Phase 1A:

# Fluctuation Modes:

Adiabatic

vs.

Isothermal / Isocurvature

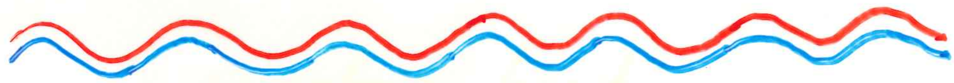
# Modes of Fluctuations

There are 2 different perturbation modes:

① **Adiabatic** (curvature) modes :

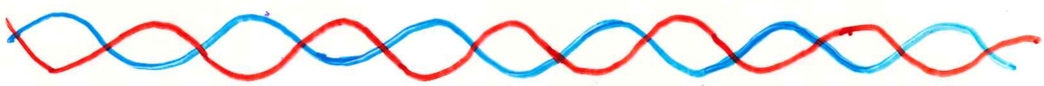
"honest-to-god" fluctuations in the energy density ($\delta\rho \neq 0$), equally shared between all components.

radiation
matter



components = radiation
matter.

② a **Isocurvature** modes

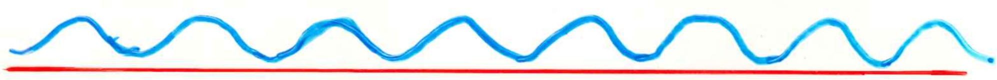


The fluctuations in radiation are such that they compensate for fluctuations in matter density, such that $\delta\rho = 0$.

Perturbations in Equation of State.

or

② b **Isothermal** modes



No fluctuations in radiation density, only in matter density :

as $\rho_r = aT^4$

The isocurvature and isothermal modes represent fluctuations in the form of the local equation of state:

- $\boxed{\text{isocurvature}}$ fluctuations accomplish this by fluctuations in the number density of species X ($n_x$) with respect to the number density of photons ($n_\gamma$):

  - $n_x$
  - $n_\gamma \propto s$ $(\propto T^3)$

  $$\left.\begin{array}{l} n_x \\ n_\gamma \propto s \end{array}\right\} \delta\left(\frac{n_x}{s}\right) \neq 0$$

  s: entropy density (in cosmology fully dominated by relativistic species, (photons & neutrinos).

  such that they compensate exactly radiation fluctuations $\delta\rho_r$:

  - $\delta\rho = \delta(\rho_x + \rho_r) = 0$

- $\boxed{\text{isothermal}}$ fluctuations:

  - no fluctuations in radiation energy density:

    $\delta\rho_r = 0 \Rightarrow \delta\rho = \delta\rho_x$.

  - $$\left.\begin{array}{l} n_\gamma \propto T^3 = cst. \\ n_x : \end{array}\right\} \delta\left(\frac{n_x}{s}\right) \neq 0.$$

modes fluct., cont'd.

- The entropy $\mathcal{S}$ per unit mass of a fluid composed of matter and radiation has a very high value because of the enormous value of the entropy per baryon:

  - $$\frac{s}{n_B} \approx \frac{1}{7}\eta^{-1} \approx 10^9$$

- Entropy in volume $V$:

$$\circledast \quad \mathcal{S} = sV = \frac{\rho_r c^2 + p_r}{T} V = \underline{\frac{4}{3}\sigma T^3 V}$$

$$\left.\begin{array}{l} \bullet\ p_r = \frac{1}{3}\rho_r c^2 \\ \bullet\ \rho_r c^2 = \frac{\sigma}{c^2} T^4 \end{array}\right\}$$

$\circledast$ per mass $\quad M = \rho_m V$:

$$\boxed{\mathcal{S}' = \frac{\mathcal{S}}{M} = \frac{4}{3}\sigma \frac{T^3}{\rho_m}} \propto \frac{\rho_r^{3/4}}{\rho_m}$$

$$\Downarrow$$

$$\boxed{\frac{\delta \mathcal{S}'}{\mathcal{S}'} = \frac{3}{4}\frac{\delta \rho_r}{\rho_r} - \frac{\delta \rho_m}{\rho_m}}$$

Entropy per mass Perturbation.

$\Longrightarrow$ Adiabatic Perturbation:

$$\frac{\delta \mathcal{S}'}{\mathcal{S}'} = 0: \qquad \delta_m \equiv \frac{\delta \rho_m}{\rho_m} = 3\frac{\delta T}{T} = \frac{3}{4}\frac{\delta \rho_r}{\rho_r} = \frac{3}{4}\delta_r$$

- adiabatic fluctuations
  - Entropy perturbations $\equiv 0$ :

$$\boxed{\delta_m = \frac{3}{4}\delta_r}$$

- The distinction between the two modes of fluctuations, adiabatic and isocurvature, is particularly relevant for very large scale perturbations (nowadays), i.e. for as long as perturbations are outside of horizon.

$\Downarrow$

Once an isocurvature mode becomes sub-horizon sized, fluctuations in local pressure ($p_r \propto \rho_r$!) can "push" energy density around and convert isocurvature perturbation $\longrightarrow$ curvature perturbation.

$\Downarrow$

- the photon fluctuations get erased in radiation-dominated phase. Thus, isocurvature fluctuations evolve into isothermal perturbations

$\Downarrow$

- once in matter-dominated regime, the only relevant fluctuations are the matter fluctuations, which then evolve further as their adiabatic equivalents.

# Coupled Perturbations

Assume gravity is only important force in medium containing several distinct components, i.e. neglect pressure force.

(1) **Continuity Equation:**

**Matter:** $p = 0.$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \left(\rho + \frac{p}{c^2}\right)\vec{v} = 0 :$$

$$\frac{\partial \rho_m}{\partial t} + \vec{\nabla} \cdot \rho_m \vec{v} = 0. \quad \underset{\text{Linear.}}{\Longrightarrow} \quad \frac{\partial \delta_m}{\partial t} + \vec{\nabla} \cdot \vec{v} = 0.$$

**Radiation:** $p = \frac{1}{3}\rho_r c^2$

$$\frac{\partial \rho_r}{\partial t} + \vec{\nabla} \cdot \left(\rho_r + \frac{p}{c^2}\right)\vec{v} = 0 :$$

$$\frac{\partial \rho_r}{\partial t} + \frac{4}{3}\vec{\nabla} \cdot \rho_r \vec{v} = 0. \quad \underset{\text{Linear.}}{\Longrightarrow} \quad \frac{\partial \delta_r}{\partial t} + \frac{4}{3}\vec{\nabla} \cdot \vec{v} = 0.$$

(2) : $$\frac{\partial}{\partial t}\vec{v} + \frac{\dot{a}}{a}\vec{v} = -\frac{1}{a}\vec{\nabla}\phi$$

$$\Downarrow$$

$$\begin{cases} \dfrac{\partial^2 \delta_m}{\partial t^2} + 2\dfrac{\dot{a}}{a}\dfrac{\partial \delta_m}{\partial t} = +\dfrac{1}{a^2}\nabla^2\phi = 4\pi G\bar{\rho}\left(\rho_m + \rho_r + \dfrac{3p}{c^2}\right. \\ \dfrac{\partial^2 \delta_r}{\partial t^2} + 2\dfrac{\dot{a}}{a}\dfrac{\partial \delta_r}{\partial t} = \dfrac{1}{a^2}\nabla^2\phi = 4\pi G\bar{\rho}\left(\rho_m + \rho_r + \dfrac{3p}{c^2}\right. \end{cases}$$

$$\begin{cases} \dfrac{\partial^2 \delta_m}{\partial t^2} + 2\dfrac{\dot a}{a}\delta_m = 4\pi G\left(\rho_m \delta_m + 2\rho_r \delta_r\right) \\[2ex] \dfrac{\partial^2 \delta_r}{\partial t^2} + 2\dfrac{\dot a}{a}\delta_r = 4\pi G\left(\frac{4}{3}\rho_m \delta_m + \frac{8}{3}\rho_r \delta_r\right) \end{cases}$$

$$\boxed{L\begin{pmatrix}\delta_m \\ \delta_r\end{pmatrix} = 4\pi G\begin{pmatrix}\rho_m & 2\rho_r \\ \frac{4\rho_m}{3} & \frac{8\rho_r}{3}\end{pmatrix}\begin{pmatrix}\delta_m \\ \delta_r\end{pmatrix}}$$

$\circledast$ $L \equiv \dfrac{\partial^2}{\partial t^2} + 2\dfrac{\dot a}{a}$   linear operator.

**Note**: if $\boxed{\delta_r = \frac{4}{3}\delta_m}$ :

evolution of matter and radiation fully coupled

"Adiabatic Mode":

entropy per baryon: $S \propto T^3 \propto \rho_r^{3/4}$

$\rho_r \propto T^4$

$$\Rightarrow \frac{\delta S}{S} = \frac{3}{4}\delta_r = \delta_m .$$

$$\boxed{\delta(\text{entropy}) = \delta(\text{matter})}$$

Note: Matter-Dark Matter System

$$\boxed{L\begin{pmatrix}\delta_b\\ \delta_d\end{pmatrix} = \frac{4\pi G\rho}{\Omega}\begin{pmatrix}\Omega_b & \Omega_d\\ \Omega_b & \Omega_d\end{pmatrix}\begin{pmatrix}\delta_b\\ \delta_d\end{pmatrix}}$$