

Cosmology,

lect. 3a

**Cosmological Principle:
the Evidence**

Cosmological Principle

Before embarking upon an assessment of the reality of the **homogeneity** and **isotropy** of our Universe, we first should pay attention to the meaning of these two concepts:

1) **Homogeneity**

The same physical circumstances prevail at any location in the Universe. In its broadest context this means:

- physical quantities like **density, temperature, pressure, ...**
- physical laws and relations

2) **Isotropy**

The Universe looks the same in whatever direction you look.

Cosmological Principle

1) Homogeneity does not imply Isotropy

In principle it is perfectly possible to have an

anisotropic & homogeneous medium:

- eg. more stretched in one direction than in the other, nonetheless exactly the same at every point in space:
- the local anisotropy may be the same everywhere

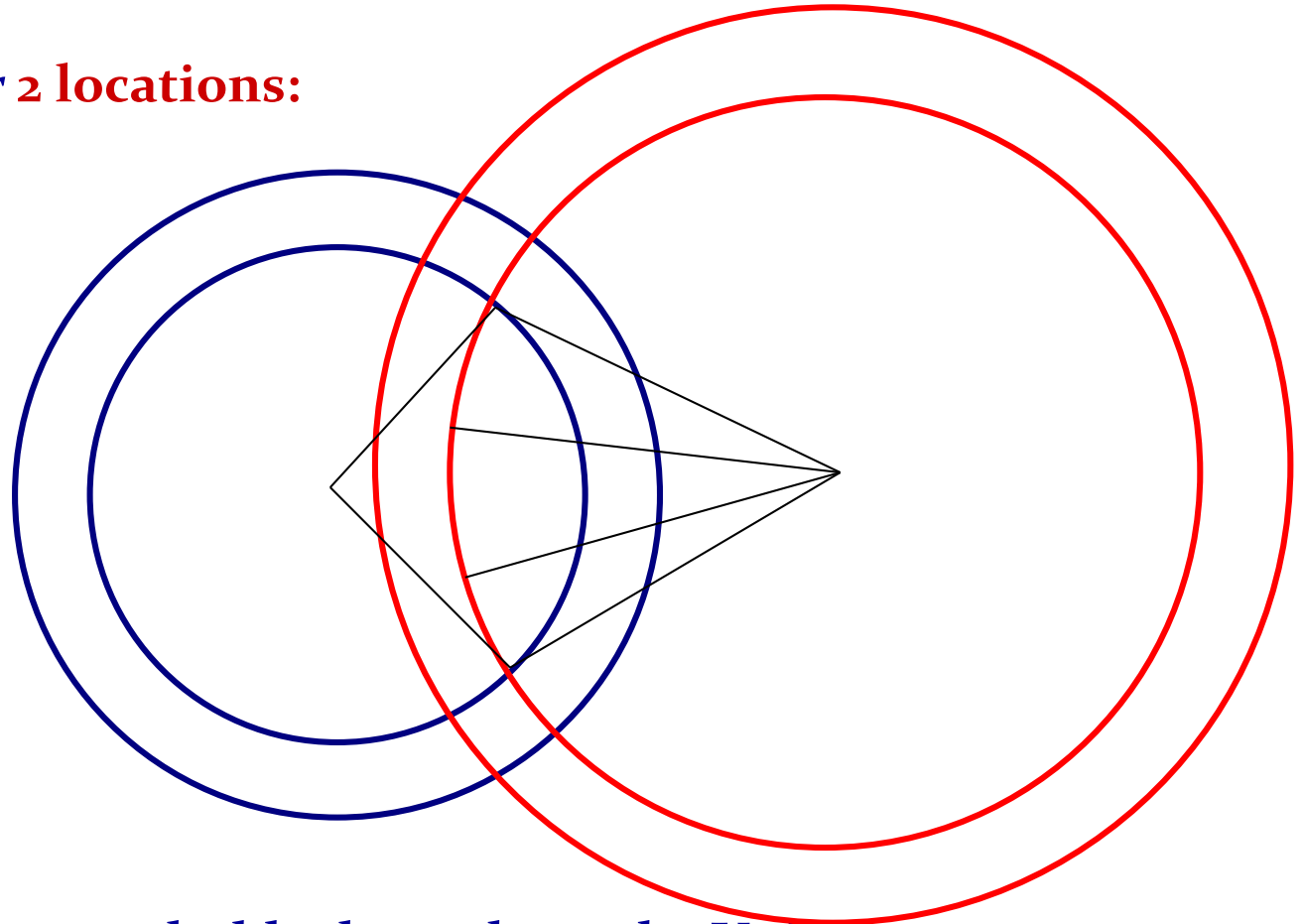
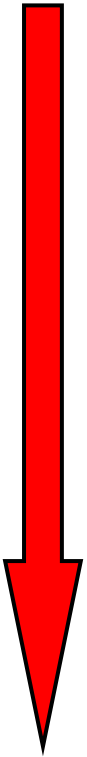
2) Isotropy does not imply Homogeneity

Given we know the Universe only from one vantage point, our cosmic location, we cannot be sure that our conclusion of isotropy is universally valid:

- however, it violates our Copernican feelings
- however, if you would know isotropy to hold for **2 locations**, homogeneity and isotropy holds throughout the Universe

Cosmological Principle

Knowing isotropy for **2 locations**:



Homogeneity and isotropy holds throughout the Universe

Cosmological Principle

While the “**homogeneity**” and “**isotropy**” of the Universe at first sight might occur like rather crude approximations of reality, there is ample evidence that on scales exceeding a few hundred Mpc it is a reasonably accurate description of reality.

In the following we will list the observational evidence for

- 1) an isotropic Universe
- 2) a homogeneous Universe

Isotropy

Prime Evidence for an Isotropic Universe:

1) **Isotropy** Cosmic Microwave Background

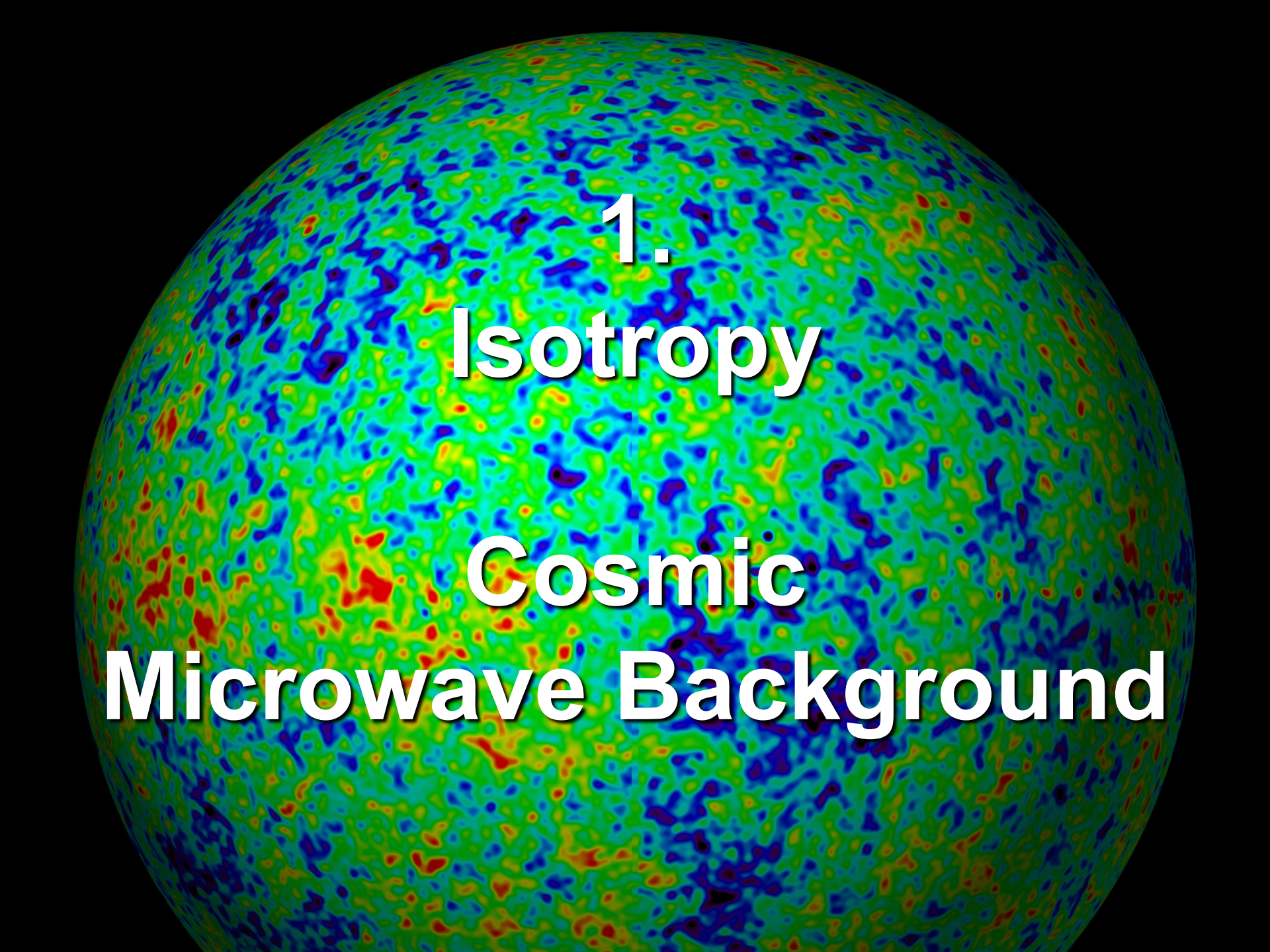
2) **Isotropy** X-ray Background

3) **Isotropy** Gamma Ray Bursts (GRBs)

4) **Isotropy** Galaxy Sky Distribution

(on cosmologically relevant scales)

5) **Isotropy** Hubble Expansion



1.

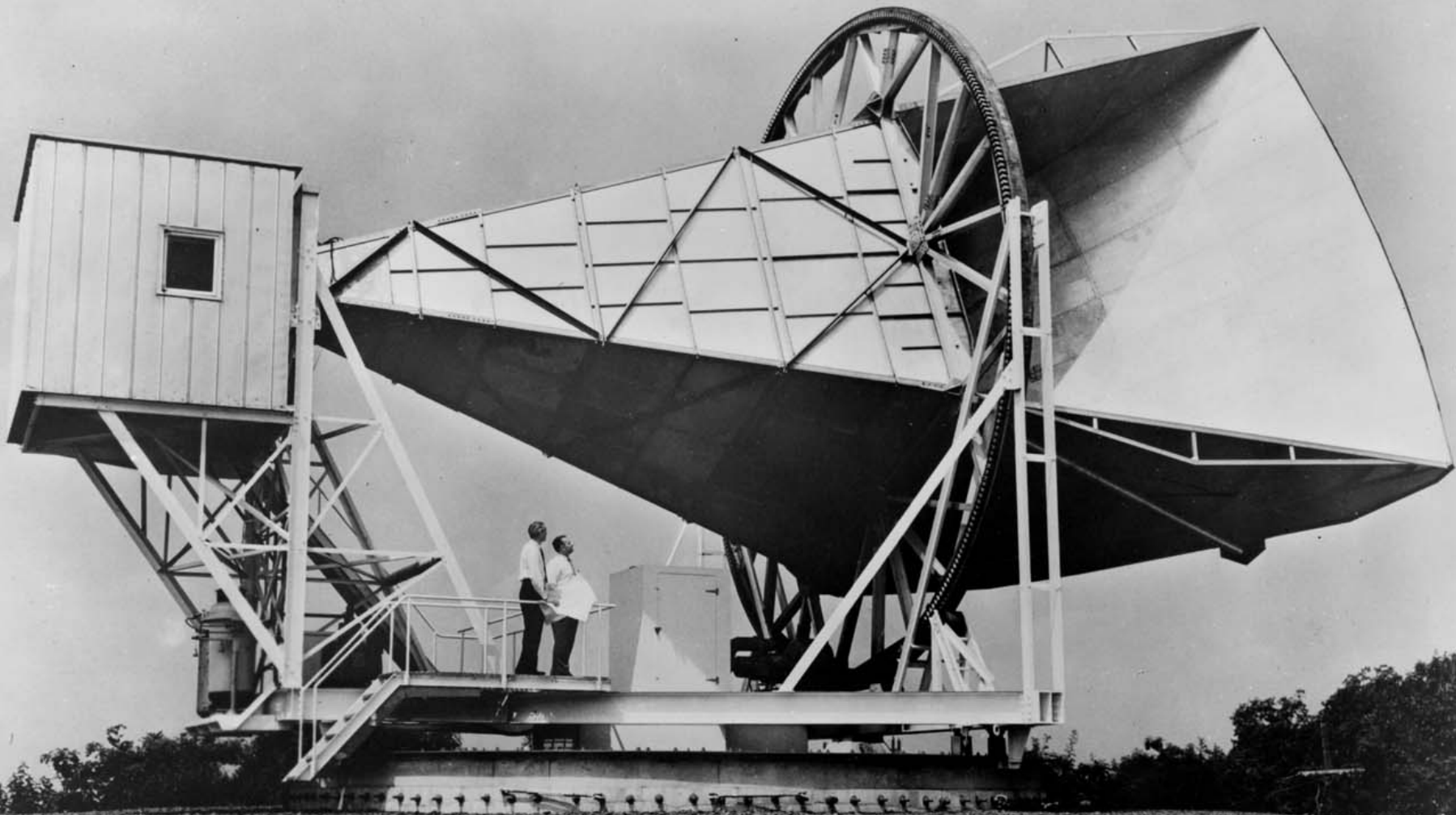
Isotropy

Cosmic

Microwave Background

Cosmic Microwave Background

discovery: 1965 (Penzias & Wilson)



Cosmic Microwave Background

Radiation Field of the Universe:

- o) Discovered in 1965 (serendipitously) by **Penzias & Wilson**,
Nobelprize 1978 !!!!!

- Thermal radiation pervading throughout the whole Universe
- As yet it has a temperature of

$$T_{\gamma} = 2.725 \text{ K}$$

- 1) By far CMB photons represent the most abundant species in the Universe:

$$n_{\gamma} \sim 415 \text{ cm}^{-3}$$

- For comparison: $n_{\gamma}/n_B \sim 1.9 \times 10^9$!!!! (second: cosmic neutrino's)
- Stellar photons: negligible !!!! (integrated over all stars at all times !)

Universe

379,000 years after Big Bang

almost perfectly smooth

Microwave Background Radiation, surface of last scattering of cosmic photons is almost perfectly isotropic, all around the same temperature:

$$T=2.725 \text{ K}$$

Cosmic Microwave Background (CMB) isotropic to almost absurdly accurate levels

- **The primary evidence for Isotropy Universe**

$$\frac{\Delta T}{T} < 10^{-5}$$

for comparison:

Planet Earth's

highest mountain

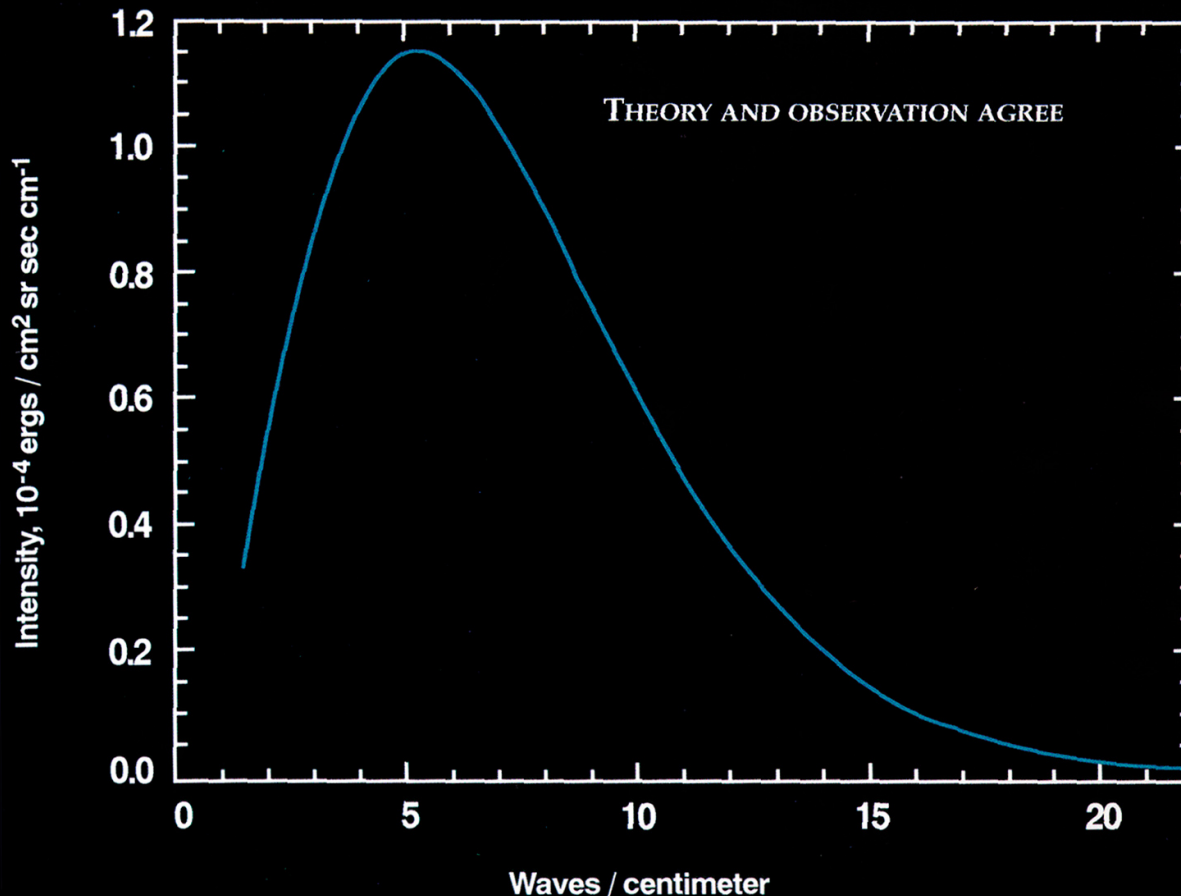
would be ~ 25-100 m !!!!!!!



CMB Blackbody Radiation Field:

Ultimate Evidence Big Bang

COSMIC MICROWAVE BACKGROUND SPECTRUM FROM COBE



- COBE-DIRBE

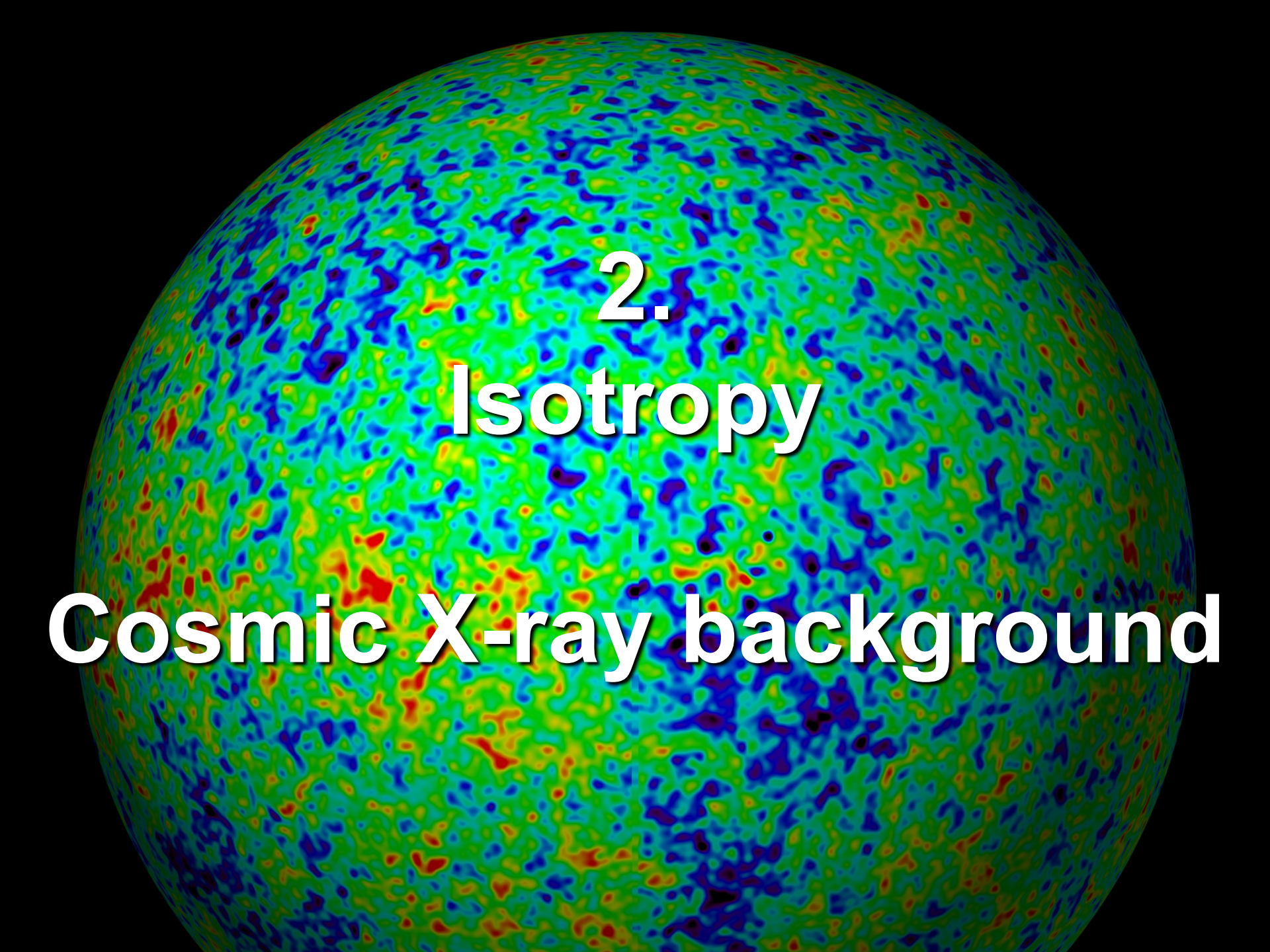
(John Mather: Nobel prize 2006):
perfect blackbody spectrum

- $T=2.735$ K

$$I_{\nu}(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

- most accurately known

Black Body radiation field in nature



2.

Isotropy

Cosmic X-ray background

X-ray background

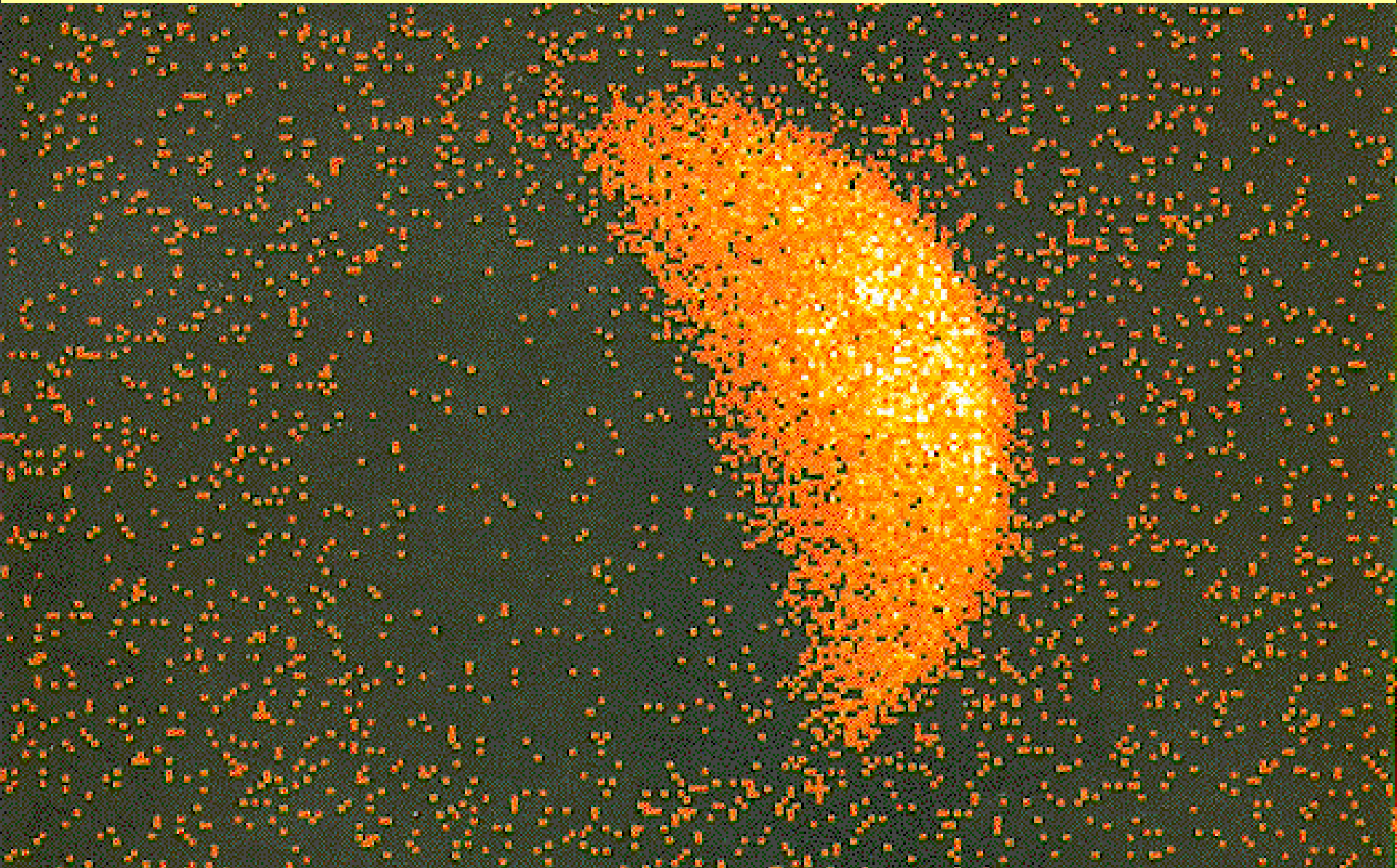
- The sky glows in X-rays in every direction, the **X-ray Background**
- discovered ~ 1962 (Giacconi), soon after launch first X-ray satellites.
- For many years its origin was not understood, possibilities were:
 - (very) **hot diffuse gas** ($T \sim 0.2 \times 10^9$ K)
 - population of unresolved **active centers of galaxies**, AGNs, thus probably involving **Massive Black Holes**, most probably at $z \sim 2-3$
- Riddle mostly solved following launch of the Chandra & XMM X-ray observatories:
 - at least 80 percent of **diffuse hard X-ray background** resolved into very many very faint (and distant) sources

!!!!!!!

X-RAY BACKGROUND HIGHLY ISOTROPIC

!!!!!!!

Moon & X-ray background

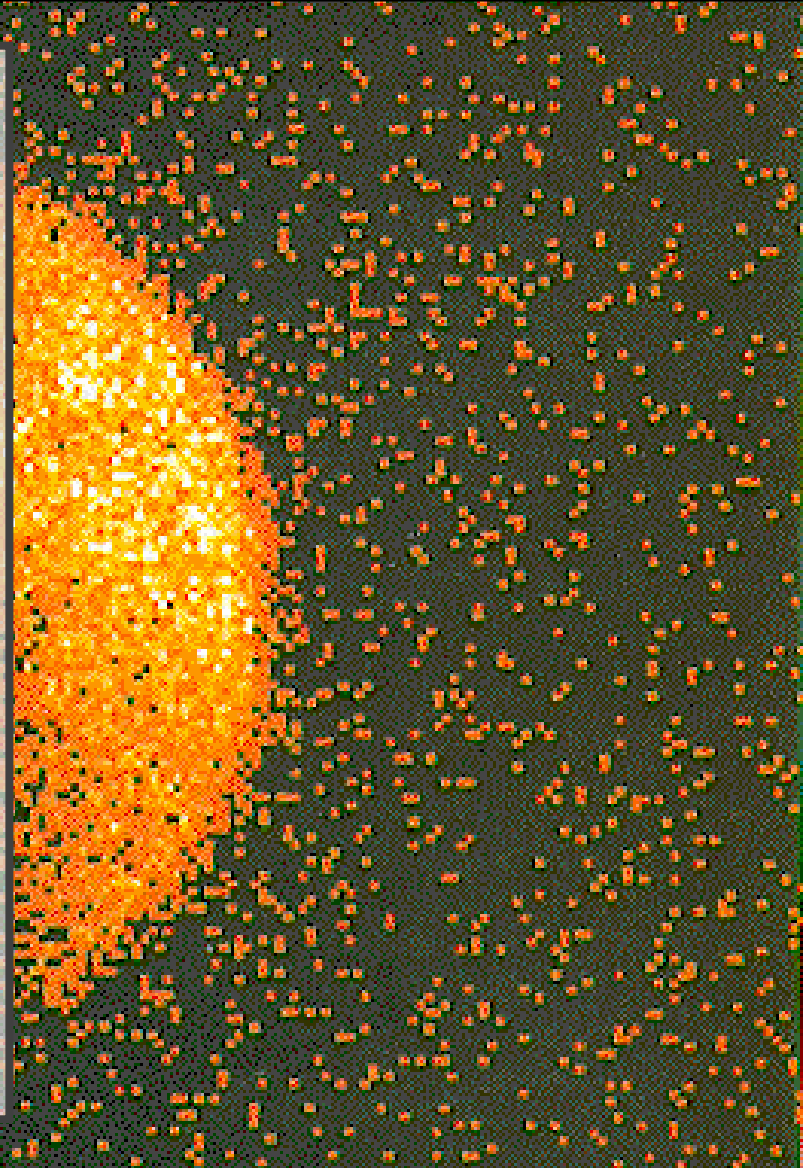


Moon & X-ray background

X-ray image of Moon by ROSAT (1990)
(pixel brightness: x-ray intensity)

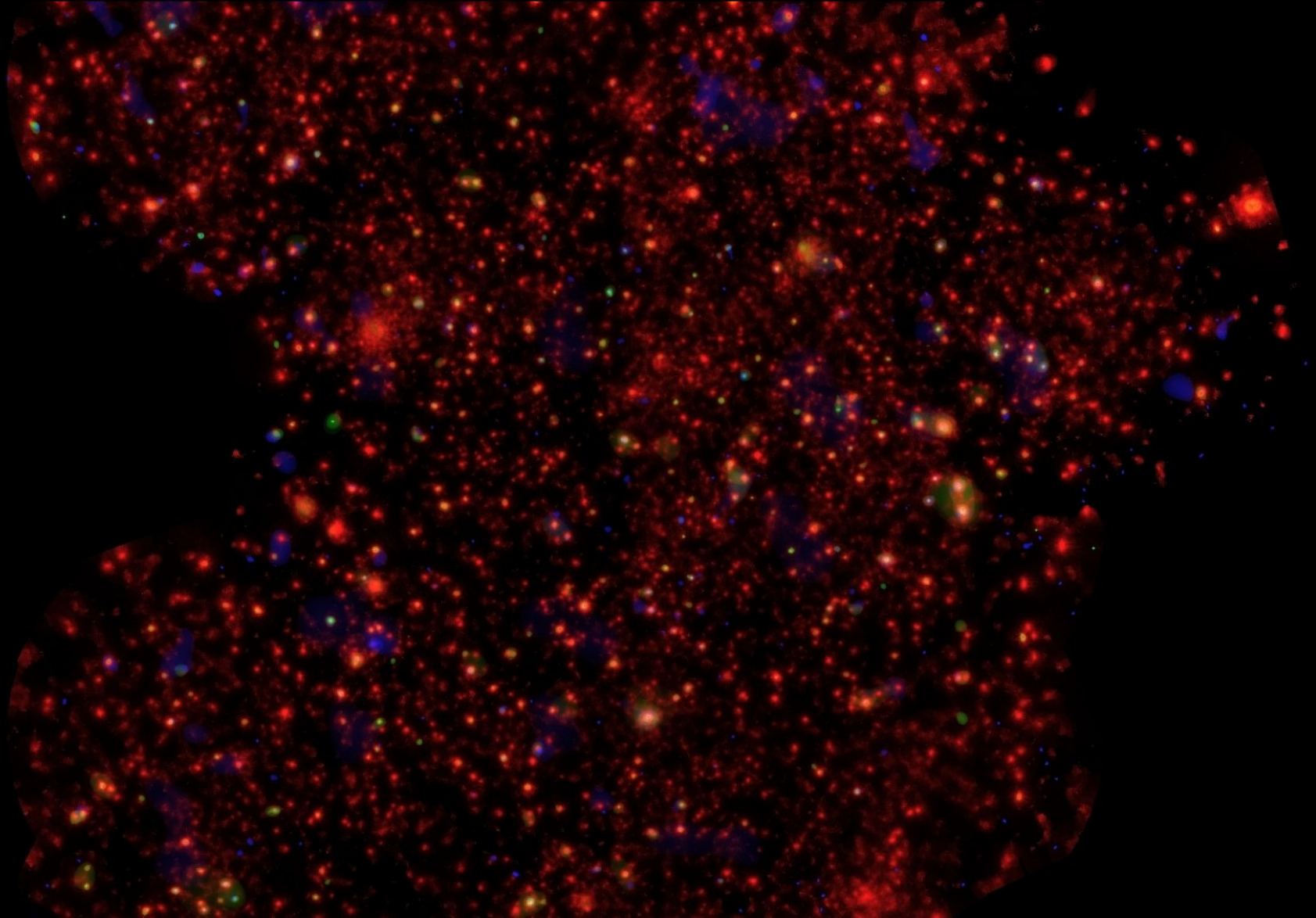
Three different parts:

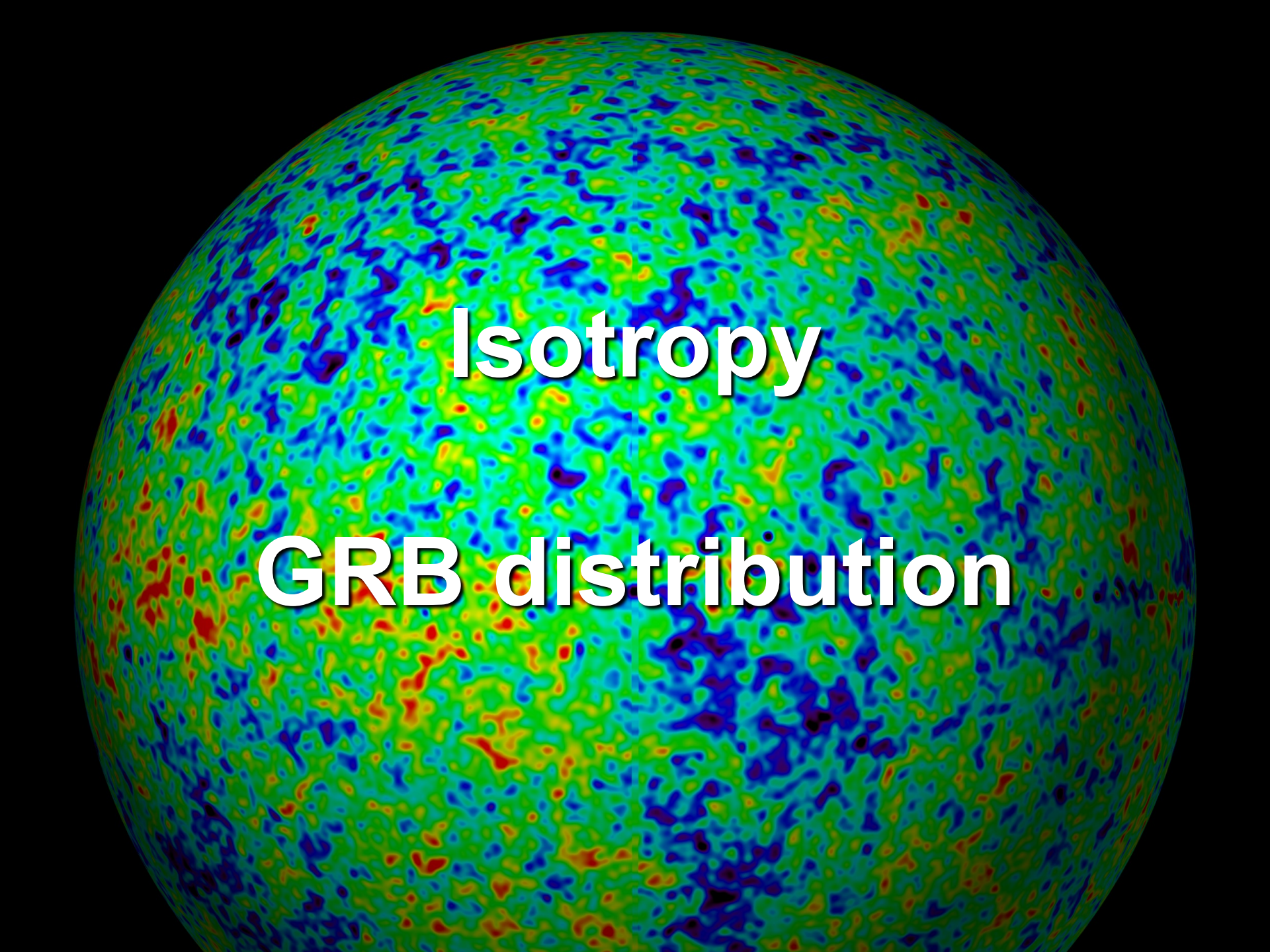
- **bright part: X-ray moon**
scattered x-rays emitted by Sun
- **dark half moon**
visible few photons originate in
Earth's geocorona & extended atmosphere
- **X-ray sky background**
myriad of distant, powerful active galaxies
unresolved in ROSAT image,
80% detected by Chandra



X-ray background:

high-z AGNs

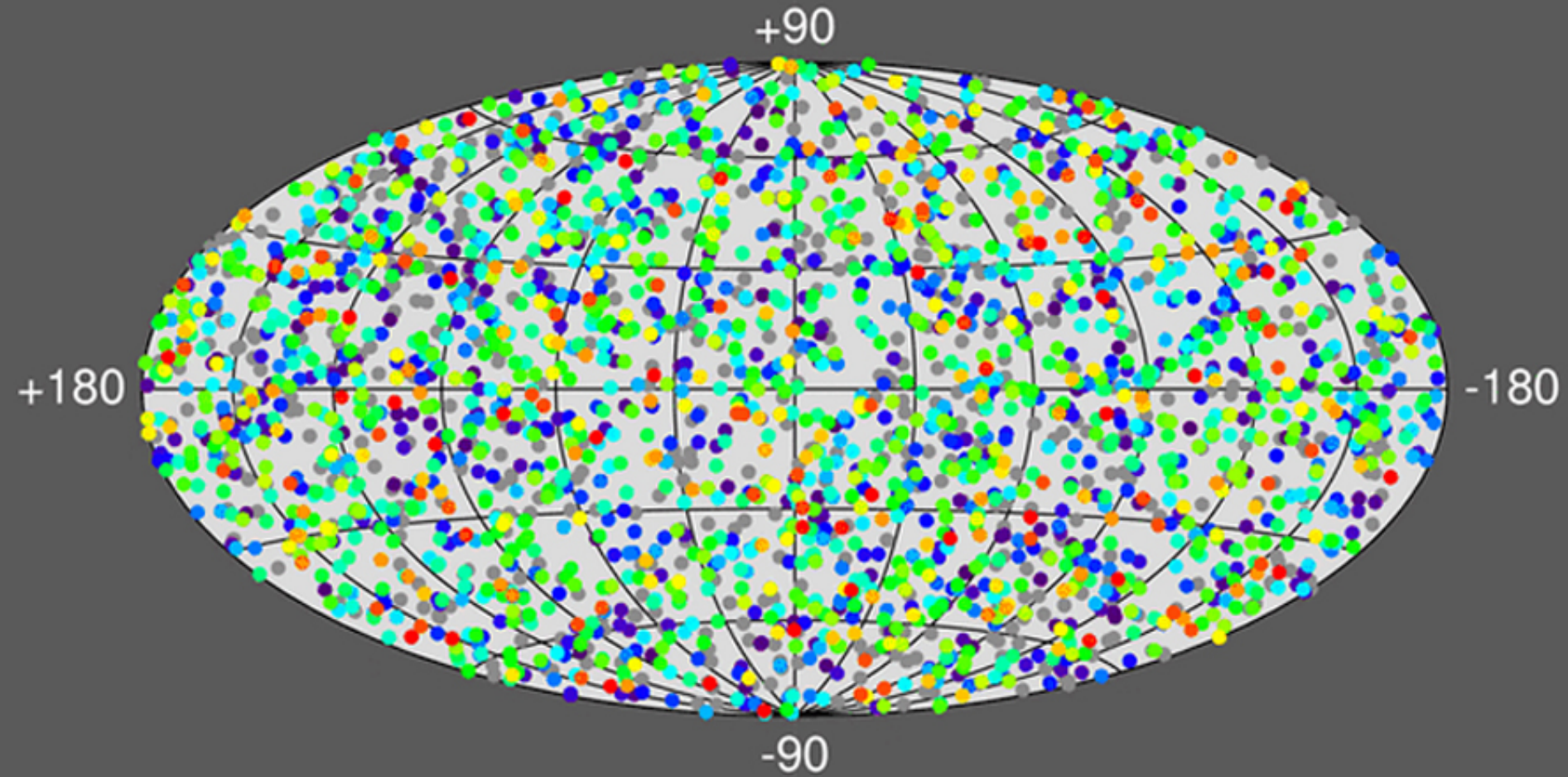




Isotropy

GRB distribution

Gamma Ray Burst (GRB) sky distribution



Angular Distribution GRBs:

2704 BATSE GRB sources

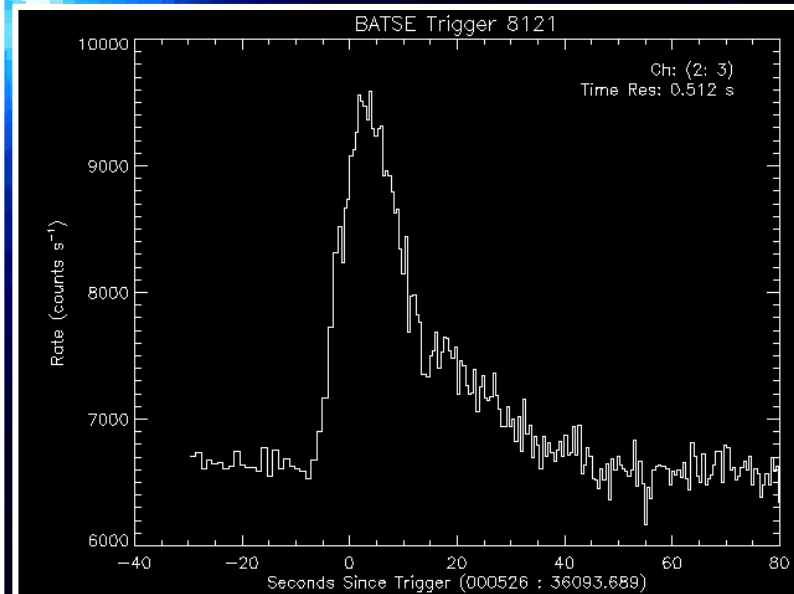
Gamma-ray Bursts

- **Brilliant flashes of γ -ray emission**,
from \sim msec – several 10s of seconds
followed by afterglows of X-ray, optical & radio emission
- **per day, \sim 1 GRB over entire Universe**
- **Most energetic events since Big Bang, $\sim 10^{54}$ ergs:**
at peak emission \sim energy all galaxies in visible Universe
- **Emission most likely highly collimated: GRB jets**

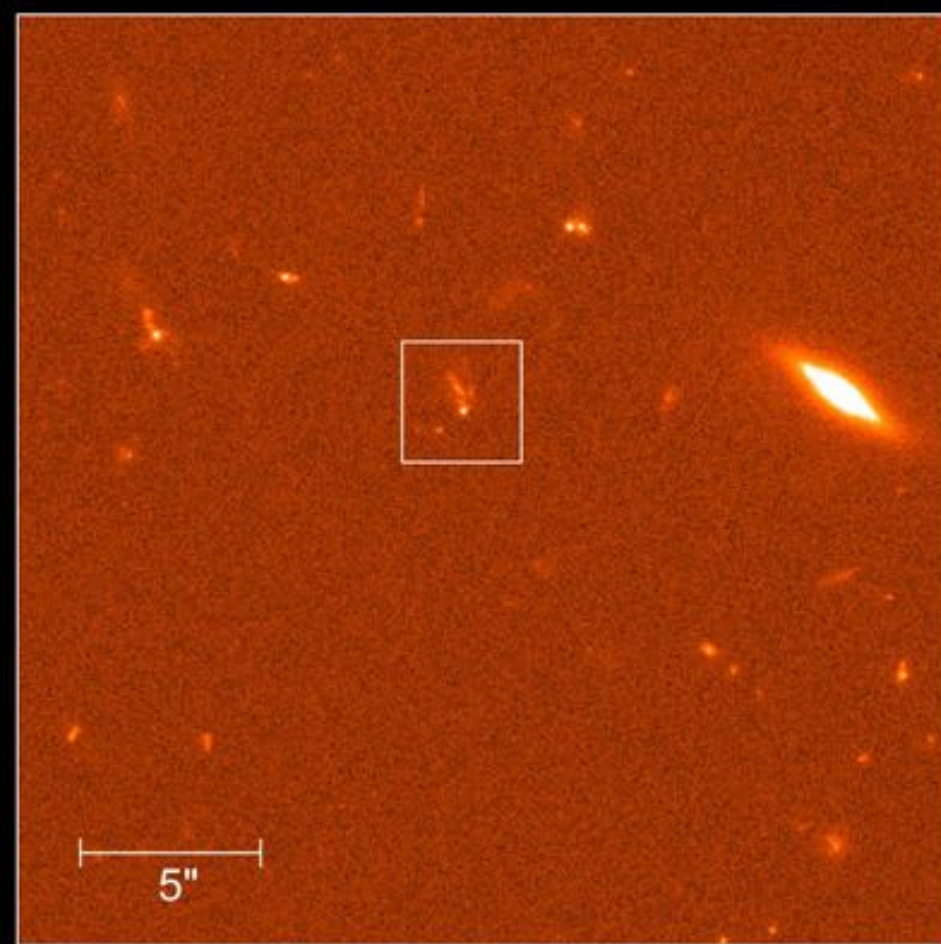
- **Discovered end of 60s by US Vela nuclear test detection satellites, testing for Soviet breaks of Nuclear Test Ban Treaty.**
Kept classified for years, public release in 1973:
cosmic origin

- **BATSE instrument on board of the Compton Gamma Ray Observatory (1990s):**

2704 GRB detections



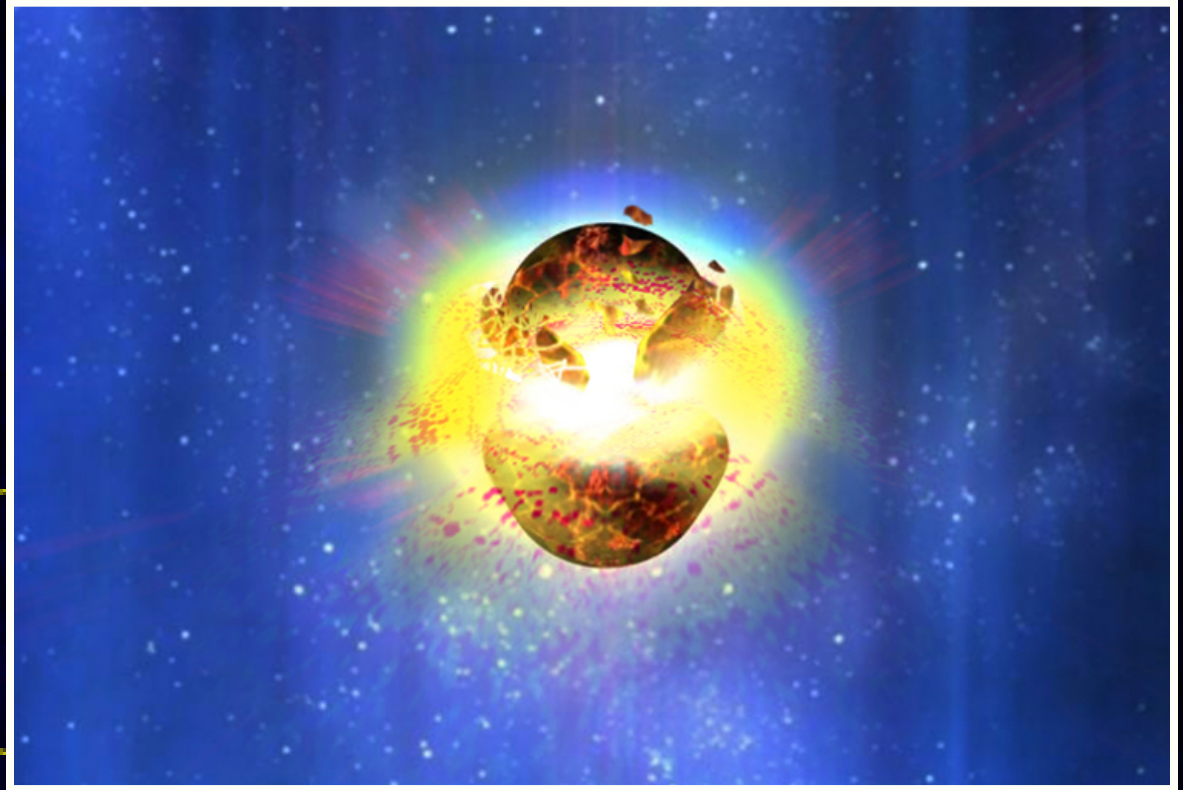
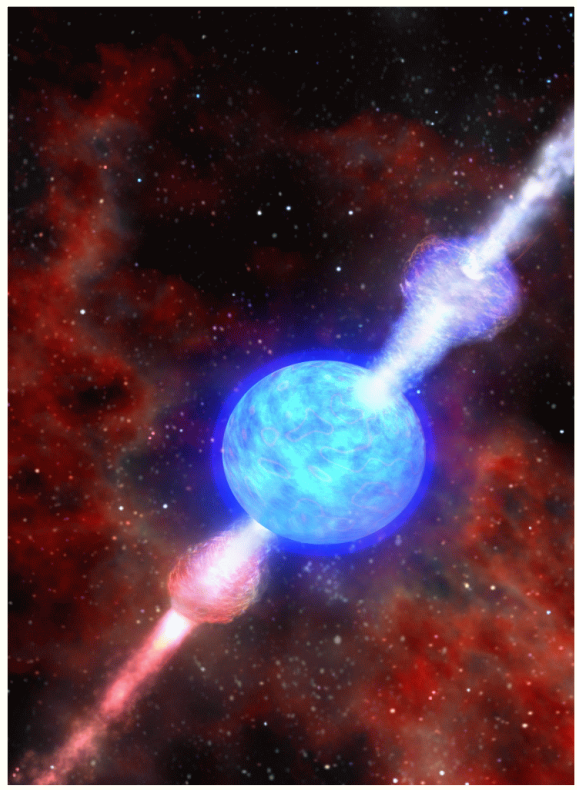
Cosmological Identity GRBs



- Final solution to the GRB riddle:
- GRB970228
- Identification of X-ray afterglow by Dutch-Italian BeppoSax satellite by van Paradijs et al. (1997)
- Followed by optical identification with remote galaxy ($z > 0.7$)
- GRBs cosmological !!!!
- typical redshift $z \sim 1$, record redshift $z > 10$
- Involved energies mind-boggling

Gamma Ray Burst GRB990123
Hubble Space Telescope • STIS

Gamma-ray Bursts



- Two classes GRBs:

long duration

> 2 sec.

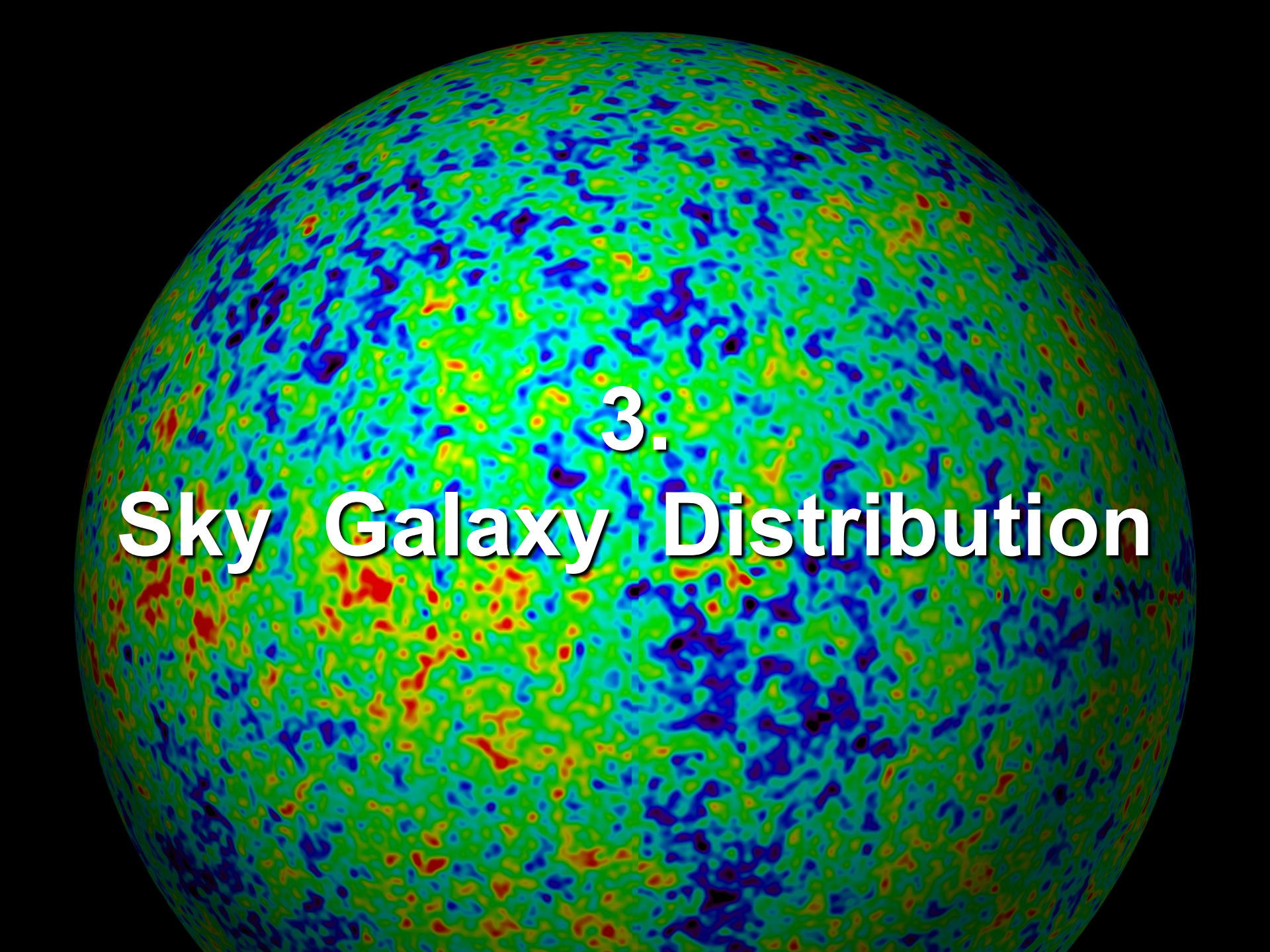
collapsar/hypernovae

short duration

msec < Δt < 2 sec.

neutron star – neutron star mergers

(short duration GRBs 10 times less bright than long duration ones)



3.

Sky Galaxy Distribution

Galaxy Sky Distribution

- Although we know that the local Universe is far from isotropic, or homogeneous, when assessing the galaxy distribution to high depths:

high level of isotropy.

- Example of a “nearby” galaxy sample:

zMASS survey

clear demonstration increasing isotropy for higher/fainter levels

- Example of “extended nearby” galaxy sample:

APM sky survey

- counting 2 million galaxies up to $m=20.5$
- local “Cosmic Web” is visible as
tenuous filamentary traces and compact dense cluster nodes
- most of the sky marked by a uniform distribution of background galaxies
- on scales > 100 Mpc hardly any noticeable structure

- Even more compelling is the evidence from deep “radio galaxy” samples:

NVVS

- almost perfectly isotropic

Increasing Depth



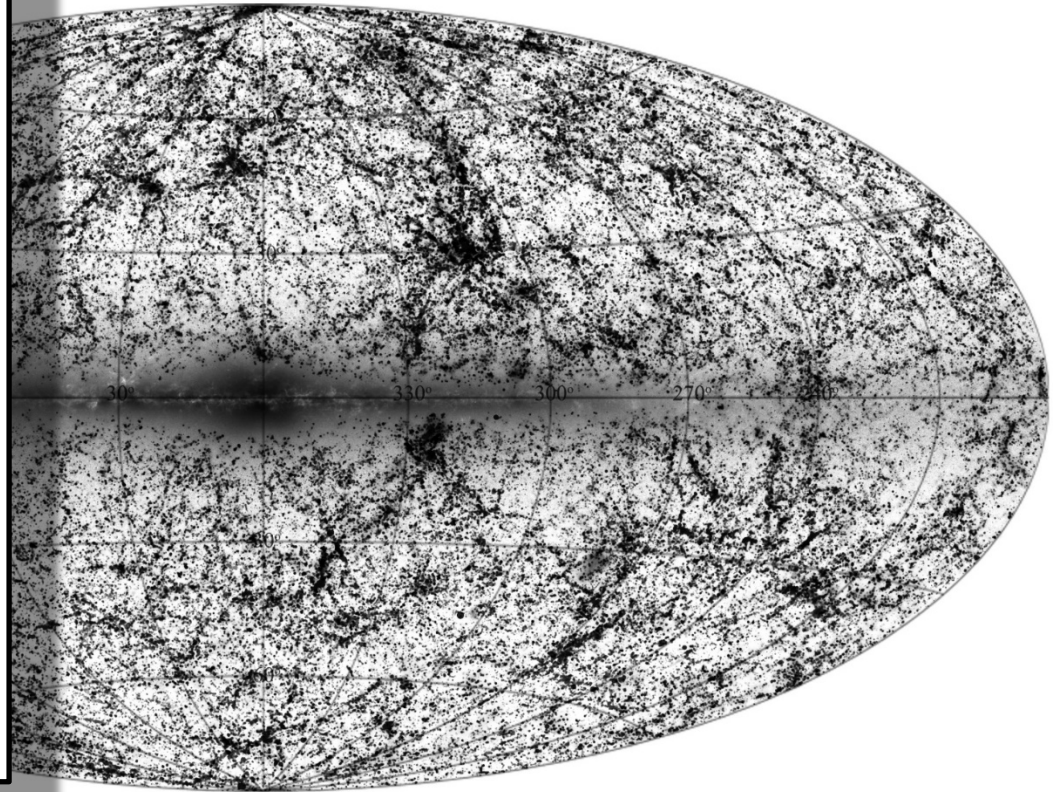
2MASS Cosmic Web

2MASS all-sky survey:

- ground-based near-infrared survey whole sky, J(1.2 μ m), H(1.6 μ m), K(2.2 μ m)
- 2MASS extended source catalog (XSC): 1.5 million galaxies
- unbiased sample nearby galaxies
- photometric redshifts: depth in 2MASS maps,
- “cosmic web” of (nearby) superclusters spanning the entire sky.

courtesy:

T. Jarrett



Looking around us we already see the unmistakable signatures of an intriguing weblike matter distribution in our immediate Cosmic Vicinity, together with an increasing isotropy as we go to fainter levels

The 2MASS Nearby Universe

2MASS Extended Sources
Integrated Flux



Ks: 8.0–10.0 mag
18'/pixel

2MASS: the integrated view

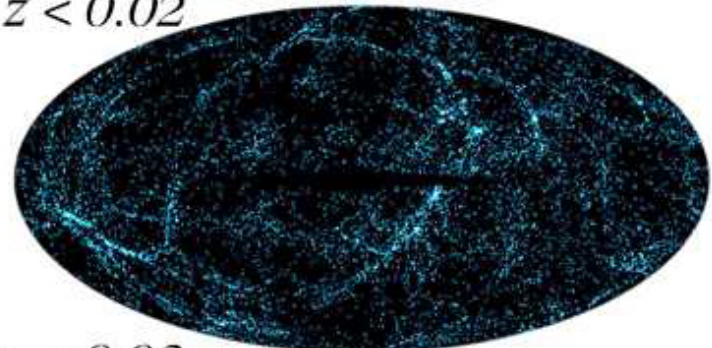
**As we move outward and fainter galaxies are projected on top the sky
the galaxy sky distribution becomes more and more uniform !**



$z < 0.01$



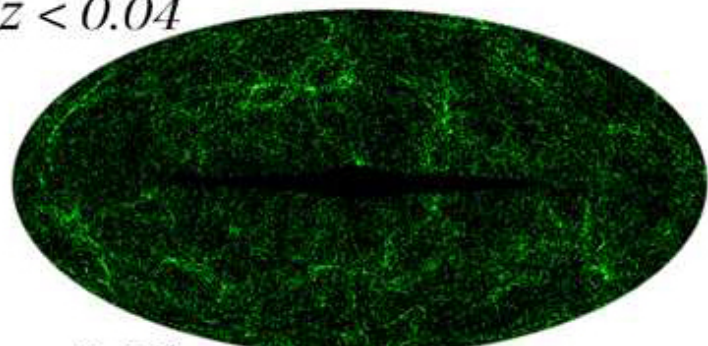
$0.01 < z < 0.02$



$0.02 < z < 0.03$



$0.03 < z < 0.04$



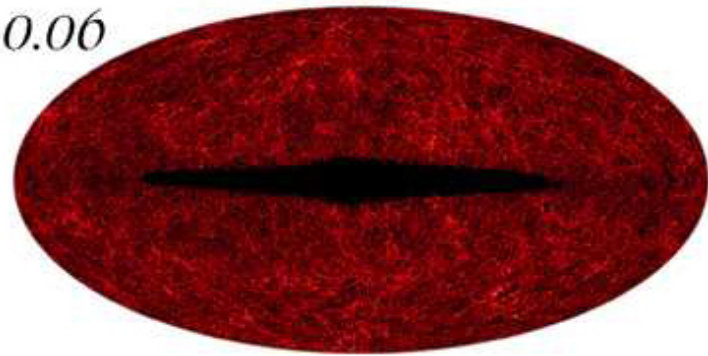
$0.04 < z < 0.05$



$0.05 < z < 0.06$



$z > 0.06$



APM Galaxy Survey

Sky map:

2×10^6 galaxies

$17 < m < 20.5$

Uniformly defined

- Sky region:
4300 sq. deg.
- 185 UK Schmidt plates,
 $6^\circ \times 6^\circ$
- Large inhomogeneities,
hints of weblike patterns,
with clusters at
densest regions.

S. Maddox, G. Efstathiou,
W. Sutherland, D. Loveday



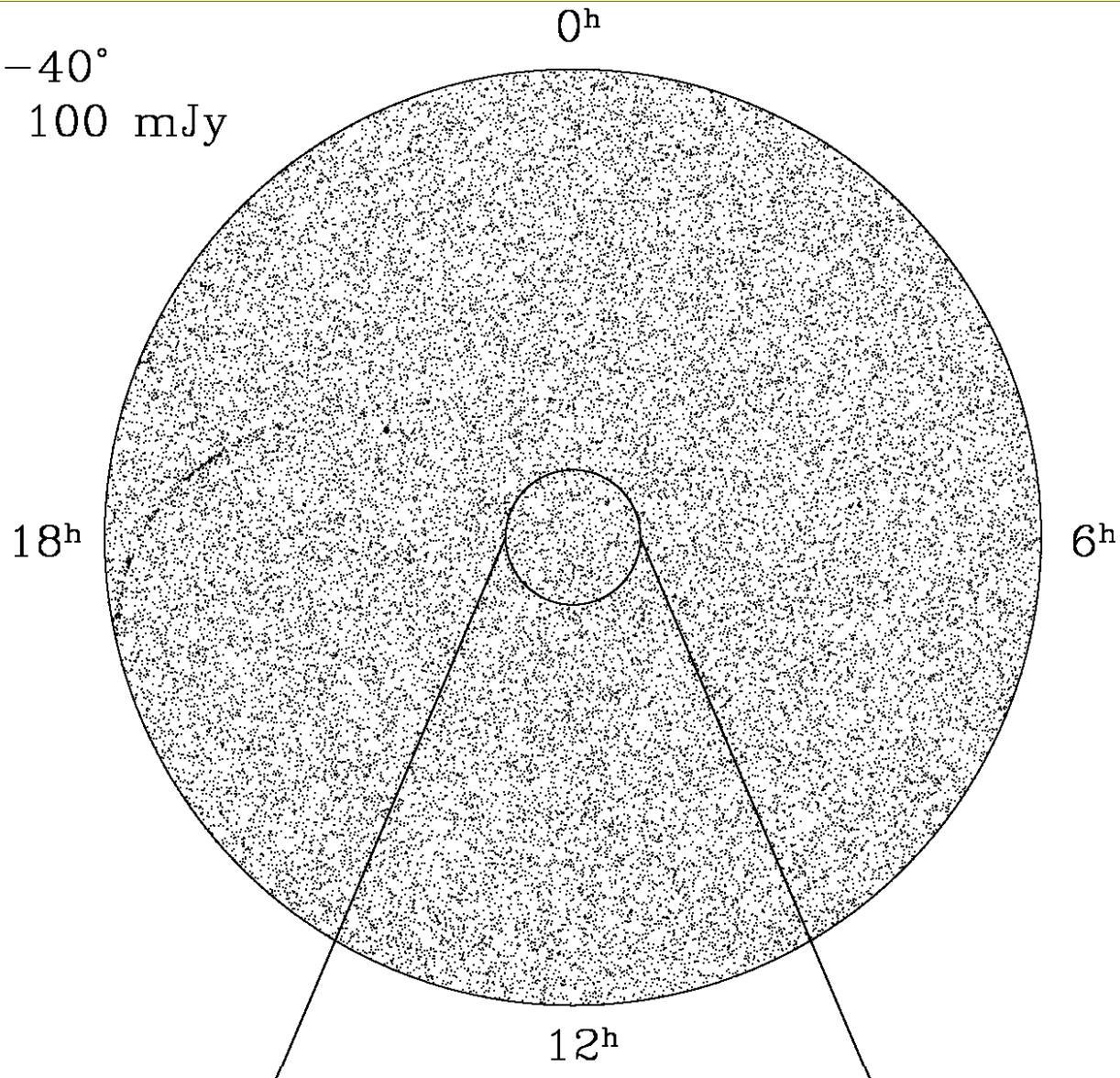
The APM Galaxy Survey
Maddox et al

Radio Galaxies: NVVS

- Good tracers of the Universe on the largest scales are radio galaxies (and other AGNs)
- The most extensive sky survey of discrete radio sources is the NRAO VLA Sky Survey (NVVS)
- Covers nearly the whole of the northern sky north of $\delta = -40^\circ$, nearly 2×10^6 sources stronger than $S = 2.5$ mJy at 1.4 GHz.
- The distribution of these discrete sources on the sky is nearly perfectly isotropic

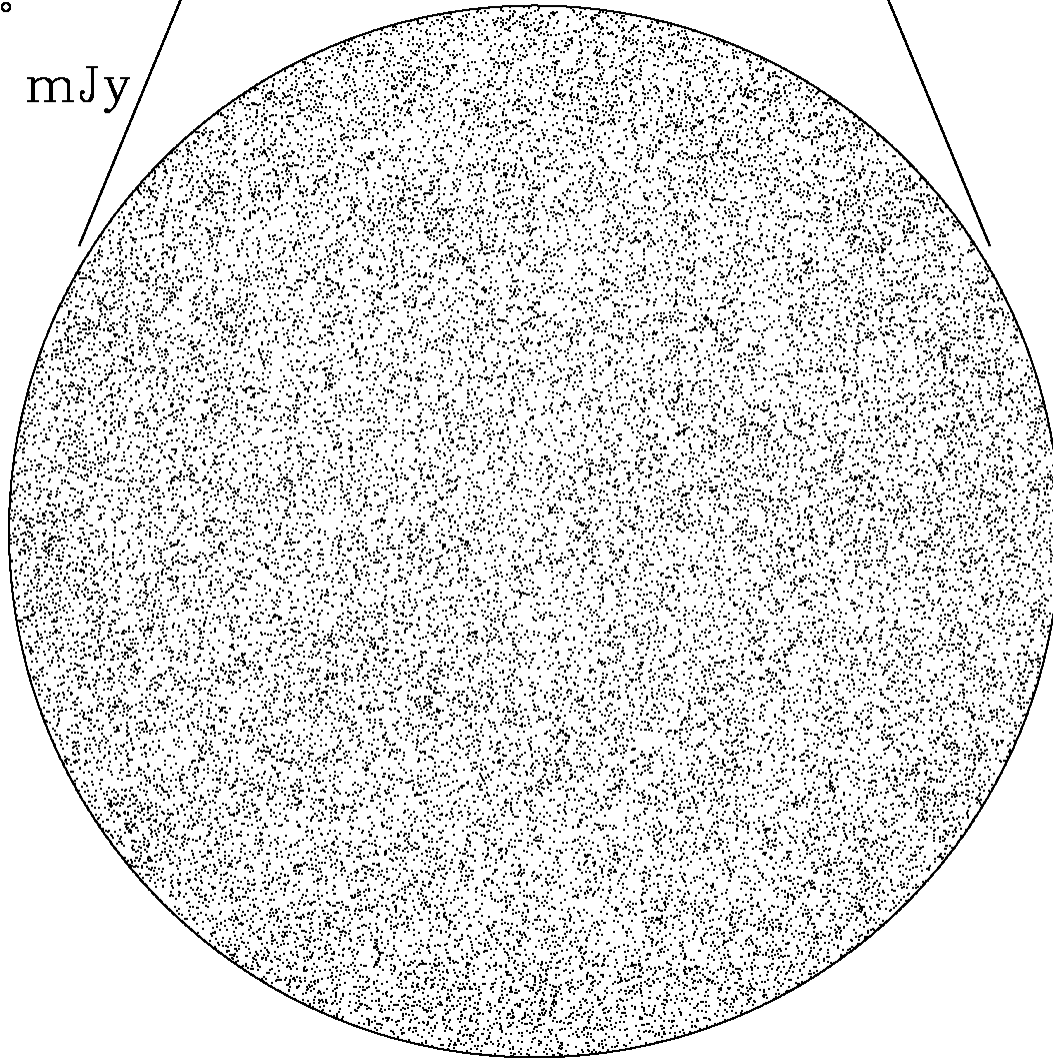
Radio Galaxies: NVVS

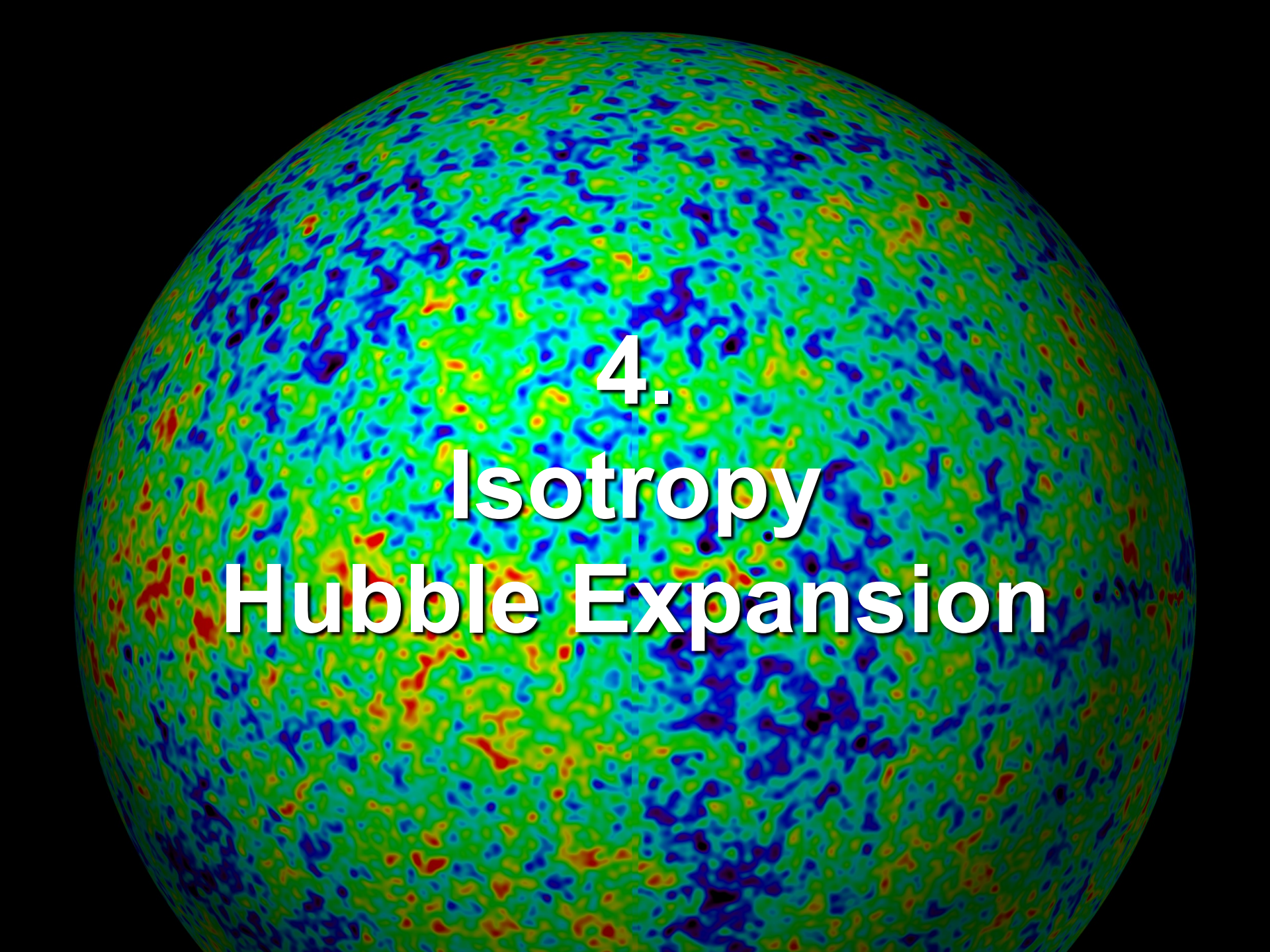
$\delta > -40^\circ$
 $S > 100$ mJy



Radio Galaxies: NVVS

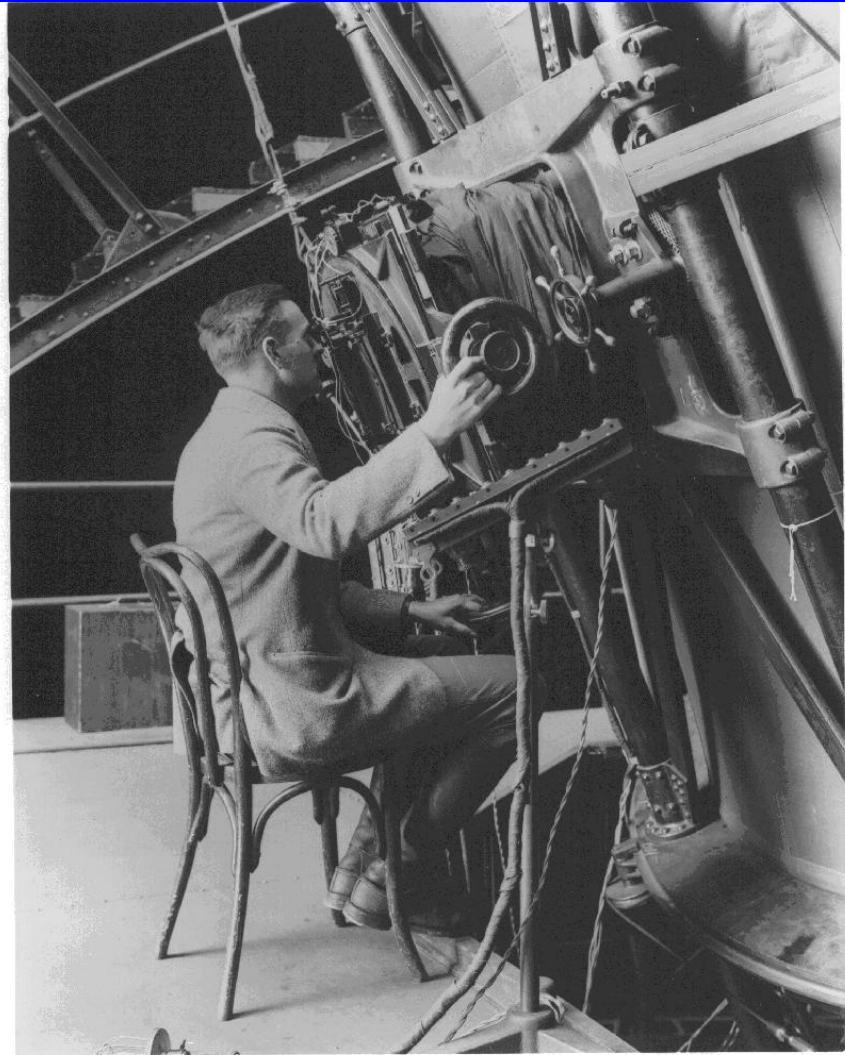
$\delta > +75^\circ$
 $S > 2.5 \text{ mJy}$





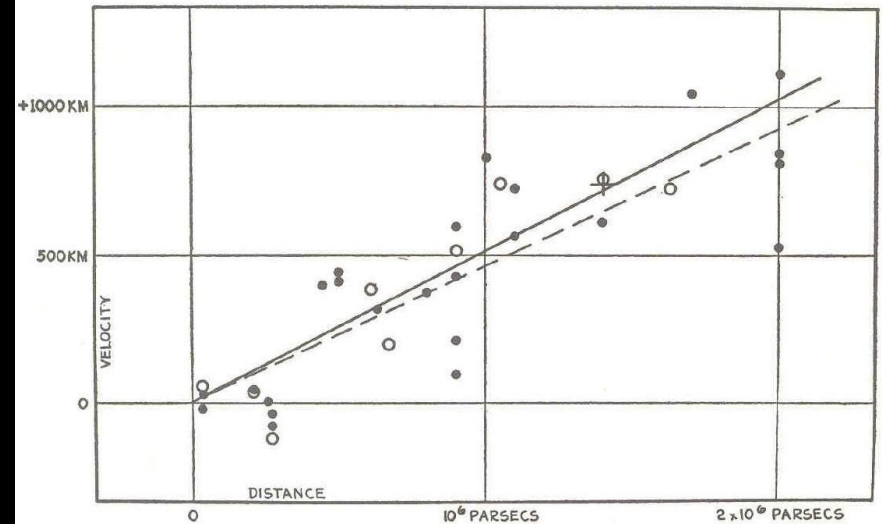
4.
Isotropy
Hubble Expansion

Hubble Expansion



Edwin Hubble

(1889-1953)

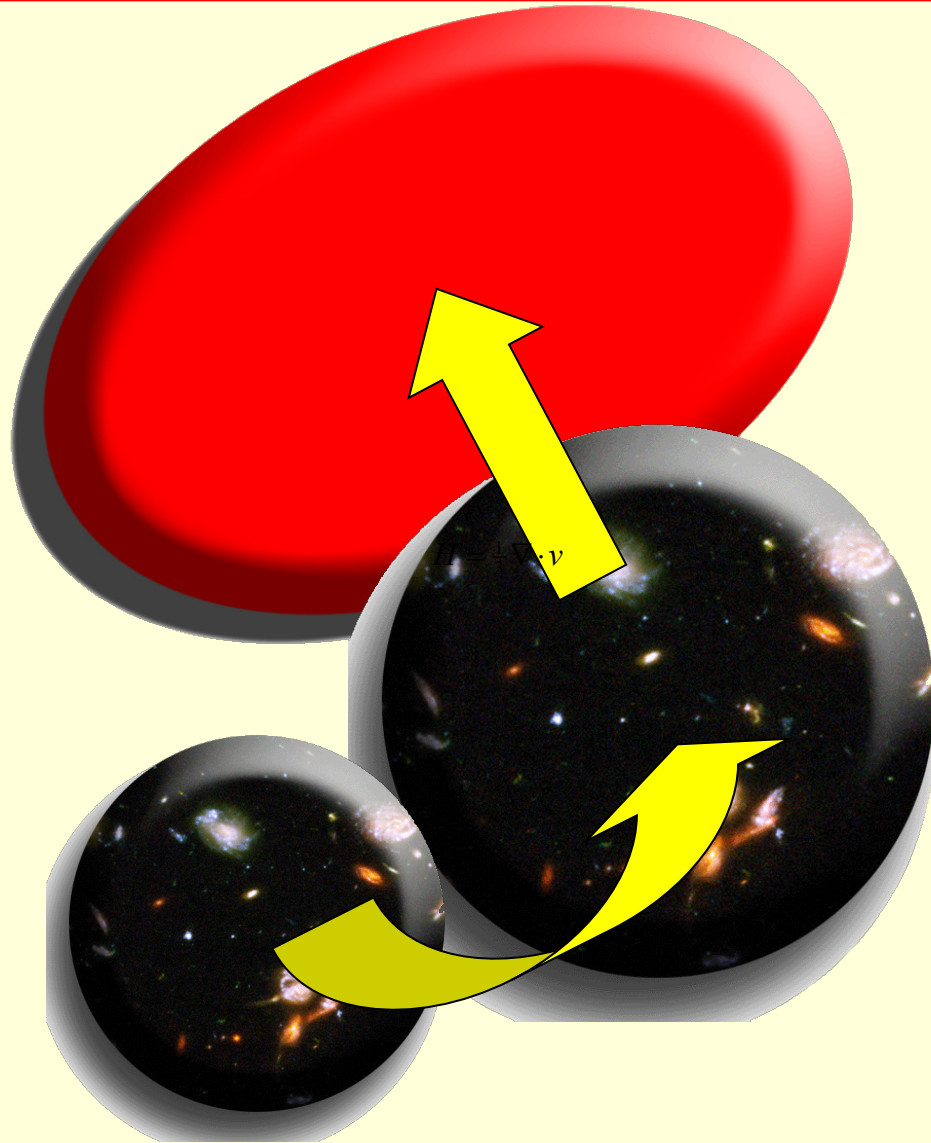


$$v = H r$$

Hubble Expansion

Deformation

Cosmic Volume Element



The evolution of a fluid element on its path through space may be specified by its velocity gradient:

$$\frac{1}{a} \frac{\partial v_i}{\partial x_j} = \frac{1}{3} \theta \delta_{ij} + \sigma_{ij} + \omega_{ij}$$

in which

θ : velocity divergence

➡ contraction/expansion

σ : velocity shear

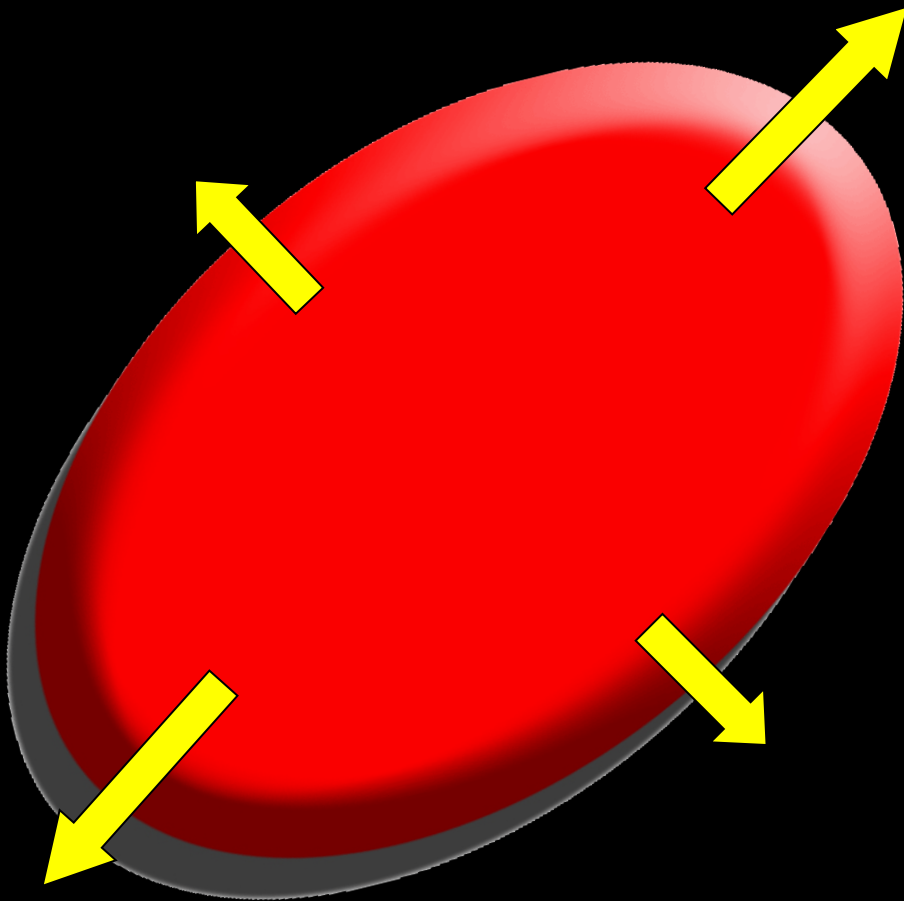
➡ deformation

ω : vorticity

➡ rotation of element

Deformation

Cosmic Volume Element



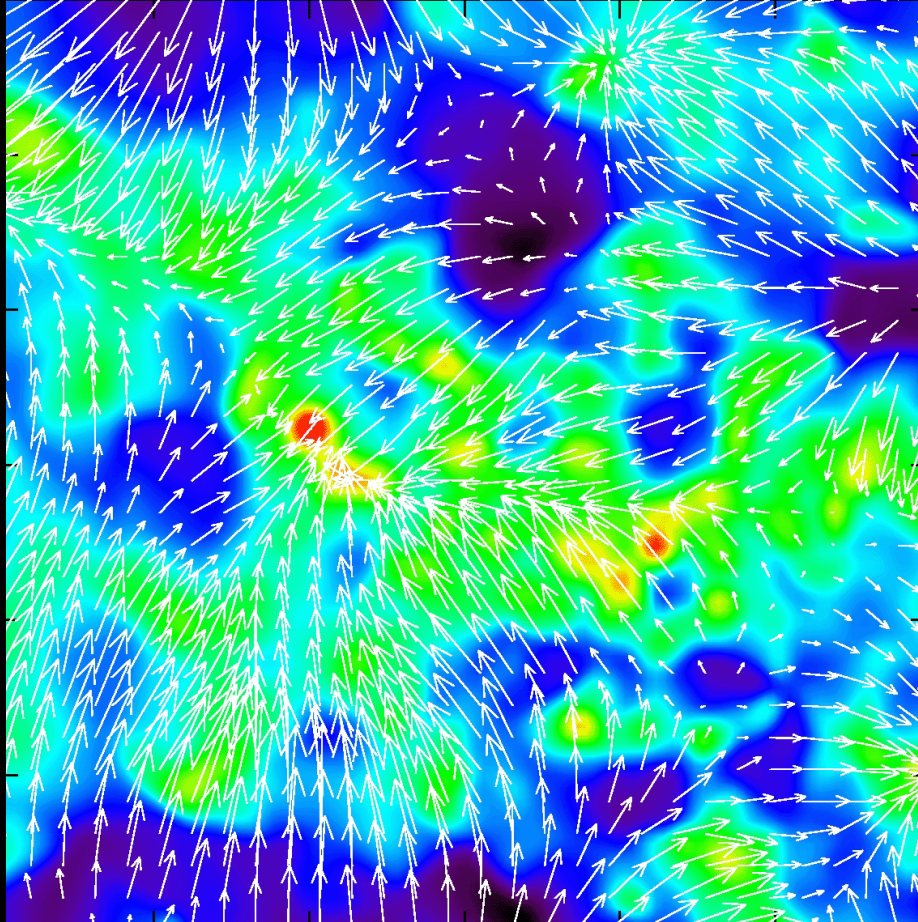
Global Anisotropic expansion/contraction

Anisotropic Relativistic Universe Models:
Bianchi I-IX Universe models

- expand anisotropically
- have to be characterized by at least 3 Hubble parameters (expansion rate different in different directions)
- Only marginal claims indicate the possibility on the basis of CMB anisotropies

Deformation

Cosmic Volume Element



Local Anisotropic Flows: “fatal” attractions

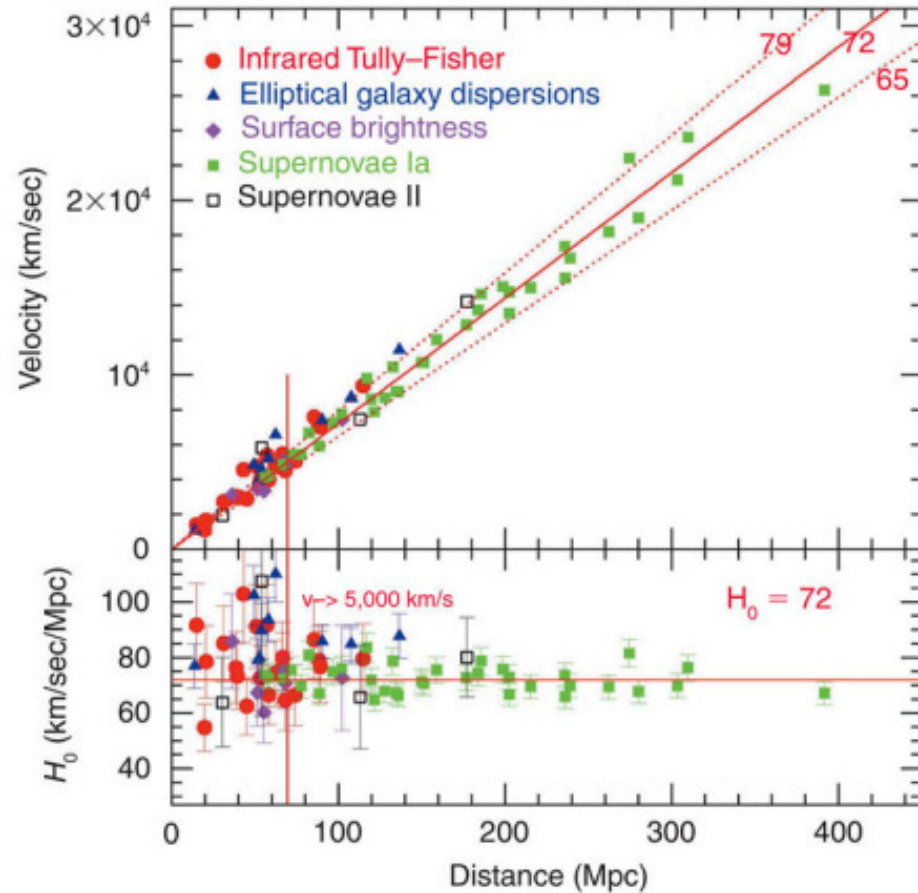
- In our local neighbourhood the cosmic flow field has a significant shear



- This shear is a manifestation of
 - infall of our Local Group into the Local Supercluster
 - motion towards the Great Attractor
 - possibly motion towards even larger mass entities: Shapley concentration
Horologium supercluster

Deformation

Cosmic Volume Element



B
 (Wendy L. Freedman, Observatories of the Carnegie Institution of Washington, and NASA)

Global Hubble Expansion

Observations over large regions of the sky, out to large cosmic depth:

- the Hubble expansion offers a very good description of the actual Universe
- the Hubble expansion is the same in whatever direction you look: isotropic

Hubble flow:

$$H = \frac{1}{3} \nabla \cdot \mathbf{v}$$

Pure expansion/contraction



Homogeneity

Prime Evidence Homogeneous Universe:

1) **The spatial galaxy distribution in galaxy redshift surveys:**

largest structures seen are $\sim 100\text{-}200$ Mpc

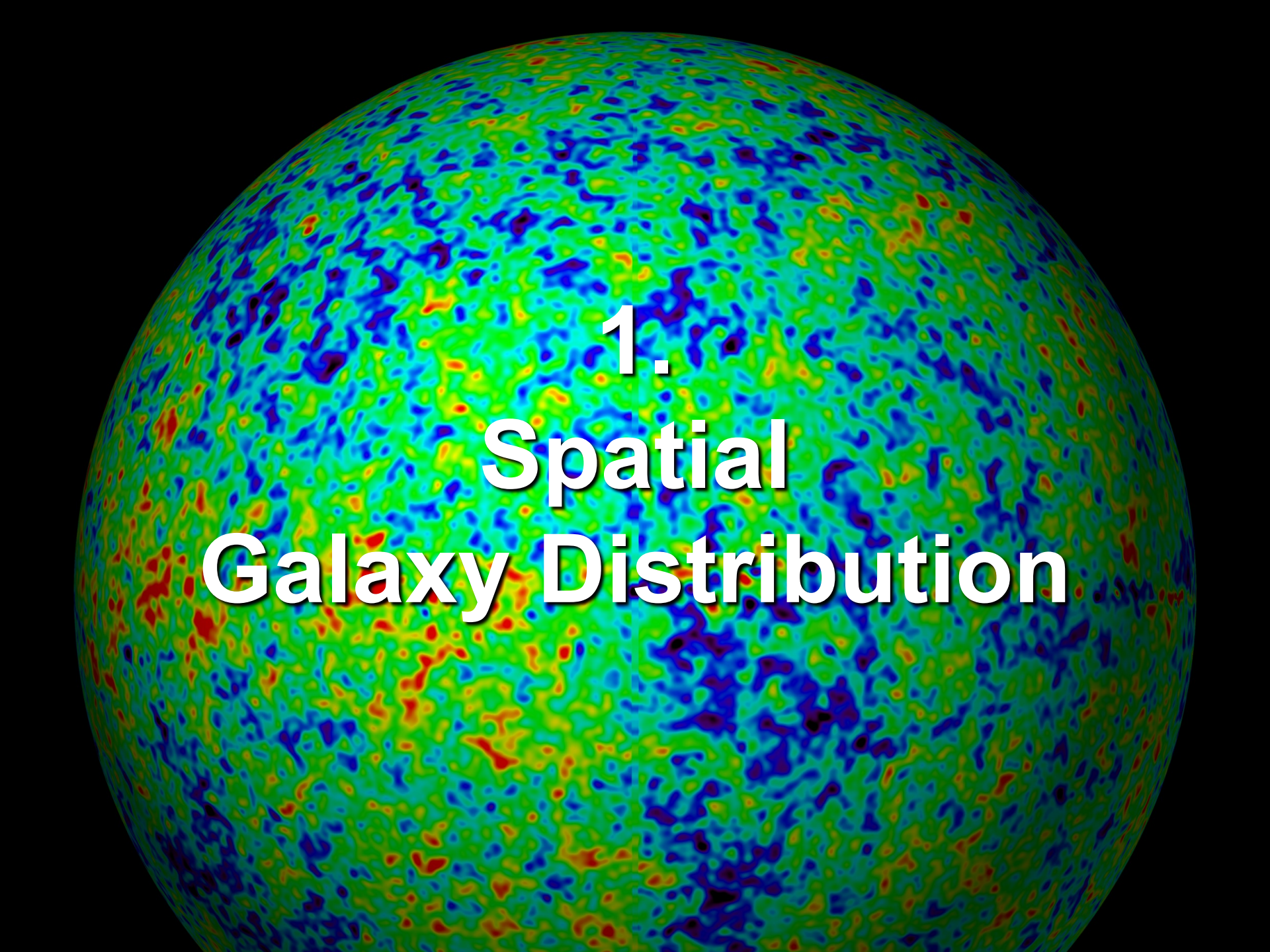
2) **Galaxy number counts:** $N(m) \sim 10^{0.6m}$

3) **Scaling of the Projected galaxy distribution:**

clustering with depth galaxy sample

4) **Cosmic Dipole:**

the convergence of the Local Group cosmic acceleration vector

A large, colorful sphere with a complex, noisy pattern of colors (red, orange, yellow, green, blue, purple) on a black background. The sphere is centered in the frame and appears to be a map of a galaxy distribution. Overlaid on the sphere is the text "1. Spatial Galaxy Distribution" in a bold, white, sans-serif font.

1.
Spatial
Galaxy Distribution

Largest Cosmic Structures

A clear obvious test for homogeneity of the Universe is to make a map of the matter distribution in the Universe and identify

- whether there is structure:
 - yes, obvious: stars, galaxies, clusters, superclusters
- what the size is of the largest structures in the Universe, ie.
 - is there a scale beyond which the Universe tends to converge to uniformity
 - or, do you see ever larger structures as you probe deeper into the Universe

The best probe for studying the Universe is (still) the distribution of galaxies

- note: in 2007 the first map of the 3-D dark matter distribution has been produced on the basis of a meticulous gravitational (weak) lensing study

Galaxy Redshift Distribution

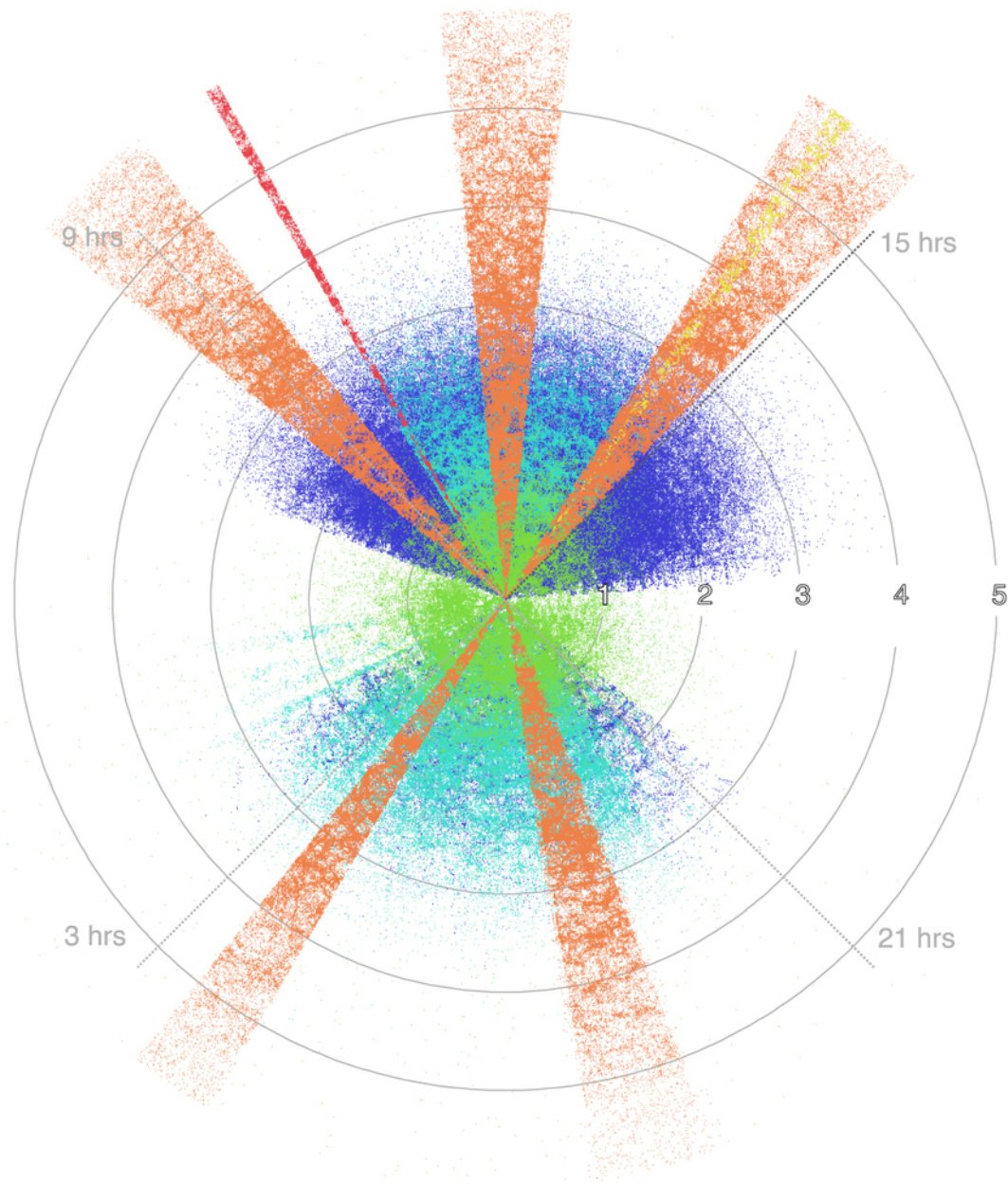
Mapping the 3-D galaxy distribution is not entirely trivial:

- We are still mostly confined to the “local” cosmic vicinity – ie. up to 500 Mpc – as galaxies quickly become too faint for a practically feasible distance estimate
- Even within our local neighbourhood it is too cumbersome to determine the real “physical” distance r of galaxies to a decent level of accuracy

- The Hubble expansion of the Universe is of great help:

Hubble relation: $c z = H r$

- Hubble parameter H_0 : 71 km/s/Mpc
- Redshift: accurately and relatively easily determined from spectrum of galaxy
- Once good spectrographs became available towards end 1970s, major mapping campaigns got started (see next page)



DEEP2



zCOSMOS



GAMA



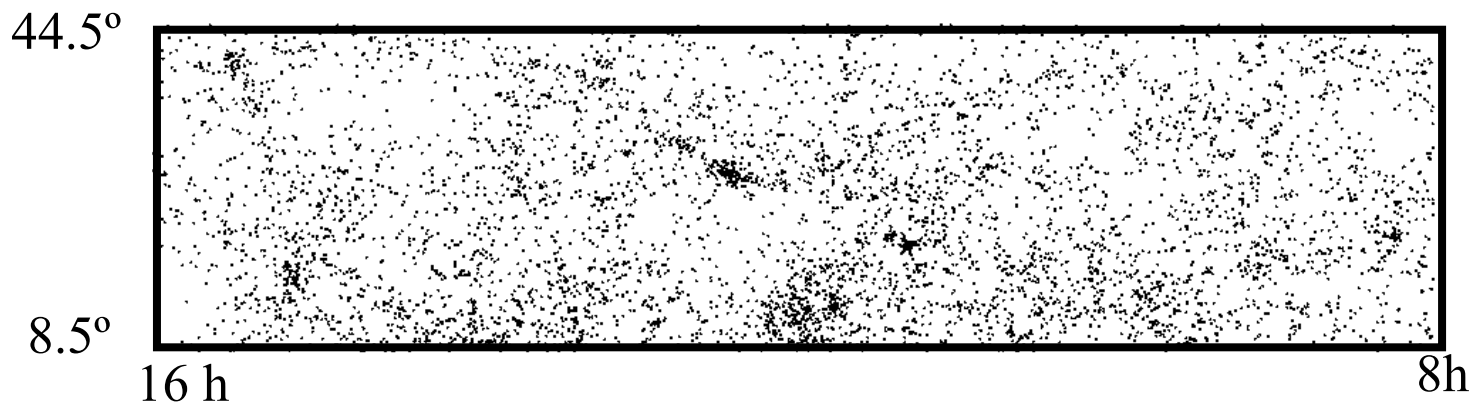
2dFGRS



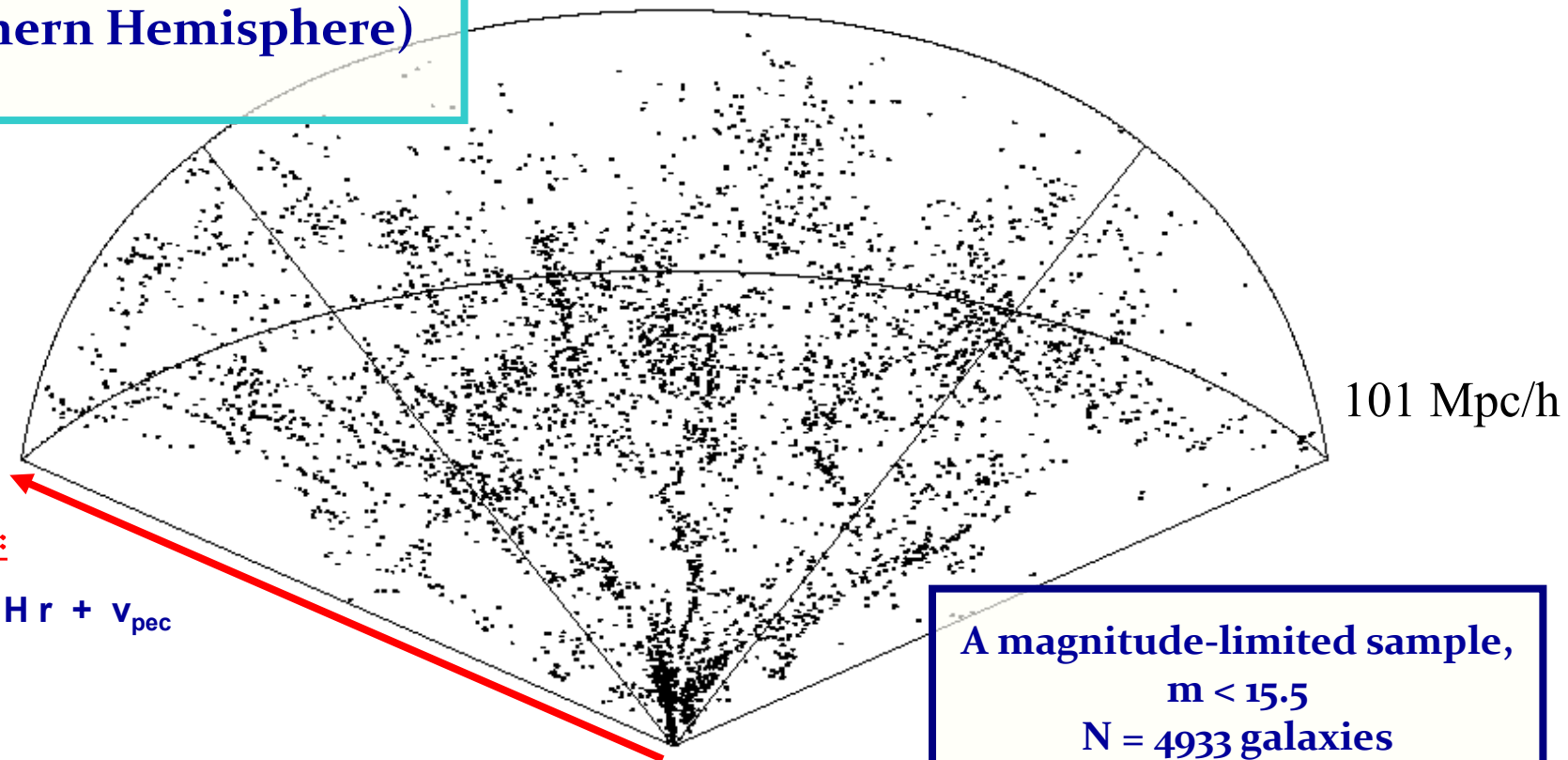
SDSS DR9



6dFGS



**CfA2 redshift survey
(Northern Hemisphere)**



**A magnitude-limited sample,
 $m < 15.5$
 $N = 4933$ galaxies**

REDSHIFT SURVEY CAMPAIGNS

Over the past two decades we have witnessed a paradigm shift in our perception of the Megaparsec scale structure in the Universe. As increasing elaborate galaxy redshift surveys charted ever larger regions in the nearby Universe, an intriguingly complex and salient foamlike network came to unfold and establish itself as the quintessential characteristic of the cosmic matter and galaxy distribution:

THE COSMIC WEB

Galaxies are grouped into a Weblike network of galaxies:

- **Sheets/Walls**
- **Filaments**
- **Clusters**
- **Voids**

These elements are spatially organized in a

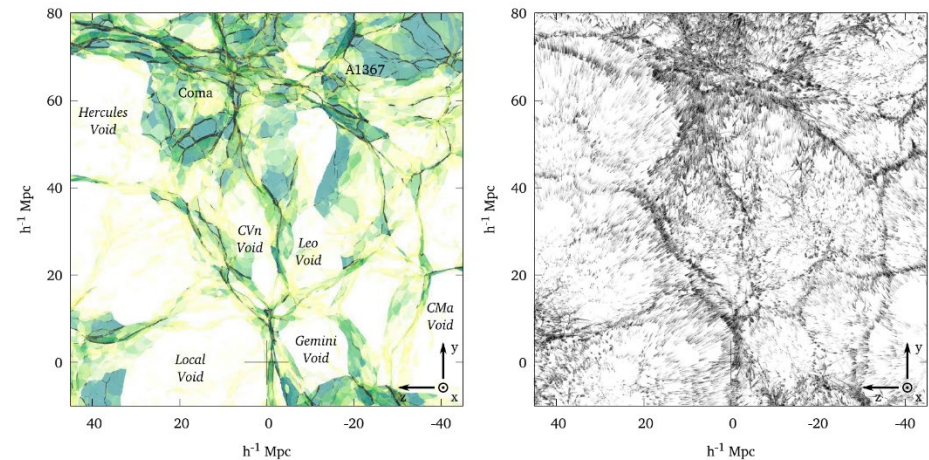
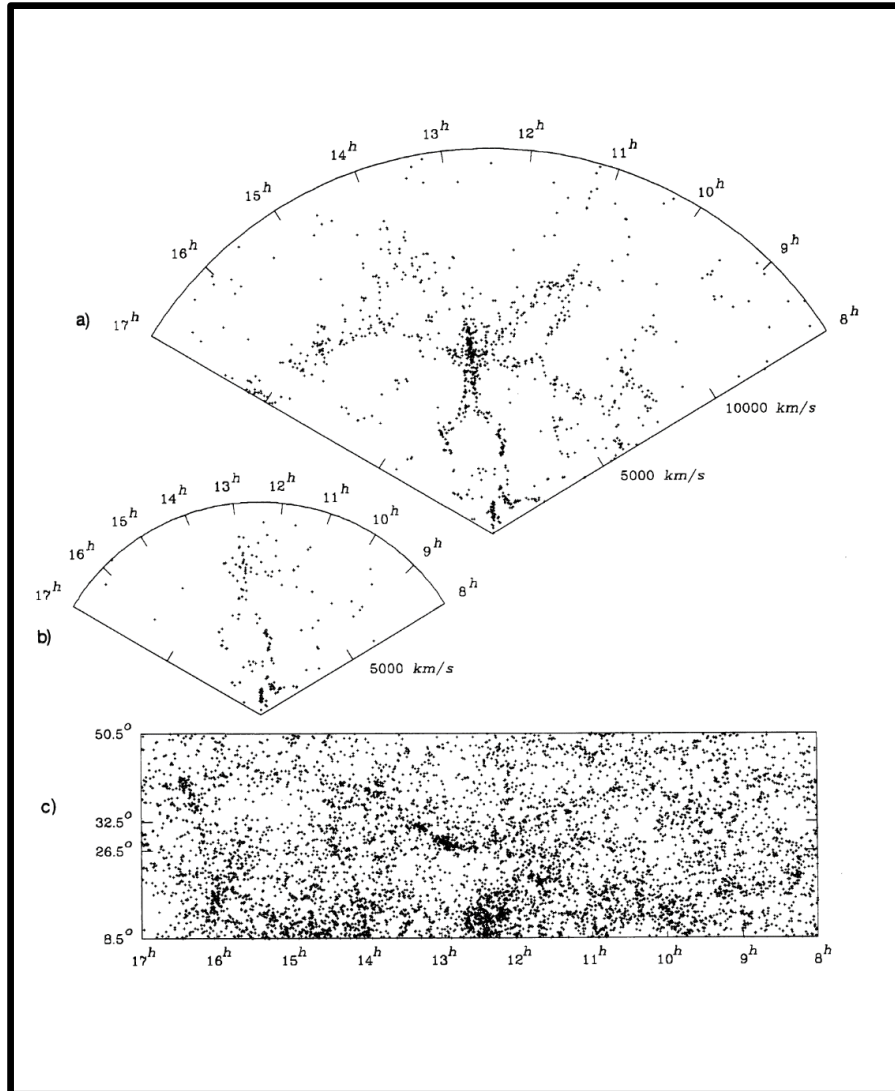
- **Non-trivial stochastic pattern of filaments and walls:**
filaments marking the dense ridges of walls, joining up at high density junctions, sites of rich clusters
- **Walls, filaments and clusters framed in a spatial network in which their spatial distribution is dictated by their location on the surface of roundish and near-empty voids**

Cosmic Web: Stickman & Discovery

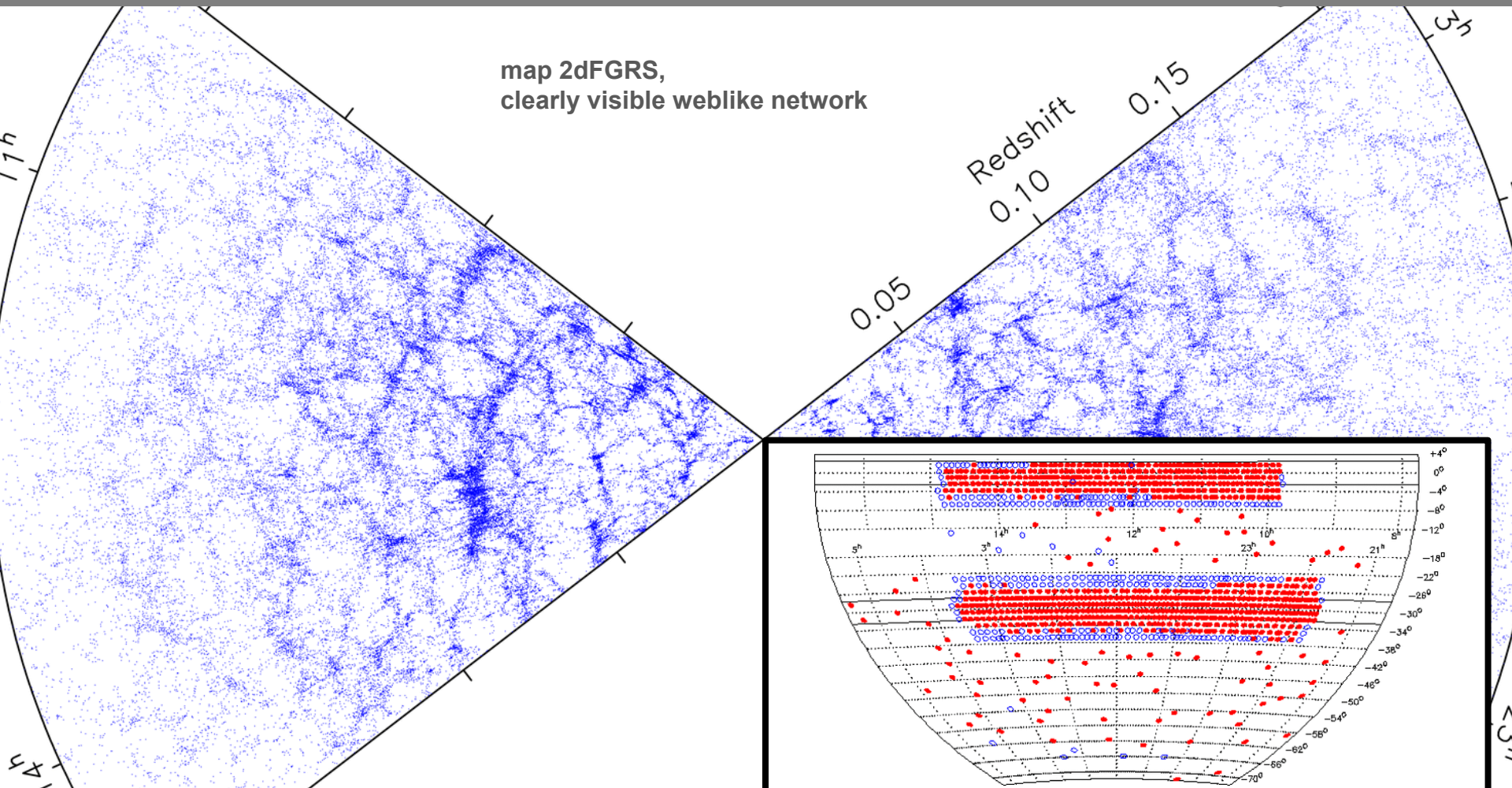
deLapparent, Geller & Huchra, 1986:

“a slice of the Universe”

Voids appear to be an integral part of a complex weblike arrangement of galaxies



2dFGRS Galaxy Survey

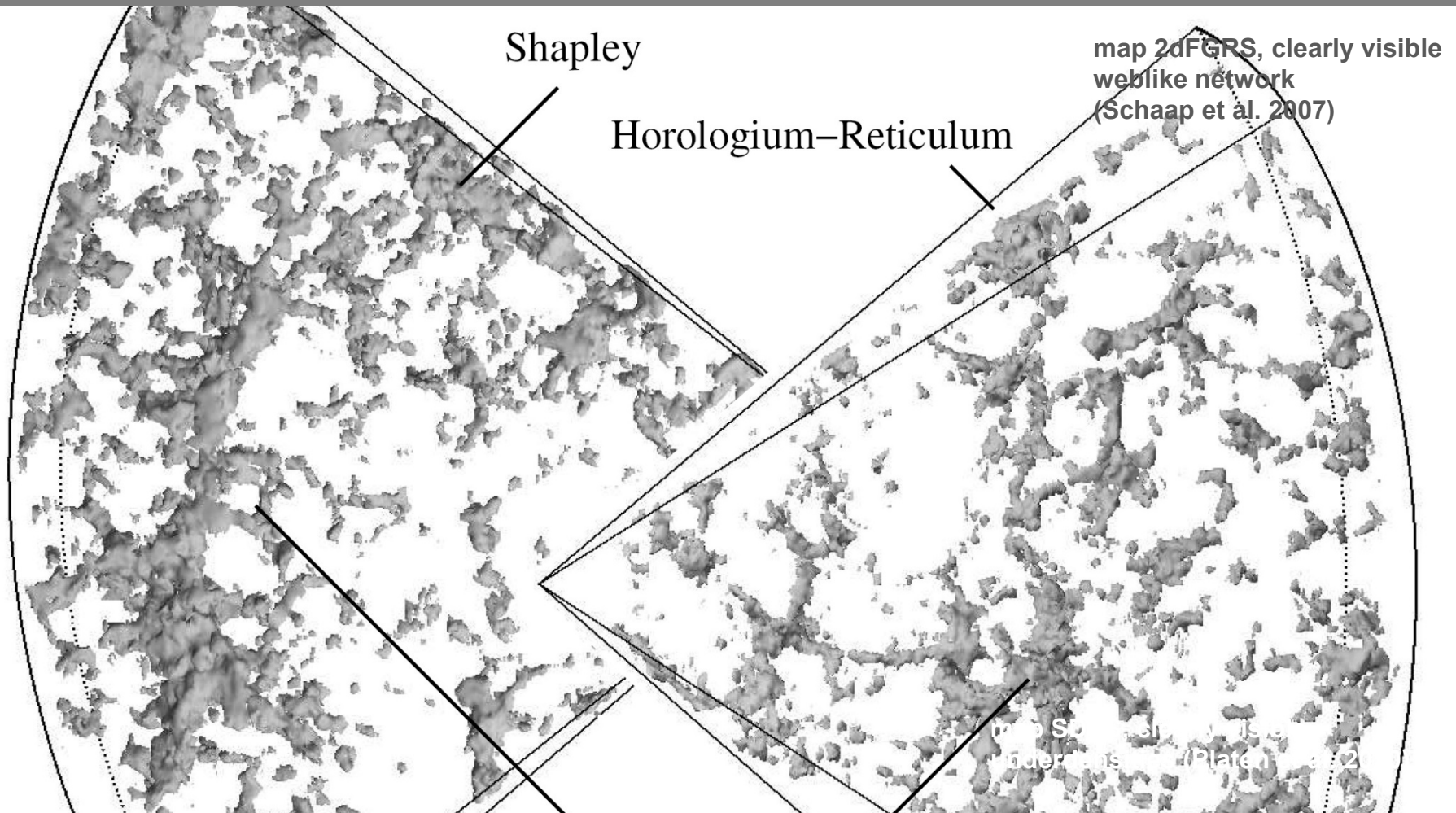


with the advent of large galaxy redshift surveys

– LCRS, 2dFGRS, SDSS, 2MRS –

voids have been recognized as one of the quintessential components of the Cosmic Web

2dFGRS Galaxy Survey

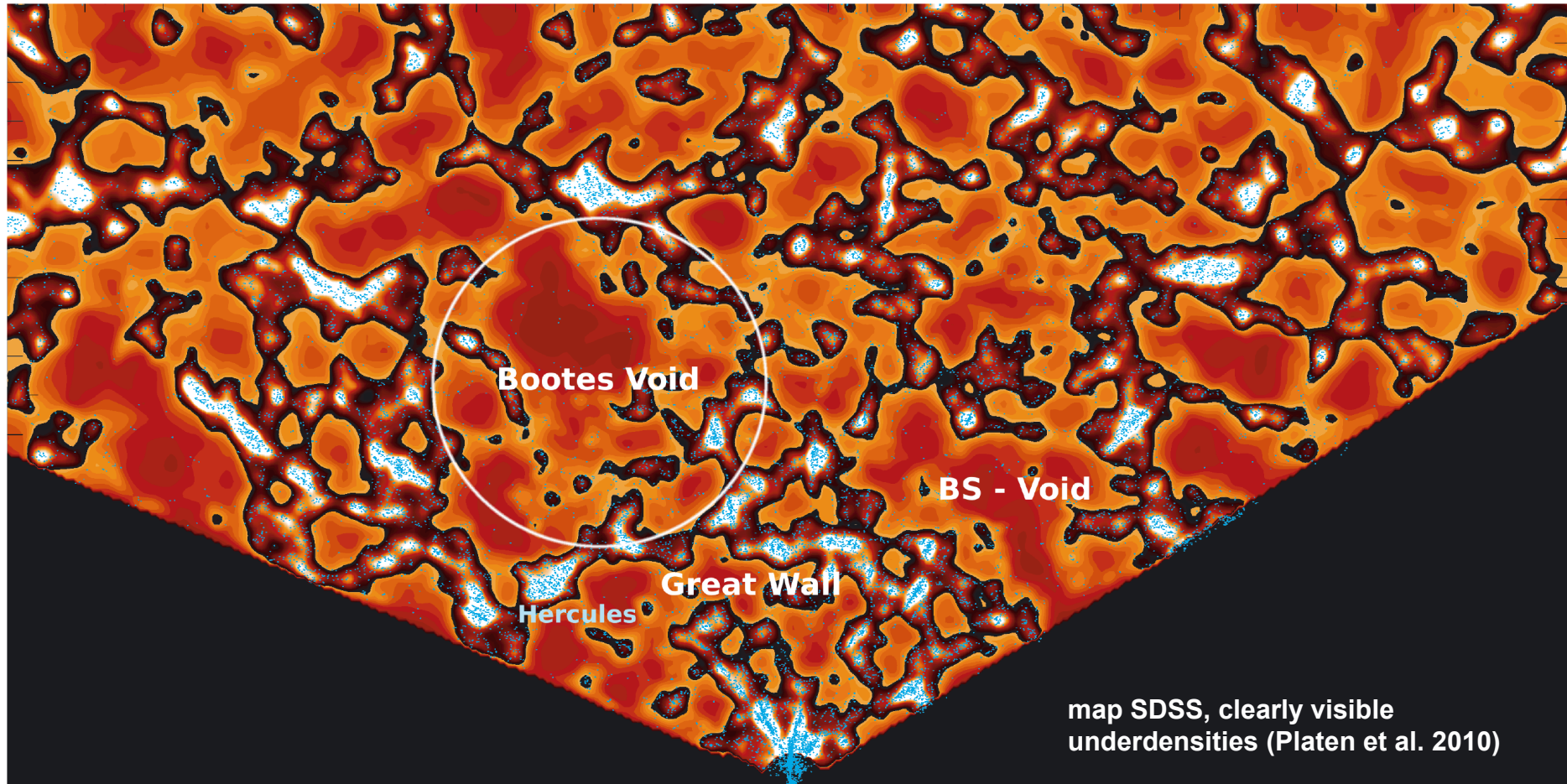


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SDSS Galaxy Survey

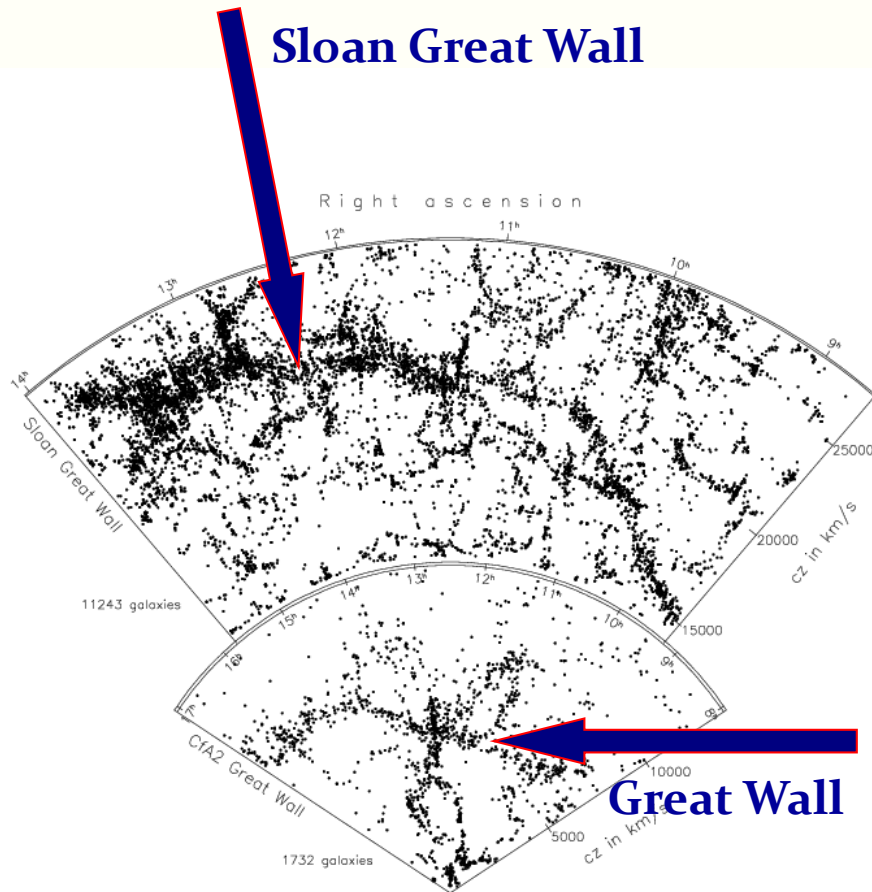


with the advent of large galaxy redshift surveys

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Largest Structures in the Universe

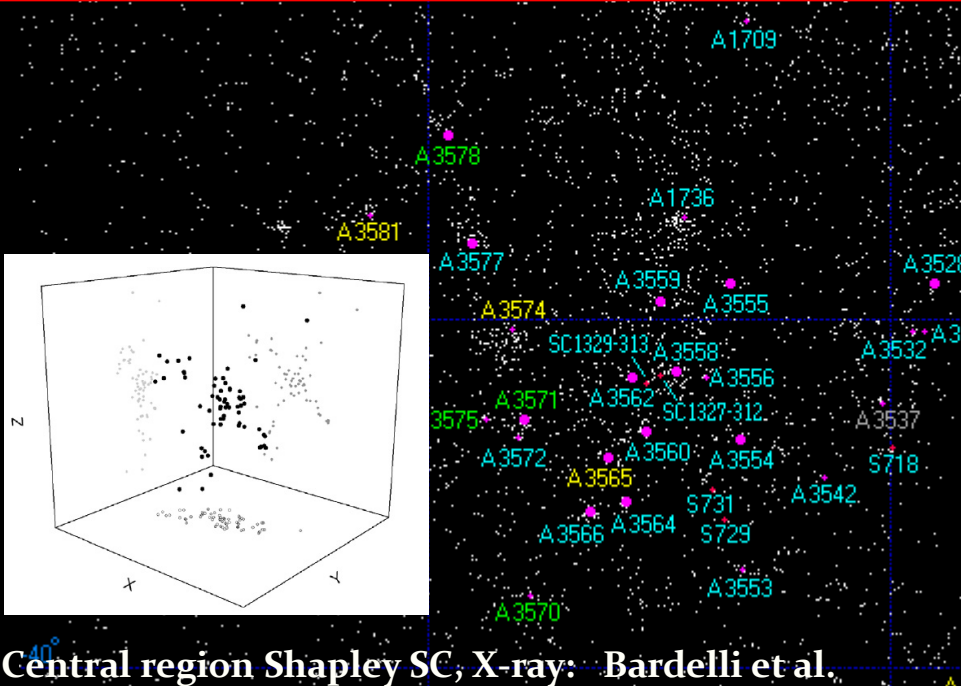


What are the largest structures that we can find in the galaxy survey maps ?

- In CfA2 a “Great Wall” of galaxies was found, dimension $\sim 180h^{-1}$ Mpc
- In SDSS an even more massive and immense structure has been identified: the “Sloan Great Wall”
- However, to some extent the Great Walls are standing out as a result of:
 - proximity to peak observ. selection funct.
 - redshift distortion effect

from: Gott et al. 2005

Largest Structures in the Universe

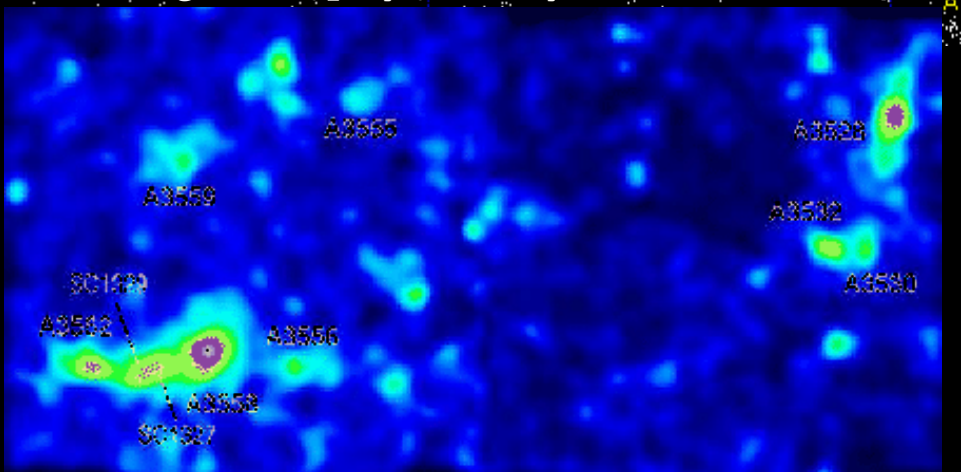


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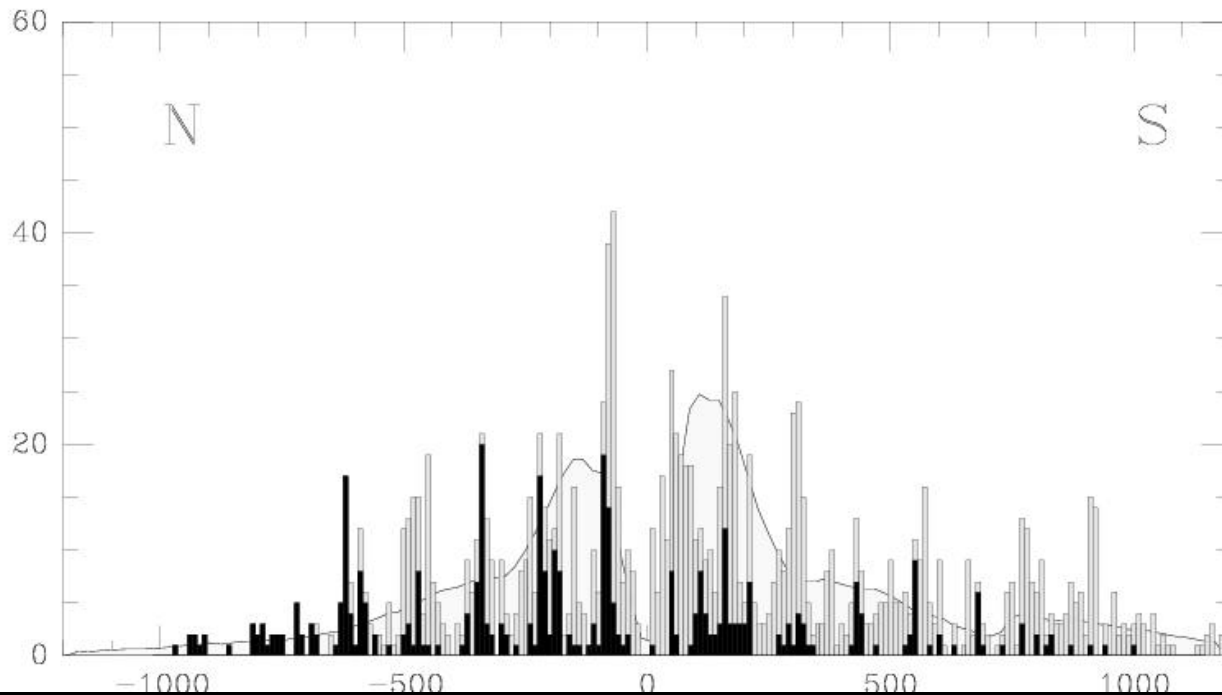
- The distribution of Clusters of Galaxies usually reveals the existence of larger scale patterns than that of galaxies.
- Clusters of galaxies tend to aggregate in superclusters, to be identified with the filaments and sheets we see in the galaxy distribution.
- The most massive superstructure in the Local Universe is the

Shapley Supercluster

- It contains 25-30 clusters of galaxies, of which at least 7 are richness I



Largest Structures in the Universe



A 1-D
skewer
through
the Cosmos

Deep pencil beam survey (Broadhurst et al):

A semi-regular pattern of redshift spikes along line of sight, indicating the passage of l.o.s. through sheets, filaments and clusters, most of which should be identified with “Great Walls”. While impressive, it also does not seem to reveal any structures larger than $\sim 120h^{-1}$ Mpc.

Does this conclude the argument on the scale of homogeneity of the Universe ?

Largest Cosmic Structures

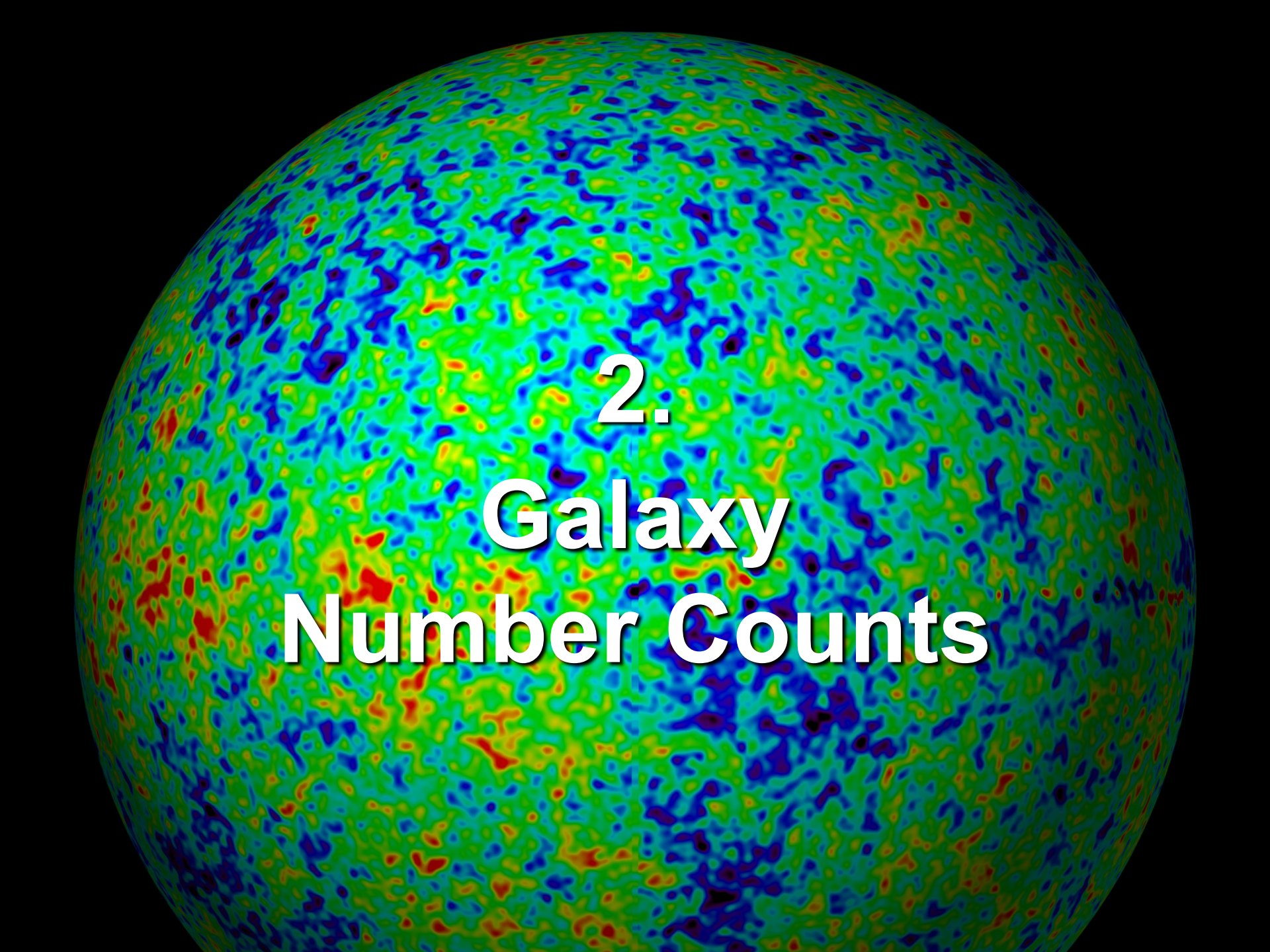
Cautious Conclusion:

While galaxy redshift surveys mapped ever larger regions of the nearby Cosmos and revealed outstanding and intriguing structures got revealed, there is

NO INDICATION

for large inhomogeneities on scales $> 150\text{-}200h^{-1}$ Mpc
(even with 2dFGRS and SDSS pressing their effective survey depth beyond this scale and therefore represent “fair samples” of the Universe).

Assumption of global cosmological homogeneity appears to be corroborated by our “cosmographic” maps.

A large, colorful sphere with a complex, noisy pattern of colors (red, orange, yellow, green, blue, purple) on a dark background. The sphere is centered in the frame and appears to be a map of galaxy number counts. The text is overlaid on the sphere.

2.
Galaxy
Number Counts

Galaxy Number Counts

Number counts of galaxies,
ie. the number of galaxies with apparent magnitude m or higher in a certain region of the sky, contain potentially a large amount of information on the structure of the Universe.

Hubble, in “The Realm of the Nebulae” (1936)
used counts of galaxies to the limit of the Mount Wilson 100-inch telescope to demonstrate that the distribution of galaxies is

HOMOGENEOUS

on the largest cosmological scales

Galaxy Number Counts

Count the number of galaxies in a magnitude limited survey:

- all galaxies brighter than apparent magnitude m
- suppose a population of objects with intrinsic luminosity function

$$n(L)dL$$

Number density objects with $L \in [L, L + dL]$

- At a distance r , the flux S for a source of luminosity $L \in [L, L + dL]$

$$S = \frac{L}{4\pi r^2}$$

Galaxy Number Counts

Survey with limiting flux density S_{lim} :

- in terms of **apparent magnitude m** :

$$m = \text{cst.} - 2.5 \log_{10} S$$

- sources of luminosity L have a flux $> S$ out to **distance r_S** :

$$r_S = \left(\frac{L}{4\pi S} \right)^{1/2}$$

- Number of sources of intrinsic luminosity L brighter than S



sources within distance r_S and within solid angle Ω

Galaxy Number Counts

For a spatially homogeneous distribution of sources:

- Number of sources of intrinsic luminosity L brighter than S



sources within distance r_s , and within solid angle Ω :

$$N(\geq S, L)dL = \frac{\Omega}{3} r_s^3 N(L)dL = \frac{\Omega}{3} \left(\frac{L}{4\pi S} \right)^{3/2} n(L)dL$$

- The total number of sources with flux higher than S is found by integration over the luminosity function of sources,

$$N(\geq S) = \frac{\Omega}{3(4\pi)^{3/2}} S^{-3/2} \int L^{3/2} n(L)dL$$

Galaxy Number Counts

- Number of sources brighter than flux S :

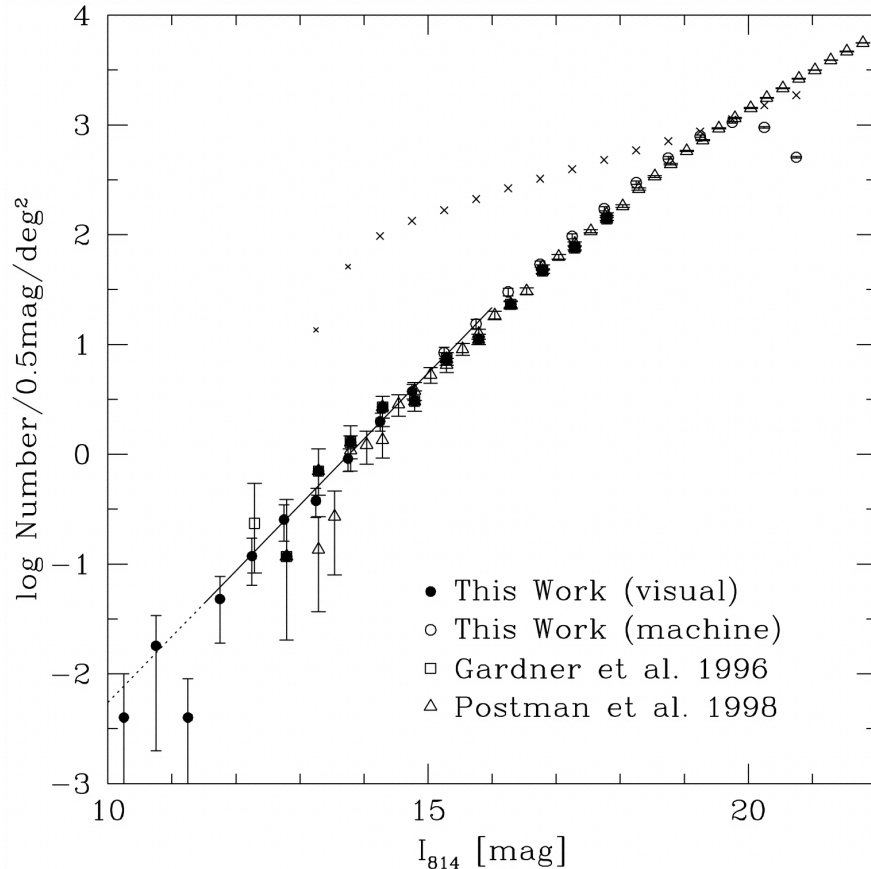
$$N(S) \approx S^{-3/2}$$

- Which in terms of magnitude m ,
translates into

$$N(m) \approx 10^{0.6m}$$

- Note that this is based upon the assumption of homogeneity !

Galaxy Number Counts



SDSS galaxy number counts:

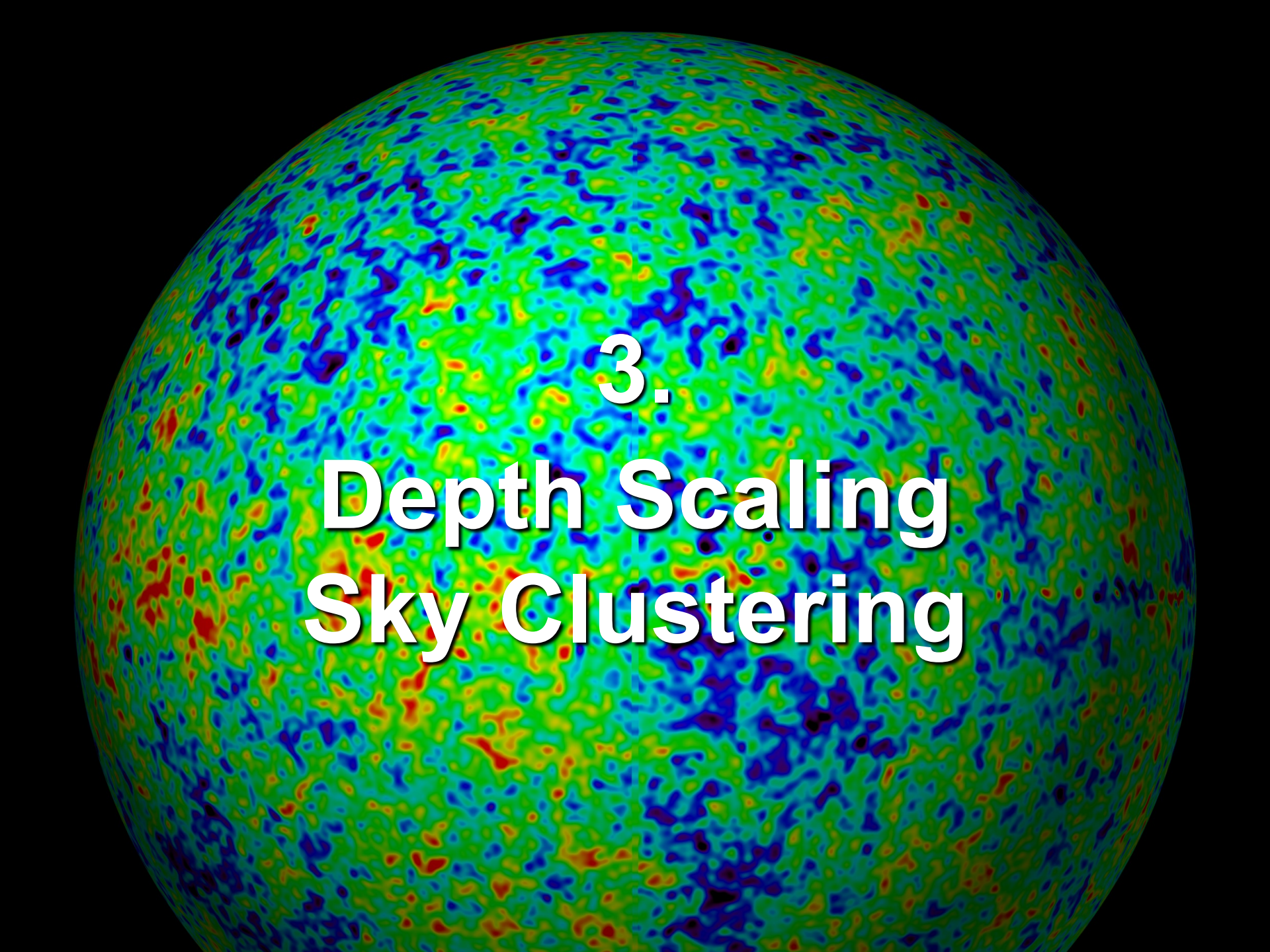
Impressive example of the accuracy of the $10^{0.6m}$ scaling for homogeneous Univ.

- **Number counts in many galaxy surveys indeed reveal the expected scaling**

$$N(m) \approx 10^{0.6m}$$

at (relatively) bright magnitudes.

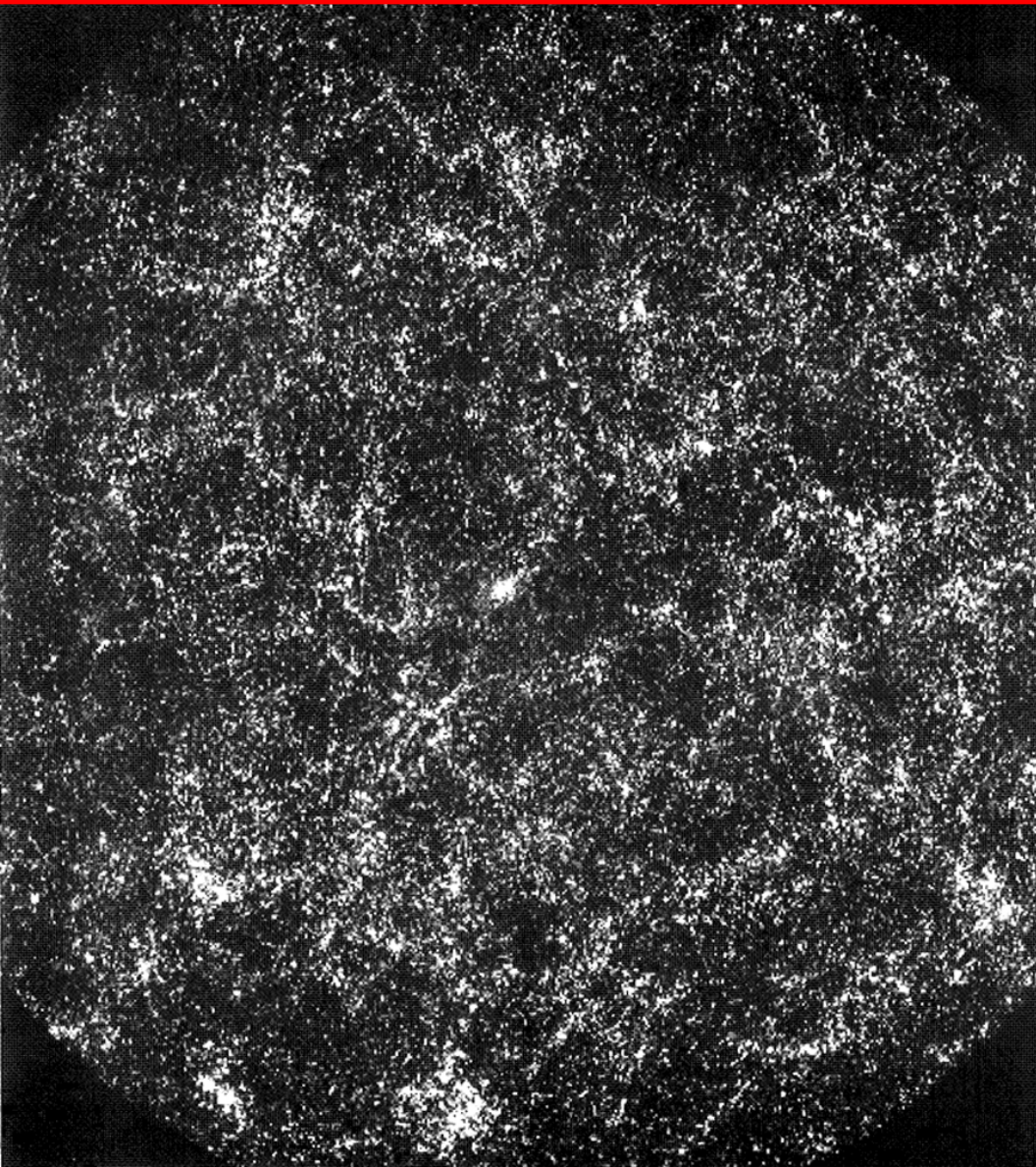
- **At faint magnitudes we are looking at galaxies at high distances, over which distances the effects of the curvature and expansion of the Universe assumes a dominant role.**

A colorful, noisy sphere representing a sky map, with a white number '3.' overlaid in the center. The sphere is covered in a complex pattern of colors including red, orange, yellow, green, cyan, blue, and purple, set against a black background.

3.

Depth Scaling Sky Clustering

Angular Clustering Scaling



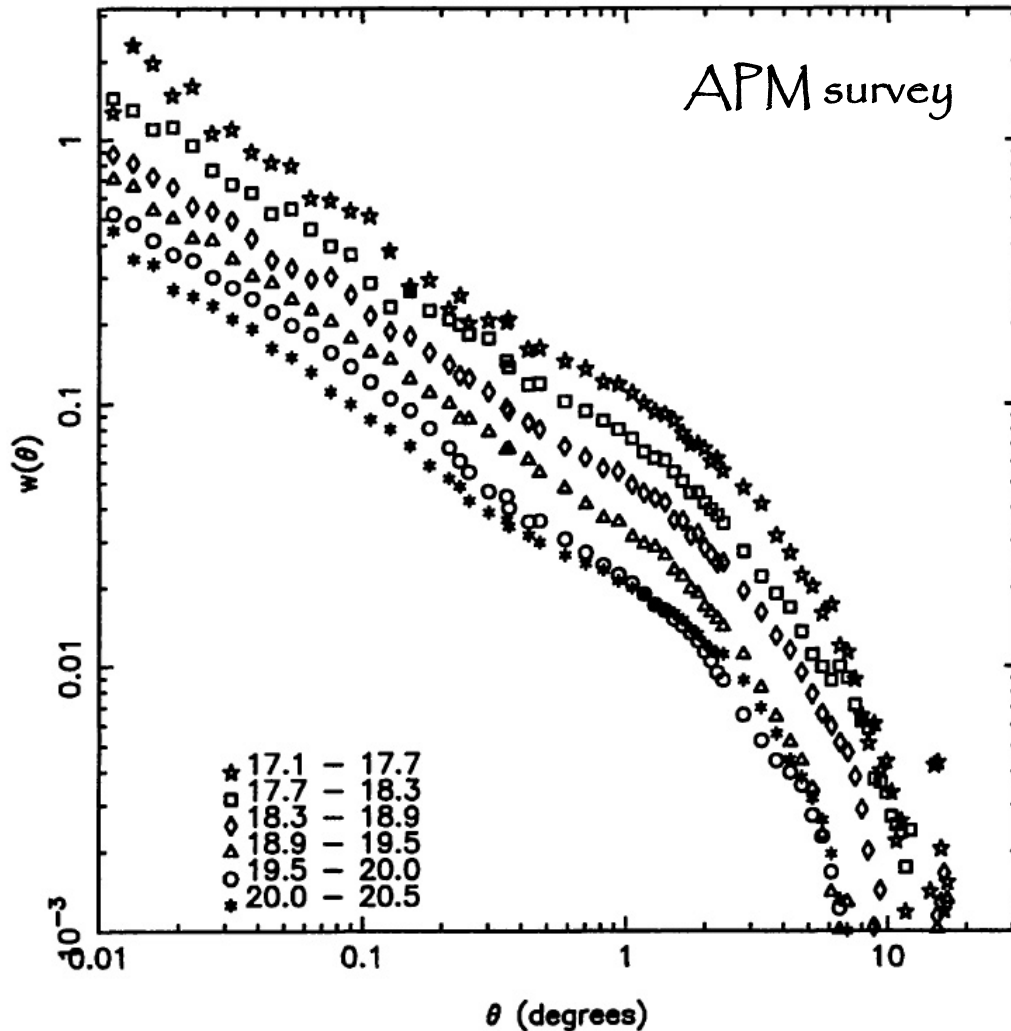
Galaxy sky distribution:

- Galaxies clustered, a projected expression of the true 3-D clustering
- Probability to find a galaxy near another galaxy higher than average (Poisson) probability
- Quantitatively expressed by 2-pt correlation function $w(\theta)$:

$$dP(\theta) = \bar{n}^2 \{1 + w(\theta)\} d\Omega_1 d\Omega_2$$

Excess probability of finding
2 gal's at angular distance θ

Angular Clustering Scaling



Two-point correlation function:

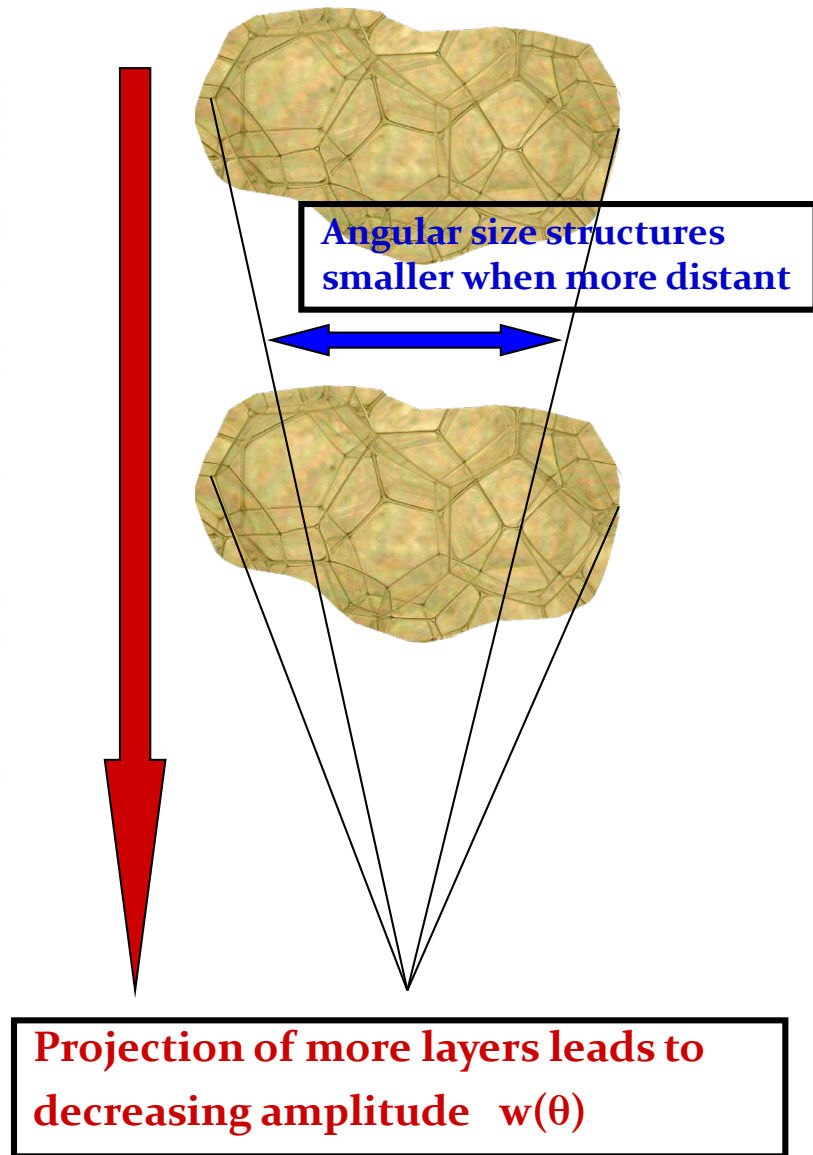
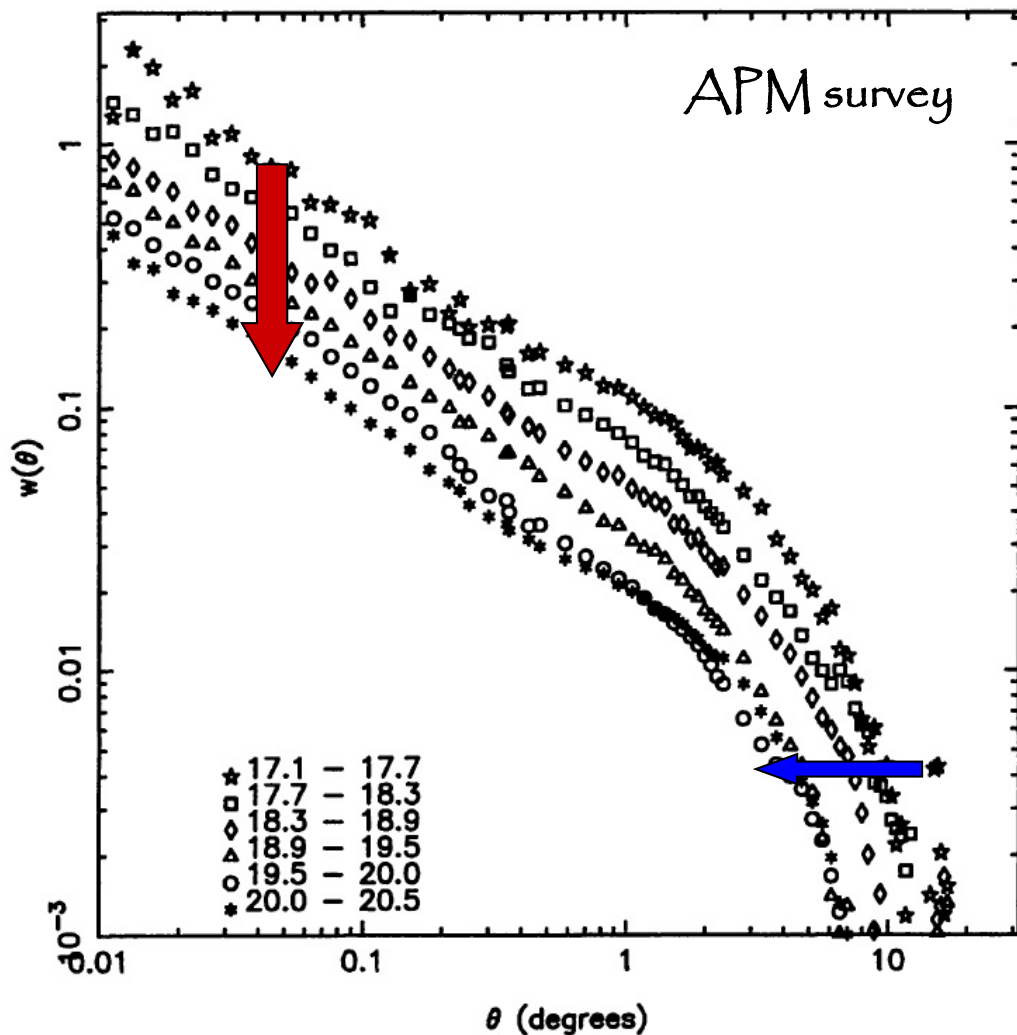
- small angles: power-law

$$w(\theta) = \left(\frac{\theta}{\theta_0} \right)^\gamma$$

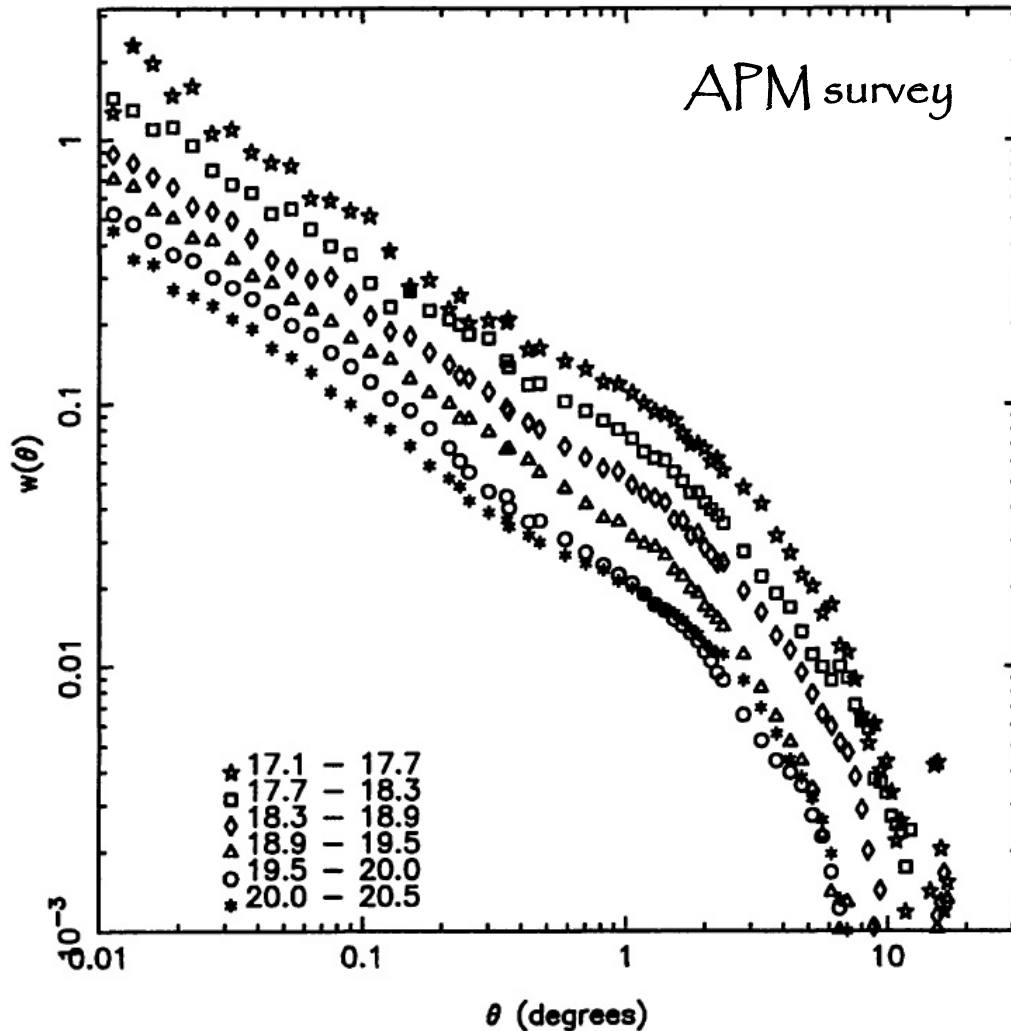
$$\gamma \approx -0.8$$

- large angles \longrightarrow o
ie. to homogeneity

Angular Clustering Scaling



Angular Clustering Scaling

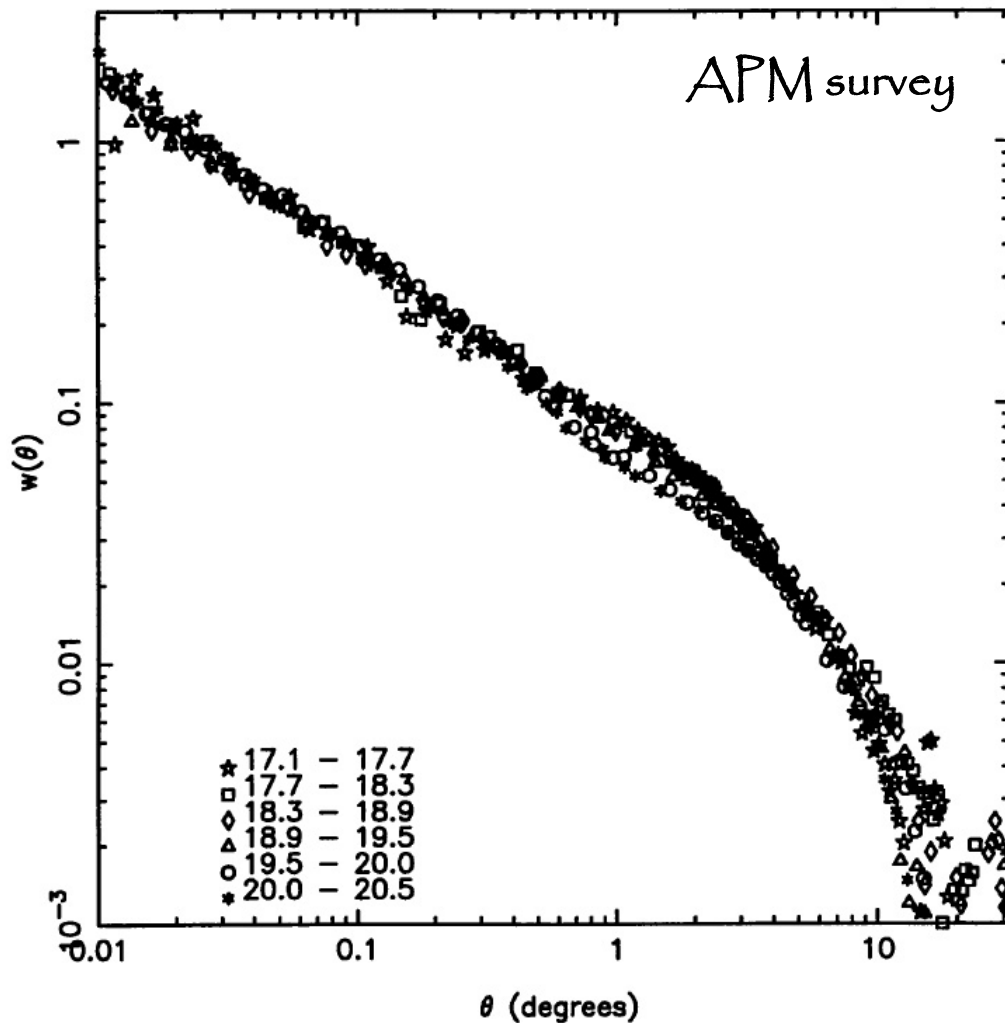


Angular size structures smaller
when more distant

Projection of
more layers
leads to
decreasing
amplitude $w(\theta)$

$$w(\theta, D_*) = \frac{1}{D_*} w(\theta D_*)$$

Angular Clustering Scaling



Angular size structures smaller
when more distant

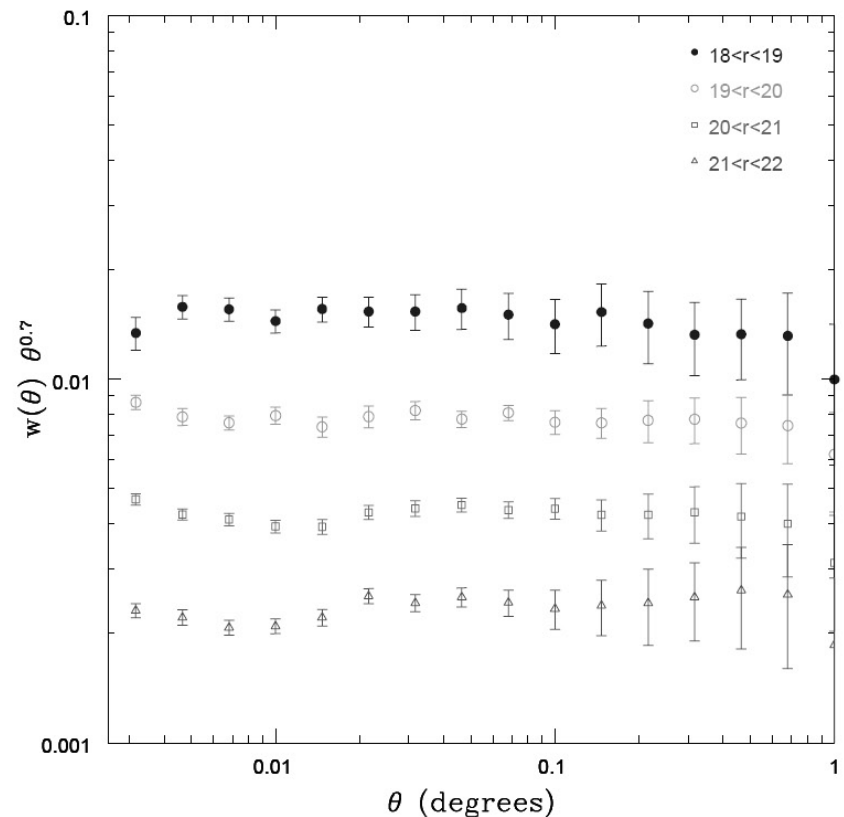
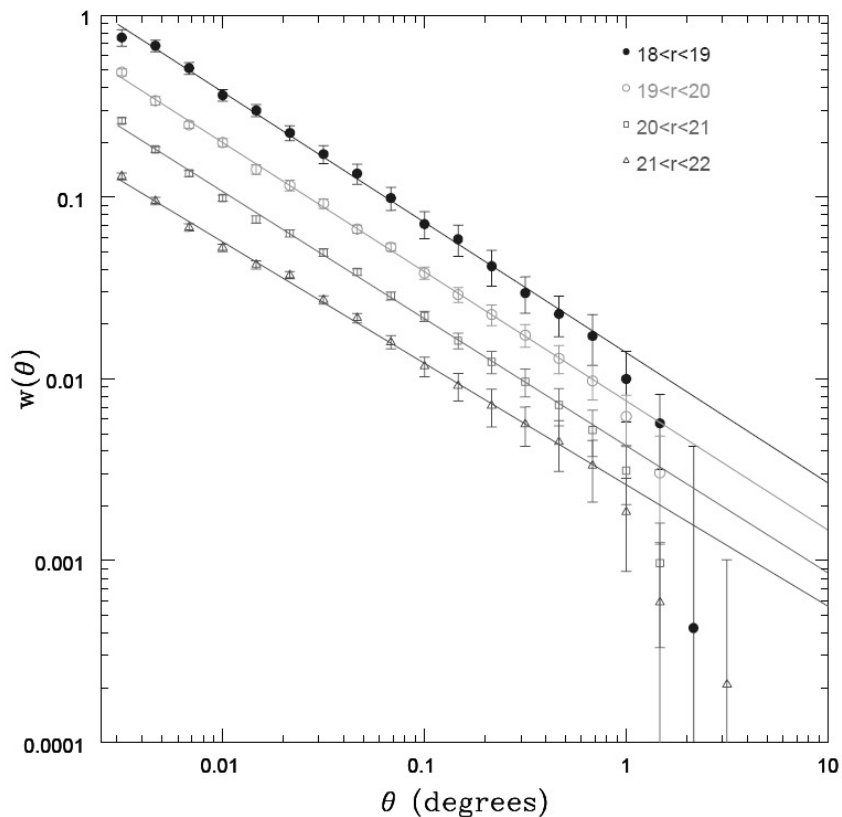
Projection of
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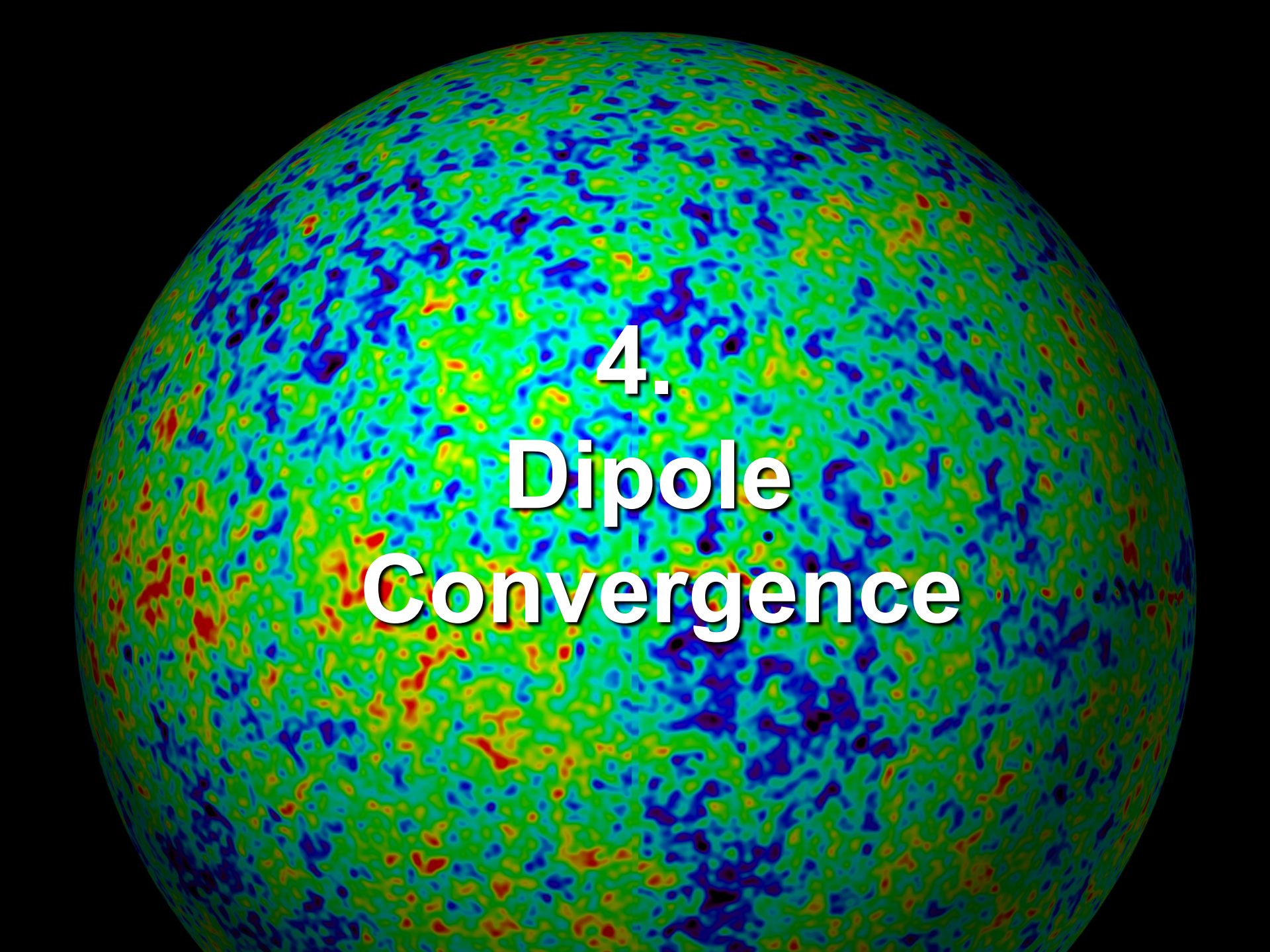
$$w(\theta, D_*) = \frac{1}{D_*} w(\theta D_*)$$

Angular Clustering Scaling

The angular scaling of $w(\theta)$ is found back to even fainter magnitudes in the SDSS survey ($m=22$)

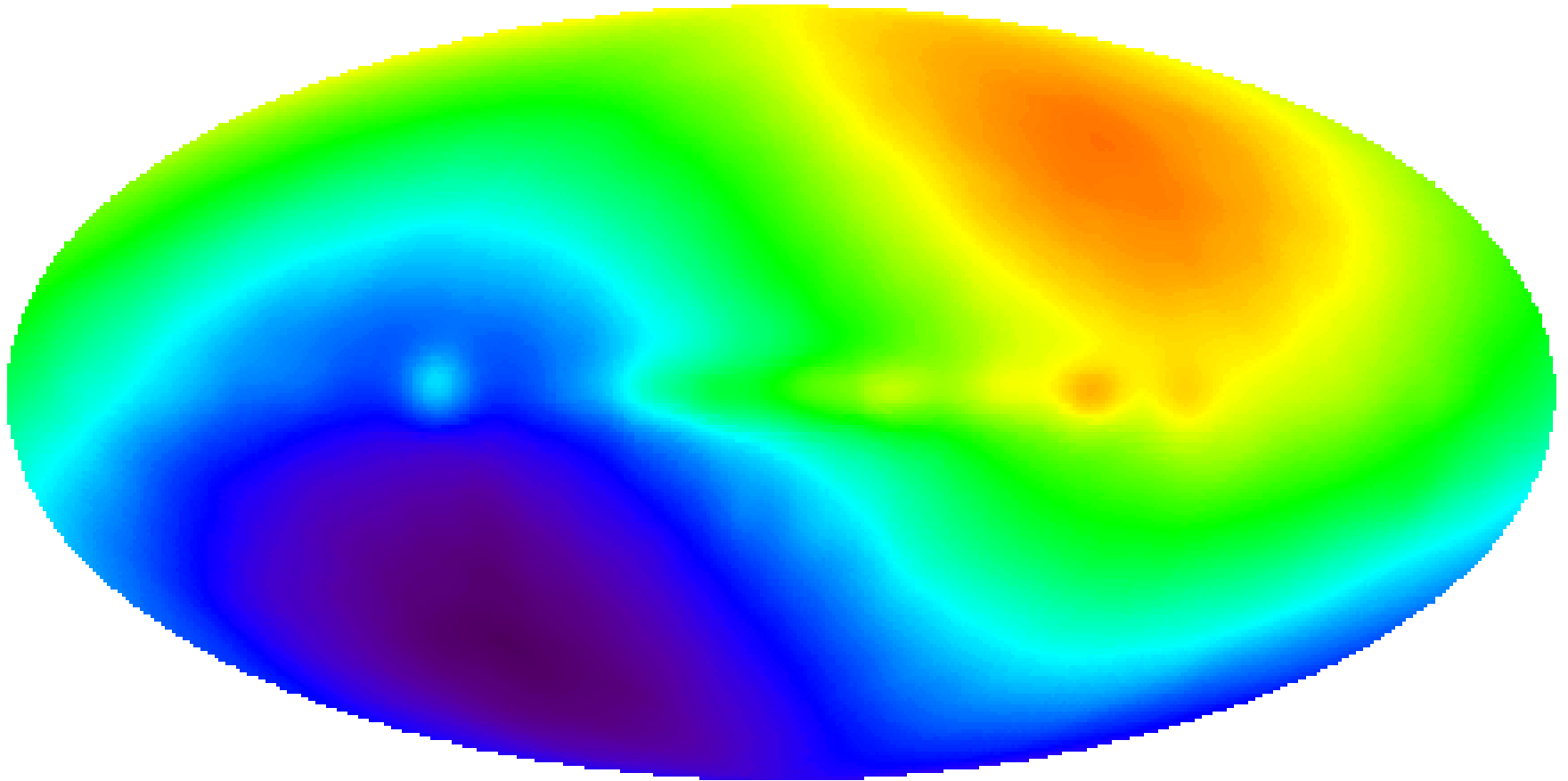
Clear evidence that there are no significant large structures on scales > 100 - 200 Mpc



A sphere covered in a complex, multi-colored pattern of noise, primarily in shades of blue, green, and red, set against a black background. The pattern appears to be a visualization of a dipole convergence process.

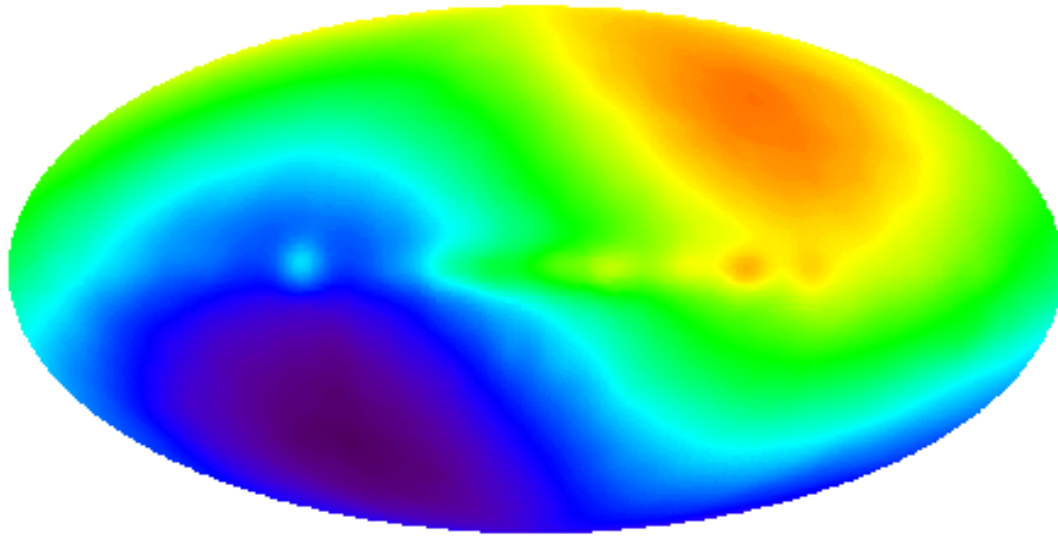
4.
Dipole
Convergence

The CMB Dipole



“Eppur si muove” (Galilei)

The CMB Dipole



The sky map of the CMB has a clear dipole anisotropy:

- Amplitude: **3.4 mK**
- Manifestation of Doppler shift CMB radiation:

result our motion with respect to the Universe.

- It reveals a velocity of the Local Group wrt. Universe of

$$v_{LG} = 627 \pm 22 \text{ km/s} \quad \text{direction: } (l, b) = (276^\circ \pm 3, 3^\circ \pm 2)$$

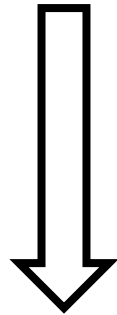
- Question is what and where this motion's origin is.
- It is the result of the gravitational attraction by surrounding matter concentrations.
- The dipole motion is embedded within a coherent shear flow, largely in the direction of the so-called "Great Attractor".

$$\mathbf{v}(\mathbf{x}, t) = \frac{H}{4\pi} \frac{f(\Omega_m)}{b} a \int d\mathbf{x}' \delta_{gal}(\mathbf{x}', t) \frac{(\mathbf{x}' - \mathbf{x})}{|\mathbf{x}' - \mathbf{x}|^3} \quad (158)$$

$$\beta \equiv \frac{f(\Omega_m)}{b} \sim \frac{\Omega_m^{0.6}}{b}$$

**Velocity
Field**

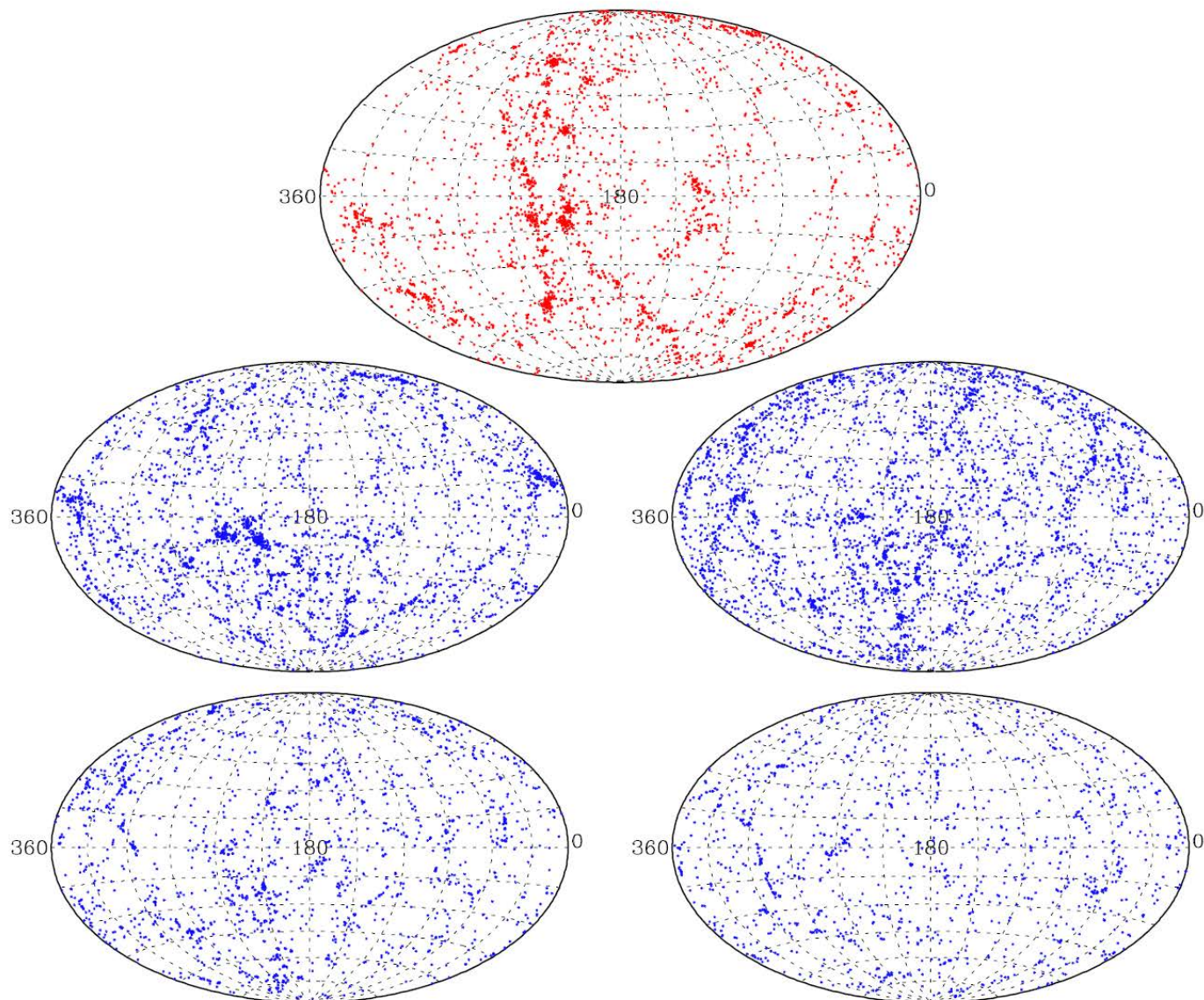
$$\mathbf{v}_{\text{LG}} = \frac{H_0\beta}{4\pi} \int_{\mathbf{r}}^{\infty} d^3\mathbf{r}' \delta_{\text{g}}(\mathbf{r}') \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^3}$$

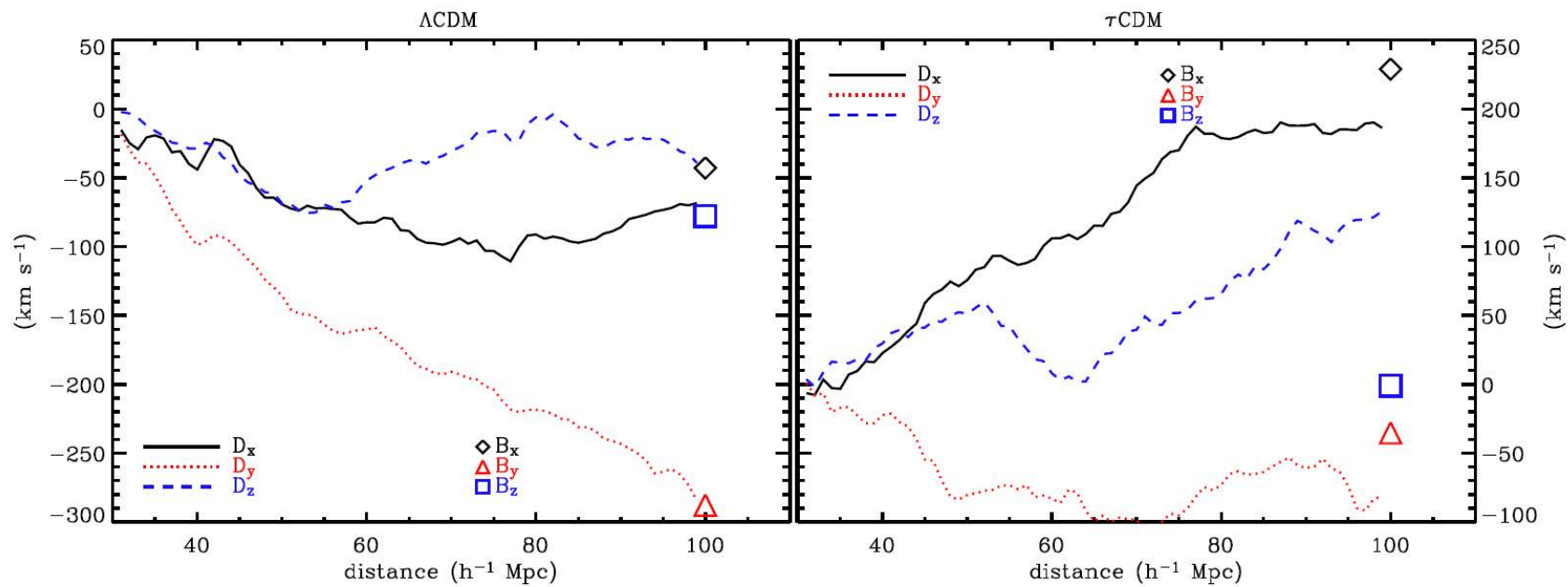
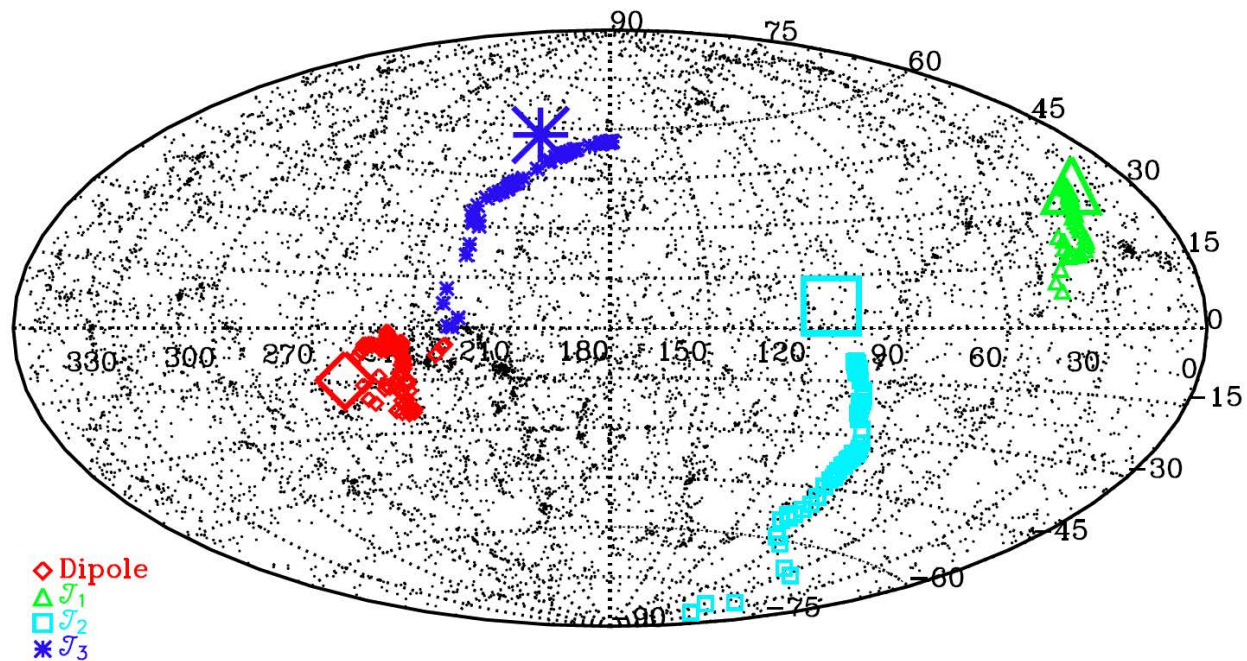


$$\mathbf{v}(\mathbf{r}) = \frac{H_0\beta}{4\pi\bar{n}} \sum_i^N \frac{w_i \hat{\mathbf{r}}_i}{r_i^2}$$

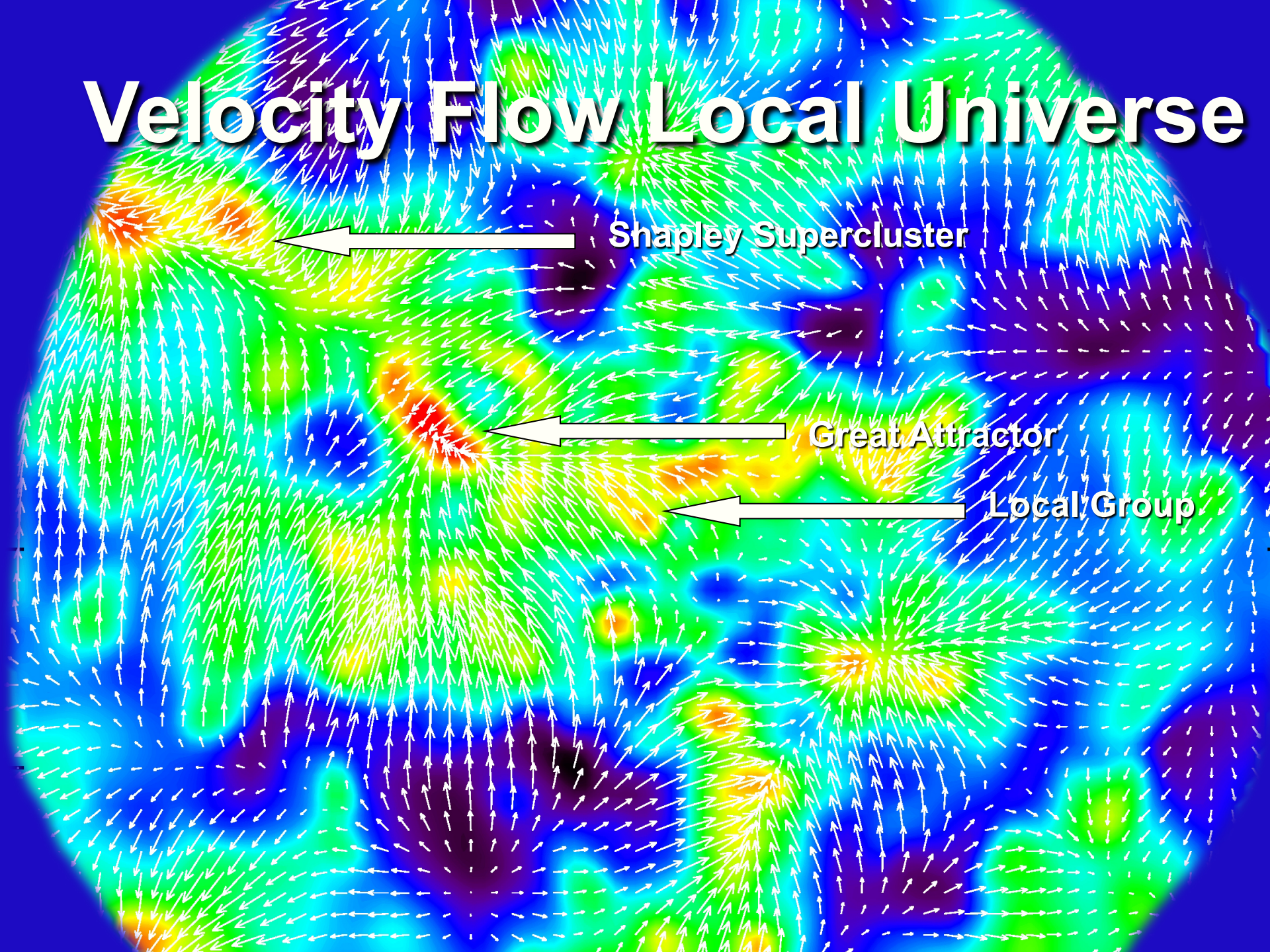
Λ CDM PSCZ+NGB mock catalog

d_{sur} : [0–30], [30–55], [55–70], [70–85], [85–100] h^{-1} Mpc





Velocity Flow Local Universe



PSCz dipole

