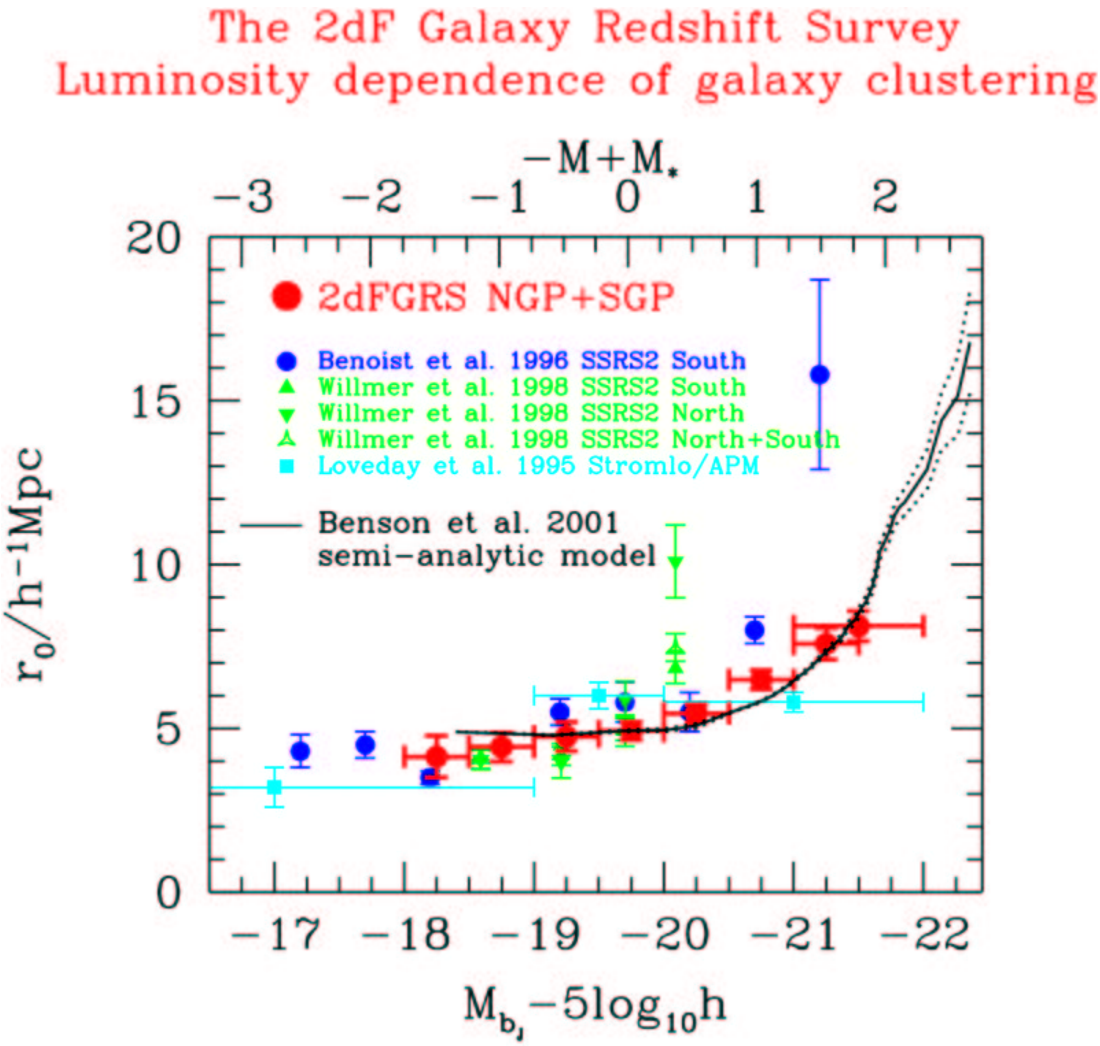


Figure 19: Dependence of the correlation length r_0 on the luminosity of the galaxy subsample in the 2dFGRS survey. From Norberg et al. 2002

132. With SDSS well on the way, an early cluster catalog has been distilled from the SDSS (Bahcall et al. 2003, ApJS, 148: astro-ph/0305202), and the recent paper by Bahcall et al. (2003, astro-ph/0307102) has analyzed the richness dependence of the resulting cluster correlation function.

133. The concept of cluster clustering is closely related to the issue of *superclusters*: the densest concentrations of clusters are often identified as superclusters. This may therefore be the location to indicate this (e.g. Bahcall 1988 (ARAA) tried to indicate the superclusters in the cluster samples). This of course refers also to the earlier section on superclusters, warranting also the remark that clusters are identified on the basis of cluster clustering, but do not only consist of clusters.

134. With respect to some of the above points, I may include a passage from a recent contribution by Van de Weygaert (2002), containing some extra references

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The clustering of clusters is considerably more pronounced than that of galaxies. The two-point correlation function $\xi_{cc}(r)$ of clusters appears to be a scaled version of the power-law

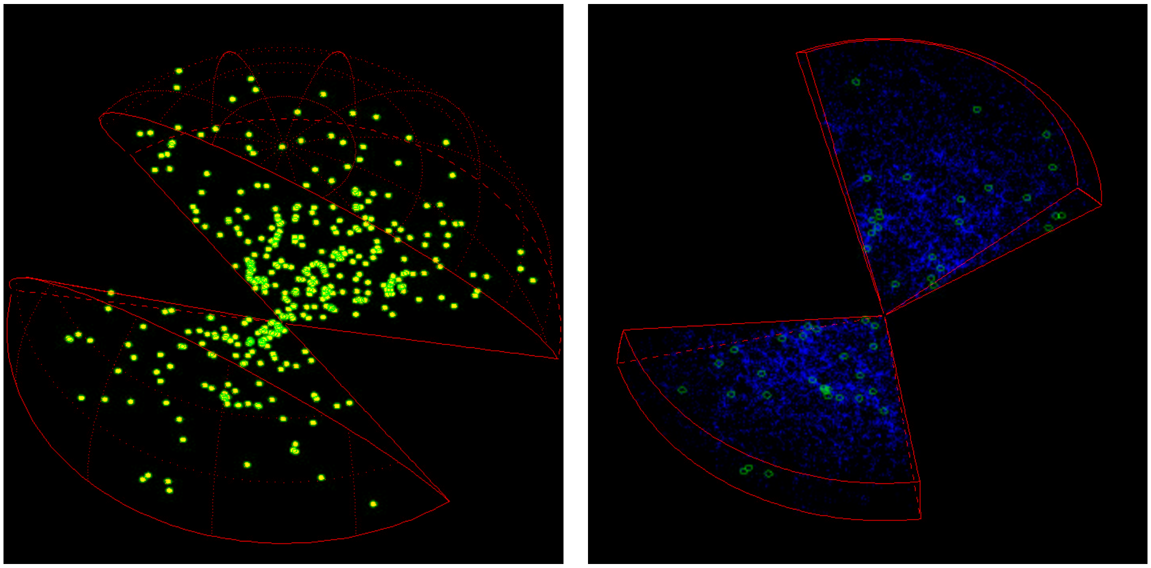

Figure 20: The spatial cluster distribution and its relation to the cosmic web. Left: The full volume of the X-ray REFLEX cluster survey within a distance of $600\mathrm{h}^{-1}$Mpc. The REFLEX galaxy cluster catalogue (Böhringer et al. 2001), contains all clusters brighter than an X-ray flux of $3 \times 10^{-12}\mathrm{erg s}^{-1}\mathrm{cm}^{-2}$ over a large part of the southern sky. The missing part of the hemisphere delineates the region highly obscured by the Galaxy. Right: The green circles mark the positions of REFLEX X-ray clusters in the northern and southern slices of the Las Campanas redshift survey (LCRS, Shectman et al. 1996), out to a maximum distance of $600\mathrm{h}^{-1}$ Mpc. Underlying, in blue, the galaxies in the LCRS delineate a foamlike distribution of filaments, walls and voids. Courtesy: Borgani & Guzzo (2001). Reproduced by permission of Nature.

galaxy-galaxy correlation function, $\xi(r) = (r_o/r)^\gamma$. Most studies agree on the same slope $\gamma \approx 1.8$ while all yield a significantly higher amplitude. The estimates of the latter differ considerably from a factor $\simeq 10 - 25$. The original value found for the "clustering length" r_o for rich $R \geq 1$ Abell clusters was $r_\mathrm{o} \approx 25h^{-1}$Mpc (Bahcall & Soneira 1983),

$$\xi_{cc}(r) \;=\; \left(\frac{r_\mathrm{o}}{r}\right)^\gamma \; ; \qquad\qquad \gamma = 1.8 \pm 0.2; \quad r_\mathrm{o} = 26 \pm 4 \; h^{-1}\mathrm{Mpc} \; , \tag{17}$$

up to a scale of $100h^{-1}$ Mpc (Bahcall 1988). Later work favoured more moderate values in the order of $15 - 20h^{-1}$Mpc (e.g. Sutherland 1988, Dalton et al. 1992, Peacock & West 1992). In terms of statistical significance, the recent clustering analysis of the cleanly defined REFLEX cluster sample has produced the currently most significant and elucidating determination of cluster-cluster correlation function (see Fig. 46, from Borgani & Guzzo 2001) and its corresponding power spectrum (Borgani & Guzzo 2001, Collins et al. 2001, Schuecker et al. 2001). As can be clearly discerned from Fig. 46, it strongly endorses the amplified cluster clustering wrt. the galaxy distribution (from the LCRS survey, Tucker et al. 1997).

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135. Given the fact that most of the issues concerning selection effects in the Abell catalogue have been rendered less relevant through the release of improved cluster samples like the REFLEX X-ray cluster set and the SDSS cluster sample (Bahcall et al. 2003), I wonder whether it would not be possible to shorten the five point listing

on page 47, on the important questions concerning the Abell clusters. I think the systematic bias issue of the Abell catalogue can almost be done with as an historic artefact.

136. In terms of visual appreciation, the image of the spatial REFLEX cluster sample and the spatial LCRS galaxy distribution in the same region is superb. I combined both figures into one, and would recommend the authors to include it in the review if they deem the cluster clustering an important issue (for scaling).

137. Also the correlation function figure from Borgani & Guzzo (2001) is amongst the clearest in showing the difference between the cluster-cluster correlation function ξ_{cc}, for the REFLEX sample, and the galaxy-galaxy correlation function ξ_{gg} (see accompanying fig.).

138. After having presented the result of the increased cluster-cluster correlation function, I would suggest a section containing the basic explanation for the "biased" clustering of clusters. This would consist of two aspects:

a) Perhaps it would be nice to mould the Jones & Jones (1985) explanation of the amplification of cluster-cluster clustering into a separate subsection instead of passing it in the running text. It would perhaps even be nice to include an illustration ?

b) Kaiser's (1984) peak bias explanation, mentioned in section V.C3 (as this is such an important concept, it might be worthwhile to devote a few more remarks to this.

139. Subsequently, a section on the richness-dependence of the cluster-cluster correlation function would be a natural follow-up. This should start with the subject of the first paragraph of section V.C.3, the Szalay & Schramm (1985) suggestion. This should be done in a somewhat more explicit fashion. I cite a paragraph from Van de Weygaert (2002):

A related second property of cluster clustering is that the differences in estimates of r_o are at least partly related to the specific selection of clusters. There appears to be a trend of an increasing clustering strength as the clusters in the sample become more rich ($\approx$ massive). On the basis of the first related studies, Szalay & Schramm (1985) even put forward the (daring) suggestion that samples of clusters selected on richness would display a 'fractal' clustering behaviour, in which the clustering scale r_o would scale linearly with the typical scale L of the cluster catalogue,

$$\xi_{cc}(r) \;=\; \beta \left(\frac{L(r)}{r}\right)^{\gamma} \;; \qquad L(R) \;=\; n^{-1/3}\,. \tag{18}$$

The typical scale $L(R)$ is then the mean separation between the clusters of richness higher than R. Although the exact scaling of $L(r)$ with mean number density n is questionable, observations seem to follow the qualitative trend of a monotonously increasing $L(R)$. It also appears to be reflected to some extent in a similar increase in clustering strength encountered in selections of model clusters in large-scope N-body simulations (e.g. Colberg 1998).

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140. With the introduction in terms of the Szalay & Schramm (1985) proposition, it would be appropriate to pass the observational "evidence" for a richness-dependent clustering, a clearcut illustration of *mass scaling.*

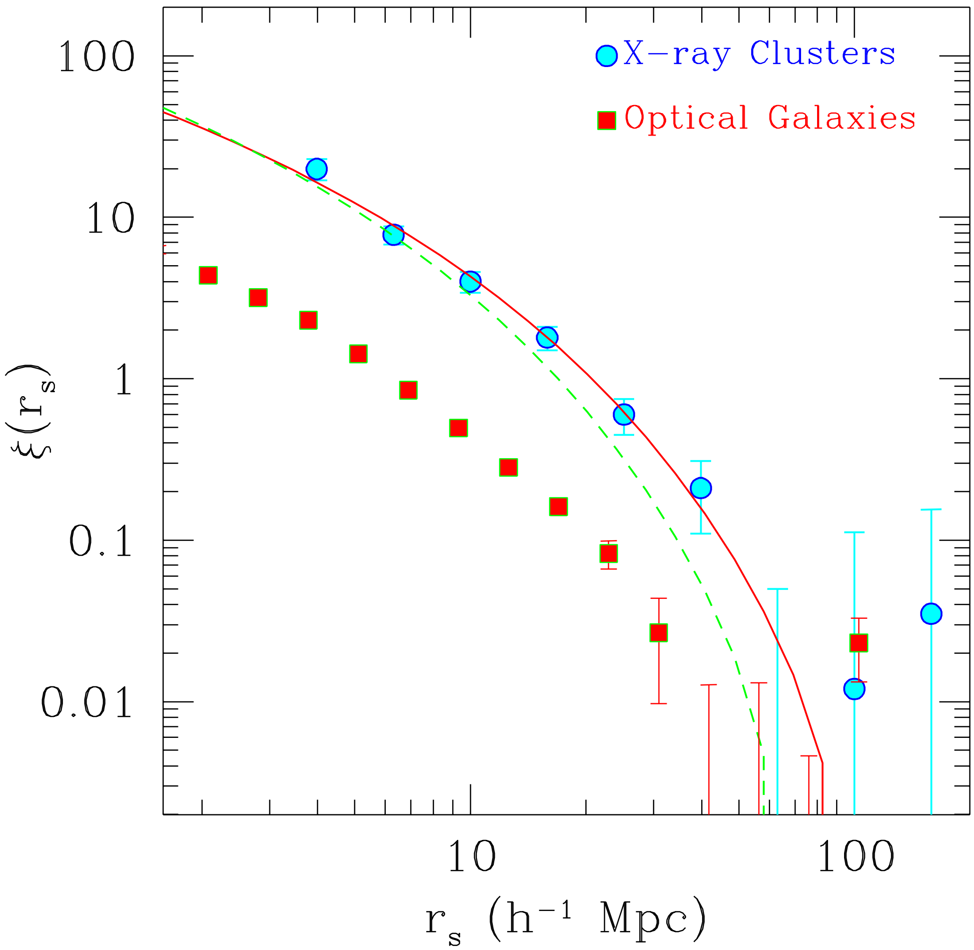

Figure 21: The two-point correlation functions ξ of galaxies (squares) and X-ray clusters of galaxies (circles), plotted as a function of (redshift space) separation r_s, computed from the Las Campanas galaxy redshift survey (Tucker et al. 1997) and the REFLEX X-ray cluster survey (Collins et al. 2001). The two curves are the predictions for 2 CDM models, both in spatially flat universes ($\Omega_m + \Omega_\lambda = 1$), one with $\Omega_m = 0.3, h = 0.7$ (solid line), the other with $\Omega_m = 0.5, h = 0.6$ (dashed line). Courtesy: Borgani & Guzzo 2001. Reproduced by permission of Nature.

141. Subsequently, the theoretical explanations for such *mass scaling* may pass the scene. This concerns the discussion in V.C.2 and the last 2 paragraphs of V.C.3.

142. With respect to the theoretical models, it may be preferrable to first concentrate on the "conventional" descriptions in terms of gravitational instability theories. That is, the analytical work concerning hierarchical clustering and Press-Schechter/excursion set formulations (Mo & White 1996, Mo, Jing & White 1996, Sheth, Mo & Tormen 2001) as well as an abundance of ever more extensive N-body simulations (e.g. Colberg et al. 1998, 2002, Governato et al. 1999, Moscardini et al. 2000).

143. I would find it more logical to include the multifractal explanation for *mass scaling* (both indicated in paragraphs of V.C.2 and V.C.3) in section VI, with the more extended treatment of fractal models).

144. I have also the (personal) suggestion to include a geometric explanation based on the foamlike/cellular patterns. Stemming from the Voronoi model of clustering a surprisingly solid mass scaling has been uncovered (Van de Weygaert 2003a,b,c, also see Van de Weygaert 2002). For reference, I have included various figures and texts in the comments on the section on statistical clustering models.

Clustering Scaling Vc: Correlation Scaling

145. Section VI.E.3 presents scaling prescriptions in terms of the correlation functions, a fundamentally analytical prescription pursued by e.g. Fry (1984a) and Balian & Schaeffer (1989a). It may be appropriate to include it in this "chapter" on clustering scaling & biasing, as one of three modes and separate it from the fractal characterizations.

   - This may or may not already include the statistical model based on this description, and described in section VII.B.7 *Balian & Schaeffer*.

VI. Fractal Characterization of Scaling

A clear CENTRAL role in the review is taken by the issue of fractal descriptors of point distributions and the relation with underlying physics. In my opinion it warrants a separate chapter. This would involve parts of chapter V. as well as most of chapter VI.

146. Start with section E intro.

147. Continue either with either the general definition of multifractal description and quantification (section VI.E.4), followed by the specific illustration of correlation dimension D_2 (sections V.B.1, V.B.4 and V.B.5), or exactly the reverse (up to preference).

148. Perhaps also interesting to indicate the relation with *wavelets* (Martínez, Paredes & Saar 1993). A highly topical reference to the issue of hierarchical clustering, scale-free correlations and wavelets is Pando et al. (1998).

  b) In this respect, there is an interesting RevModPhys article by Bowman & Newell 1998) which discusses the relation between complex pattern formation in physical phenomena and wavelet descriptors.

149. Perhaps preferrable to have VI.E.6 *multifractality* before VI.E. *intermittency.*

150. With respect to VI.E.6, would it not be good to also include a reference to some other cosmological publications which did study multifractal characterizations of 'standard' models of structure formation (e.g. Colombi, Bouchet & Schaeffer 1992, Yepes et al. 1992).

151. Given the rather complex nature of the multifractal description, it may help to have

  a) figures of e.g. fractal dimensions D_q and/or $f(\alpha)$.

  b) a more intuitive description of generalized dimensions (as e.g. in Martínez et al. 1990), in particular the difference between D_0 and D_2. It would help to get an intuitive idea for the fact that fractal dimensions do not need to be the same (as quite often anyone not familiar with the subject presumes).

152. The section VI.E.5 on *intermittency* would also benefit greatly from an illustrative explanation of e.g. eqn. (42), as well as from somewhat more explanation. The section as it stands now may be rather obscure for many, given the context of scaling.

  - As for eqn. (42), the text should explain L (only l is explained).

  a) in the 3^{rd} paragraph it is remarked that higher order moment will dominate the 2^{nd} and 3^{rd} order moments in patterns like the cosmic foam. This need not necessarily be the case ... (e.g. in initial conditions of N-body simulations, the weblike pattern is readily seen, while the distribution is yet close to Gaussian). Thus, it is a matter of how far evolution has progressed.

  b) Later, when mentioning the Schrödinger equation in VII.E.5, the contribution by Coles (2002) is mentioned. It certainly also would be in place in this section on intermittency.

153. Section VI.E.1 may be rather heavily worded. It may be nice to ameliorate some of the wording, most of the text restricted to the first paragraph. In this, also include references to the study by Coleman, Pietronero & Sanders 1988.

154. Perhaps list shortly the number of counterarguments against simple fractal clustering. In addition to the mentioned fact that there is no indication that the Universe is not homogeneous on scales exceeding $\approx 150 - 200h^{-1}$Mpc, and the fact that there is no convincing evidence for the breakdown of Newtonian/Einsteinian gravity (but there is MOND !), there is what in my view is always the most convincing evidence: the perfect depth scaling of the angular two-point correlation function. The latter may be convincingly presented by means of a figure of a few sky projected galaxy distributions.

VIa. Cluster Scaling: Multifractal Model

155. Following up on the issue of the amplified clustering of the cluster population (Clustering Scaling II, see above), it may be good to spend a (sub)section on multifractal "hierarchical" models (for which fig. 13 is an excellent illustration.). References: Martinez et al. 1990; Paredes et al. 1995, Martinez et al. 1995).

156. It may be good to specify here what is indicated as "hierarchical", a concept that has been covering rather different kinds of concepts. Perhaps an illustration of the work by Martinez et al. (1995) could be considered.

157. A possibly interesting model, and likely (?) providing a particular illustration/realization of the multi-fractal explanation is the Voronoi vertex model (Van de Weygaert 2003b, also 2002). Although by all means its proper place is in section B. on "statistical models", the recent result may be mentioned here, elaborated upon in the chapter on statistical models.

VIII. Statistical Models

Instead of 1 chapter VII on "Clustering Models", it may be considered to shift the section on "A. N-body simulations" to the earlier (background) chapter on Gravitational Instability (in these comments, this section is also discussed in this context). Subsequently, the section on Statistical Models may become an independent chapter, while part of "C. Dynamical Models" (in particular "Pancake and adhesion models") is combined with the ultimate section "D. Hydrodynamic Models" and "E. Nonlinear dynamic models" in the closing chapter. This would put more emphasis on that which should be the most important part of the review, the theoretical interpretation and explanation of the uncovered scalings within the framework of a Lagrangian description of gravitational clustering, culminating in e.g Random Heat Equation.

Also, as discussed at the beginning of the review, it may put the statistical models in particular historic perspective when the current section **III.D "Hierarchical Models"** is either included in the chapter on statistical models or precedes it. Following the extensive discussions on the theory of gravitational clustering, on the observational reality, clustering measures and the existence of clustering scaling, this may be a nice 'punchline'.

VIIIa. General

158. Statistical Models (VII.B) provides a nice overview, with a few side remarks:

     - given its cosmohistorical importance, its success in recovering the principal features of the one-million galaxy Lick counts map, the review may elaborate somewhat more on the Soneira & Peebles model. Possibly including an illustration (from e.g. Soneira & Peebles 1978).

     - section B.7 on "Balian & Schaeffer" models should perhaps be combined with section VI.E.3 on "scaling properties of counts in cells" and included in the earlier mentioned Clustering Scaling III. section.

     - as indicated earlier, for a geometrical model of *mass scaling*, the recent results on Voronoi tessellations may provide a nice illustration. It may be relegated to the last section of this chapter IX. on Statistical Models. Below I provide some texts and figures from a recent "review" (Van de Weygaert 2002), in the hope some of it may be of use for the review.

VIIIb. Addition: Voronoi Model

Following section VII.B.4 on *Voronoi Tessellations*, I would like to suggest to include some additional material.

159. As for the physical basis of the model, recent work by Sheth & Van de Weygaert (2003) has demonstrated that a self-similarly evolving peaked void distribution is the natural result of gravitational clustering in hierarchical structure formation scenarios starting from Gaussian initial conditions. The natural asymptotic configuration would be that of a packing of equal-sized and equally fast expanding voids with matter assembling in the filamentary and wall-like interstices. Geometrically this is a Voronoi tessellation, and on the basis of these considerations one may understand why Voronoi tessellations seemed to be so, surprisingly, succesfull in reproducing clustering results.

160. Following up on the mention of power-law clustering of Voronoi vertices, I included a figure of a spatial vertex distribution, with graphs of the 2 pt correlation function,

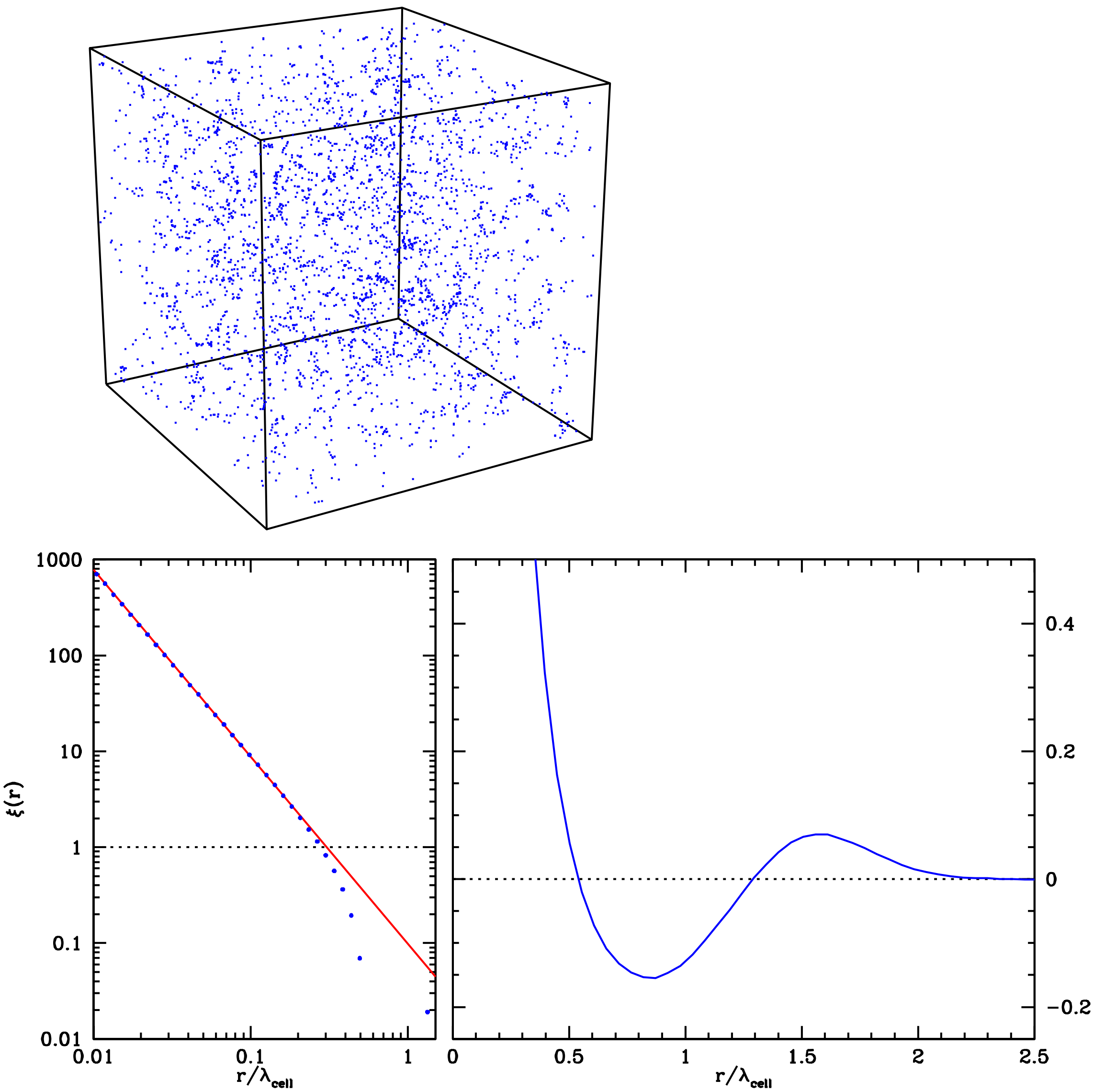

Figure 22: Two-point correlation function analysis of a (full, non-selected) set of Voronoi vertices. Top frame: a spatial 3-D depiction of Voronoi vertex distribution. Upon close attention, the underlying cellular geometry may be discerned. Bottom left: a log-log plot of the $\xi(r)$, with distance r in units of the basic cellsize $\lambda_{\rm cell}$. The power-law character of ξ up to $r \sim 0.3\lambda_c$ is evident. Bottom right: a lin-lin plot of ξ. The beautiful ringing behaviour out to scales $r \sim 2\lambda_{\rm cell}$ has been amply recovered. From: Van de Weygaert 2002, 2003b.

both for the small power-law range (log-log) as well as the large range non-power-law regime where the vertex distribution relaxes towards homogeneity $\xi \approx 0$.

161. Main addition concerns the issue of *mass scaling*, the tendency of more massive cosmic objects to be more strongly clustered. Additional work (Van de Weygaert 2003b, 2003c), has uncovered a geometric *mass scaling* in Voronoi distributions. In the following I cite some passages from Van de Weygaert (2002). Perhaps they may be of use:

162. Samples of more "massive" Voronoi vertices are progressively stronger clustered:

... In reality, not every vertex will represent sufficient mass, or a sufficiently deep potential well, to be identified with a true compact galaxy cluster. If we take the Voronoi model as an asymptotic approximation to the true galaxy distribution, its vertices will comprise a range of "masses". Dependent on the specific geometrical setting of each vertex – the size of the corresponding cells, walls and edges, the proximity of nearby vertices, etc. – the total mass acquired by a vertex will span a wide range of values. Brushing crudely over the details of the temporal evolution, each Voronoi vertex may be assigned a "mass" estimate by equating that to the total amount of matter ultimately will flow towards that vertex. These may be computed exactly on the basis of pure geometric considerations.

Selections of Voronoi vertices are depicted in the accompanying figure with vertex distributions in a central slice, for 6 different vertex mass selections. Each subsample consists of the same number of vertices, randomly selected from samples of ever richer vertices from top left to bottom right. Top left: random selection from complete sample of vertices. Bottom right: 0.25% richest vertices. Notice the continuous increase in clustering strength, and the stark contrast between the mild clustering of the full sample and that amongst the richest vertices.

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163. At least three aspects concerning the more pronounced clustering of the more massive cluster/vertex samples may be discerned:

- *Stronger clustering*
  The clustering itself is stronger, expressing itself in tighter and more compact point concentrations.

- *Increased clustering scale*
  The clustering extends over a substantially larger spatial range. Structures, clumps and huge voids, subtending several elementary cell scales are clearly visible (see in particular centre and right bottom slices).

- *Anisotropic extensions*
  The subtended large scale features appear to become more distinctly anisotropic, wall-like or filamentary, for more massive samples (note the huge filamentary complexes in lower righthand frame Fig. 50).

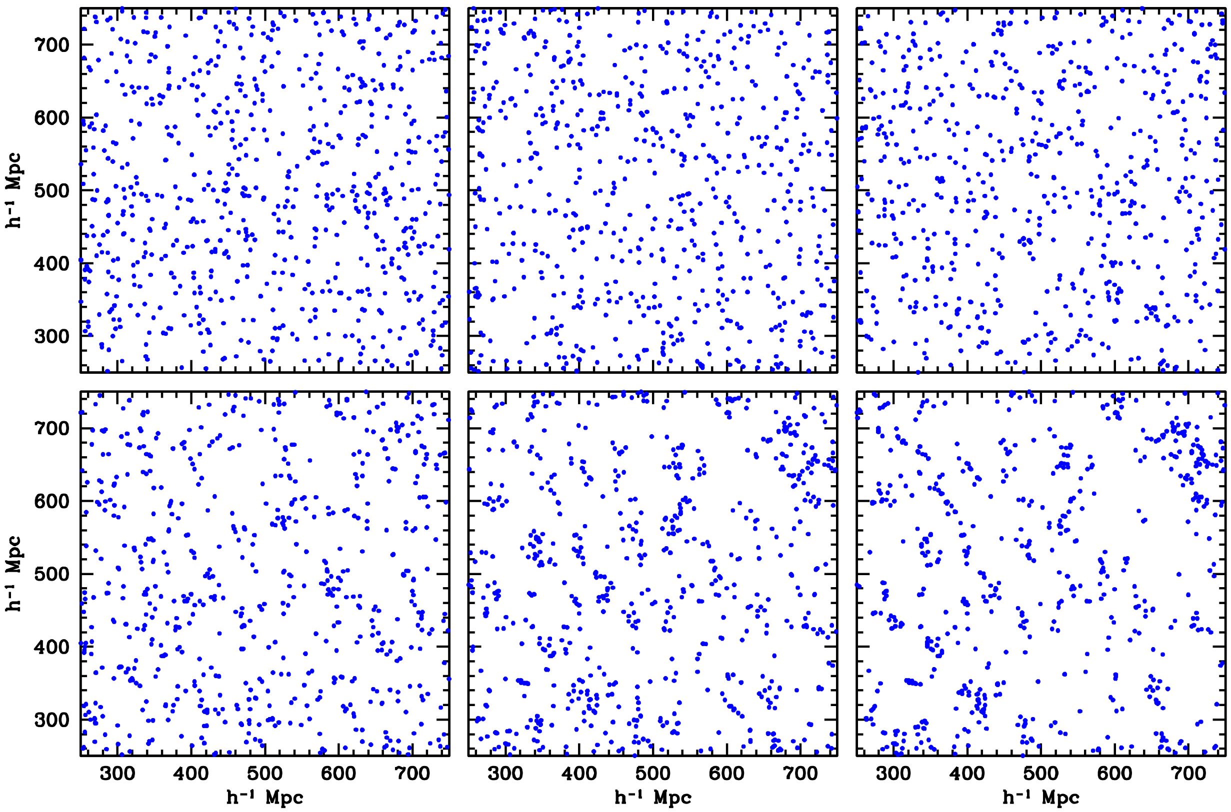

Figure 23: Selections of Voronoi vertices. Each subsample consists of the same number of vertices, randomly selected from samples of ever richer vertices from top left to bottom right. Top left: random selection from complete sample of vertices. Bottom right: 0.25% richest vertices. Notice the continuous increase in clustering strength, and the stark contrast between the mild clustering of the full sample and that amongst the richest vertices. From: Van de Weygaert 2002, 2003b.

164. The qualitative impression of a gradually stronger, more pronounced and richer pattern of clustering becomes even more striking upon quantitatively analyzing correlation function systematics. A thorough numerical study of vertex clustering patterns disclosed an unexpected and surprising "self-similarity":

a) In the accompanying figure the resulting scaling of the two-point correlation function of Voronoi vertices is shown. For a variety of subsamples selected on the basis of "richness", ranging from samples with the complete population of vertices down to subsamples containing the 2.5% most massive vertices, we see a systematic "scaling" behaviour.

b) Notice that the "scaling" does not only involve an increase in amplitude, but also a shift in spatial range of the clustering pattern. We have therefore found that richer objects not only cluster more strongly, but also out to a larger range. Thus, large coherent filaments are a direct consequence of an underlying cellular geometry.

c) The impression of stronger clustering is indeed confirmed through a systematic, linear, increase in the value of the "clustering length" r_o. Possibly more surprising is the equally systematic increase of the "correlation length" r_a, the quantitative expression for the observed impression of point clustering extending over larger regions of space.

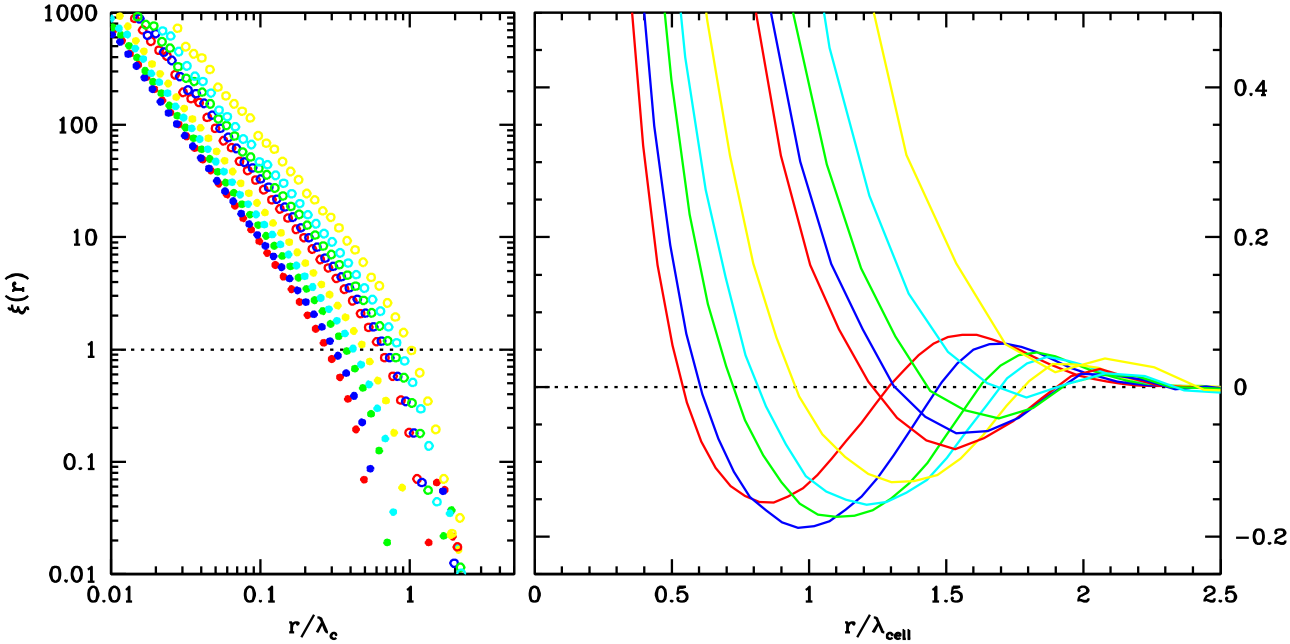

Figure 24: Left: log-log plot of $\xi(r)$ against r/λ_c, with λ_c the basic tessellation cellsize ($\equiv$ intranucleus distance). Notice the upward shift of $\xi(r)$ for subsamples with more massive vertices. Right: lin-lin plot of $\xi(r)$ against r/λ_c. Notice the striking rightward shift of the "beating" pattern as richness of the sample increases. From: Van de Weygaert 2002,2003b.

d) Quantitatively the systematic scaling of vertex clustering is worded by the accompanying figure and text:

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165. Properties Voronoi vertex clustering scaling (from Van de Weygaert 2002):

+ *Two-point correlation function*
  The two-point correlation functions of selected massive cluster samples display a behaviour similar to that found for unbiased samples (Fig. 52): an almost perfect power-law at short range which beyond its coherence scale changes gradually into a oscillating behaviour between positive and negative correlations, swiftly decaying within a few "ringings" to zero level.

+ *Parameters* $\xi(r)$
  The parameters characterizing the generic behaviour of ξ – amplitude, coherence scale and power-law slope – are subject to systematic scaling behaviour.

+ *Correlation amplitude*
  The amplitude of the correlation functions increases with rising vertex sample richness. The "clustering length" r_o increases almost perfectly linear as a function of the characteristic intra-vertex distance λ_v of the particular richness selected vertex sample.

+ *Correlation range*
  The large-scale (lin-lin) behaviour of ξ_{vv} extends out to larger and larger distances

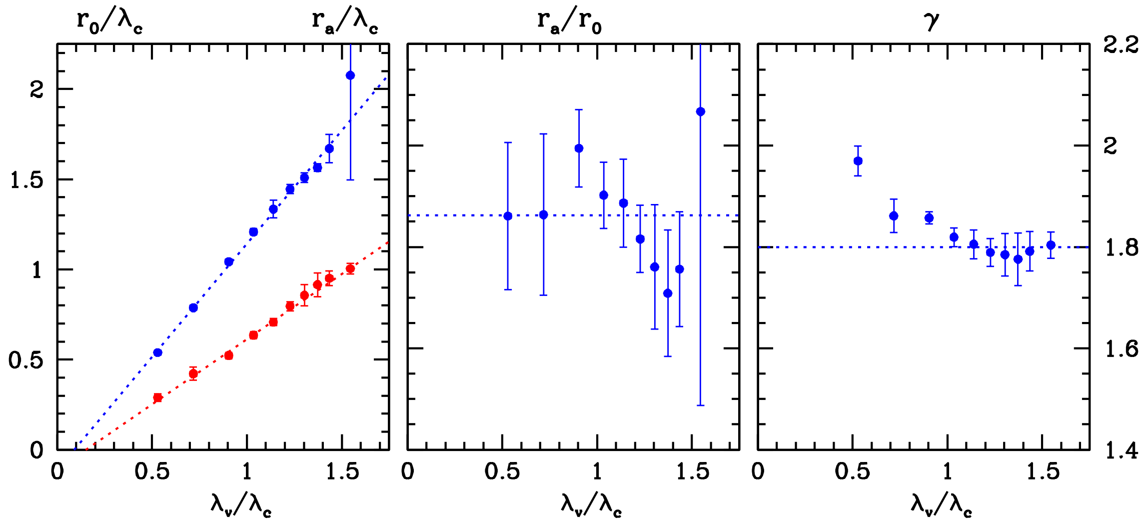

Figure 25: Scaling of Voronoi vertex two-point correlation function parameters for vertex subsamples over a range of "richness"/"mass". Left: the clustering length r_0 (red, $\xi(r_0) \equiv 1.0$) and the correlation (coherence) length r_a (blue, $\xi(r_a) \equiv 0$) as a function of average spatial separation between vertices in (mass) selected subsample, λ_v/λ_c. Centre: the ratio between clustering length r_0 and coherence length r_a as function of subsample intravertex distance λ_v/λ_c. Right: the power-law slope γ as function of λ_v/λ_c. From Van de Weygaert 2002, 2003b.

with increasing sample richness. As in the case of r_o the "correlation (coherence) scale" r_a possesses an almost perfectly linear relation as function of the average sample vertex distance λ_v.

+ *Clustering and coherence scaling*
  Therefore, combining the behaviour of r_o and r_a a striking "self-similar" scaling behaviour is revealed: the ratio of correlation versus clustering length is virtually constant for all vertex samples, $r_\text{a}/r_\text{o} \approx 1.86$ (for Poisson Voronoi tessellations).

+ *Correlation function slope*
  At the short power-law range, the correlation functions have rather similar slopes. Nonetheless, a slight and significant trend in the power-law slope has been found, involving an gradually increasing tilt. Interestingly, we see a gradual change from a slope $\gamma \approx 1.95$ for the full sample to a robust (and suggestive) $\gamma \approx 1.8$ for the selected samples.

All in all, these intrinsically geometrical properties hint at a scaling behaviour which may befittedly be called "geometrical biasing". It is be qualitatively different from the more conventional "peak biasing" picture (Kaiser 1984) in that it involves an effect of spatial extending clustering.

IX. Dynamical Models

Except for the summary and concluding remarks chapter, it might be best to combine the current section VII.C.3 "Pancake and adhesion models", VII.D. "Hydrodynamic Models" and VII.E. "Nonlinear dynamic models" into one chapter. A first part of this chapter might be devoted to *Lagrangian Dynamical Models*, as a lot of material covered in this chapter relates to Lagrangian descriptions.

IXa. Lagrangian Theory and Models

166. Start with basic Lagrangian perturbation theory: Personally, I find one of the most illuminating expose's on Lagrangian theory the contribution by Bertschinger (1992, see above). Also, a basic summary was given in the 2002 review by Van de Weygaert in "Modern Theoretical and Observational Cosmology" (eds. Plionis & Cotsakis). This involves the Lagrangian equivalents of the continuity equation, the Euler equation and Poisson equation, involving e.g. the Raychaudhury equation.

167. As for the formalism of Lagrangian perturbation theory, one should also refer to Bouchet et al. 1995.

168. On the basis of the work by Bertschinger & Jain (1994), Hui & Bertschinger (1996) and very much so Giavalisco et al. 1996 one is lead automatically to the Zel'dovich approximation and the basic approximations it entails:

    - Zel'dovich approximation adheres to Euler and Poisson equation

    - Zel'dovich approximation does not adhere to mass/continuity equation.

169. Having arrived at the Zel'dovich approximation:

    - I think the Zel'dovich equation should be included in the text, too important not to quote it.

    - one should try to find an appropriate illustration for the formalism. It being so essential in this discussion on dynamical models, and in the field of structure formation, it would be a compelling addition.

170. The Zel'dovich approximation has found widespread application as formalism to set initial conditions. Recently, Scoccimarro (1998) extended this by means of a perturbation scheme into a highly improved initial conditions description. Its success was demonstrated in Scoccimarro & Sheth (2002), in the context of generating mock galaxy distributions.

171. Also mention the application of the Zel'dovich approximation towards the dynamical analysis of the large-scale local Universe and the related cosmic flows (quasi-linear regime !). Instrumental have been the contributions by Nusser & Dekel (1992, 1993), Nusser et al. 1991.

172. The illustration on the Zel'dovich approximation may be combined with e.g. a fitting illustration of an adhesion approximation realization, and/or any of the Super-Zel'dovich approximations.

173. Perhaps the *adhesion approximation* might be best included in this section. Also this may profit from both illustration as well as key equation(s).

174. Recently, Novikov et al. (2003) published a similar nonlinear Lagrangian model they coined *cosmic skeleton*.

IXb. Nonlinear Dynamic Models

175. Include, one after the other, the section D.2 "Bernouilli equation" and section E.2, E.3, E.4 on the "Random heat Equation" in this section, they logically hang together.

176. With respect to the Schrödinger equation approximation, again an image may do wonders. Recently, there has been a paper by Szapudi & Kaiser (2003), extending the formalism towards the development of a cosmological perturbation theory.

177. It may be more appropriate to insert the remarks in the 2 last paragraphs of section E.6 "General Remarks", on the forced adhesion model, in the sections on the adhesion approximation. The reference to Jones's external field approximation may then be a forward one.

X. Summary and Concluding Remarks

With respect to the closing chapter, a few minor remarks:

178. The section starting with "Most of the techniques ..." would be an ideal closing paragraph ... in particular its last sentence ...

179. Perhaps the last paragraph on simulations may be worked into the more extensive section on N-body simulations.

180. Perhaps nice also to not only summarize the statistical arguments for scaling, but also the close connection to gravitational growth of structure, while perhaps also the relevant observational background (the cosmic fossils) should be recalled.