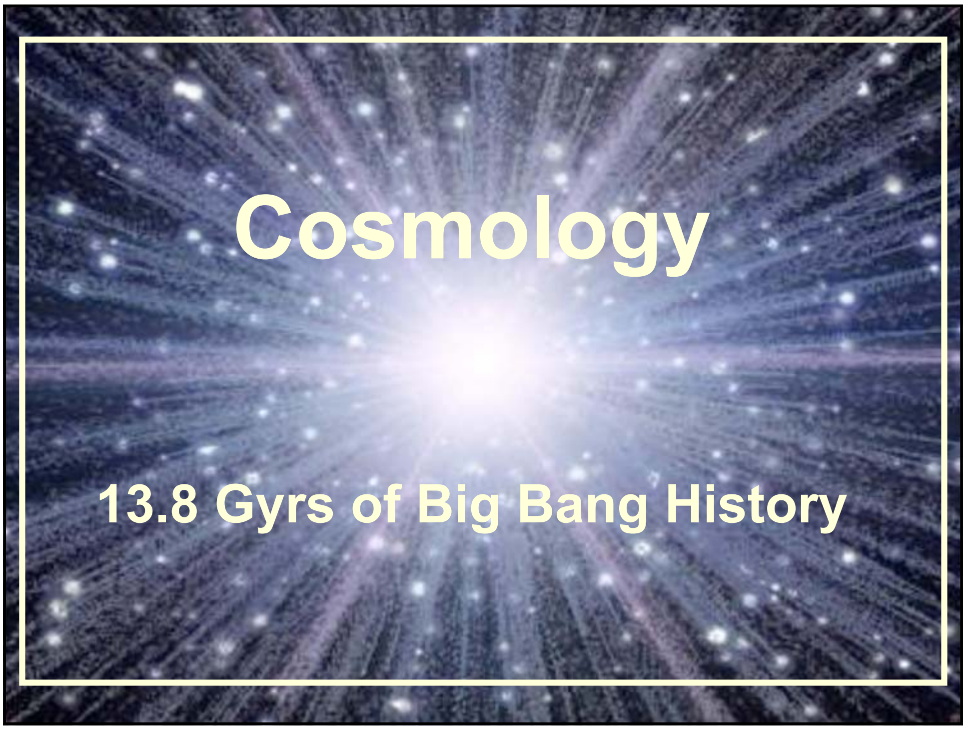


Gravity

Ruler of the Universe

Four Fundamental Forces of Nature

- **Strong Nuclear Force**

  Responsible for holding particles together inside the nucleus.
  The nuclear strong force carrier particle is called the gluon.
  The nuclear strong interaction has a range of 10^{-15} m (diameter of a proton).

- **Electromagnetic Force**

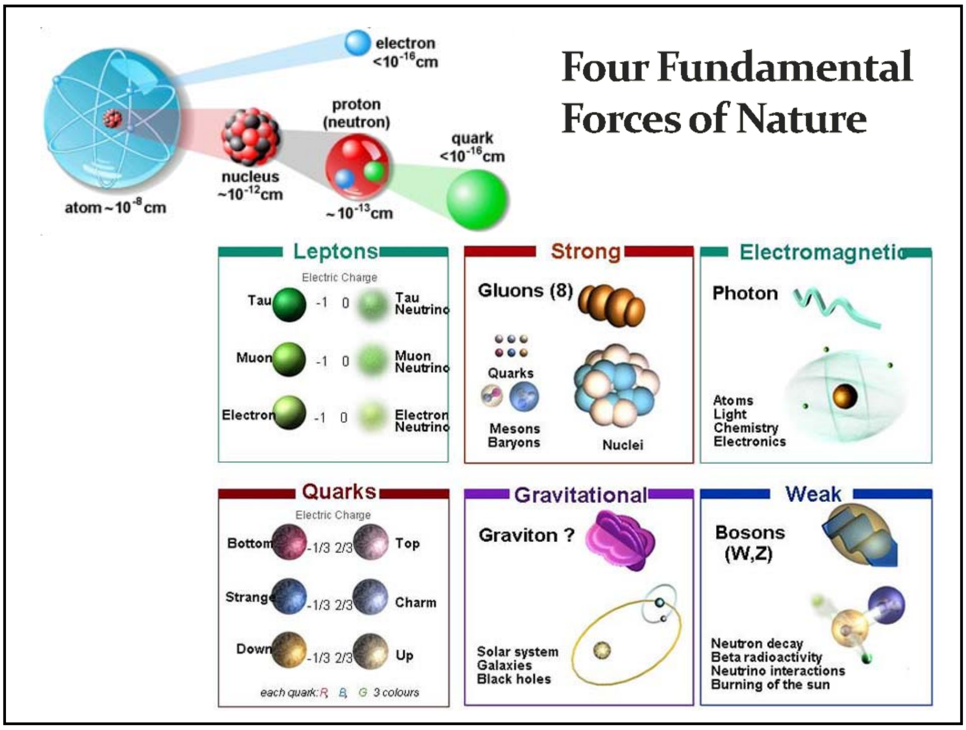
  Responsible for electric and magnetic interactions, and determines structure of atoms and molecules.
  The electromagnetic force carrier particle is the photon (quantum of light)
  The electromagnetic interaction range is infinite.

- **Weak Force**

  Responsible for (beta) radioactivity.
  The weak force carrier particles are called weak gauge bosons (Z, W^+, W^-).
  The nuclear weak interaction has a range of 10^{-17} m (1% of proton diameter).

- **Gravity**

  Responsible for the attraction between masses. Although the gravitational force carrier
  The hypothetical (carrier) particle is the graviton.
  The gravitational interaction range is infinite.
  By far the weakest force of nature.

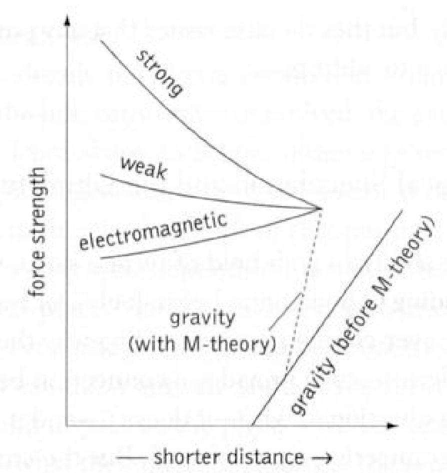


Interaction	Current Theory	Mediators	Relative Strength[1]	Long-Distance Behavior	Range(m)
Strong	Quantum chromodynamics (QCD)	gluons	10^{38}	1 (see discussion below)	10^{-15}
Electromagnetic	Quantum electrodynamics (QED)	photons	10^{36}	$\frac{1}{r^2}$	infinite
Weak	Electroweak Theory	W and Z bosons	10^{25}	$\frac{e^{-m_{W,Z}r}}{r}$	10^{-18}
Gravitation	General Relativity (GR)	gravitons	1	$\frac{1}{r^2}$	infinite

The weakest force, by far, rules the Universe ...

Gravity has dominated its evolution, and determines its fate ...

Grand Unified Theories (GUT)



Grand Unified Theories

* describe how
  - Strong
  - Weak
  - Electromagnetic

  Forces are manifestations of the same underlying GUT force ...

* This implies the strength of the forces to diverge from their uniform GUT strength

* Interesting to see whether gravity at some very early instant unifies with these forces ???

Newton’s Static Universe

The Unchanging Universe

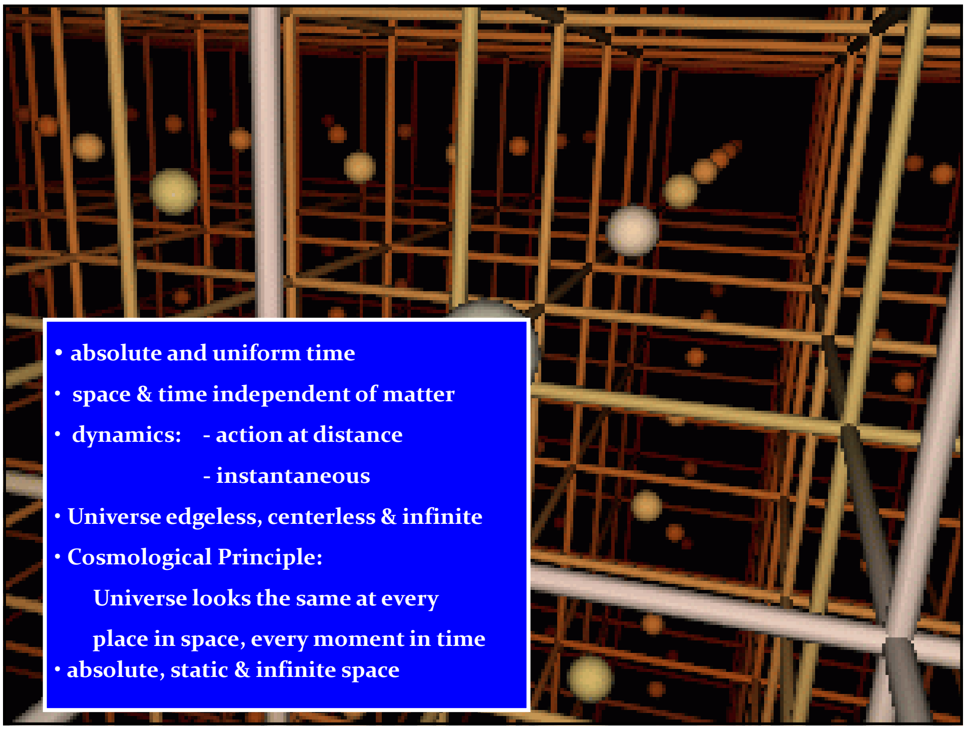
- In two thousand years of astronomy, no one ever guessed that the universe might be expanding.
- To ancient Greek astronomers and philosophers, the universe was seen as the embodiment of perfection, the heavens were truly heavenly:
  – unchanging, permanent, and geometrically perfect.
- In the early 1600s, Isaac Newton developed his law of gravity, showing that motion in the heavens obeyed the same laws as motion on Earth.

Newton's Universe

- However, Newton ran into trouble when he tried to apply his theory of gravity to the entire universe.
- Since gravity is always attractive, his law predicted that all the matter in the universe should eventually clump into one big ball.
- Newton knew this was not the case, and assumed that the universe had to be static
- So he conjectured that:

the Creator placed the stars such that they were

``at immense distances from one another.”



- absolute and uniform time
- space & time independent of matter
- dynamics: - action at distance
  - instantaneous
- Universe edgeless, centerless & infinite
- Cosmological Principle:
  Universe looks the same at every place in space, every moment in time
- absolute, static & infinite space

Einstein's

Dynamic & Geometric Universe

Einstein's Universe

In 1915,
Albert Einstein completed his General Theory of Relativity.

- General Relativity is a "metric theory":
  gravity is a manifestation of the geometry, curvature, of space-time.

- Revolutionized our thinking about the nature of space & time:
  - no longer Newton's static and rigid background,
  - a dynamic medium, intimately coupled to the universe's content of matter and energy.

- All phrased into perhaps
  the most beautiful and impressive scientific equation
  known to humankind, a triumph of human genius,

Einstein Field Equations

... Spacetime becomes a dynamic continuum,
integral part of the structure of the cosmos ...
curved spacetime becomes force of gravity

$$R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R = -\frac{8\pi G}{c^4} T^{\alpha\beta}$$

... its geometry rules the world,
the world rules its geometry...

Albert Einstein

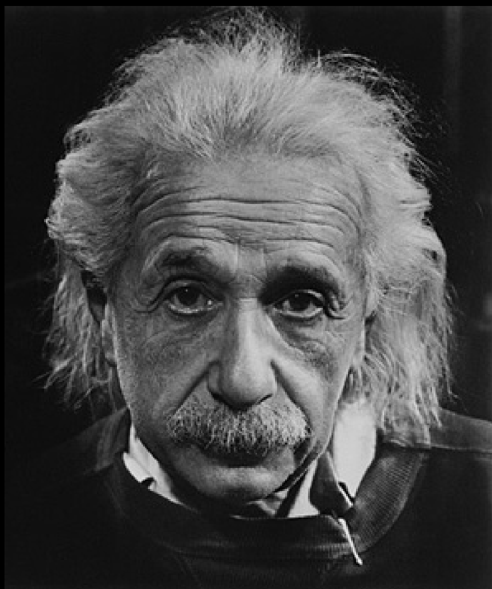
Albert Einstein
(1879-1955; Ulm-Princeton)

father of
General Relativity (1915),

opening the way towards
Physical Cosmology

The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction.

(Albert Einstein, 1954)



General Relativity:

Einstein Field Equations

Einstein Field Equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

Metric tensor: $g_{\mu\nu}$

Energy-Momentum tensor: $T_{\mu\nu}$

$$T_{\mu\nu} = \left(\rho + \frac{p}{c^2} \right) U^{\mu} U^{\nu} - p g^{\mu\nu}$$

Einstein Field Equation

Einstein Tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

$$G_{\mu\nu;\nu} = T_{\mu\nu;\nu} = 0$$

⇓ Einstein Tensor only rank 2 tensor for which this holds:

$$G_{\mu\nu} \propto T_{\mu\nu}$$

Einstein Field Equation

also: $$g_{\mu\nu;\nu} = 0$$

⇓ Freedom to add a multiple of metric tensor to Einstein tensor:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

Λ : Cosmological Constant

Einstein Field Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

↓ Dark Energy

$$G^{\mu\nu} = -\frac{8\pi G}{c^4}\left(T^{\mu\nu} + T^{\mu\nu}{}_{vac}\right)$$

$$T^{\mu\nu}{}_{vac} \equiv \frac{\Lambda c^4}{8\pi G} g^{\mu\nu}$$

Einstein Field Equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Curved Space:

Cosmological Principle
Friedmann-Robertson Metric

General Relativity

A crucial aspect of any particular configuration is the geometry of spacetime: because Einstein's General Relativity is a metric theory, knowledge of the geometry is essential.

Einstein Field Equations are notoriously complex, essentially 10 equations. Solving them for general situations is almost impossible.

However, there are some special circumstances that do allow a full solution. The simplest one is also the one that describes our Universe. It is encapsulated in the

Cosmological Principle

On the basis of this principle, we can constrain the geometry of the Universe and hence find its dynamical evolution.

Cosmological Principle: the Universe Simple & Smooth

"God is an infinite sphere whose centre is everywhere and its circumference nowhere"
Empedocles, 5th cent BC

Cosmological Principle:

Describes the symmetries in global appearance of the Universe:

- Homogeneous → The Universe is the same everywhere:
  - physical quantities (density, T,p,…)
- Isotropic → The Universe looks the same in every direction
- Universality → Physical Laws same everywhere
- Uniformly Expanding → The Universe "grows" with same rate in
  - every direction
  - at every location

"all places in the Universe are alike"
Einstein, 1931

Geometry of the Universe

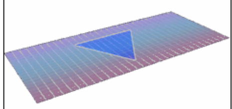
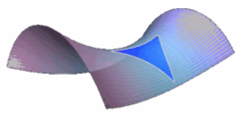
Fundamental Tenet

of (Non-Euclidian = Riemannian) Geometry

There exist no more than THREE uniform spaces:

1)	Euclidian (flat) Geometry	Euclides
2)	Hyperbolic Geometry	Gauß, Lobachevski, Bolyai
3)	Spherical Geometry	Riemann

uniform=
homogeneous & isotropic
(cosmological principle)

Property	Closed	Euclidean	Open
Spatial Curvature	Positive	Zero	Negative
Circle Circumference	$< 2\pi R$	$2\pi R$	$> 2\pi R$
Sphere Area	$< 4\pi R^2$	$4\pi R^2$	$> 4\pi R^2$
Sphere Volume	$< {}^4/_3\ \pi R^3$	${}^4/_3\ \pi R^3$	$> {}^4/_3\ \pi R^3$
Triangle Angle Sum	$> 180°$	$180°$	$< 180°$
Total Volume	Finite ($2\pi^2 R^3$)	Infinite	Infinite
Surface Analog	Sphere 	Plane 	Saddle 

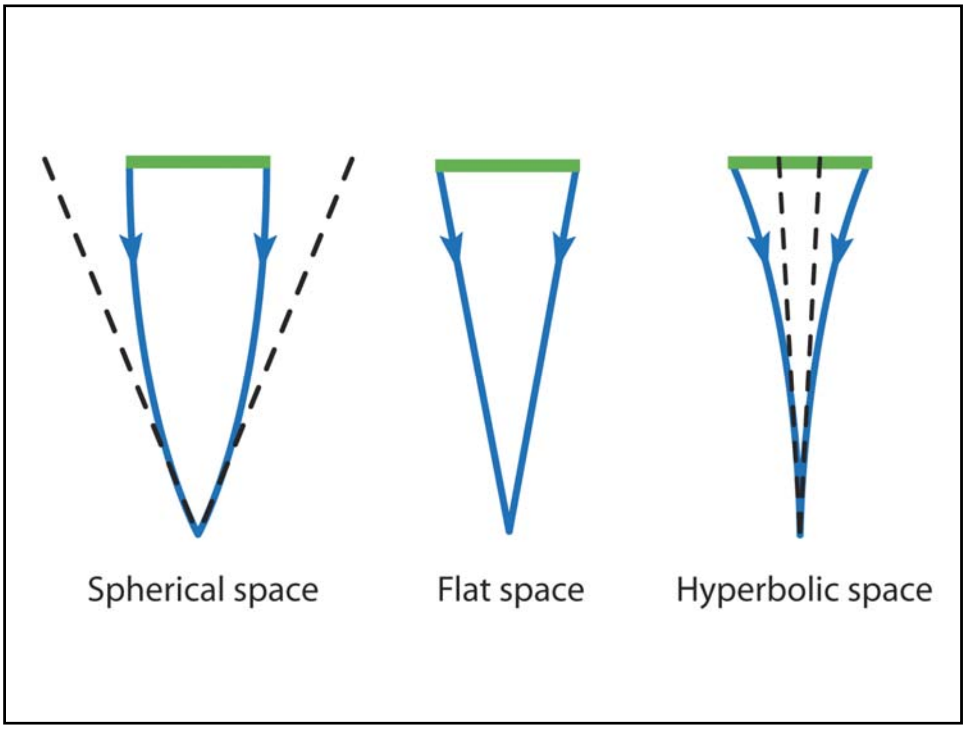
Robertson-Walker Metric

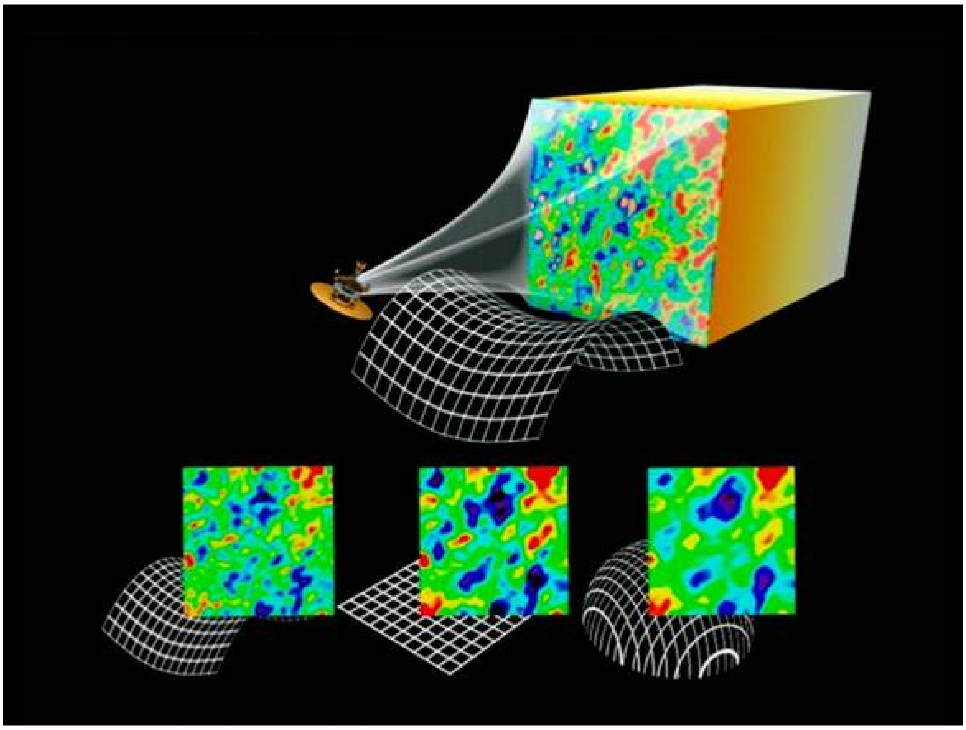
Distances in a uniformly curved spacetime is specified in terms of the Robertson-Walker metric. The spacetime distance of a point at coordinate (r,θ,ϕ) is:

$$ds^2 = c^2dt^2 - a(t)^2\left\{dr^2 + R_c{}^2 S_k{}^2\left(\frac{r}{R_c}\right)\left[d\theta^2 + \sin^2\theta\, d\phi^2\right]\right\}$$

where the function $S_k(r/R_c)$ specifies the effect of curvature on the distances between points in spacetime

$$S_k\left(\frac{r}{R_c}\right) = \begin{cases} \sin\left(\frac{r}{R_c}\right) & k = +1 \\ \frac{r}{R_c} & k = 0 \\ \sinh\left(\frac{r}{R_c}\right) & k = -1 \end{cases}$$


Spherical space
Flat space
Hyperbolic space



Friedmann-Robertson-Walker-Lemaitre (FRLW)

Universe

Einstein Field Equation

$$G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

$$g_{\mu\nu,RW} \Rightarrow \Gamma^{\mu}{}_{\lambda\nu} \Rightarrow R_{\mu\nu} \;,\; R$$

$$T_{\mu\nu} = \left(\rho + \frac{p}{c^2}\right) U^{\mu} U^{\nu} - p g^{\mu\nu}$$
$$= diag\left(\rho c^2, p, p, p\right)$$

Einstein Field Equation

$$G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

$$G^0{}_0 \quad \rightarrow \quad G^0{}_0 = 3\left(\dot{R}^2 + kc^2\right) / R^2 = \frac{8\pi G}{c^2} \rho c^2$$

$$G^1{}_1 \quad \rightarrow \quad G^1{}_1 = \left(2R\ddot{R} + \dot{R}^2 + kc^2\right) / R^2 = -\frac{8\pi G}{c^2} p$$

Friedmann-Robertson-Walker-Lemaitre Universe

$$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) R + \frac{\Lambda}{3} R$$

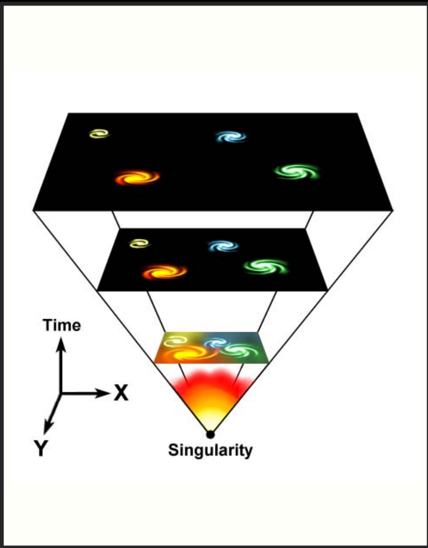
$$\dot{R}^2 = \frac{8\pi G}{3} \rho R^2 - kc^2 + \frac{\Lambda}{3} R^2$$

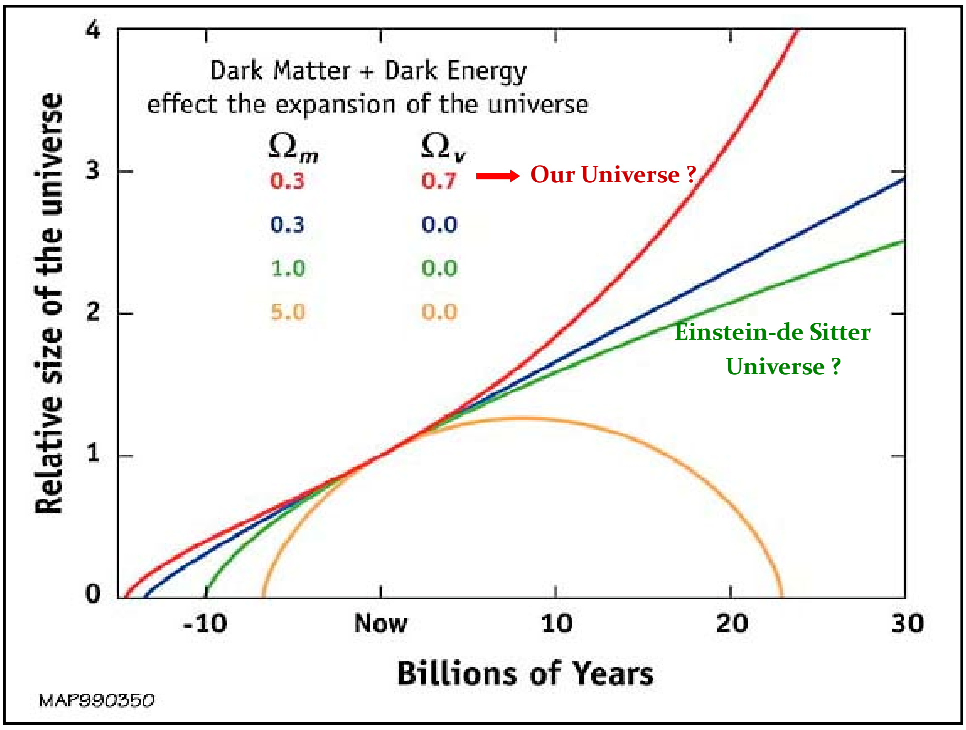
Cosmic Expansion Factor

$$a(t) = \frac{R(t)}{R_0}$$

- Cosmic Expansion is a uniform expansion of space

$$\vec{r}(t) = a(t)\vec{x}$$





Friedmann-Robertson-Walker-Lemaitre Universe

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a + \frac{\Lambda}{3}a$$

density pressure cosmological constant curvature term

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0^2} + \frac{\Lambda}{3}a^2$$

Friedmann-Robertson-Walker-Lemaitre Universe

Because of General Relativity, the evolution of the Universe is fully determined by four factors:

- density $\rho(t)$
- pressure $p(t)$
- curvature kc^2 / R_0^2 — $k = 0, +1, -1$; R_0: present curvature radius
- cosmological constant Λ

• Density & Pressure:
- in relativity, energy & momentum need to be seen as one physical quantity (four-vector)
- pressure = momentum flux

• Curvature:
- gravity is a manifestation of geometry spacetime

• Cosmological Constant:
- free parameter in General Relativity
- Einstein's "biggest blunder"
- mysteriously, since 1998 we know it dominates the Universe

Friedmann-Robertson-Walker-Lemaitre Universe

Relativistic Cosmology

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a + \frac{\Lambda}{3}a$$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0^{\,2}} + \frac{\Lambda}{3}a^2$$

Newtonian Cosmology

$$\ddot{a} = -\frac{4\pi G}{3}\rho a$$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 + E$$

$-kc^2 / R_0^{\,2}$	Curvature
Λ	Cosmological Constant
p	Pressure

⟺

E	Energy
	———
	———

Cosmological Constant & FRW equations

Friedmann-Robertson-Walker-Lemaitre Universe

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a + \frac{\Lambda}{3}a$$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0^{\,2}} + \frac{\Lambda}{3}a^2$$

Dark Energy & Energy Density

$$\tilde{\rho} = \rho + \rho_\Lambda$$

$$\tilde{p} = p + p_\Lambda$$

$$\rho_\Lambda = \frac{\Lambda}{8\pi G}$$

$$p_\Lambda = -\frac{\Lambda c^2}{8\pi G}$$

Friedmann-Robertson-Walker-Lemaitre Universe

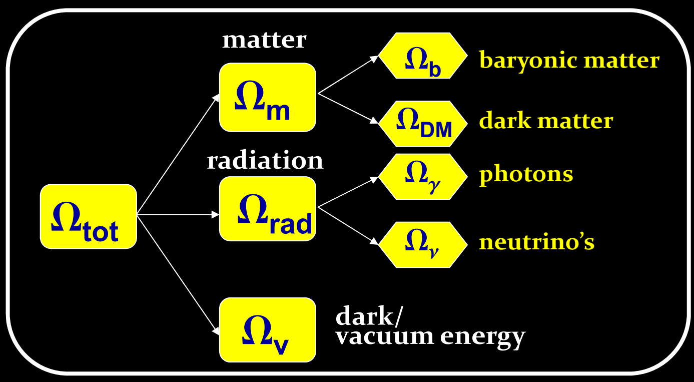
$$\ddot{a} = -\frac{4\pi G}{3}\left(\tilde{\rho} + \frac{3\tilde{p}}{c^2}\right)a$$

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\tilde{\rho} - \frac{kc^2/R_0{}^2}{a^2}$$

Cosmic Constituents

Cosmic Constituents

The total energy content of Universe made up by various constituents, principal ones:

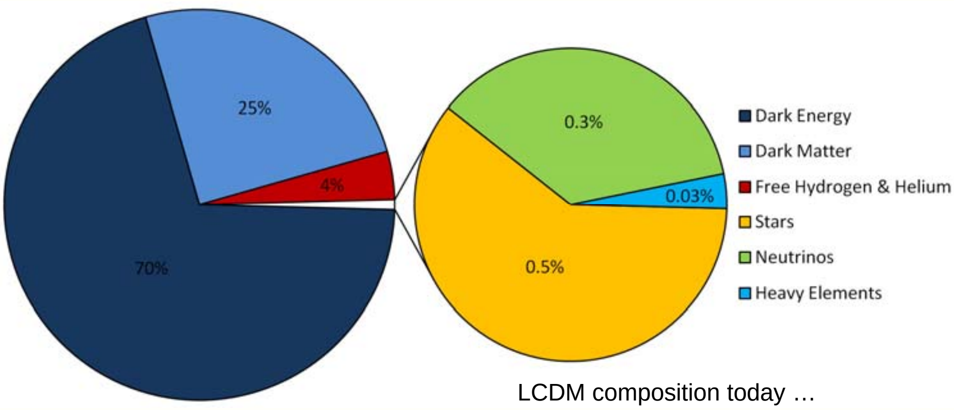


In addition, contributions by
- gravitational waves
- magnetic fields,
- cosmic rays ...

Poor constraints on their contribution: henceforth we will not take them into account !

LCDM Cosmology

- Concordance cosmology
  - model that fits the majority of cosmological observations
  - universe dominated by Dark Matter and Dark Energy



LCDM composition today …

Cosmic Energy Inventory

1	dark sector			0.954 ± 0.003
1.1	dark energy		0.72 ± 0.03	
1.2	dark matter		0.23 ± 0.03	
1.3	primeval gravitational waves		$\lesssim 10^{-10}$	
2	primeval thermal remnants			0.0010 ± 0.0005
2.1	electromagnetic radiation		$10^{-4.3\pm0.0}$	
2.2	neutrinos		$10^{-2.9\pm0.1}$	
2.3	prestellar nuclear binding energy		$-10^{-4.1\pm0.0}$	
3	baryon rest mass			0.045 ± 0.003
3.1	warm intergalactic plasma		0.040 ± 0.003	
3.1a	virialized regions of galaxies	0.024 ± 0.005		
3.1b	intergalactic	0.016 ± 0.005		
3.2	intracluster plasma		0.0018 ± 0.0007	
3.3	main sequence stars	spheroids and bulges	0.0015 ± 0.0004	
3.4		disks and irregulars	0.00055 ± 0.00014	
3.5	white dwarfs		0.00036 ± 0.00008	
3.6	neutron stars		0.00005 ± 0.00002	
3.7	black holes		0.00007 ± 0.00002	
3.8	substellar objects		0.00014 ± 0.00007	
3.9	HI + HeI		0.00062 ± 0.00010	
3.10	molecular gas		0.00016 ± 0.00006	
3.11	planets		10^{-6}	
3.12	condensed matter		$10^{-5.6\pm0.3}$	
3.13	sequestered in massive black holes		$10^{-5.4}(1+\epsilon_n)$	
4	primeval gravitational binding energy			$-10^{-6.1\pm0.1}$
4.1	virialized halos of galaxies		$-10^{-7.2}$	
4.2	clusters		$-10^{-6.9}$	
4.3	large-scale structure		$-10^{-6.2}$	

Fukugita & Peebles 2004

Cosmic Constituents:

Evolving Energy Density

FRW Energy Equation

To infer the evolving energy density ρ(t) of each cosmic component, we refer to the cosmic energy equation. This equation can be directly inferred from the FRW equations

$$\dot{\rho} + 3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{a}}{a} = 0$$

The equation forms a direct expression of the adiabatic expansion of the Universe, ie.

$U = \rho c^2 V$ Internal energy

$V \propto a^3$ Expanding volume

$$dU = -p dV$$

FRW Energy Equation

To infer ρ(t) from the energy equation, we need to know the pressure p(t) for that particular medium/ingredient of the Universe.

$$\dot{\rho} + 3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{a}}{a} = 0$$

To infer p(t), we need to know the nature of the medium, which provides us with the equation of state,

$$p = p(\rho, S)$$

Cosmic Constituents:
Evolution of Energy Density

- **Matter:** $\rho_m(t) \propto a(t)^{-3}$
- **Radiation:** $\rho_{rad}(t) \propto a(t)^{-4}$
- **Dark Energy:** $\rho_v(t) \propto a(t)^{-3(1+w)} \quad \Leftarrow \quad p = w\rho_v c^2$

$$\Downarrow \quad w = -1$$

$$\rho_\Lambda(t) = cst.$$

Dark Energy:

Equation of State

Einstein Field Equation

curvature side

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu}$$

energy-momentum side

Equation of State

$$T^{\mu\nu}{}_{vac} \equiv \frac{\Lambda c^4}{8\pi G} g^{\mu\nu}$$

restframe ⇒

$$T^{\mu\nu}{}_{vac} \equiv \frac{\Lambda c^4}{8\pi G} \eta^{\mu\nu}$$

$$\eta^{00} = 1, \quad \eta^{ii} = -1$$

$$T_{\mu\nu} = \left(\rho + \frac{p}{c^2}\right) U^{\mu} U^{\nu} - p g^{\mu\nu}$$

restframe:

$$\left. \begin{array}{l} T^{00}{}_{vac} = \rho_{vac} c^2 \\ T^{ii}{}_{vac} = p \end{array} \right\} \Rightarrow \begin{array}{l} \rho_{vac} c^2 = \frac{\Lambda c^4}{8\pi G} \\ p = -\frac{\Lambda c^4}{8\pi G} \end{array}$$

Equation of State

$$\rho_{vac}c^2 = \frac{\Lambda c^4}{8\pi G}$$

$$p = -\frac{\Lambda c^4}{8\pi G}$$

$$p_{vac} = -\rho_{vac}c^2$$

Dynamics

Relativistic Poisson Equation:

$$\nabla^2\phi = 4\pi G\left(\rho + \frac{3p}{c^2}\right)$$

$$\rho_{vac} + \frac{3p_{vac}}{c^2} = -2\rho_{vac} < 0; \qquad \rho_{vac} = \frac{\Lambda}{8\pi G}$$

⇓

$$\nabla^2\phi < 0$$ **Repulsion !!!**

Dark Energy & Cosmic Acceleration

Nature Dark Energy:

(Parameterized) Equation of State

$$p(\rho) = w\rho c^2$$

Cosmic Acceleration:

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a$$

Gravitational Repulsion:

$$p = w\rho c^2 \quad \Leftrightarrow \quad w < -\frac{1}{3} \quad \Rightarrow \quad \ddot{a} > 0$$

Dark Energy & Cosmic Acceleration

DE equation of State

$$p(\rho) = w\rho c^2$$

$$\rho_w(a) = \rho_w(a_0)\, a^{-3(1+w)}$$

Cosmological Constant:

$$\Lambda: \quad w = -1 \quad \rho_w = cst.$$

-1/3 > w > -1:

$$\rho_w \propto a^{-3(1+w)} \quad 1 + w > 0$$ **decreases with time**

Phantom Energy:

$$\rho_w \propto a^{-3(1+w)} \quad 1 + w < 0$$ **increases with time**

Dynamic Dark Energy

DE equation of State

$$p(\rho) = w\rho c^2$$

Dynamically evolving dark energy, parameterization:

$$w(a) = w_0 + (1-a)w_a \approx w_\phi(a)$$

$$\rho_w(a) = \rho_w(a_0)\ \exp\left\{-3\int_1^a \frac{1+w_\phi(a')}{a'}da'\right\}$$

Critical Density & Omega

FRW Dynamics

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0^{\ 2}}$$

Critical Density:

- **For a Universe with Λ=o**
- **Given a particular expansion rate H(t)**
- **Density corresponding to a flat Universe (k=o)**

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

FRW Dynamics

In a FRW Universe,
densities are in the order of the critical density,

$$\rho_{crit} = \frac{3H_0^{\ 2}}{8\pi G} = 1.8791 h^2 \times 10^{-29}\, g\, cm^{-3}$$

$$\begin{aligned}\rho_0 &= 1.8791\times 10^{-29}\,\Omega h^2\ \ g\,cm^{-3}\\ &= 2.78\times 10^{11}\,\Omega h^2 \quad M_\odot Mpc^{-3}\end{aligned}$$

FRW Dynamics

**In a matter-dominated Universe,
the evolution and fate of the Universe entirely determined
by the (energy) density in units of critical density:**

$$\Omega \equiv \frac{\rho}{\rho_{crit}} = \frac{8\pi G\rho}{3H^2}$$

Arguably, Ω is the most important parameter of cosmology !!!

Present-day Cosmic Density:

$$\rho_0 = 1.8791\times10^{-29}\,\Omega h^2 \; g\,cm^{-3}$$
$$= 2.78\times10^{11}\,\Omega h^2 \quad M_{\odot}Mpc^{-3}$$

FRW Dynamics

- **The individual contributions to the energy density of the Universe can be figured into the Ω parameter:**

- radiation

$$\Omega_{rad} = \frac{\rho_{rad}}{\rho_{crit}} = \frac{\sigma T^4 / c^2}{\rho_{crit}} = \frac{8\pi G\sigma T^4}{3H^2c^2}$$

- matter

$$\Omega_m = \Omega_{dm} + \Omega_b$$

- dark energy/ cosmological constant

$$\Omega_{\Lambda} = \frac{\Lambda}{3H^2}$$

$$\Omega = \Omega_{rad} + \Omega_m + \Omega_{\Lambda}$$

Critical Density

There is a 1-1 relation between the total energy content of the Universe and its curvature. From FRW equations:

$$k = \frac{H^2 R^2}{c^2}(\Omega - 1) \qquad \Omega = \Omega_{rad} + \Omega_m + \Omega_\Lambda$$

$\Omega < 1$	$k = -1$	*Hyperbolic*	*Open Universe*
$\Omega = 1$	$k = 0$	*Flat*	*Critical Universe*
$\Omega > 1$	$k = +1$	*Spherical*	*Close Universe*

FRW Universe: Curvature

There is a 1-1 relation between the total energy content of the Universe and its curvature. From FRW equations:

$$k = \frac{H^2 R^2}{c^2}(\Omega - 1) \qquad \Omega = \Omega_{rad} + \Omega_m + \Omega_\Lambda$$

$\Omega < 1$	$k = -1$	*Hyperbolic*	*Open Universe*
$\Omega = 1$	$k = 0$	*Flat*	*Critical Universe*
$\Omega > 1$	$k = +1$	*Spherical*	*Close Universe*

Radiation, Matter & Dark Energy

The individual contributions to the energy density of the Universe can be figured into the Ω parameter:

- radiation $\Omega_{rad} = \frac{\rho_{rad}}{\rho_{crit}} = \frac{\sigma T^4 / c^2}{\rho_{crit}} = \frac{8\pi G \sigma T^4}{3H^2 c^2}$
- matter $\Omega_m = \Omega_{dm} + \Omega_b$
- dark energy/ cosmological constant $\Omega_\Lambda = \frac{\Lambda}{3H^2}$

$$\Omega = \Omega_{rad} + \Omega_m + \Omega_\Lambda$$

Hubble Expansion

Hubble Expansion

- Cosmic Expansion is a uniform expansion of space
- Objects do not move themselves:
  they are like beacons tied to a uniformly expanding sheet:

$$\vec{r}(t) = a(t)\vec{x}$$

$$\dot{\vec{r}}(t) = \dot{a}(t)\vec{x} = \frac{\dot{a}}{a}a\vec{x} = H(t)\vec{r}$$

$$H(t) = \frac{\dot{a}}{a}$$

Hubble Expansion

- Cosmic Expansion is a uniform expansion of space
- Objects do not move themselves:
  they are like beacons tied to a uniformly ex

Comoving Position

Hubble Parameter:

Hubble "constant": $H_o \equiv H(t=t_o)$

$$\vec{r}(t) = a(t)\vec{x}$$

$$\dot{\vec{r}}(t) = \dot{a}(t)\vec{x} = \frac{\dot{a}}{a}a\vec{x} = H(t)\vec{r}$$

$$H(t) = \frac{\dot{a}}{a}$$

Hubble Parameter

- For a long time, the correct value of the Hubble constant H_o was a major unsettled issue:

$H_o = 50$ km s^{-1} Mpc^{-1} ⟷ $H_o = 100$ km s^{-1} Mpc^{-1}

- This meant distances and timescales in the Universe had to deal with uncertainties of a factor 2 !!!

- Following major programs, such as Hubble Key Project, the Supernova key projects and the WMAP CMB measurements,

$$H_0 = 71.9^{+2.6}_{-2.7}\, km\, s^{-1} Mpc^{-1}$$

Hubble Time

$$t_H = \frac{1}{H}$$

⇒

$$H_0 = 100h\ km\, s^{-1} Mpc^{-1}$$
$$\Downarrow$$
$$t_0 = 9.78h^{-1}\ Gyr$$

Hubble Distance

Just as the Hubble time sets a natural time scale for the universe,
one may also infer a natural distance scale of the universe, the

Hubble Distance

$$R_H = \frac{c}{H_0} \approx 2997.9 h^{-1} Mpc$$

Acceleration Parameter

FRW Dynamics: Cosmic Acceleration

Cosmic acceleration quantified by means of dimensionless deceleration parameter q(t):

$$q = -\frac{a\ddot{a}}{\dot{a}^2}$$

$$q = \frac{\Omega_m}{2} + \Omega_{rad} - \Omega_\Lambda$$

$$q \approx \frac{\Omega_m}{2} - \Omega_\Lambda$$

Examples:

$\Omega_m = 1; \quad \Omega_\Lambda = 0;$
$q = 0.5$

$\Omega_m = 0.3; \; \Omega_\Lambda = 0.7;$
$q = -0.65$

Dynamics

FRW Universe

General Solution Expanding FRW Universe

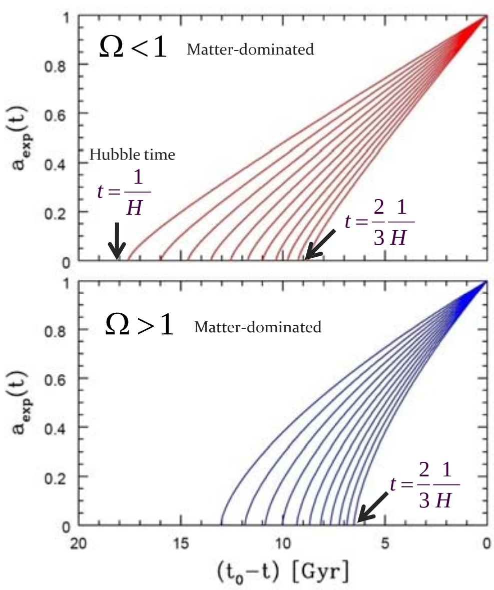
From the FRW equations:

$$\frac{H(t)^2}{H_0^{\,2}} = \frac{\Omega_{rad,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

$a(t)$ Expansion history Universe

$$H_0 t = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{rad,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{\Lambda,0} a^2 + (1-\Omega_0)}}$$

Age of the Universe



Age of a FRW universe at Expansion factor a(t)

$$H\,t = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{rad}}{a^2} + \frac{\Omega_m}{a} + \Omega_\Lambda a^2 + (1-\Omega\,)}}$$

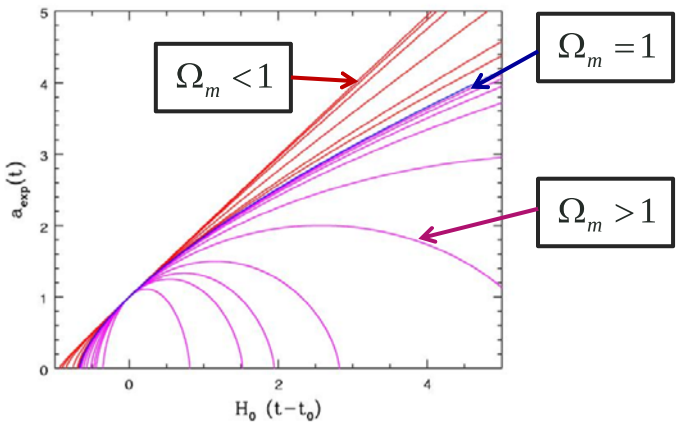
Specific Solutions
FRW Universe

While general solutions to the FRW equations is only possible by numerical integration, analytical solutions may be found for particular classes of cosmologies:

- Single-component Universes:
  - empty Universe
  - flat Universes, with only radiation, matter or dark energy
- Matter-dominated Universes
- Matter+Dark Energy flat Universe

Matter-Dominated Universes

- Assume radiation contribution is negligible: $\Omega_{rad,0} \approx 5\times10^{-5}$
- Zero cosmological constant: $\Omega_\Lambda = 0$
- Matter-dominated, including curvature



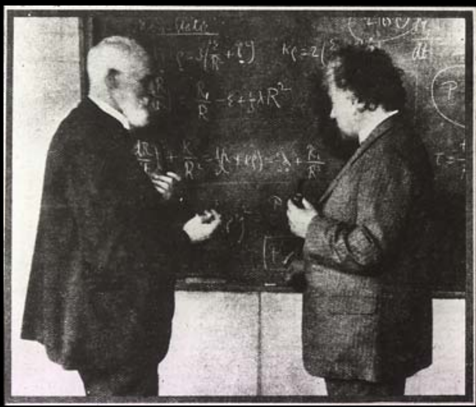
Einstein-de Sitter Universe

$$\left.\begin{array}{l}\Omega_m = 1 \\ \Omega_\Lambda = 0\end{array}\right\} k = 0$$

FRW: $\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 = \frac{8\pi G \rho_0}{3}\frac{1}{a}$

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

Age EdS Universe: $t_0 = \frac{2}{3}\frac{1}{H_0}$



Albert Einstein and Willem de Sitter discussing the Universe. In 1932 they published a paper together on the Einstein-de Sitter universe, which is a model with flat geometry containing matter as the only significant substance.

Free Expanding "Milne" Universe

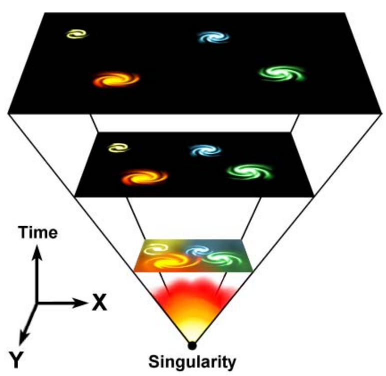
$$\left.\begin{array}{l}\Omega_m = 0 \\ \Omega_\Lambda = 0\end{array}\right\} k = -1$$

Empty space is curved

FRW: $\dot{a}^2 = -\frac{kc^2}{R_0^2} = cst.$

$$a(t) = \left(\frac{t}{t_0}\right)$$

Age Empty Universe: $t_0 = \frac{1}{H_0}$



De Sitter Expansion

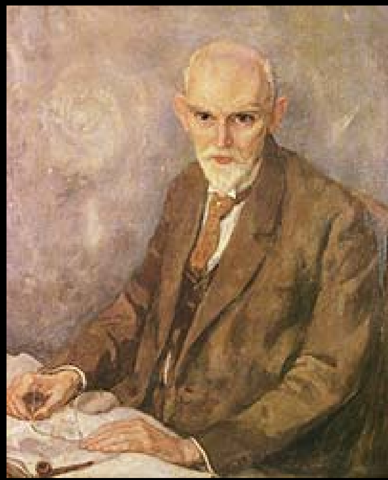
$$\left.\begin{array}{l}\Omega_m = 0 \\ \Omega_\Lambda = 1\end{array}\right\} k = 0$$

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^{\;2}} \quad \Rightarrow \quad H_0 = \sqrt{\frac{\Lambda}{3}}$$

FRW: $\dot{a}^2 = \frac{\Lambda}{3} a^2 \quad \Rightarrow \quad \dot{a} = H_0 a$

$$a(t) = e^{H_0 (t - t_0)}$$

Age
De Sitter Universe: infinitely old



Willem de Sitter (1872-1934; Sneek-Leiden)
director Leiden Observatory
alma mater: Groningen University

Expansion Radiation-dominated Universe

$$\left.\begin{array}{l}\Omega_{rad} = 1 \\ \Omega_m = 0 \\ \Omega_\Lambda = 0\end{array}\right\} k = 0$$

In the very early Universe, the energy density is completely dominated by radiation. The dynamics of the very early Universe is therefore fully determined by the evolution of the radiation energy density:

FRW: $\dot{a}^2 = \frac{8\pi G}{3} \rho a^2 = \frac{8\pi G \rho_0}{3} \frac{1}{a^2}$ ⟵ $\rho_{rad}(a) \propto \frac{1}{a^4}$

$$a(t) = \left(\frac{t}{t_0}\right)^{1/2}$$

Age
Radiation
Universe: $t_0 = \frac{1}{2} \frac{1}{H_0}$

General Flat FRW Universe

$$k = 0$$

$$\rho_v(t) \propto a(t)^{-3(1+w)} \quad \Leftarrow \quad p = w\rho_v c^2$$

FRW:

$$a(t) \propto t^{\frac{2}{3+3w}}$$

Cosmological Transitions

Dynamical Transitions

Because radiation, matter, dark energy (and curvature) of the Universe evolve differently as the Universe expands, at different epochs the energy density of the Universe is alternately dominated by these different ingredients.

As the Universe is dominated by either radiation, matter, curvature or dark energy, the cosmic expansion a(t) proceeds differently.

We therefore recognize the following epochs:

- radiation-dominated era
- matter-dominated era
- curvature-dominated expansion
- dark energy dominated epoch

The different cosmic expansions at these eras have a huge effect on relevant physical processes

Dynamical Transitions

- Radiation Density Evolution

$$\rho_{rad}(t) = \frac{1}{a^4}\rho_{rad,0}$$

- Matter Density Evolution

$$\rho_m(t) = \frac{1}{a^3}\rho_{m,0}$$

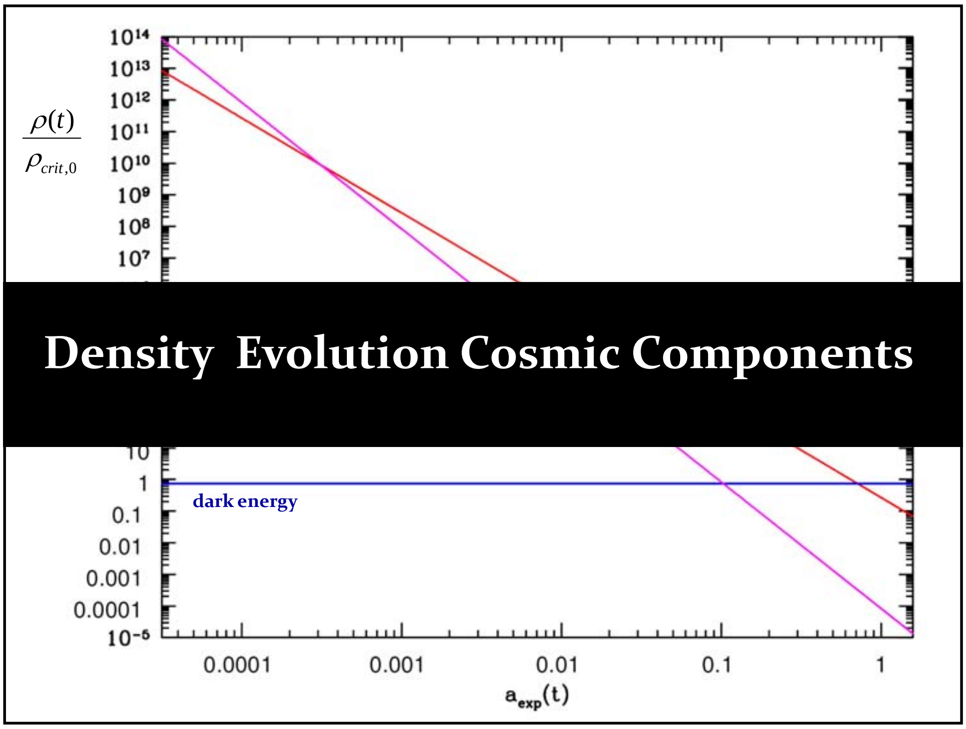
- Curvature Evolution

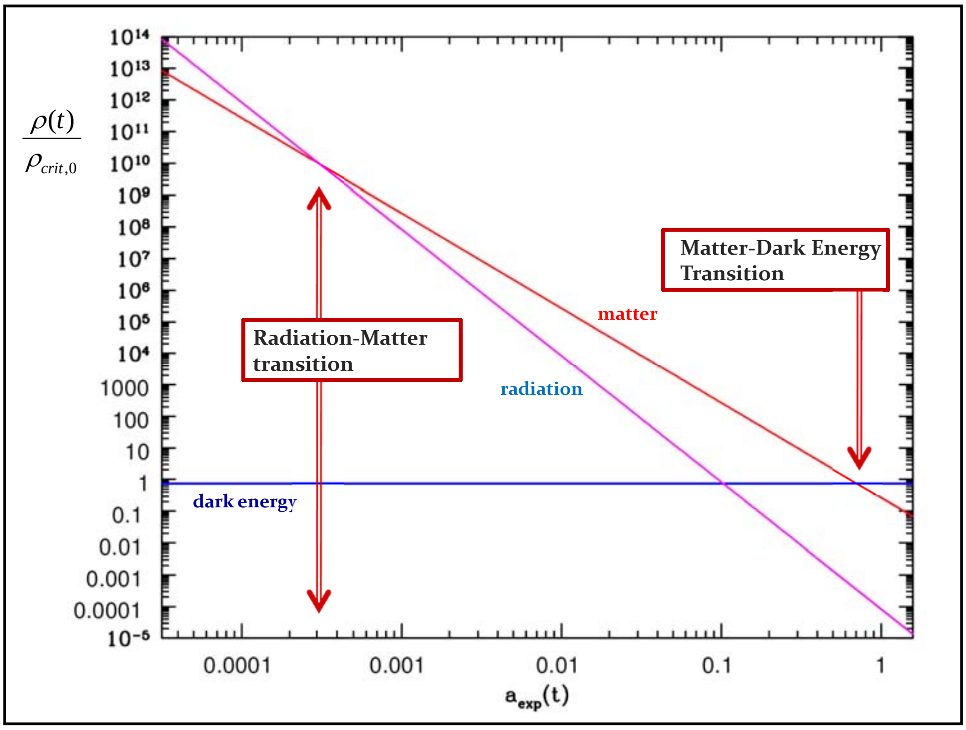
$$\frac{kc^2}{R(t)^2} = \frac{1}{a^2}\frac{kc^2}{R_0^{\ 2}} = \frac{1}{a^2}(1-\Omega_0)$$

- Dark Energy (Cosmological Constant) Evolution

$$\rho_\Lambda(t) = cst. = \rho_{\Lambda 0}$$

Density Evolution Cosmic Components





Radiation-Matter Transition

- Radiation Density Evolution

$$\rho_{rad}(t) = \frac{1}{a^4}\rho_{rad,0}$$

- Matter Density Evolution

$$\rho_m(t) = \frac{1}{a^3}\rho_{m,0}$$

• Radiation energy density decreases more rapidly than matter density: this implies radiation to have had a higher energy density before a particular cosmic time:

$$a_{rm} = \frac{\Omega_{rad,0}}{\Omega_{m,0}} \Leftarrow \frac{\rho_{m,0}}{a^3} = \frac{\rho_{rad,0}}{a^4}$$

$a < a_{rm}$ Radiation dominance

$a > a_{rm}$ Matter dominance

Matter-Dark Energy Transition

- Matter Density Evolution

$$\rho_m(t) = \frac{1}{a^3}\rho_{m,0}$$

- Dark Energy Density Evolution

$$\rho_\Lambda(t) = cst. = \rho_{\Lambda 0}$$

• While matter density decreases due to the expansion of the Universe, the cosmological constant represents a small, yet constant, energy density. As a result, it will represent a higher density after

$$a_{m\Lambda} = \sqrt[3]{\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}} \Leftarrow \frac{\rho_{m,0}}{a^3} = \rho_{\Lambda,0}$$

$a < a_{m\Lambda}$ Matter dominance

$a > a_{m\Lambda}$ Dark energy dominance

Matter-Dark Energy Transition

$$a_{m\Lambda} = \sqrt[3]{\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}}$$

$\Omega_{\Lambda,0} = 0.27$, $\Omega_{m,0} = 0.73$ $\Rightarrow$ $a_{m\Lambda} = 0.72$, $a_{m\Lambda}{}^{\dagger} = 0.57$

Flat Universe:

$$a_{m\Lambda} = \sqrt[3]{\frac{\Omega_{m,0}}{1-\Omega_{m,0}}}$$

Note: a more appropriate characteristic transition is that at which the deceleration turns into acceleration:

$$a_{m\Lambda}{}^{\dagger} = \sqrt[3]{\frac{\Omega_{m,0}}{2\Omega_{\Lambda,0}}} = \sqrt[3]{\frac{\Omega_{m,0}}{2(1-\Omega_{m,0})}}$$

Evolution

Cosmological Density Parameter

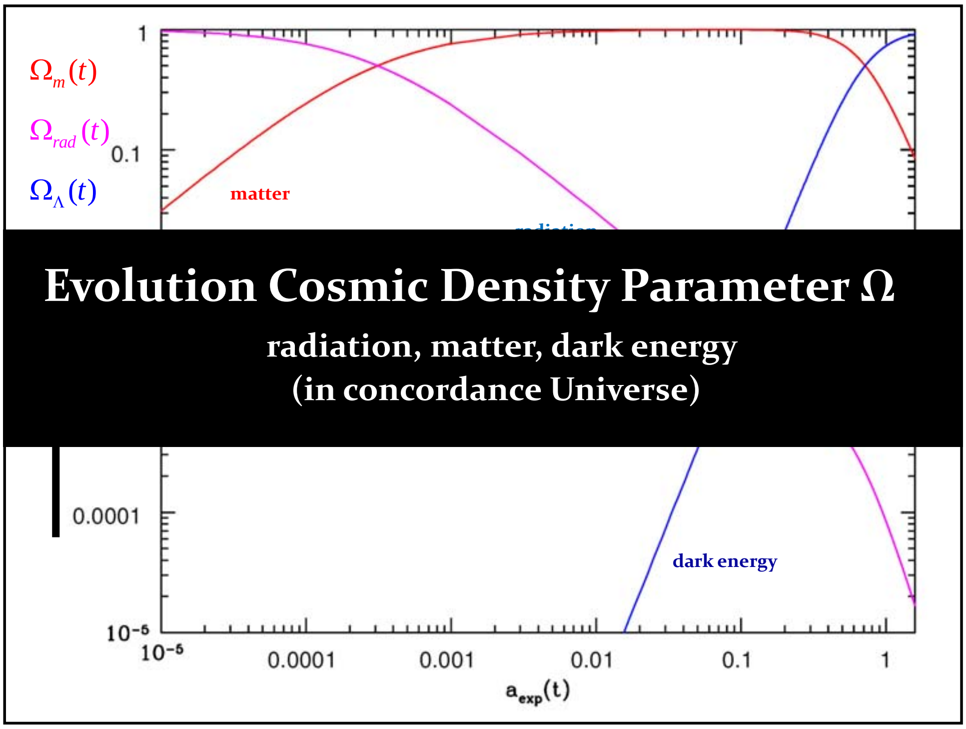
Limiting ourselves to a flat Universe
(and discarding the contribution by and evolution of curvature):

to appreciate the dominance of radiation, matter and dark energy in the subsequent cosmological eras, it is most illuminating to look at the evolution of the cosmological density parameter of these cosmological components:

$$\Omega_{rad}(t) \longleftrightarrow \Omega_m(t) \longleftrightarrow \Omega_\Lambda(t)$$

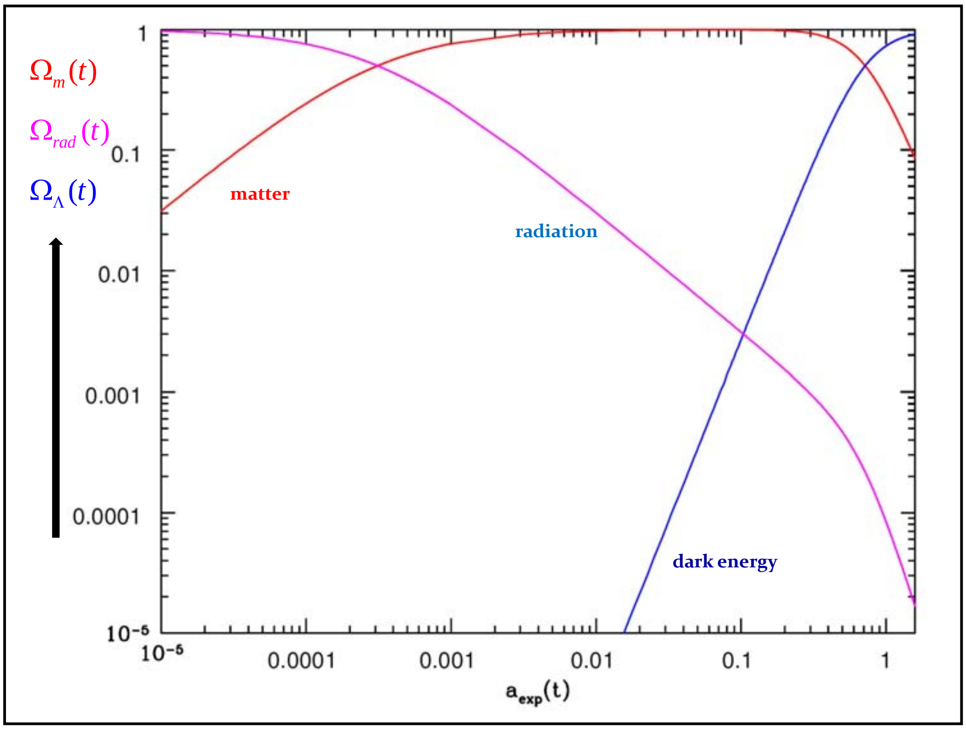
e.g.

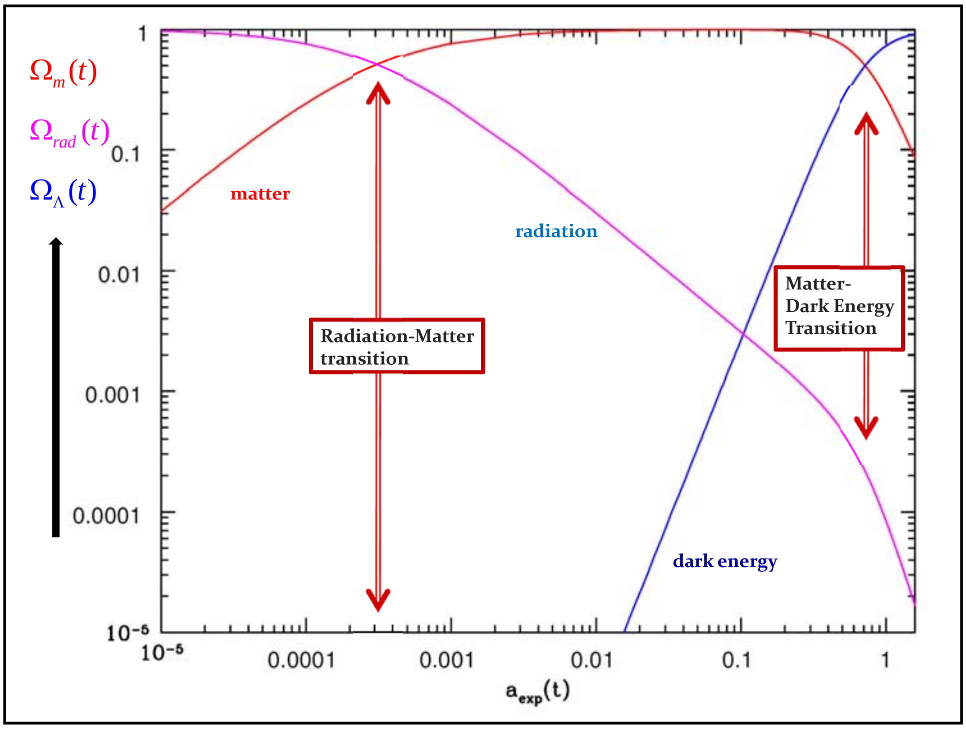
$$\Omega_m(t) = \frac{\Omega_{m,0}a^4}{\Omega_{rad,0} + \Omega_{m,0}a + \Omega_{\Lambda,0}a^4}$$



Evolution Cosmic Density Parameter Ω

radiation, matter, dark energy
(in concordance Universe)





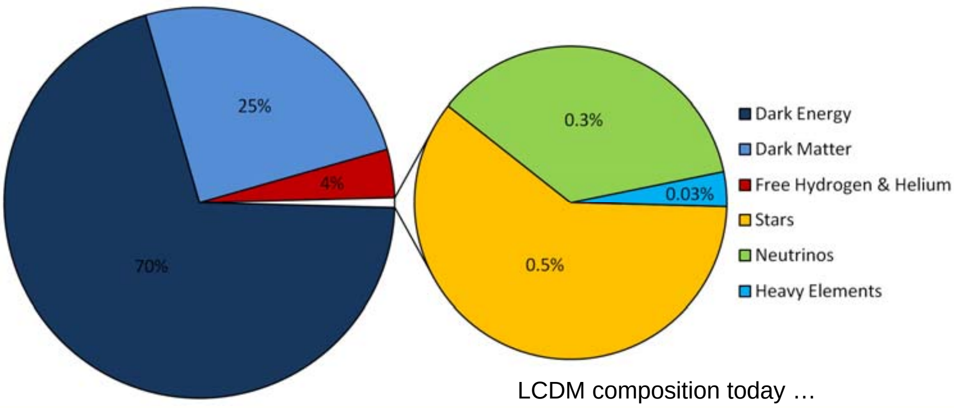
Concordance Universe

Concordance Universe Parameters

Hubble Parameter		$H_0 = 71.9 \pm 2.6\ km\, s^{-1}\, Mpc^{-1}$	
Age of the Universe		$t_0 = 13.7 \pm 0.12\, Gyr$	
Temperature CMB		$T_0 = 2.725 \pm 0.001 K$	
Matter	Baryonic Matter Dark Matter	$\Omega_m = 0.27$	$\Omega_b = 0.0456 \pm 0.0015$ $\Omega_{dm} = 0.228 \pm 0.013$
Radiation	Photons (CMB) Neutrinos (Cosmic)	$\Omega_{rad} = 8.4 \times 10^{-5}$	$\Omega_\gamma = 5 \times 10^{-5}$ $\Omega_\nu = 3.4 \times 10^{-5}$
Dark Energy		$\Omega_\Lambda = 0.726 \pm 0.015$	
Total		$\Omega_{tot} = 1.0050 \pm 0.0061$	

LCDM Cosmology

- Concordance cosmology
  - model that fits the majority of cosmological observations
  - universe dominated by Dark Matter and Dark Energy



LCDM composition today …

Concordance Expansion

$$H_0 t = \frac{2}{3\sqrt{1-\Omega_{m,0}}} \ln\left\{ \left(\frac{a}{a_{m\Lambda}}\right)^{3/2} + \sqrt{1+\left(\frac{a}{a_{m\Lambda}}\right)^{3}} \right\}$$

transition epoch:

matter-dominate to
Λ dominated

$a_{m\Lambda}$~0.75

$$a_{m\Lambda} = \sqrt[3]{\frac{\Omega_{m0}}{1-\Omega_{m0}}}$$

Concordance Expansion

We can recognize two extreme regimes:

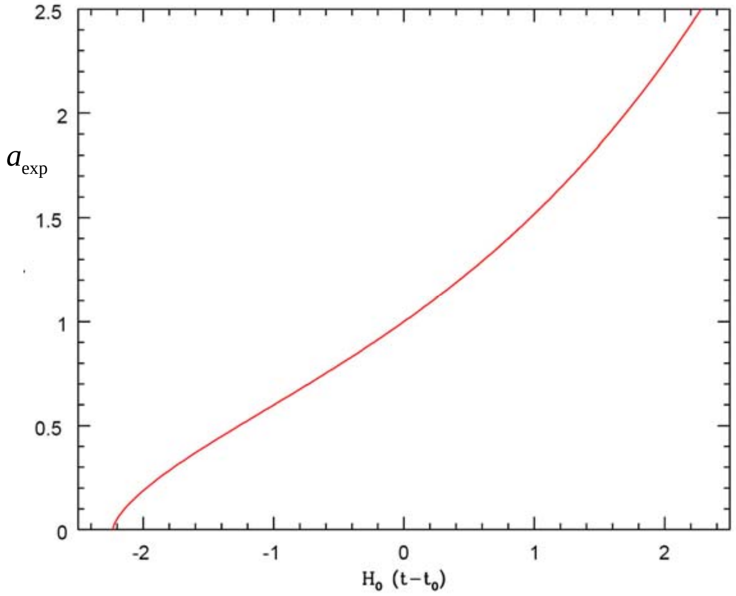
- $a \ll a_{m\Lambda}$ very early times
  matter dominates the expansion, and $\Omega_m \approx 1$: Einstein-de Sitter expansion,

$$a(t) \approx \left(\frac{3}{2}\sqrt{\Omega_{m,0}}\, H_0 t\right)^{2/3}$$

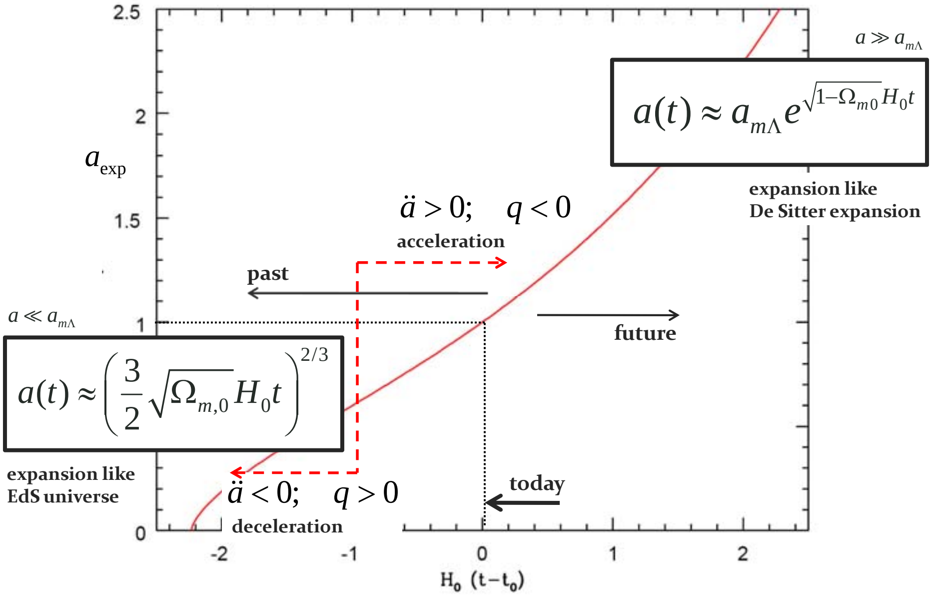
- $a \gg a_{m\Lambda}$ very late times
  matter has diluted to oblivion, and $\Omega_m \approx 0$: de Sitter expansion driven by dark energy

$$a(t) \approx a_{m\Lambda} e^{\sqrt{1-\Omega_{m0}}H_0 t}$$

Concordance Expansion



Concordance Expansion



Matter-Dark Energy Transition

$$a_{m\Lambda} = \sqrt[3]{\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}}$$

$\Rightarrow$

$$\left.\begin{array}{l}\Omega_{\Lambda,0} = 0.27 \\ \Omega_{m,0} = 0.73\end{array}\right\} \quad \begin{array}{l} a_{m\Lambda} = 0.72 \\ a_{m\Lambda}^{\dagger} = 0.57\end{array}$$

Flat Universe

$$a_{m\Lambda} = \sqrt[3]{\frac{\Omega_{m,0}}{1-\Omega_{m,0}}}$$

Note: a more appropriate characteristic transition is that at which the deceleration turns into acceleration:

$$a_{m\Lambda}^{\dagger} = \sqrt[3]{\frac{\Omega_{m,0}}{2\Omega_{\Lambda,0}}} = \sqrt[3]{\frac{\Omega_{m,0}}{2(1-\Omega_{m,0})}}$$

Key Epochs Concordance Universe

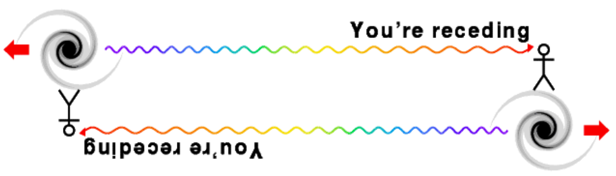
Radiation-Matter Equality		$a_{eq} = 2.8\times10^{-4}$	$t_{eq} = 4.7\times10^{4}\ yr$
Recombination/ Decoupling		$a_{rec} \approx 1/1091$ $z_{rec} = 1090.88 \pm 0.72$	$t_{rec} = 3.77 \pm 0.03\times10^{5}\ yrs$
Reionization	Optical Depth Redshift	$\tau_{reion} = 0.084 \pm 0.016$ $z_{reion} = 10.9 \pm 1.4$	$t_{reion} = 432^{+90}_{-67}\times10^{6}\ yrs$
Matter-Dark Energy Transition	Acceleration Energy	$a_{m\Lambda}^{\dagger} \approx 0.60;\ z_{m\Lambda}^{\dagger} \approx 0.67$ $a_{m\Lambda} \approx 0.75;\ z_{m\Lambda} \approx 0.33$	$t_{m\Lambda} = 9.8\ Gyr$
Today		$a_0 = 1$	$t_{eq} = 13.72 \pm 0.12\ Gyr$

Observational Cosmology in FRW Universe

Cosmic Redshift

$$1 + z = \frac{1}{a} \quad \Longleftarrow \quad \begin{cases} \lambda_{em} = \lambda_0 \\ \lambda_{obs} = \dfrac{a(t_{obs})}{a(t_{em})} \lambda_0 \end{cases}$$

$$z \equiv \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$



RW Distance Measure

In an (expanding) space with Robertson-Walker metric,

$$ds^2 = c^2dt^2 - a(t)^2 \left\{ dr^2 + R_c^{\,2} S_k^{\,2}\left(\frac{r}{R_c}\right)\left[d\theta^2 + \sin^2\theta\, d\phi^2\right]\right\}$$

there are several definitions for distance, dependent on how you measure it.

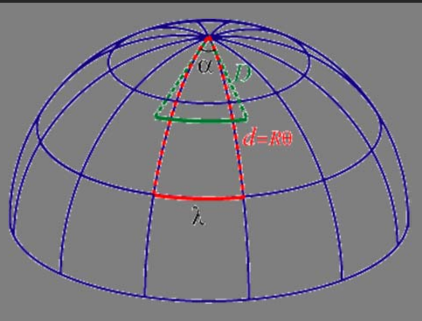
They all involve the central distance function, the *RW Distance Measure,*

$$D(r) = R_c S_k\left(\frac{r}{R_c}\right)$$

Angular Diameter Distance

Imagine an object of *proper size d,* at redshift z, its angular size $\Delta\theta$ is given by

$$d = a(t) R_c S_k\left(\frac{r}{R_c}\right)\Delta\theta \quad \Rightarrow \quad \Delta\theta = \frac{d\,(1+z)}{D} = \frac{d}{D_A}$$



Angular Diameter distance:

$$D_A = \frac{D}{1+z}$$

Luminosity Distance

Imagine an object of luminosity $L(\nu_e)$, at redshift z, its flux density at observed frequency ν_o is

$$S(\nu_o) = \frac{L(\nu_e)}{4\pi D^2 (1+z)} \Rightarrow S_{bol} = \frac{L_{bol}}{4\pi D^2 (1+z)^2} = \frac{L_{bol}}{4\pi D_L{}^2}$$

Luminosity distance:

$$D_L = D(1+z)$$

FRW Redshift-Distance

Observing in a FRW Universe, we locate galaxies in terms of their redshift z. To connect this to their true physical distance, we need to know what the coordinate distance r of an object with redshift z,

$$R_0 dr = \frac{c}{H(z)} dz$$

In a FRW Universe, the dependence of the Hubble expansion rate H(z) at any redshift z depends on the content of matter, dark energy and radiation, as well ss its curvature. This leads to the following explicit expression for the redshift-distance relation,

$$R_0 dr = \frac{c}{H_0} \left\{ (1-\Omega_0)(1+z)^2 + \Omega_{\Lambda,0} + \Omega_{m,0}(1+z)^3 + \Omega_{rad,0}(1+z)^4 \right\}^{-1/2} dz$$

Matter-Dominated FRW Universe

in a matter-dominated Universe, the redshift-distance relation is

$$R_0 dr = \frac{c}{H_0}\left\{(1-\Omega_0)(1+z)^2 + \Omega_0 (1+z)^3\right\}^{-1/2} dz$$

from which one may find that

$$R_0 r = \frac{c}{H_0}\int_0^z \frac{dz'}{(1+z')\sqrt{1+\Omega_0 z'}}$$

Mattig's Formula

The integral expression

$$R_0 r = \frac{c}{H_0}\int_0^z \frac{dz'}{(1+z')\sqrt{1+\Omega_0 z'}}$$

can be evaluated by using the substitution: $u^2 = \frac{k(\Omega_0 - 1)}{\Omega_0 (1+z)}$

This leads to Mattig's formula:

$$D(z) = R_c S_k\left(\frac{r}{R_c}\right) = \frac{2c}{H_0}\frac{\Omega_0 z + (\Omega_0 - 2)\left\{\sqrt{1+\Omega_0 z} - 1\right\}}{\Omega_0^{\,2}(1+z)}$$

This is one of the very most important and most useful equations in observational cosmology.

Mattig's Formula

$$D(z) = R_c S_k\left(\frac{r}{R_c}\right) = \frac{2c}{H_0}\frac{\Omega_0 z + (\Omega_0 - 2)\left\{\sqrt{1+\Omega_0 z} - 1\right\}}{\Omega_0^{\,2}(1+z)}$$

In a low-density Universe, it is better to use the following version:

$$D(z) = R_c S_k\left(\frac{r}{R_c}\right) = \frac{c}{H_0}\frac{z}{1+z}\frac{1+\sqrt{1+\Omega_0 z}}{1+\sqrt{1+\Omega_0 z} + \Omega_0 z/2}$$

For a Universe with a cosmological constant, there is not an easily tractable analytical expression (a Mattig's formula). The comoving Distance r has to be found through a numerical evaluation of the fundamental dr/dz expression.

Distance-Redshift Relation, 2nd order

For all general FRW Universe, the second-order distance-redshift relation is identical, only depending on the *deceleration parameter* q_0:

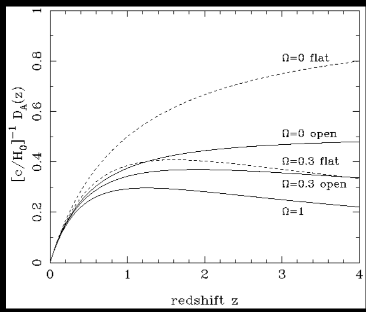
$$D(z) = R_c S_k\left(\frac{r}{R_c}\right) \simeq \frac{c}{H_0}\left(z - \frac{1}{2}(1+q_0)z^2\right)$$

q_0 can be related to Ω_0 once the *equation of state* is known.

Angular Diameter Distance

matter-dominated FRW Universe

$$D_A = \frac{D}{1+z} = \frac{1}{1+z} R_c S_k \left(\frac{r}{R_c} \right)$$



In a matter-dominated Universe, the angular diameter distance as function of redshift is given by:

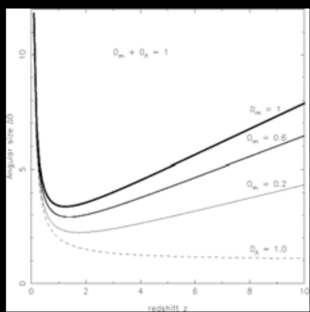
$$D_A(z) = \frac{1}{1+z} R_c S_k \left(\frac{r}{R_c} \right) = \frac{2c}{H_0} \frac{1}{\Omega_0^{\,2} (1+z)^2} \left\{ \Omega_0 z + (\Omega_0 - 2)\left(\sqrt{1+\Omega_0 z} - 1 \right) \right\}$$

Angular Size - Redshift

FRW Universe

$$\theta(z) = \frac{\ell}{D_A}$$

The angular size $\theta(z)$ of an object of physical size l at a redshift z displays an interesting behaviour. In most FRW universes is has a minimum at a medium range redshift – z=1.25 in an Ω_m=1 EdS universe – and increases again at higher redshifts.



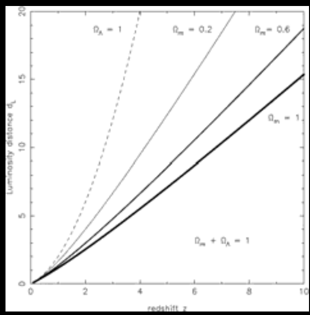
In a matter-dominated Universe, the angular diameter distance as function of redshift is given by:

$$D_A(z) = \frac{1}{1+z} R_c S_k \left(\frac{r}{R_c} \right) = \frac{2c}{H_0} \frac{1}{\Omega_0^{\,2} (1+z)^2} \left\{ \Omega_0 z + (\Omega_0 - 2)\left(\sqrt{1+\Omega_0 z} - 1 \right) \right\}$$

Luminosity Distance

matter-dominated FRW Universe

$$D_L = D(1+z) = (1+z)R_c S_k\left(\frac{r}{R_c}\right)$$

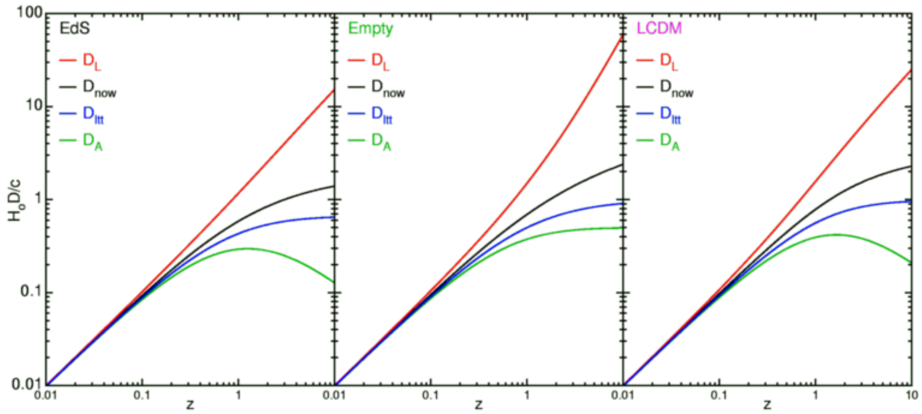


In a matter-dominated Universe, the luminosity distance as function of redshift is given by:

$$D_L(z) = (1+z)R_c S_k\left(\frac{r}{R_c}\right) = \frac{2c}{\Omega_0^{\,2} H_0}\left\{\Omega_0 z + (\Omega_0 - 2)\left(\sqrt{1+\Omega_0 z} - 1\right)\right\}$$

FRW Universe Distances

summary



FRW

Thermodynamics

FRW Dynamics

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a + \frac{\Lambda}{3}a$$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0^2} + \frac{\Lambda}{3}a^2$$

To find solutions a(t) for the expansion history of the Universe, for a particular FRW Universe ,

one needs to know how the density ρ(t) and pressure p(t) evolve as function of a(t)

FRW equations are implicitly equivalent to a third Einstein equation, the energy equation,

$$\dot{\rho} + 3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{a}}{a} = 0$$

FRW Dynamics: Adiabatic Cosmic Expansion

Important observation:
the energy equation,

$$\dot{\rho} + 3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{a}}{a} = 0$$

is equivalent to stating that the change in internal energy

$$U = \rho c^2 V$$

of a specific co-expanding volume V(t) of the Universe, is due to work by pressure:

$$dU = -p\,dV$$

Friedmann-Robertson-Walker-Lemaitre expansion of the Universe is

→ Adiabatic Expansion ←

FRW Dynamics: Thermal Evolution

Adiabatic Expansion of the Universe:

- **Implication for Thermal History**
- **Temperature Evolution of cosmic components**

For a medium with adiabatic index γ:

$$TV^{\gamma-1} = cst$$

Radiation (Photons)	$\gamma = \frac{4}{3}$	$T = \frac{T_0}{a}$
Monatomic Gas (hydrogen)	$\gamma = \frac{5}{3}$	$T = \frac{T_0}{a^2}$

FRW Dynamics: Thermal Evolution

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Radiation & Matter

Cosmic Radiation

The Universe is filled with thermal radiation, the photons that were created in The Big Bang and that we now observe as the Cosmic Microwave Background (CMB).

The CMB photons represent the most abundant species in the Universe, by far !

The CMB radiation field is PERFECTLY thermalized, with their energy distribution representing the most perfect blackbody spectrum we know in nature. The energy density $u_\nu(T)$ is therefore given by the Planck spectral distribution,

$$u_\nu(T) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

At present, the temperature T of the cosmic radiation field is known to impressive precision,

$$T_0 = 2.725 \pm 0.001 K$$

Cosmic Radiation

With the energy density $u_\nu(T)$ of CMB photons with energy $h\nu$ given, we know the number density $n_\nu(T)$ of such photons:

$$n_\nu(T) = \frac{u_\nu(T)}{h\nu} = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

The total number density $n_\gamma(T)$ of photons in the Universe can be assessed by integrating the number density $n_\nu(T)$ of photons with frequency ν over all frequencies,

$$n_\gamma(T) = \int_0^\infty n_\nu(T)d\nu = \int_0^\infty \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT}-1}d\nu = 60.4\left(\frac{kT}{hc}\right)^3$$

$$T = 2.725 K$$

⬇

$$n_\gamma(T) = 412\ cm^{-3}$$

Baryon-Photon Ratio

Having determined the number density of photons, we may compare this with the number density of baryons, $n_b(T)$. That is, we wish to know the PHOTON-BARYON ratio,

$$\eta \equiv \frac{n_\gamma}{n_B}$$

$$n_b = \frac{\rho_B}{m_p} = \frac{\Omega_B\, \rho_{crit}}{m_p}$$

The baryon number density is inferred from the baryon mass density. here, for simplicity, we have assumed that baryons (protons and neutrons) have the same mass, the proton mass $m_p \sim 1.672 \times 10^{-24}$ g. At present we therefore find

$$n_b = 1.12\times10^{-5}\ \Omega_b h^2\ g\, cm^{-3}$$

⬇

$$\eta_0 = \frac{n_\gamma}{n_B} \approx 3.65\times10^{7}\ \frac{1}{\Omega_b h^2}\ \ g\, cm^{-3}$$

We know that $\Omega_b \sim 0.044$ and $h \sim 0.72$:

$$\eta_0 = \frac{n_\gamma}{n_b} \approx 1.60\times10^{9}$$

Baryon-Photon Ratio

From simple thermodynamic arguments, we find that the number of photons is vastly larger than that of baryons in the Universe.

$$\eta_0 = \frac{n_\gamma}{n_b} \approx 1.60\times10^{9}$$

In this, the Universe is a unique physical system, with tremendous repercussions for the thermal history of the Universe. We may in fact easily find that the cosmic photon-baryon ratio remains constant during the expansion of the Universe,

$$n_b(t) = \frac{n_{b,0}}{a^3}$$

$$n_\gamma(t) \propto T(t)^3 \propto \frac{1}{a^3} \Rightarrow n_\gamma(t) = \frac{n_{\gamma 0}}{a^3}$$

➡

$$\eta = \frac{n_\gamma(t)}{n_b(t)} = \frac{n_{\gamma,0}}{n_{b,0}} = \eta_0$$

Entropy of the Universe

The photon-baryon ratio in the Universe remains constant during the expansion of the Universe, and has the large value of

$$\eta = \frac{n_\gamma(t)}{n_b(t)} = \frac{n_{\gamma,0}}{n_{b,0}} = \eta_0 = 1.60 \times 10^9$$

This quantity is one of the key parameters of the Big Bang. The baryon-photon ratio quantifies the ENTROPY of the Universe, and it remains to be explained why the Universe has produced such a system of extremely large entropy !!!!!

The key to this lies in the very earliest instants of our Universe !

Hot Big Bang

Key Observations

Big Bang Evidence

- Olber's paradox:
  the night sky is dark
  → finite age Universe (13.7 Gyr)

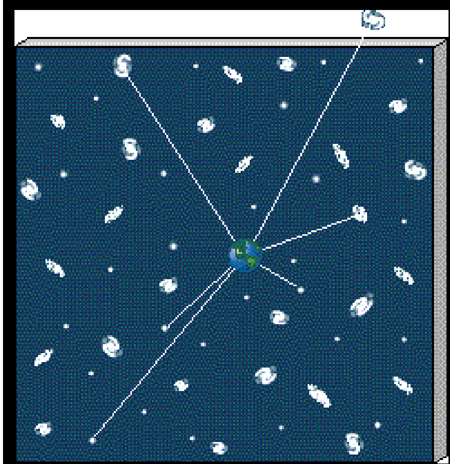
- Hubble Expansion
  uniform expansion, with
  expansion velocity ~ distance: v = H r

- Explanation Helium Abundance 24%:
  light chemical elements formed (H, He, Li, ...)
  after ~3 minutes ...

- The Cosmic Microwave Background Radiation:
  the 2.725K radiation blanket, remnant left over
  hot ionized plasma → neutral universe
  (379,000 years after Big Bang)

- Distant, deep Universe indeed looks different ...

1. Olber's Paradox



In an infinitely large, old and unchanging Universe each line of sight would hit a star:

→

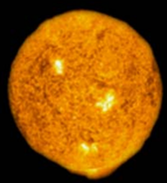
Sky would be as bright as surface of star:

Night sky as bright as
Solar Surface, yet
the night sky is dark

→

finite age of Universe (13.7 Gyr)

1. Paradox van Olbers



In an infinitely large, old and unchanging Universe each line of sight would hit a star:

→

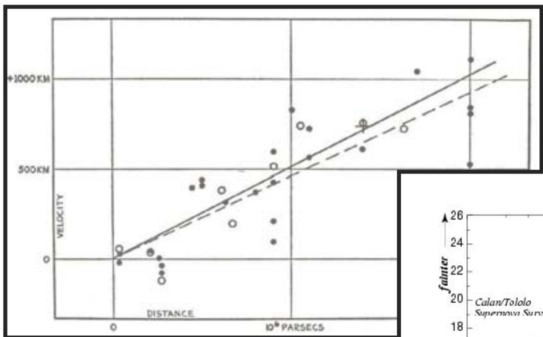
Sky would be as bright as surface of star:

Night sky as bright as Solar Surface, yet the night sky is dark

→

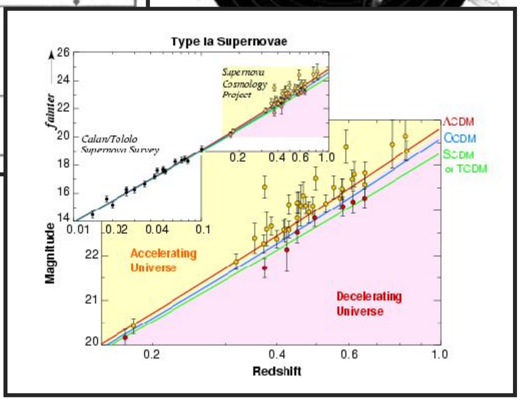
finite age of Universe (13.8 Gyr)

2. Hubble Expansion

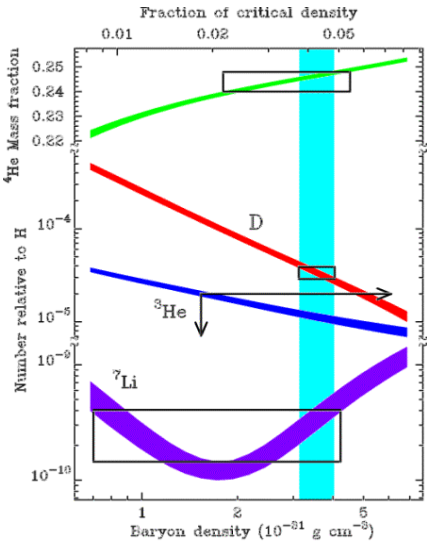


Hubble Diagram: ↑

- Hubble 1929: Universe expands !!!!
- Supernova Projects (1998) Cosmic Expansion is accelerating →

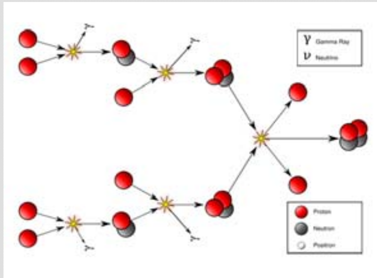


3. Light Element Abundance



Mass Fraction Light Elements

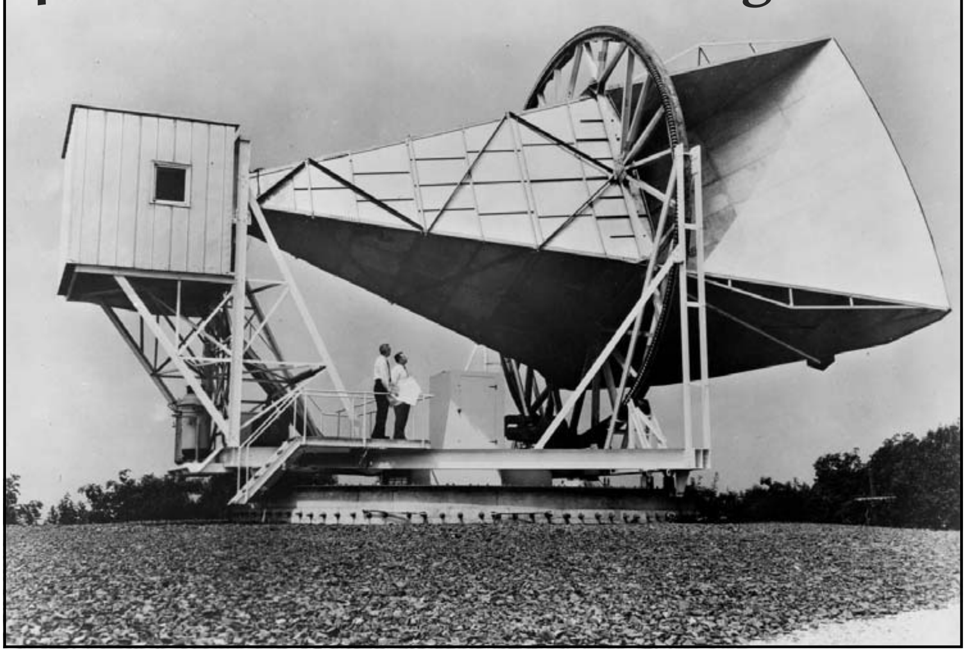
24%	^{4}He	nuclei
traces	D, ^{3}He, ^{7}Li	nuclei
75%	H	nuclei (protons)

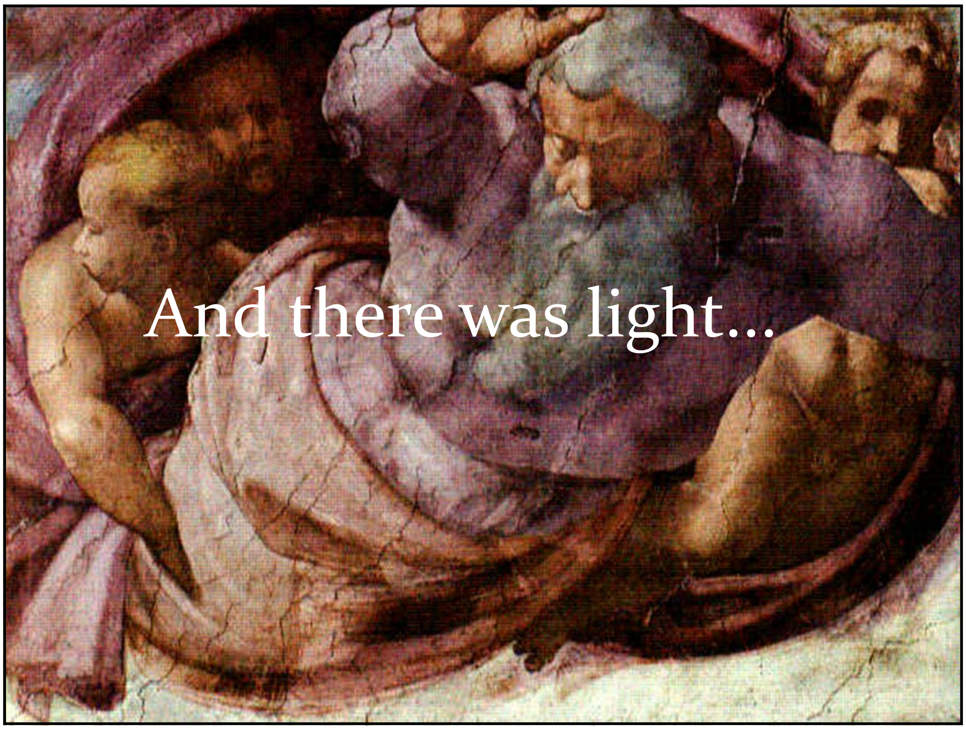


Between 1-200 seconds after Big Bang, temperature dropped to 10^9 K:

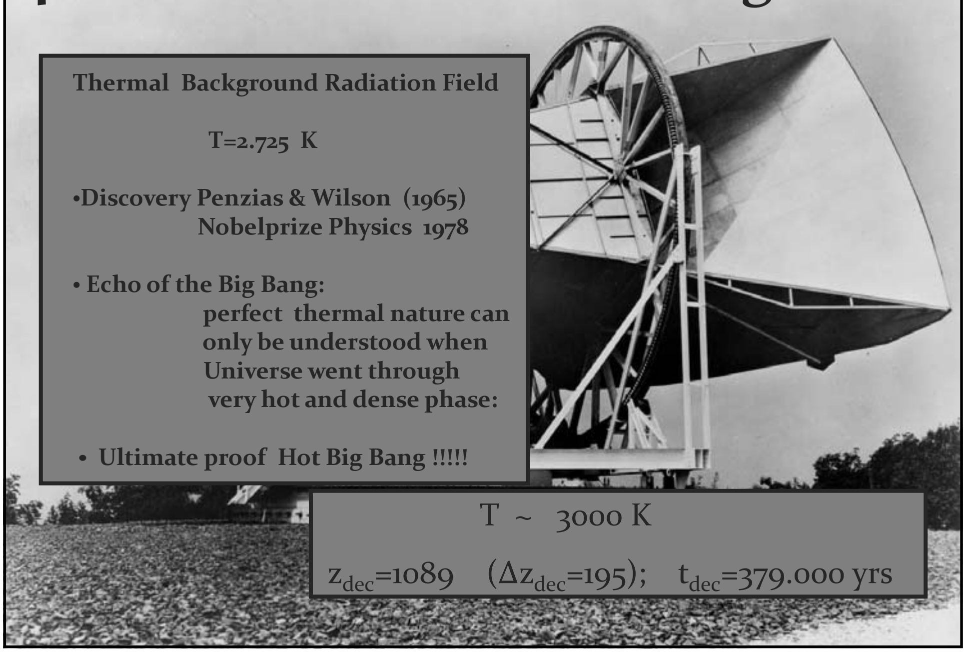
Fusion protons & neutrons into light atomic nuclei

4. Cosmic Microwave Background





4. Cosmic Microwave Background



Thermal Background Radiation Field

T=2.725 K

•Discovery Penzias & Wilson (1965)
Nobelprize Physics 1978

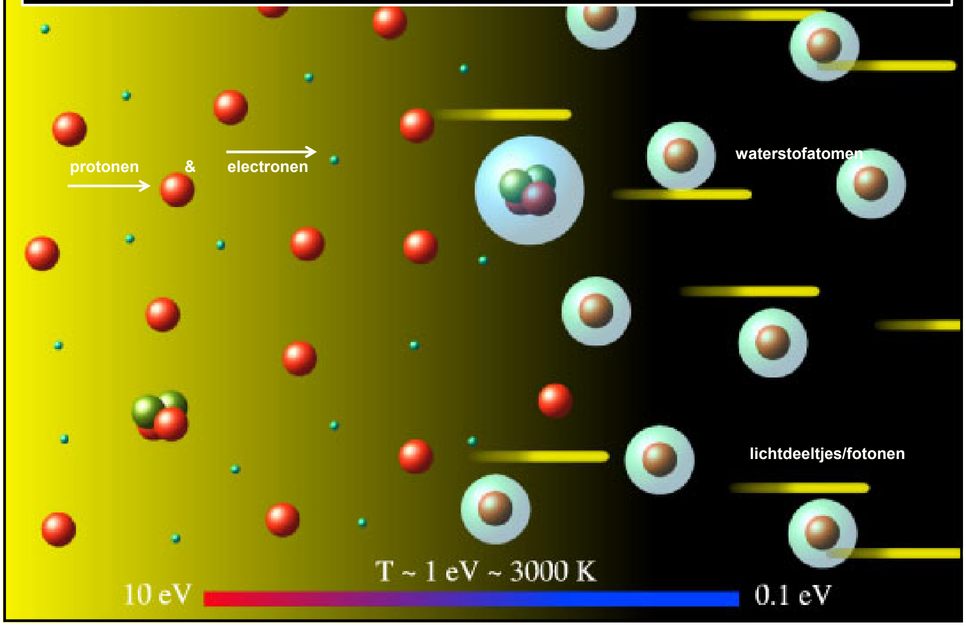
• Echo of the Big Bang:
perfect thermal nature can only be understood when Universe went through very hot and dense phase:

• Ultimate proof Hot Big Bang !!!!!

T ~ 3000 K

z_{dec}=1089 (Δz_{dec}=195); t_{dec}=379.000 yrs

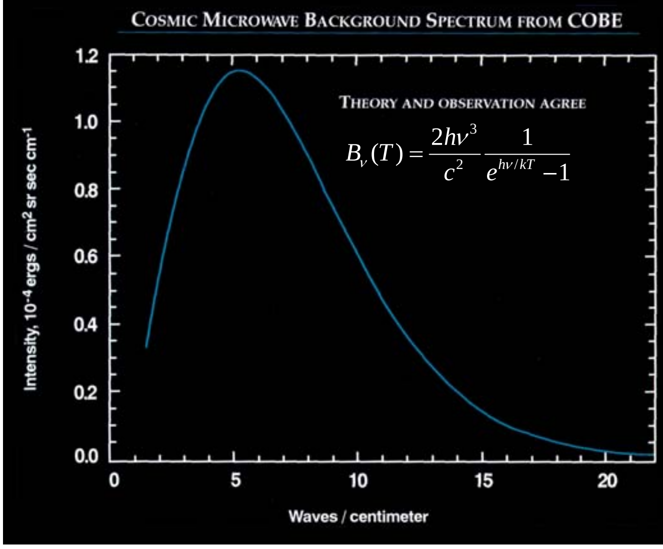
Recombination & Decoupling



Cosmic Light (CMB): the facts

- Ontdekt in 1965 door Penzias & Wilson, Nobelprijs 1978 !!!!!
- Kosmisch Licht dat het gehele heelal uniform vult
- Temperatuur: $T_\gamma = 2.725$ K
- Fotonen veruit het meest voorkomende deeltje in de natuur: $n_\gamma \sim 415\ cm^{-3}$
- Per atoom in het Heelal: $n_\gamma / n_B \sim 1.9 \times 10^9$
- Ultieme Bewijs van de Big Bang !!!!!!!!!!!!!!!!!!

CMB Radiation Field
Blackbody Radiation



- COBE-DIRBE: temperature, blackbody
- T = 2.725 K
- John Mather Nobelprize physics 2006
- Most accurately measured Black Body Spectrum Ever !!!!!

CMB Photons

Note:

far from being an exotic faraway phenomenon, realize that the CMB nowadays is counting for approximately 1% of the noise on your tv set …

Courtesy: W. Hu

Cosmic Light (CMB): most abundant species

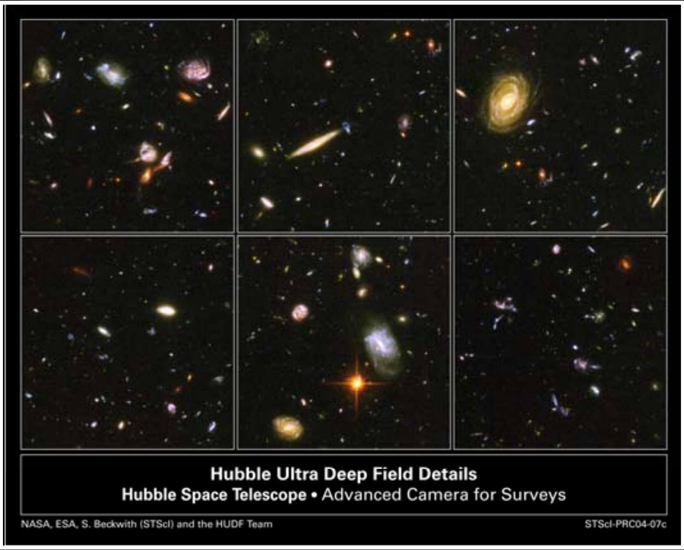
By far,

the most abundant particle species

in the Universe

$n_{\gamma}/n_{B} \sim 1.9$ billion

5. Changing Universe



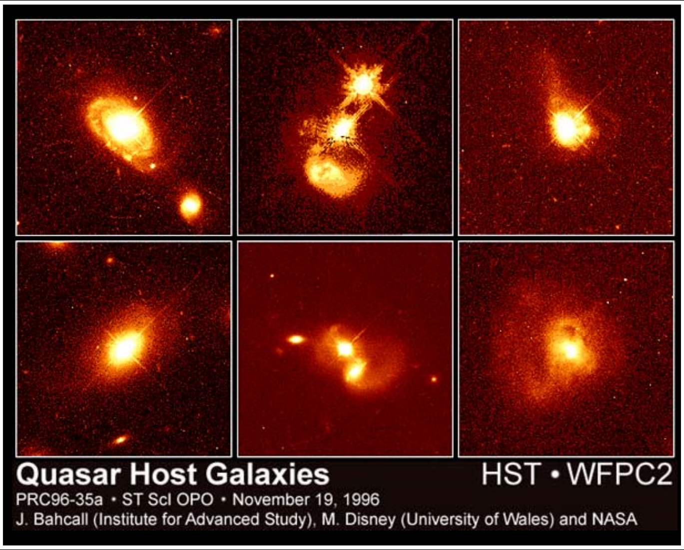
The appearance of the Universe does change when looking deeper into the Universe:

Depth=Time

→

Galaxies in Hubble Ultra Deep Field

5. Changing Universe



The appearance of the Universe does change when looking deeper into the Universe:

Depth=Time

→

Quasars

(very high z)

How Much ?

Cosmic Curvature

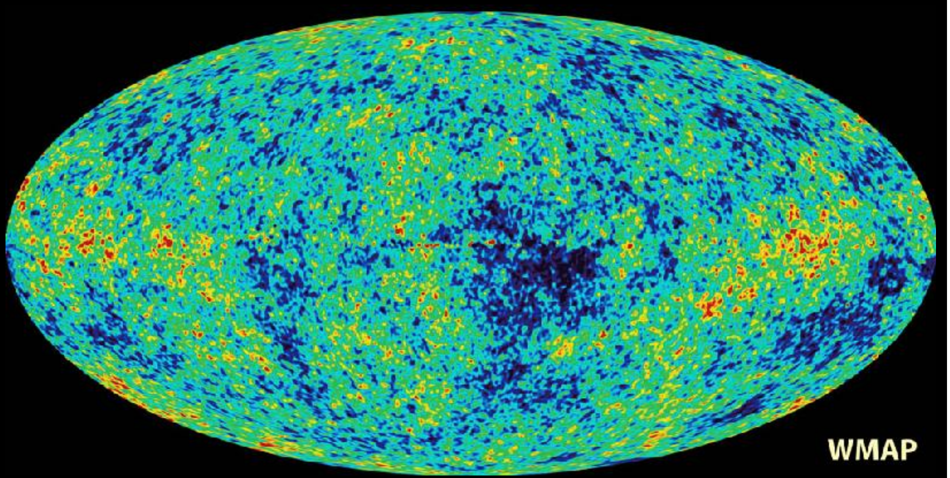
FRW Universe: Curvature

There is a 1-1 relation between the total energy content of the Universe and its curvature. From FRW equations:

$$k = \frac{H^2 R^2}{c^2}(\Omega - 1) \qquad \Omega = \Omega_{rad} + \Omega_m + \Omega_\Lambda$$

$\Omega < 1$	$k = -1$	*Hyperbolic*	*Open Universe*
$\Omega = 1$	$k = 0$	*Flat*	*Critical Universe*
$\Omega > 1$	$k = +1$	*Spherical*	*Close Universe*

Cosmic Microwave Background



Map of the Universe at Recombination Epoch (WMAP, 2003):

- 379,000 years after Big Bang
- Subhorizon perturbations: primordial sound waves
- $\Delta T/T < 10\text{-}5$

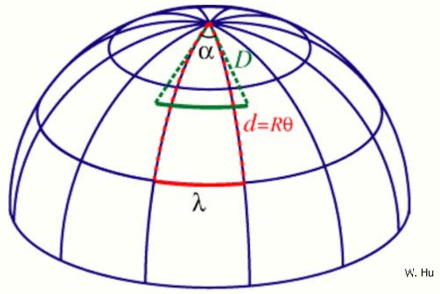
Measuring Curvature

Measuring the Geometry of the Universe:

- Object with known physical size, at large cosmological distance
- Measure angular extent on sky
- Comparison yields light path, and from this the curvature of space

⬇

Geometry of Space



In a FRW Universe: lightpaths described by Robertson-Walker metric

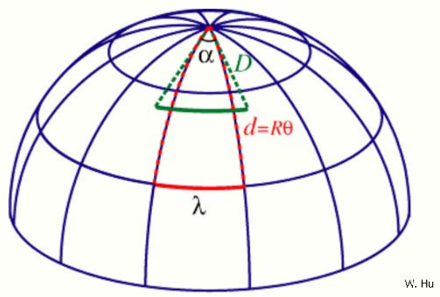
$$ds^2 = c^2dt^2 - a(t)^2\left\{dr^2 + R_c^{\,2}S_k^{\,2}\left(\frac{r}{R_c}\right)\left[d\theta^2 + \sin^2\theta\, d\phi^2\right]\right\}$$

Measuring Curvature

- Object with known physical size, at large cosmological distance:
- Sound Waves in the Early Universe !!!!

⬇

Temperature Fluctuations CMB



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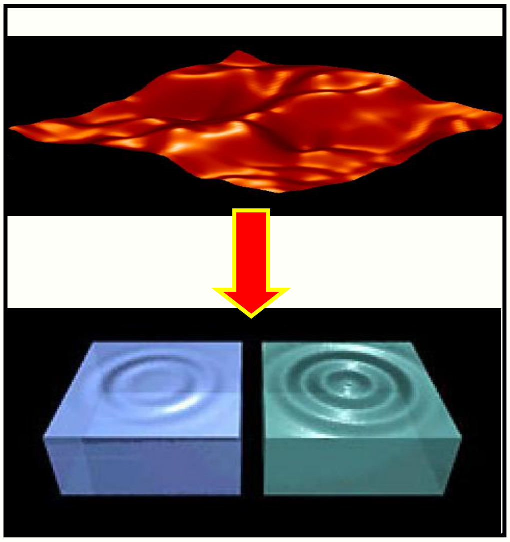
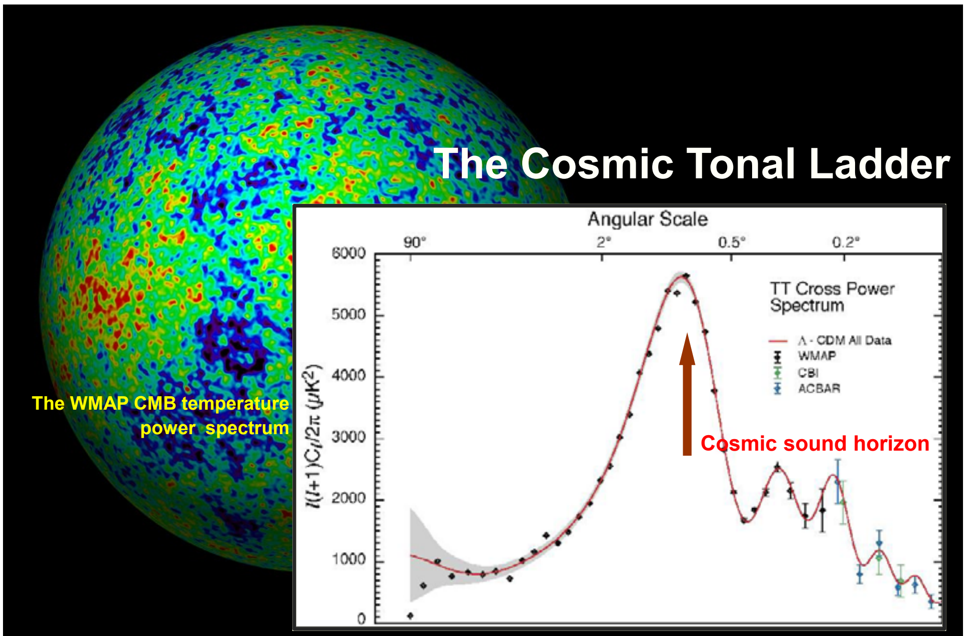
Music of the Spheres

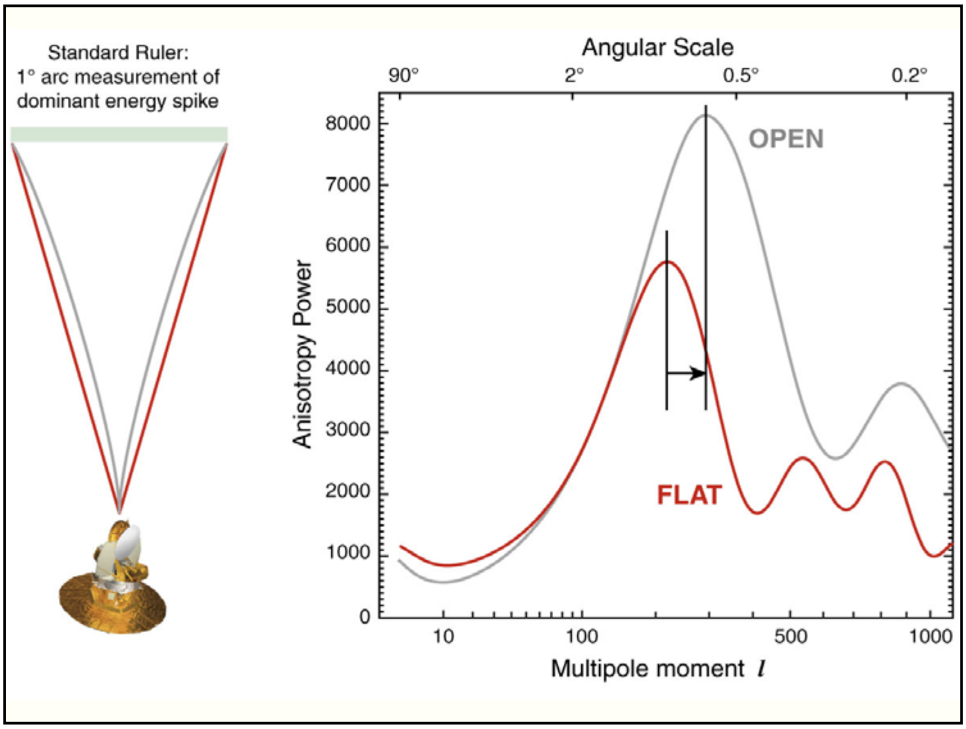
- small ripples in
  primordial matter & photon distribution
- gravity:
  - compression primordial photon gas
  - photon pressure resists
- compressions and rarefactions
  in photon gas: sound waves
- sound waves not heard, but seen:
  - compressions: (photon) T higher
  - rarefactions: lower
- fundamental mode sound spectrum
  - size of "instrument":
  - (sound) horizon size last scattering
- Observed, angular size: θ~1°
  - exact scale maximum compression, the
  "cosmic fundamental mode of music"

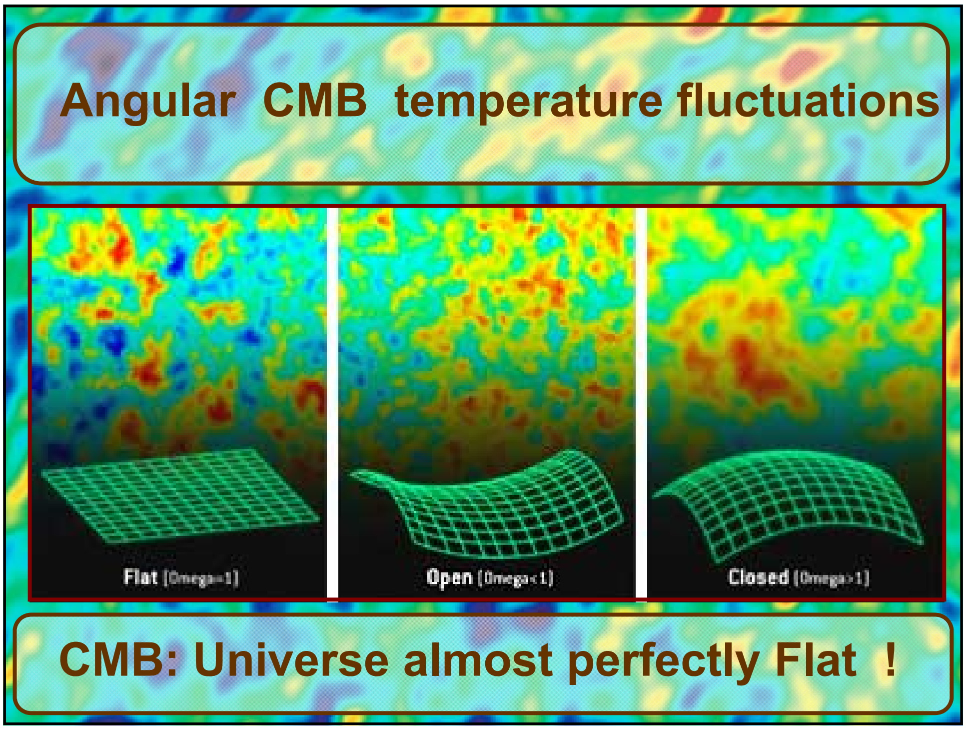
W. Hu





The Cosmic Microwave Background Temperature Anisotropies: Universe is almost perfectly flat


Standard Ruler:
1° arc measurement of
dominant energy spike
Angular Scale
90°
2°
0.5°
0.2°
OPEN
FLAT
Anisotropy Power
8000
7000
6000
5000
4000
3000
2000
1000
0
10
100
500
1000
Multipole moment l


Angular CMB temperature fluctuations
Flat (Omega=1)
Open (Omega<1)
Closed (Omega>1)
CMB: Universe almost perfectly Flat !

Cosmic Constraints

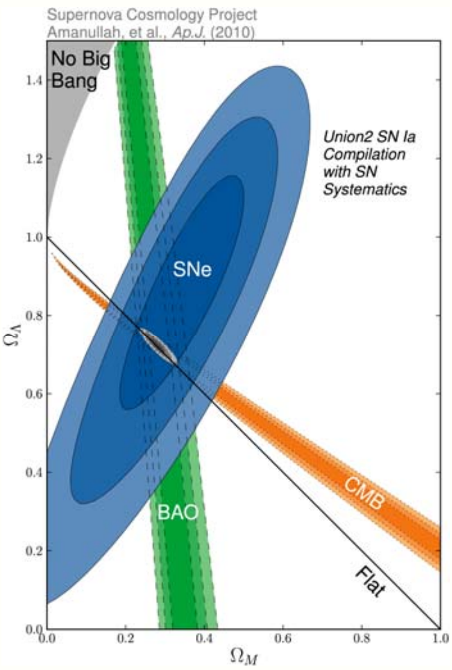
Ω_m vs. Ω_Λ

$$q \approx \frac{\Omega_m}{2} - \Omega_\Lambda$$

$$k = \frac{H^2 R^2}{c^2}(\Omega_m + \Omega_\Lambda - 1)$$

SCP Union2 constraints (2010)

on values of matter density Ω_m
dark energy density Ω_Λ



Standard Big Bang:

what it cannot explain

- **Flatness Problem**
  the Universe is remarkably flat, and was even (much) flatter in the past
- **Horizon Problem**
  the Universe is nearly perfectly isotropic and homogeneous, much more so in the past
- **Monopole Problem:**
  There are hardly any magnetic monopoles in our Universe
- **Fluctuations, seeds of structure**
  Structure in the Universe: origin

Flatness Problem

Flatness Problem

FRW Dynamical Evolution:

Going back in time, we find that the Universe was much flatter than it is at the present.

Reversely, that means that any small deviation from flatness in the early Universe would have been strongly amplified nowadays …

We would therefore expect to live in a Universe that would either be almost $\Omega=0$ or $\Omega\sim\infty$;

Yet, we find ourselves to live in a Universe that is almost perfectly flat … $\Omega_{tot}\sim 1$

How can this be ?

Evolution Ω

From the FRW equations, one can infer that the evolution of Ω goes like
(for simplicity, assume matter-dominated Universe),

$$\left(\frac{1}{\Omega}-1\right)=a(t)\left(\frac{1}{\Omega_0}-1\right) \quad \Longleftrightarrow \quad \Omega(z)=\frac{\Omega_0(1+z)}{1+\Omega_0 z}$$

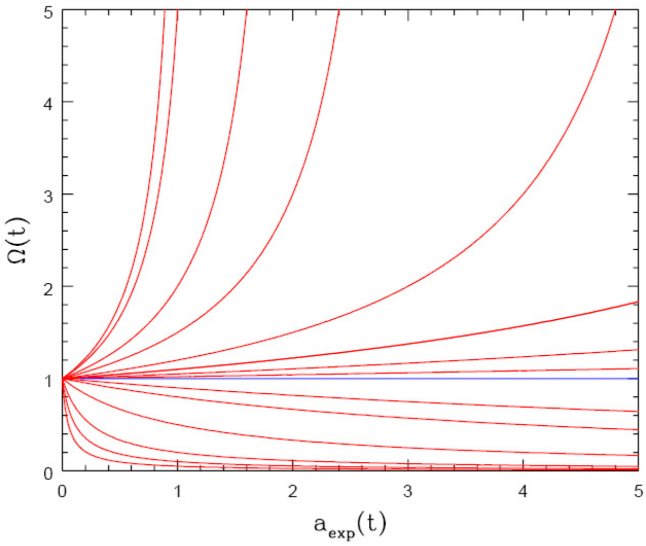
These equations directly show that

$$a\downarrow 0 \quad \Longrightarrow \quad \Omega \rightarrow 1$$

$$k=\frac{H^2R^2}{c^2}(\Omega-1)$$

implying that the early Universe was very nearly flat ...

Flatness Evolution



$$\left(\frac{1}{\Omega}-1\right)=a(t)\left(\frac{1}{\Omega_0}-1\right)$$

- At radiation-matter equiv.
  $$\left|1-\Omega_{rm}\right|\leq 2\times 10^{-4}$$
- Big Bang nucleosynthesis
  a_{nuc}~3.6×10^{-8}
  $$\left|1-\Omega_{nucl}\right|\leq 3\times 10^{-14}$$
- Planck time
  $$\left|1-\Omega_{P}\right|\leq 1\times 10^{-60}$$

Horizon Problem

Cosmic Horizons

Light travel in an expanding Universe:

- **Robertson-Walker metric:** $ds^2 = c^2 dt^2 - a(t)^2 dr^2$
- **Light:** $ds^2 = 0$

$$d_{Hor} = \int_0^t \frac{c\,dt'}{a(t')}$$

Horizon distance in comoving space

⇔

$$R_{Hor} = a(t)\int_0^t \frac{c\,dt'}{a(t')}$$

Horizon distance in physical space

Horizon of the Universe:
distance that light travelled since the Big Bang

Cosmic Horizons

$$R_{Hor} = a(t)\int_0^t \frac{c\,dt'}{a(t')}$$

Horizon distance in physical space

$$R_{Hor} = 3ct$$

In an Einstein-de Sitter Universe

Horizon of the Universe:
distance that light travelled since the Big Bang

Cosmic Horizons

$$R_{Hor} = a(t)\int_0^t \frac{c\,dt'}{a(t')}$$

Horizon distance in physical space

$$R_{Hor} = 3ct$$

In an Einstein-de Sitter Universe

The horizon distance at recombination/decoupling (CMB),

angular size on the sky:

$$\theta_{Hor} \approx 1.74^{\circ}\,\Omega_0^{1/2}\left(\frac{z_{dec}}{1100}\right)^{-1/2}$$

Horizon of the Universe:
distance that light travelled since the Big Bang

Cosmic Horizons

The horizon distance at recombination/decoupling (CMB),

angular size on the sky:

$$\theta_{Hor} \approx 1.74^{\circ}\, \Omega_0^{1/2} \left(\frac{z_{dec}}{1100} \right)^{-1/2}$$

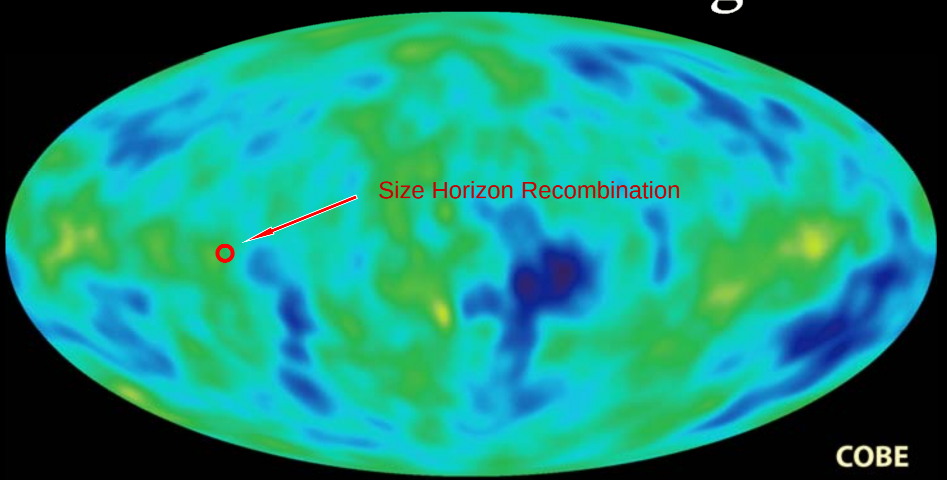
⇓

$\theta \gg 1^{\circ}$ Large angular scales: NOT in physical contact

$\theta \ll 1^{\circ}$ Small angular scales: In physical (thus, also thermal) contact

Horizon of the Universe:
distance that light travelled since the Big Bang

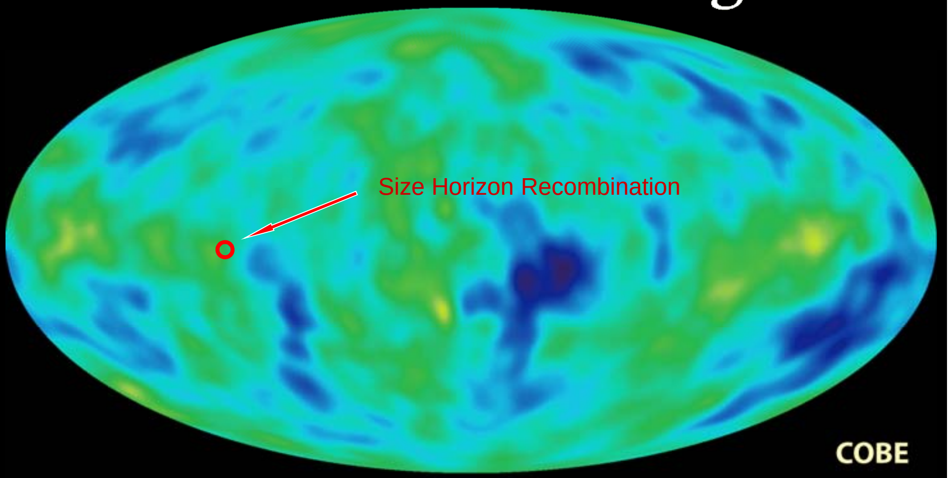
Cosmic Microwave Background



COBE measured fluctuations: > 7°
Size Horizon at Recombination spans angle ~ 1°

How can it be that regions totally out of thermal contact have the same temperature ?

Cosmic Microwave Background



COBE measured fluctuations: > 7°
Size Horizon at Recombination spans angle ~ 1°

COBE proved that superhorizon fluctuations do exist: **prediction Inflation !!!!!**

Horizon Problem

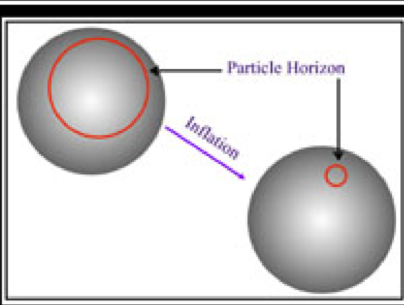
The horizon distance at recombination/decoupling (CMB),

angular size on the sky:

$$\theta_{Hor} \approx 1.74^{\circ}\, \Omega_0^{1/2} \left(\frac{z_{dec}}{1100} \right)^{-1/2}$$

Angular scales: $\theta_{Hor} \approx 1^{\circ}$

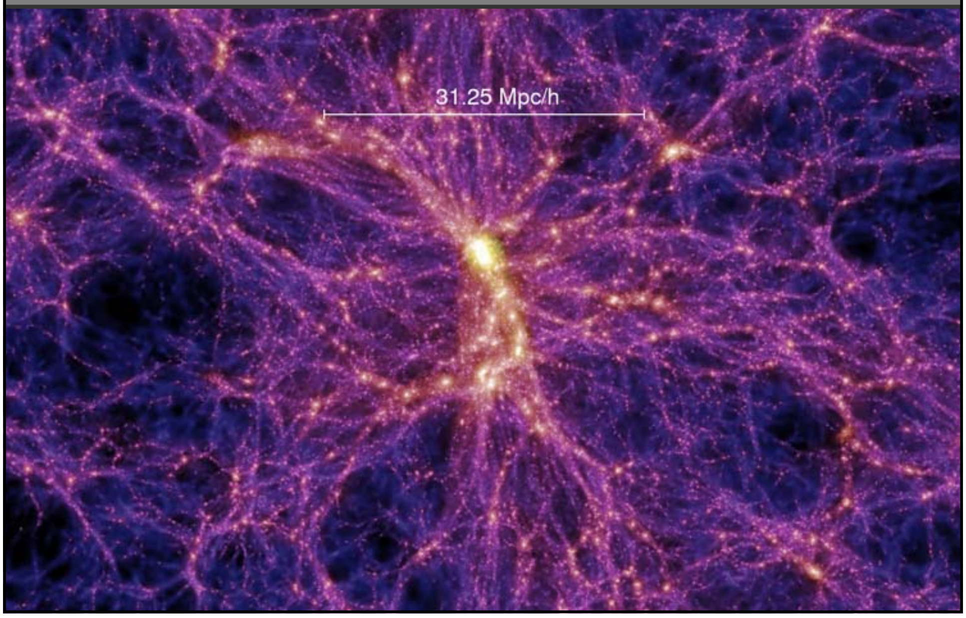
How can it be that regions that were never in thermal still have almost exactly the same temperature T~2.725 K



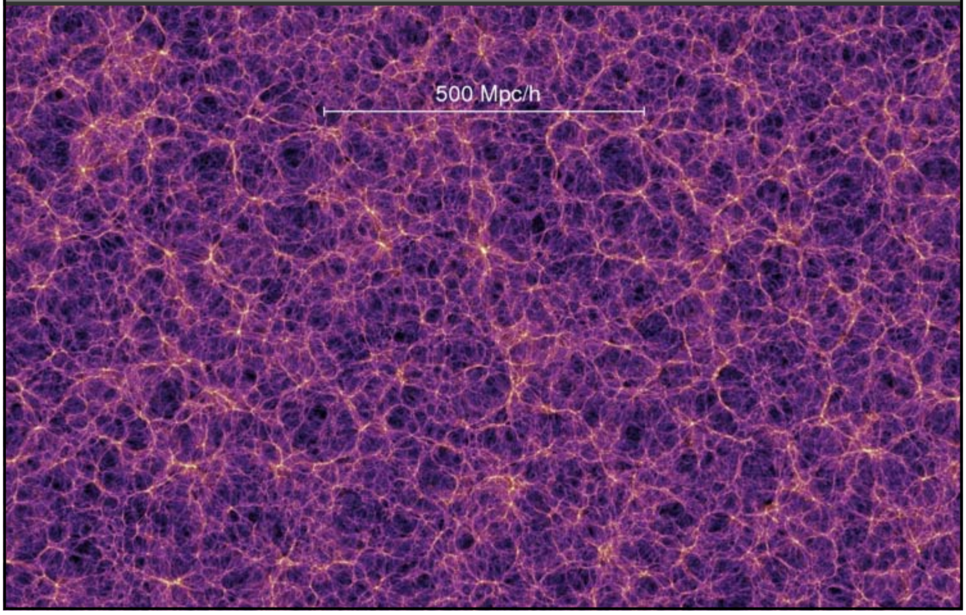
Horizon of the Universe:
distance that light travelled since the Big Bang

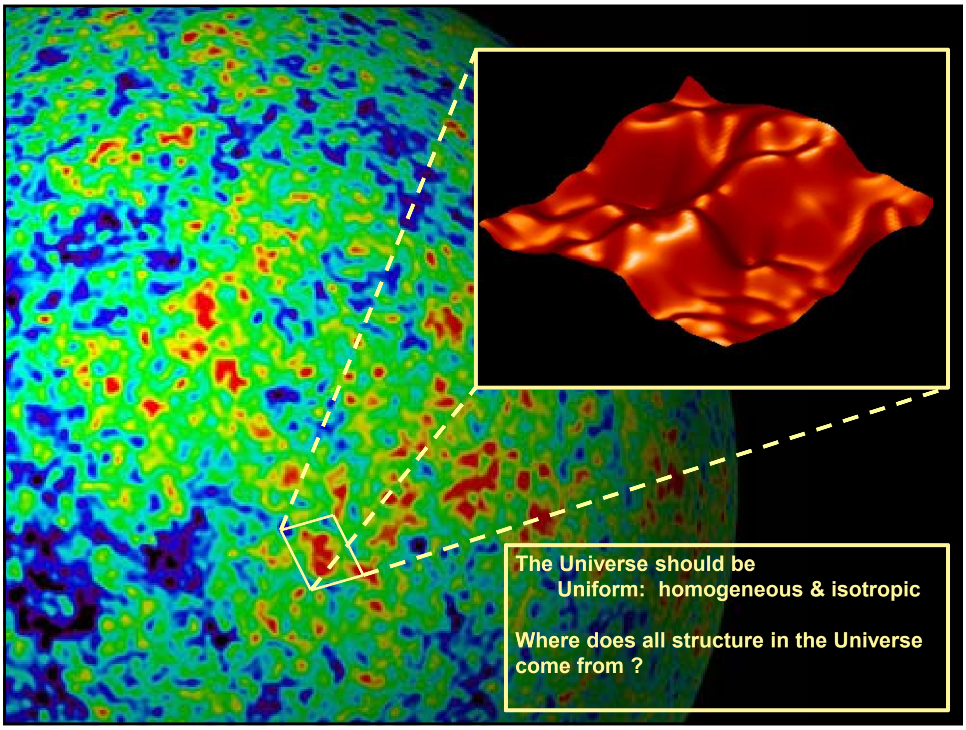
Structure Problem

Millennium Simulation



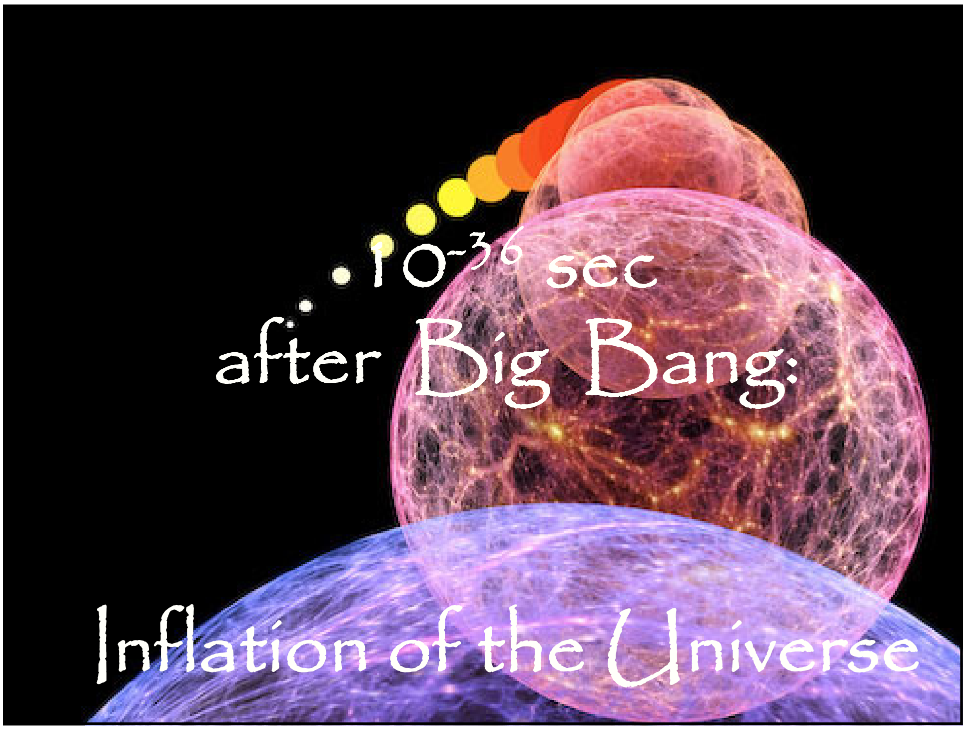
Millennium Simulation





Monopole Problem

Inflationary Universe



FRW Big Bang extended: Inflationary Universe

Essential
Ingredient/Extension
Standard Cosmology

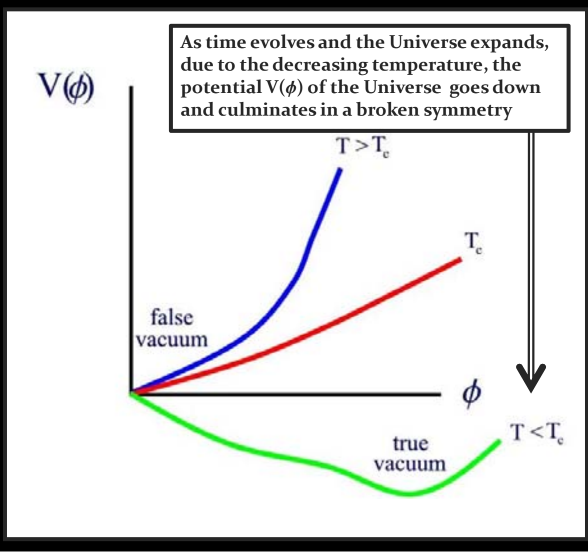
Inflationary Universe

Phase transition
Early Universe

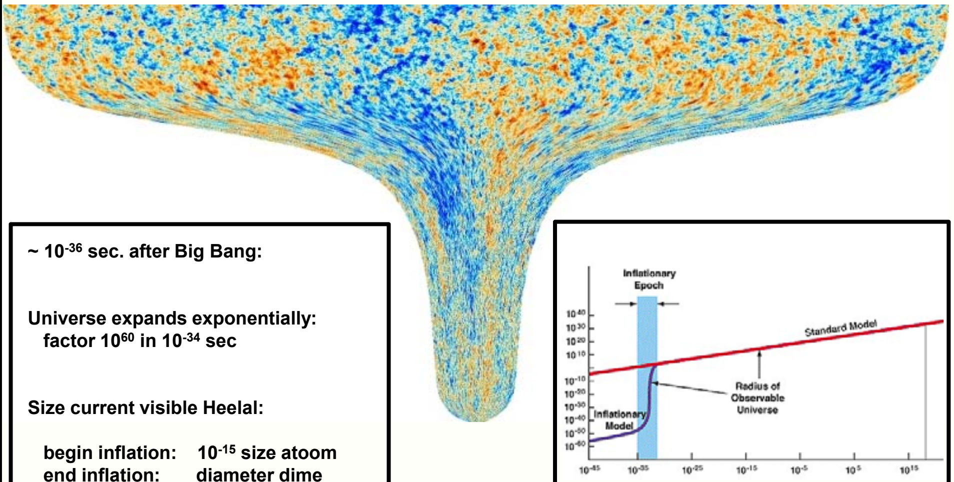
- GUT transition :
  $t \sim 10^{-36}$ sec ??

- (false) vacuum potential
  induces exponential
  (de Sitter) expansion

Universe blows up by factor
$N > 10^{60}$



Cosmic Inflation



~ 10^{-36} sec. after Big Bang:

Universe expands exponentially:
factor 10^{60} in 10^{-34} sec

Size current visible Heelal:

begin inflation: 10^{-15} size atoom
end inflation: diameter dime

FRW Big Bang extended: Inflationary Universe

Essential Ingredient/Extension Standard Cosmology

Inflationary Universe

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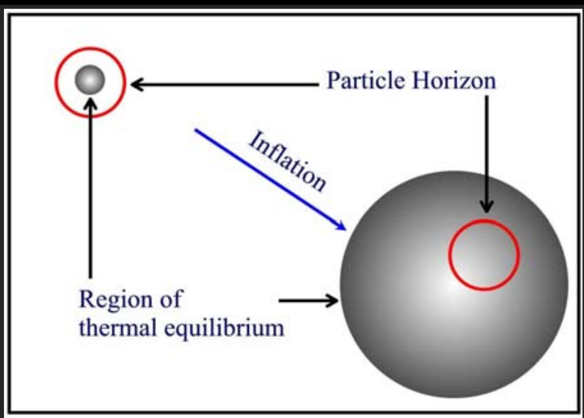

Inflation explains the horizon problem in a natural fashion:
a region that initially was much smaller than the pre-inflation horizon, rapidly blew to a superhorizon size: any region on the CMB sky was in thermal contact before inflation !

FRW Big Bang extended: Inflationary Universe

Essential Ingredient/Extension Standard Cosmology

Inflationary Universe

Phase transition Early Universe

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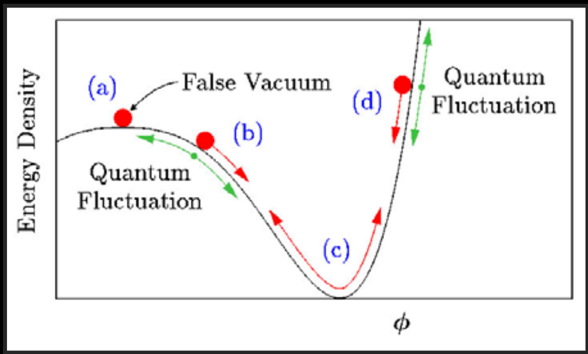
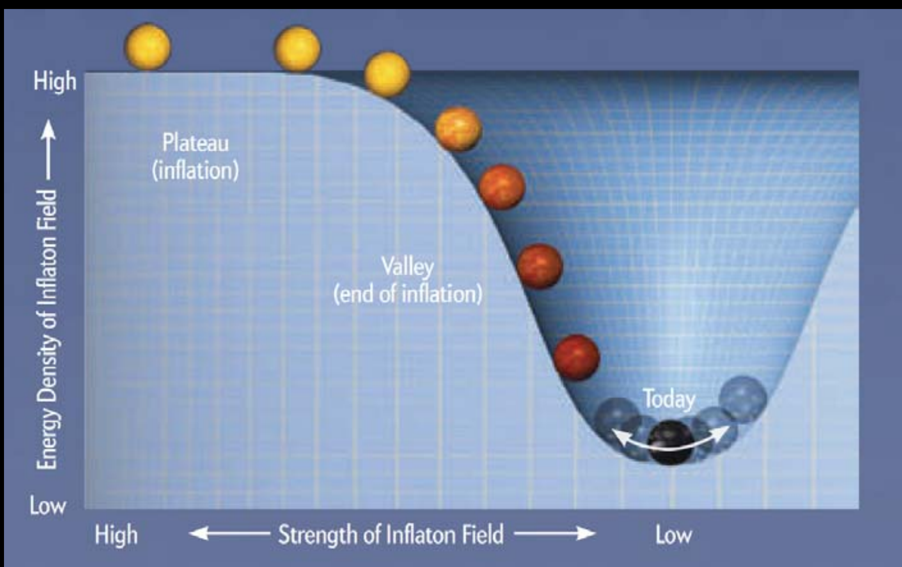
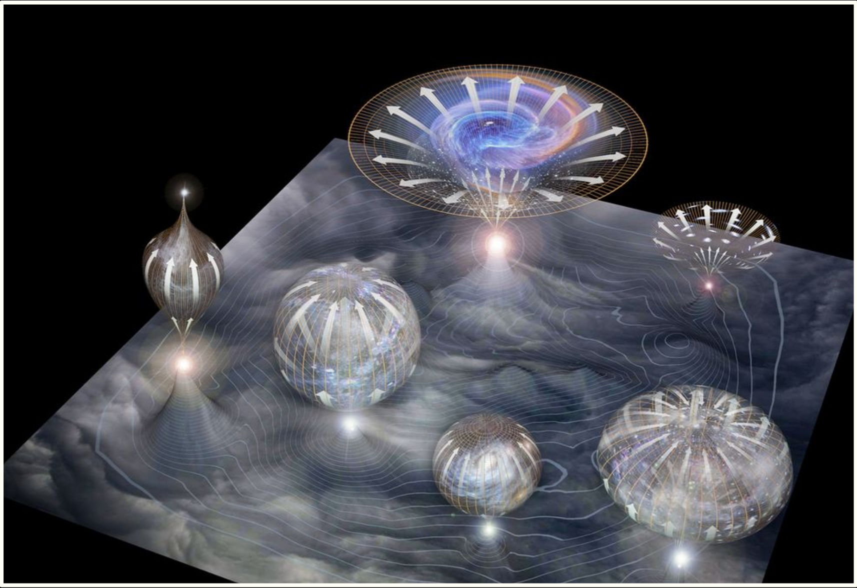


Fluctuation Generation during Inflation:
Quantum fluctuations around the potential get magnified to macroscopic scales.

Propelling Inflation: Inflaton



Inflatie & Multiverse

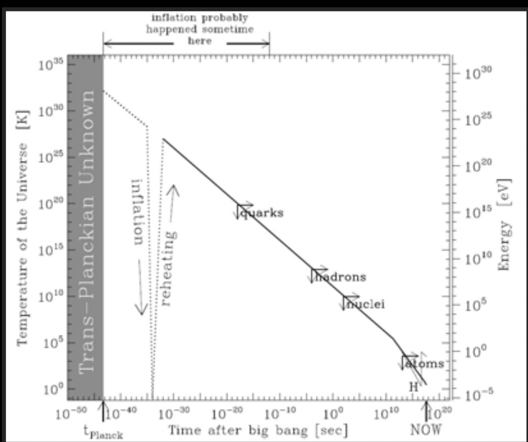


FRW Big Bang extended: Inflationary Universe

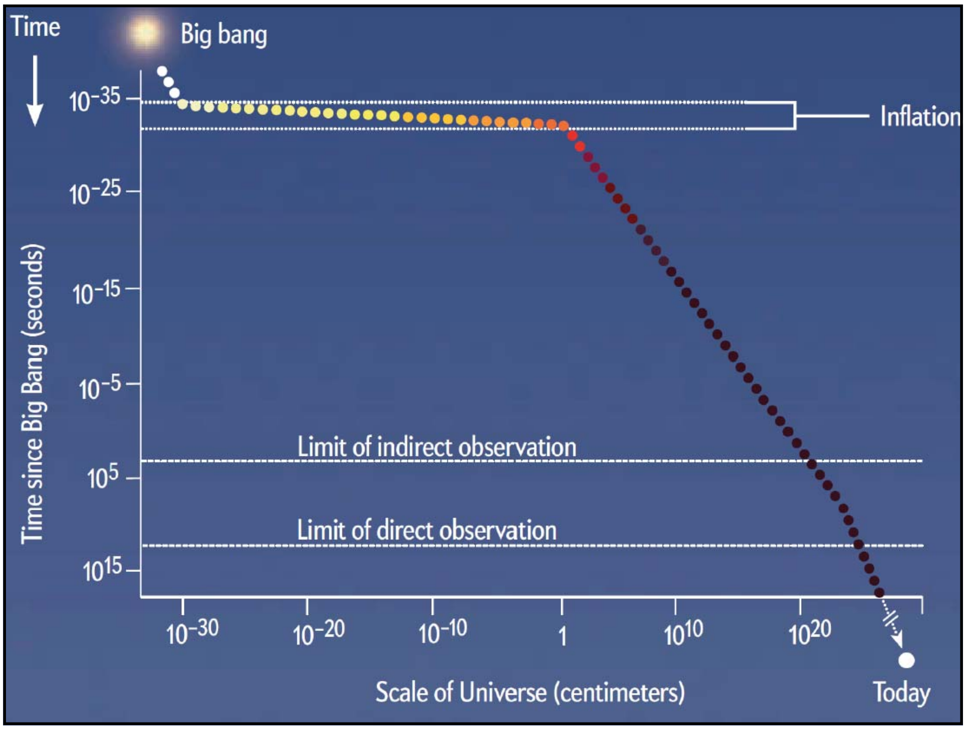
Inflationary Universe

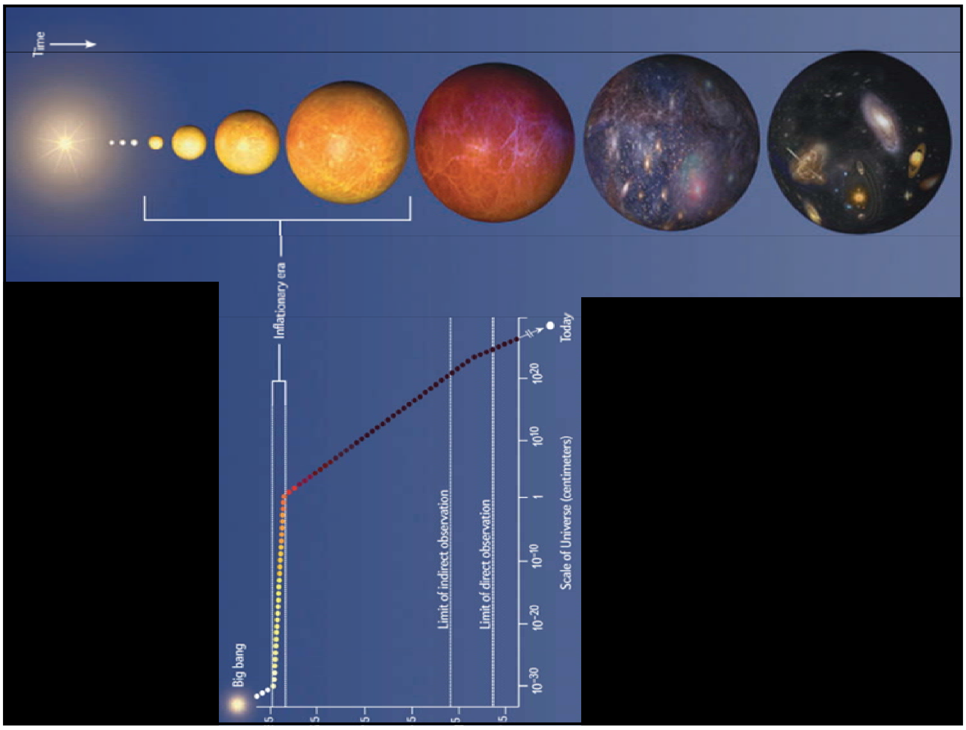
Explains:
- Horizon Problem
- Flatness Problem
- Monopole Problem

And ...
- Origin of Structure



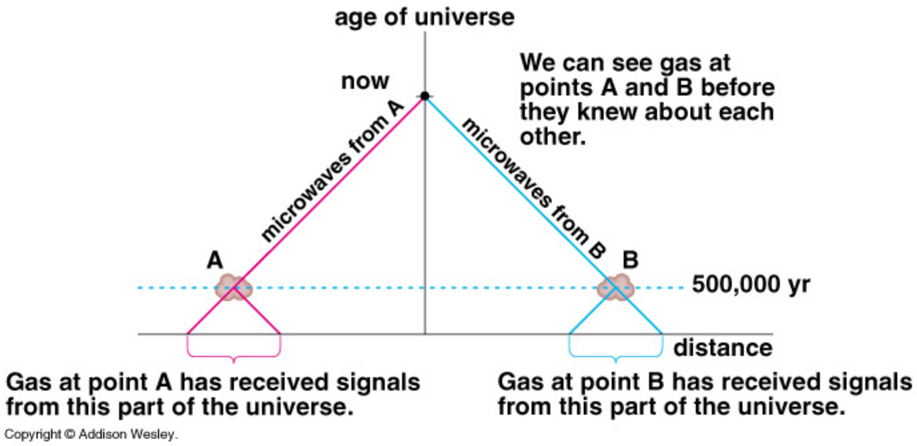
Towards the end of inflation, the Universe converts the tremendous amount of vacuum energy that drives inflation into a surge of newly created radiation and particles (“latent heat”). This is the Reheating Phase of inflation.


Time
Big bang
Inflation
Time since Big Bang (seconds)
10−35
10−25
10−15
10−5
105
1015
Limit of indirect observation
Limit of direct observation
10−30
10−20
10−10
1
1010
1020
Scale of Universe (centimeters)
Today


Time
Inflationary era
Big bang
Today
Limit of indirect observation
Limit of direct observation
Scale of Universe (centimeters)

Cosmic Future & Cosmic Horizons

Cosmic Horizons



Particle Horizon of the Universe:
distance that light travelled since the Big Bang

Cosmic Horizons

Fundamental Concept for our understanding of the physics of the Universe:

- Physical processes are limited to the region of space with which we are or have ever been in physical contact.
- What is the region of space with which we are in contact ?
  Region with whom we have been able to exchange photons
  (photons: fastest moving particles)
- From which distance have we received light.
- Complication: - light is moving in an expanding and curved space
  - fighting its way against an expanding background
- This is called the

Horizon of the Universe

Cosmic Particle Horizon

Light travel in an expanding Universe:

- **Robertson-Walker metric:** $ds^2 = c^2 dt^2 - a(t)^2 dr^2$
- **Light:** $ds^2 = 0$

$$d_{Hor} = \int_0^t \frac{c\,dt'}{a(t')}$$

Horizon distance in comoving space

⇔

$$R_{Hor} = a(t) \int_0^t \frac{c\,dt'}{a(t')}$$

Horizon distance in physical space

Particle Horizon of the Universe:
distance that light travelled since the Big Bang

Cosmic Particle Horizon

Particle Horizon of the Universe:
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$$R_{Hor} = a(t)\int_0^t \frac{c\,dt'}{a(t')}$$

Horizon distance in physical space

In a spatially flat Universe, the horizon distance has a finite value for w>-1/3

$$a(t) \propto t^{\frac{2}{3+3w}}$$

⇒

$$d_{Hor}(t_0) = ct_0 \frac{2}{1+3w}$$

Cosmic Particle Horizon

Particle Horizon of the Universe:
distance that light travelled since the Big Bang

In a spatially flat Universe, the horizon distance has a finite value for w>-1/3

$$a(t) \propto t^{\frac{2}{3+3w}}$$

⇒

$$d_{Hor}(t_0) = ct_0 \frac{2}{1+3w}$$

$d_{Hor}(t_0) = 3ct_0$ flat, matter-dominated universe

$d_{Hor}(t_0) = 2ct_0$ flat, radiation-dominated universe

Infinite Particle Horizon

Particle Horizon of the Universe:
distance that light travelled since the Big Bang

In a spatially flat Universe, the horizon distance is infinite for w<-11/3

$$a(t) \propto t^{\frac{2}{3+3w}} \Rightarrow d_{Hor}(t_0) = ct_0 \frac{2}{1+3w}$$

In such a universe, all of space is causally connected to observer:

In such a universe, you could see every point in space.

Cosmic Event Horizon

Light travel in an expanding Universe:

- **Robertson-Walker metric:** $ds^2 = c^2 dt^2 - a(t)^2 dr^2$
- **Light:** $ds^2 = 0$

$$d_{event} = \int_t^\infty \frac{c\,dt'}{a(t')}$$

Event Horizon distance in comoving space

$\Leftrightarrow$

$$R_{event} = a(t) \int_t^\infty \frac{c\,dt'}{a(t')}$$

Event Horizon distance in physical space

Event Horizon of the Universe:
the distance over which one may still communicate …

Cosmic Event Horizon

Event Horizon of the Universe:
distance light may still travel in Universe.

$$d_{EHor} = \int_t^{\infty} \frac{c\,dt'}{a(t')}$$

Event Horizon distance comoving space

⇔

$$R_{EHor} = a(t)\int_t^{\infty} \frac{c\,dt'}{a(t')}$$

Event Horizon distance physical space

In a spatially flat Universe, the event horizon is:

$$a(t) \propto t^{\frac{2}{3+3w}} \Rightarrow d_{EHor}(t_0) \propto \left[t^{\frac{1+3w}{3+3w}} \right]_t^{\infty}$$

Cosmic Event Horizon

Event Horizon of the Universe:
distance light may still travel in Universe.

$$a(t) \propto t^{\frac{2}{3+3w}} \Rightarrow d_{EHor}(t_0) \propto \left[t^{\frac{1+3w}{3+3w}} \right]_t^{\infty}$$

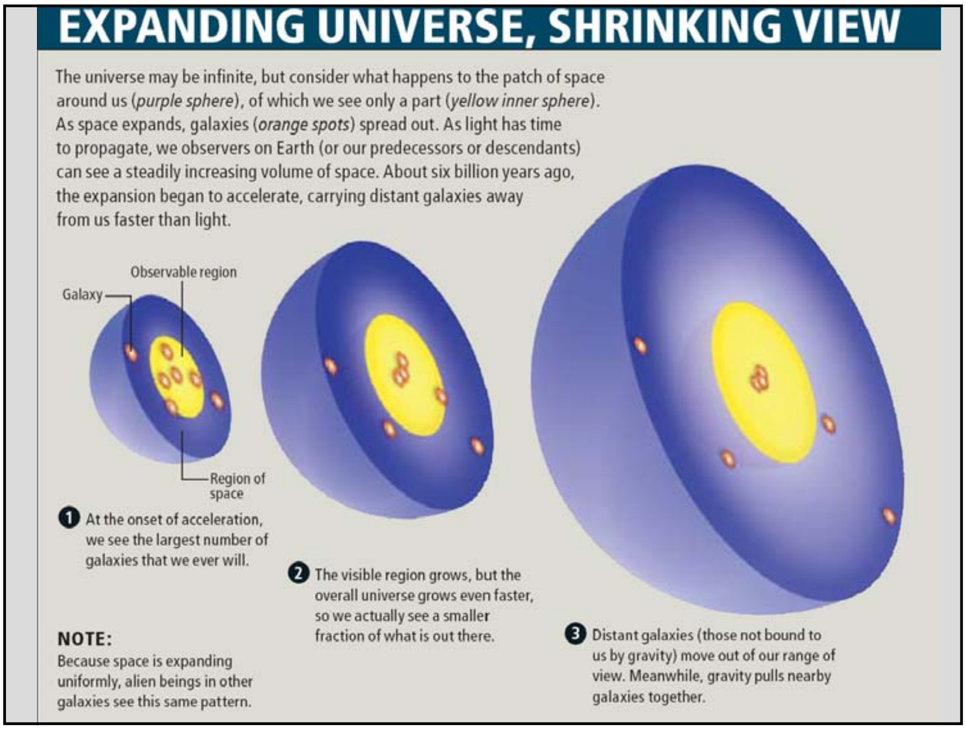
In a spatially flat Universe, the event horizon is:

$w > -1/3 \quad \rightarrow \quad d_{EHor} = \infty$

$w < -1/3 \quad \rightarrow \quad d_{EHor}$ finite

$$d_{EHor}(t_0) \propto t^{\frac{1+3w}{3+3w}}$$

shrinking event horizon:



Cosmic Fate

100 Gigayears:
the end of Cosmology