the Big Bang

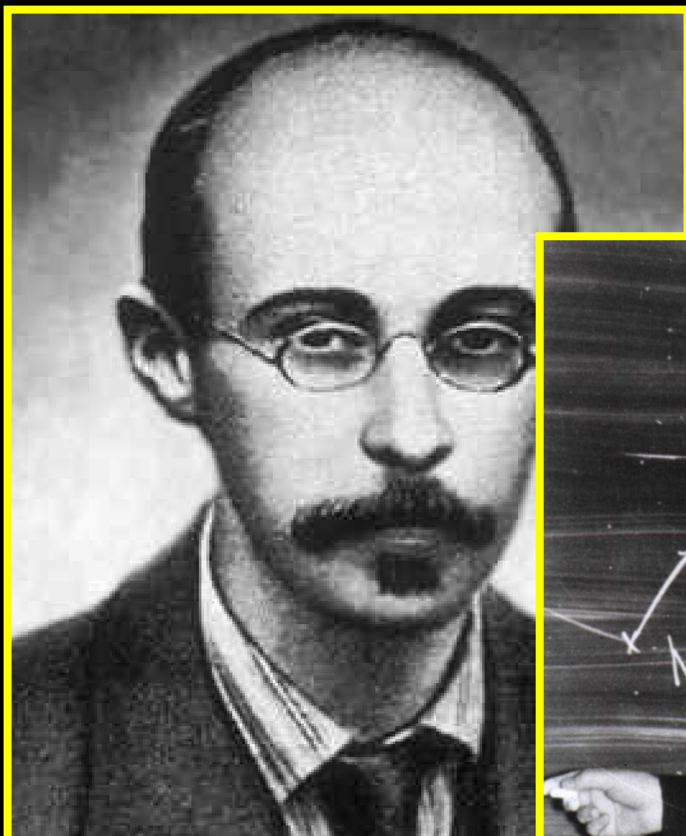
Friedmann, Lemaitre

&

Cosmic Expansion History

Friedmann & Lemaitre

Alexander Friedmann (1888 -1925)
George Lemaitre (1894-1966)



They discovered (independently) theoretically the expansion of the Universe as a solution to the Theory of General Relativity.

... and derived the equations that describe the expansion and evolution of the universe,

the foundation for all of modern Cosmology:

Friedmann-Lemaitre Equation

Evolving Universe

- Einstein, de Sitter, Friedmann and Lemaitre all realized that in General Relativity, there cannot be a stable and static Universe:

- The Universe either expands, or it contracts …

- Expansion Universe encapsulated in a

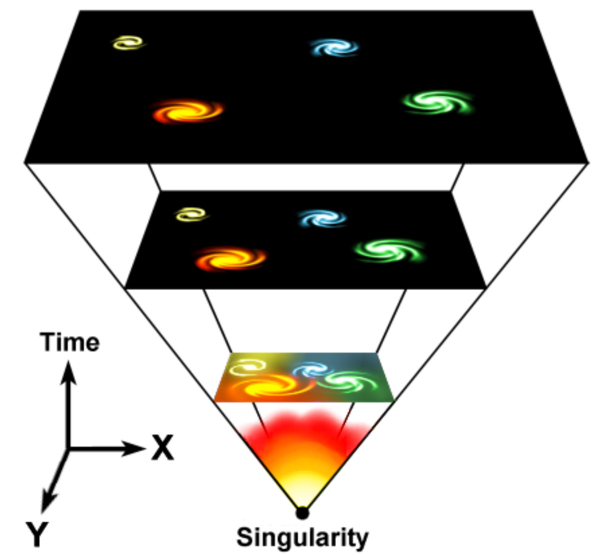
GLOBAL expansion factor a(t)

- All distances/dimensions of objects uniformly increase by a(t):

  at time t, the distance between two objects i and j has increased to

$$\vec{r}_i - \vec{r}_j = a(t)\left(\vec{r}_{i,0} - \vec{r}_{j,0}\right)$$

- Note: by definition we chose $a(t_0)=1$, i.e. the present-day expansion factor



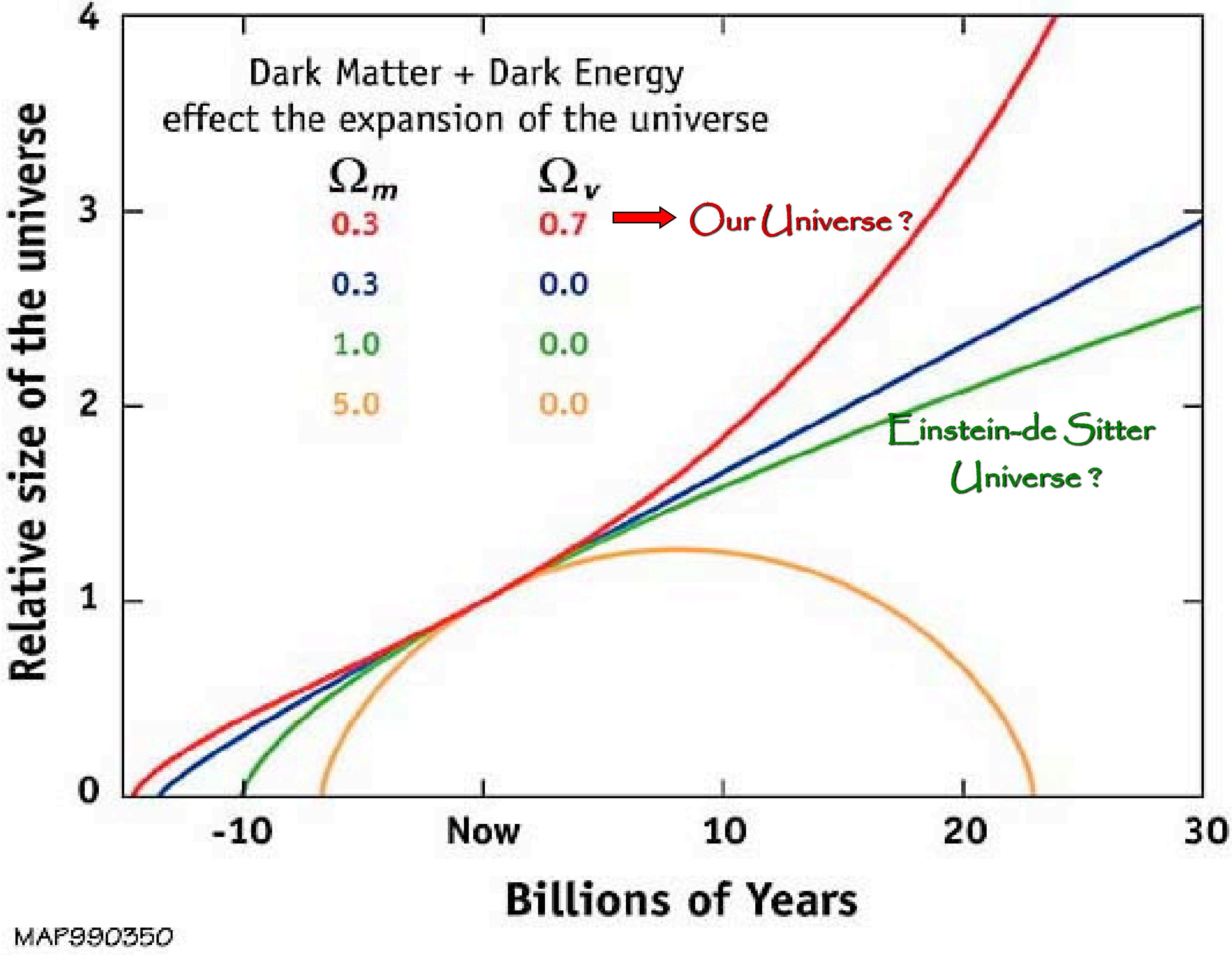
Evolution & Fate
Friedmann-Robertson-Walker-Lemaitre Universe

Completely determined by 3 factors:

- energy and matter content
  (density and pressure)

- geometry of the Universe
  (curvature)

- Cosmological Constant


Dark Matter + Dark Energy
effect the expansion of the universe
Ω_m
Ω_v
0.3 0.7
Our Universe ?
0.3 0.0
1.0 0.0
5.0 0.0
Einstein-de Sitter
Universe ?
Relative size of the universe
0
1
2
3
4
-10
Now
10
20
30
Billions of Years
MAP990350

Friedmann-Robertson-Walker-Lemaitre Universe

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a + \frac{\Lambda}{3}a$$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0{}^2} + \frac{\Lambda}{3}a^2$$

Friedmann-Robertson-Walker-Lemaitre Universe

Because of General Relativity, the evolution of the Universe is determined by four factors:

- density $\rho(t)$
- pressure $p(t)$
- curvature $kc^2 / R_0^{\ 2}$ $\quad k = 0, +1, -1$
  R_0: present curvature radius
- cosmological constant Λ

- Density & Pressure:
  - in relativity, energy & momentum need to be seen as one physical quantity (four-vector)
  - pressure = momentum flux
- Curvature:
  - gravity is a manifestation of geometry spacetime
- Cosmological Constant:
  - free parameter in General Relativity
  - Einstein's "biggest blunder"
  - mysteriously, since 1998 we know it dominates the Universe

FRWL Dynamics

In a Friedmann-Robertson-Walker-Lemaitre Universe, densities are in the order of the critical density, the density at which the Universe has a flat curvature

$$\rho_{crit} = \frac{3H_0^{\;2}}{8\pi G} = 1.8791 h^2 \times 10^{-29}\, g\, cm^{-3}$$

$$H_0 = 100h\ km\, s^{-1}\, Mpc^{-1}$$

expansion rate Universe

$$\rho_0 = 1.8791 \times 10^{-29}\, \Omega h^2\ g\, cm^{-3}$$

$$= 2.78 \times 10^{11}\, \Omega h^2 \quad M_\odot Mpc^{-3}$$

FRWL Dynamics

In a matter-dominated Universe,
the evolution and fate of the Universe entirely determined by the (energy) density in units of critical density:

$$\Omega \equiv \frac{\rho}{\rho_{crit}}$$

Arguably, ⍰ is the most important parameter of cosmology !!!

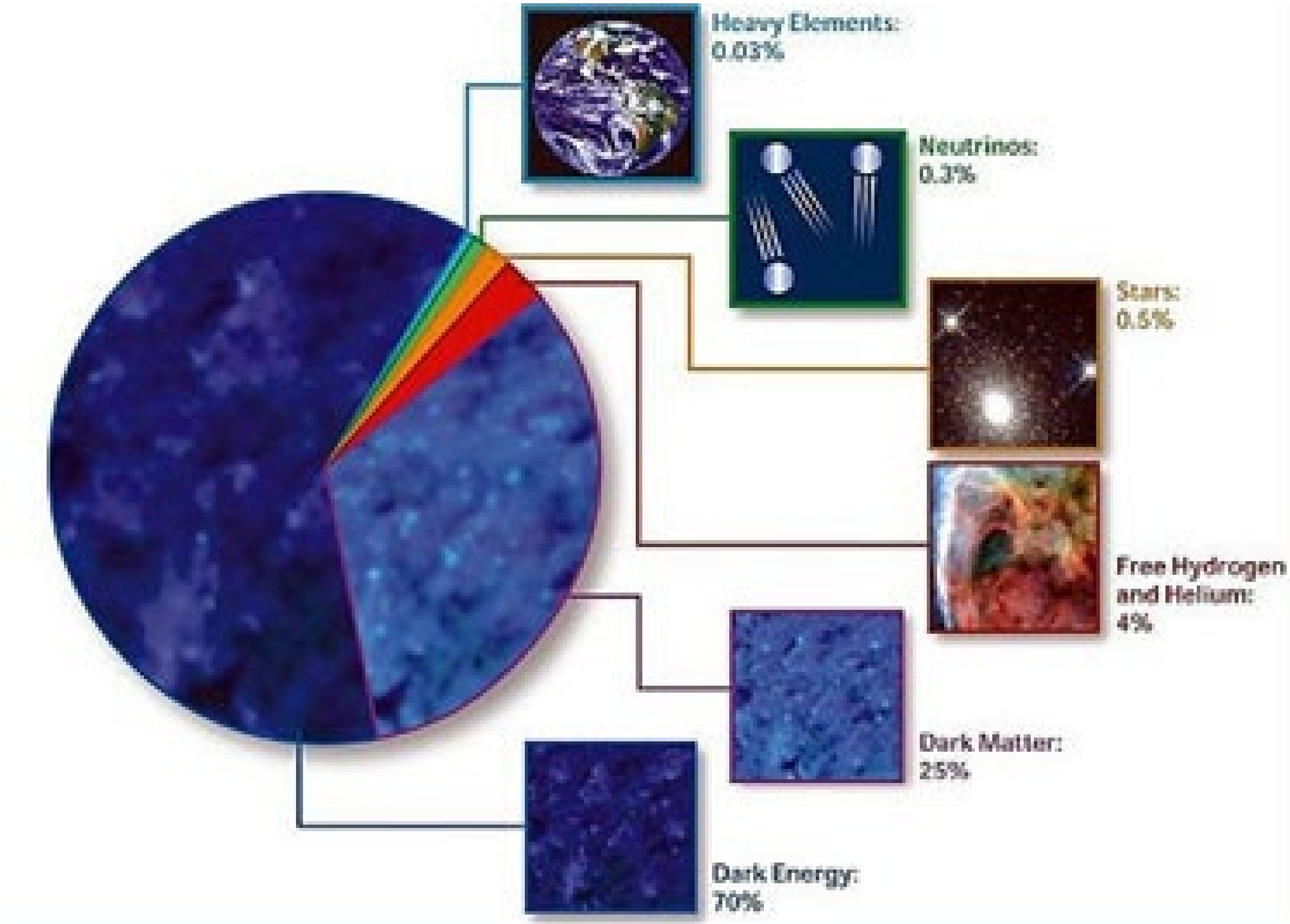
Present-day Cosmic Density:

$$\rho_0 = 1.8791 \times 10^{-29}\, \Omega h^2 \;\; g\, cm^{-3}$$
$$= 2.78 \times 10^{11}\, \Omega h^2 \qquad M_{\odot} Mpc^{-3}$$

what the Universe exists of:

Cosmic Constituents

Cosmic Components



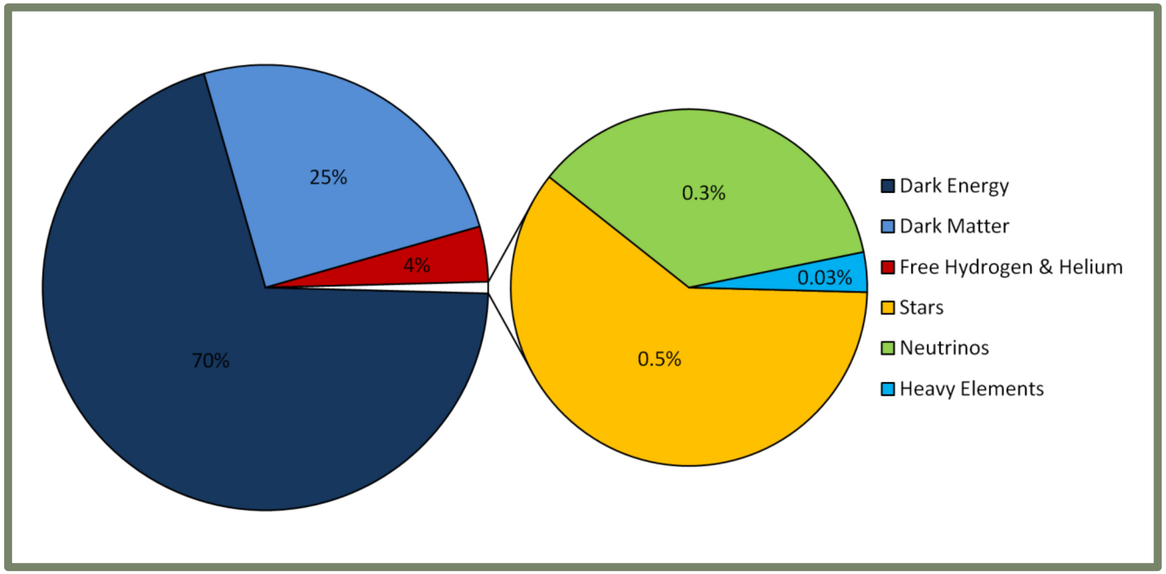
Cosmic Energy Inventarisation

1	dark sector			0.954 ± 0.003
1.1	dark energy		0.72 ± 0.03	
1.2	dark matter		0.23 ± 0.03	
1.3	primeval gravitational waves		$\lesssim 10^{-10}$	
2	primeval thermal remnants			0.0010 ± 0.0005
2.1	electromagnetic radiation		$10^{-4.3\pm0.0}$	
2.2	neutrinos		$10^{-2.9\pm0.1}$	
2.3	prestellar nuclear binding energy		$-10^{-4.1\pm0.0}$	
3	baryon rest mass			0.045 ± 0.003
3.1	warm intergalactic plasma		0.040 ± 0.003	
3.1a	virialized regions of galaxies	0.024 ± 0.005		
3.1b	intergalactic	0.016 ± 0.005		
3.2	intracluster plasma		0.0018 ± 0.0007	
3.3	main sequence stars	spheroids and bulges	0.0015 ± 0.0004	
3.4		disks and irregulars	0.00055 ± 0.00014	
3.5	white dwarfs		0.00036 ± 0.00008	
3.6	neutron stars		0.00005 ± 0.00002	
3.7	black holes		0.00007 ± 0.00002	
3.8	substellar objects		0.00014 ± 0.00007	
3.9	HI + HeI		0.00062 ± 0.00010	
3.10	molecular gas		0.00016 ± 0.00006	
3.11	planets		10^{-6}	
3.12	condensed matter		$10^{-5.6\pm0.3}$	
3.13	sequestered in massive black holes		$10^{-5.4}(1+\epsilon_n)$	
4	primeval gravitational binding energy			$-10^{-6.1\pm0.1}$
4.1	virialized halos of galaxies		$-10^{-7.2}$	
4.2	clusters		$-10^{-6.9}$	
4.3	large-scale structure		$-10^{-6.2}$	

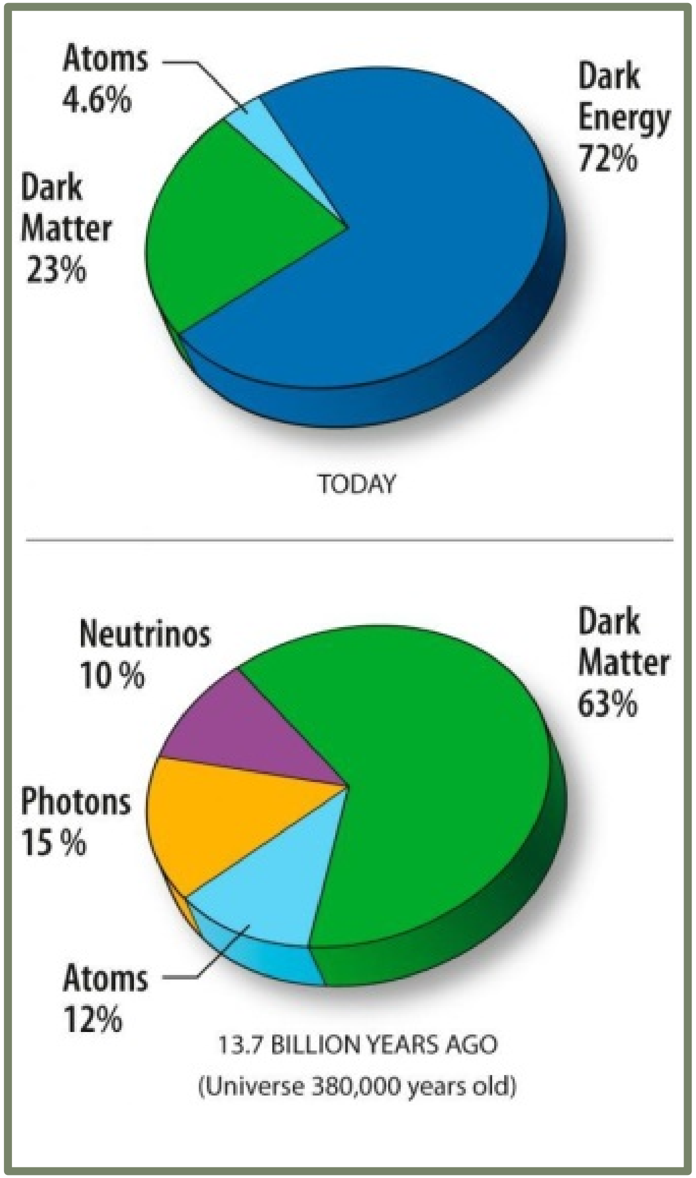
sterren slechts ~0.1% energie Heelal

Fukugita & Peebles 2004

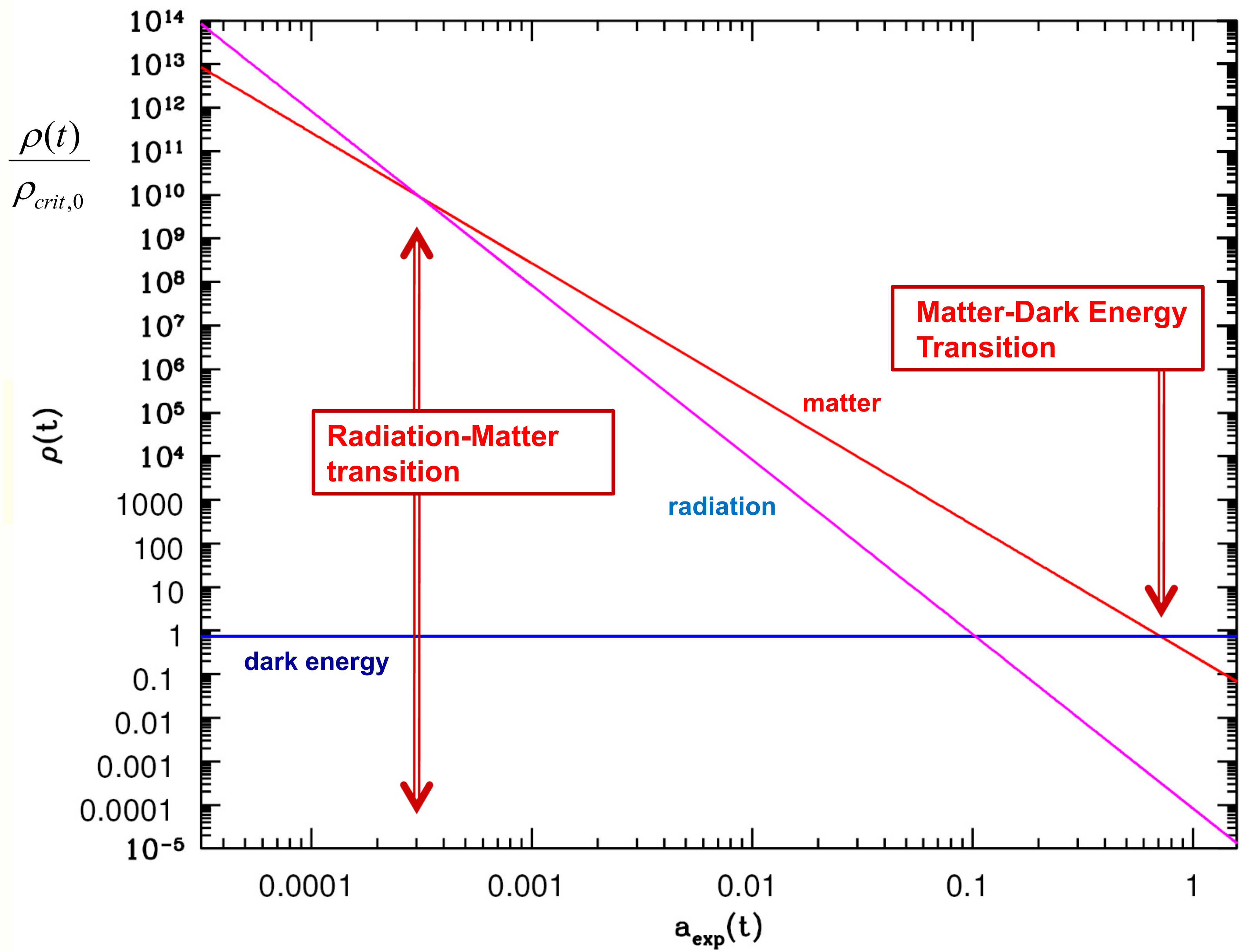
Cosmic Constitution



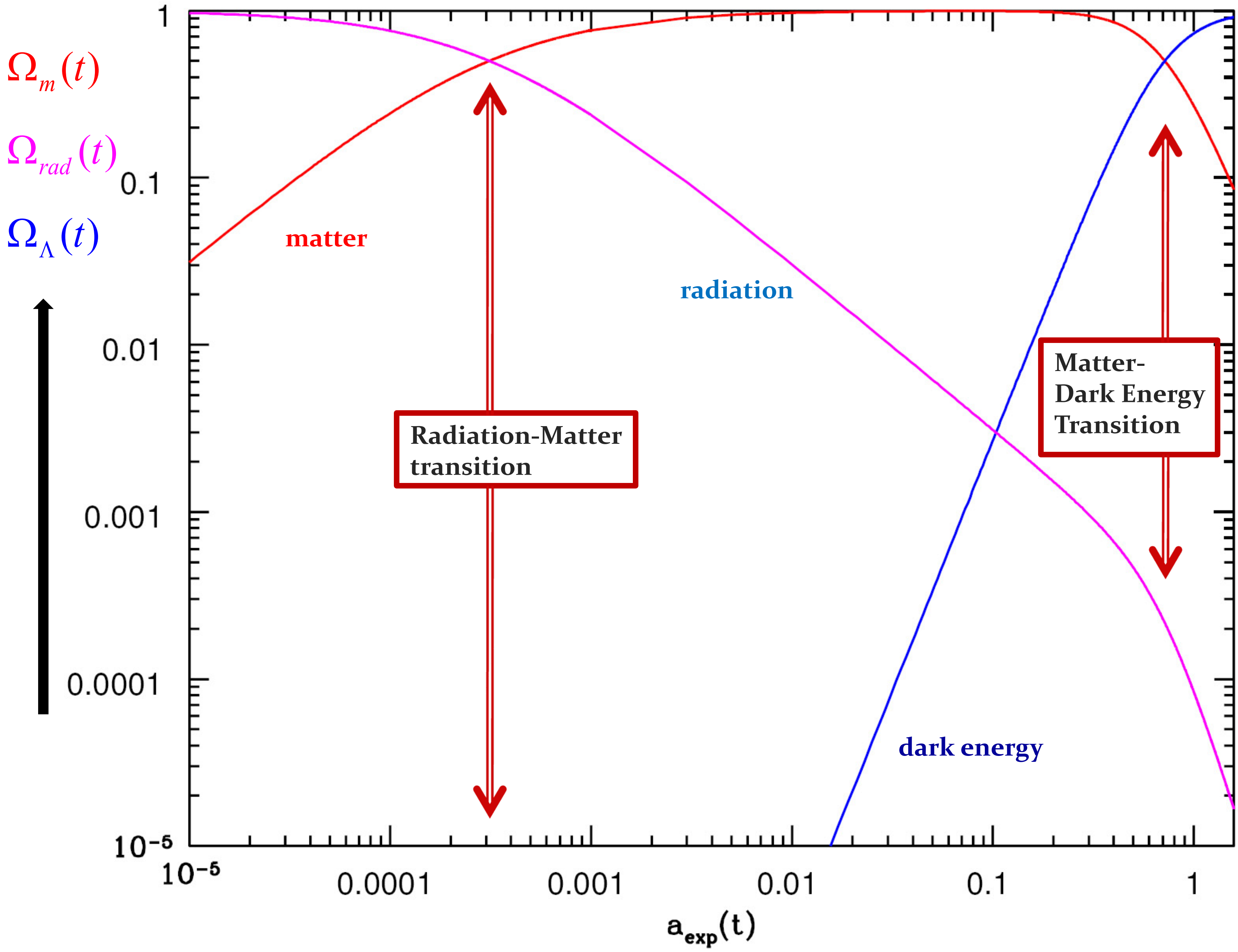
Cosmic Pie Diagram



Changes in Time:



$\frac{\rho(t)}{\rho_{crit,0}}$
ρ(t)
10¹⁴
10¹³
10¹²
10¹¹
10¹⁰
10⁹
10⁸
10⁷
10⁶
10⁵
10⁴
1000
100
10
1
0.1
0.01
0.001
0.0001
10⁻⁵
0.0001
0.001
0.01
0.1
1
$a_{exp}(t)$
Radiation-Matter transition
Matter-Dark Energy Transition
matter
radiation
dark energy



$\Omega_m(t)$
$\Omega_{rad}(t)$
$\Omega_\Lambda(t)$
1
0.1
0.01
0.001
0.0001
10^{-5}
matter
radiation
dark energy
Radiation-Matter transition
Matter-Dark Energy Transition
10^{-5}
0.0001
0.001
0.01
0.1
1
$a_{exp}(t)$

General FRWL cxpansion histories:

cosmic “phase diagram”

Cosmological Evolution Modes

It is interesting to inspect the possible expansion histories for generic FRWL cosmologies with matter & cosmological constant.

- The expansion histories entirely determined by 2 parameters:

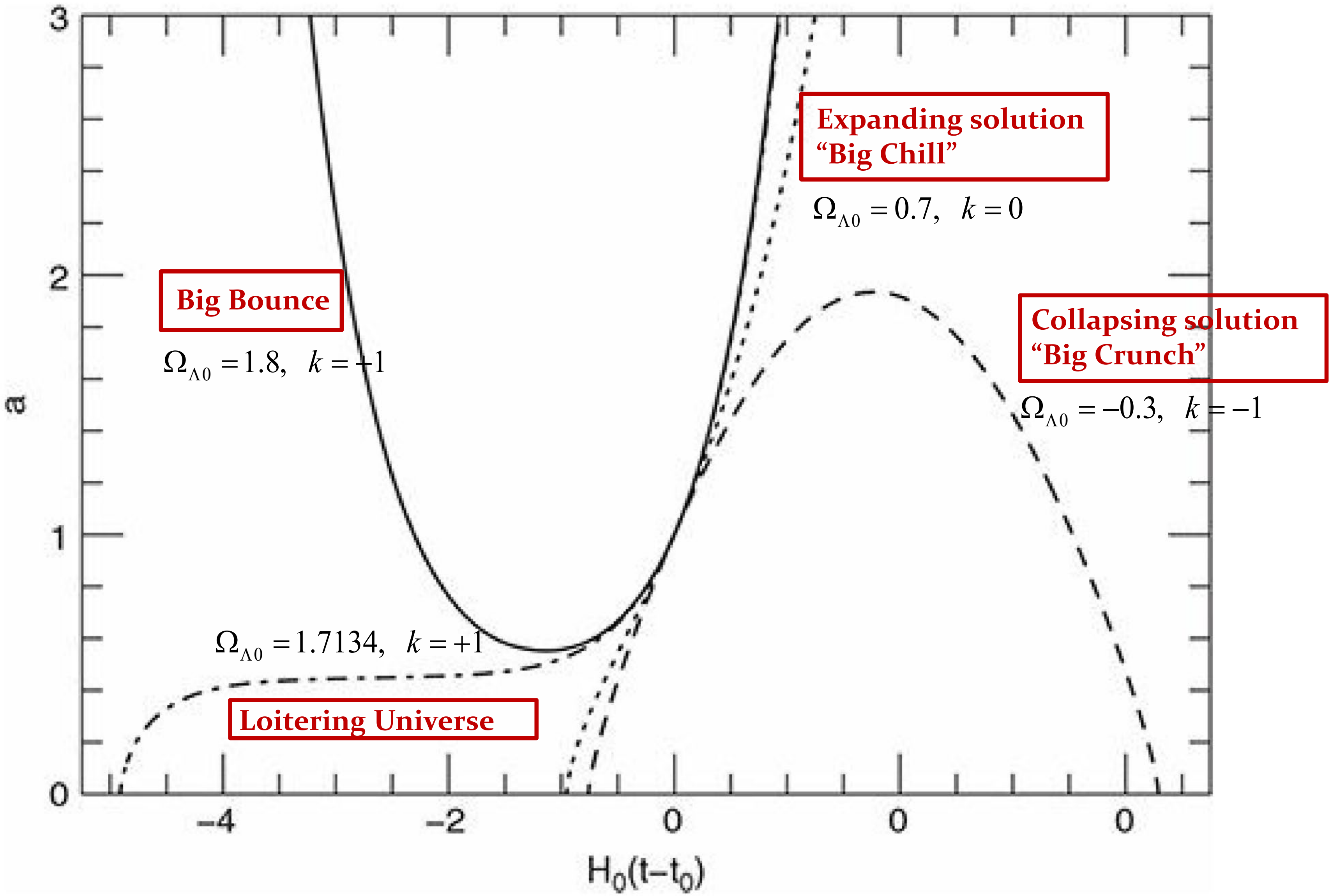
matter density $\Omega_{m,0}$

cosmological constant $\Omega_{\Lambda,0}$

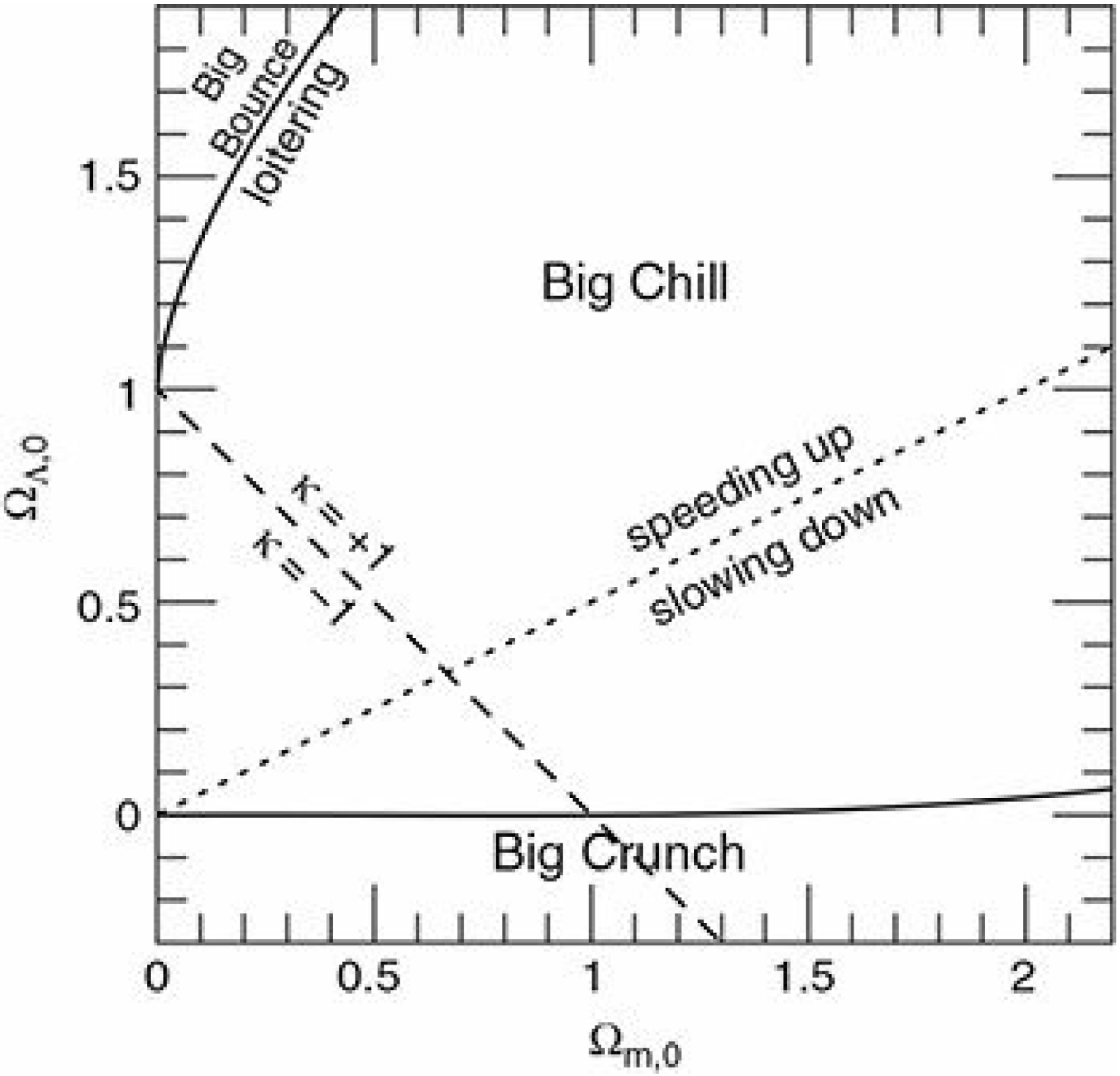
- 4 (qualitatively) different and possible modes of cosmic evolution:

1) Bouncing universe
2) Collapsing universe "Big Crunch"
3) Loitering universe
4) Expansion (only) universe

Cosmological Evolution Modes



Cosmological Evolution Modes



$$\Omega_{\Lambda 0} \leftrightarrow \Omega_{m0}$$

Expansion Modes:

different combinations Ω_{m0} and $\Omega_{\Lambda 0}$

In the diagram you can identify regions of

- **curvature**

$$k = \frac{H_0^{\,2} R_0^{\,2}}{c^2}(\Omega_{m0} + \Omega_{\Lambda 0} - 1)$$

- **acceleration**

$$q = \frac{\Omega_{m0}}{2} - \Omega_{\Lambda 0}$$

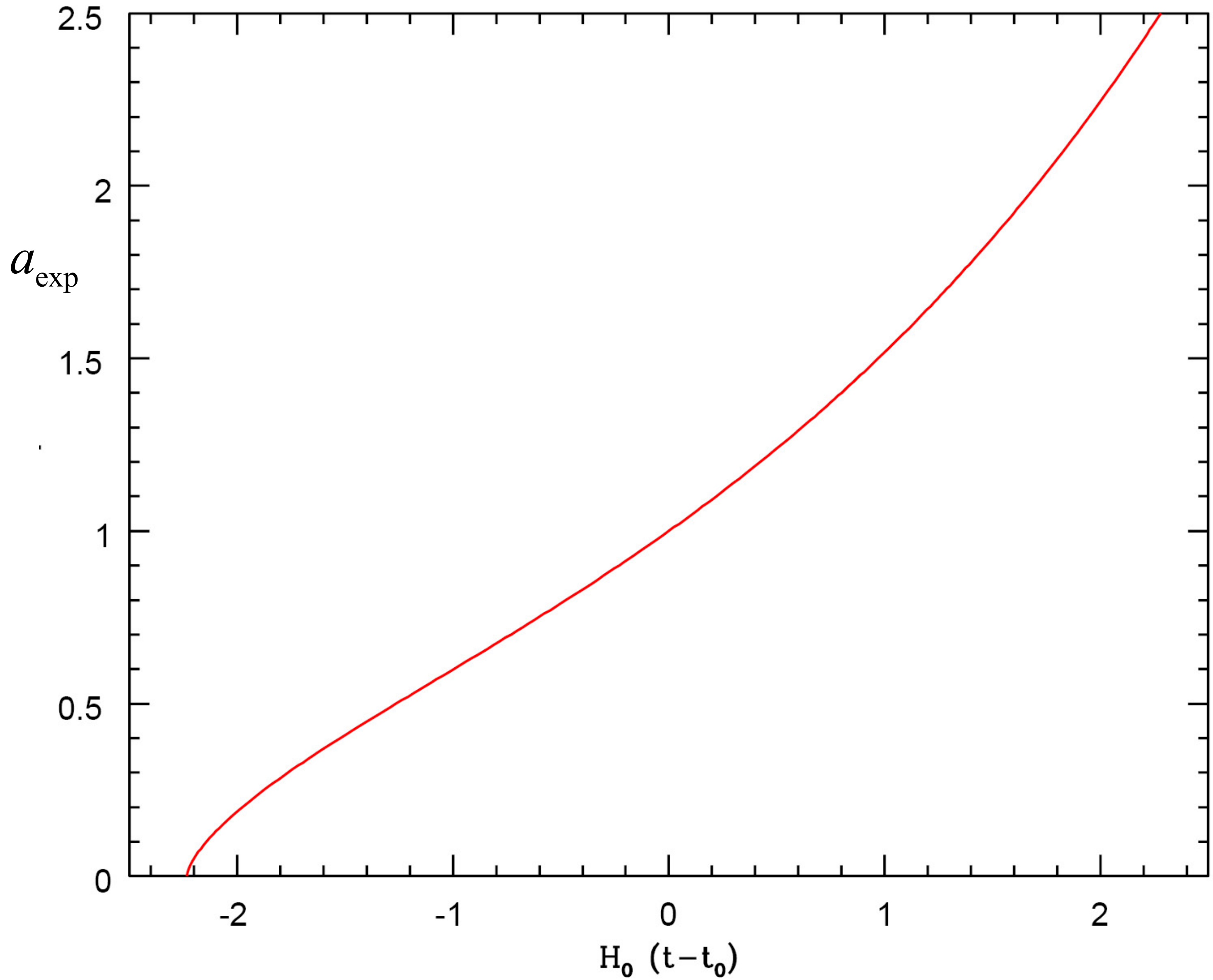
Our Universe:

the Concordance Cosmos

Concordance Universe Parameters

Hubble Parameter		$H_0 = 71.9 \pm 2.6\ km\ s^{-1}\ Mpc^{-1}$	
Age of the Universe		$t_0 = 13.8 \pm 0.1\ Gyr$	
Temperature CMB		$T_0 = 2.725 \pm 0.001\ K$	
Matter	**Baryonic Matter** **Dark Matter**	$\Omega_m = 0.27$	$\Omega_b = 0.0456 \pm 0.0015$ $\Omega_{dm} = 0.228 \pm 0.013$
Radiation	**Photons (CMB)** **Neutrinos (Cosmic)**	$\Omega_{rad} = 8.4 \times 10^{-5}$	$\Omega_\gamma = 5 \times 10^{-5}$ $\Omega_\nu = 3.4 \times 10^{-5}$
Dark Energy		$\Omega_\Lambda = 0.726 \pm 0.015$	
Total		$\Omega_{tot} = 1.0050 \pm 0.0061$	

Concordance Expansion

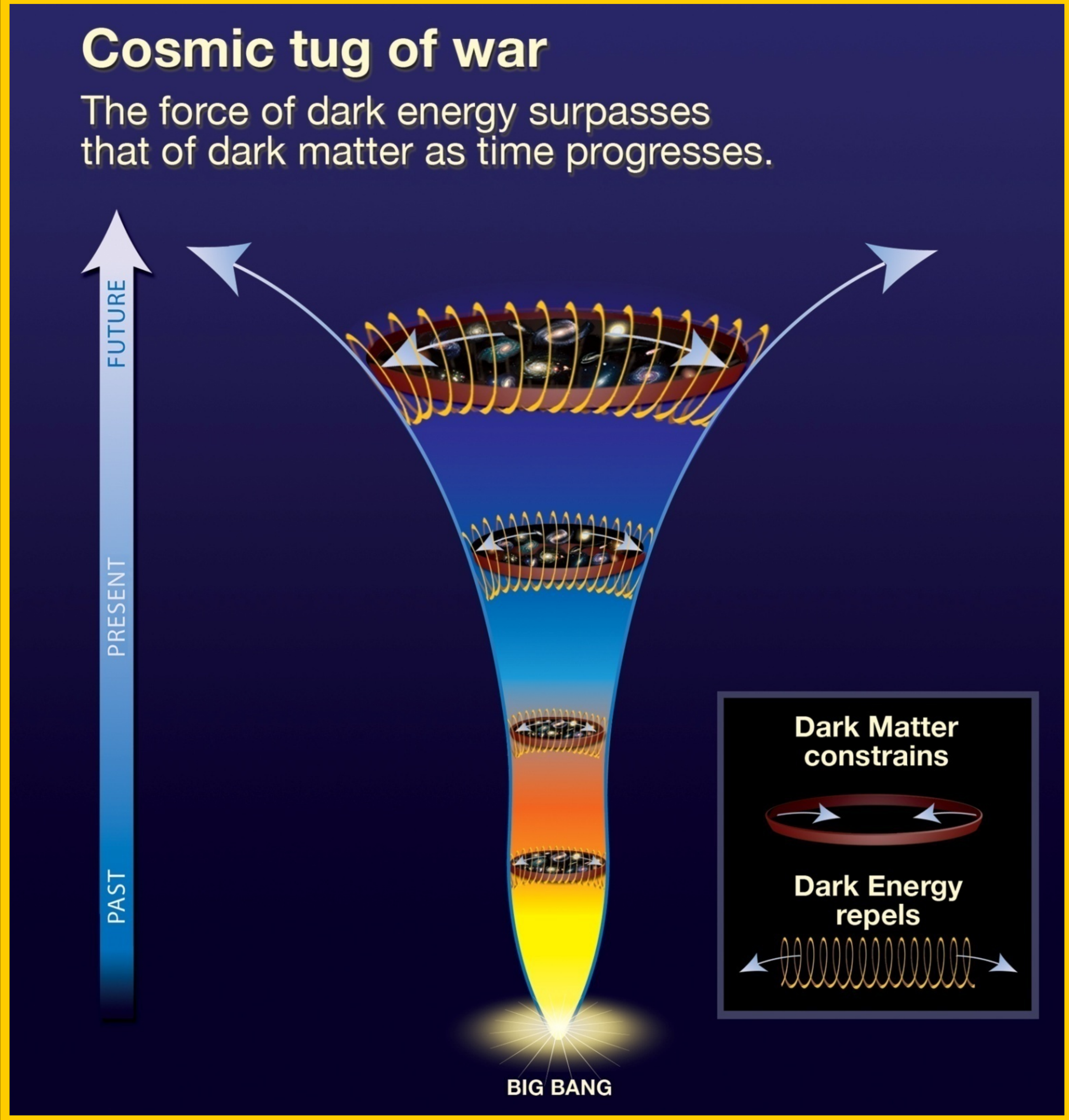


Heden & Toekomst:

VERSNELLING

Vroeger:

VERTRAGING



Age of the Universe

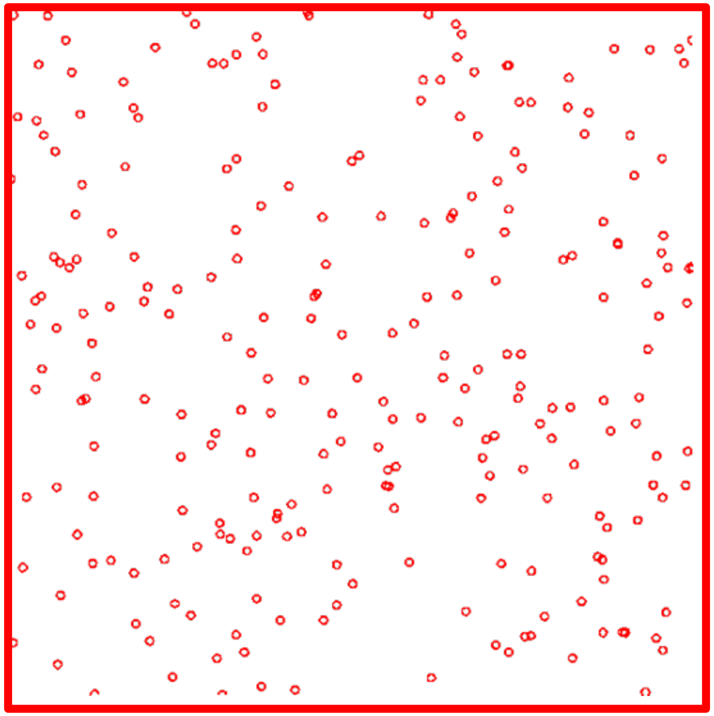
Hubble Expansion

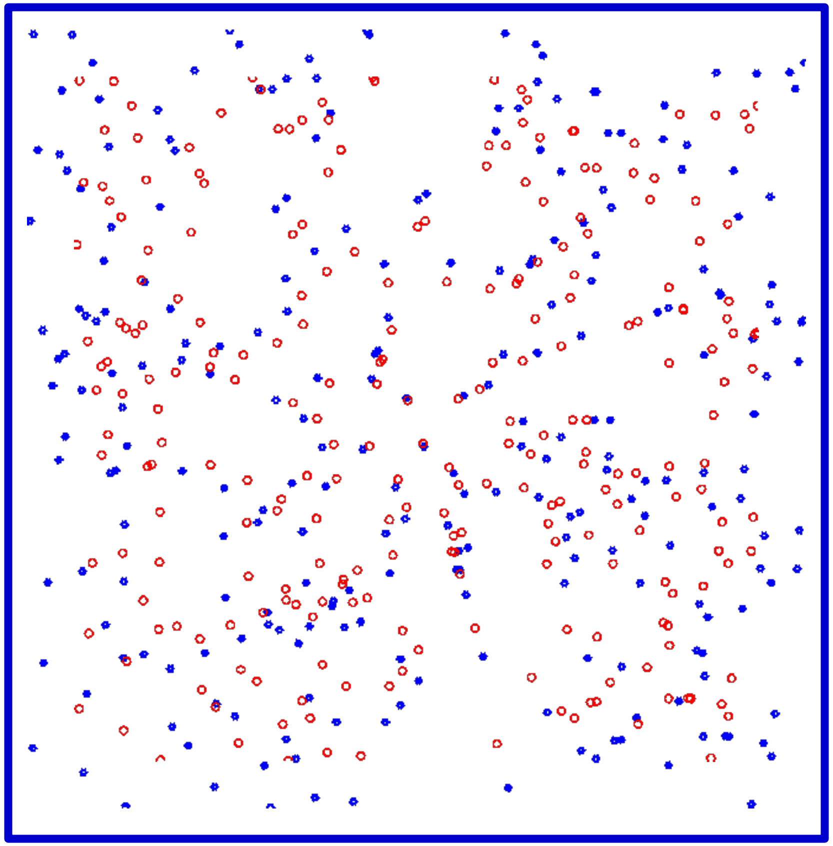
Space expands:

displacement - distance

$$v = H\ r$$

Hubble law: velocity - distance





Hubble Time

- By simple extrapolation back in time we find that at some instant the objects will have touched upon each other, i.e. $r(t_H)=0$. If we assume for simplicity that the expansion rate did remain constant (which it did not !), we find a direct measure for the age of the universe, the

Hubble Time:

$$t_H = \frac{1}{H}$$

$$H_0 = 100h \ km\, s^{-1} Mpc^{-1}$$

$$\Downarrow$$

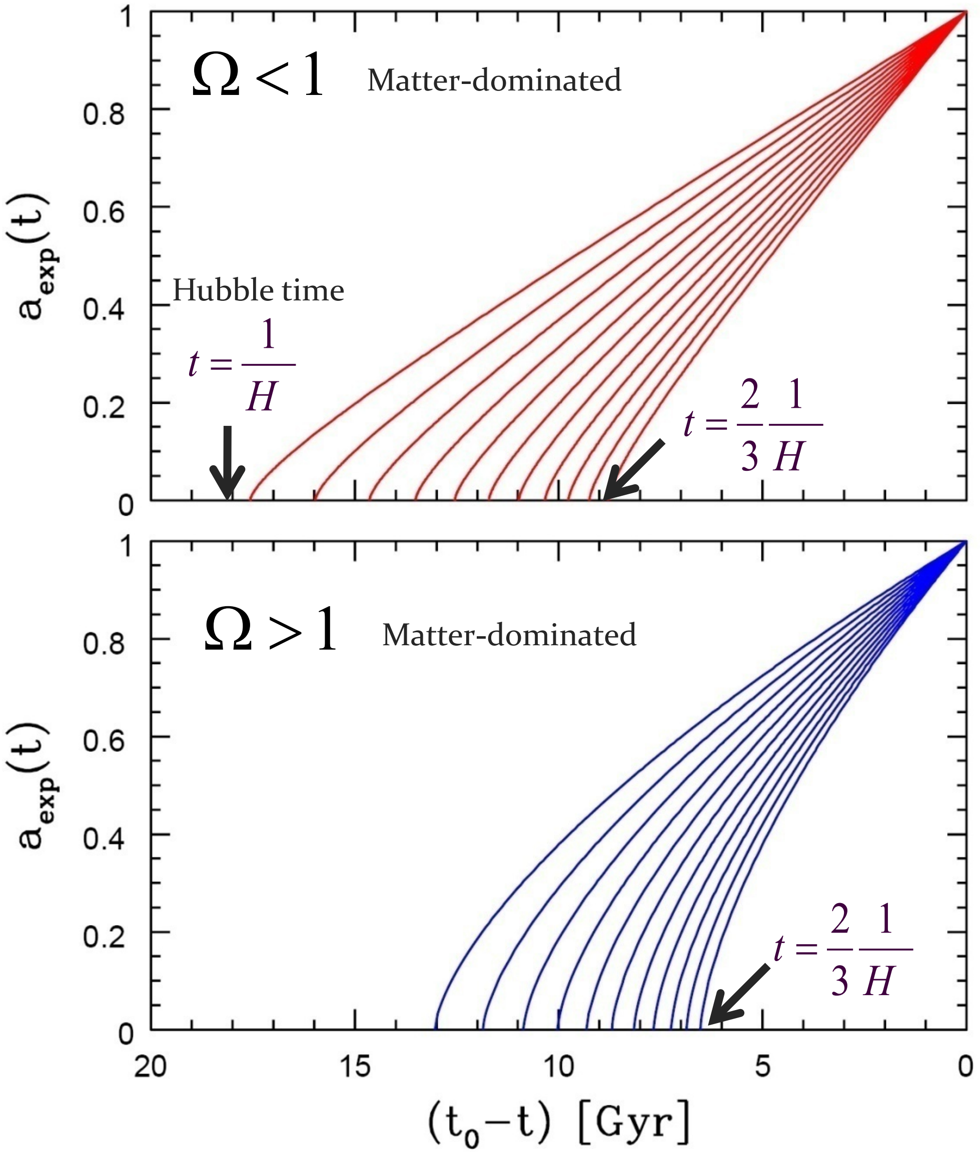
$$t_0 = 9.78 h^{-1} \ Gyr$$

The Hubble parameter is usually stated in units of km/s/Mpc.

It's customary to express it in units of 100 km/s/Mpc, expressing the real value in terms of the dimensionless value $h=H_0/[100 \text{ km/s/Mpc}]$.

The best current estimate is H_0=72 km/s/Mpc. This sets t_0~10 Gyr.

Age of the Universe



Age of a FRW universe at
Expansion factor a(t):

Universes of lower density – ie.
Universes of lower Ω - are older !

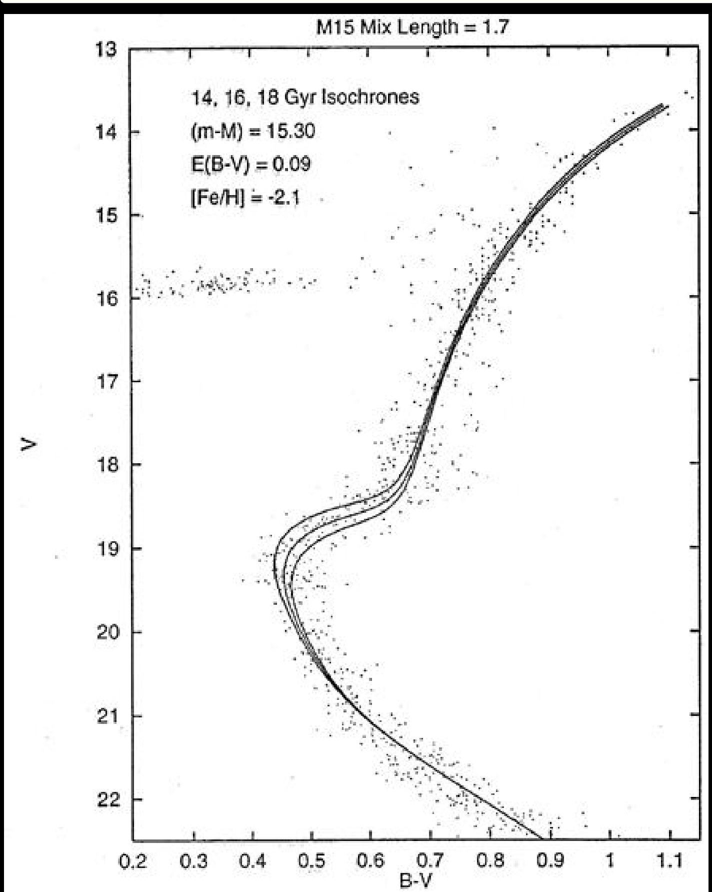
Cosmic Age

estimated age of the oldest stars in Universe far in excess of estimated age of matter-dominated FRW Universe:

Globular cluster stars:	**13-15 Gyr**
Universe:	**10-12 Gyr**

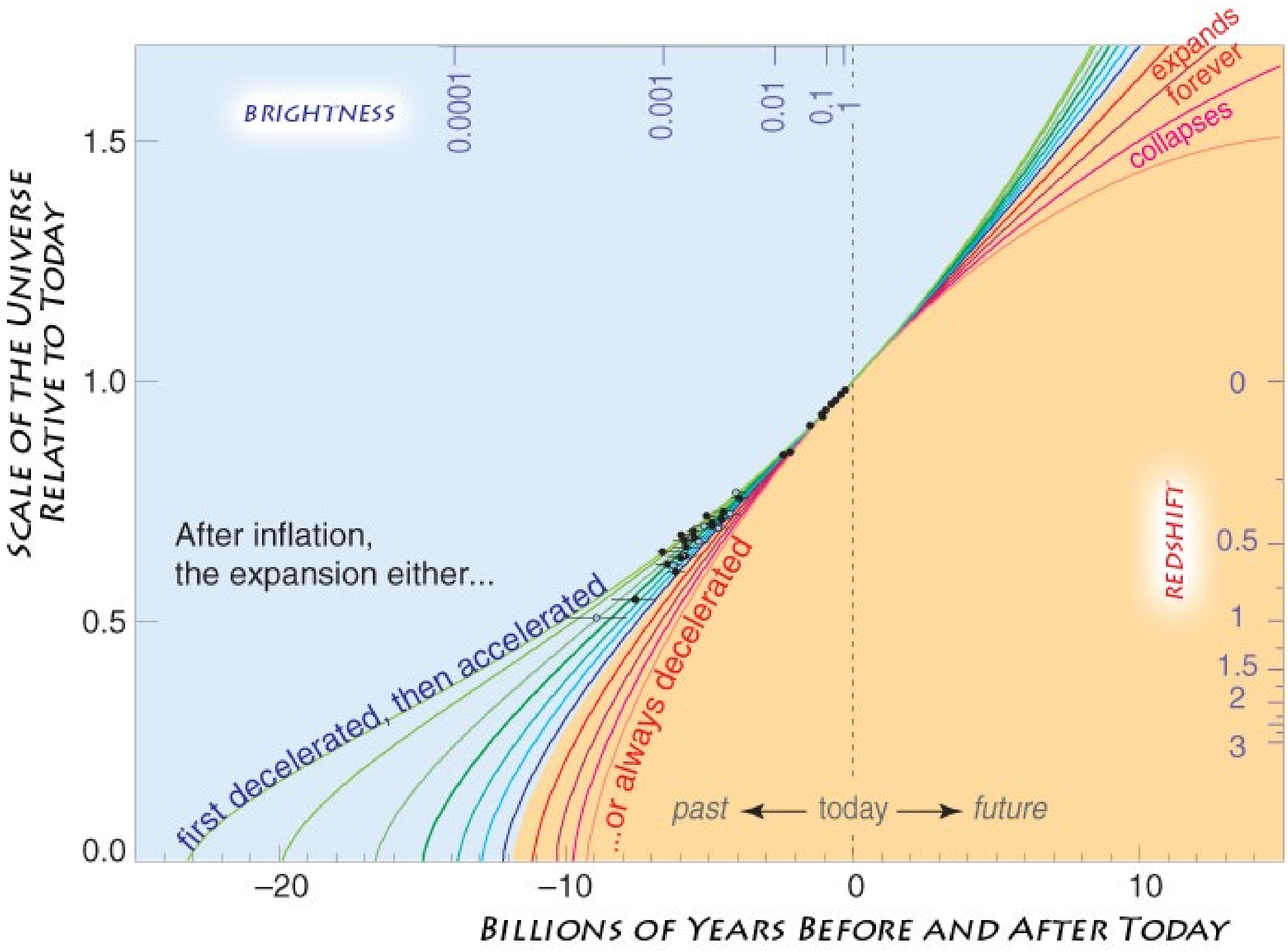
Globular Clusters

- Roughly spherical assemblies of 100,000-200,000 stars
- Radius ~ 20-50 pc: extremely high star density
- Globulars are very old, amongst oldest objects in local Universe
- Stars formed around same time: old, red, population
- Colour-magnitude diagram characteristic: accurate age determination on the basis of stellar evolution theories.



Typical 1980-1990s isochrone fit

Concordance Expansion



Adiabatic Expansion

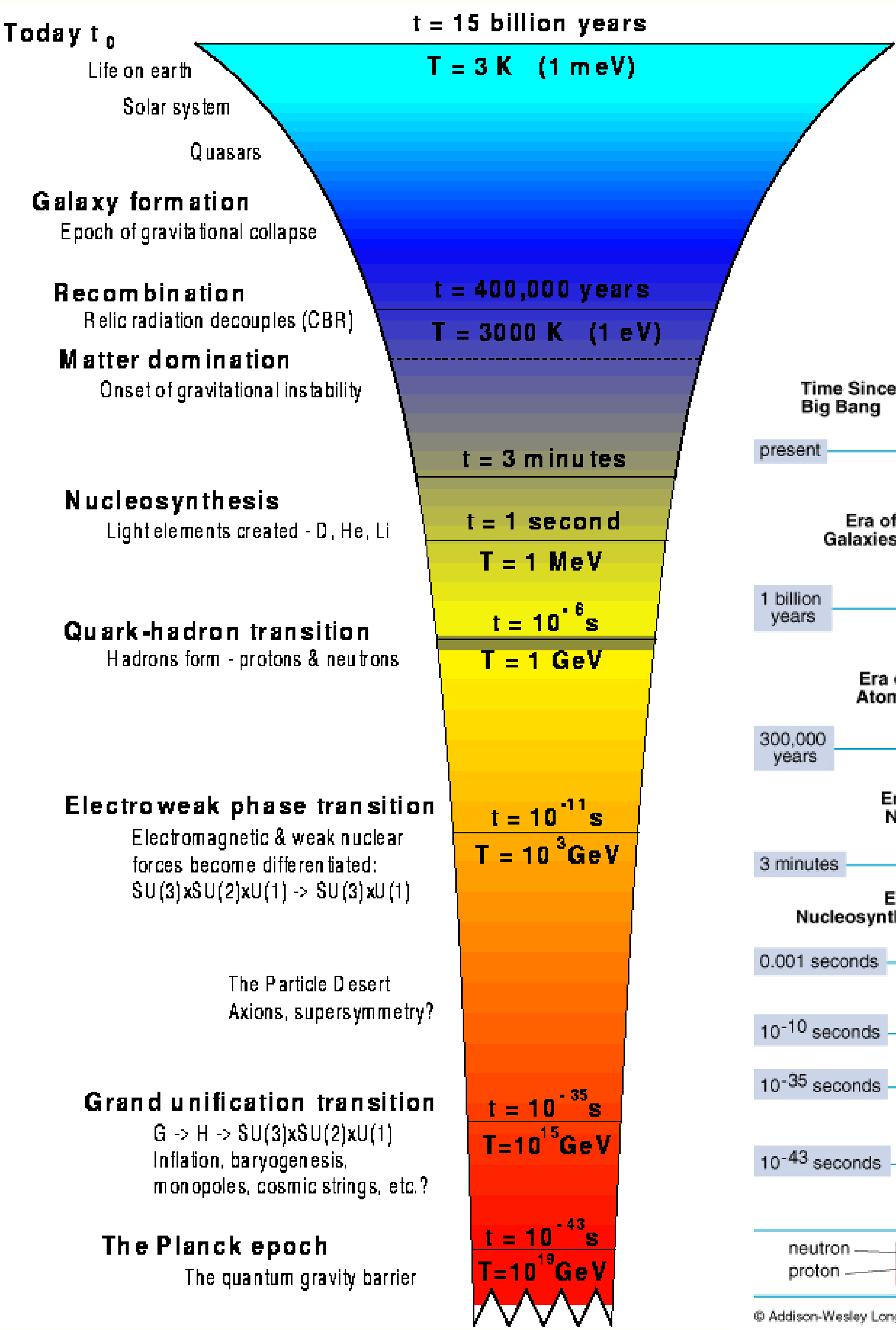
Adiabatic Expansion

- The Universe of Einstein, Friedmann & Lemaitre expands *adiabatically*

- Energy of the expansion of the Universe corresponds to the decrease in the energy of its constituents

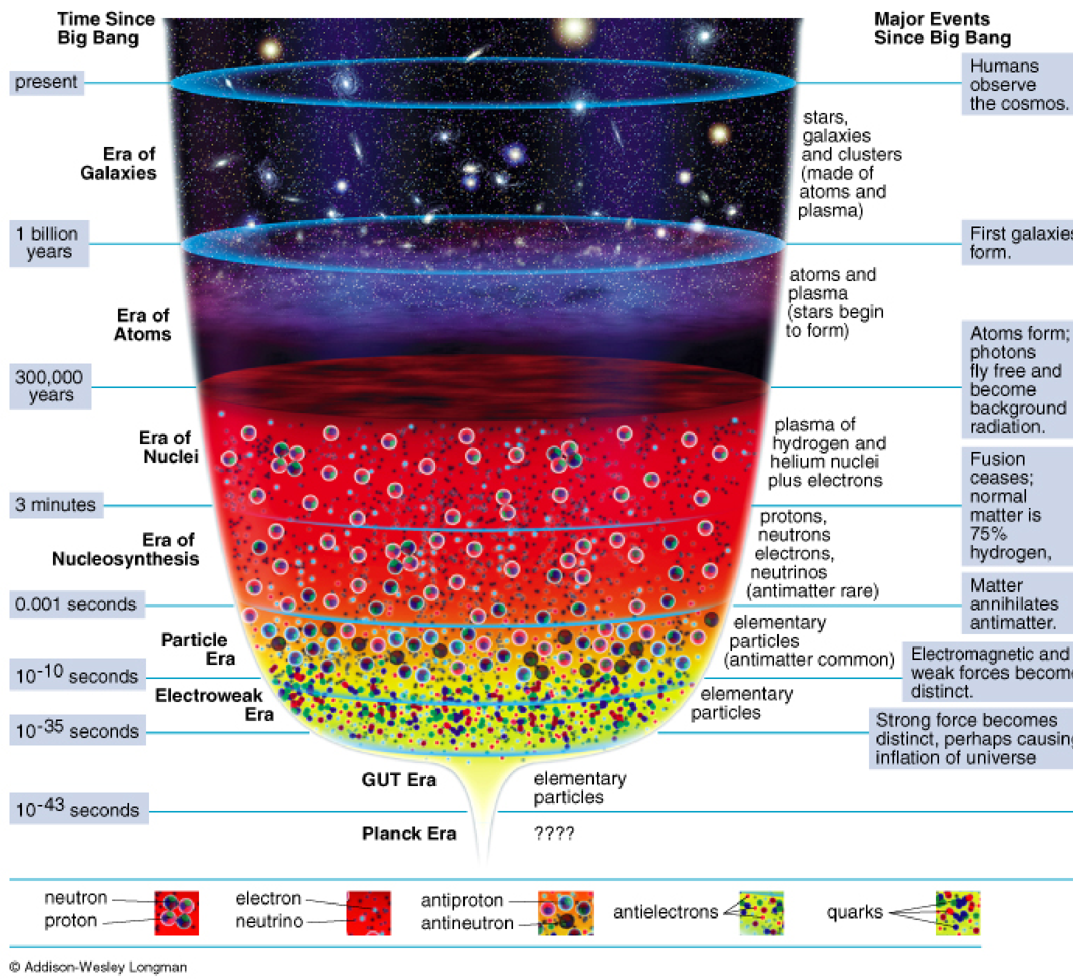
- *The Universe COOLS as a result of its expansion !*

$$T(t) \propto 1/a(t)$$



Adiabatic Expansion

reconstruction
Thermal History
of the Universe



Big Bang:

the Evidence

Big Bang Evidence

- **Olber's paradox:**
  the night sky is dark
  → finite age Universe (13.7 Gyr)

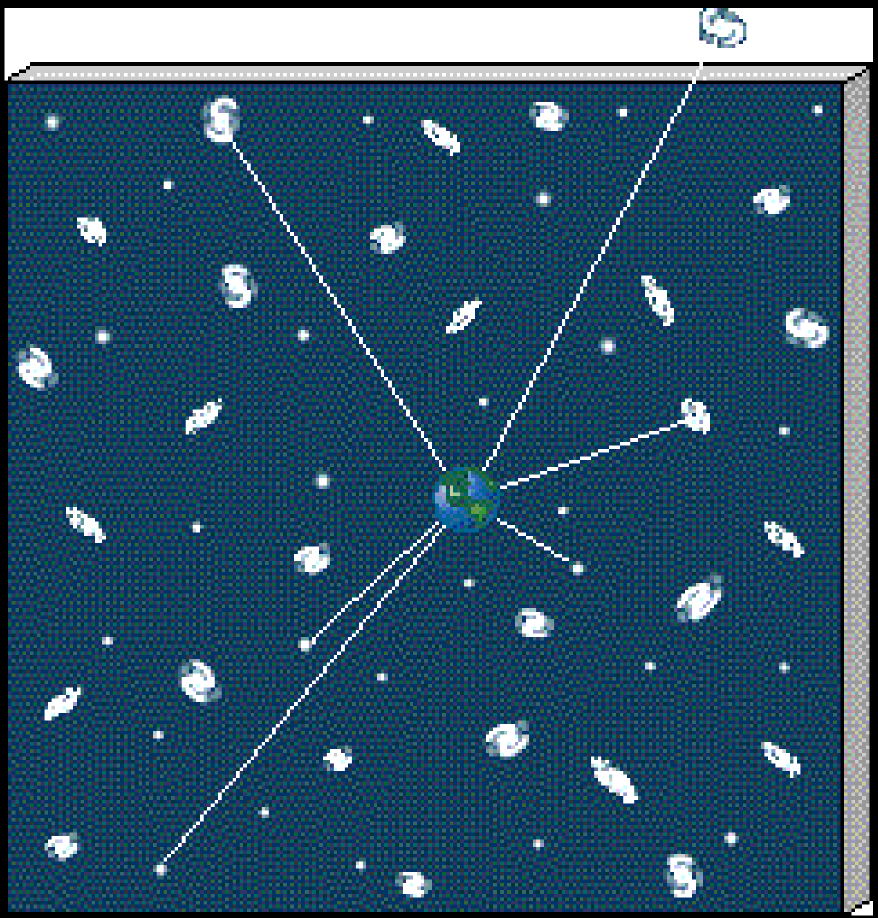
- **Hubble Expansion**
  uniform expansion, with
  expansion velocity ~ distance: $v = H r$

- **Explanation Helium Abundance 24%:**
  light chemical elements formed (H, He, Li, …)
  after ~3 minutes …

- **The Cosmic Microwave Background Radiation:**
  the 2.725K radiation blanket, remnant left over
  hot ionized plasma → neutral universe
  (379,000 years after Big Bang)

- **Distant, deep Universe indeed looks different …**

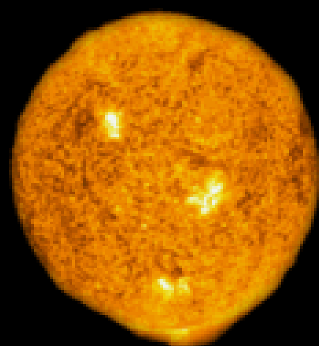
1. Olber's Paradox



In an infinitely large, old and unchanging Universe each line of sight would hit a star:

Sky would be as bright as surface of star:

1. Olber's Paradox



In an infinitely large, old and unchanging Universe each line of sight would hit a star:

→

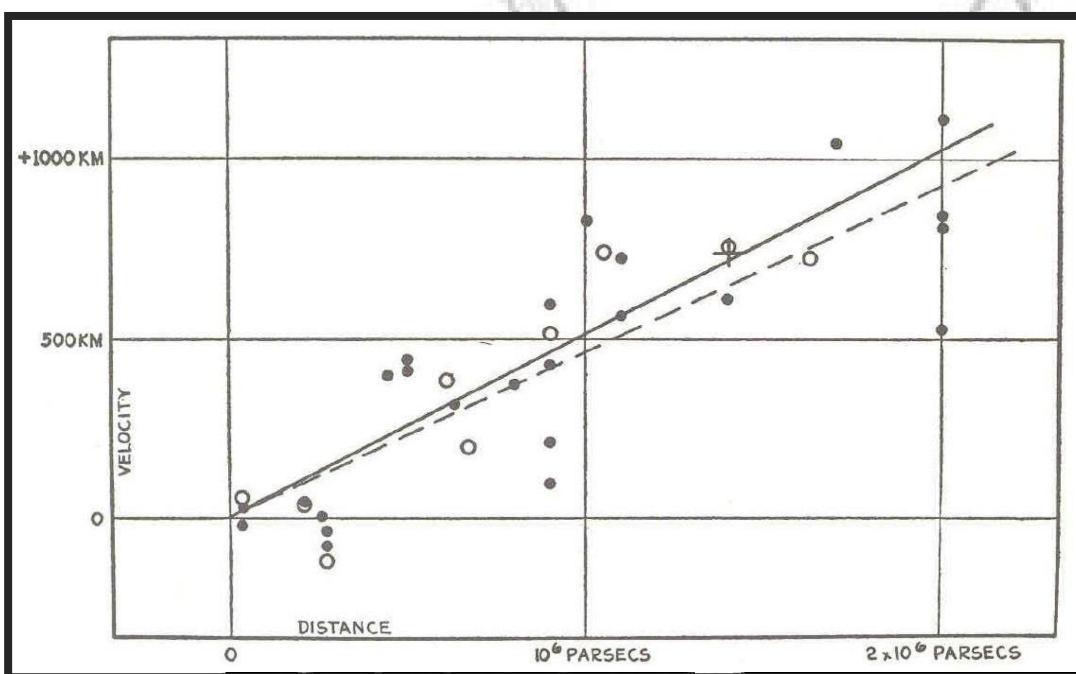
Sky would be as bright as surface of star:

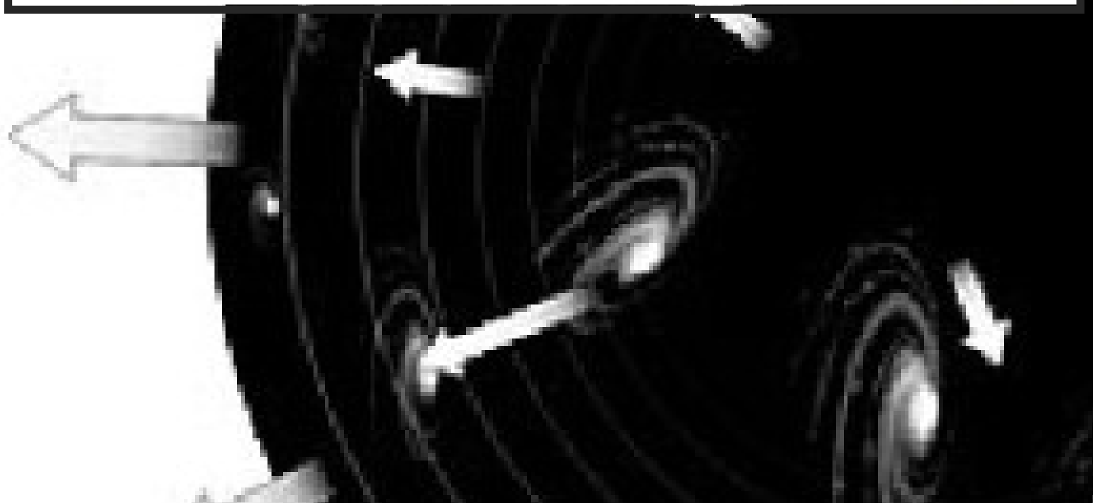
Night sky as bright as
Solar Surface, yet
the night sky is dark

→

finite age of Universe (13.8 Gyr)

2. Hubble Expansion



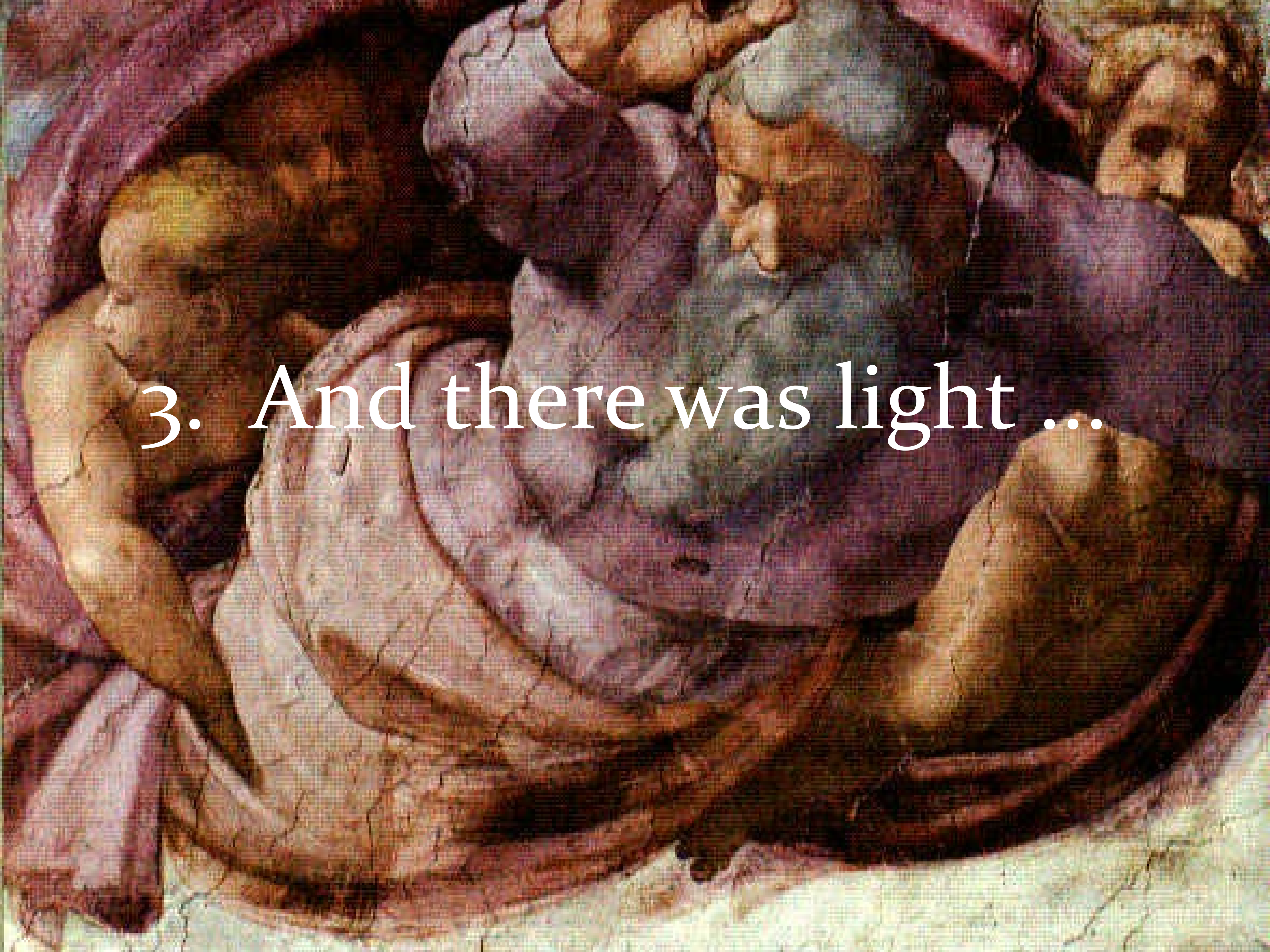


$$v_{rad} = cz = H_0 r$$

H_0 : **Hubble constant**

specifies expanssion rate of the Universe

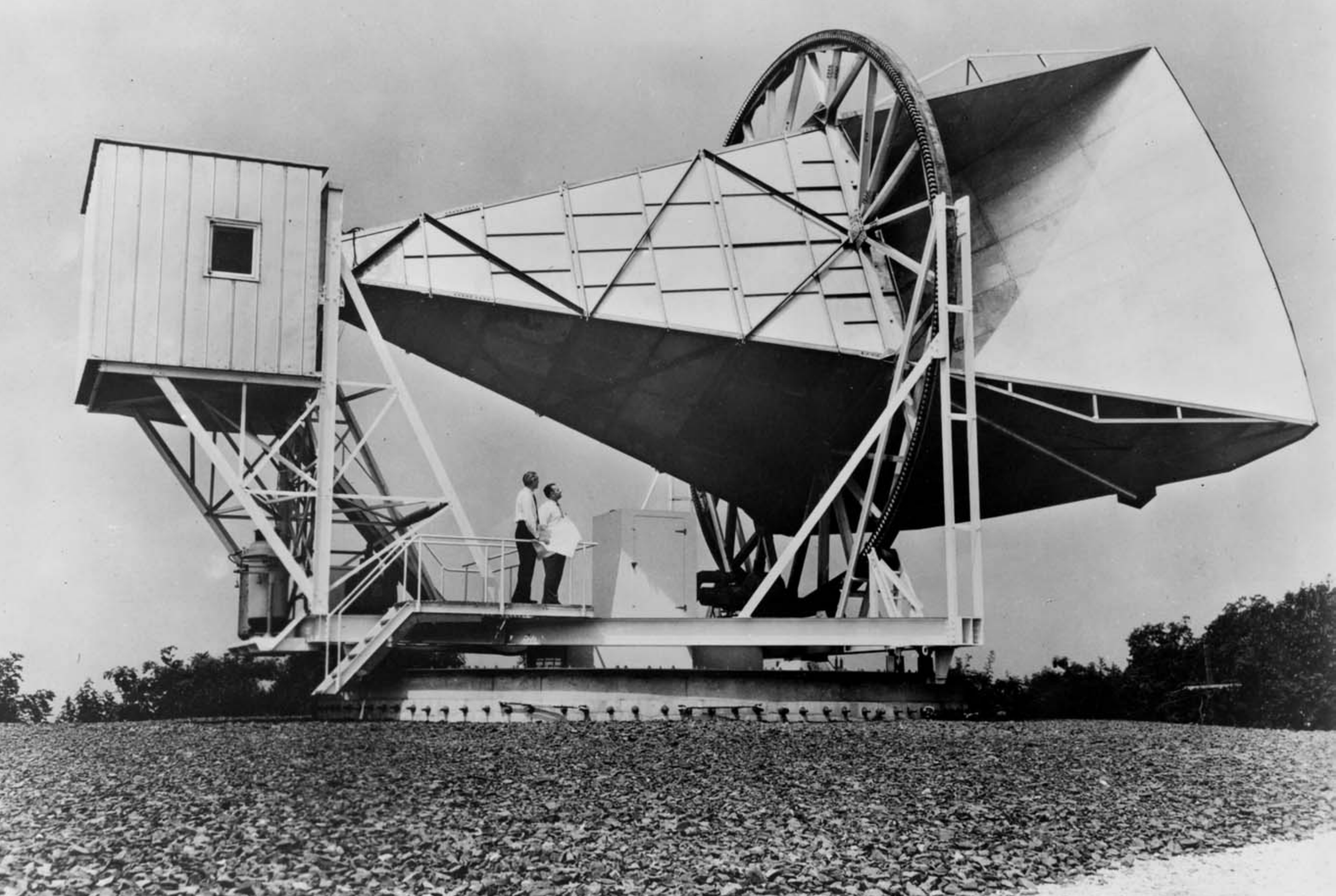
3. And there was light …



... and there was light ...

379.000 years after the Big Bang

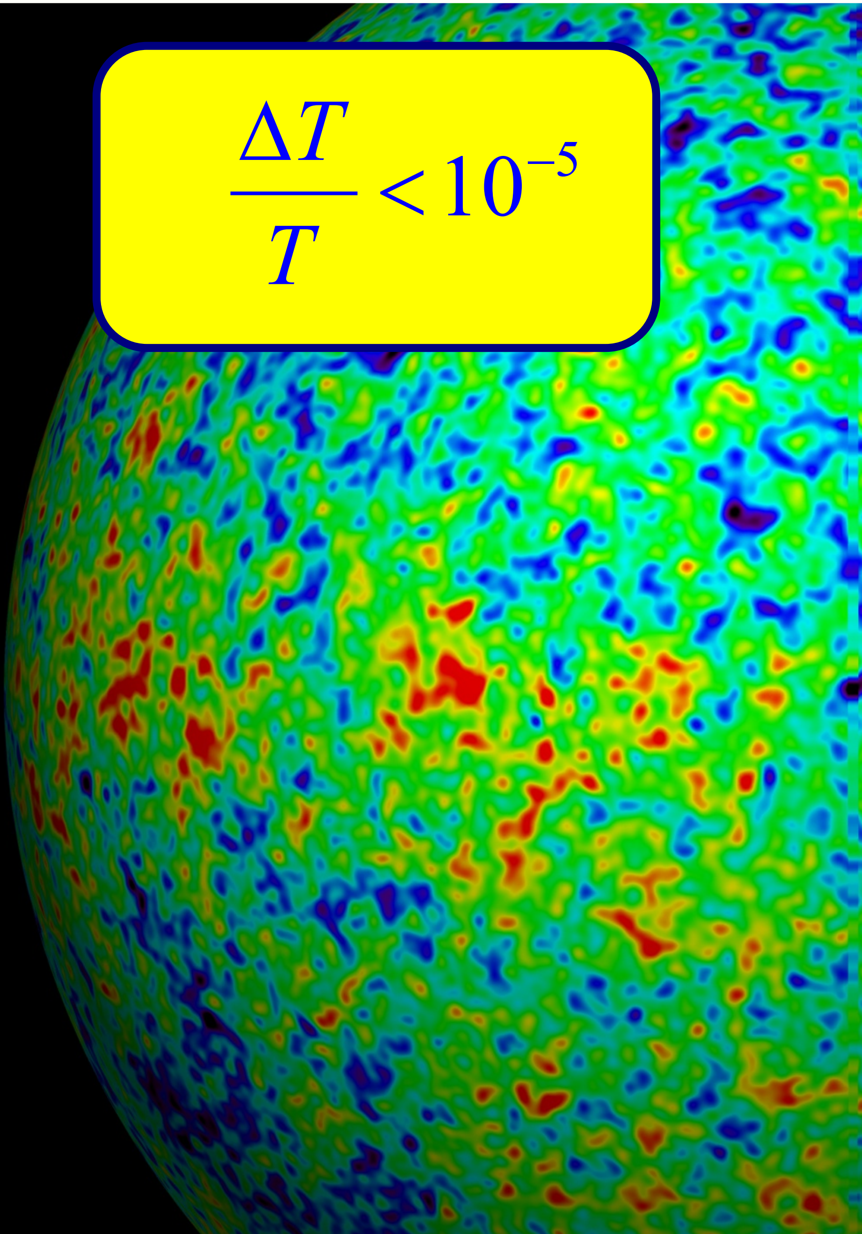
3. Cosmic Microwave Background Radiation



Cosmic Light (CMB): the facts

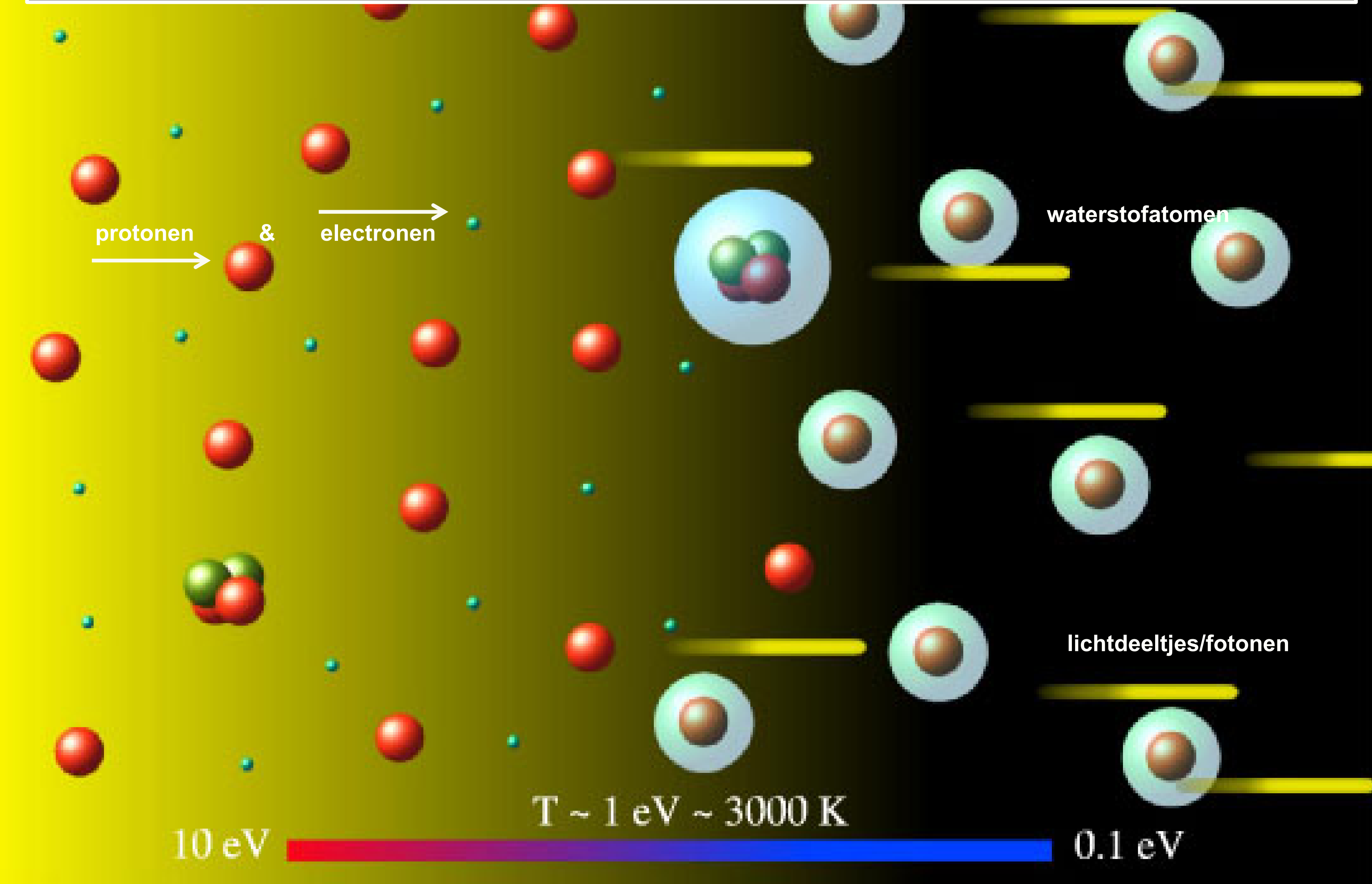
- Discovered serendipitously in 1965 — Penzias & Wilson, Nobelprize 1978 !!!!!
- Cosmic Licht that fills up the Universe uniformly
- Temperature: T_γ=2.725 K
- (CMB) photons most abundant particle in the Universe: $n_\gamma \sim 415\ cm^{-3}$
- Per atom in the Universe: $n_\gamma/n_B \sim 1.9 \times 10^9$
- Ultimate evidence of the Big Bang !!!!!!!!!!!!!!!!!!!

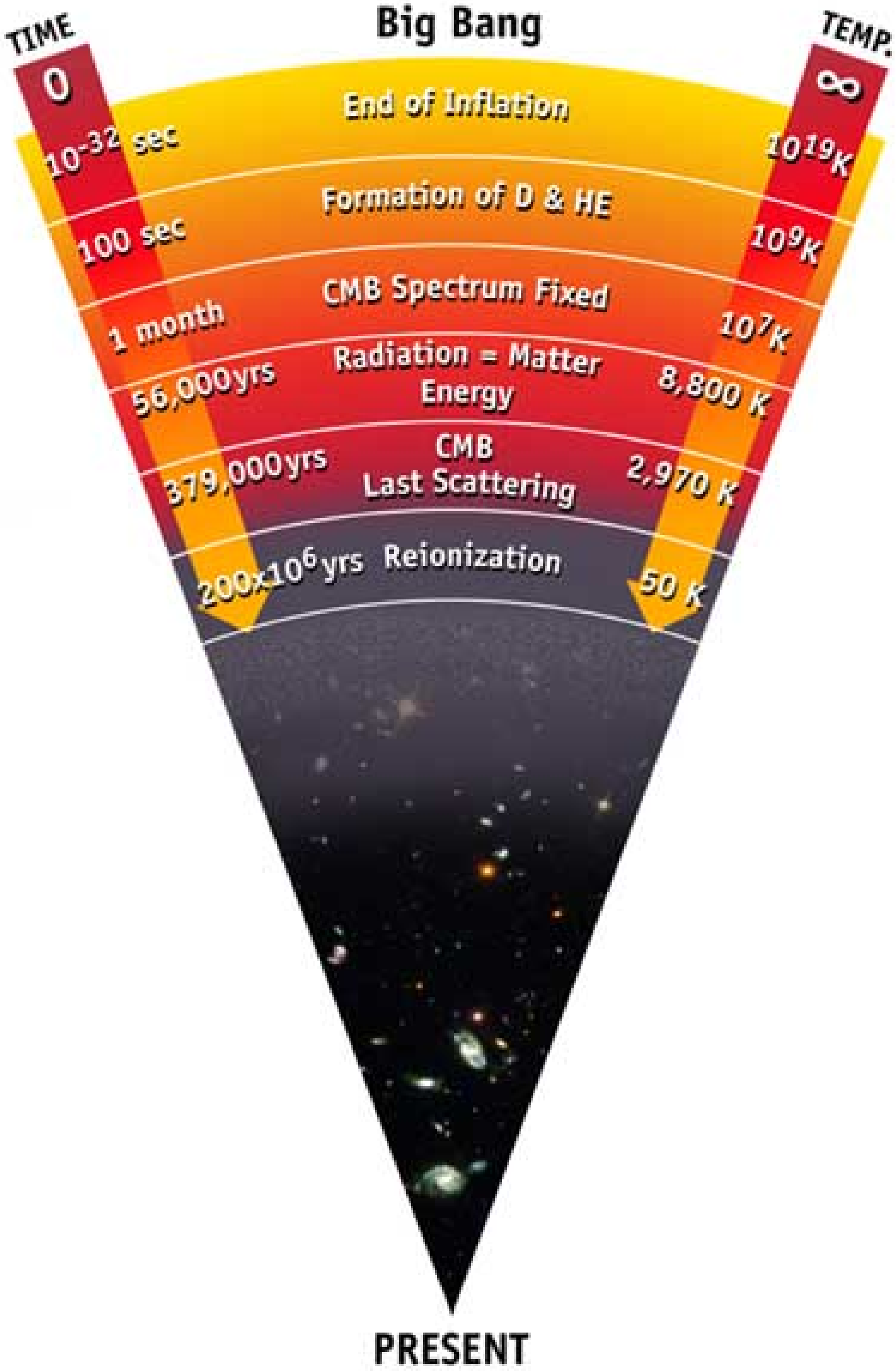
Extremely Smooth Radiation Field

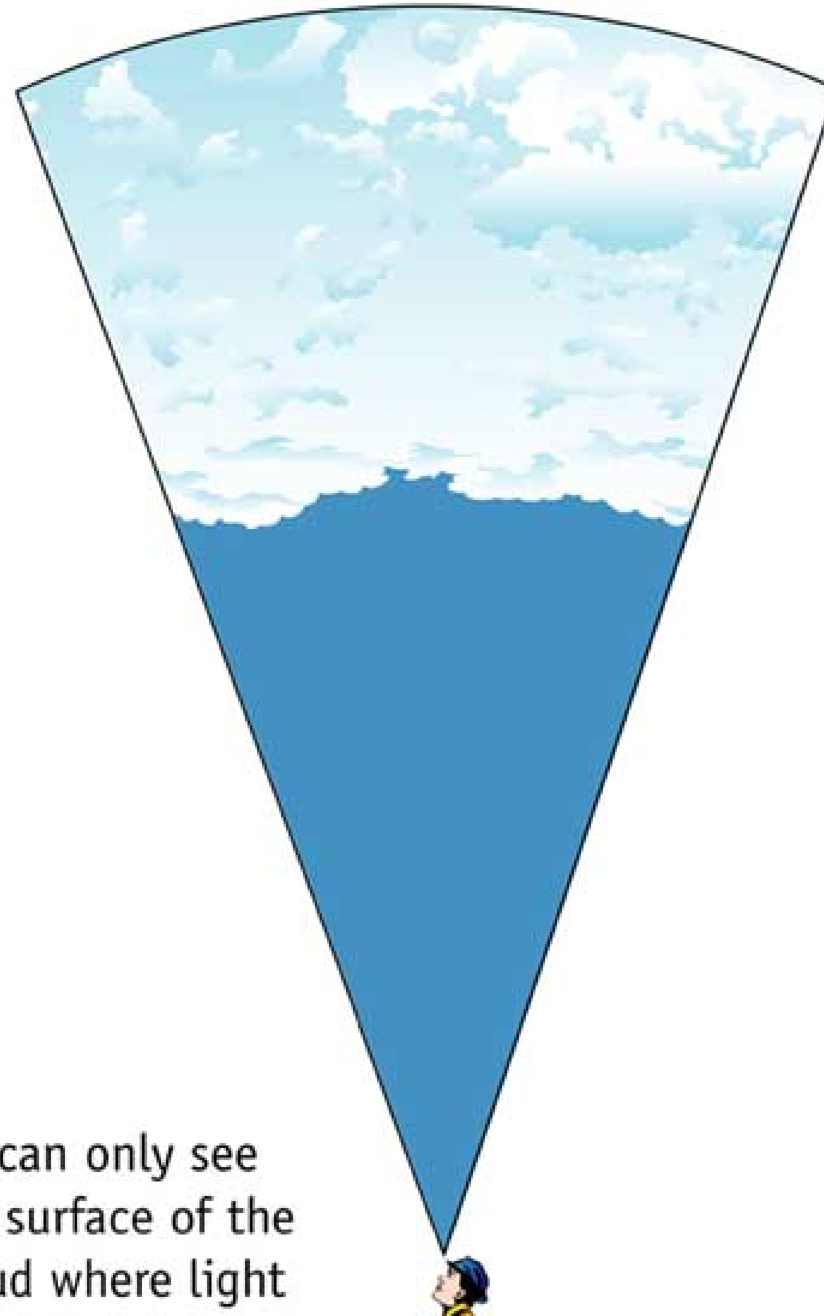


$$\frac{\Delta T}{T} < 10^{-5}$$

Recombination & Decoupling







We can only see the surface of the cloud where light

the Cosmic TV Show

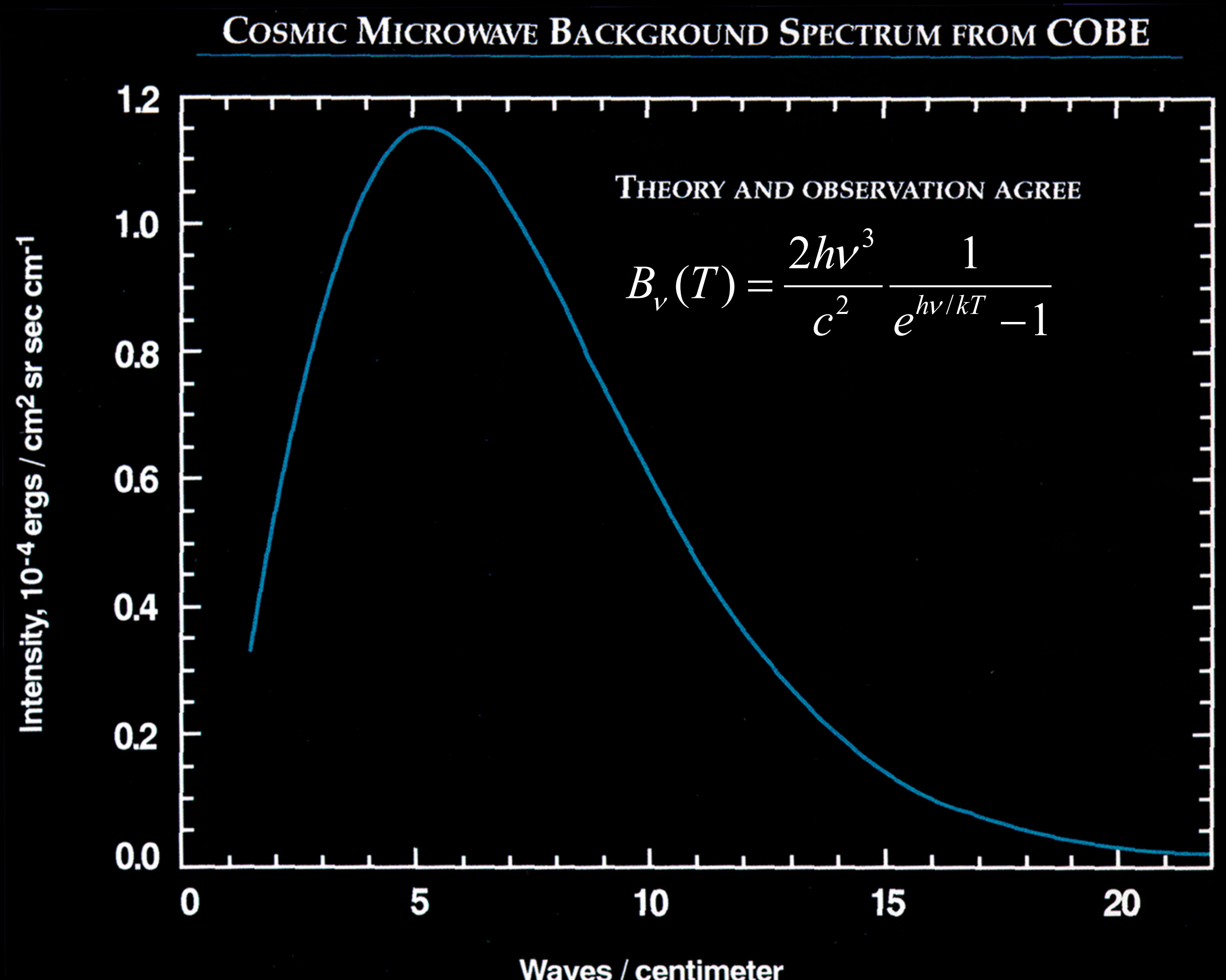
__Note:__

far from being an exotic faraway phenomenon, realize that the CMB nowadays is counting for approximately 1% of the noise on your (camping) tv set ...

!!!! Live broadcast from the Big Bang !!!!

Courtesy: W. Hu

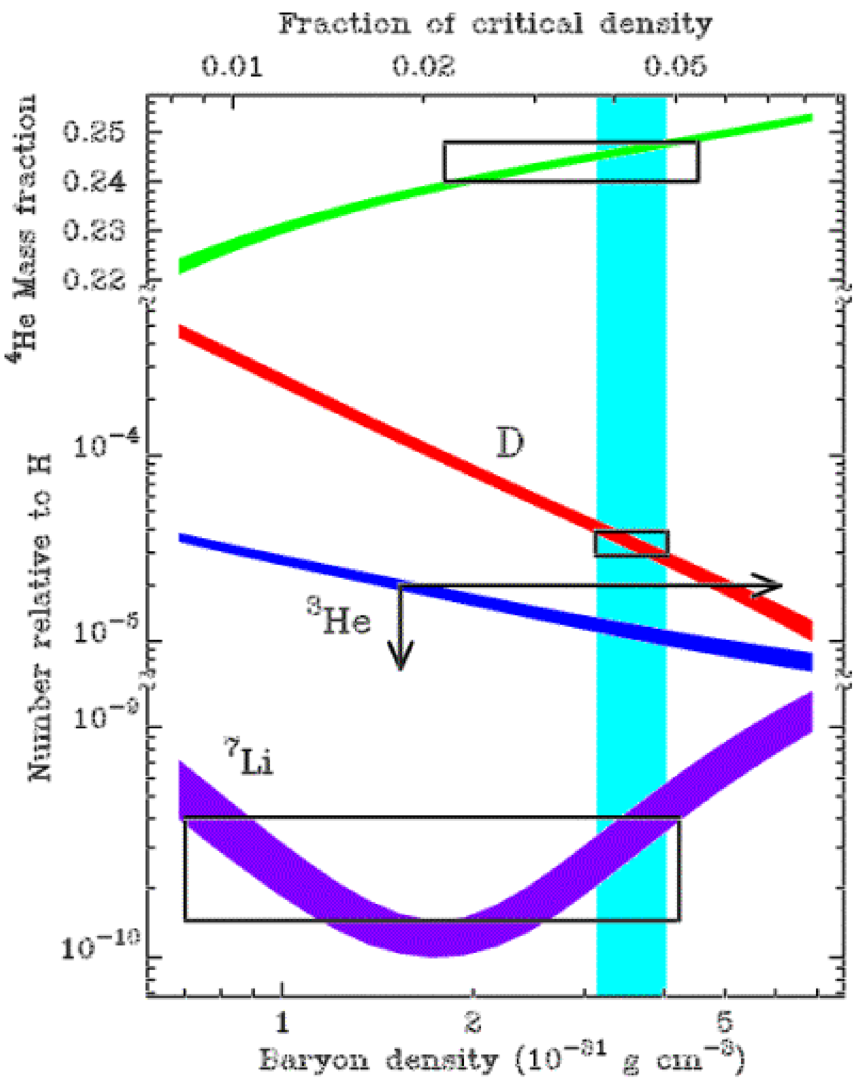
Energy Spectrum Cosmic Light



- COBE-DIRBE:
  temperatureT = 2.725 K
- John Mather
  Nobelprize physics
  2006
- Most perfect
  Black Body
  Spectrum ever seen !!!!

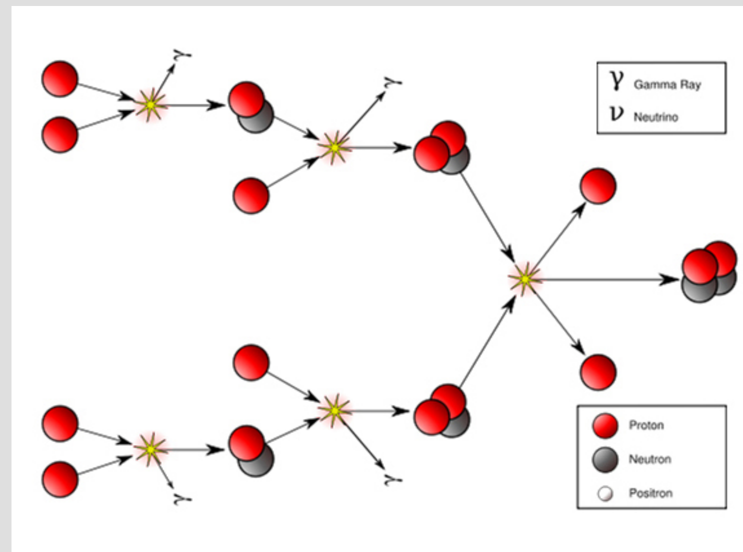
4. Proton-Neutron & Helium

p/n ~1/7: 1 min na BB



Mass Fraction Light Elements

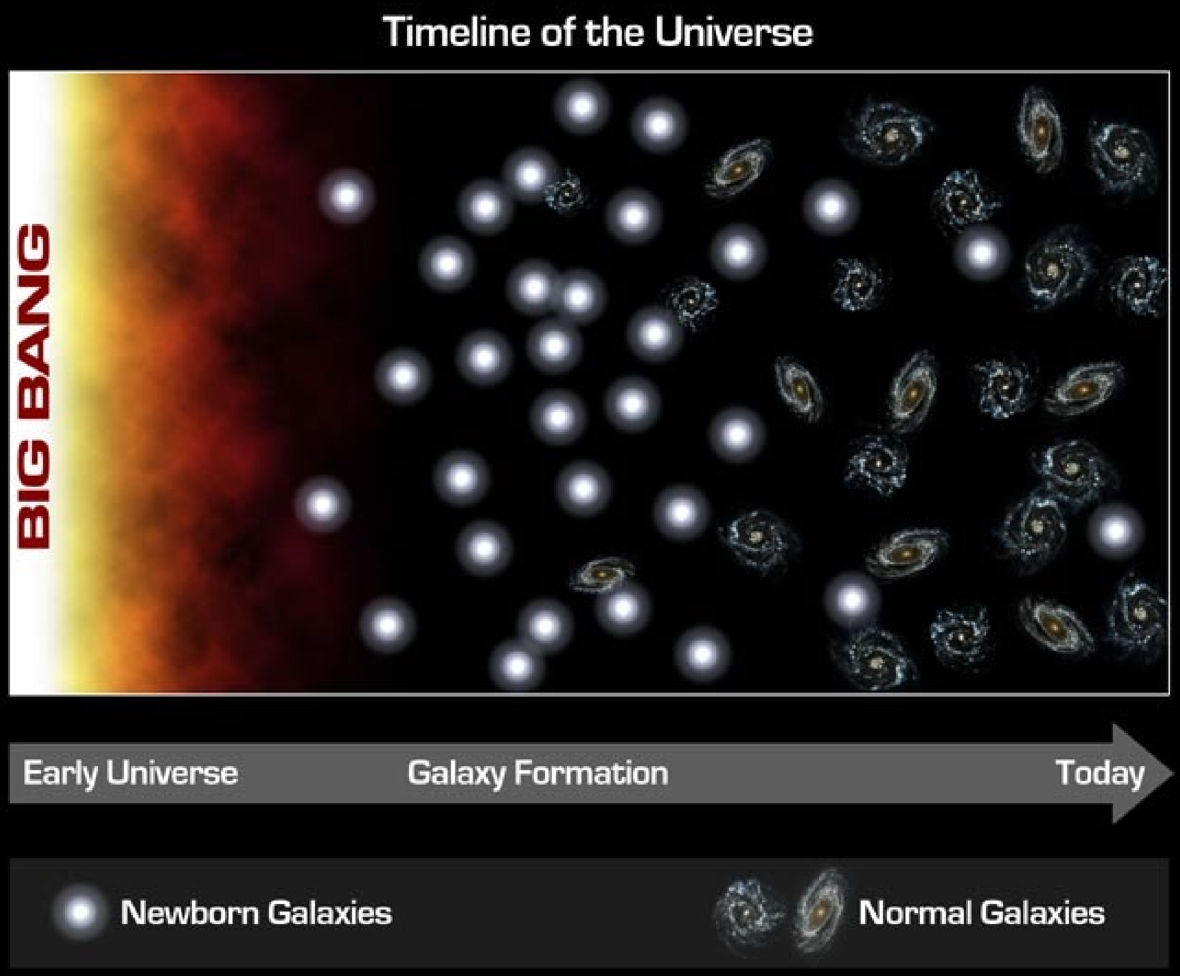
24%	^{4}He	nuclei
traces	D, ^{3}He, ^{7}Li	nuclei
75%	H	nuclei (protons)



Between 1-200 seconds after Big Bang, temperature dropped to 10^9 K:

Fusion protons & neutrons into light atomic nuclei

5. the Changing Universe



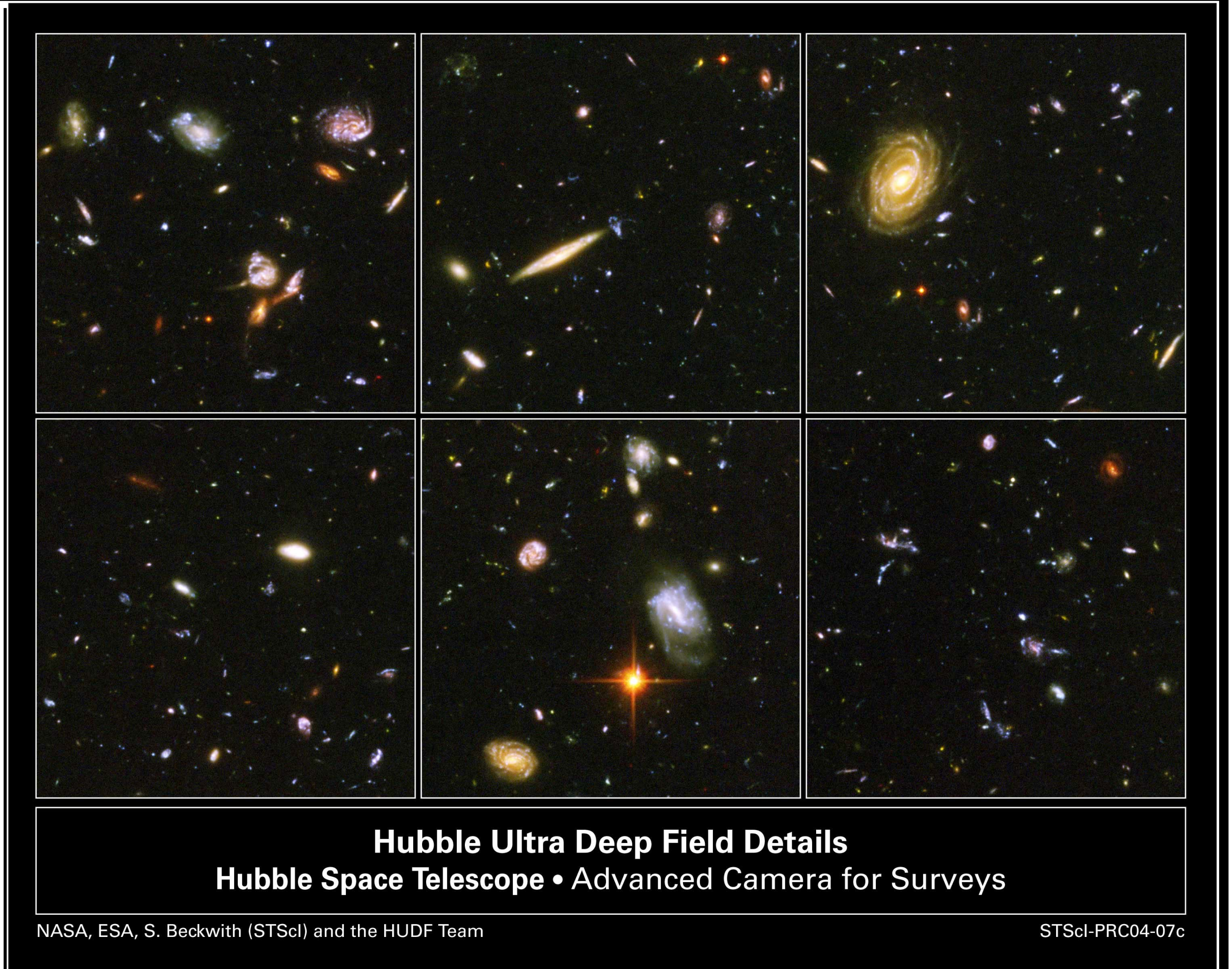
At great depths the Universe looks completely different

- and thus long ago :

Depth= Time

Galaxies in Hubble Ultra Deep Field

5. the Changing Universe



At great depths the Universe looks completely different

- and thus long ago : Depth= Time

→

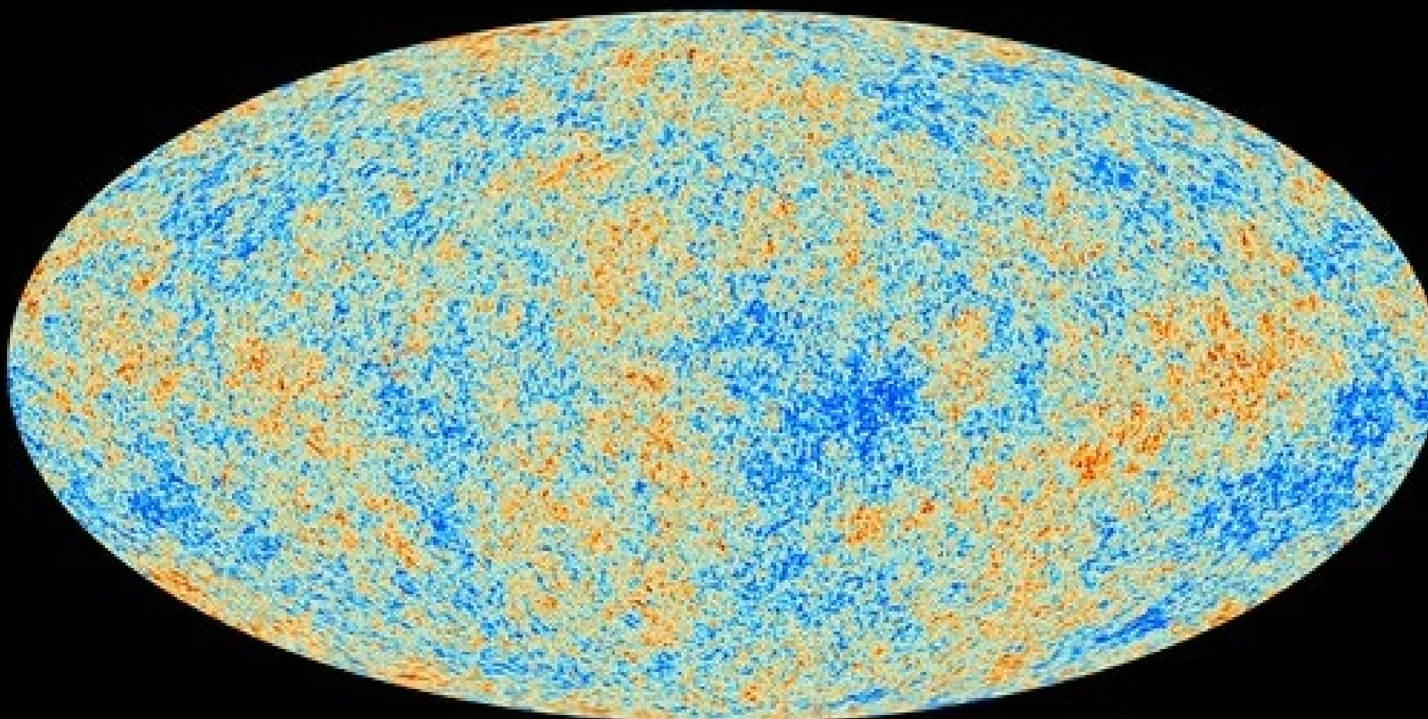
Galaxies in Hubble Ultra Deep Field

Cosmic Curvature

How Much ?

Cosmic Curvature

Cosmic Microwave Background



Map of the Universe at Recombination Epoch (Planck, 2013):

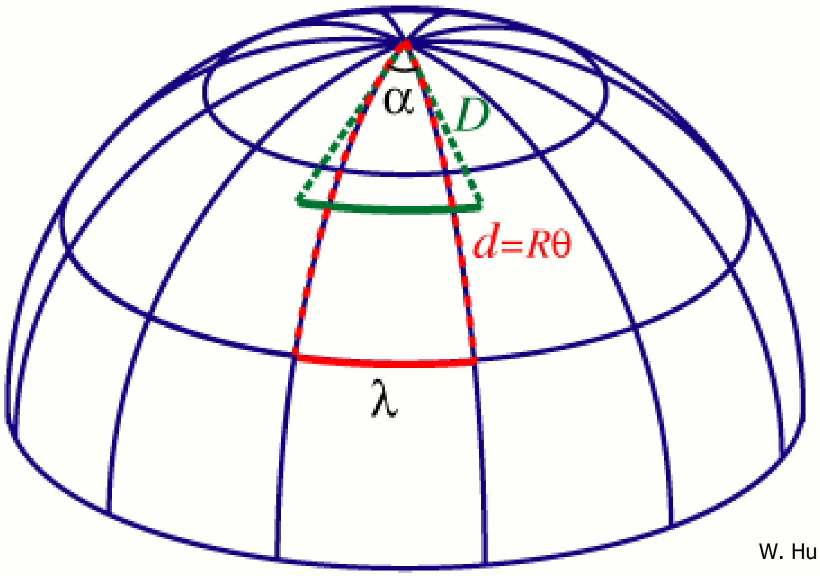
- **379,000 years after Big Bang**
- **Subhorizon perturbations: primordial sound waves**
- **$\Delta T/T$ < 10-5**

Measuring Curvature

Measuring the Geometry of the Universe:

- **Object with known physical size, at large cosmological distance**
- **Measure angular extent on sky**
- **Comparison yields light path, and from this the curvature of space**

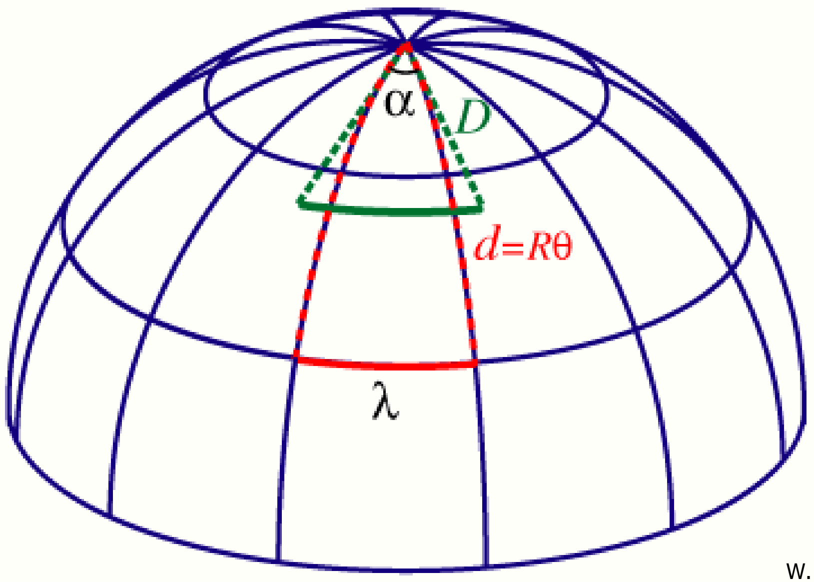
Geometry of Space



Measuring Curvature

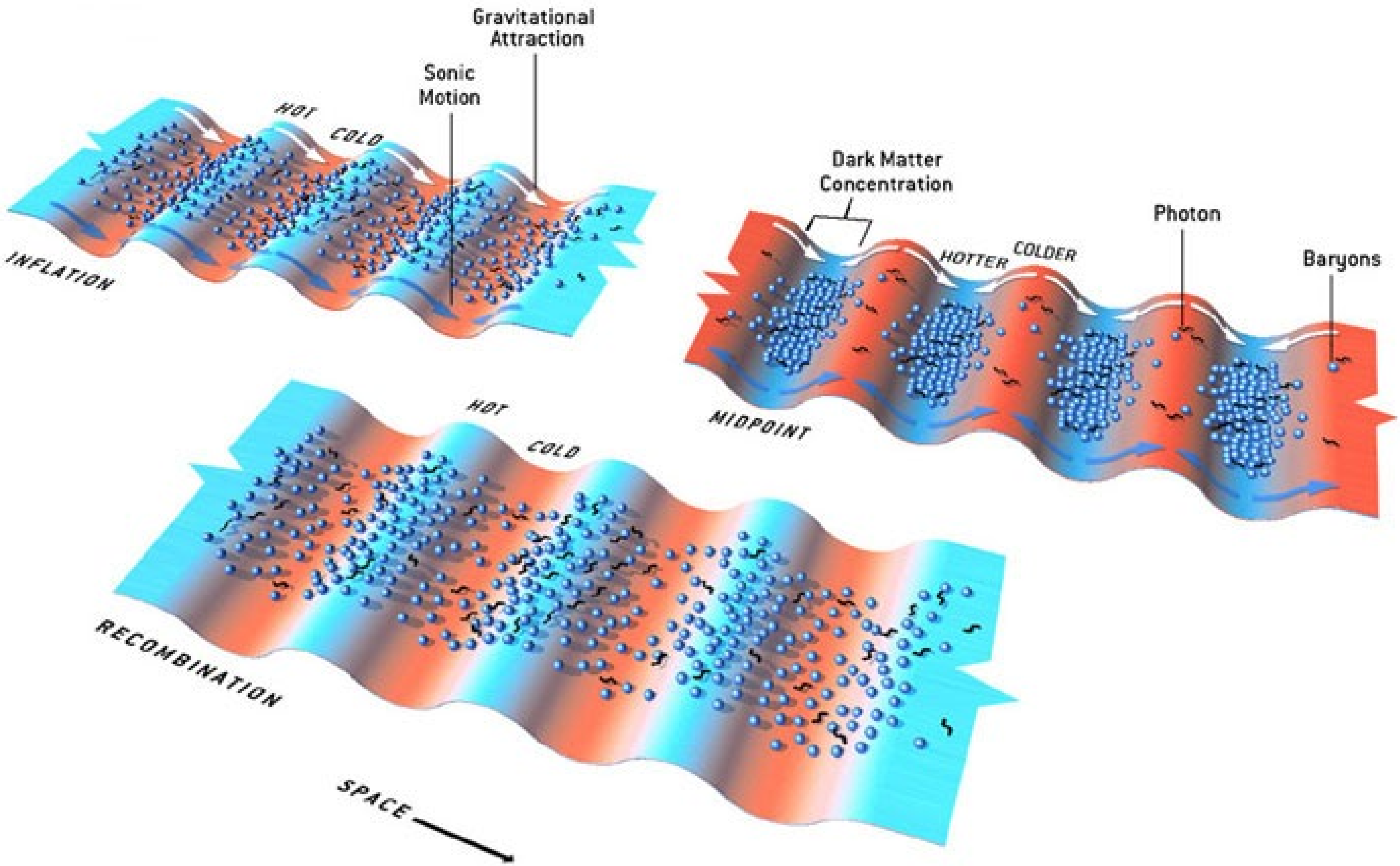
- **Object with known physical size, at large cosmological distance:**
- **Sound Waves in the Early Universe !!!!**

Temperature Fluctuations
CMB



W. Hu

Fluctuations-Origin



Music of the Spheres

- small ripples in
  primordial matter & photon   distribution
- gravity:
  - compression primordial photon gas
  - photon pressure resists

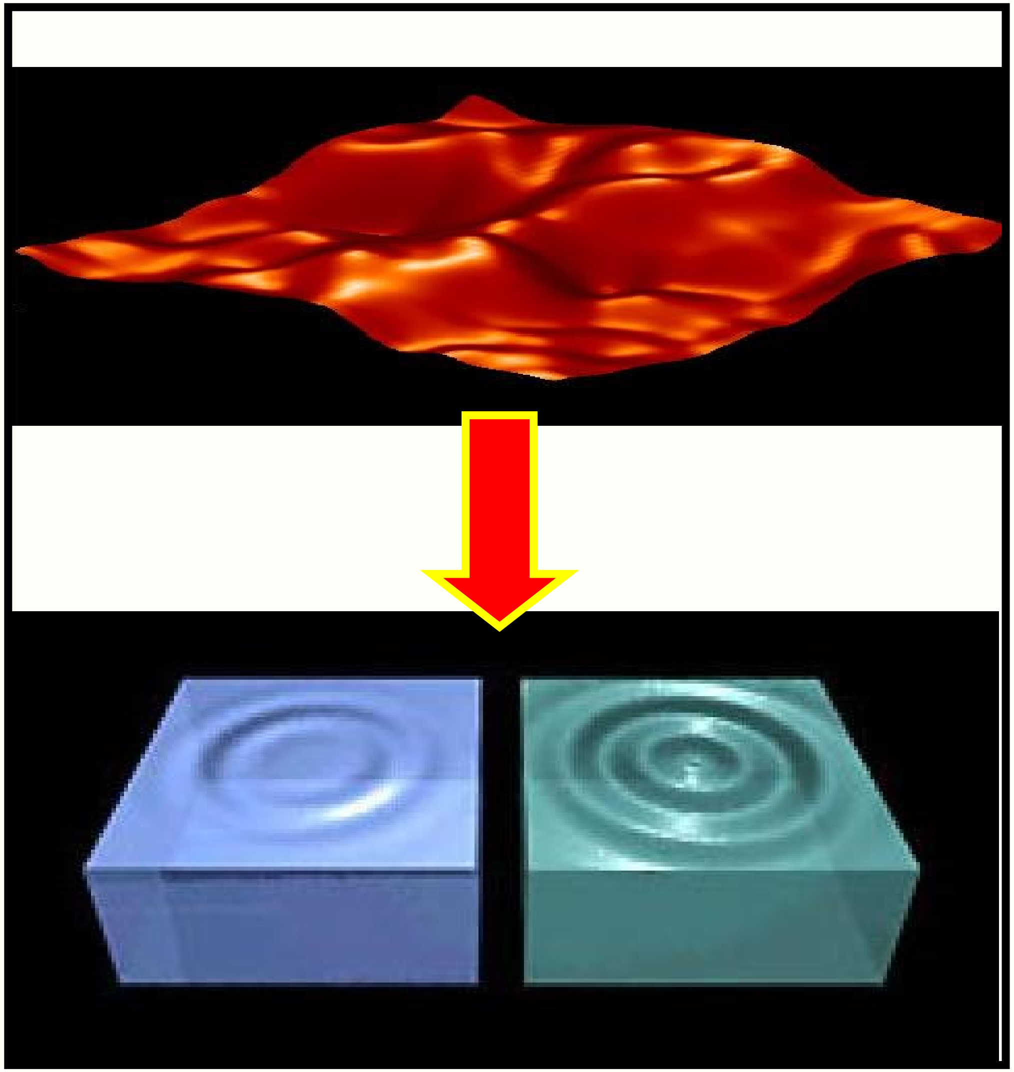
- compressions and rarefactions
  in photon gas: sound waves

- sound waves not heard, but seen:
  - compressions: (photon) T higher
  - rarefactions: lower

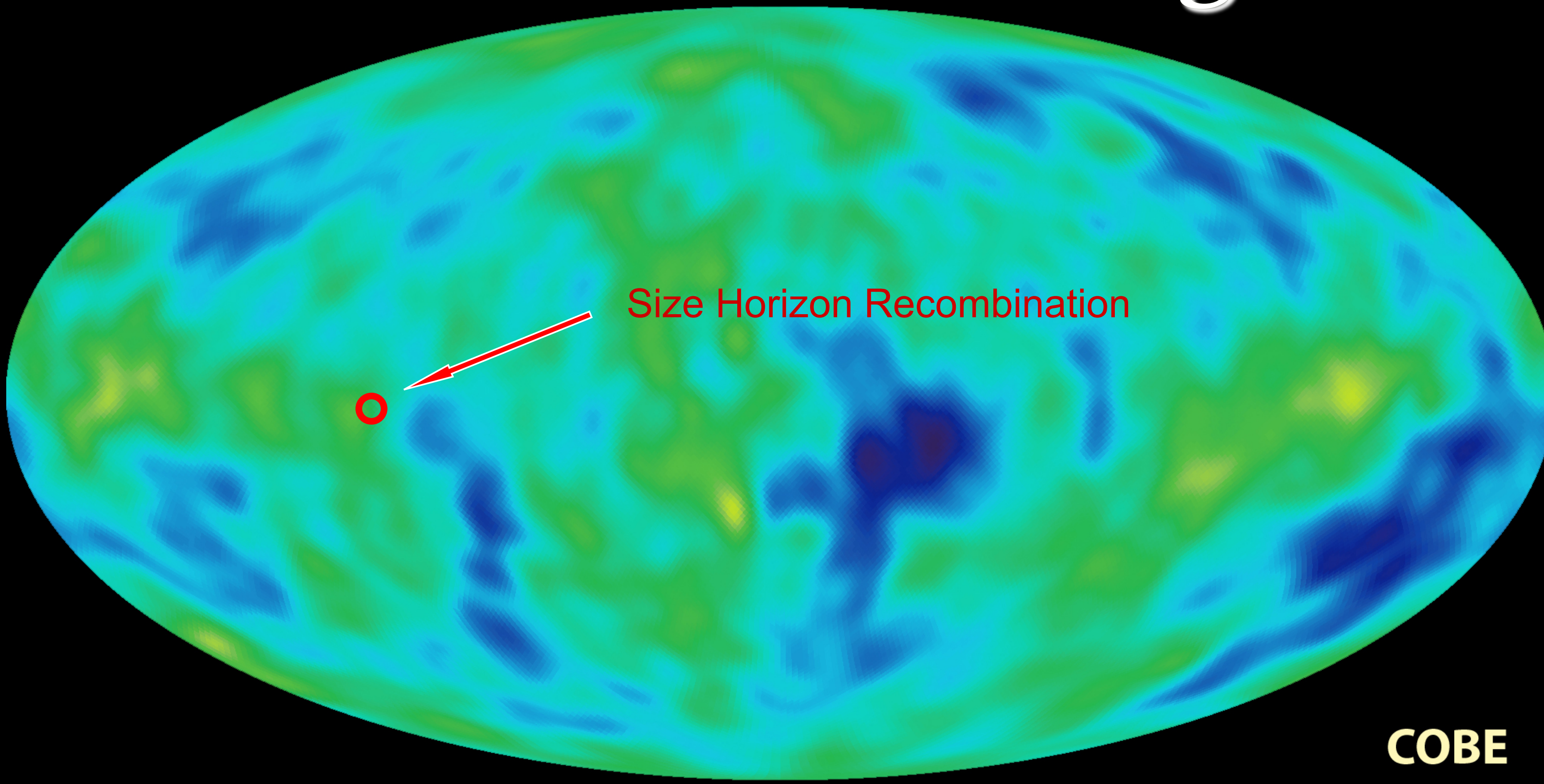
- fundamental mode sound spectrum
  - size of "instrument":
  - (sound) horizon size last scattering

- Observed, angular size: θ~1º
  - exact scale maximum compression, the
    "cosmic fundamental mode of music"

W. Hu



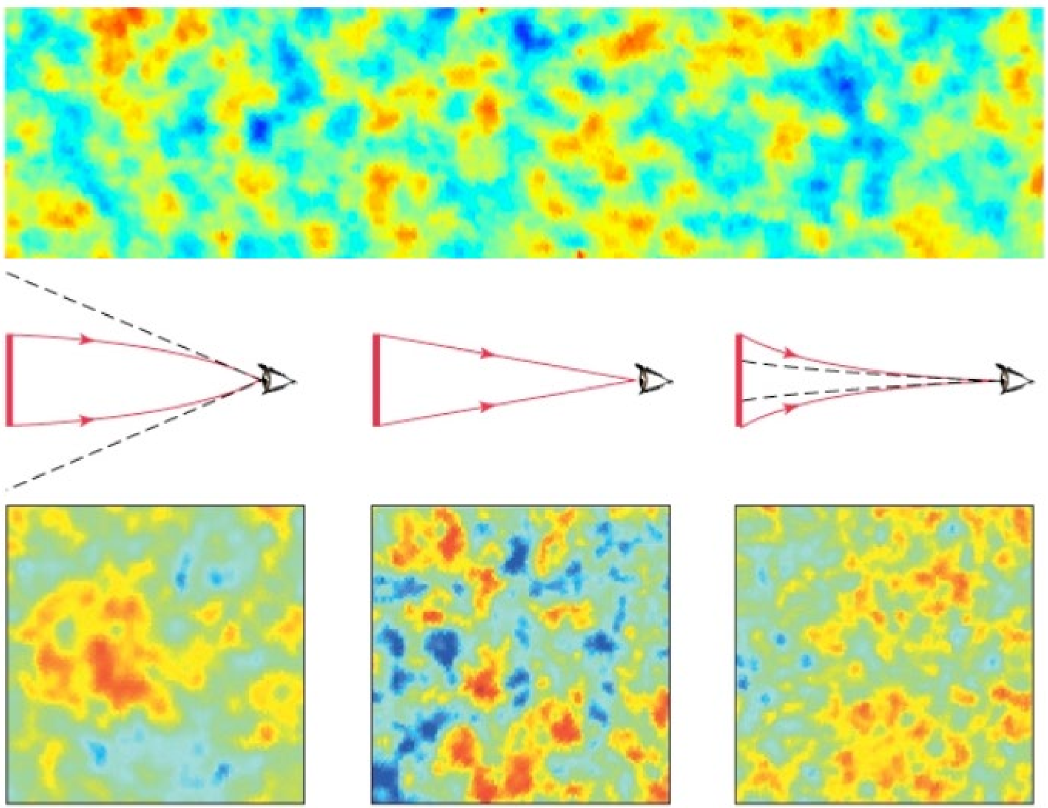
Cosmic Microwave Background



COBE measured fluctuations: $> 7^\circ$
Size Horizon at Recombination spans angle $\sim 1^\circ$

Flat universe from CMB

- **First peak: flat universe**



Closed:
hot spots
appear larger

Flat:
appear as big
as they are

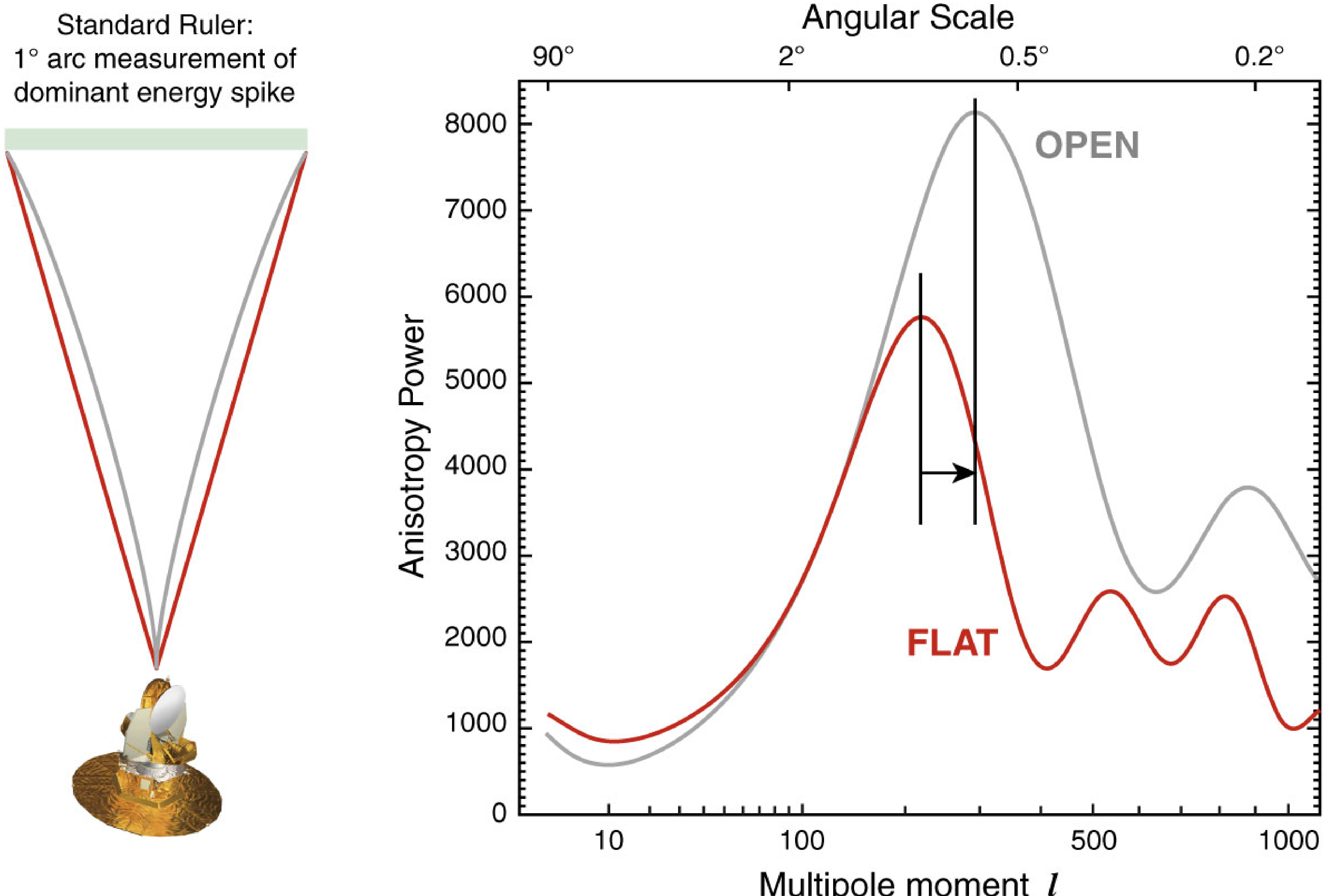
Open:
spots appear
smaller

We know the redshift and the time it took for the light to reach us:

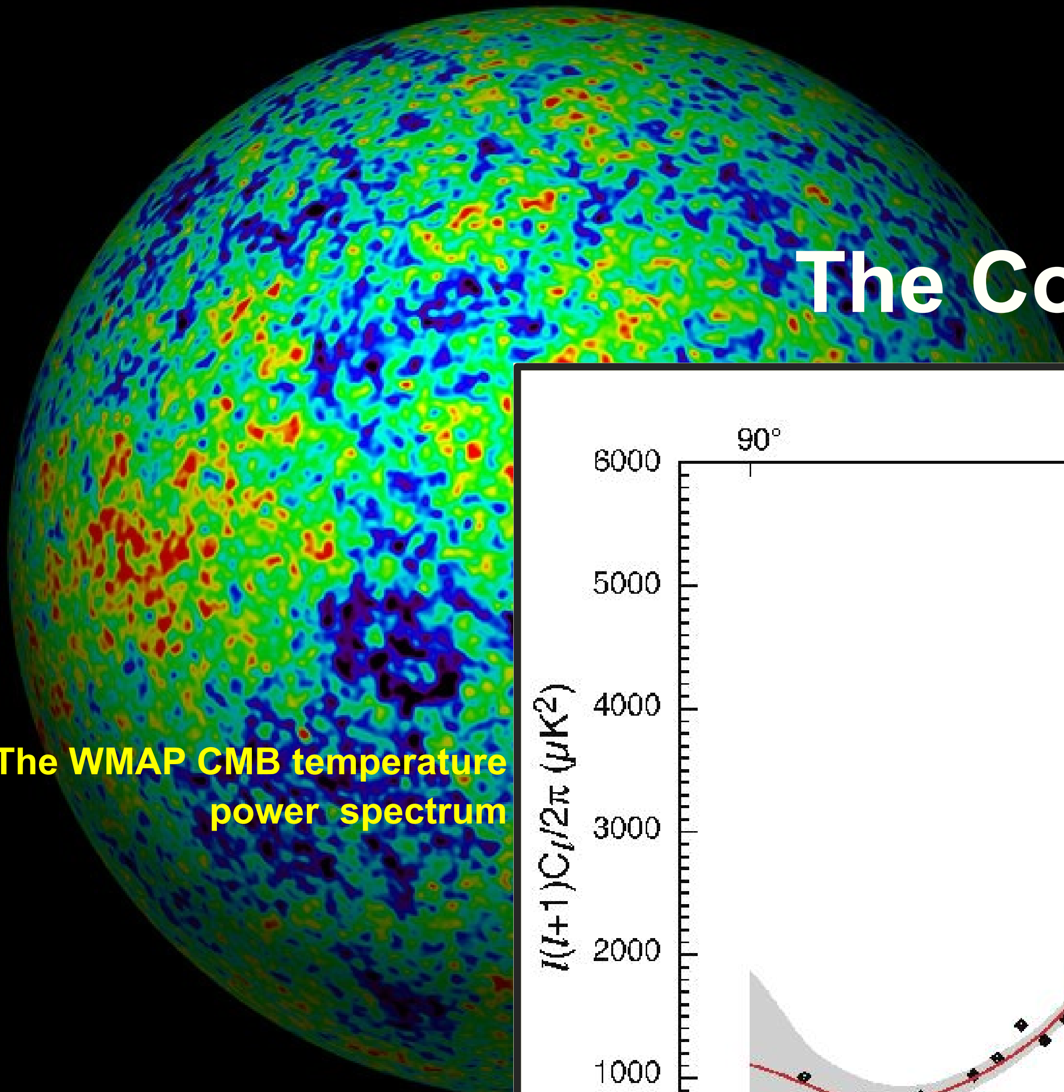
from this we know the
- length of the legs of the triangle
- the angle at which we are measuring the sound horizon.

$$v \approx \frac{c}{\sqrt{3}}$$

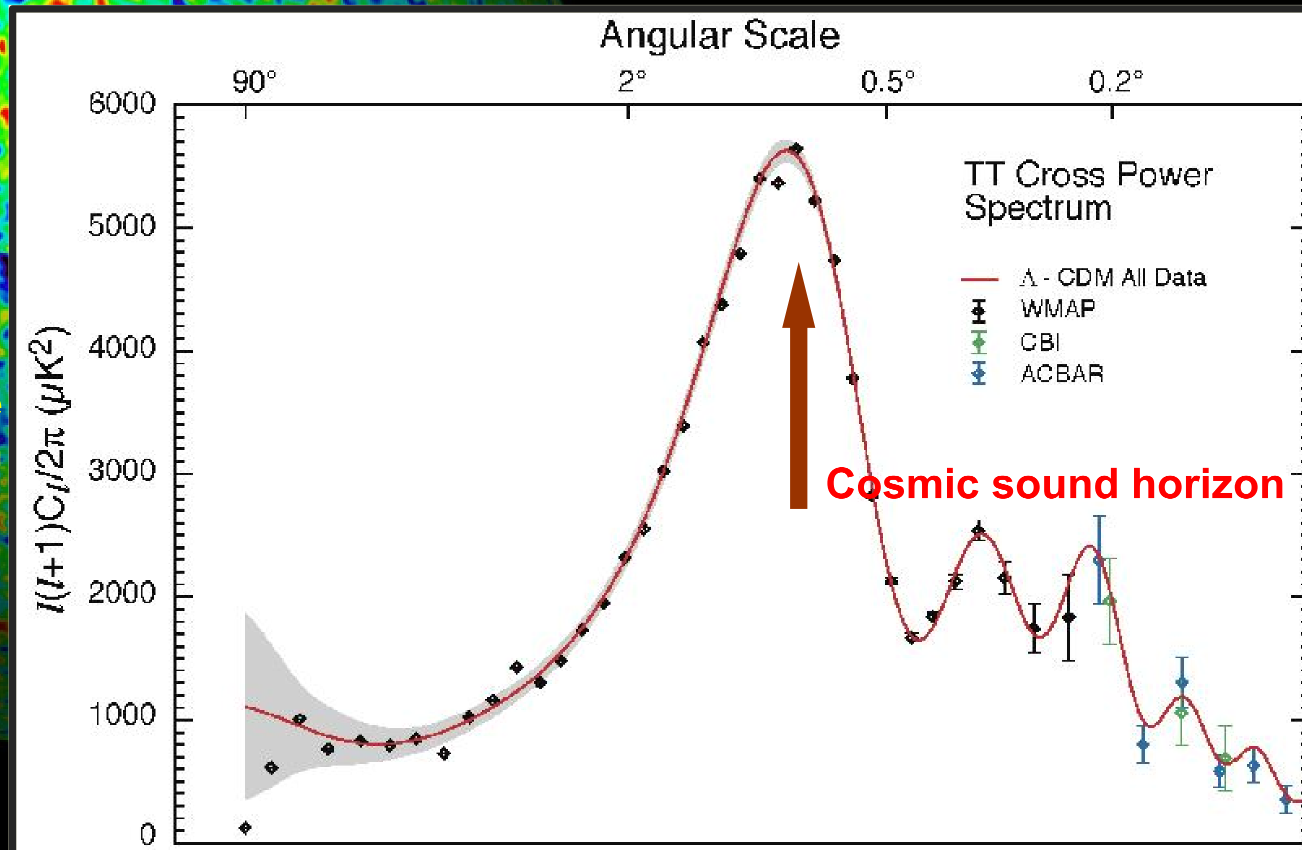
$$\ell \approx 200/\sqrt{1-\Omega_k}$$


Standard Ruler:
1° arc measurement of
dominant energy spike
Angular Scale
90°
2°
0.5°
0.2°
OPEN
FLAT
Anisotropy Power
8000
7000
6000
5000
4000
3000
2000
1000
0
10
100
500
1000
Multipole moment l

The Cosmic Tonal Ladder

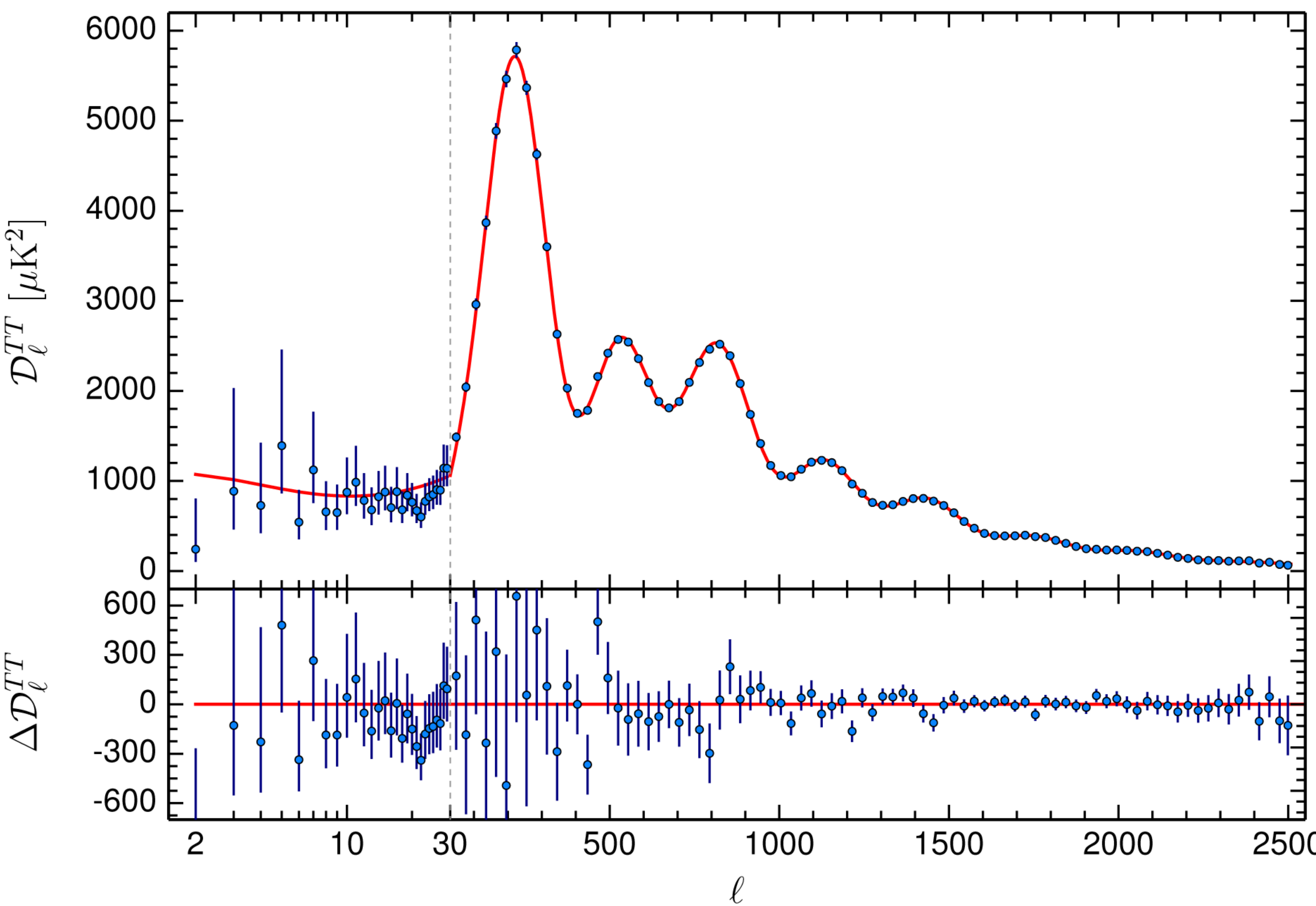


The WMAP CMB temperature power spectrum



The Cosmic Microwave Background Temperature Anisotropies: Universe is almost perfectly FLAT !!!!

CMB - Fluctuations



Cosmic Horizons

Cosmic Horizons

Fundamental Concept for our understanding of the physics of the Universe:

- Physical processes are limited to the region of space with which we are or have ever been in physical contact.

- What is the region of space with which we are in contact ?
  Region with whom we have been able to exchange photons
  (photons: fastest moving particles)

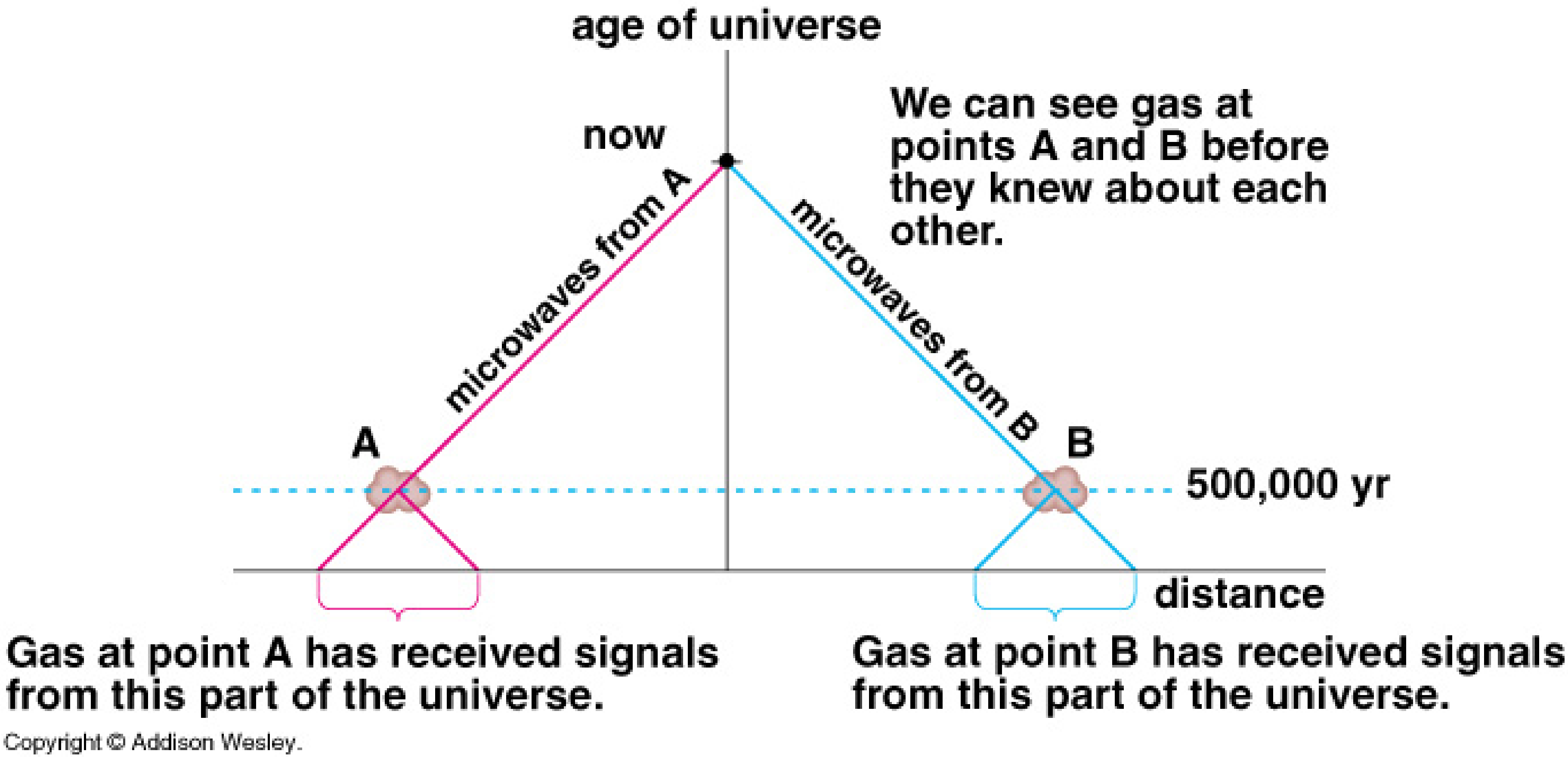
- From which distance have we received light.

- Complication: - light is moving in an expanding and curved space
  - fighting its way against an expanding background

- This is called the

Horizon of the Universe

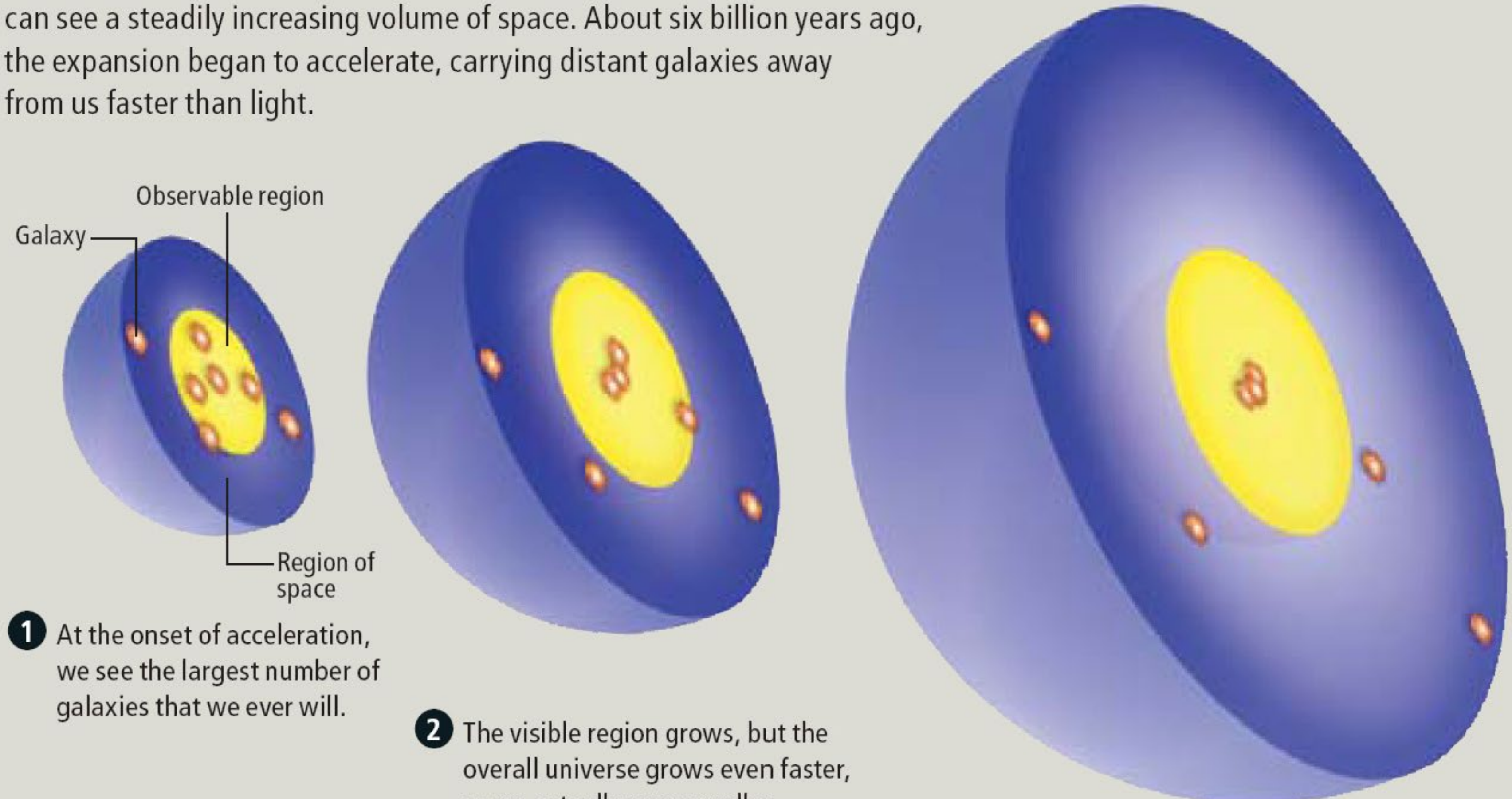
Cosmic Horizons



Horizon of the Universe:
distance that light travelled since the Big Bang

EXPANDING UNIVERSE, SHRINKING VIEW

The universe may be infinite, but consider what happens to the patch of space around us (*purple sphere*), of which we see only a part (*yellow inner sphere*). As space expands, galaxies (*orange spots*) spread out. As light has time to propagate, we observers on Earth (or our predecessors or descendants) can see a steadily increasing volume of space. About six billion years ago, the expansion began to accelerate, carrying distant galaxies away from us faster than light.



1 At the onset of acceleration, we see the largest number of galaxies that we ever will.

2 The visible region grows, but the overall universe grows even faster, so we actually see a smaller fraction of what is out there.

3 Distant galaxies (those not bound to us by gravity) move out of our range of view. Meanwhile, gravity pulls nearby galaxies together.

NOTE:
Because space is expanding uniformly, alien beings in other galaxies see this same pattern.

Cosmic Future

Cosmic Fate

100 Gigayears:
the end of Cosmology