PHASE 36.

FULL NONLINEARITY:

ELLIPSOIDAL MODEL

&

FORMATION COSMIC FOAM

Dynamics of Cosmic Foam Formation

While a primordial overdensity decouples from cosmic expansion, entering its nonlinear evolution regime and turns around to collapse

1. it will start to collapse first along its shortest axis
2. then follows its medium axis,
3. and finally its longest axis

⇒ Spherical ⇒ Elongated (filament) ⇒ Flattened (wall)

) this anisotropic collapse may be separated into 2 separate parts:

- Internally induced flattening
  as a consequence of the nonspherical shape of object:
  typical example: collapsing ellipsoid.

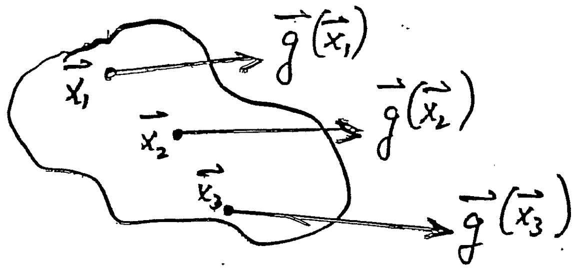
- Externally induced flattening (external tidal field)
  due to anisotropic force field (tidal field) induced by inhomogeneous external

even makes perfect sphere collapse,,

~~Collapse including external tidal forces:~~ general density field.

Anisotropic Collapse Overdensity:

- Anisotropic Force field induced by random density fluctuation field.



$$\boxed{\vec g_i(\vec x) = \underbrace{\vec g_i(\vec x_0)}_{\substack{\text{overall}\\ \text{acceleration}\\ (\text{"bulk flow"})}} + \left\{\sum_{j=1}^{3} \underbrace{\tfrac{1}{3}(\vec\nabla\cdot\vec g)\,\delta_{ij}}_{\substack{\text{contraction/}\\ \text{expansion.}}} - \underbrace{T_{ij}}_{\substack{\text{differential/anisotropic}\\ \text{term}}}\right\}(x_j - x_{0,j}) + \dots}$$

$\Downarrow$

T_{ij} : "tidal force"

$= T_{ij}^{(int)} + T_{ij}^{(ext)}$.

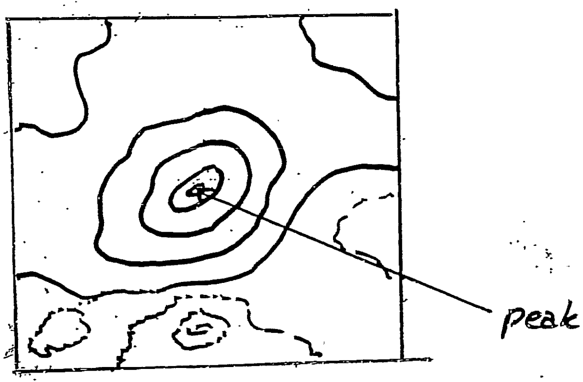
Distinguish "sources" anisotropic force field

1. internal flattening: cf. isolated collapsing ellipsoid
2. $T_{ij}^{(ext)}$: external tidal influence.

Homogeneous Ellipsoidal Density Perturbations

Next step after spherical model :

* Dropping assumption of sphericity:

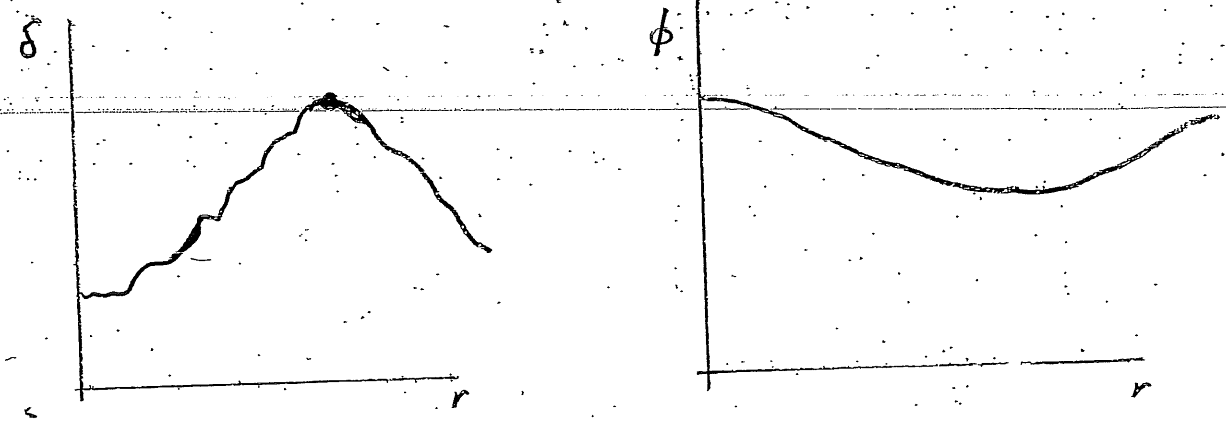


peaks in random field are not spherical !!

* simplified model for formation of flattened or filamentary objects like superclusters.

evolution flattened object can be studied analytically / numerically making additional assumption:

- density in object homogeneous or, rather, potential in object quadratic in $\vec{r}$



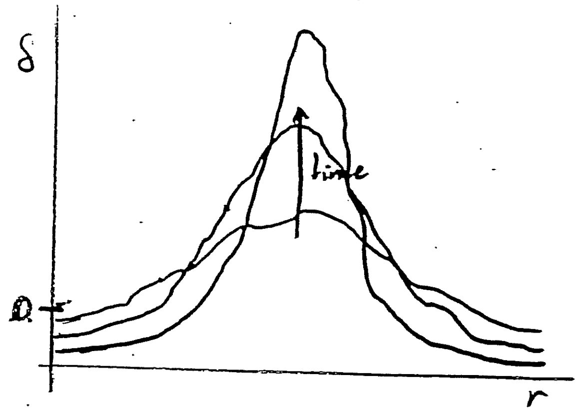
$$f_G(\vec{r}) = f_G(\vec{r}_{pk}) + \frac{1}{2}\sum \frac{\partial^2 f_G}{\partial r_i \partial r_j}(r_i - r_{pk_i})(r_j - r_{pk_j})$$

Density field around peak.

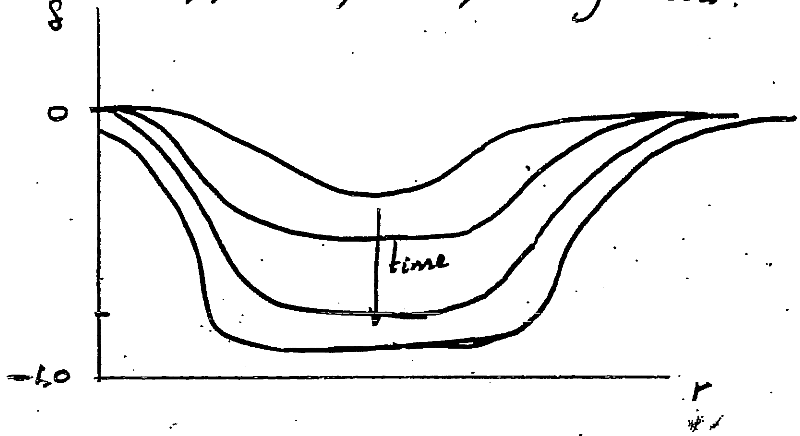
focus on region where the term $\nabla^2 f$ negligible

$P(k)$

• Note: • as overdense peak collapses it becomes steeper:



⇒ homogeneous density region becomes smaller,
⇒ the reverse happens for expanding void:



2. Assumption of isolated object:

• no (neighbouring) objects that would influence evolution.

• no influence on (homogeneous) background, nor back reaction of background on object.

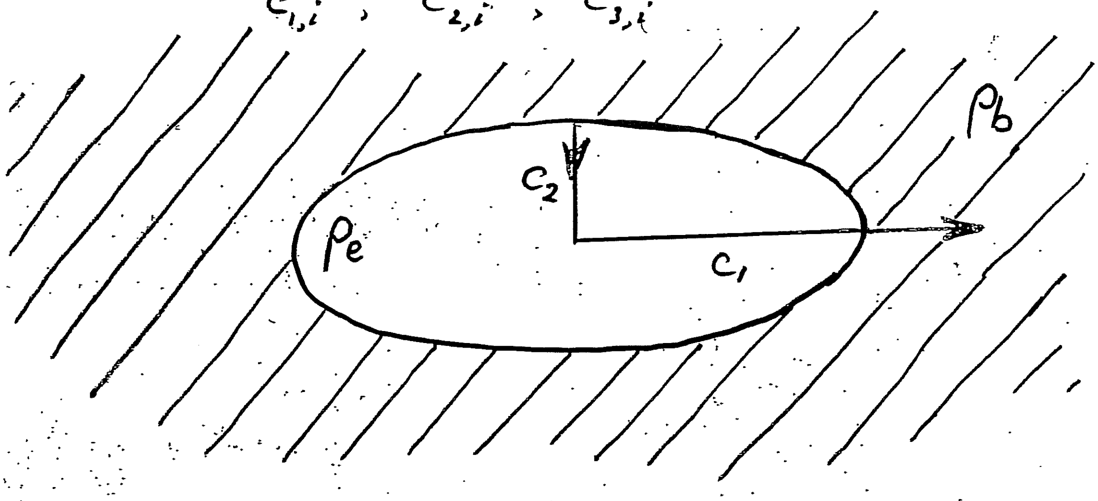
• (in case of underdense ellipsoids):
no formation of (anisotropic) ridges around voids, and therefore neglect of their influence

• do allow for external tidal field induced by far removed density fluctuations (artificial)

4) Leads to configuration of homogeneous ellipsoid

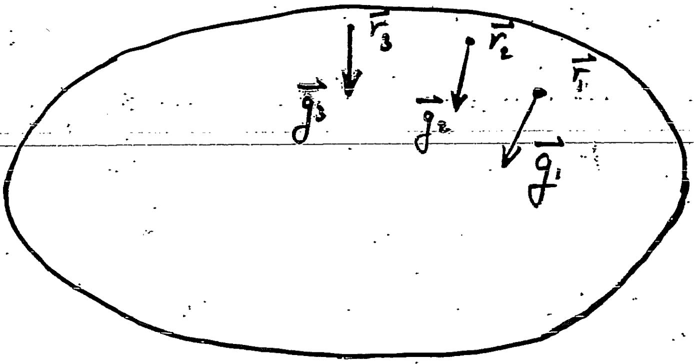
- Isolated ellipsoidal object of homogeneous density ρ_e, embedded in a uniform background of density ρ_b
- At some initial time t_i the ellipsoid's flattening is characterized by 3 axes:

$$c_{1,i}\ ,\ c_{2,i}\ ,\ c_{3,i}$$

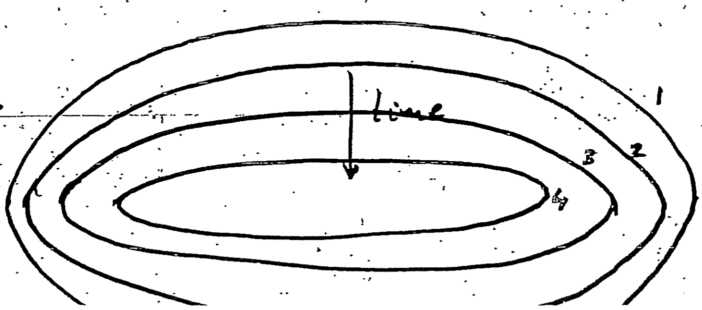


5) Consider location $\vec{r}$ within the ellipsoid

...celeration at
...fferent locations



...ollapse sequence
... ellipsoid.



Homogeneous Ellipsoid Dynamics

Gravitational Potential $\Phi(\vec{r})$:

- quadratic function of location $\vec{r}$
- separable into 3 constituents

$$\boxed{\Phi = \underbrace{\Phi_b(\vec{r})}_{\substack{\text{Cosmic Background Term} \\ \downarrow \\ \text{FRW cosmology}}} + \underbrace{\Phi^{(int,ell)}(\vec{r})}_{\substack{\text{Internal Ellipsoidal Term} \\ \downarrow \\ \text{ellipsoidal shape}}} + \underbrace{\Phi^{(ext,ell)}(\vec{r})}_{\substack{\text{External Tidal Field} \\ \downarrow \\ \text{inhomogeneous, external matter distribution.}}}}$$

Cosmic Background Contribution:

$$\Phi_b(r) = \frac{2}{3}\pi G \rho^b \left(r_1^2 + r_2^2 + r_3^2\right)$$

Internal Ellipsoidal Potential $\Phi^{(int,ell)}(\vec{r})$:

$$\Phi^{(int,ell)}(\vec{r}) = \frac{2}{3}\pi G\left(\rho^{ell} - \rho^{b}\right)\left(r_1^2 + r_2^2 + r_3^2\right) + \frac{1}{2}\sum_{m,n} T_{mn}^{(int)}\, r_m r_n \qquad (m,n = 1,2,3)$$

$$T_{mn}^{(int)} = \frac{\partial^2 \Phi}{\partial r\, \partial r} - \frac{1}{3}\nabla^2 \Phi\, \delta_{mn}$$

Consider some position $\vec{r}$ within the ellipsoid:

* Total potential $\Phi(\vec{r})$:

$$\boxed{\Phi(\vec{r}) = \underbrace{\frac{2}{3}\pi G\bar{\rho}\sum_m r_m^2}_{(1)} + \underbrace{\frac{1}{2}\sum_{m,n}\Phi_{mn}^{ell}\, r_m r_n}_{(2)} + \underbrace{\frac{1}{2}\sum_{m,n} E_{mn}\, r_m r_n}_{(3)}}$$

homogeneous background of density $\bar{\rho}$ (1)

corresponding to ellipsoidal perturbation of density $\rho_e - \bar{\rho}$ (2)

external tidal ter (3)

(1) $\Phi_b(\vec{r}) = \frac{2}{3}\pi G\bar{\rho}\,(r_1^2 + r_2^2 + r_3^2)$

(2) $\Phi_{ell}(\vec{r}) = \frac{1}{2}\sum_{m,n}\Phi_{mn}^{ell}\, r_m r_n$

In principal axis system of ellipsoid:

$$\Phi_{ell}(\vec{r}) = \pi G(\rho_e - \bar{\rho})\sum_m \alpha_m r_m^2$$

$$\alpha_m = c_1 c_2 c_3 \int_0^\infty \frac{1}{(\lambda + c_m^2)} \prod_{n=1}^{3} \frac{1}{\sqrt{\lambda + c_n^2}}\, d\lambda$$

$$\sum \alpha_m = 2.$$

(3) $\Phi_{ext}(\vec{r}) = \frac{1}{2}\sum_{m,n} E_{mn}\, r_m r_n$

externally induced term.

$$\Rightarrow \quad \Phi^{(int, ell)}(\vec{r}) = \pi G(\rho^{ell} - \rho^{b}) \sum_m \alpha_m r_m^2$$

α_m quantifies anisotropy potential!
$\alpha_m = \frac{2}{3}$ for sphere

$$\alpha_m = c_1 c_2 c_3 \int_0^\infty \frac{1}{(\lambda + c_m^2)} \prod_{i=1}^{3} \frac{1}{\sqrt{c_i^2 + \lambda}} \, d\lambda$$

Internal Tidal Force

$$\boxed{T_{mn}^{(int)} = 2\pi G(\rho^{ell} - \rho^{b})(\alpha_m - \tfrac{2}{3})\delta_{mn}}$$

• note: $T_{mn}^{(int)} = 0$ if $\alpha_m = \frac{2}{3}$: sphere !!

$\Rightarrow$ Equations of motion ellipsoid :

$$\frac{d^2 r_m}{dt^2} = -\frac{4\pi}{3} G \rho^{b} r_m(t) - \sum_m \Phi_{mn}^{(int,ell)} r_n(t) - \sum_n T_{mn}^{(ext)} r_n(t)$$

$$= \underbrace{-\frac{4\pi}{3} G \rho^{ell} r_m(t)}_{\text{isotropic collapse term}} - \underbrace{\sum_m T_{mn}^{(int)} r_n(t)}_{\text{internal "shape-induced" anisotropic collapse}} - \underbrace{\sum_n T_{mn}^{(ext)} r_n(t)}_{\text{external tidal force term.}}$$

$$r_m(t) = \sum_k R_{mk}(t)\, r_{k,i} \;\hat{=}\; R_m'(t)\, r_{m,i}$$

(for suitable choice axes in absence

Acceleration of matter element at $\vec{r}$:

$$\boxed{\frac{d^2 r_m}{dt^2} = -\frac{4\pi}{3} G \bar{\rho} r_m(t) - \sum_n \Phi^{ell}_{mn} r_n(t) - \sum_n E_{mn} r_n(t)}$$

- ⇒ if initial position $(r_{1,i}, r_{2,i}, r_{3,i})$:

$$r_m(t) = \sum_k R_{mk}(t)\, r_{k,i} \qquad \text{linear relation.}$$

$$\boxed{\frac{d^2 R_{mk}}{dt^2} = -\frac{4\pi}{3}\bar{\rho} G R_{mk} - \sum_n \Phi^{ell}_{mn} R_{nk} - \sum_n E_{mn} R_{nk}}$$

Further assumption:

* tidal tensor aligned along principal axes system

- no coupling of R_{ij} terms: <u>no rotation.</u>

$$E_{mn} = E_{mm}\,\delta_{mn}.$$

(but generically: off-di terms (tidal torques) induce rotation)

$$\boxed{\frac{d^2 R_{mk}}{dt^2} = -2\pi G\left[\alpha_m \rho_e + \left(\frac{2}{3} - \alpha_m\right)\bar{\rho}\right] R_{mk} - E_{mm} R_{mk}}$$

$$\Rightarrow \quad R_{mk}(t) = R_m(t)\,\delta_{mn}$$

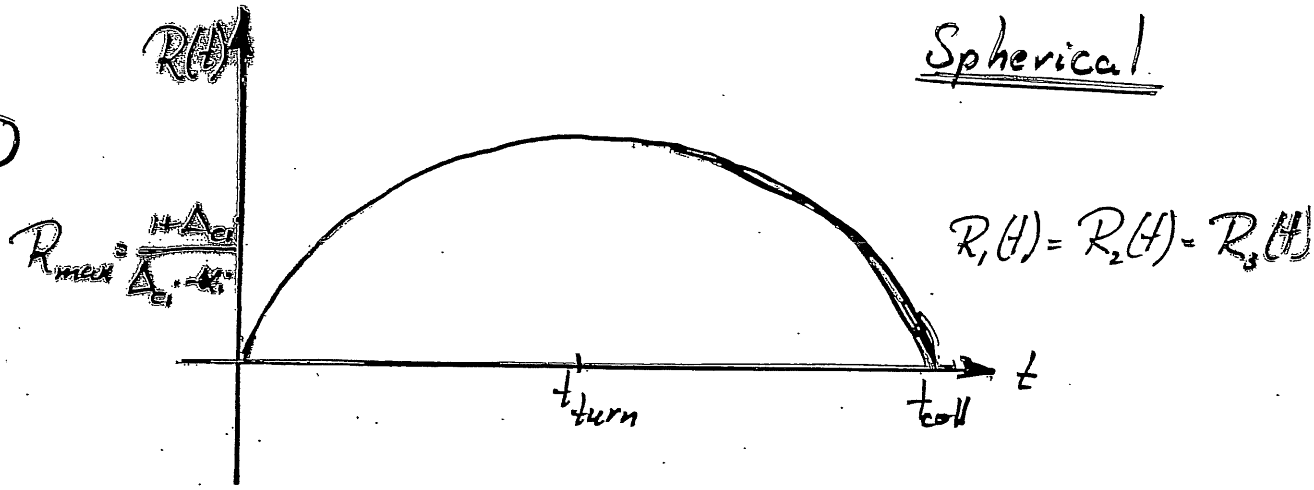
$$\boxed{\frac{d^2 R_m}{dt^2} = -2\pi G\left[\alpha_m \rho_e + \left(\frac{2}{3} - \alpha_m\right)\bar{\rho}\right] R_m - E_{mm} R_m}$$

Axis Evolution Overdense Ellipsoids.

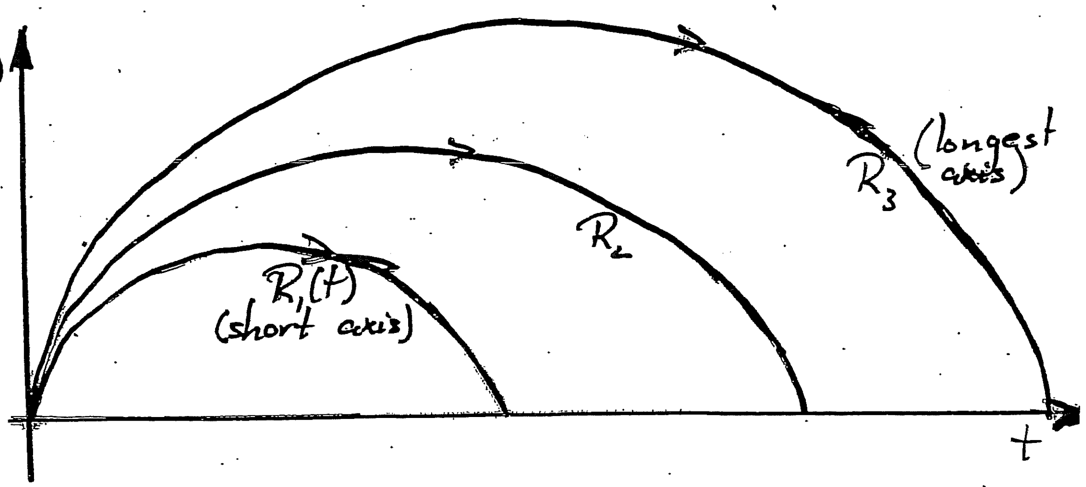
- Comparison axis evolution of spherical overdensity and ellipsoidal overdensity, both with δ_i at t_i

Spherical

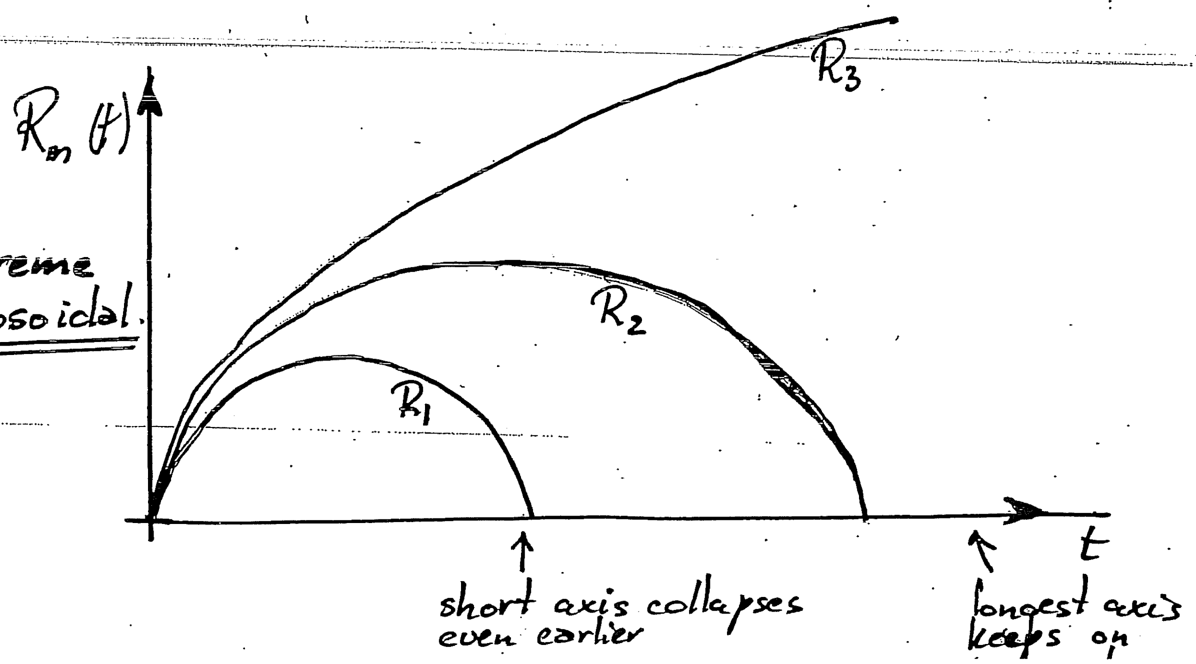
①



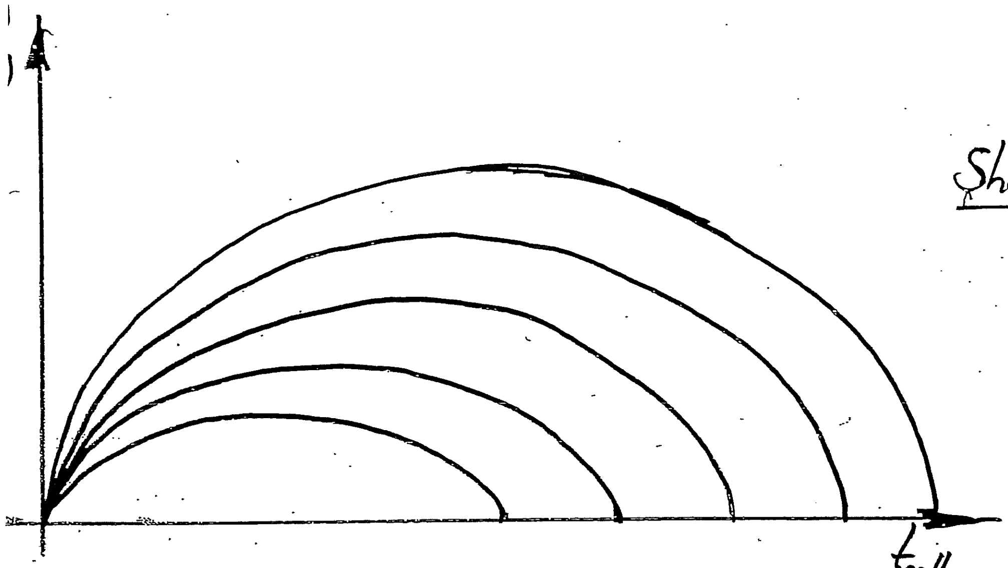
② Ellipsoidal



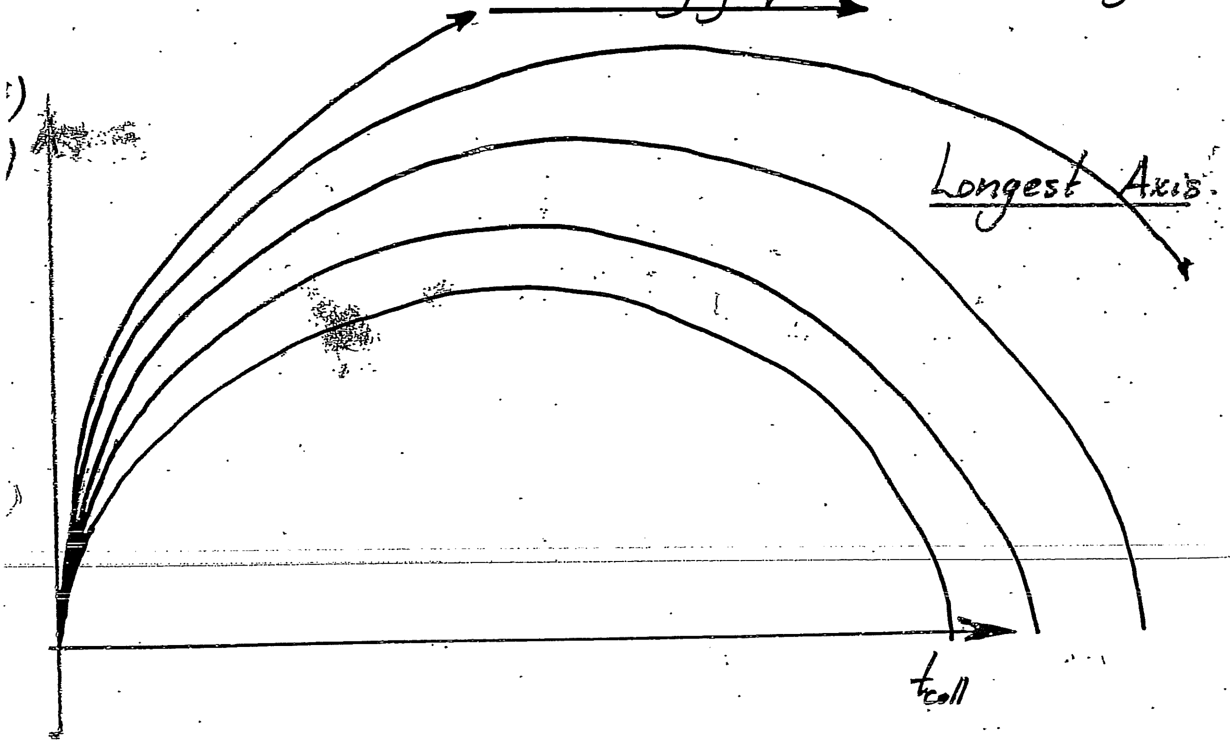
Extreme Ellipsoidal.



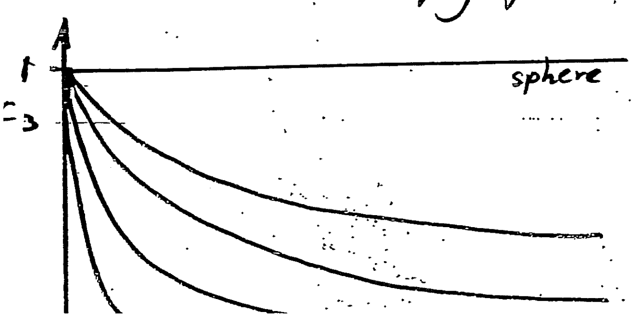
Comparison Evolution of Sphere and various Ellipsoids



Increasingly flattened initially



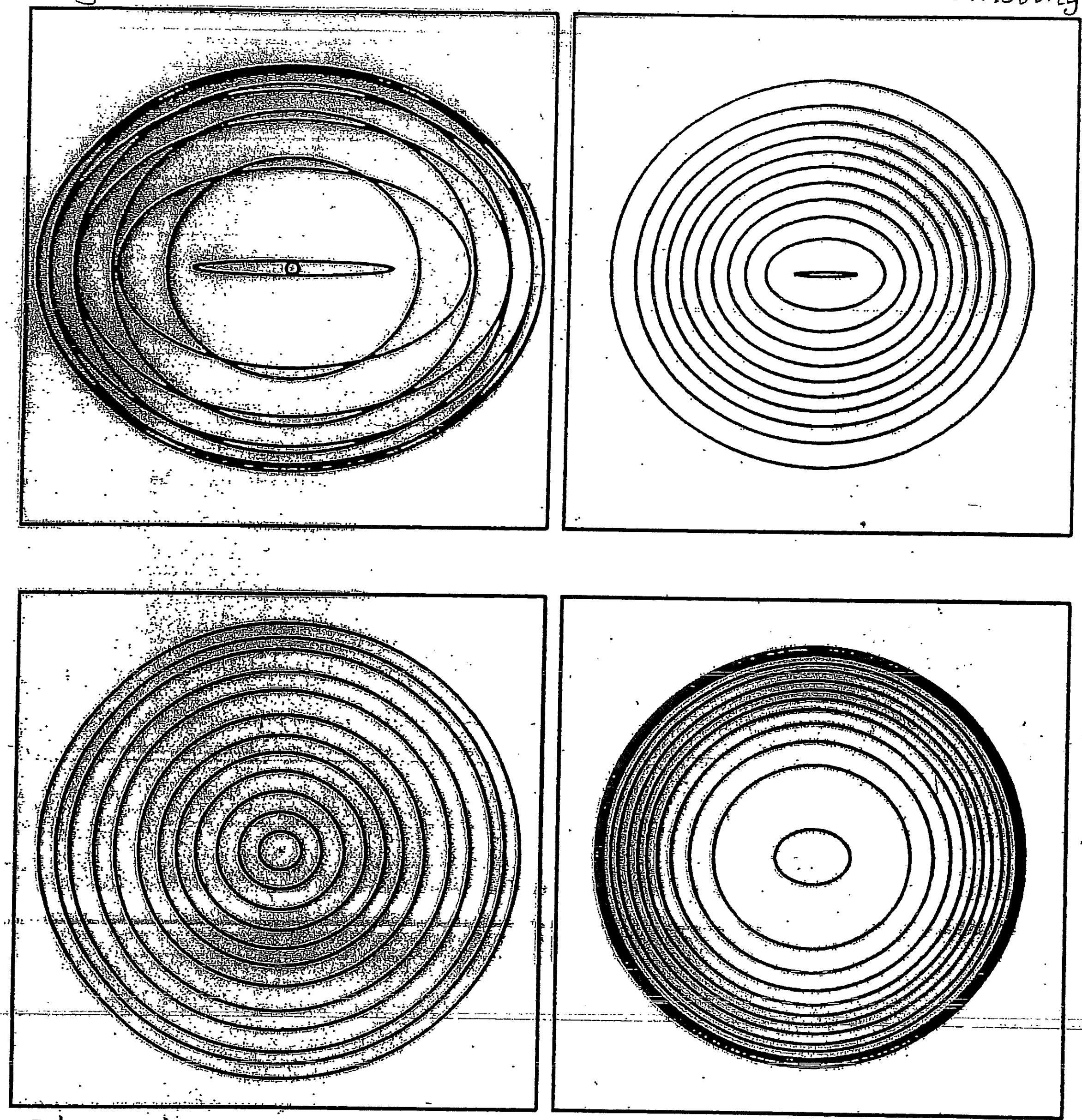
What does this imply for the axis ratios (the flattening)



Ellipsoidal Collapse

Overdensity

Physical Comoving



Physical Comoving

Underdensity

Ellipsoidal Expansion