the Cosmic Web:

Lecture 3: dynamics & structure

Rien van de Weijgaert,
Cosmic Web, Caput Course, Oct. 2017

Cosmic Web

Dynamics & Formation:

Program

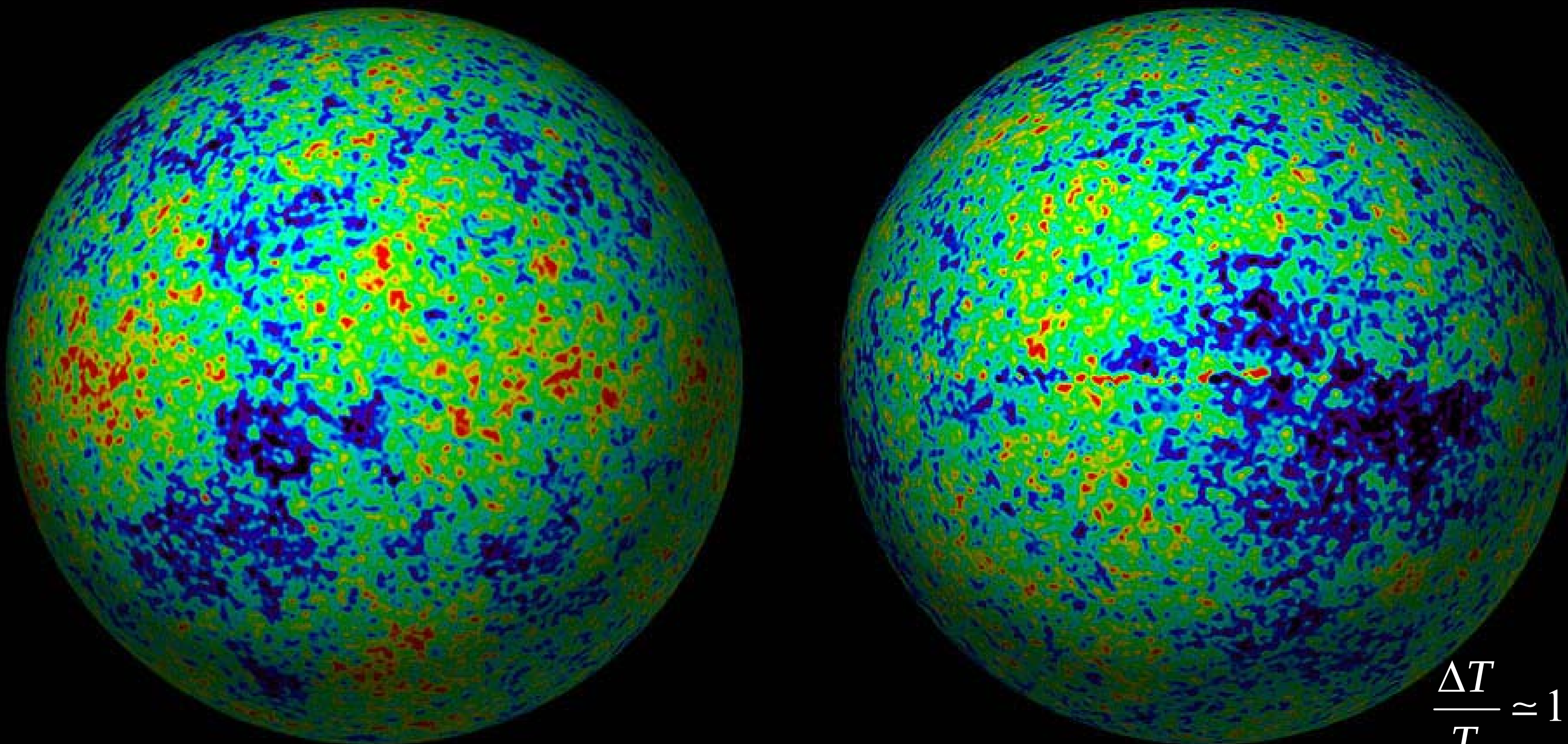
Cosmic Web – Formation & Dynamics

- the Mechanism - Gravitational Instability
- Anisotropic Collapse - Formation of filaments and walls
- Weaving the Web - Connection Clusters, Filaments and Walls
- Hierarchical Formation - from small to the Megaparsec Cosmic Web
- Anisotropy & Hierarchy - the Adhesion formalism
- Phase Space - Multistream structure

Cosmic Structure Formation:

Gravitational Instability

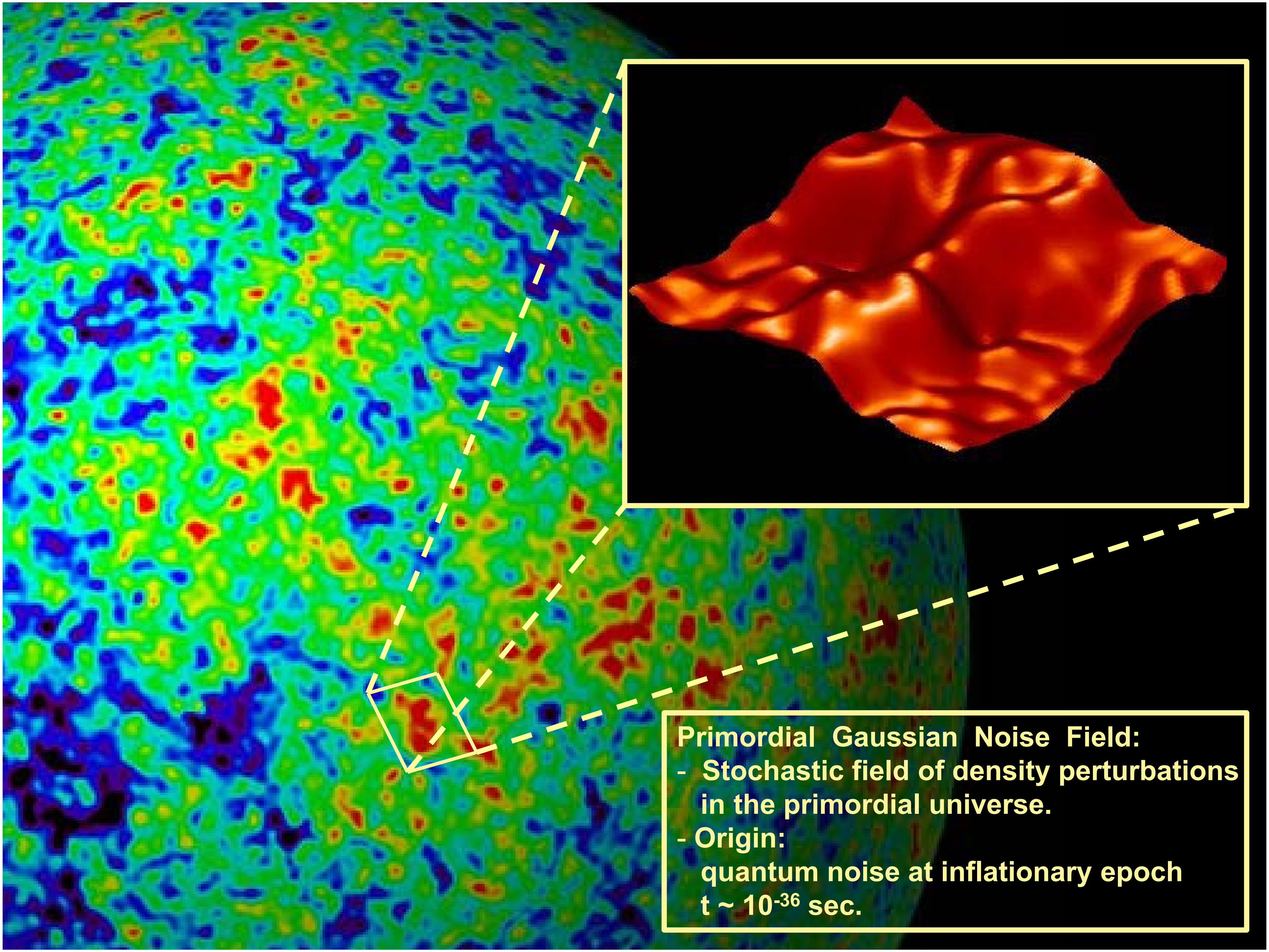
Primordial Universe

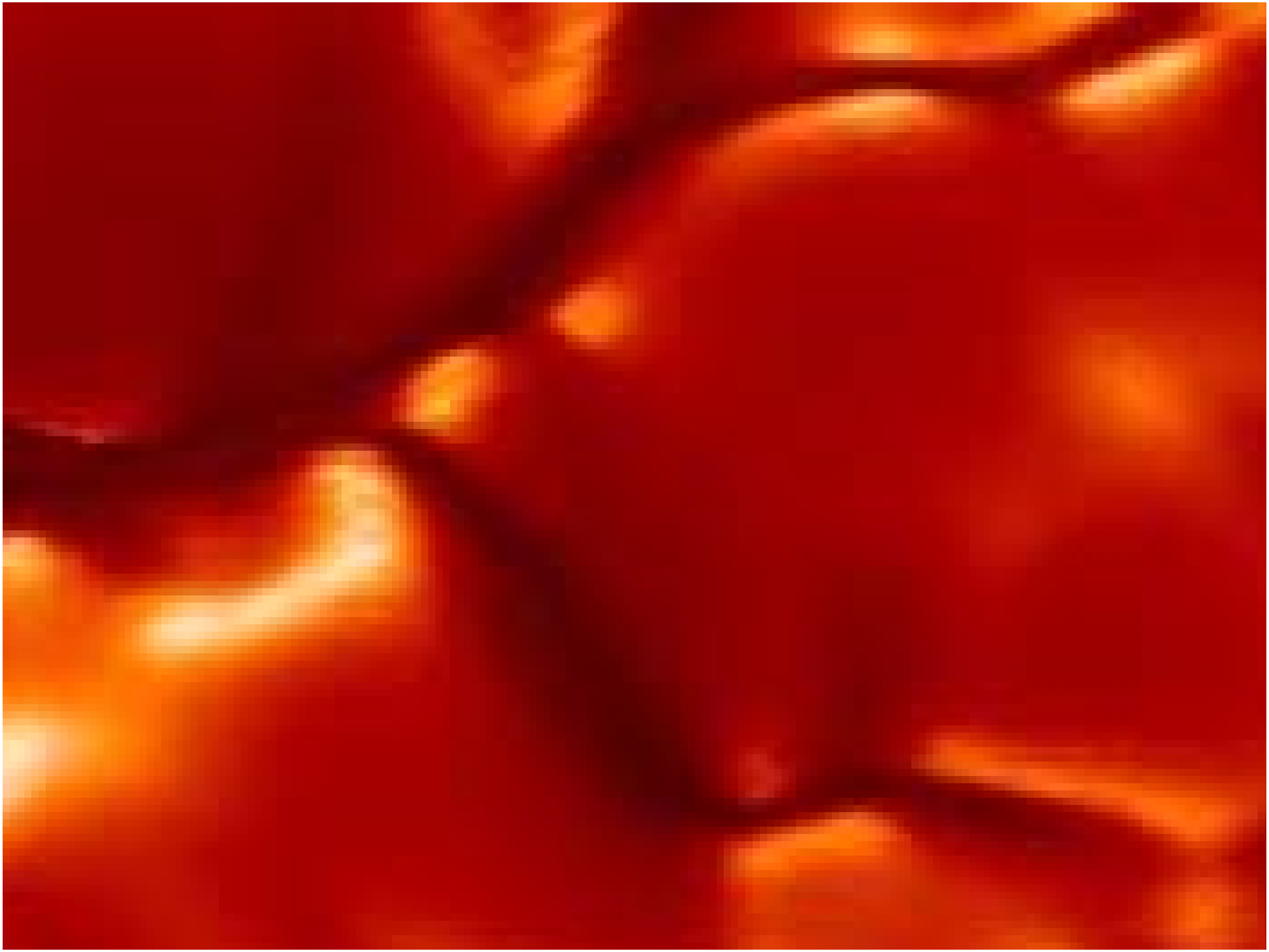


$$\frac{\Delta T}{T} \simeq 10^{-5}$$

global representation cosmic surface last scattering: the world inside out

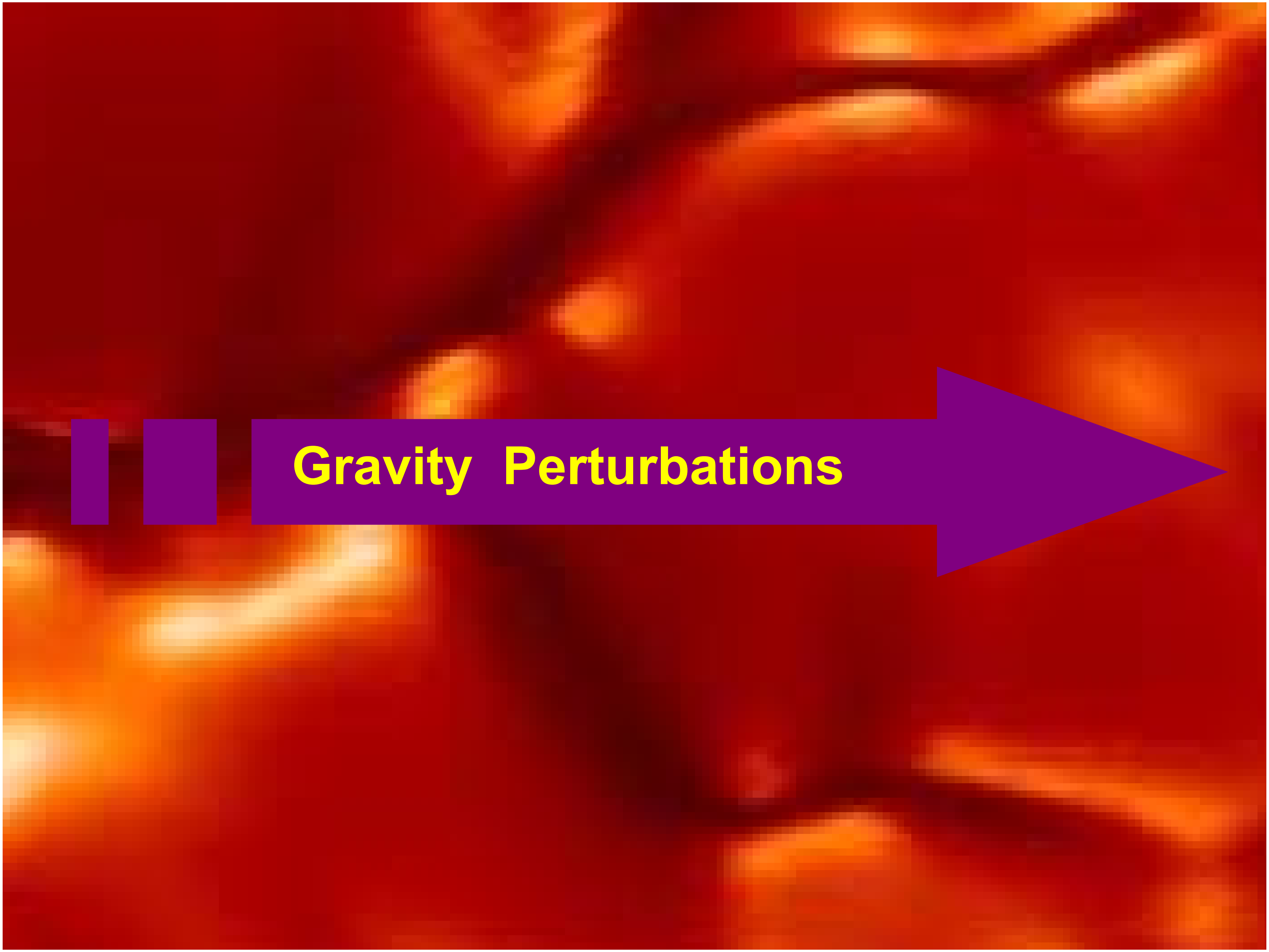
Temperature Map CMB radiation:
Tiny variations in primordial temperature, reflecting tiny inhomogeneities in
energy density of ?~10^{-5} K at
recombination epoch, 379,000 yrs after Big Bang


Primordial Gaussian Noise Field:
- Stochastic field of density perturbations in the primordial universe.
- Origin:
quantum noise at inflationary epoch
t ~ 10^{-36} sec.



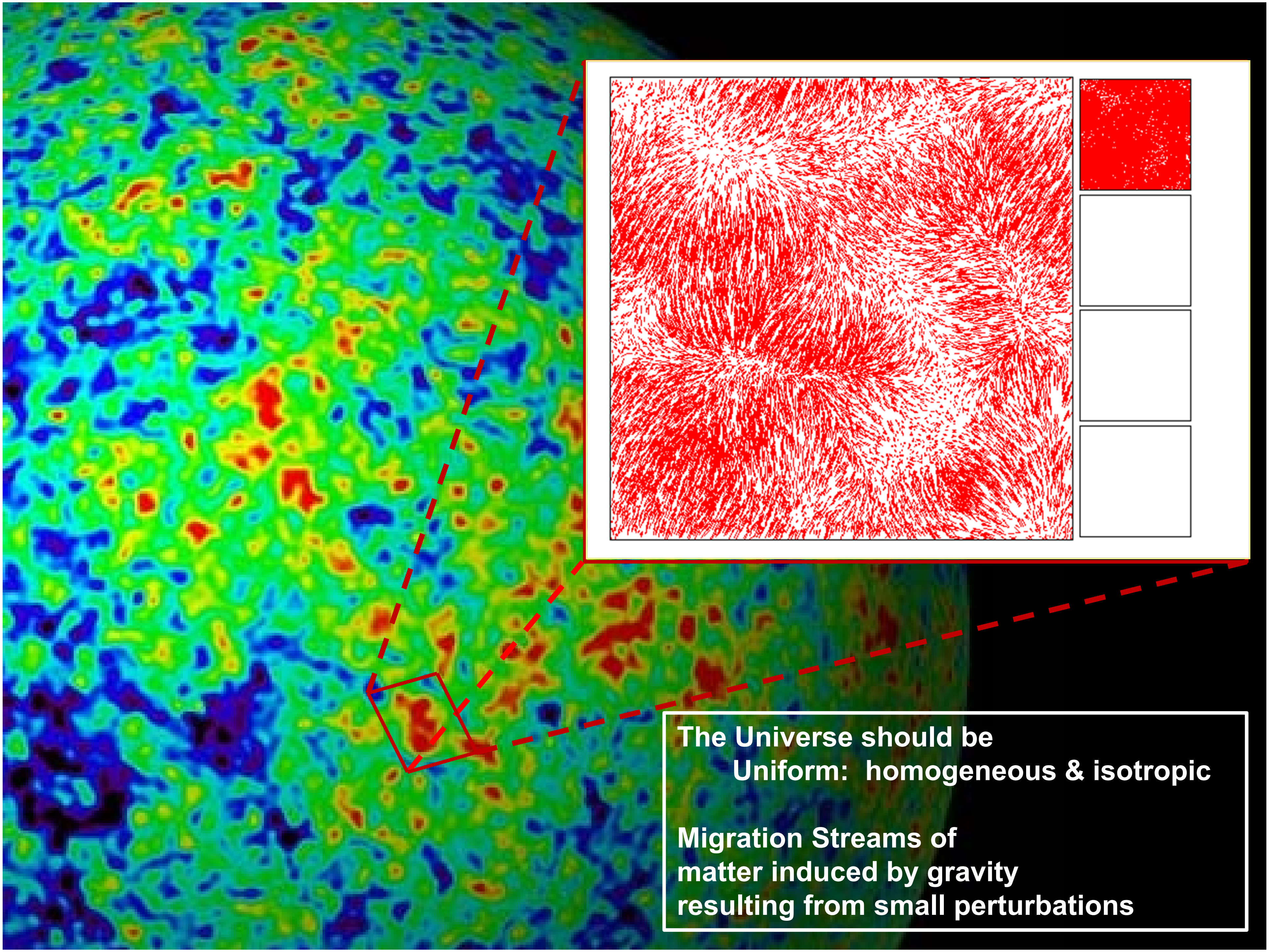
Density Perturbation Field:

$$\delta(\vec{x},t) = \frac{\rho(x,t) - \rho_u(t)}{\rho_u(t)}$$


Gravity Perturbations

Gravity Perturbations

$$\mathbf{g}(\mathbf{r},t) = -\frac{1}{a}\nabla\phi = \frac{3\Omega H^2}{8\pi}\int \mathrm{d}\mathbf{x}'\,\delta(\mathbf{x}',t)\,\frac{(\mathbf{x}'-\mathbf{x})}{|\mathbf{x}'-\mathbf{x}|^3}$$


The Universe should be
Uniform: homogeneous & isotropic
Migration Streams of
matter induced by gravity
resulting from small perturbations

Cosmic Structure Formation

(Energy) Density Perturbations

↓

Gravity Perturbations

↓

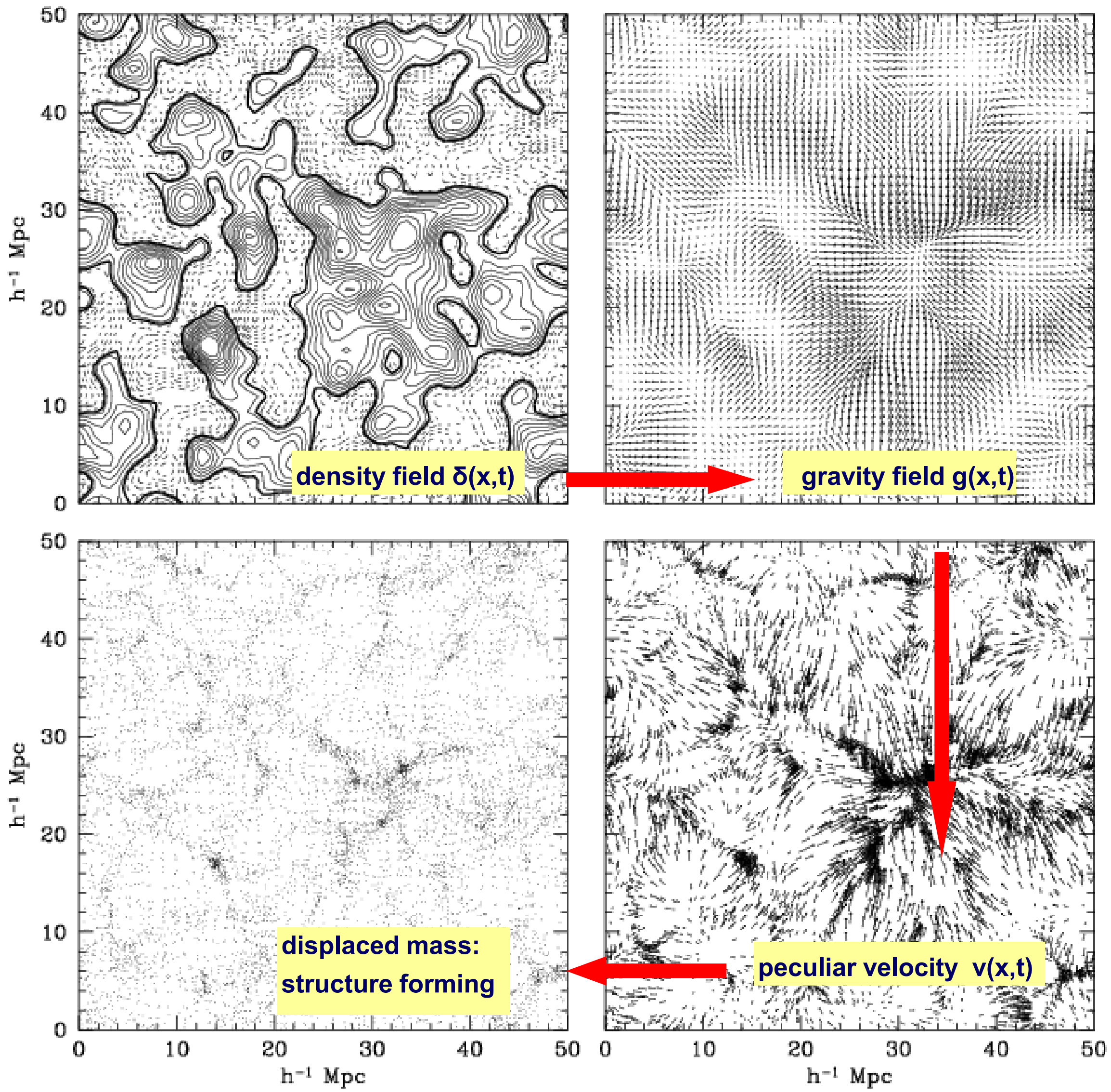
(Cosmic) Flows of (Energy) & Matter:

- towards high density regions:
  - assemble more and more matter
  - their expansion comes to a halt
  - turn around and collapse
- evacuating void regions
  - low-density regions expand
  - matter moves out of region
  - turn into prominent empty voids

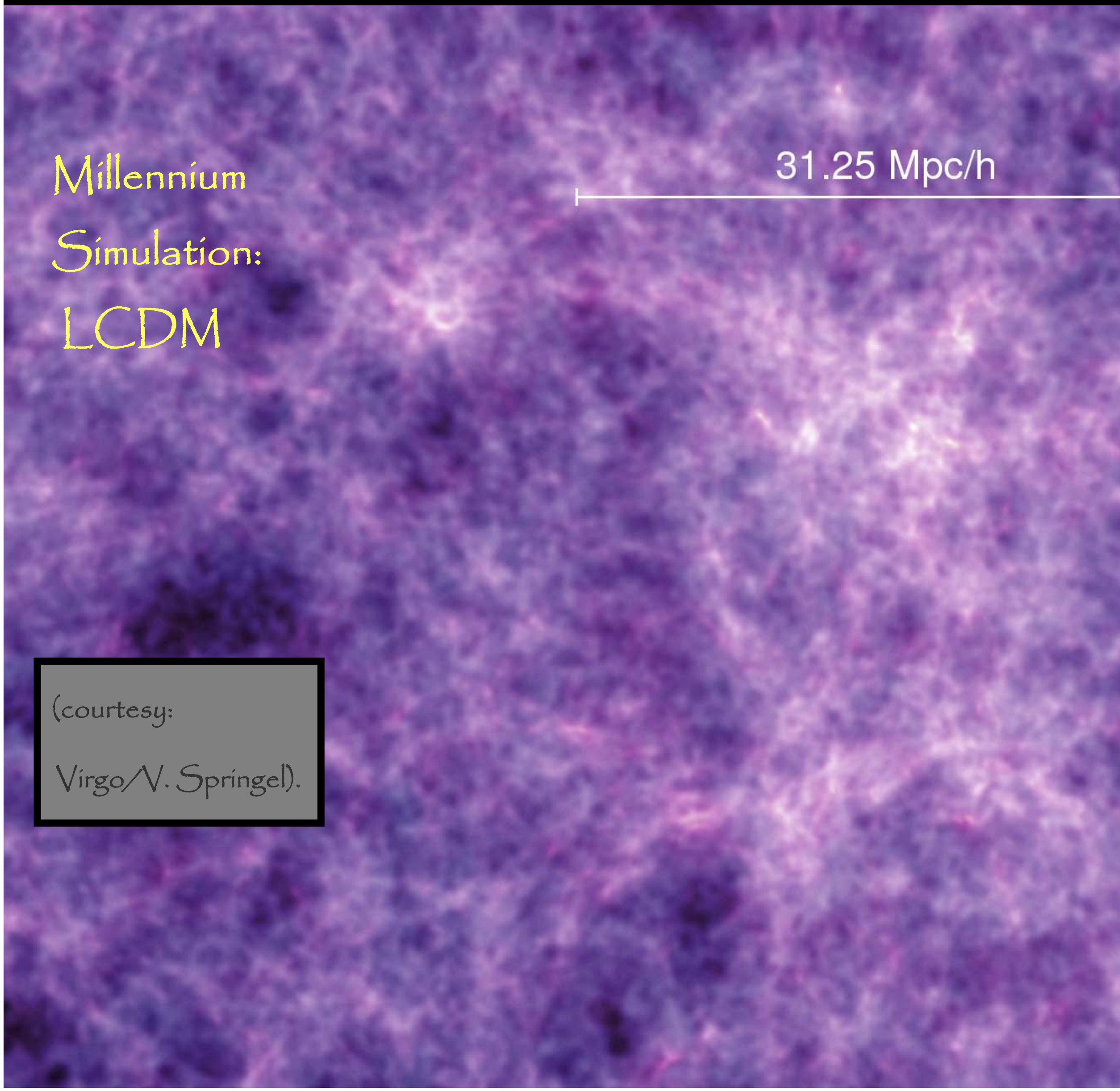
↓

Emergence of cosmic structures

- Computer Simulations
  - succesfull confrontation with observational reality



Cosmic Structure Formation



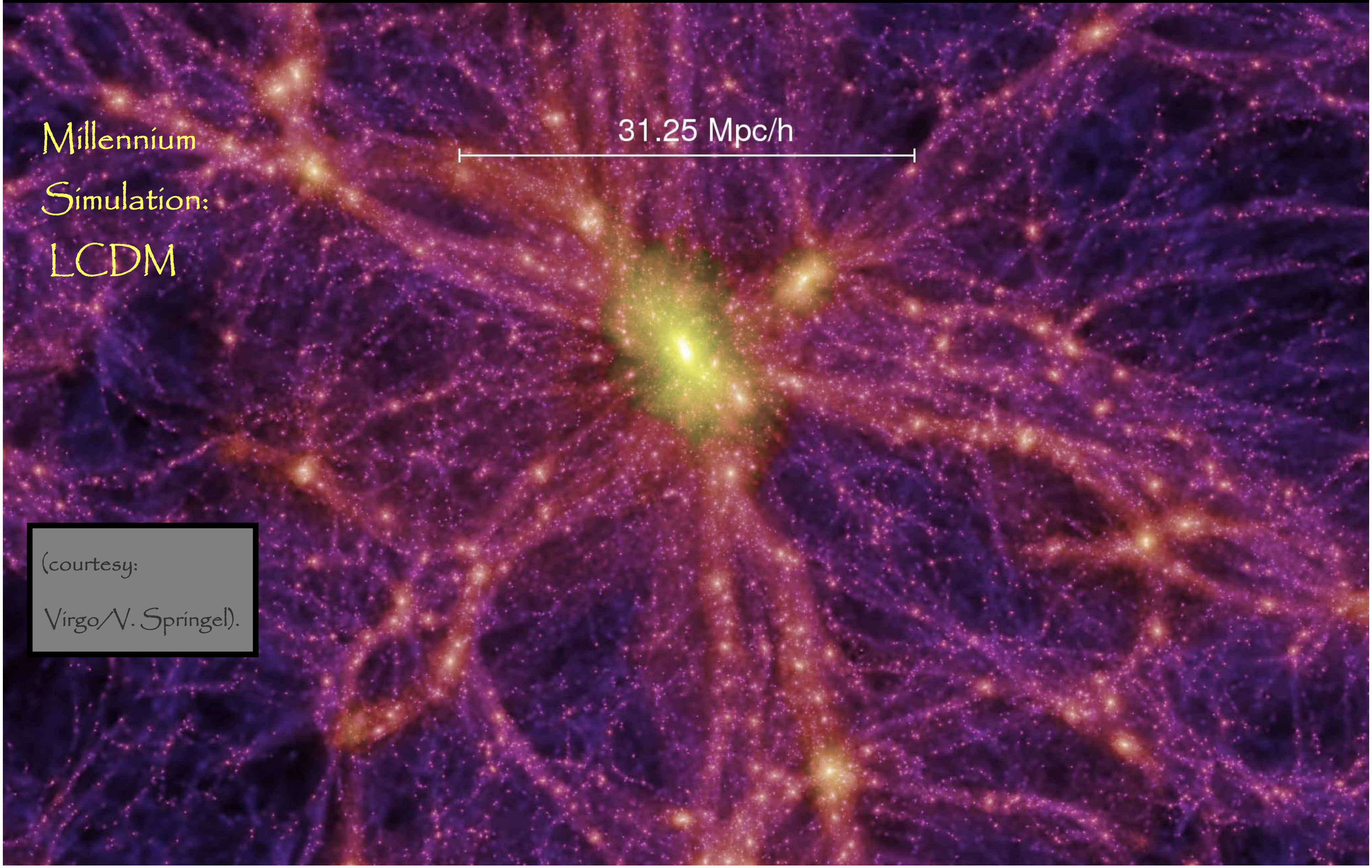
Dark Matter,
(~ 5.5x more than
baryonic matter)

⬇

without: not enough time to form structure in the Universe in 13.8 Gyrs

(cosmic web, clusters, galaxies, stars, ...)

Cosmic Structure Formation



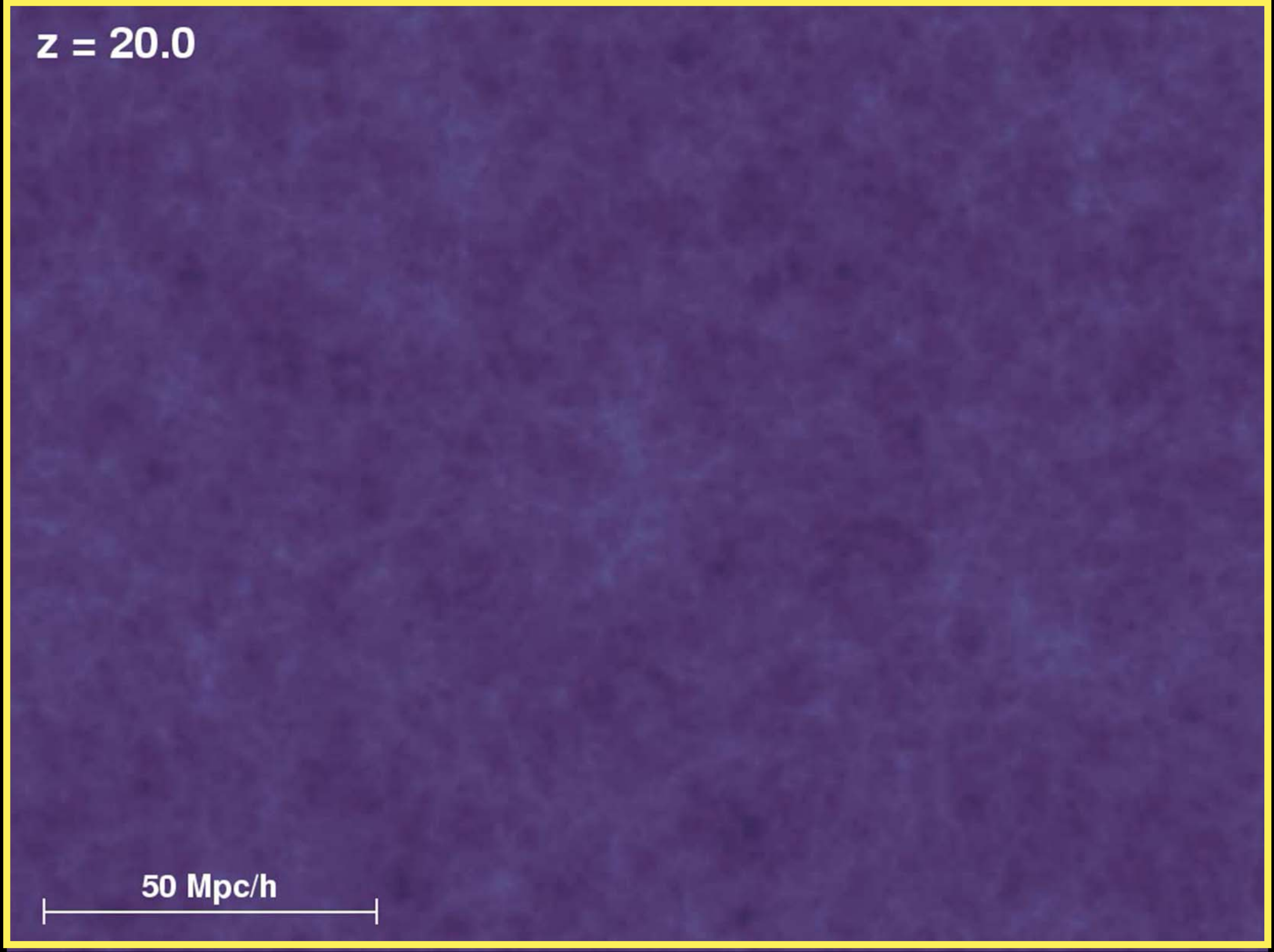
Cosmic Structure Formation

Formation
Cosmic Web:

simulation
sequence

(cold)
dark matter

(courtesy:
Virgo/V. Springel).



Dynamical Evolution Cosmic Web

- hierarchical structure formation
- anisotropic collapse
- establishing the connectivity
- void formation:
  asymmetry
  overdense vs. underdense

Dynamics of the Cosmic Web:

Anisotropic Collapse & Zeldovich Formalism

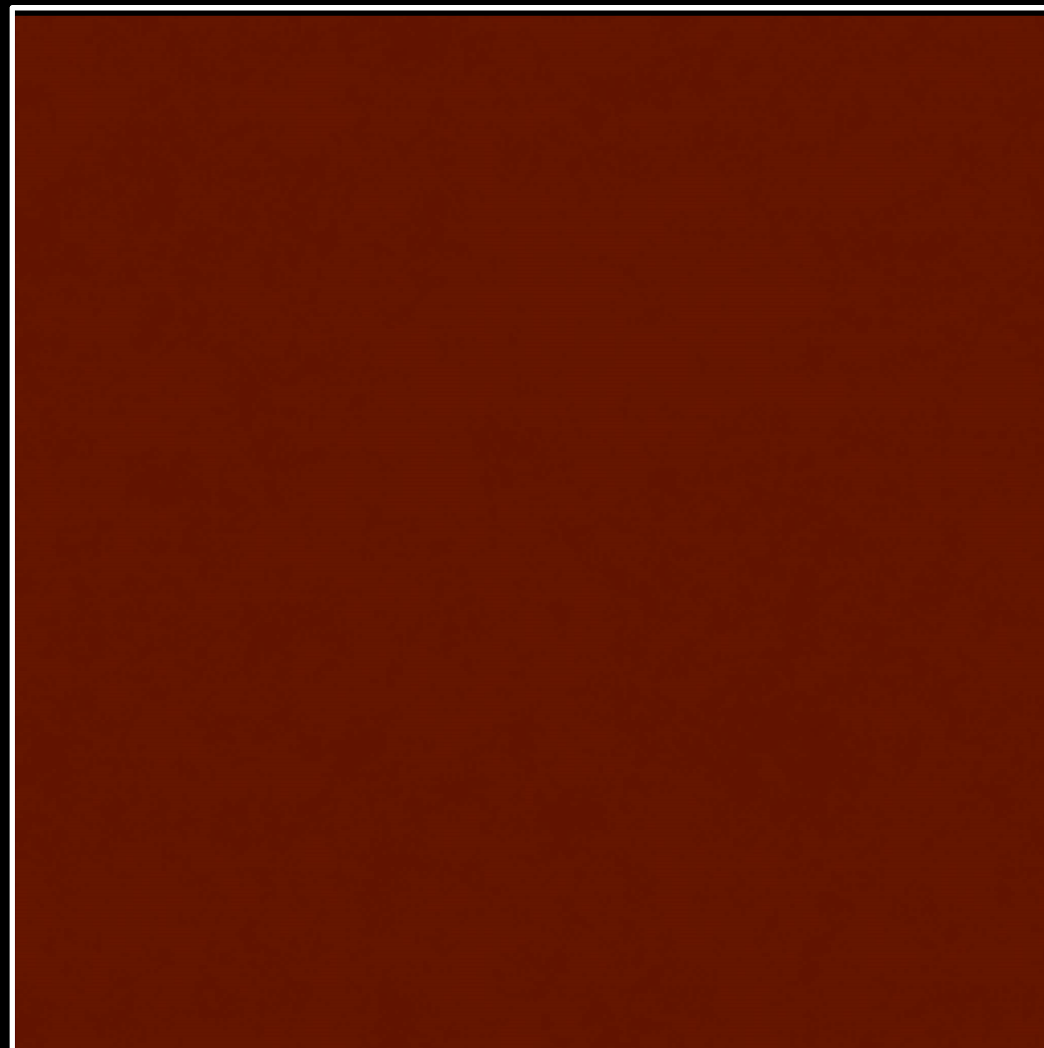
Yakov Borisovich Zel'dovich

Zel'dovich Approximation

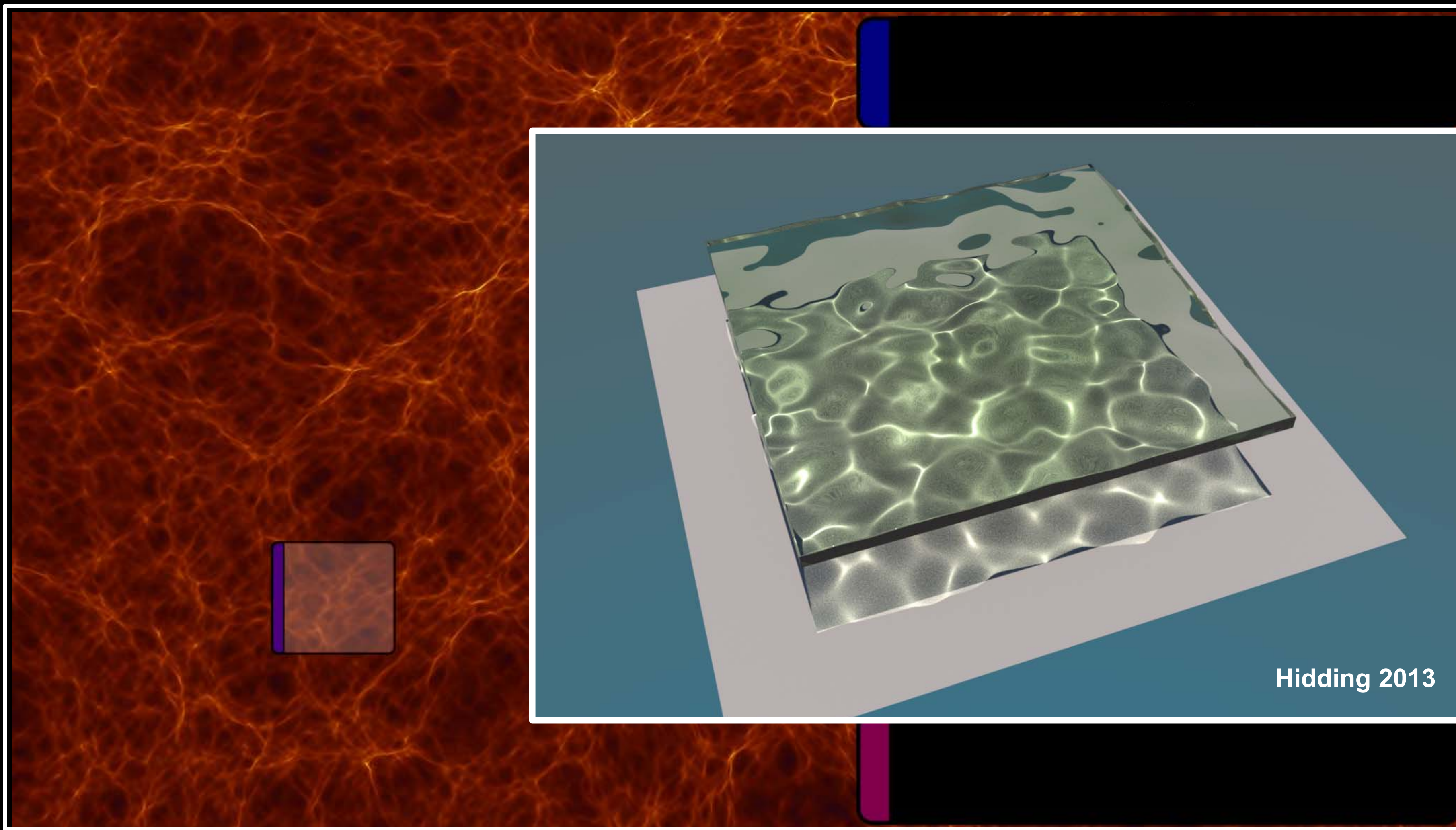
$$\vec{x} = \vec{q} + D(t)\,\vec{u}(\vec{q})$$

$$\vec{u}(\vec{q}) = -\vec{\nabla}\Phi(\vec{q})$$

$$\Phi(\vec{q}) = \frac{2}{3Da^2H^2\Omega}\phi_{lin}\left(\vec{q}\right)$$



Zel'dovich Approximation



Zel'dovich Approximation

$$\vec{x} = \vec{q} + D(t)\,\vec{u}(\vec{q})$$

$$\vec{u}(\vec{q}) = -\vec{\nabla}\Phi(\vec{q})$$

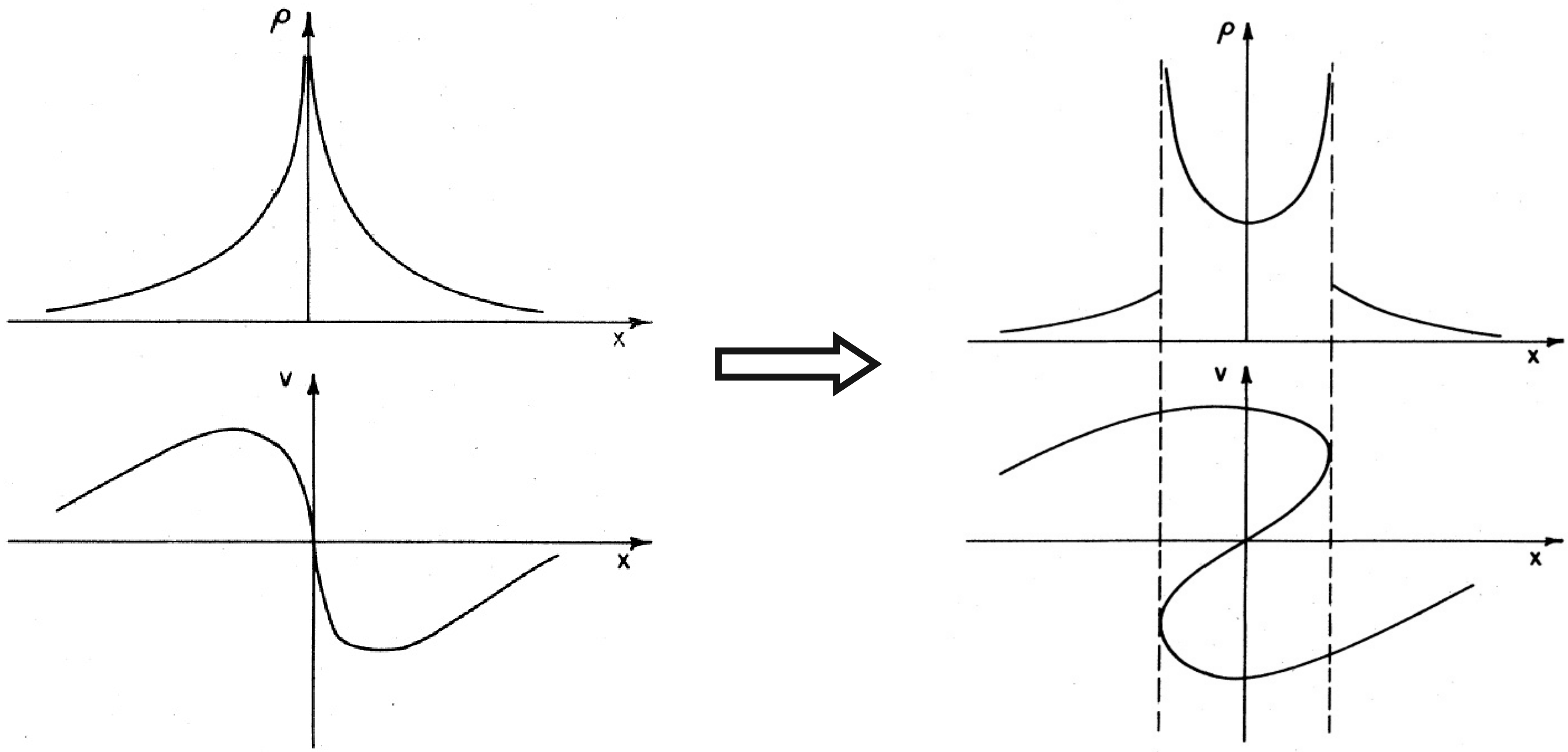
$$d_{ij} = -\frac{\partial u_i}{\partial q_j}$$

$$\rho(\vec{q},t) = \frac{\rho_u(t)}{\left(1 - D(t)\lambda_1(\vec{q})\right)\left(1 - D(t)\lambda_2(\vec{q})\right)\left(1 - D(t)\lambda_3(\vec{q})\right)}$$

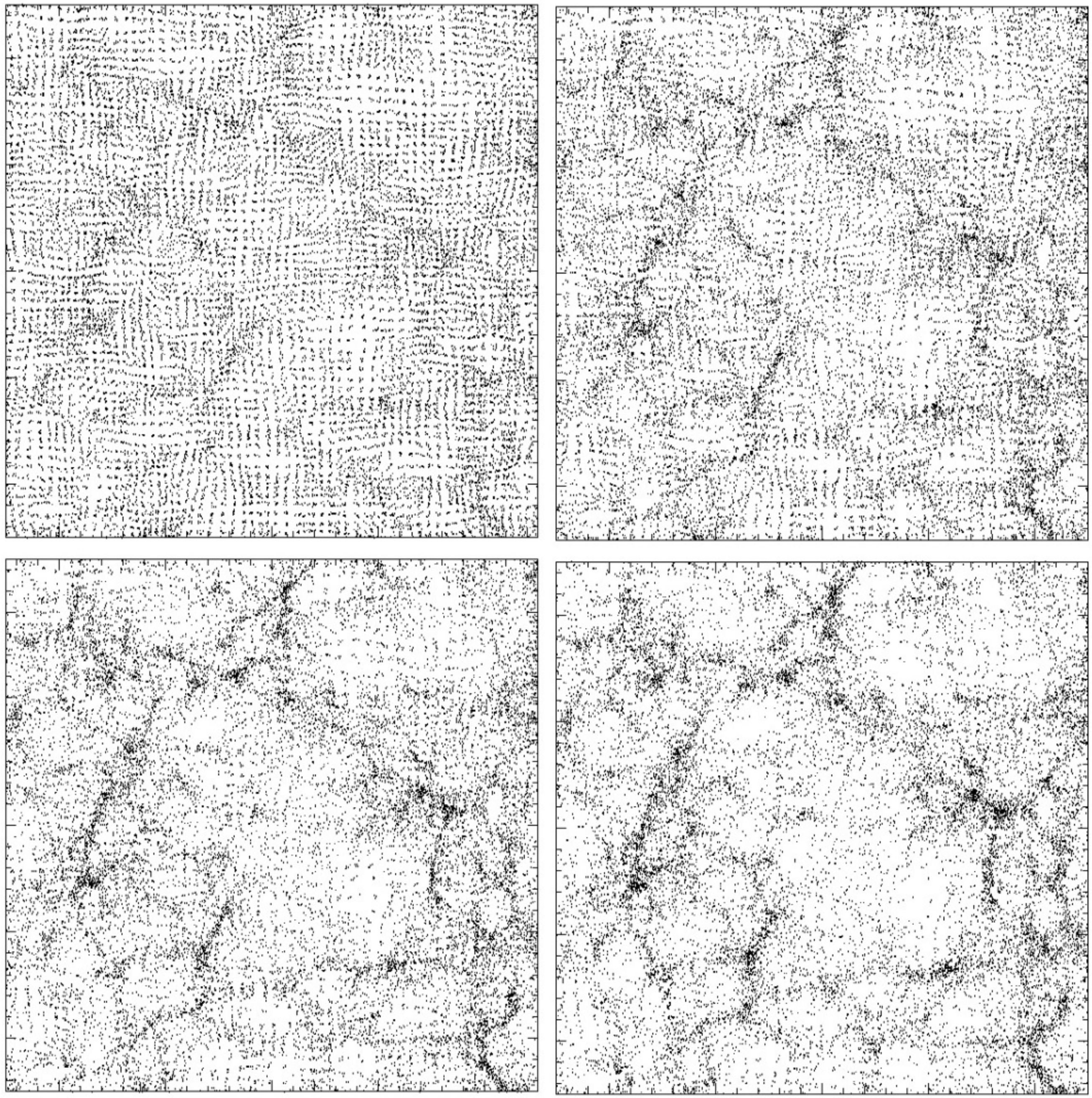
structure of the cosmic web determined by the spatial field of eigenvalues

$$\lambda_1, \lambda_2, \lambda_3$$

Zel'dovich Formalism: Density Evolution



Density Profile through pancake, at moment of formation and shortly thereafter (multistream)



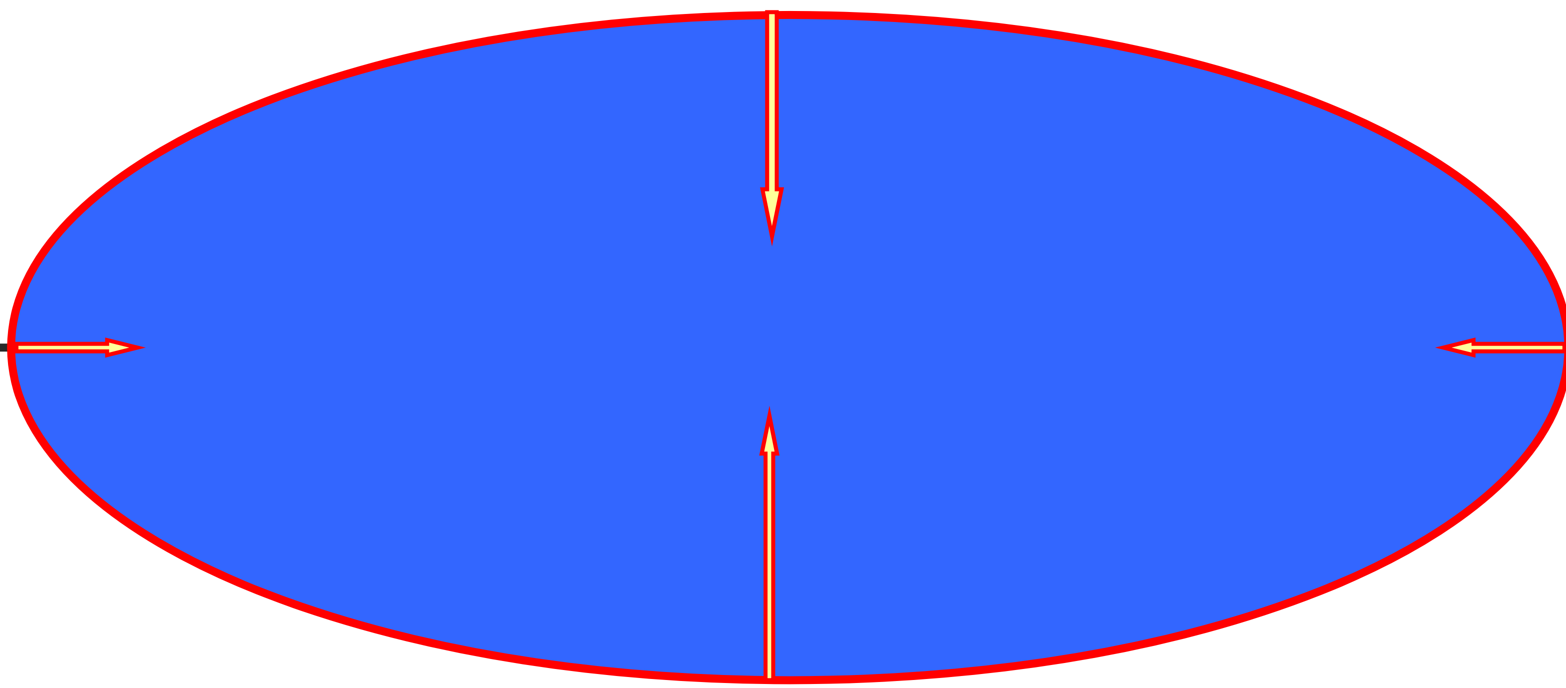
Cosmic Web Connectivity:

weaving the Cosmic Tapestry

Anisotropic

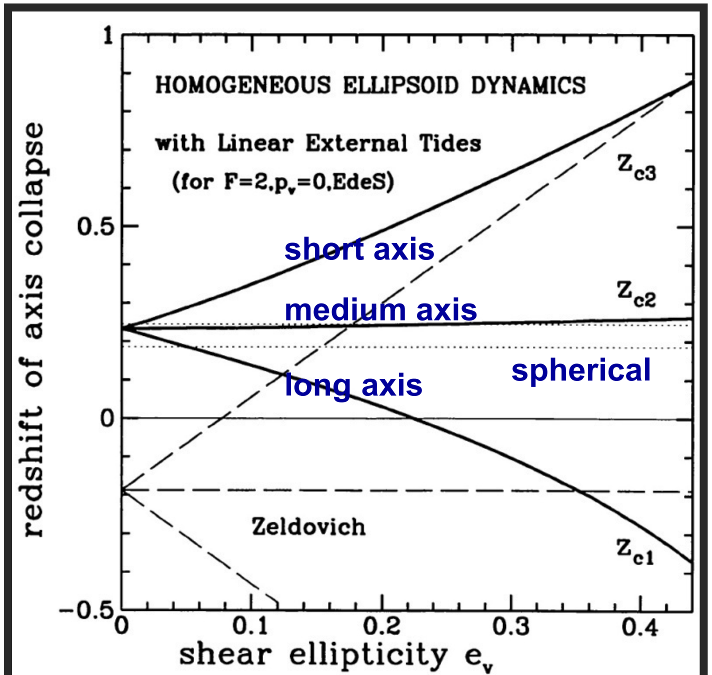
Gravitational Collapse

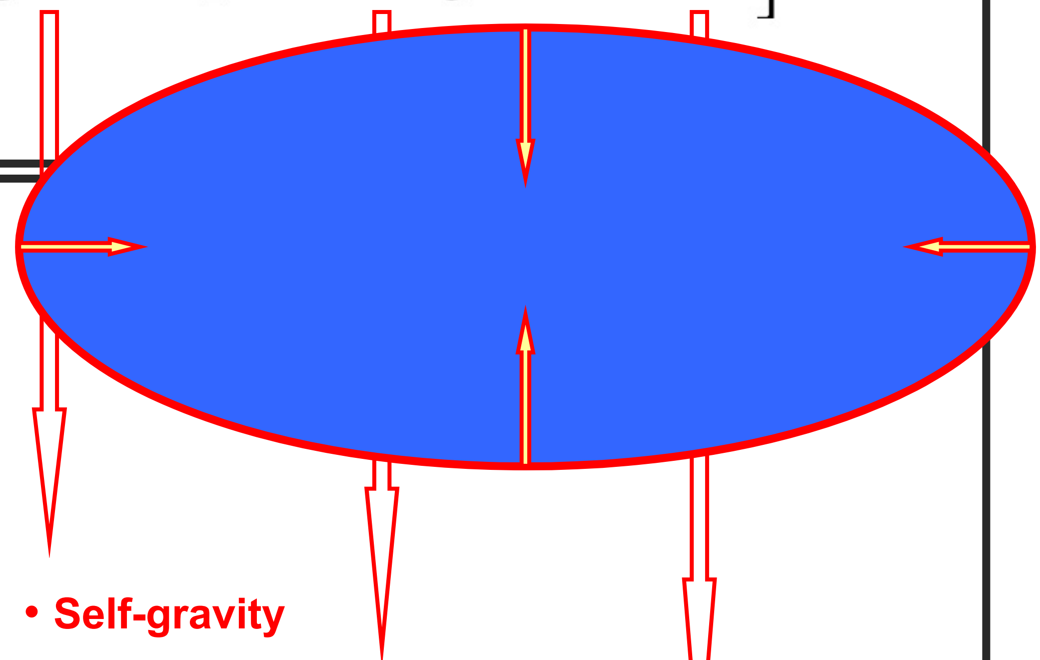
Amplification
small perturbations in gravity along different directions (tidal forces)



Anisotropic (Ellipsoidal) Collapse

$$\frac{d^2\mathcal{R}_\mathrm{m}}{dt^2} = -4\pi G\rho_\mathrm{u}(t)\left[\frac{1+\delta}{3} + \frac{1}{2}\left(\alpha_m - \frac{2}{3}\right)\delta + \lambda'_\mathrm{vm}\right]\mathcal{R}_\mathrm{m}$$





- Self-gravity
- Internal tidal shear (due to shape)
- External tidal shear

Formative agent of the Cosmic Web:

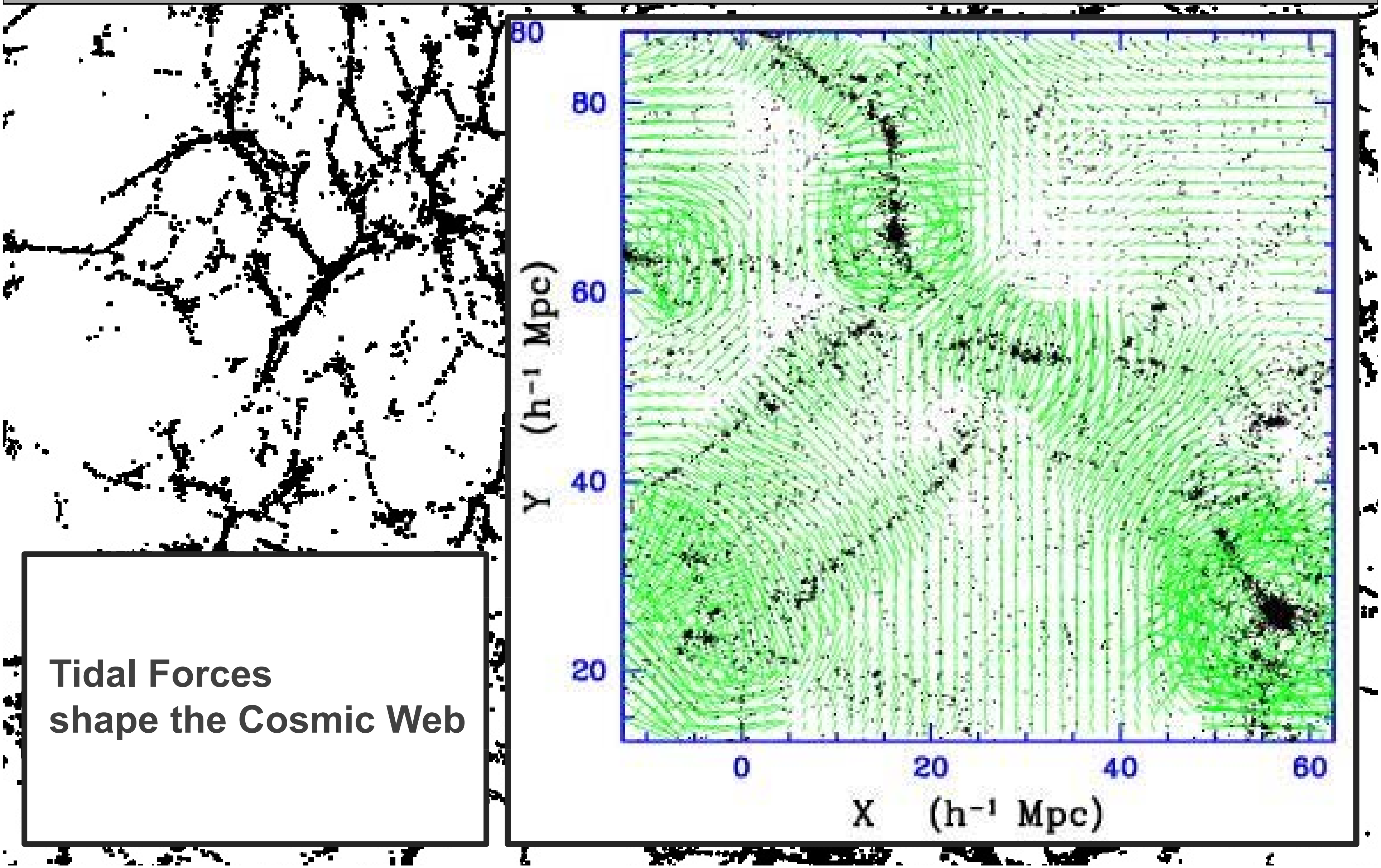
Tidal strain induced my the Megaparsec Matter Distribution:

- **anisotropic collapse of structures**
- **connection clusters-filaments:**

  clusters main agent for stretching filaments

$$T_{ij}(\vec{r},t) = \frac{3\Omega H^2}{8\pi} \int d\vec{x}\ \delta(\vec{x},t) \left\{ \frac{3(x_i - r_i)(x_j - r_j) - |\vec{x}-\vec{r}|^2\, \delta_{ij}}{|\vec{x}-\vec{r}|^5} \right\} - \frac{1}{2}\Omega H^2 \delta(\vec{r},t)\ \delta_{ij}$$

Tidal Shaping of the Cosmic Web

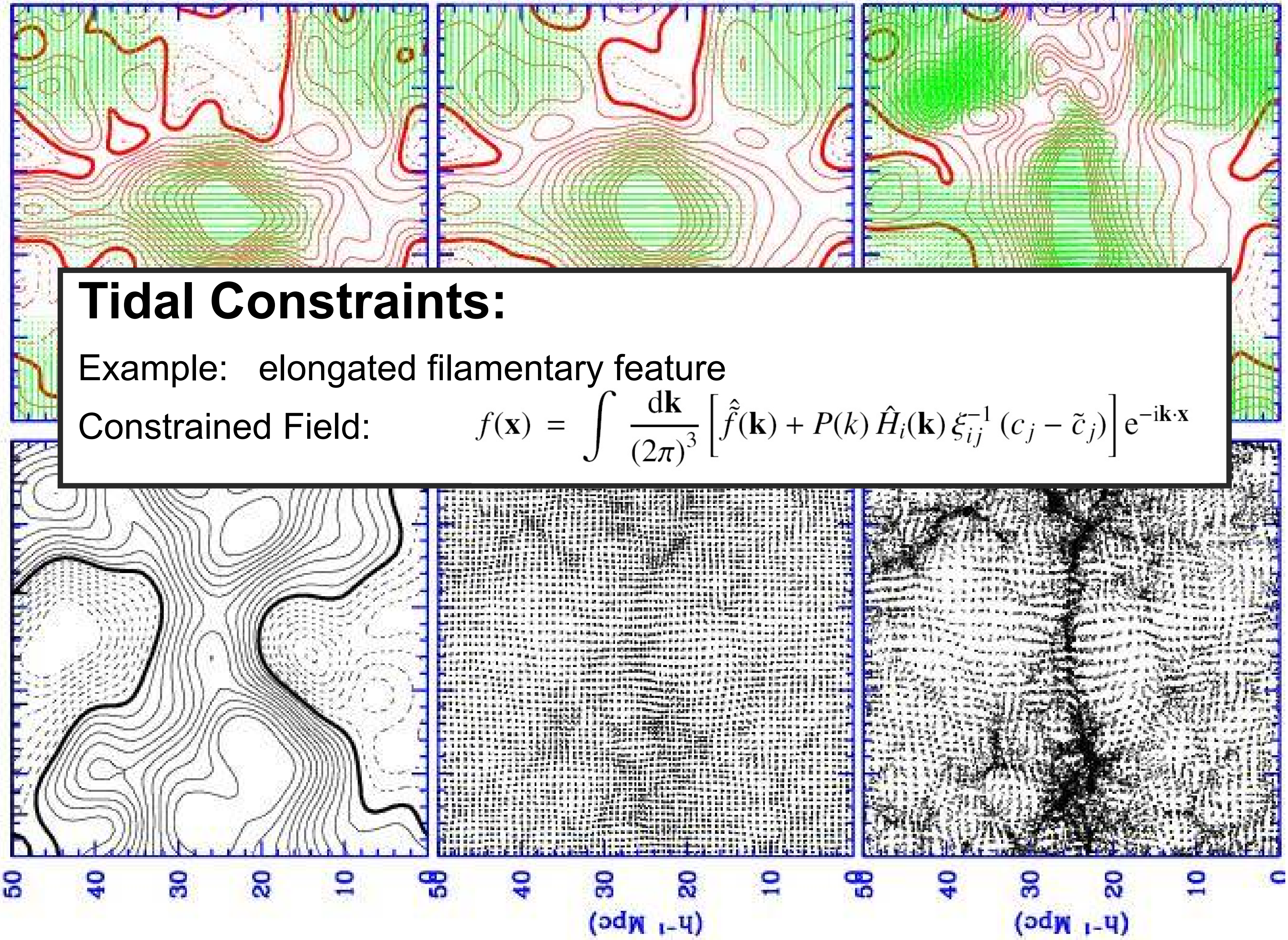


Tidal Forces
shape the Cosmic Web

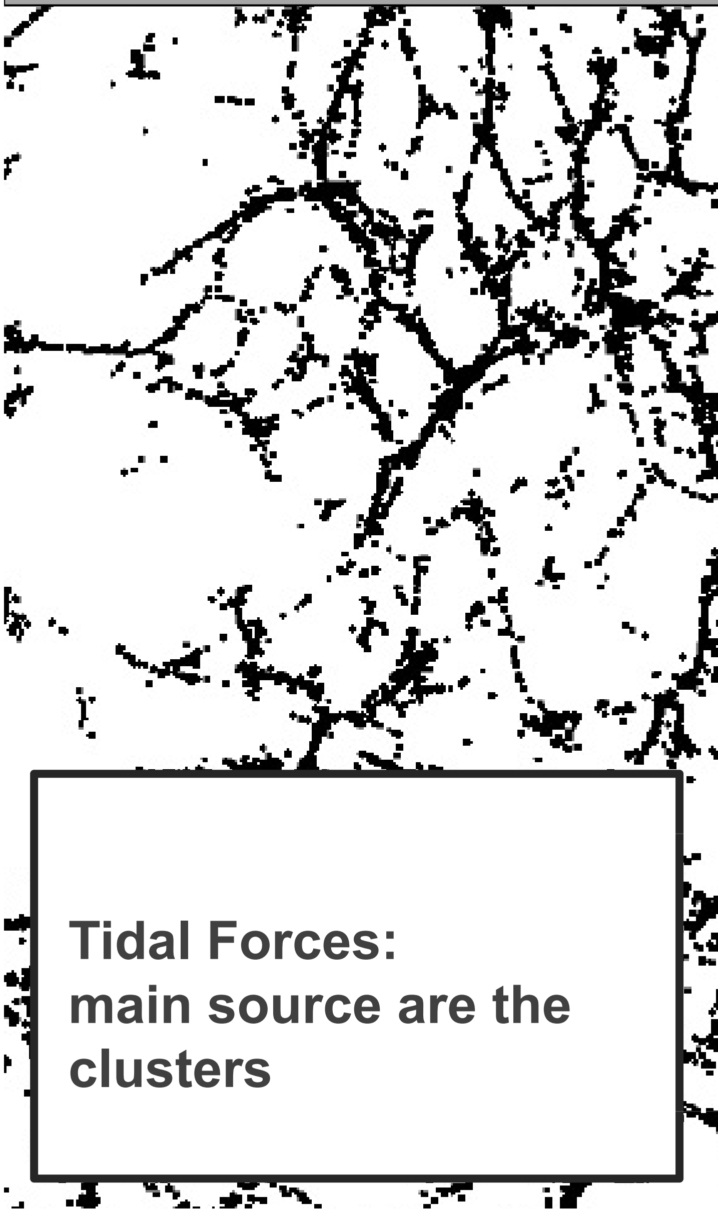
Tidal Constraints:

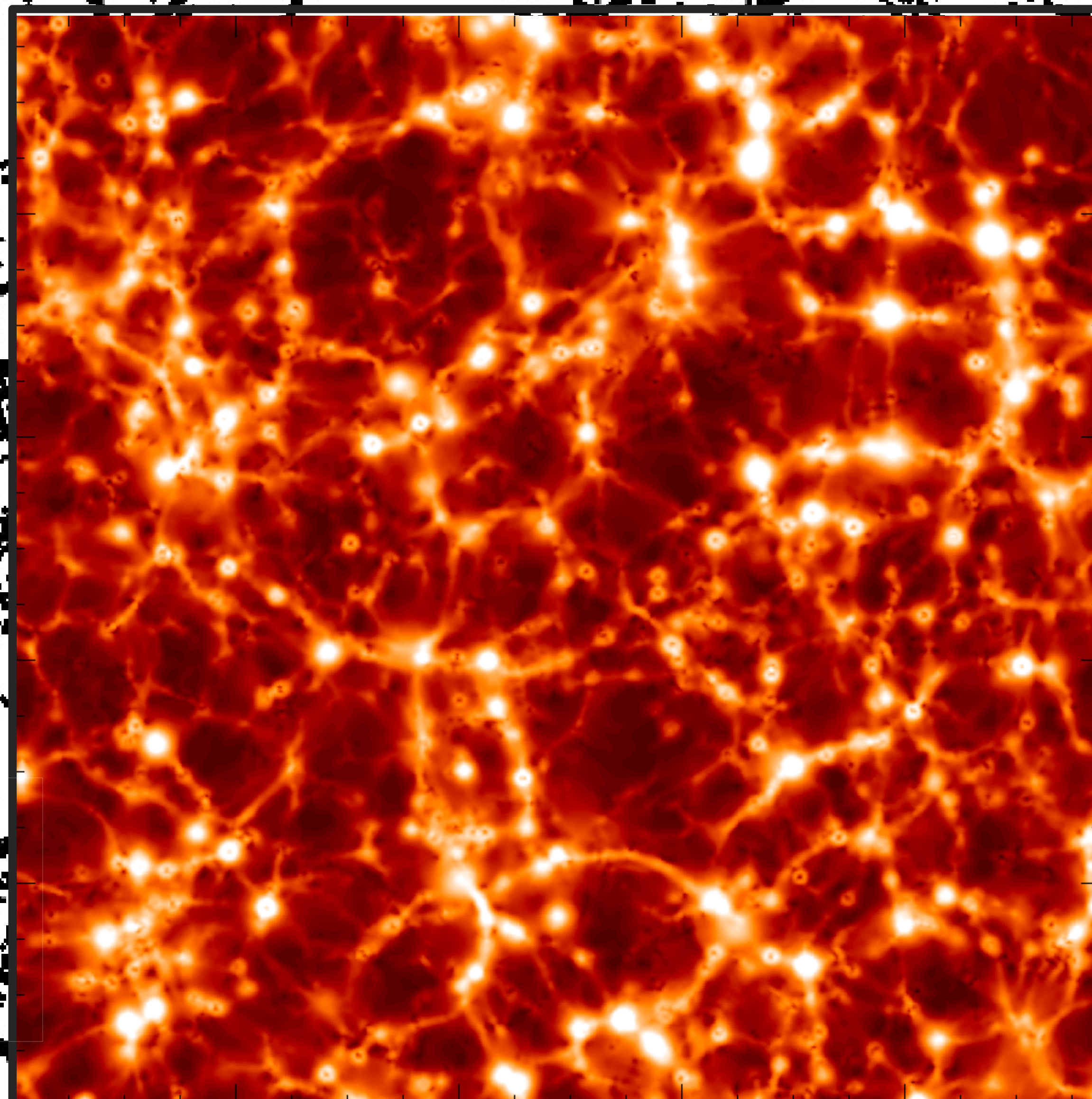
Example: elongated filamentary feature

Constrained Field: $$f(\mathbf{x}) = \int \frac{d\mathbf{k}}{(2\pi)^3} \left[\hat{\tilde{f}}(\mathbf{k}) + P(k)\, \hat{H}_i(\mathbf{k})\, \xi_{ij}^{-1}\, (c_j - \tilde{c}_j) \right] e^{-i\mathbf{k}\cdot\mathbf{x}}$$



Tidal Shaping of the Cosmic Web



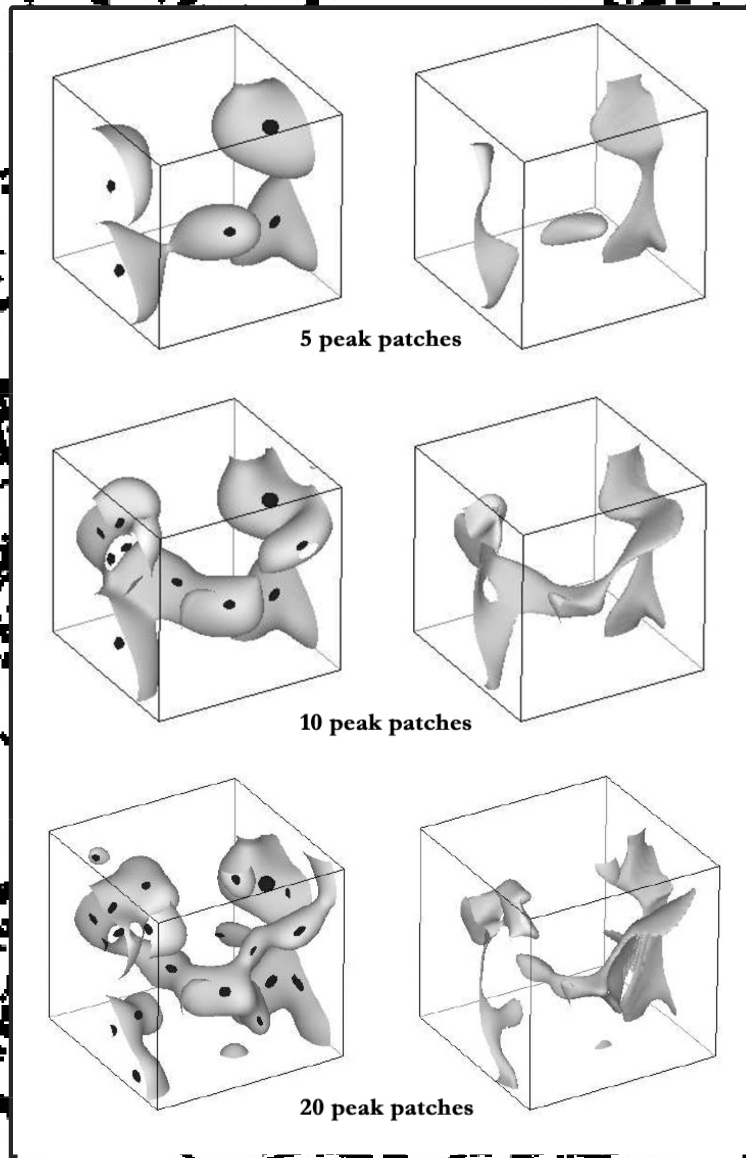


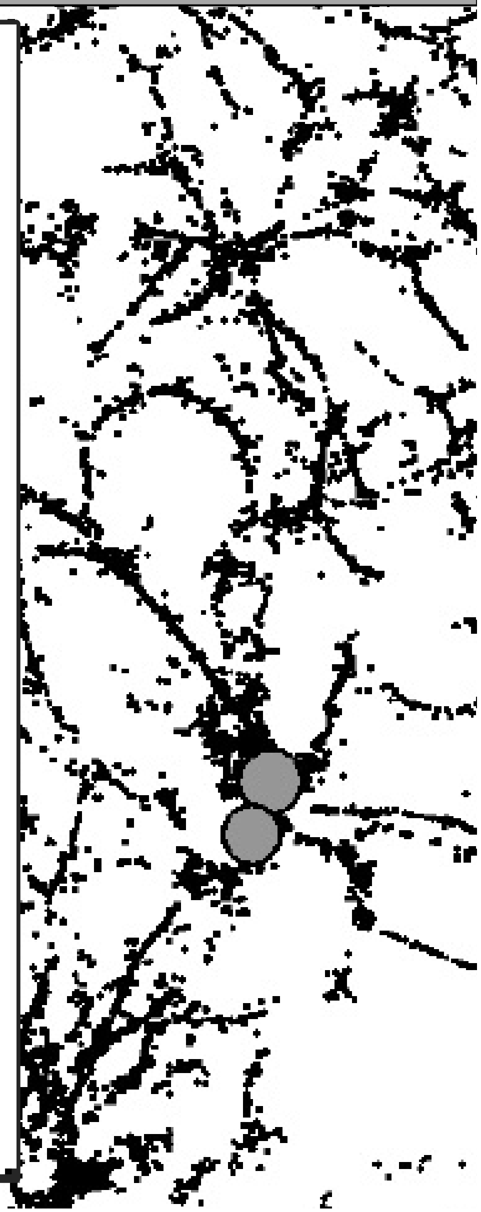
Tidal Forces:
main source are the clusters

Tidal Shaping of the Cosmic Web

Cosmic Web Theory

Bond, Kofman & Pogosyan 1996



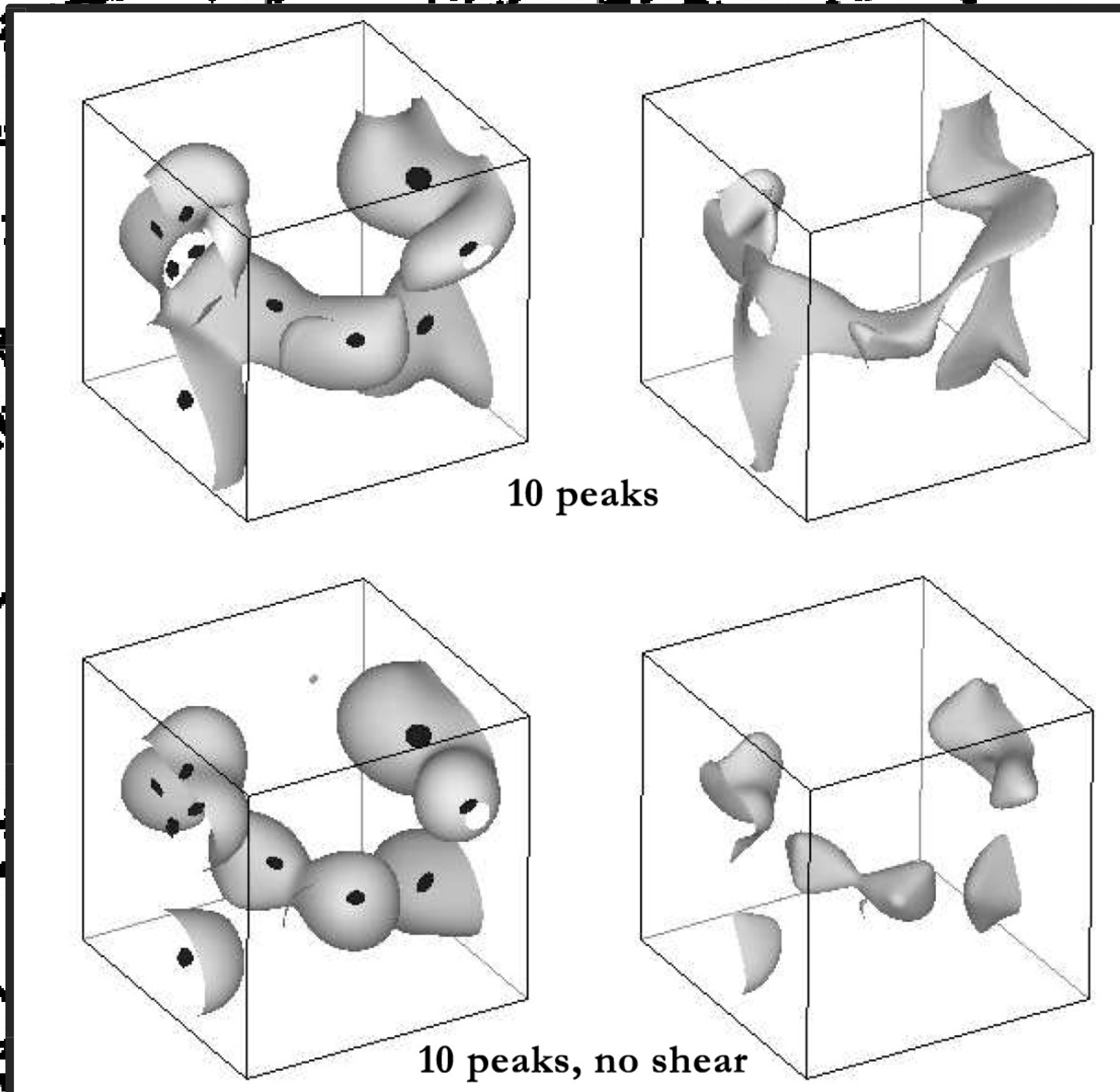


Tidal Forces:
main source are the clusters

Tidal Shaping of the Cosmic Web

Cosmic Web Theory

Bond, Kofman & Pogosyan 1996

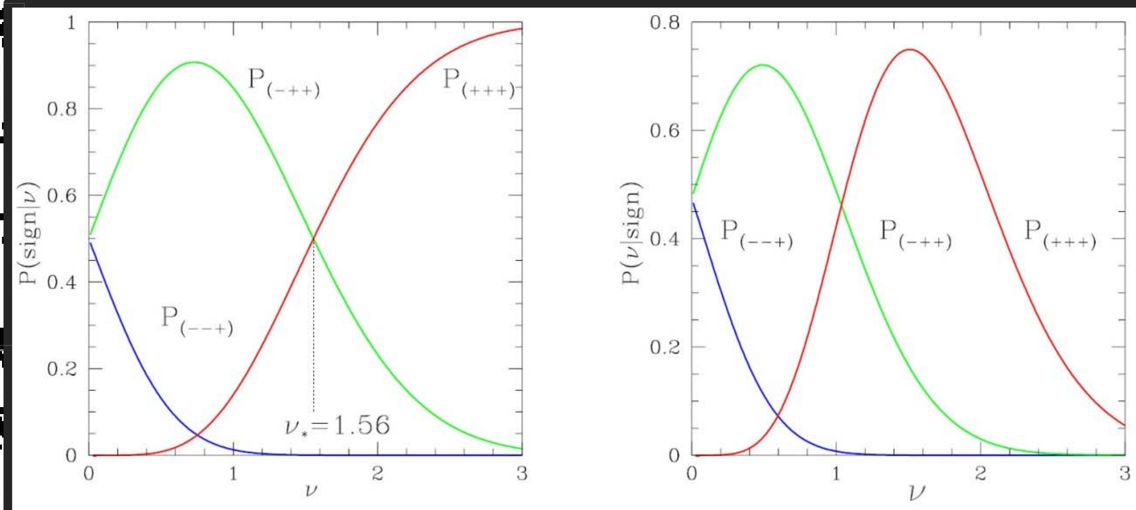


Tidal Forces:
main source are the clusters

Tidal Shaping of the Cosmic Web

Cosmic Web Theory

Bond, Kofman & Pogosyan 1996



Conditional Statistics
Tidal Shear eigenvalues in Gaussian field:

in overdense regions:
most prominent structures are FILAMENTS

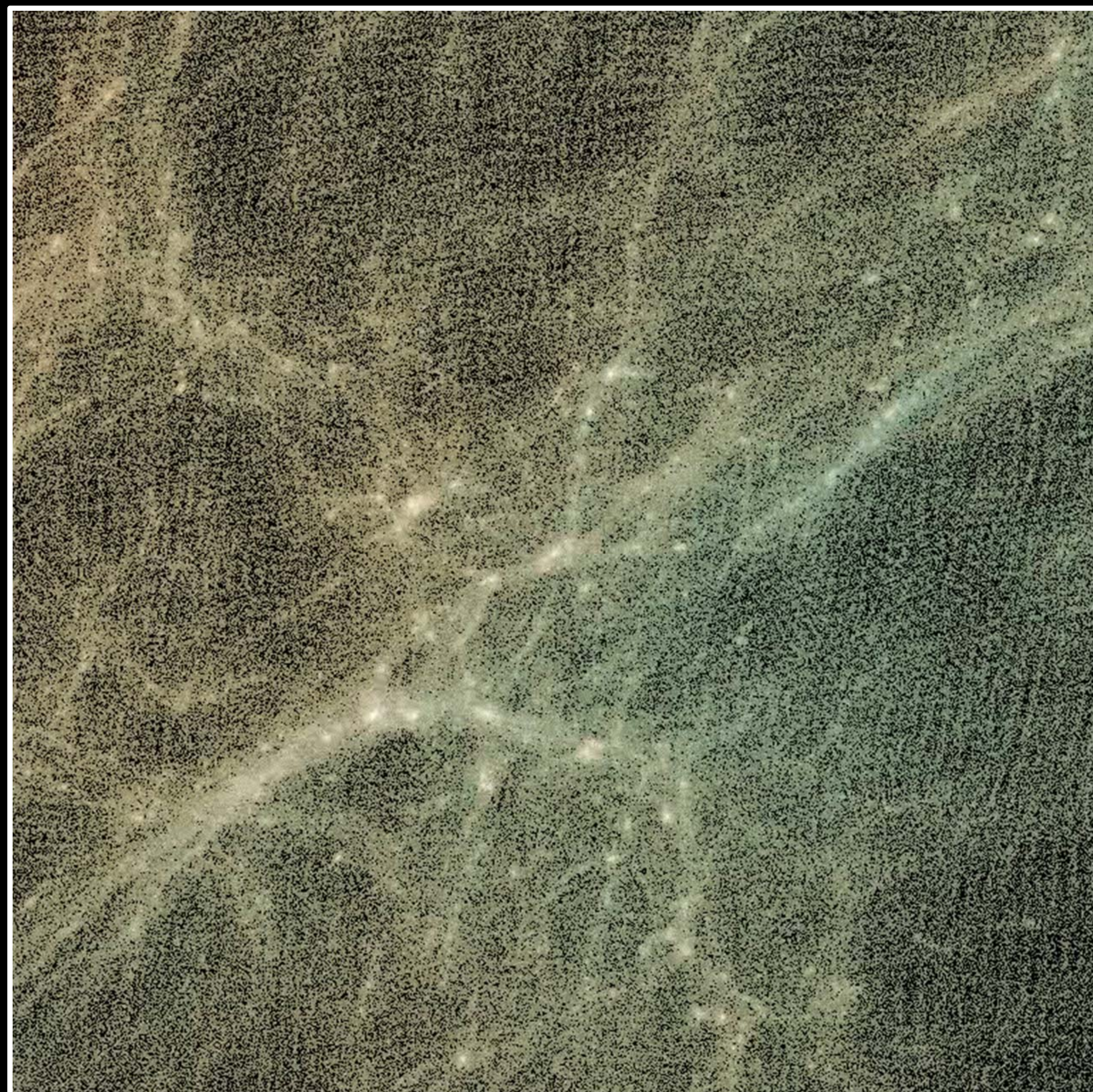
in underdense regions:
most prominent structure are WALLS

Tidal Forces:
main source are the clusters

Cosmic Web

Hierarchical Evolution

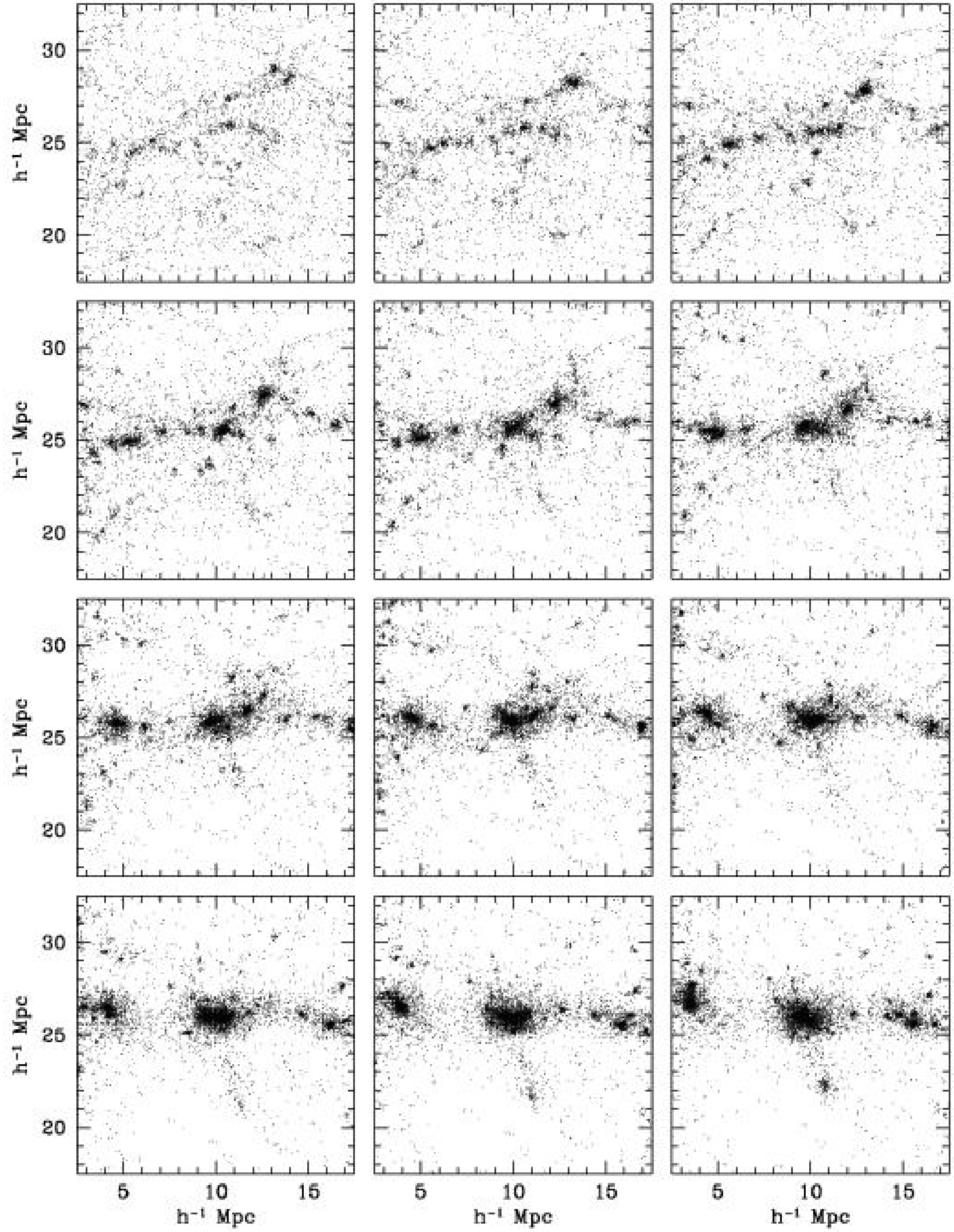
CGV halo & web evolution



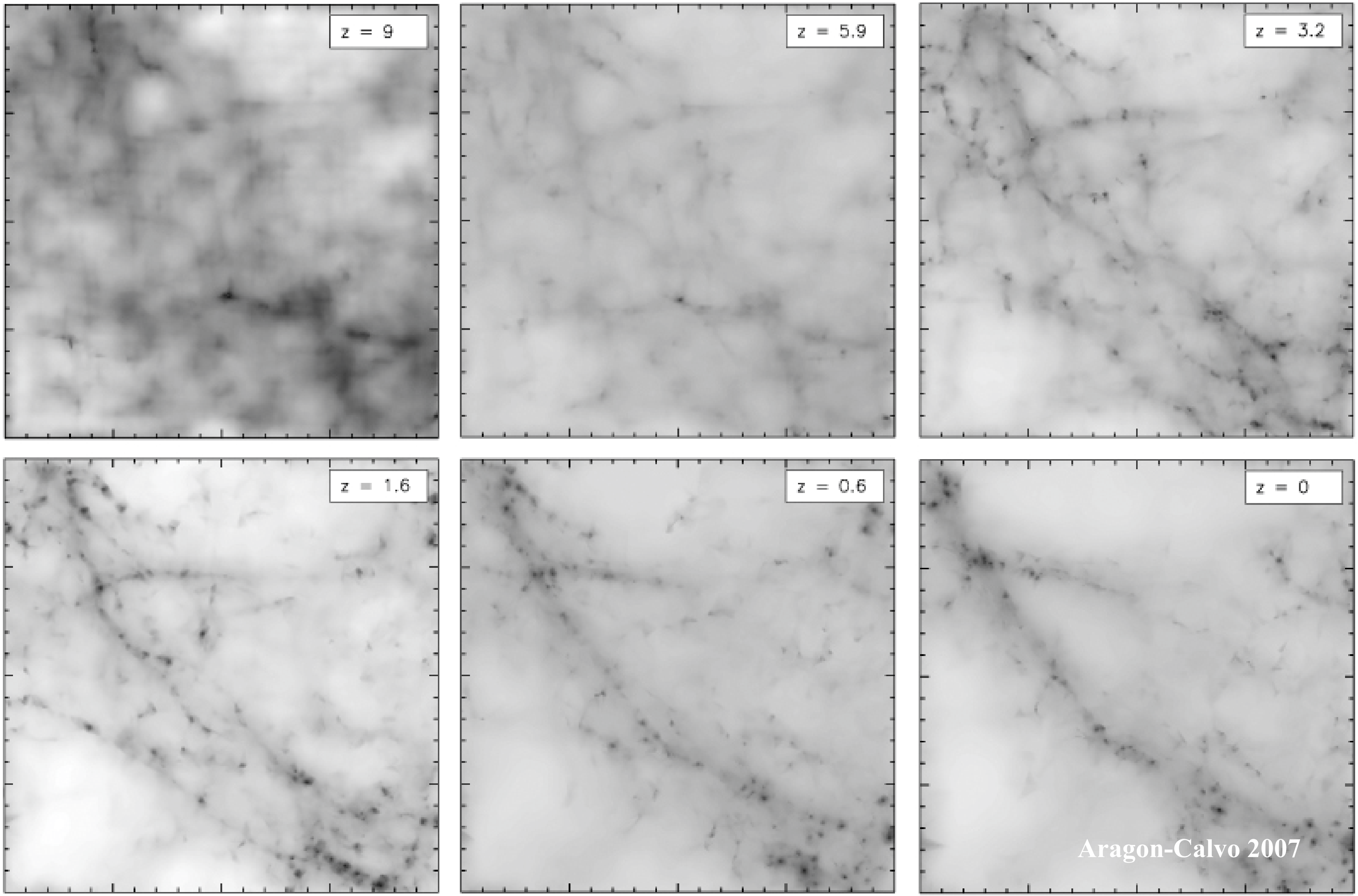
Rieder et al. 2013

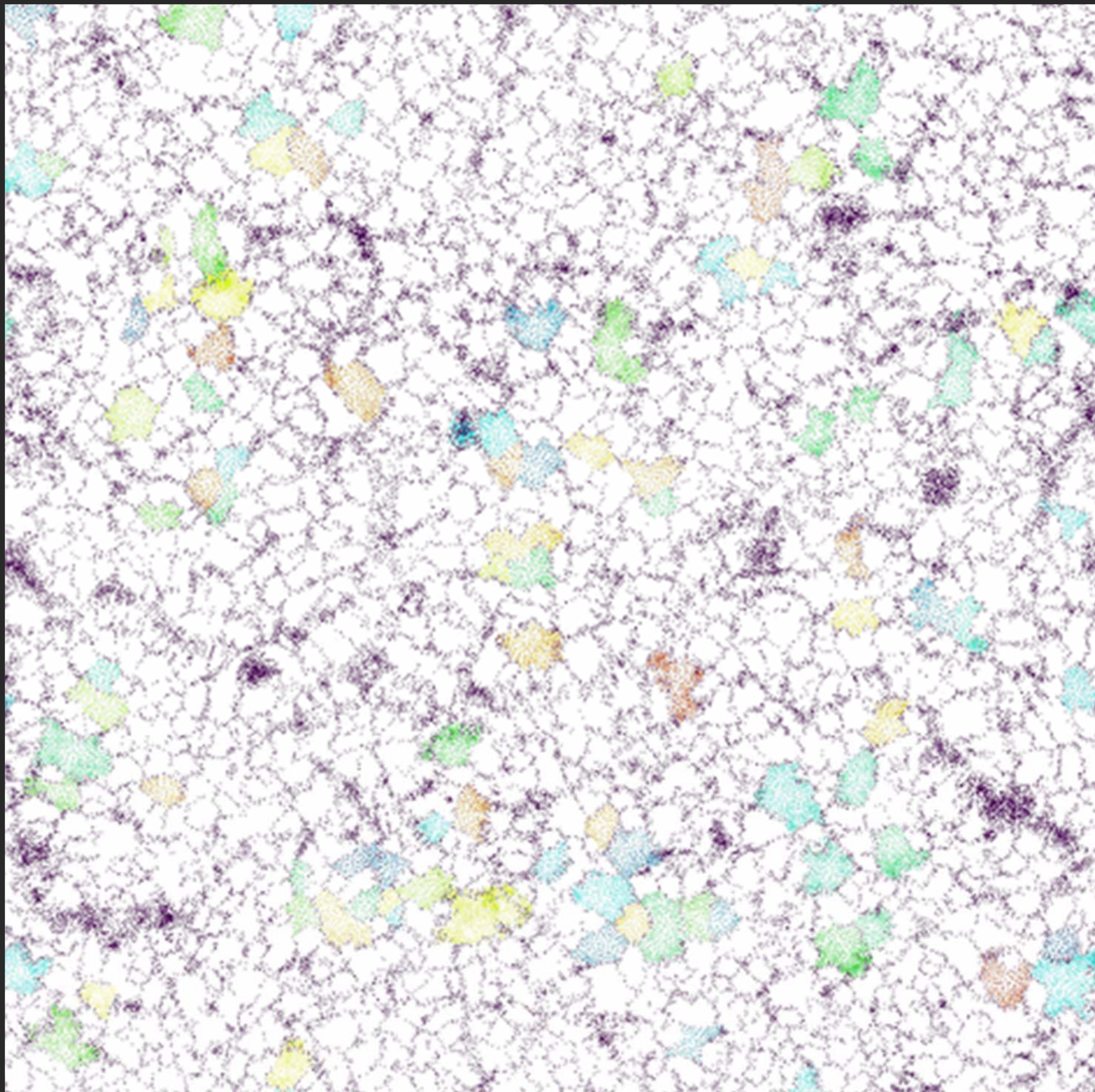
Structures in the Universe form by gradual hierarchical assembly:

- ❖ **small objects emerge & collapse first,**
- ❖ **then merge with other clumps**
- ❖ **while forming larger objects in hierarchy**



Hierarchical Filament Formation





Void Hierarchy:

"Lagrangian" view:

void-dominated
hierarchical
development
Cosmic Web

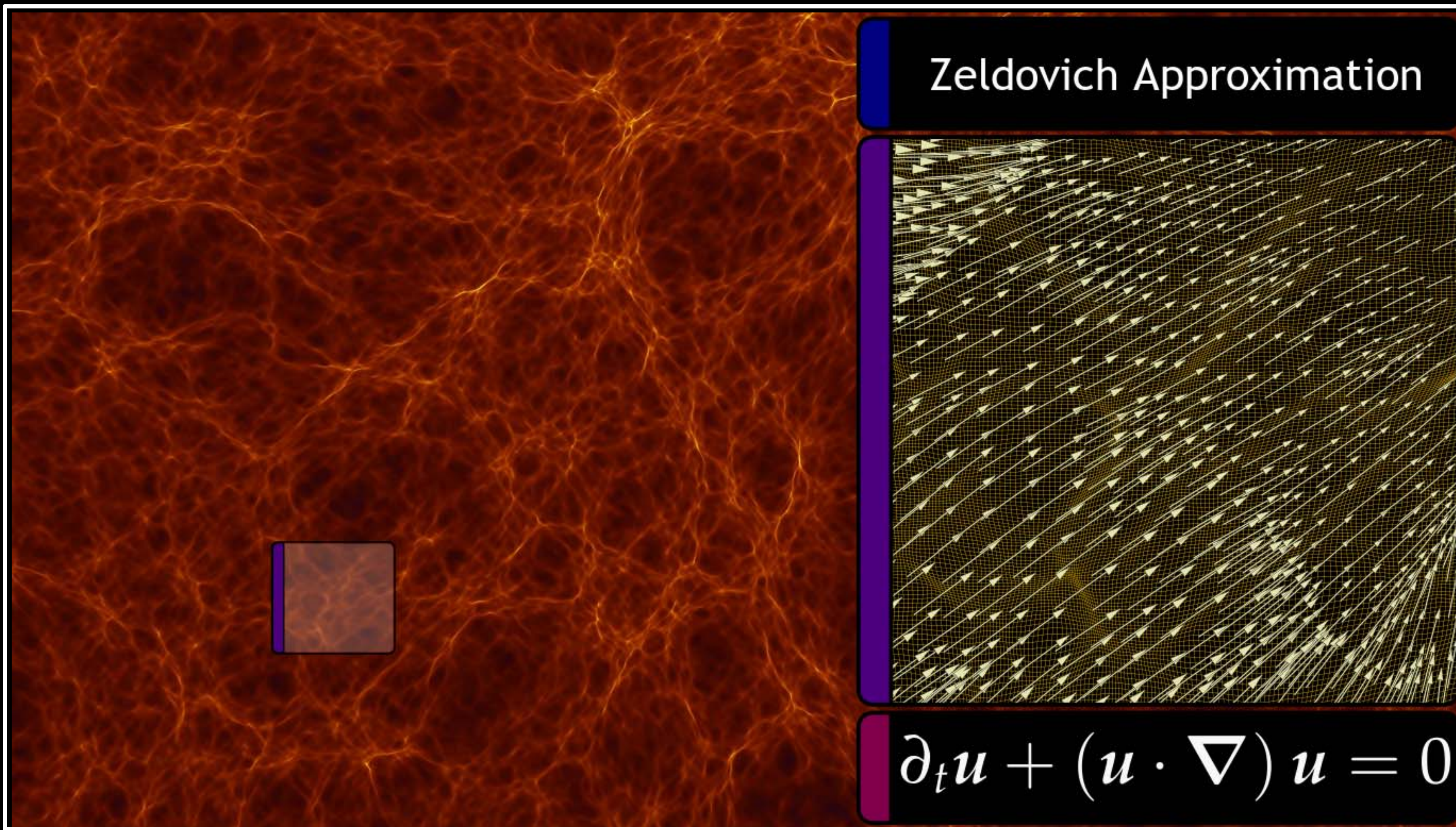
Platen & vdW 2004
Sheth & vdW 2004
Aragon-Calvo & Szalay 2012

Hierarchical Dynamics of the Cosmic Web:

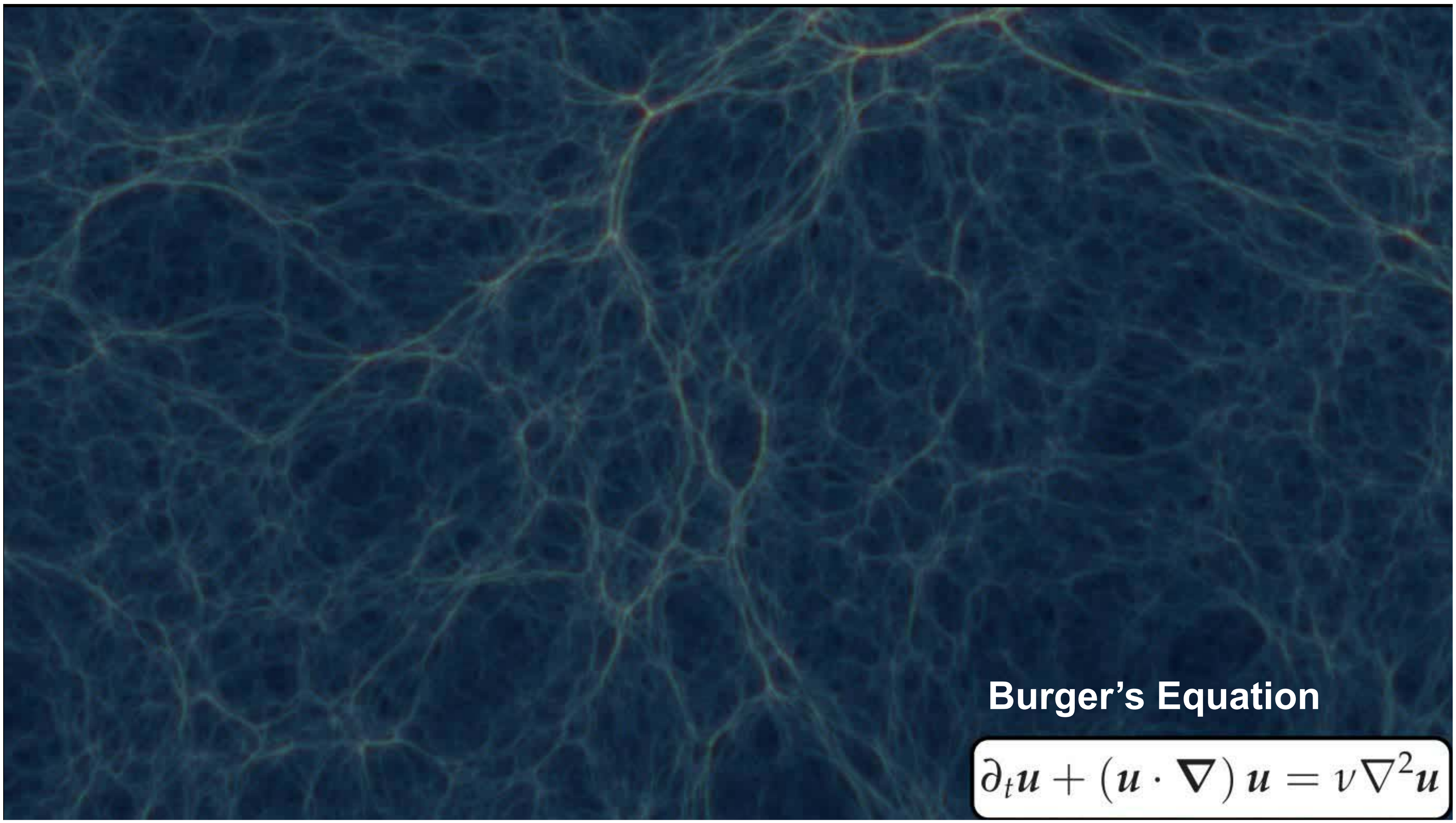
Adhesion

Hidding, vdW et al. 2012,
Hidding, vdW et al. 2016
Hidding, vdW et al. 2018

Zel'dovich Approximation



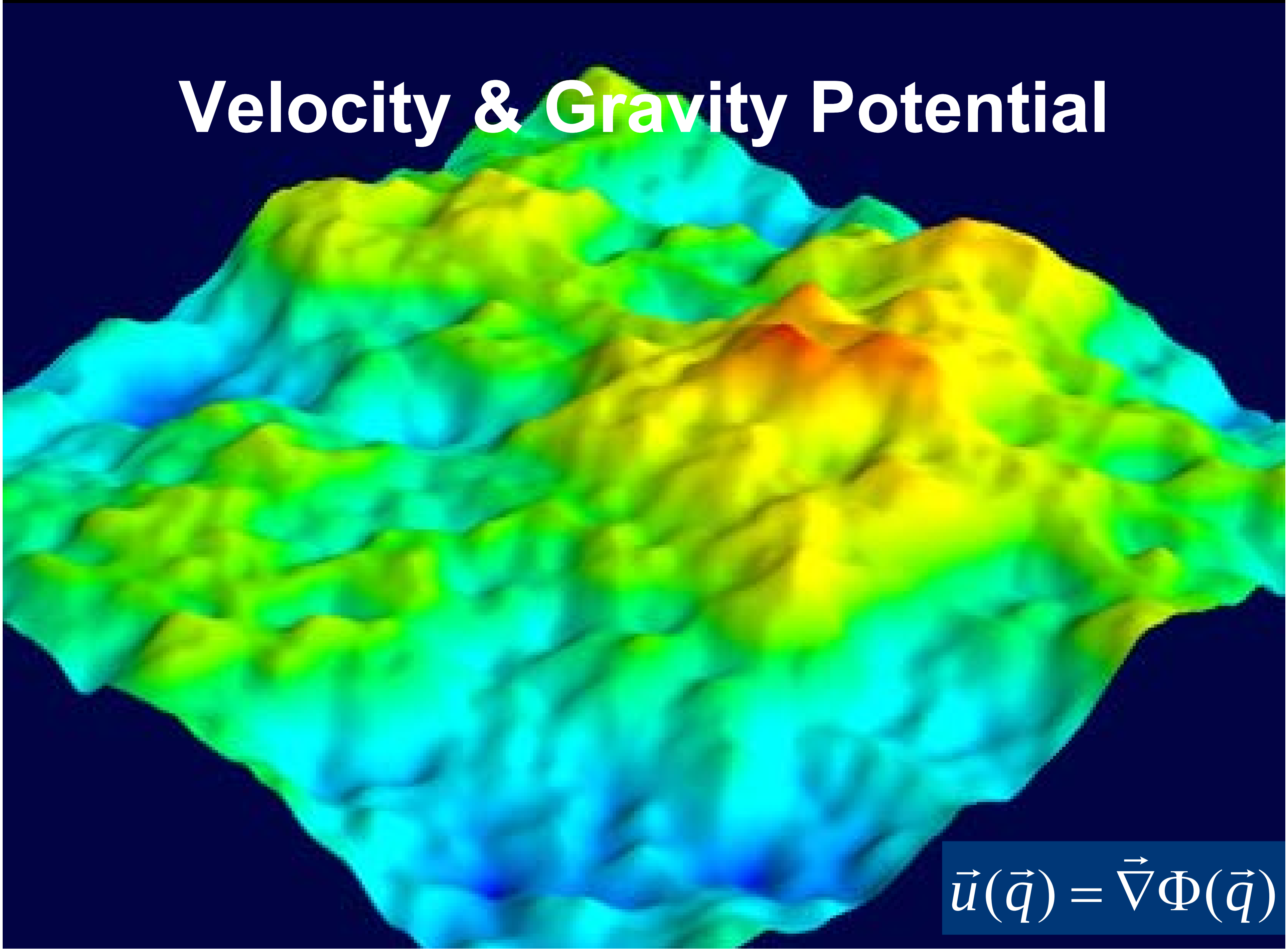
Adhesion Approximation



Burger's Equation

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\, \boldsymbol{u} = \nu \nabla^2 \boldsymbol{u}$$

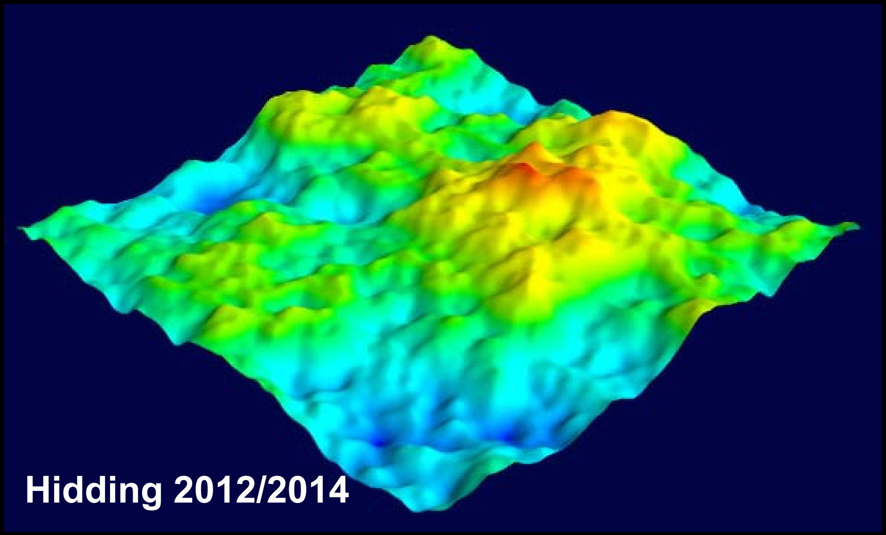
Velocity & Gravity Potential

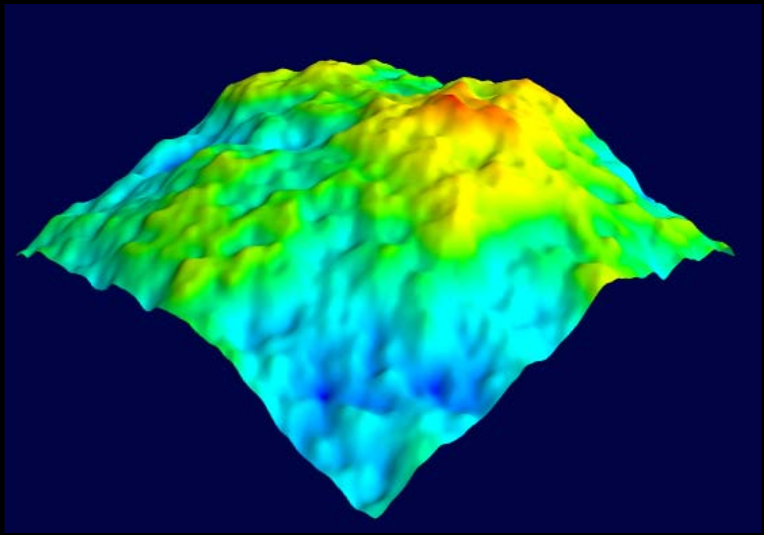


$$\vec{u}(\vec{q}) = \vec{\nabla}\Phi(\vec{q})$$

Burger's Equation: Hopf Solution

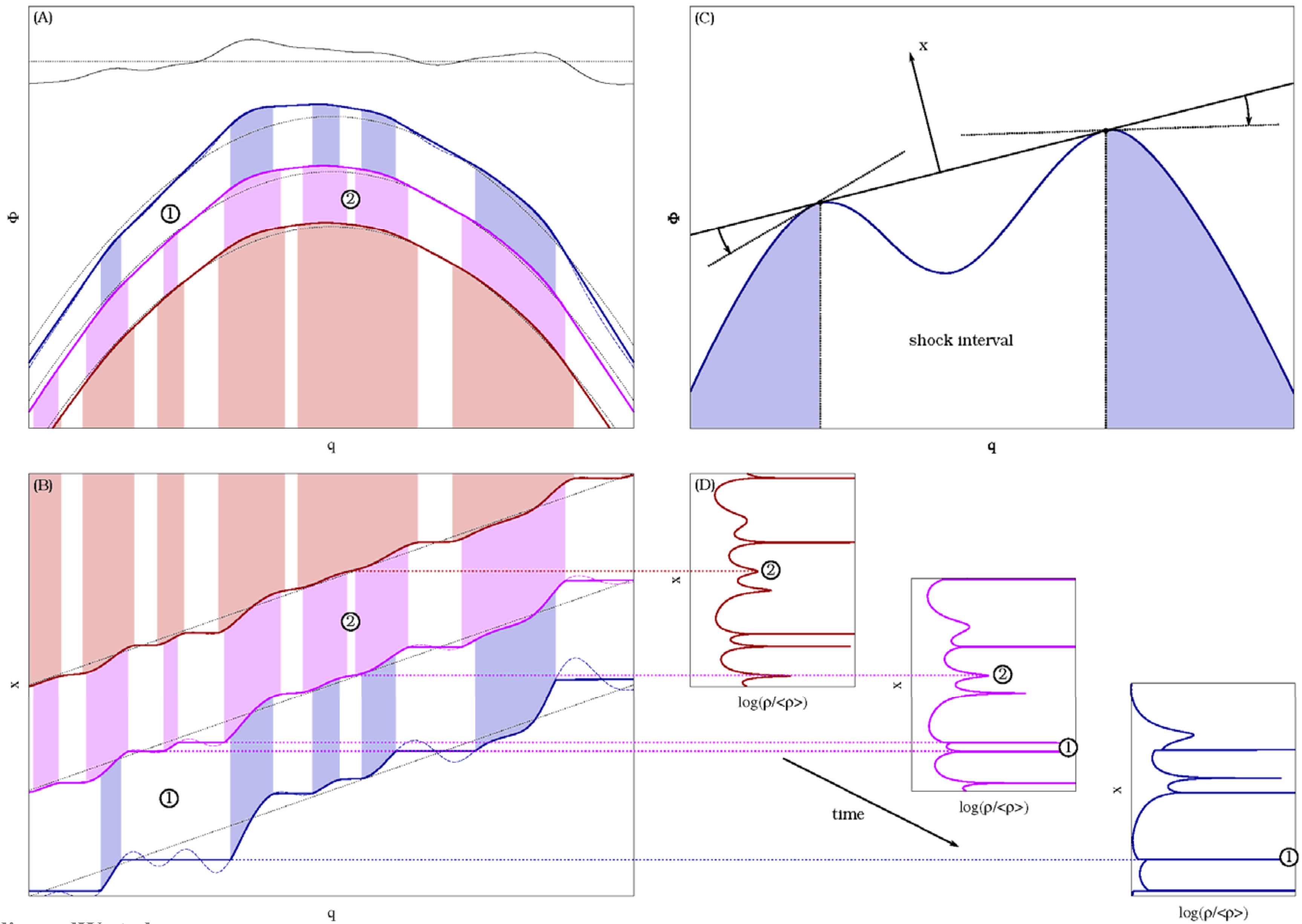
$$\frac{\partial \vec{u}}{\partial t} + \left(\vec{u} \cdot \vec{\nabla}\right)\vec{u} = \nu \nabla^2 \vec{u}$$



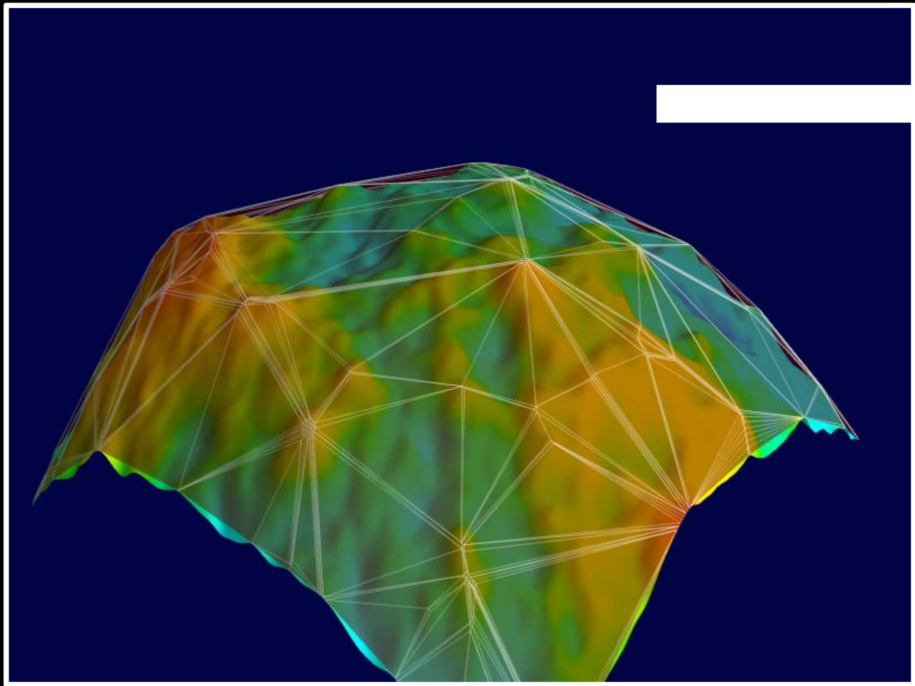


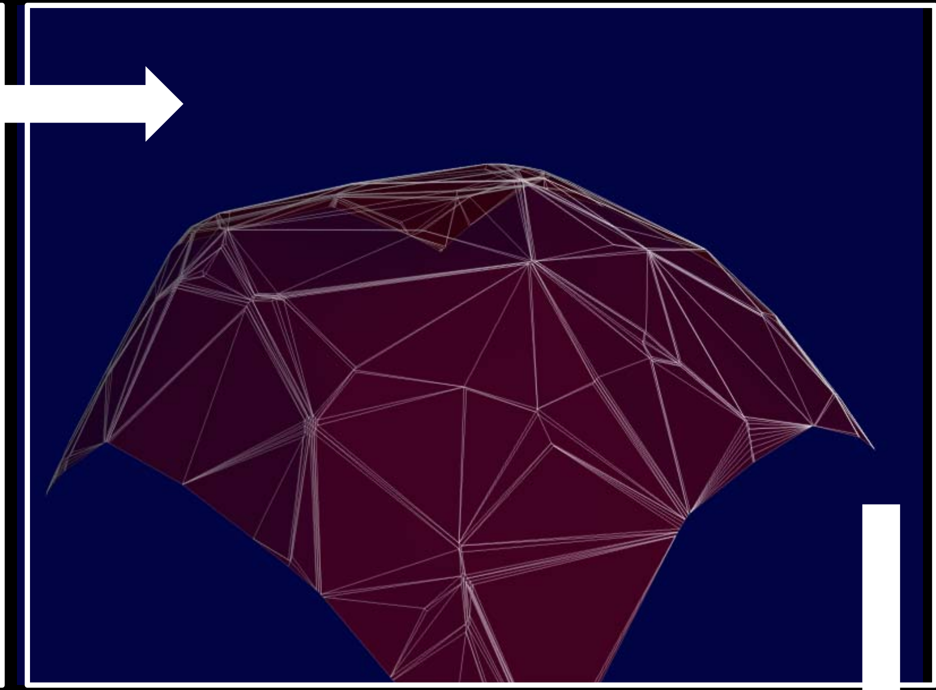
$$\Phi(\vec{x},t) + \frac{x^2}{2} = \max_q \left[\left(t\Phi_0(q) - \frac{q^2}{2} \right) + \vec{x} \cdot \vec{q} \right]$$

Burger's Equation: Hopf Solution



Hidding, vdW et al.

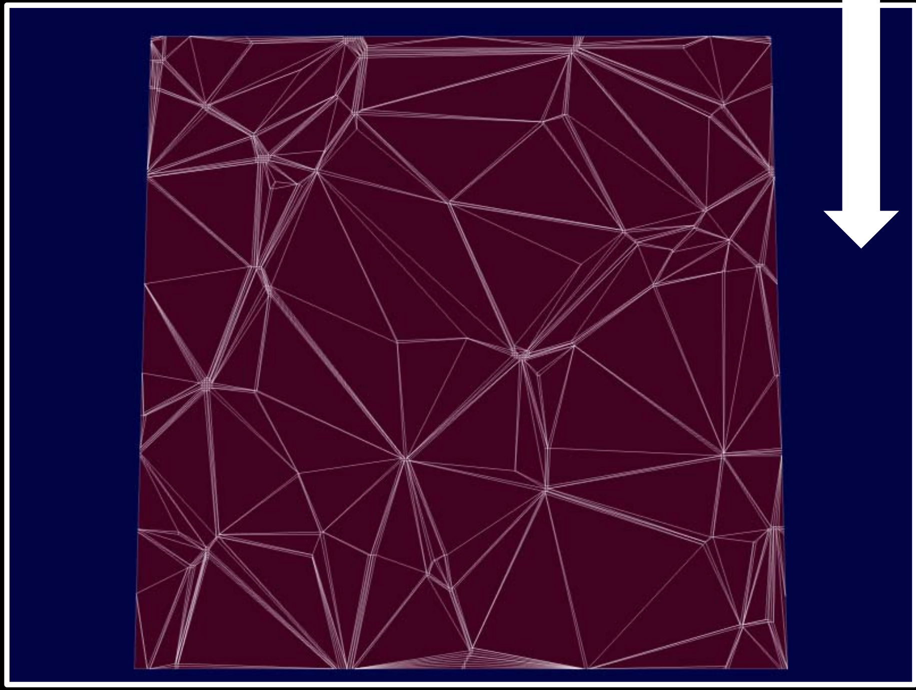




Hidding 2012/2014

Convex Hull
quadratically lifted potential field

Delaunay tessellation
generated by maxima potential field

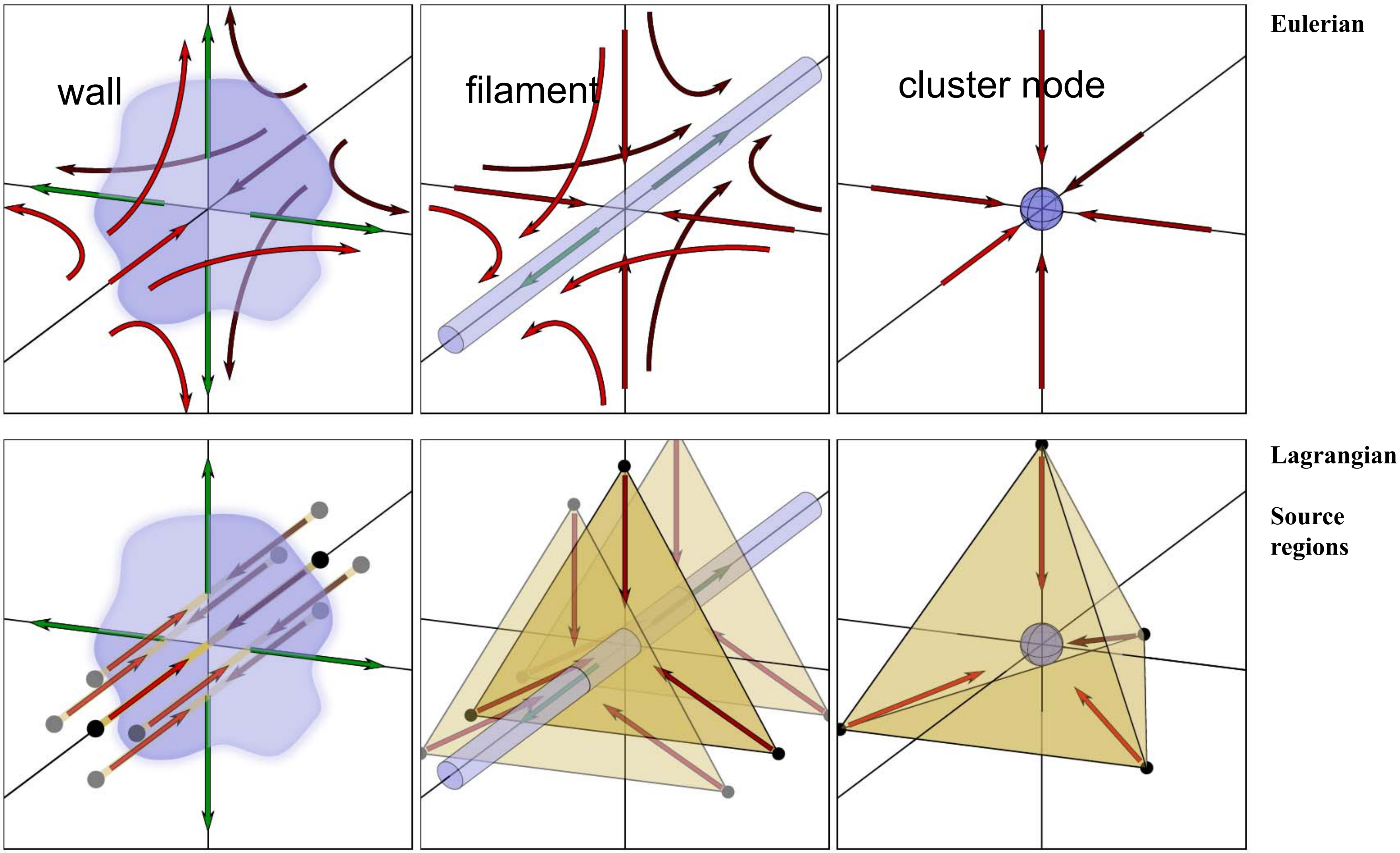


Hierarchical Web Evolution:

Adhesion simulation buildup Cosmic Web

Johan Hidding
2012

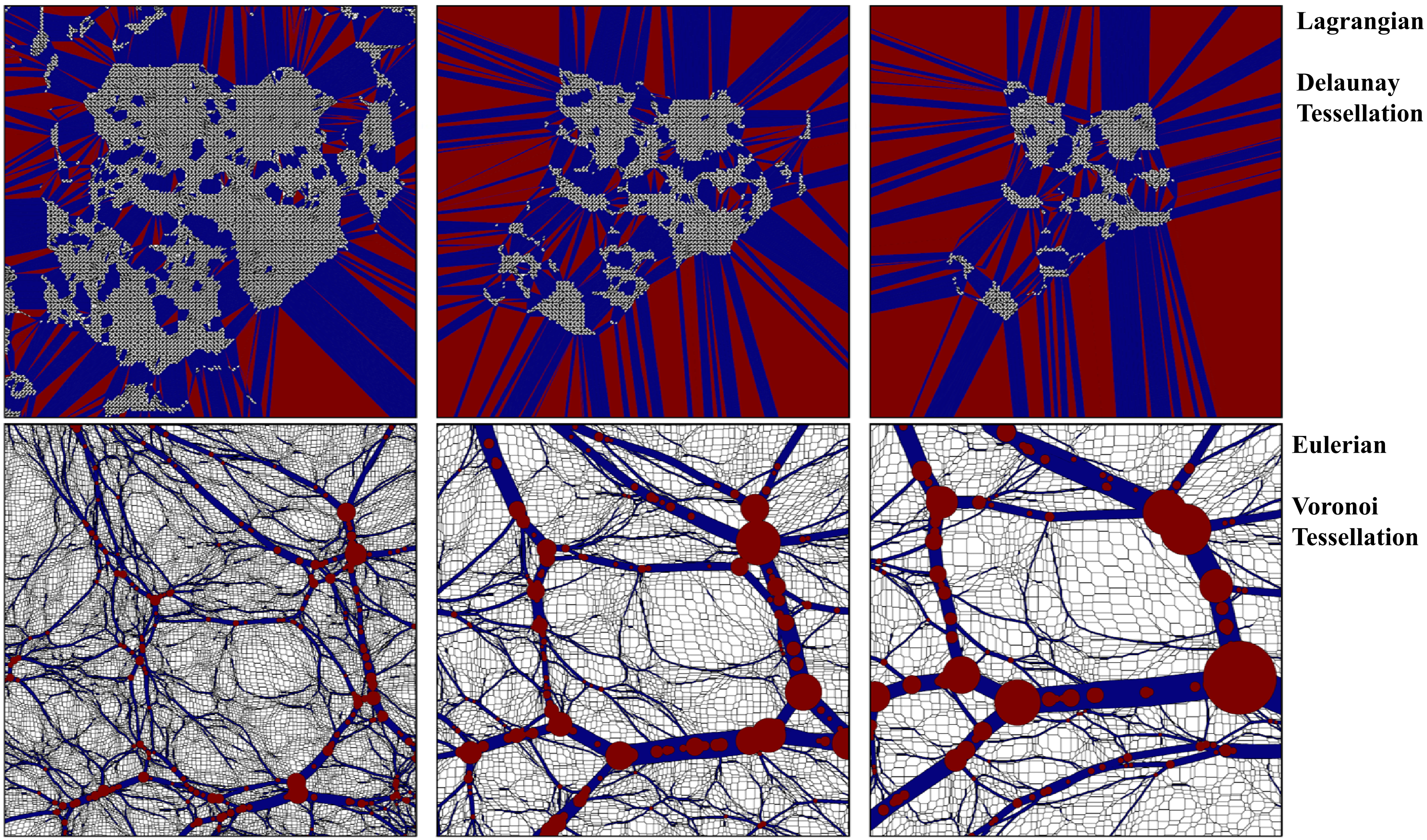
Euierian vs. Lagrangian
weblike geometry



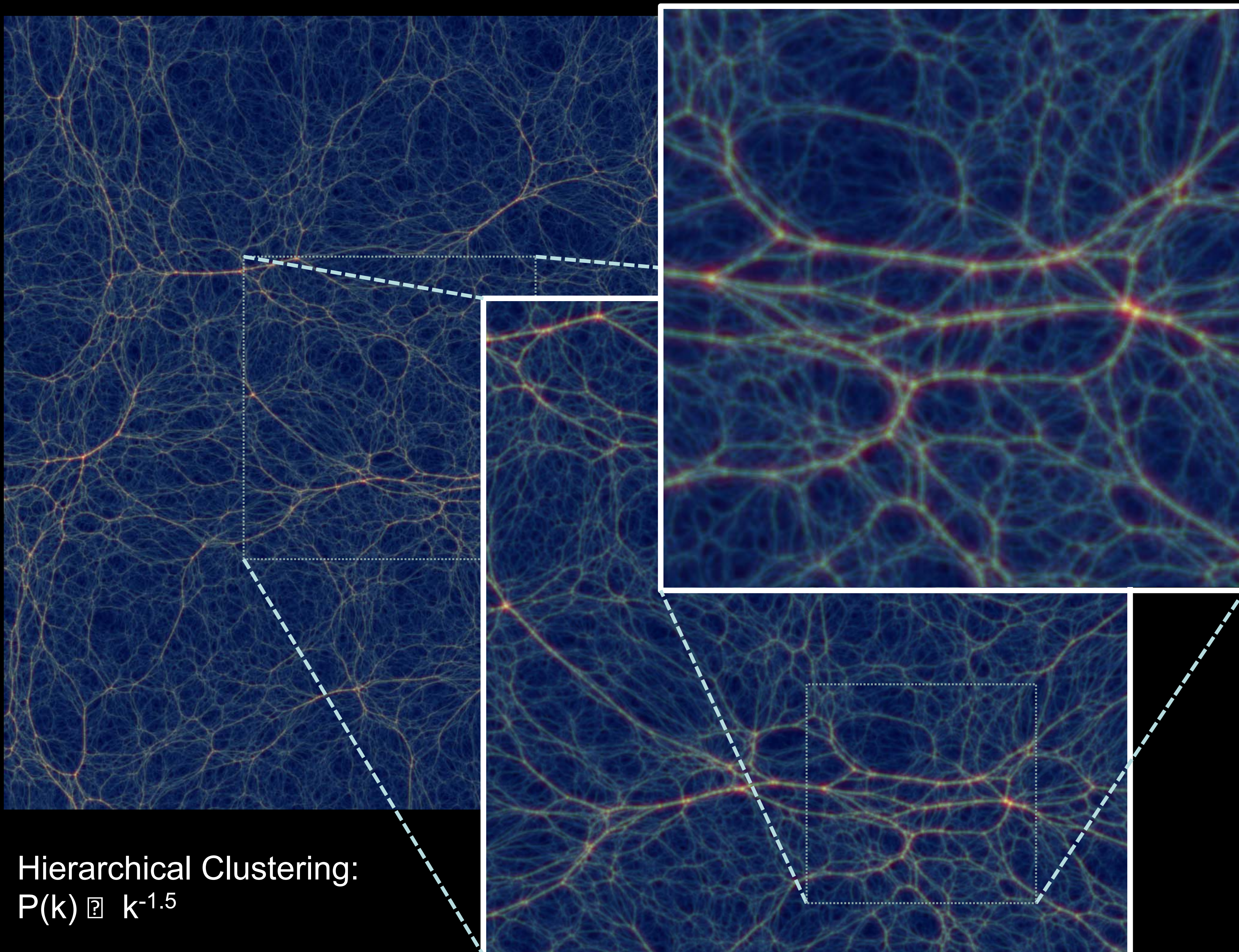
Hidding, vdW et al.

Lagrangian – Eulerian Cosmic Web

Delaunay- Voronoi Tessellations



Hidding, vdW et al.



Hierarchical Clustering:
$P(k) \propto k^{-1.5}$

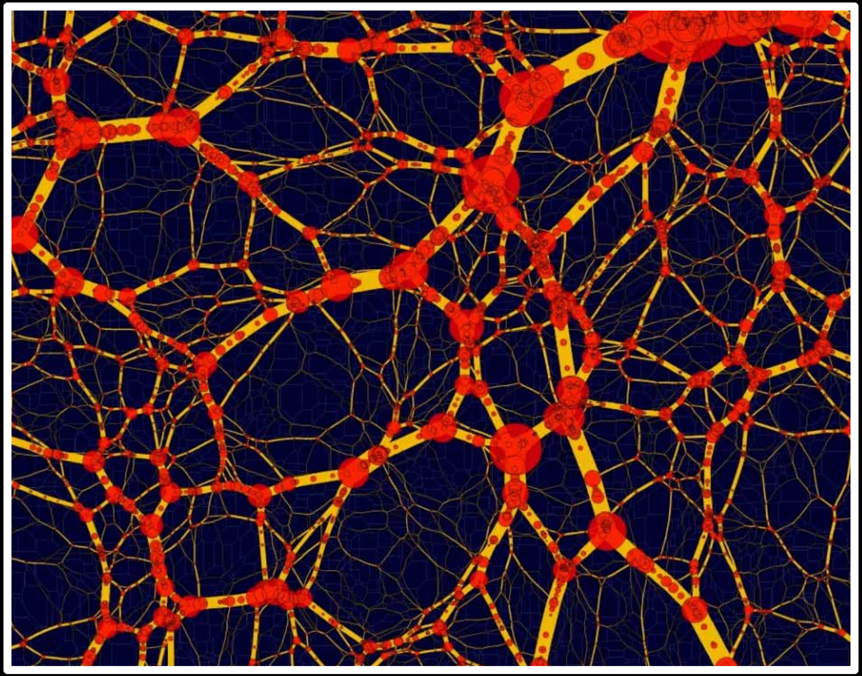
Cosmological Sensitivity Cosmic Web

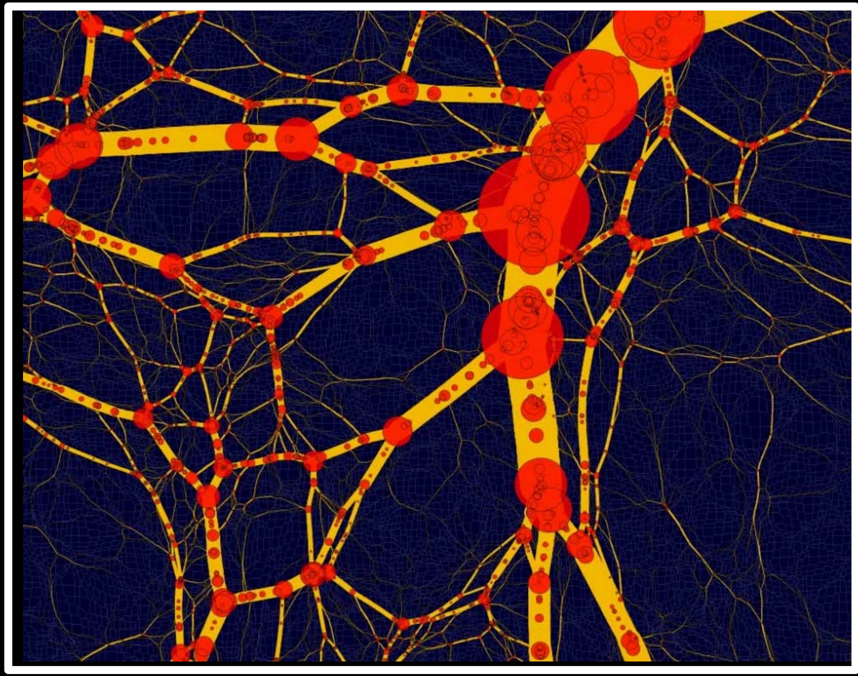
the morphology of the weblike network is
highly sensitive to the underlying cosmology

$P(k) \sim k^{-1.5}$

Hidding 2012/2014

$P(k) \sim k^{-2.0}$





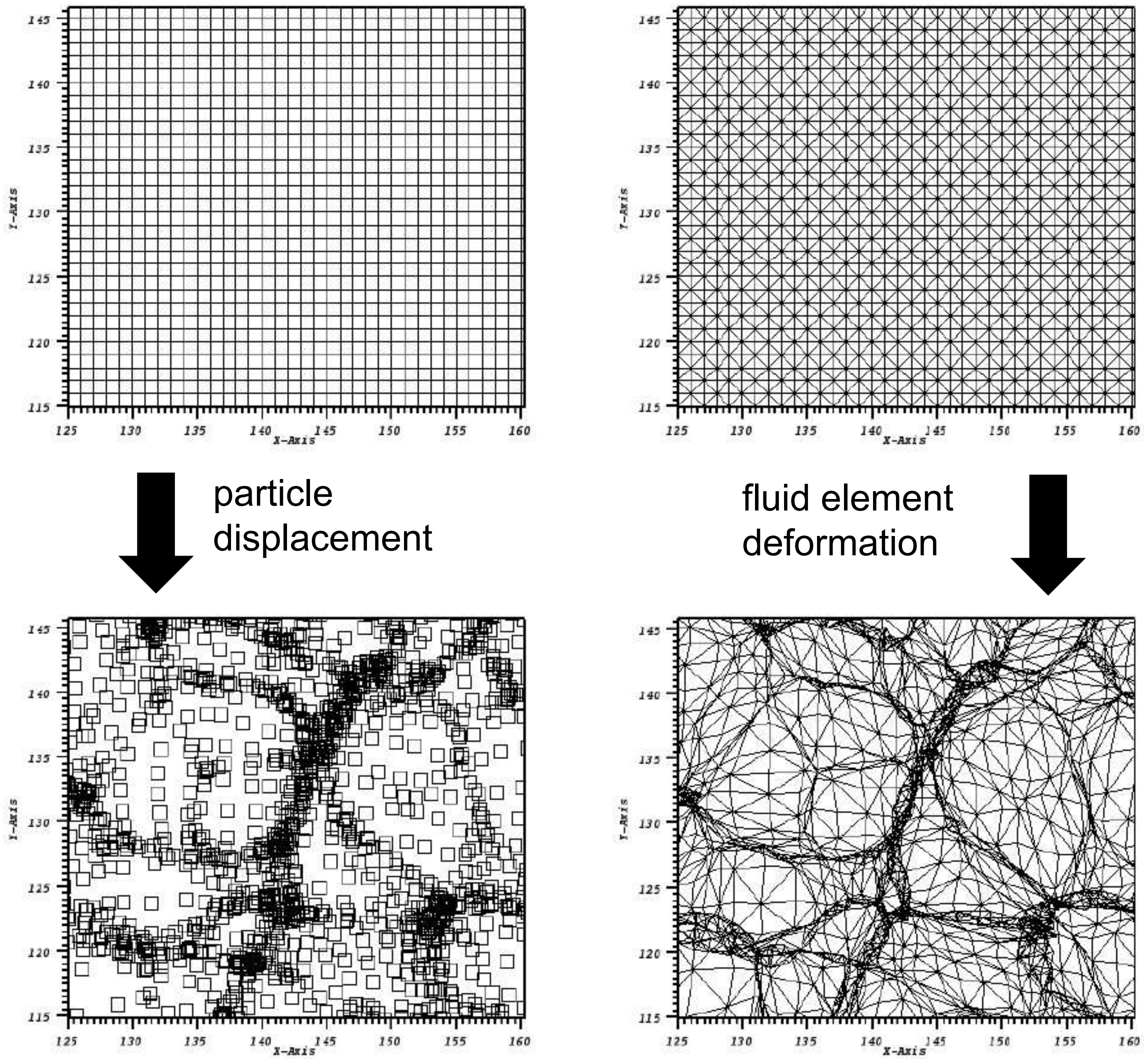
Cosmic Web

Phase-Space Evolution

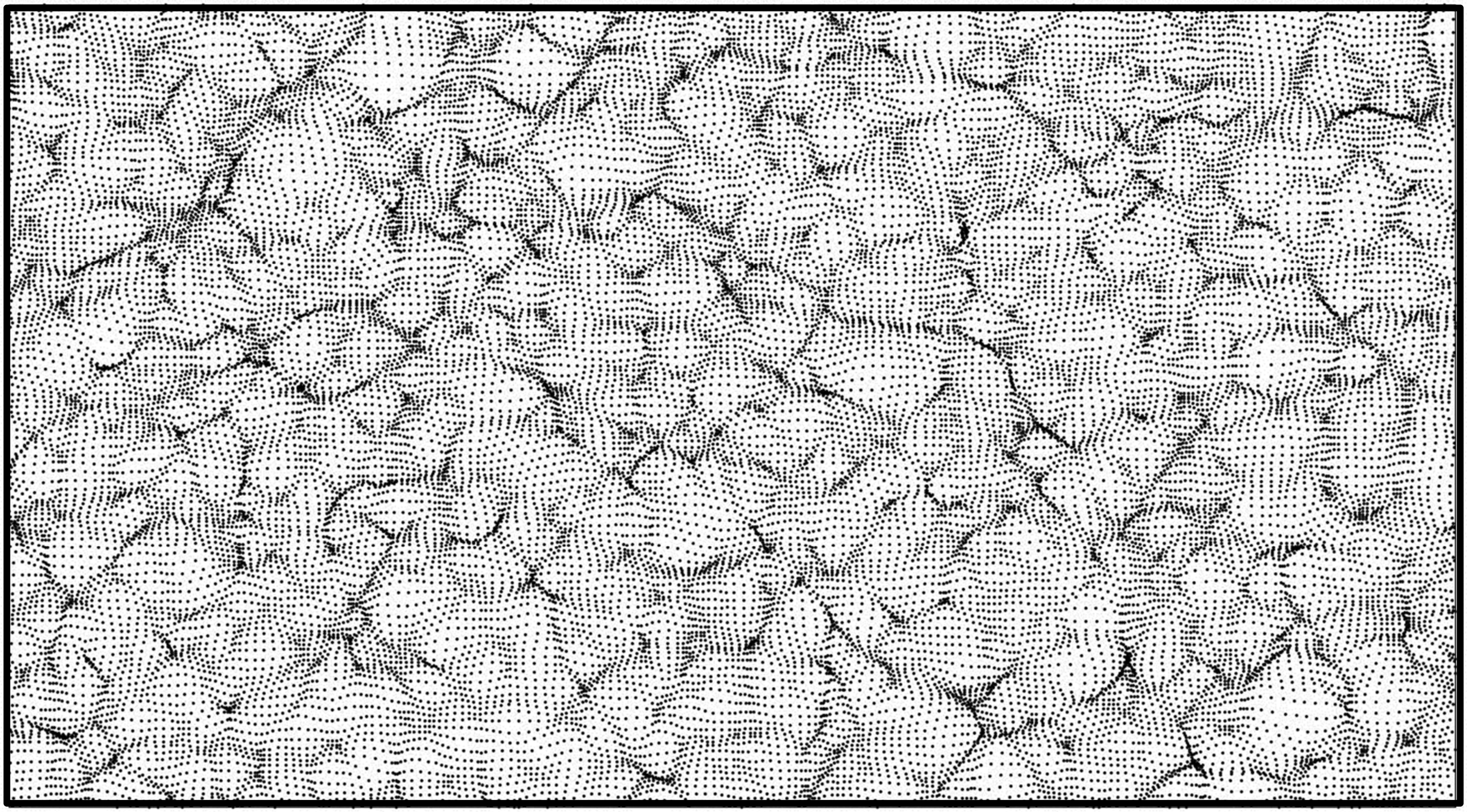
Tessellation Deformation & Phase Space Projection

Translation towards Multi-D space:

- **Look at deformation of initial tessellation**
- **each tessellation cell represents matter cell**
- **evolution deforms cell**
- **once cells start to overlap, manifestation of different phase-space matter streams**



Simulation – Discrete Particles



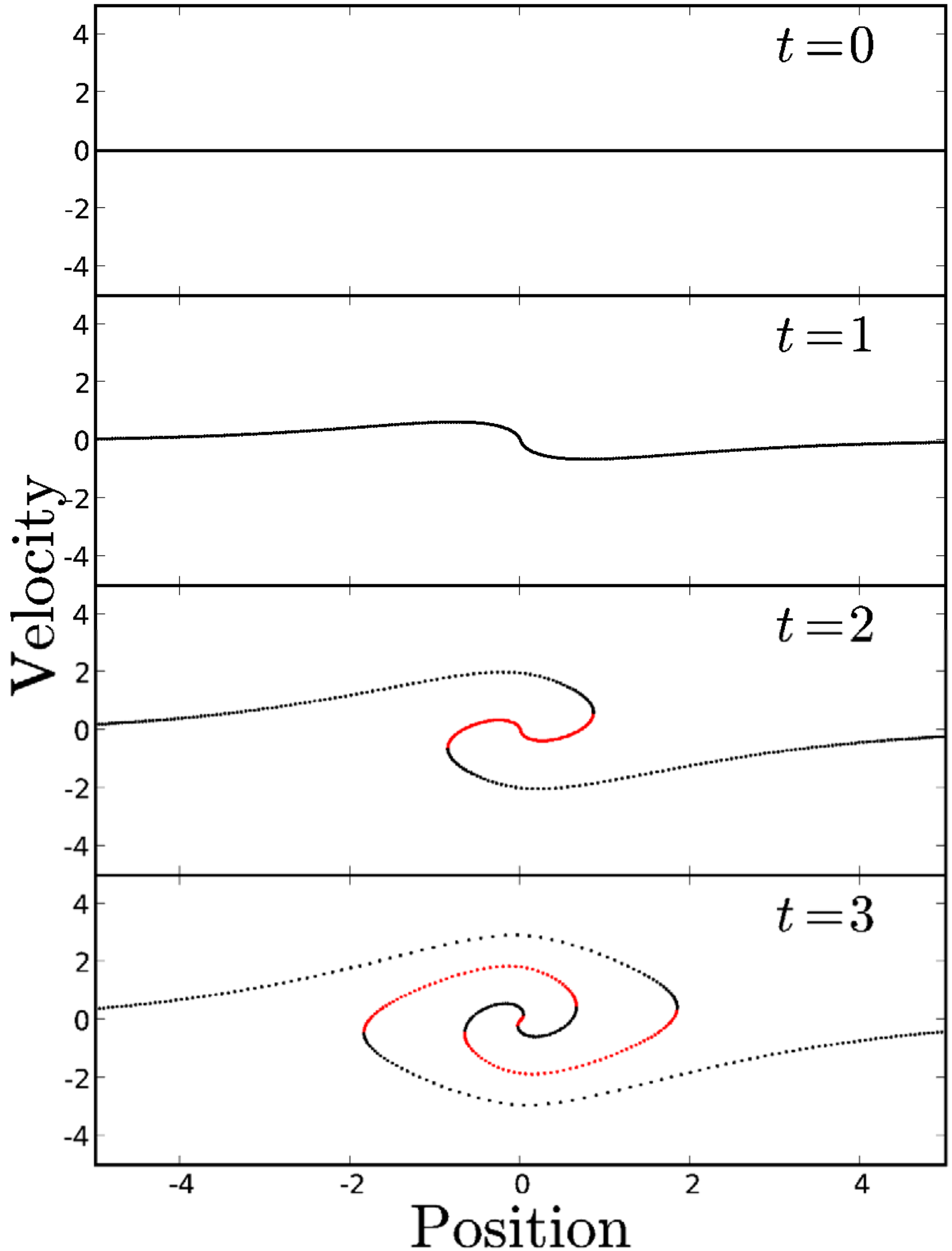
Simulation – Mass Elements

Phase Space Evolution

Dark Matter Phase Space sheet:

3-D structure projection of a folding DM phase space sheet In 6-D phase space

- **Shandarin 2010, 2011**
- **Neyrinck et al. 2011, 2012 Origami**
- **Abel et al. 2011**



Evolving matter distribution in position-velocity space – 1D

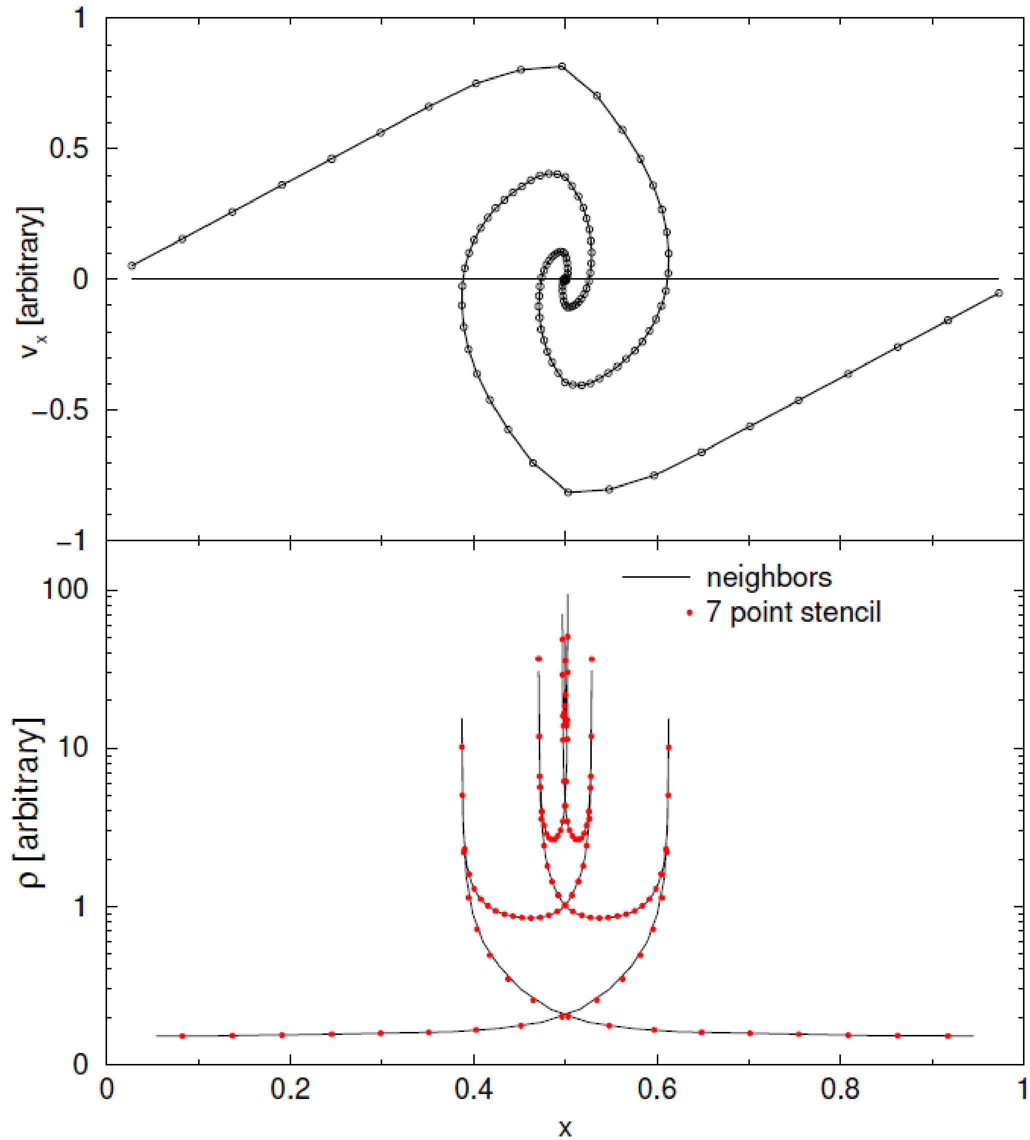
Phase Space Evolution

Phase space:

Velocity vs. Position

Density:

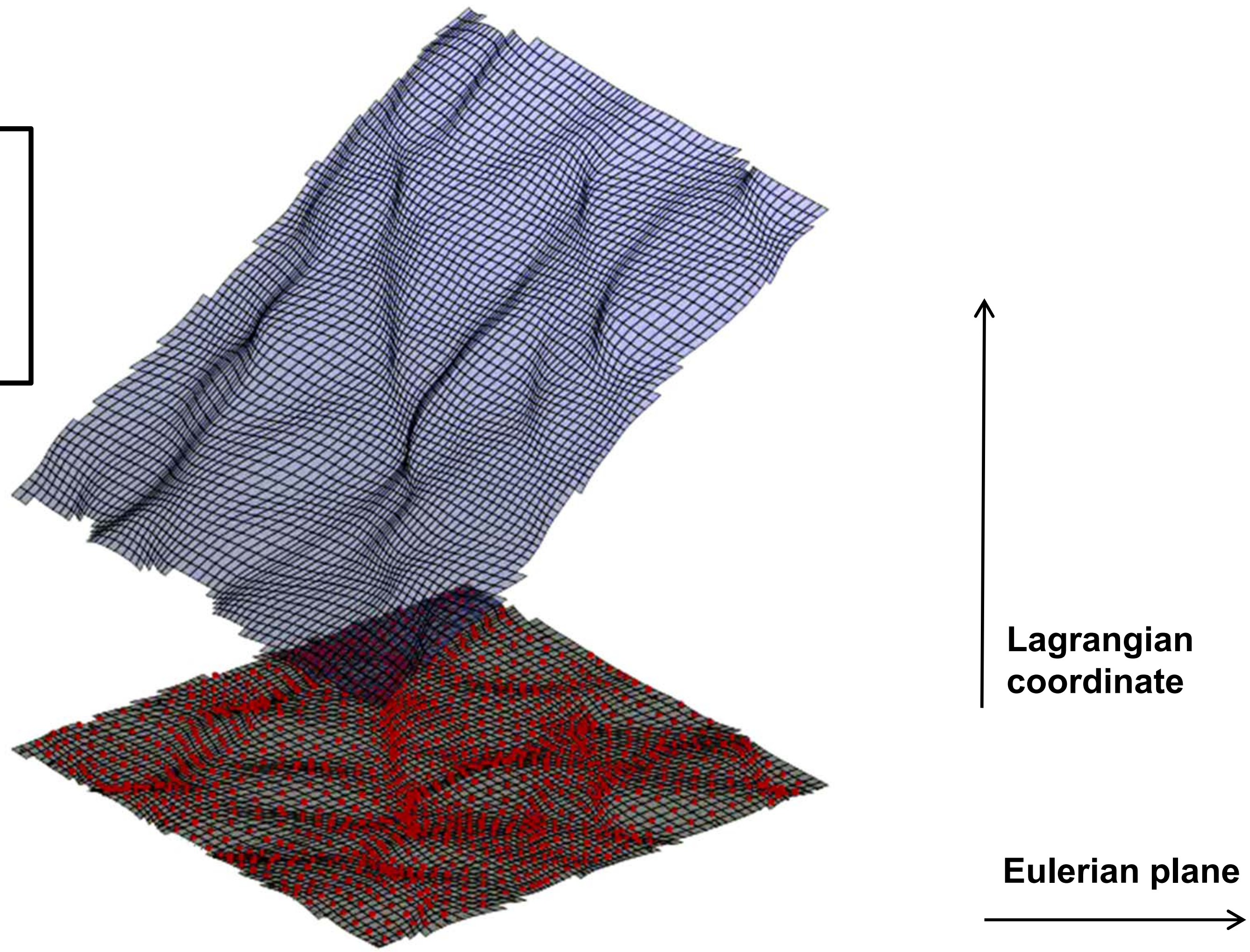
$$\rho(\vec{x},t) = \int f\left(\vec{x},\vec{v},t\right) d\vec{v}$$



Phase-Space Evolution

Dynamical Evolution:

folding the phase-space sheet {q,x}



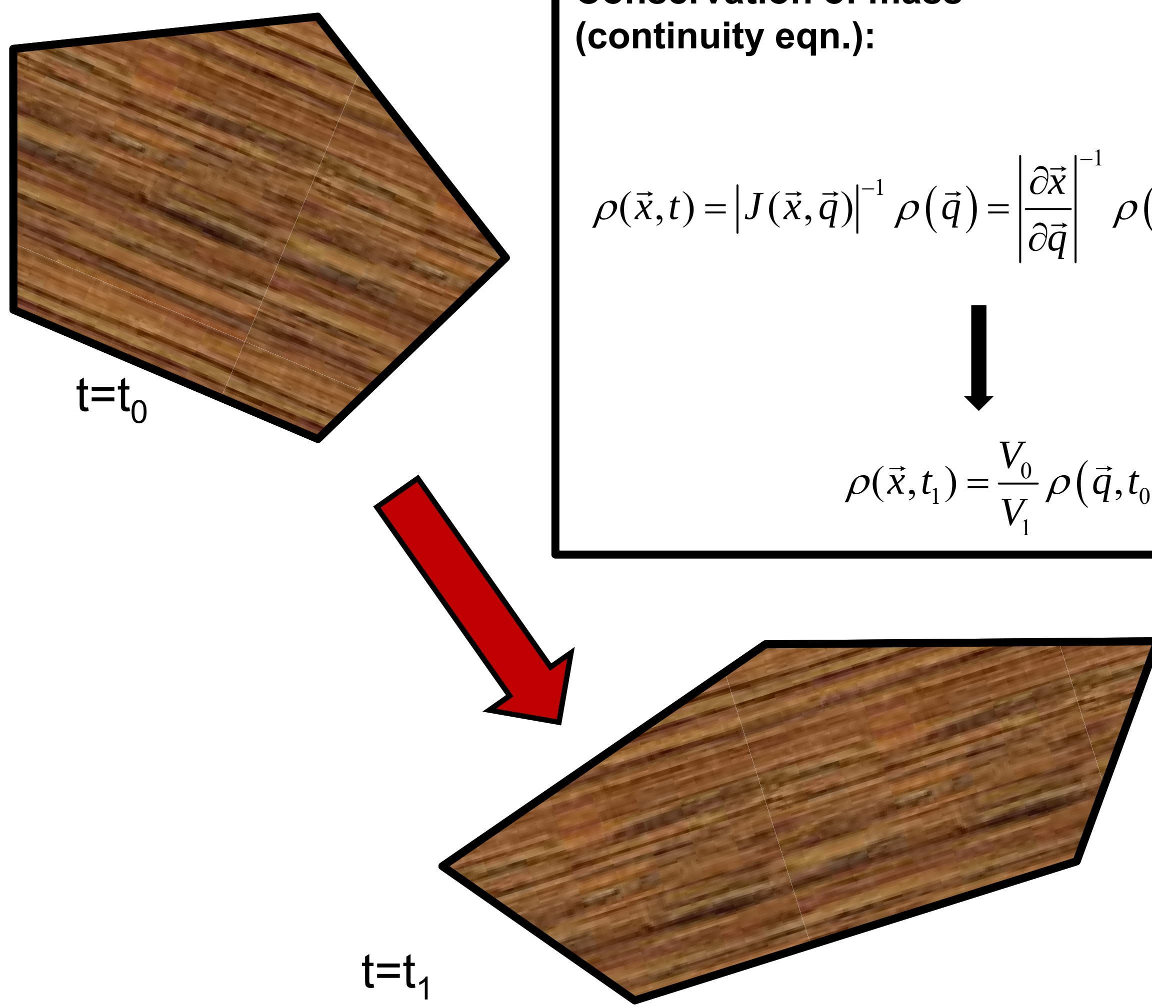
Hidding 2014

Tessellation Deformation & Phase Space Projection

Translation towards Multi-D space:

- Look at deformation of initial tessellation
- each tessellation cell represents matter cell
- evolution deforms cell

Monostream Density Evolution



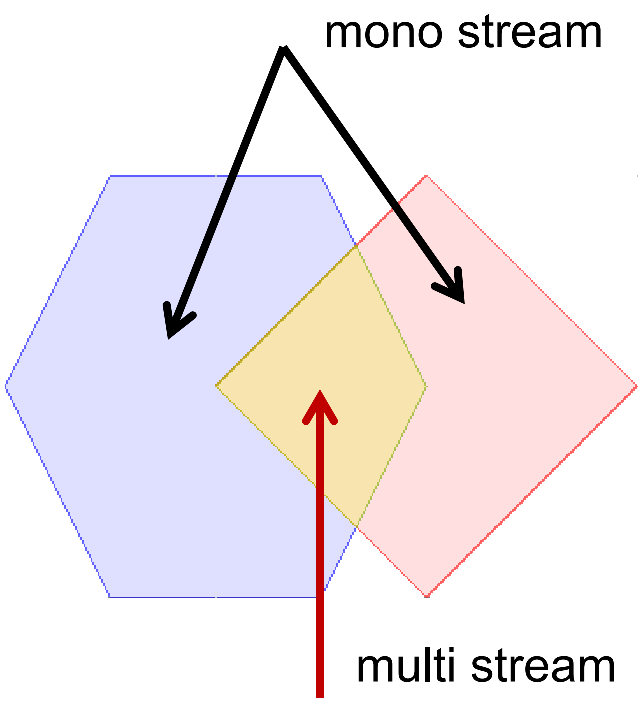
Conservation of mass (continuity eqn.):

$$\rho(\vec{x},t) = \left|J(\vec{x},\vec{q})\right|^{-1} \rho(\vec{q}) = \left|\frac{\partial \vec{x}}{\partial \vec{q}}\right|^{-1} \rho(\vec{q})$$

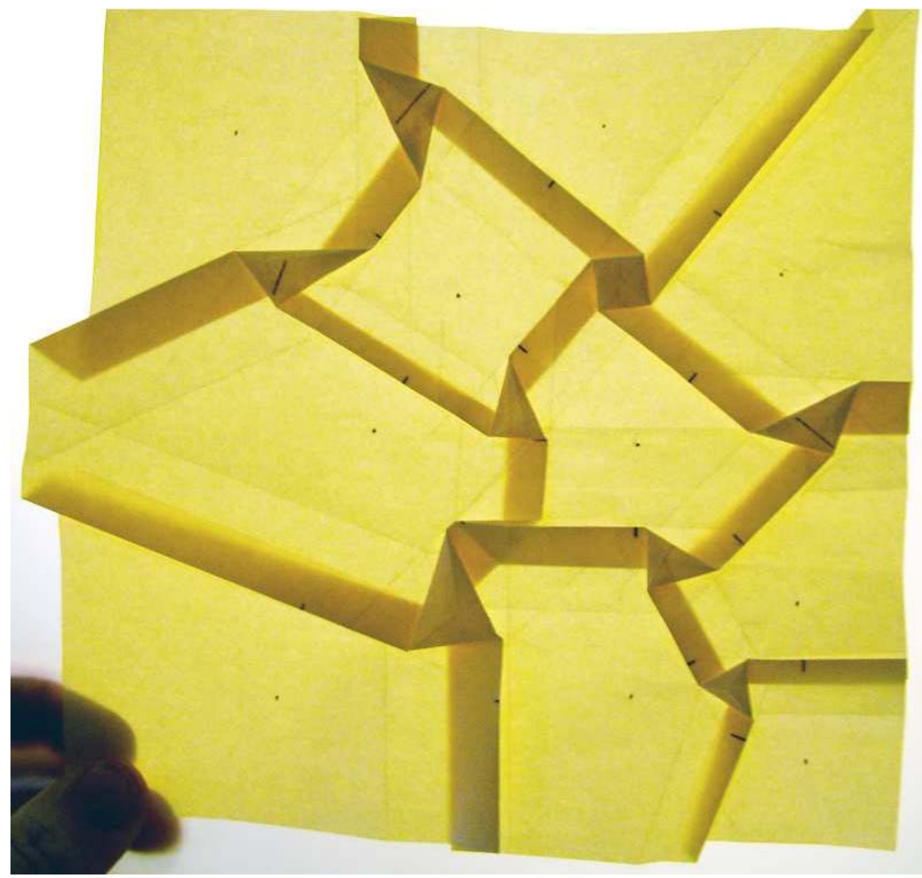
$$\Downarrow$$

$$\rho(\vec{x},t_1) = \frac{V_0}{V_1}\,\rho(\vec{q},t_0)$$

(Cosmic) ORIGAMI



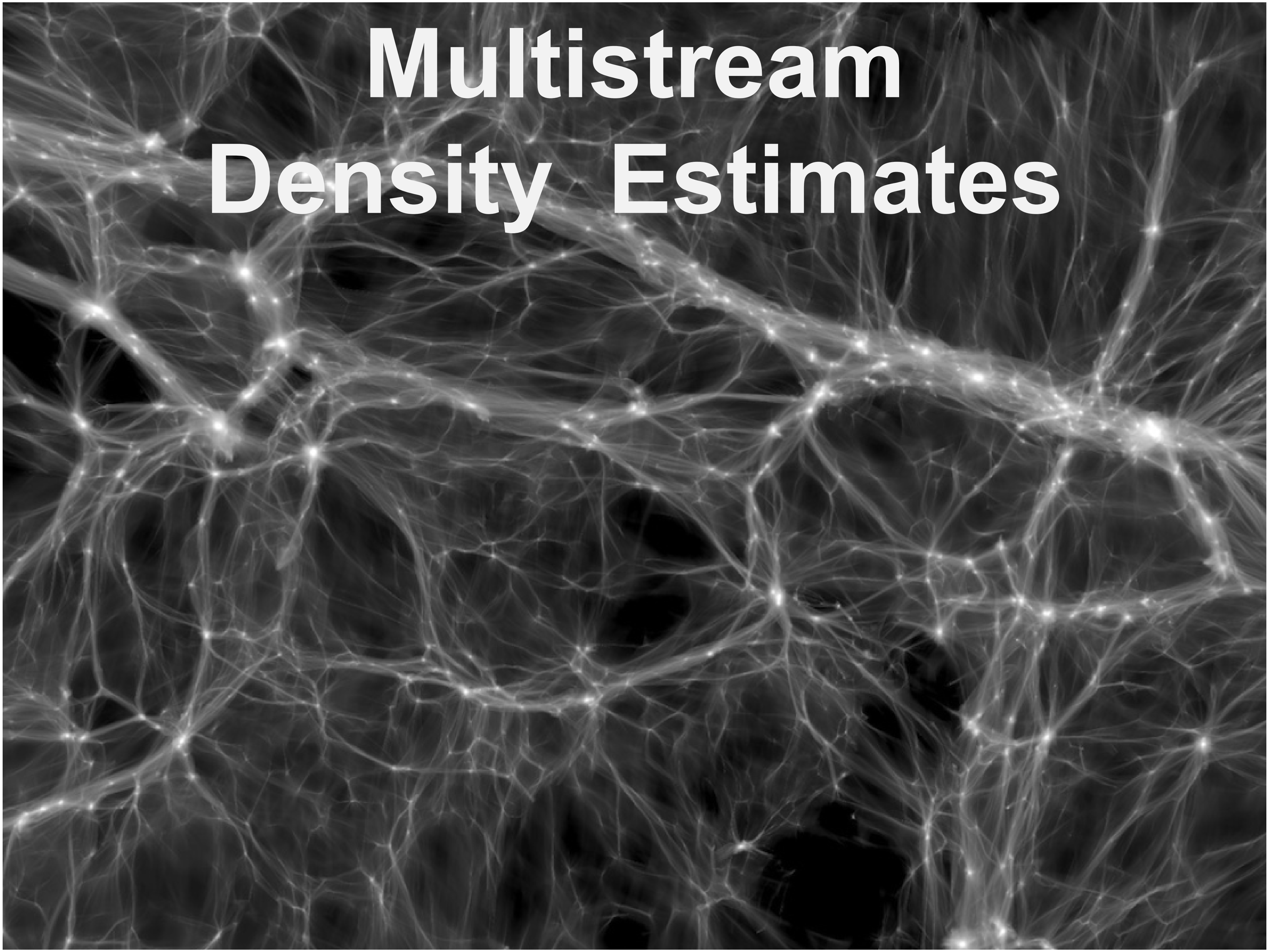
Evolution of dynamical system:
Phase-space folding – Cosmic Origami



$$\rho_{total}(\vec{x},t_1) = \sum_i \frac{V_{0i}}{V_{1i}} \rho(\vec{q}_i,t_0)$$

Mark Neyrinck, Bridget Falck

Multistream Density Estimates



Cosmic Web Stream Density

Translation towards Multi-D space:

Density of dark matter streams:

- # phase space folds

=

 # locally overlapping tessellation cells

Shandarin 2012
Abel, Hahn & Kaehler 2012
Falck, Neyrinck et al. 2012