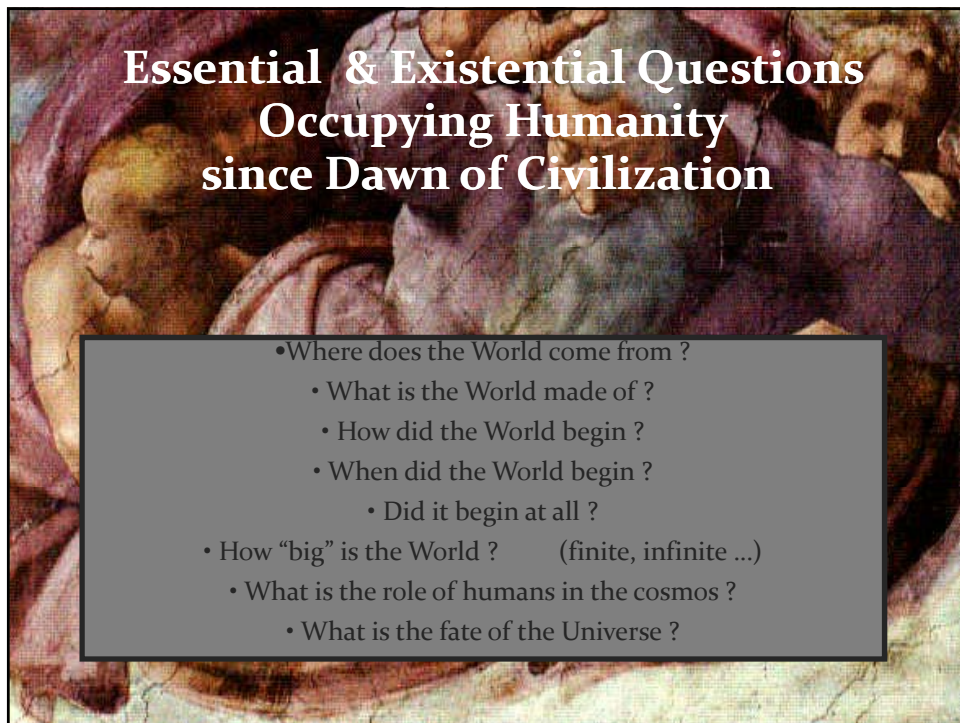


# Cosmology

## Science of the Universe



**Essential & Existential Questions  
Occupying Humanity  
since Dawn of Civilization**

- Where does the World come from ?
  - What is the World made of ?
  - How did the World begin ?
  - When did the World begin ?
    - Did it begin at all ?
- How “big” is the World ? (finite, infinite ...)
- What is the role of humans in the cosmos ?
- What is the fate of the Universe ?



# Energy: Content of the Universe

- What are the components of the Universe ?
- How does each influence the evolution of the Universe ?  
... and ...
- How is each influenced by the evolution of the Universe ?

# A unique time ...

- The past century, since 1915, marks a special epoch
- For the first time in human history, we are able to address the great questions of Cosmology ...
- scientifically ...



# Cosmology

Modern Questions ...

## What is the Universe ?

- Space & Time...
- Energy

# Space & Time ?

- **Geometry ?**
  - Rigid & Flat
  - Dynamic & Curved
- **Extent ?**
  - Open: spatially  $\infty$
  - Closed: spatially finite
- **Topology ?**
  - Simple Euclidian
  - Complex Connectivity

# What do you mean, Cosmic Time ?

- **GR: time locally determined**
- **What about universal time ?**  
Weyl's Theorem

# Cosmology

Questions not yet answerable...

## Cosmological Riddles

- **Is our Universe unique, or are there many other Universes (multiverse) ... ?**
- **What made the Universe originate ?**

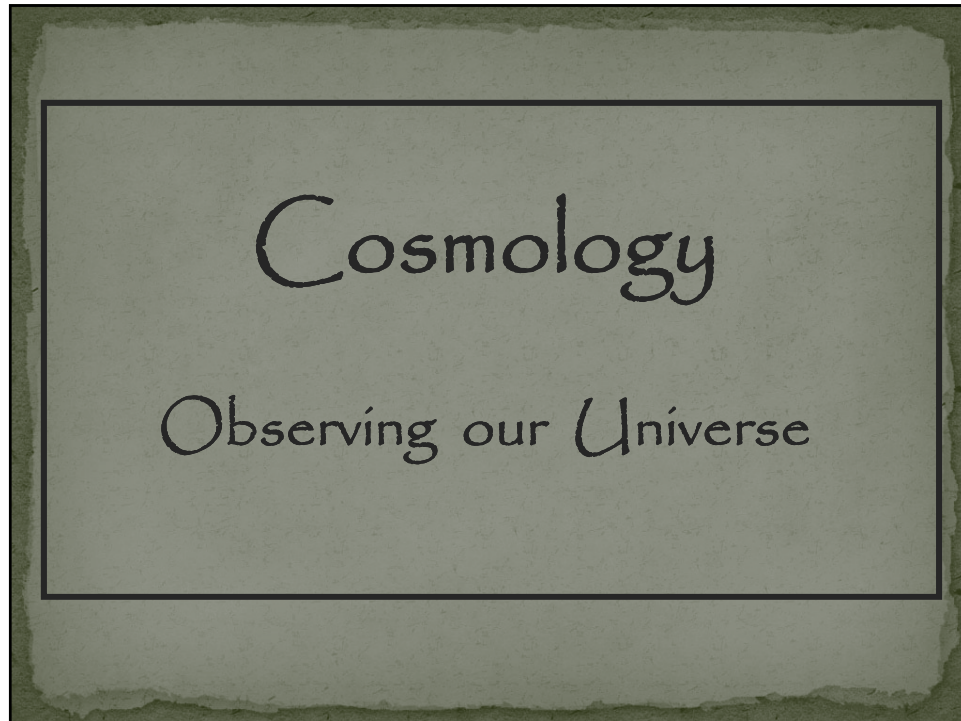


# Cosmological Riddles

- **Why are the physical laws as they are ?**  
Do they need to be ?
- **How many dimensions does the Universe have?**  
More than 1 timelike + 3 spacelike ?

# Cosmological Riddles

- ... and ...
- **Are our brains sufficiently equipped to understand and answer the ultimate questions ... ?**



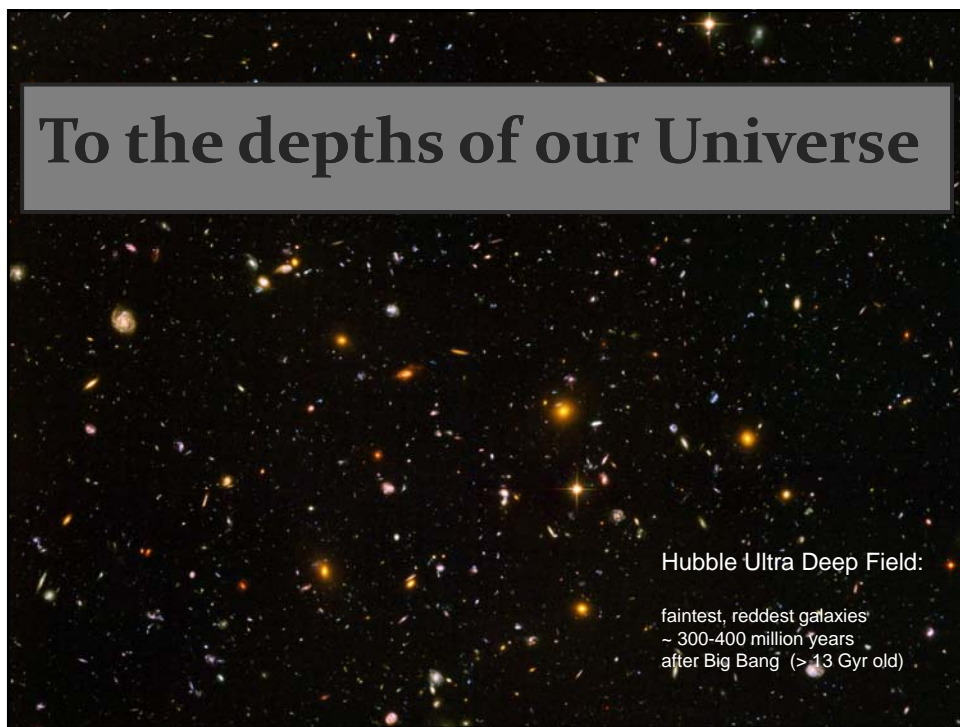
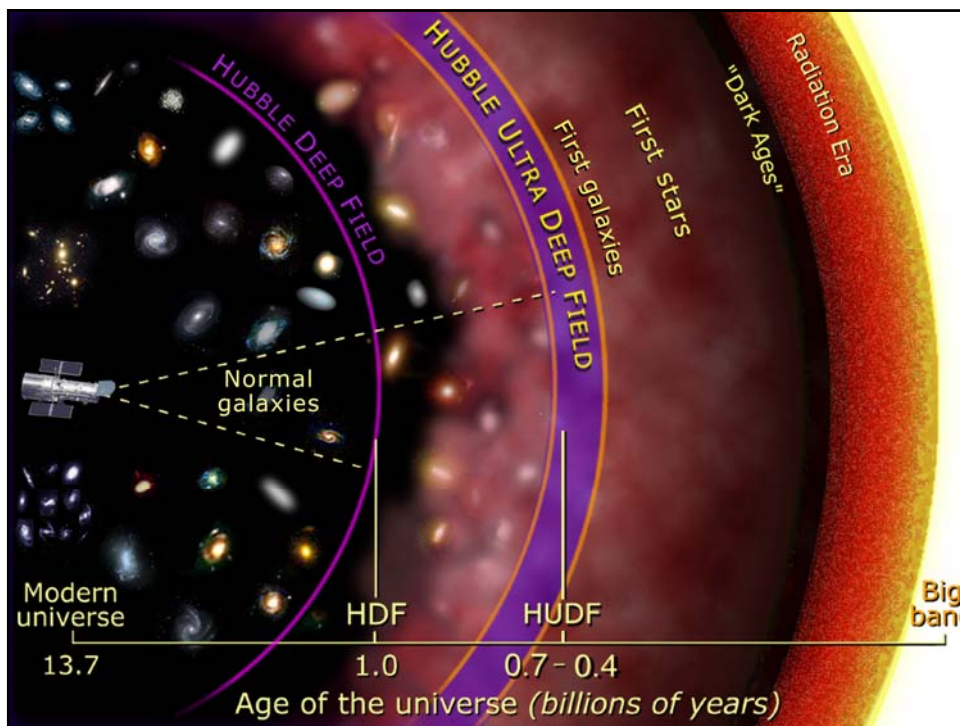
## **Cosmology: exploring Space & Time**

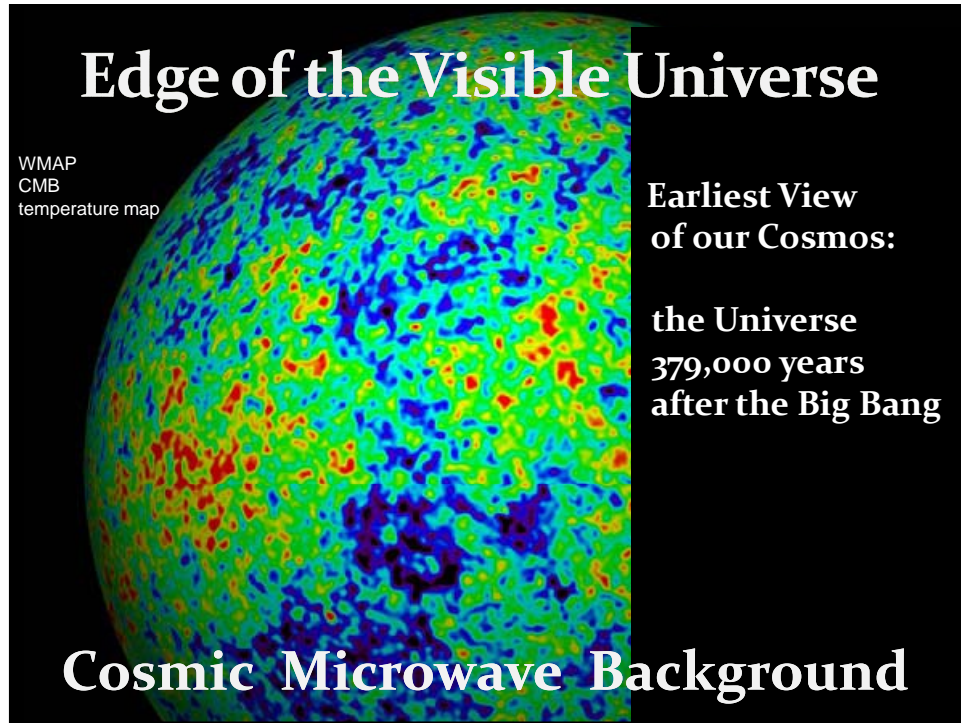
Cosmology is a unique science:

not only it looks out to the deepest realms and  
largest scales of our Universe

on cosmological scales,  
the finite velocity of light becomes a critical factor ...

thus, it also looks back in time, to the earliest moments,  
and thus is the ultimate archaeological science





# the Universe: a Unique Astrophysical Object

- There is only one (visible) Universe ...
- Finite velocity of light,  $c$ :  
... a look in depth = a look back in time ...
- $c$  & implications for space-time:  
observational cosmology limited to only  
a minor thin "shell" of all of spacetime ...

# Hot Big Bang

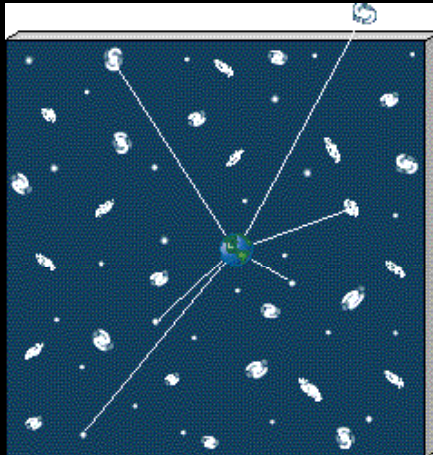
## Key Observations

# Big Bang Evidence

- Olber's paradox:  
the night sky is dark  
||————→ finite age Universe (13.7 Gyr)
- Hubble Expansion  
uniform expansion, with  
expansion velocity ~ distance:  $v = H r$
- Explanation Helium Abundance 24%:  
light chemical elements formed (H, He, Li, ...)  
after ~3 minutes ...
- The Cosmic Microwave Background Radiation:  
the 2.725K radiation blanket, remnant left over  
hot ionized plasma ||————→ neutral universe  
(379,000 years after Big Bang)
- Distant, deep Universe indeed looks different ...



# 1. Olber's Paradox



In an infinitely large, old and unchanging Universe each line of sight would hit a star:



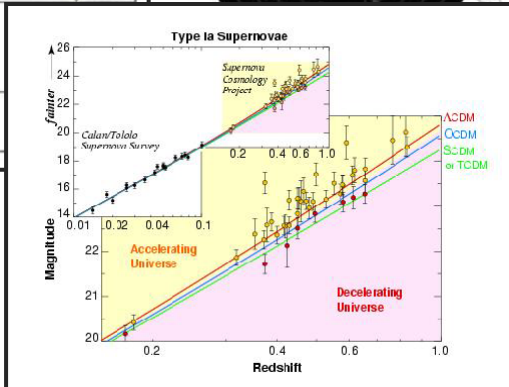
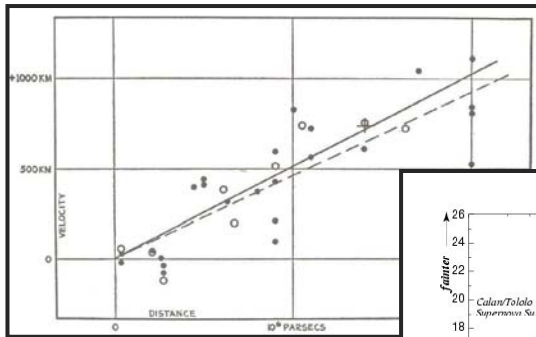
Sky would be as bright as surface of star:

Night sky as bright as Solar Surface, yet the night sky is dark



finite age of Universe (13.7 Gyr)

# 2. Hubble Expansion

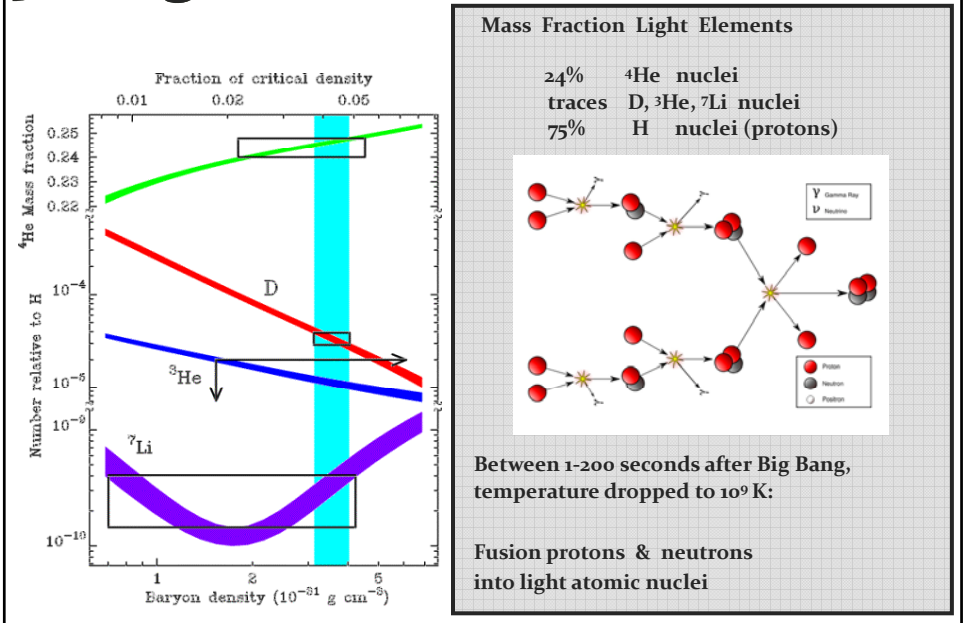


Hubble Diagram:

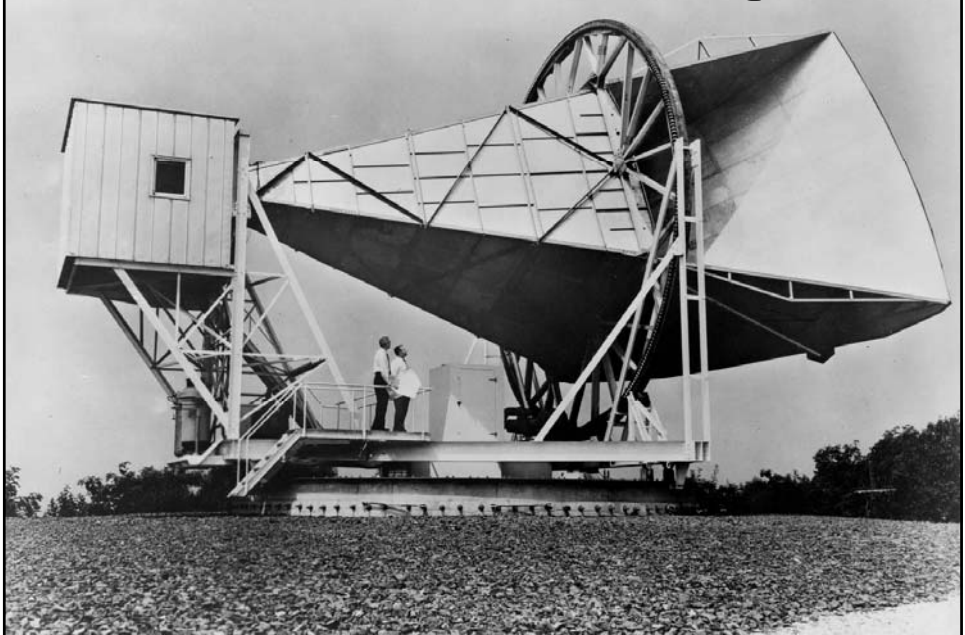
- Hubble 1929: Universe expands !!!!
- Supernova Projects (1998) Cosmic Expansion is accelerating



### 3. Light Element Abundance



### 4. Cosmic Microwave Background



## 4. Cosmic Microwave Background

Thermal Background Radiation Field

$$T=2.725 \text{ K}$$

- Discovery Penzias & Wilson (1965)  
Nobelprize Physics 1978
- Echo of the Big Bang:  
perfect thermal nature can  
only be understood when  
Universe went through  
very hot and dense phase:
- Ultimate proof Hot Big Bang !!!!!

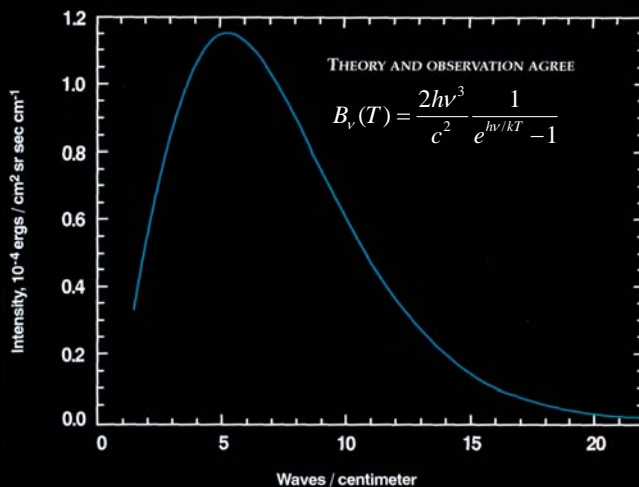


$$T \sim 3000 \text{ K}$$

$$z_{\text{dec}}=1089 \quad (\Delta z_{\text{dec}}=195); \quad t_{\text{dec}}=379.000 \text{ yrs}$$

## CMB Radiation Field Blackbody Radiation

COSMIC MICROWAVE BACKGROUND SPECTRUM FROM COBE



- COBE-DIRBE:  
temperature, blackbody
- $T = 2.725 \text{ K}$
- John Mather  
Nobelprize physics 2006
- Most accurately measured  
Black Body Spectrum  
Ever !!!!!

# CMB Photons

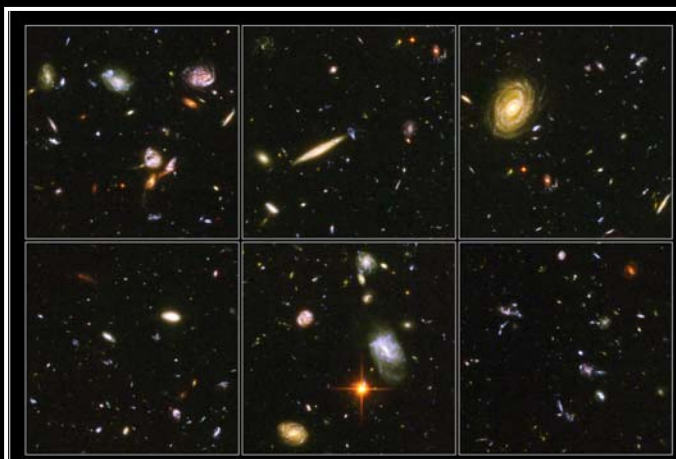


Note:

far from being an exotic faraway phenomenon, realize that the CMB nowadays is counting for approximately 1% of the noise on your tv set ...

Courtesy: W. Hu

## 5. Changing Universe



The appearance of the Universe does change when looking deeper into the Universe:

Depth=Time



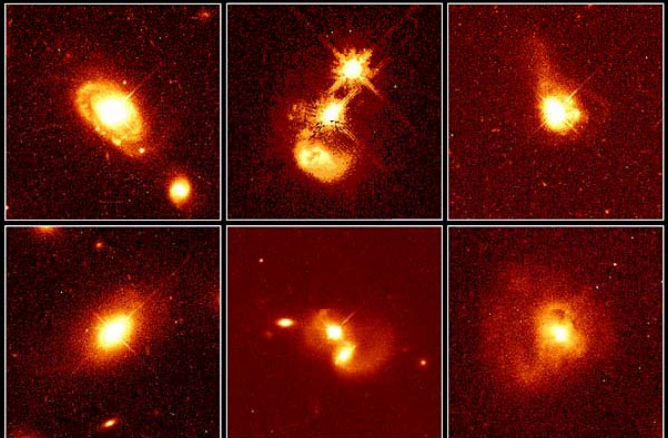
Galaxies in Hubble Ultra Deep Field

Hubble Ultra Deep Field Details  
Hubble Space Telescope • Advanced Camera for Surveys

NASA, ESA, S. Beckwith (STScI) and the HUDF Team

STScI-PRC04-07c

# 5. Changing Universe



The appearance of the Universe does change when looking deeper into the Universe:

Depth=Time  
→

Quasars  
(very high  $z$ )

**Quasar Host Galaxies** HST • WFPC2  
PRC96-35a • ST ScI OPO • November 19, 1996  
J. Bahcall (Institute for Advanced Study), M. Disney (University of Wales) and NASA

Gravity:  
Ruler of the Universe

# Four Fundamental Forces of Nature

- **Strong Nuclear Force**

Responsible for holding particles together inside the nucleus.  
 The nuclear strong force carrier particle is called the gluon.  
 The nuclear strong interaction has a range of  $10^{-15}$  m (diameter of a proton).

- **Electromagnetic Force**

Responsible for electric and magnetic interactions, and determines structure of atoms and molecules.  
 The electromagnetic force carrier particle is the photon (quantum of light)  
 The electromagnetic interaction range is infinite.

- **Weak Force**

Responsible for (beta) radioactivity.  
 The weak force carrier particles are called weak gauge bosons ( $Z, W^+, W^-$ ).  
 The nuclear weak interaction has a range of  $10^{-17}$  m (1% of proton diameter).

- **Gravity**

Responsible for the attraction between masses. Although the gravitational force carrier  
 The hypothetical (carrier) particle is the graviton.  
 The gravitational interaction range is infinite.  
 By far the weakest force of nature.

## Four Fundamental Forces of Nature

The diagram illustrates the scale of matter and the four fundamental forces. On the left, a scale shows an atom (~10<sup>-8</sup> cm), a nucleus (~10<sup>-12</sup> cm), a proton/neutron (~10<sup>-13</sup> cm), and a quark (<10<sup>-16</sup> cm). On the right, a scale shows an electron (<10<sup>-16</sup> cm) and a quark (<10<sup>-16</sup> cm).

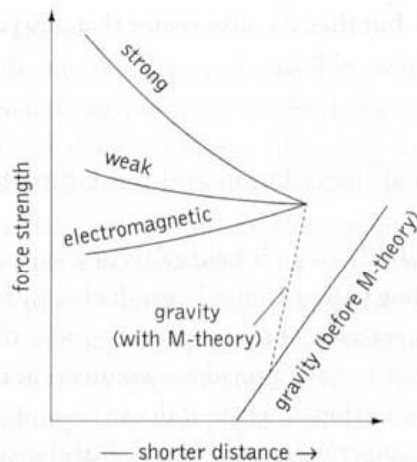
Leptons	Strong	Electromagnetic
<p>Electric Charge</p> <p>Tau: -1, 0, Tau Neutrino: 0</p> <p>Muon: -1, 0, Muon Neutrino: 0</p> <p>Electron: -1, 0, Electron Neutrino: 0</p>	<p>Gluons (8)</p> <p>Quarks</p> <p>Mesons, Baryons</p> <p>Nuclei</p>	<p>Photon</p> <p>Atoms, Light, Chemistry, Electronics</p>
Quarks	Gravitational	Weak
<p>Electric Charge</p> <p>Bottom: -1/3, 2/3, Top: 2/3</p> <p>Strange: -1/3, 2/3, Charm: 2/3</p> <p>Down: -1/3, 2/3, Up: 2/3</p> <p>each quark: R, B, G 3 colours</p>	<p>Graviton ?</p> <p>Solar system, Galaxies, Black holes</p>	<p>Bosons (W, Z)</p> <p>Neutron decay, Beta radioactivity, Neutrino interactions, Burning of the sun</p>



Interaction	Current Theory	Mediators	Relative Strength <sup>[1]</sup>	Long-Distance Behavior	Range(m)
Strong	Quantum chromodynamics (QCD)	gluons	$10^{38}$	1 (see discussion below)	$10^{-15}$
Electromagnetic	Quantum electrodynamics (QED)	photons	$10^{36}$	$\frac{1}{r^2}$	infinite
Weak	Electroweak Theory	W and Z bosons	$10^{25}$	$\frac{e^{-m_{W,Z}r}}{r}$	$10^{-18}$
Gravitation	General Relativity (GR)	gravitons	1	$\frac{1}{r^2}$	infinite

**The weakest force, by far, rules the Universe ...**  
**Gravity has dominated its evolution, and determines its fate ...**

## Grand Unified Theories (GUT)



### Grand Unified Theories

- \* describe how
  - Strong
  - Weak
  - Electromagnetic

Forces are manifestations of the same underlying GUT force ...
- \* This implies the strength of the forces to diverge from their uniform GUT strength
- \* Interesting to see whether gravity at some very early instant unifies with these forces ???



## Newton's Static Universe

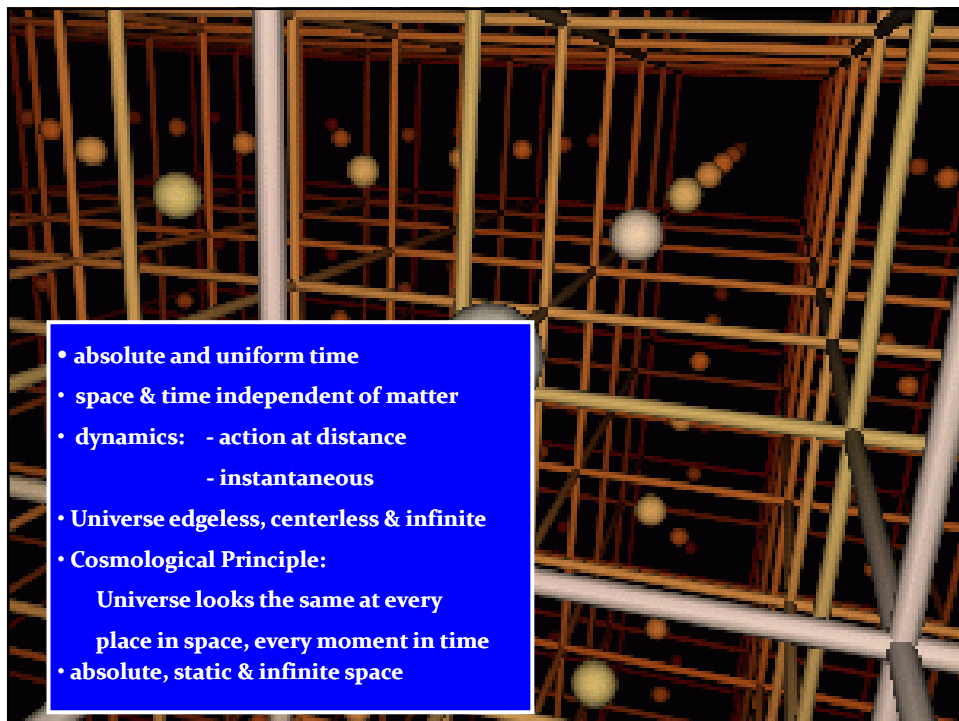
### The Unchanging Universe

- In two thousand years of astronomy, no one ever guessed that the universe might be expanding.
- To ancient Greek astronomers and philosophers, the universe was seen as the embodiment of perfection, the heavens were truly heavenly:
  - unchanging, permanent, and geometrically perfect.
- In the early 1600s, Isaac Newton developed his law of gravity, showing that motion in the heavens obeyed the same laws as motion on Earth.

# Newton's Universe

- However, Newton ran into trouble when he tried to apply his theory of gravity to the entire universe.
- Since gravity is always attractive, his law predicted that all the matter in the universe should eventually clump into one big ball.
- Newton knew this was not the case, and assumed that the universe had to be static
- So he conjectured that:

the Creator placed the stars such that they were  
 "at immense distances from one another."



# Einstein's Dynamic & Geometric Universe

## Einstein's Universe

In 1915,  
Albert Einstein completed his General Theory of Relativity.

- General Relativity is a “metric theory”:  
gravity is a manifestation of the geometry, curvature, of space-time.
- Revolutionized our thinking about the nature of space & time:
  - no longer Newton's static and rigid background,
  - a dynamic medium, intimately coupled to the universe's content of matter and energy.
- All phrased into perhaps the most beautiful and impressive scientific equation known to humankind, a triumph of human genius,

## Einstein Field Equations

*... Spacetime becomes a dynamic continuum,  
integral part of the structure of the cosmos ...  
curved spacetime becomes force of gravity*

$$R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R = -\frac{8\pi G}{c^4} T^{\alpha\beta}$$

*... its geometry rules the world,  
the world rules its geometry...*

# Albert Einstein

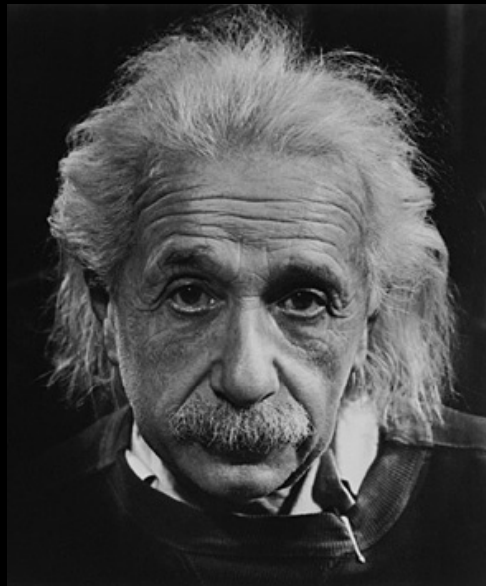
**Albert Einstein**  
(1879-1955; Ulm-Princeton)

**father of**  
**General Relativity (1915),**

**opening the way towards**  
**Physical Cosmology**

The supreme task of the physicist is  
to arrive at those universal  
elementary laws from which the  
cosmos can be built up by pure  
deduction.

(Albert Einstein, 1954)



# General Relativity

A crucial aspect of any particular configuration is the geometry of spacetime: because Einstein's General Relativity is a metric theory, knowledge of the geometry is essential.

Einstein Field Equations are notoriously complex, essentially 10 equations. Solving them for general situations is almost impossible.

However, there are some special circumstances that do allow a full solution. The simplest one is also the one that describes our Universe. It is encapsulated in the

## Cosmological Principle

On the basis of this principle, we can constrain the geometry of the Universe and hence find its dynamical evolution.

## Cosmological Principle: the Universe Simple & Smooth

"God is an infinite sphere whose centre is everywhere and its circumference nowhere"  
Empedocles, 5<sup>th</sup> cent. BC

### Cosmological Principle:

Describes the symmetries in global appearance of the Universe:

- **Homogeneous** → The Universe is the same everywhere:  
- physical quantities (density,  $T, p, \dots$ )
- **Isotropic** → The Universe looks the same in every direction
- **Universality** → Physical Laws same everywhere
- **Uniformly Expanding** → The Universe "grows" with same rate in  
- every direction  
- at every location

"all places in the Universe are alike"  
Einstein, 1931

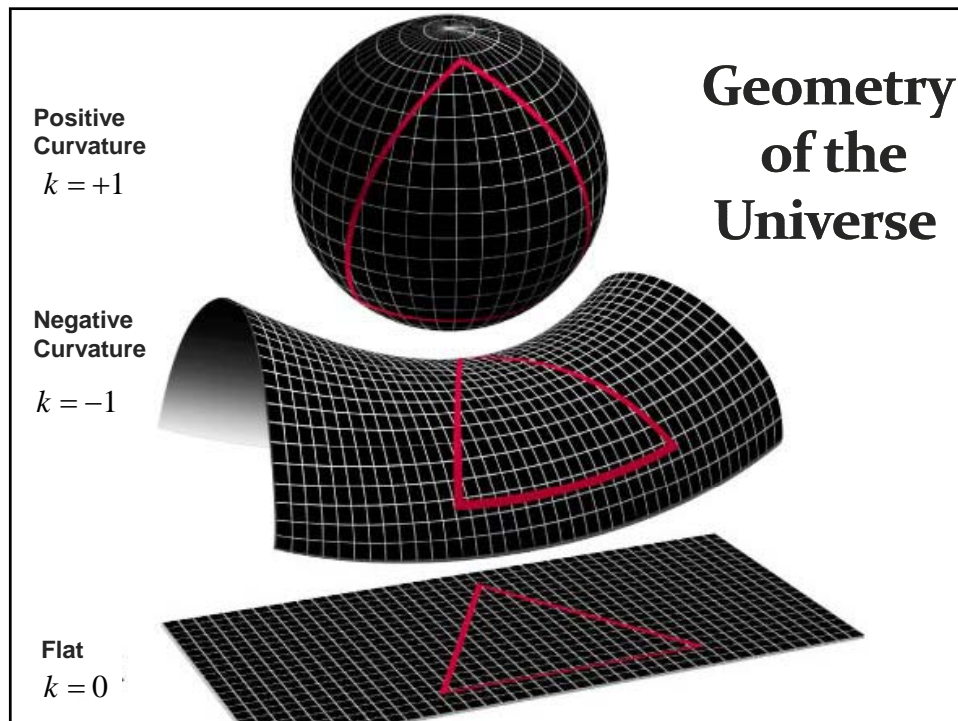
# Geometry of the Universe

## Fundamental Tenet of (Non-Euclidian = Riemannian) Geometry

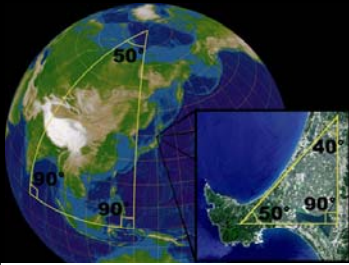
There exist no more than **THREE** uniform spaces:

- |    |                           |                           |
|----|---------------------------|---------------------------|
| 1) | Euclidian (flat) Geometry | Euclides                  |
| 2) | Hyperbolic Geometry       | Gauß, Lobachevski, Bolyai |
| 3) | Spherical Geometry        | Riemann                   |

uniform=  
homogeneous & isotropic  
(cosmological principle)







# Uniform Spaces: Geometric Characteristics

	Parallel Lines	Triangular Angles $\alpha + \beta + \gamma$	Circumference Circle $x \equiv \frac{S}{2r}$	Curvature $k$	Extent	Boundary
Flat Space	parallels: 1 never intersects	$\pi$	$\pi$	0	open: infinite	unbounded
Spherical Space	parallels: $\infty$ along great circles, all intersect	$> \pi$	$< \pi$	$1/R^2$ $> 0$	closed: finite	unbounded
Hyperbolic Space	parallels: $\infty$ diverge & never intersect	$< \pi$	$> \pi$	$-1/R^2$ $< 0$	open: infinite	unbounded

## Robertson-Walker Metric

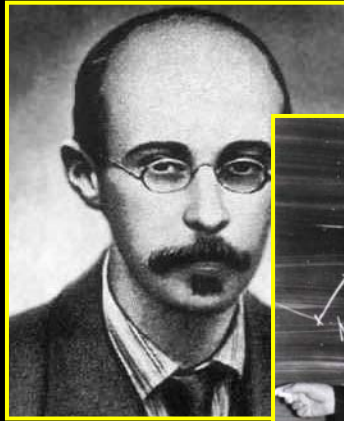
Distances in a uniformly curved spacetime is specified in terms of the Robertson-Walker metric. The spacetime distance of a point at coordinate  $(r, \theta, \phi)$  is:

$$ds^2 = c^2 dt^2 - a(t)^2 \left\{ dr^2 + R_c^2 S_k^2 \left( \frac{r}{R_c} \right) \left[ d\theta^2 + \sin^2 \theta d\phi^2 \right] \right\}$$

where the function  $S_k(r/R_c)$  specifies the effect of curvature on the distances between points in spacetime

$$S_k \left( \frac{r}{R_c} \right) = \begin{cases} \sin \left( \frac{r}{R_c} \right) & k = +1 \\ \frac{r}{R_c} & k = 0 \\ \sinh \left( \frac{r}{R_c} \right) & k = -1 \end{cases}$$

# Friedmann & Lemaitre



Alexander Friedmann (1888 -1925)  
George Lemaitre (1894-1966)



They discovered (independently) theoretically the expansion of the Universe as a solution to the Theory of General Relativity.

... and derived the equations that describe the expansion and evolution of the universe,

the foundation for all of modern Cosmology:

## Friedmann-Lemaitre Equation

# Expanding Universe

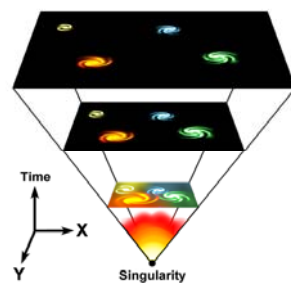
- Einstein, de Sitter, Friedmann and Lemaitre all realized that in General Relativity, there cannot be a stable and static Universe:
- The Universe either expands, or it contracts ...

- Expansion Universe encapsulated in a **GLOBAL expansion factor  $a(t)$**
- All distances/dimensions of objects uniformly increase by  $a(t)$ :

at time  $t$ , the distance between two objects  $i$  and  $j$  has increased to

$$\vec{r}_i - \vec{r}_j = a(t) (\vec{r}_{i,0} - \vec{r}_{j,0})$$

- Note: by definition we chose  $a(t_0)=1$ , i.e. the present-day expansion factor



## Friedmann-Robertson-Walker-Lemaitre Universe

Because of General Relativity, the evolution of the Universe is determined by four factors:

- density  $\rho(t)$
- pressure  $p(t)$
- curvature  $kc^2 / R_0^2$   $k = 0, +1, -1$   
 $R_0$ : present curvature radius
- cosmological constant  $\Lambda$

- Density & Pressure:
  - in relativity, energy & momentum need to be seen as one physical quantity (four-vector)
  - pressure = momentum flux
- Curvature:
  - gravity is a manifestation of geometry spacetime
- Cosmological Constant:
  - free parameter in General Relativity
  - Einstein's "biggest blunder"
  - mysteriously, since 1998 we know it dominates the Universe

## Friedmann-Robertson-Walker-Lemaitre Universe

$$\ddot{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) a + \frac{\Lambda}{3} a$$

$$\dot{a}^2 = \frac{8\pi G}{3} \rho a^2 - \frac{kc^2}{R_0^2} + \frac{\Lambda}{3} a^2$$

## Friedmann-Robertson-Walker-Lemaitre Universe

$$\ddot{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) a + \frac{\Lambda}{3} a$$

$$\dot{a}^2 = \frac{8\pi G}{3} \rho a^2 - \frac{kc^2}{R_0^2} + \frac{\Lambda}{3} a^2$$

## Friedmann-Robertson-Walker-Lemaitre Universe

Relativistic Cosmology

$$\ddot{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) a + \frac{\Lambda}{3} a$$

$$\dot{a}^2 = \frac{8\pi G}{3} \rho a^2 - \frac{kc^2}{R_0^2} + \frac{\Lambda}{3} a^2$$

Newtonian Cosmology

$$\ddot{a} = -\frac{4\pi G}{3} \rho a$$

$$\dot{a}^2 = \frac{8\pi G}{3} \rho a^2 + E$$

$-kc^2 / R_0^2$   
 $\Lambda$   
 $p$

Curvature

Cosmological Constant

Pressure



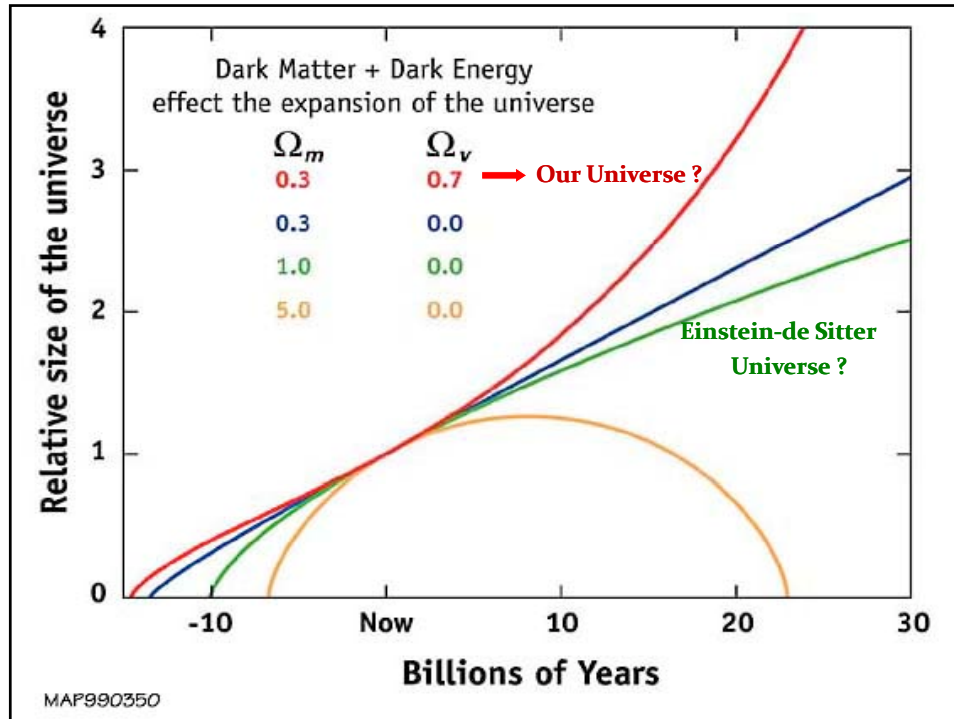
$E$

Energy

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## Evolution & Fate Friedmann-Robertson-Walker-Lemaitre Universe

Fully determined by three factors:

- energy content of the Universe  
(density & pressure)
- geometry of the Universe  
(curvature term)
- cosmological constant