



the Universe Mechanized:

the Antikythera Mechanism,

ancient Greek astronomical computer

**A tiny device pregnant with the world, a portable sky,
a compendium of the universe, a mirror of nature
which reflects the heavens.**

– Cassiodorus, 6th century AD

National Archaeological Museum, Athens



Fragment C, Fragment A, Fragment B

Mike Edmunds (astronomer & PI of AMRP):

“This device is just extraordinary, the only thing of its kind, The design is beautiful, the astronomy is exactly right ...

In terms of historical and scarcity value, I have to regard this mechanism as being more valuable than the Mona Lisa”

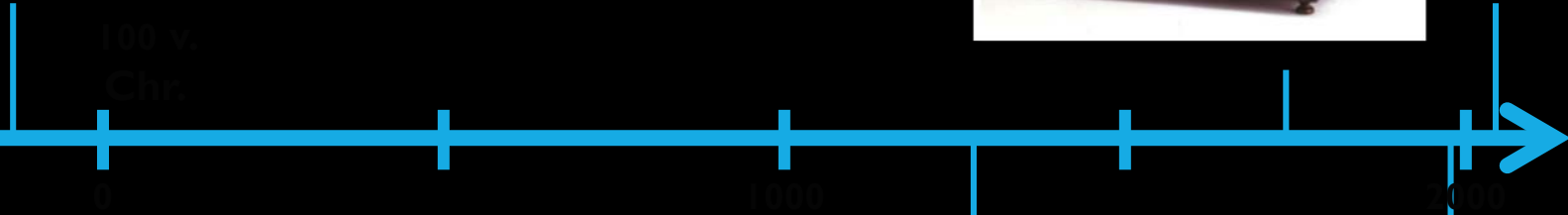


**Most sophisticated and intricate piece of technology
for over almost 1400 years !**

**Up to the appearance of mechanical astronomical
clocks towards the end of the 13th century, we do
not know anything as complex ...**



1670



1400 jaar!!




1300



- **Where did this technology come from ?**
who invented this ... ?
- **Testimony and Manifestation of Hellenistic Scientific Revolution ?**
- **Innovation:**
Why did this not propagate into economic and social applications?
- **What happened with this knowledge ?**
 - Disappeared ? Implications for our idea of progress & advance
 - or, is there a direct link over the many centuries to our clocks ... ?

a Hellenistic Scientific Revolution ?

- Known was that Greeks pondered deeply on the workings and laws of nature ... Greek natural philosophy
- Known was that the Hellenistic Greeks managed to combine sophisticated geometric models of the heavens with observational data (mostly Babylonian), into a genuine theory of nature.
- However,
we never imagined they would be able to translate this model into a sophisticated mechanical device,
translating mathematical theory into a mechanical representation of reality.
- What does this imply ?
 - testing theories by computation ? This is true science in modern sense !
 - how did it affect their view of the world ? Mechanical Worldview ?

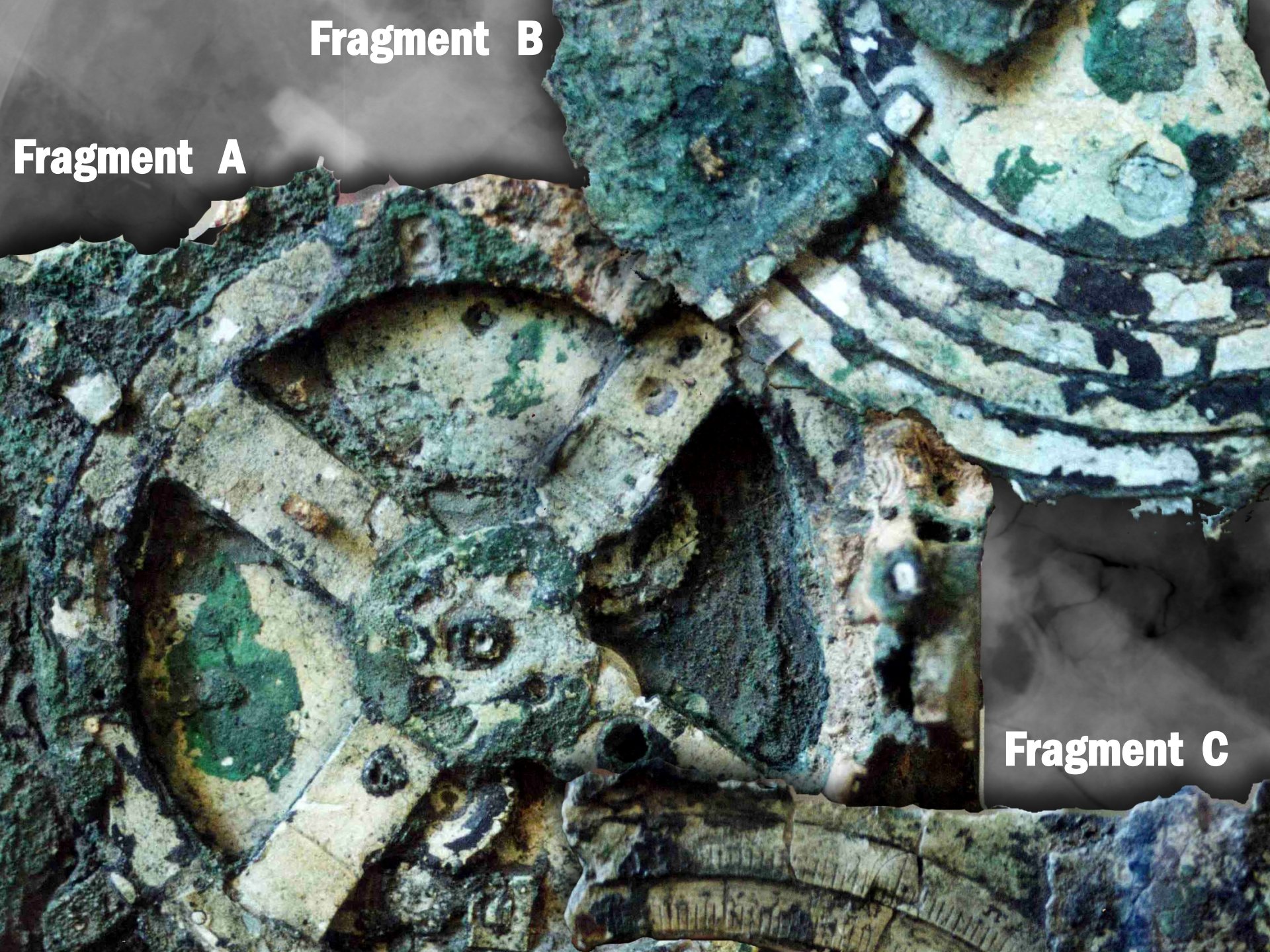
The background is a complex, multi-colored abstract pattern. It features a mix of bright colors including cyan, magenta, yellow, and dark blue, set against a black base. The colors are arranged in irregular, organic shapes that resemble marbled paper or a microscopic view of a mineral. A thin white border frames the entire image. In the center, the text "The Fragments" is written in a white, serif font with a slight drop shadow.

The Fragments

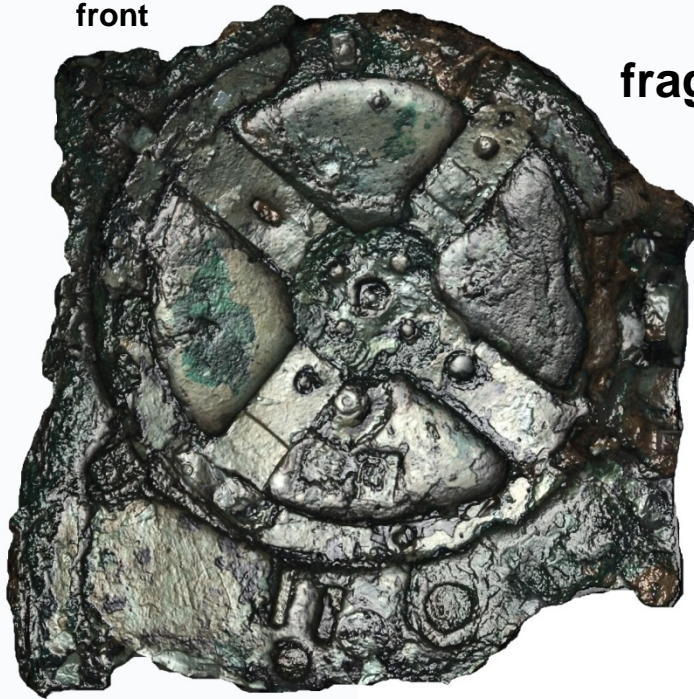
Fragment B

Fragment A

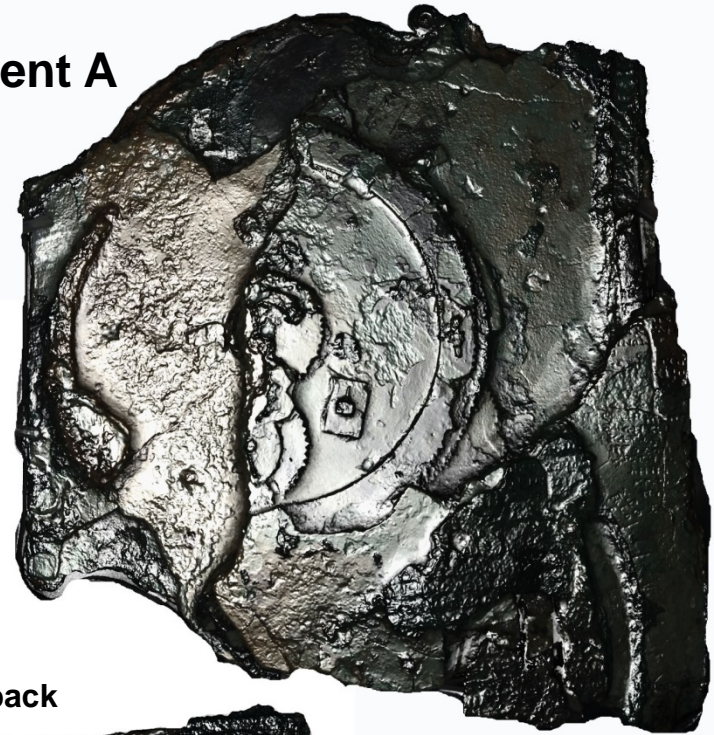
Fragment C



front



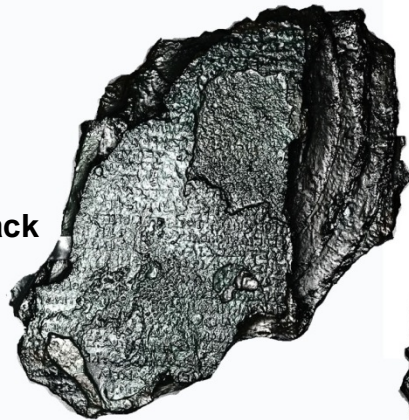
fragment A



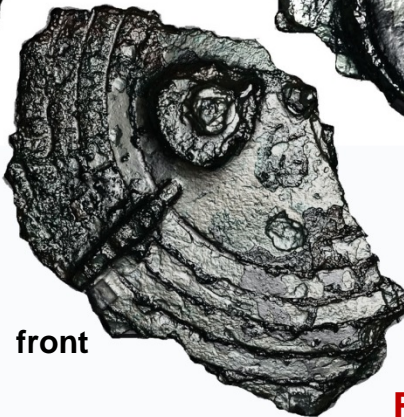
back

back

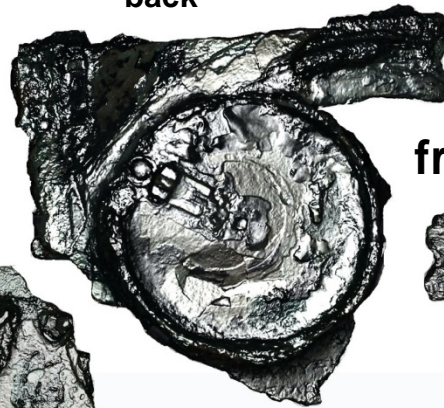
back



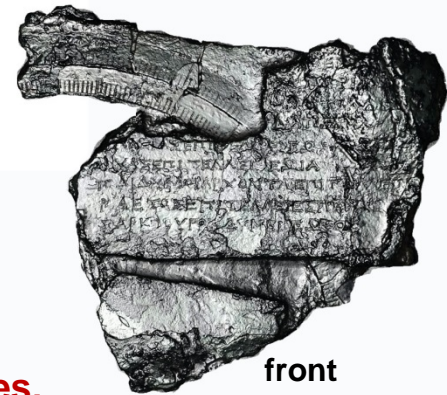
fragment B



front

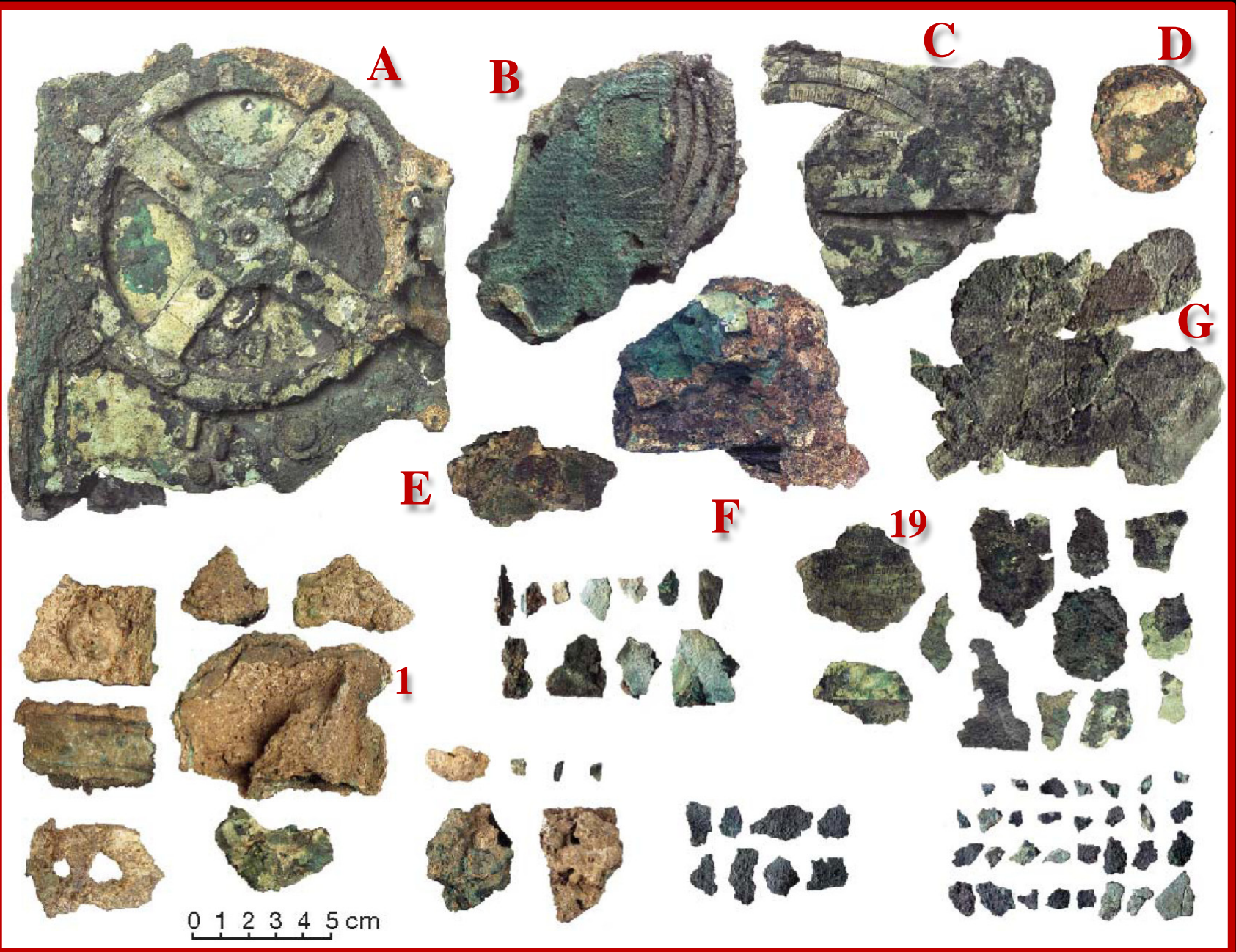


fragment C



front

PTM images,
Freeth & Jones 2012



**Most sophisticated technological artefact of antiquity,
more complex than ANY device for at least Millennium afterwards !!!!**

☒ **Bronze Mechanism**

☒ 82 fragments identified (major fragments A,B,C,D,E,F,G sure)

☒ Contains at least 32 gear wheels (30 identified !!!!)

☒ Sophisticated internal gearhouse (central processing unit)

☒ Pin-Slot mechanism for lunar epicyclic orbit (Hipparcos) !!!!

☒ Calculating Panhellenic Games (incl. Olympic games)

☒ Originally housed in wooden-framed case

☒ Size: 315x190x100 mm (laptop size)

☒ Front and Back doors

☒ Astronomical Inscriptions covering much of the exterior

☒ Probably Hand-driven



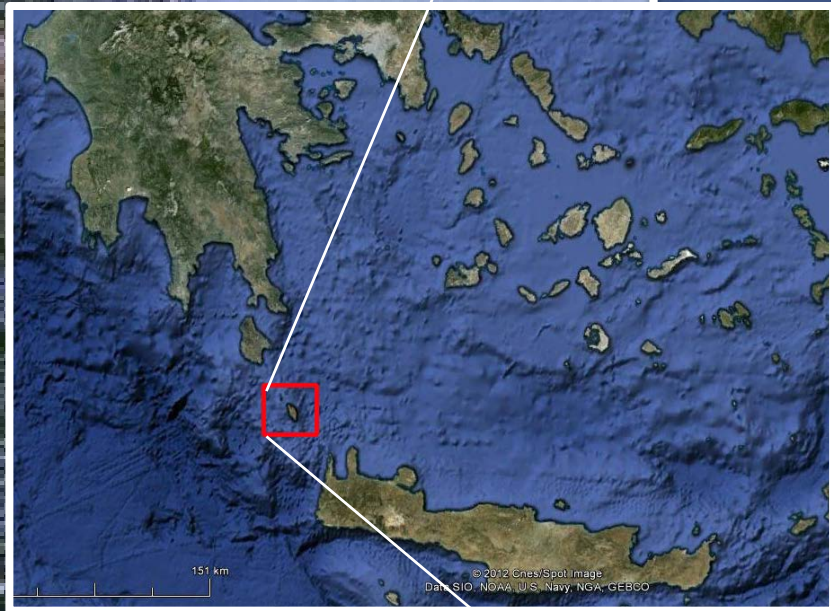
image courtesy: Tony Freeth/Images First Ltd.



Antikythera

&

the ship's discovery



October 1900:

☐ **Group (sponge) divers,**

lead by **Elias Stadiatos**

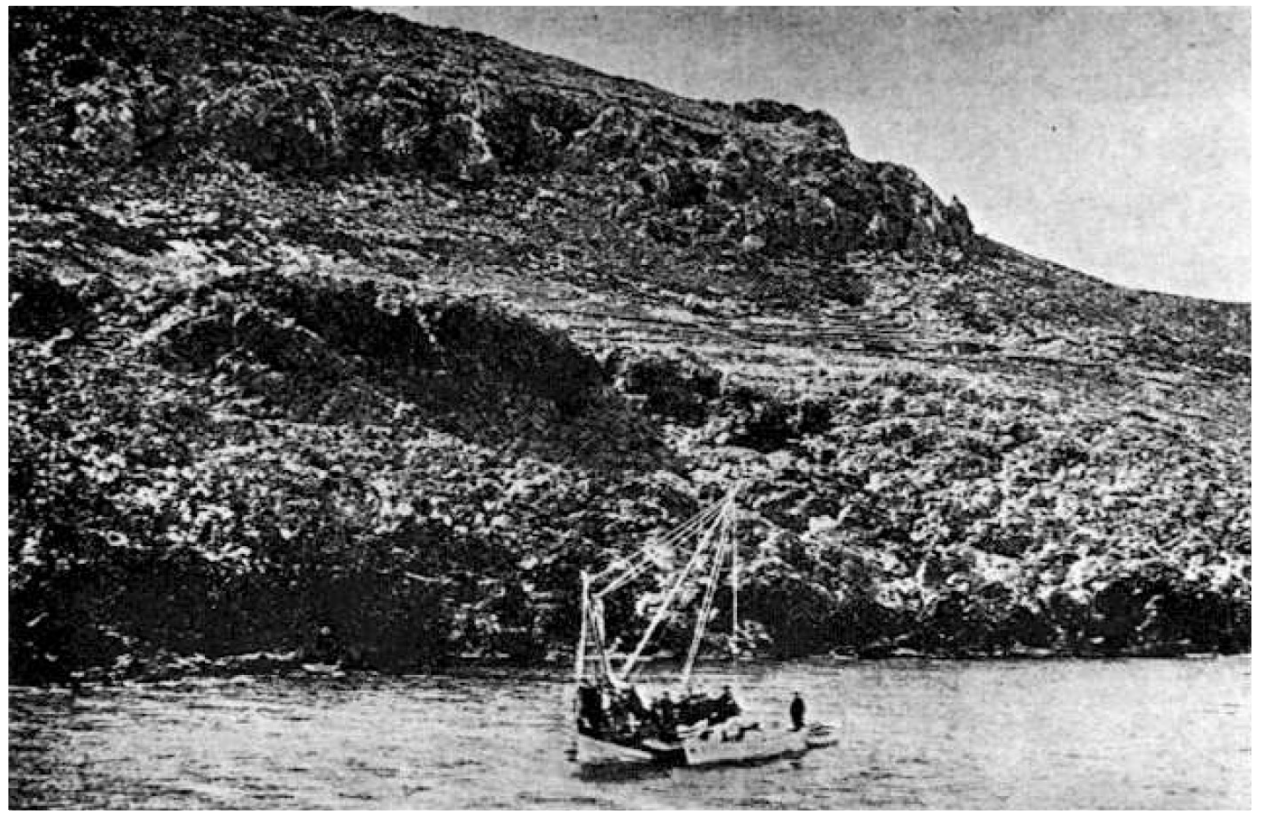
☐ **Shipwreck 43m deep :**

50 m long, 30 tons

15-25 m off Cape Glyphadia

Until 1902:

☐ **Salvaging numerous artefacts ...**



- 1) **Ephebe of Antikythera ~ 340 BC**
- 2) **Hercules, marble bull, bronze lyre**
- 3) **Philosopher Antikythera**
- 4) **Golden jewellery, utensils, statues**
- 5) **May 17, 1902, Valerios Stais:**
Antikythera Mechanism

Note: - In those days no scuba diving:

- cold water, currents, $p > 5$ atm.

**- 9 min dive, 4 min descent+ascent,
5 min bottom time**

- 10 divers,

1 diver died, 2 permanently disabled

2012-2015: Brendan Foley



image: Alexandros Sotiriou

A painting of a wooden sailing ship with large yellow sails on a stormy sea. The ship is dark brown and appears to be struggling against the waves. The sky is dark and cloudy, and the sea is turbulent with white foam. The overall mood is dramatic and somber.

the Antikythera shipwreck

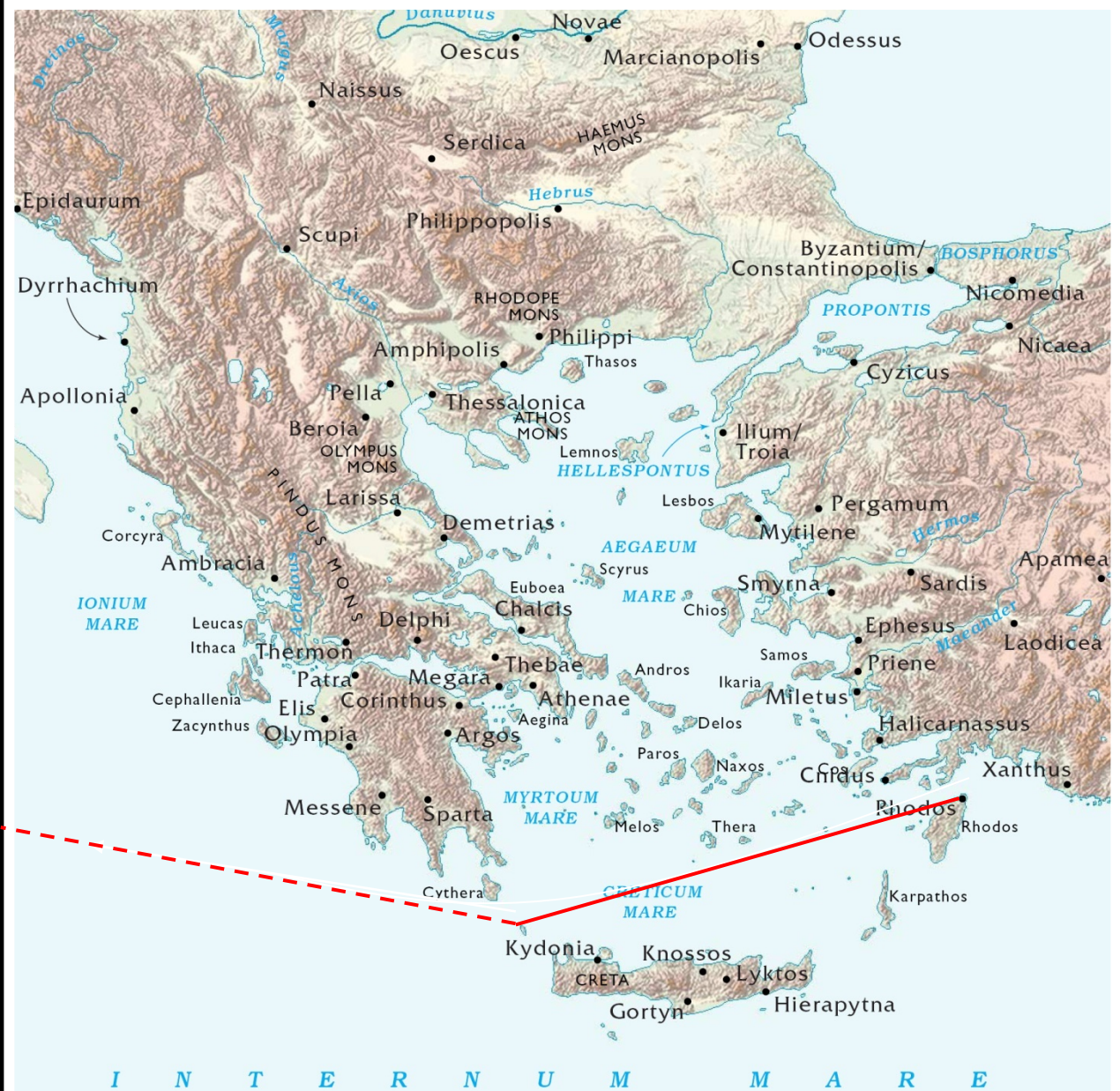
Roman Shipwreck

Dating Ship nontrivial:

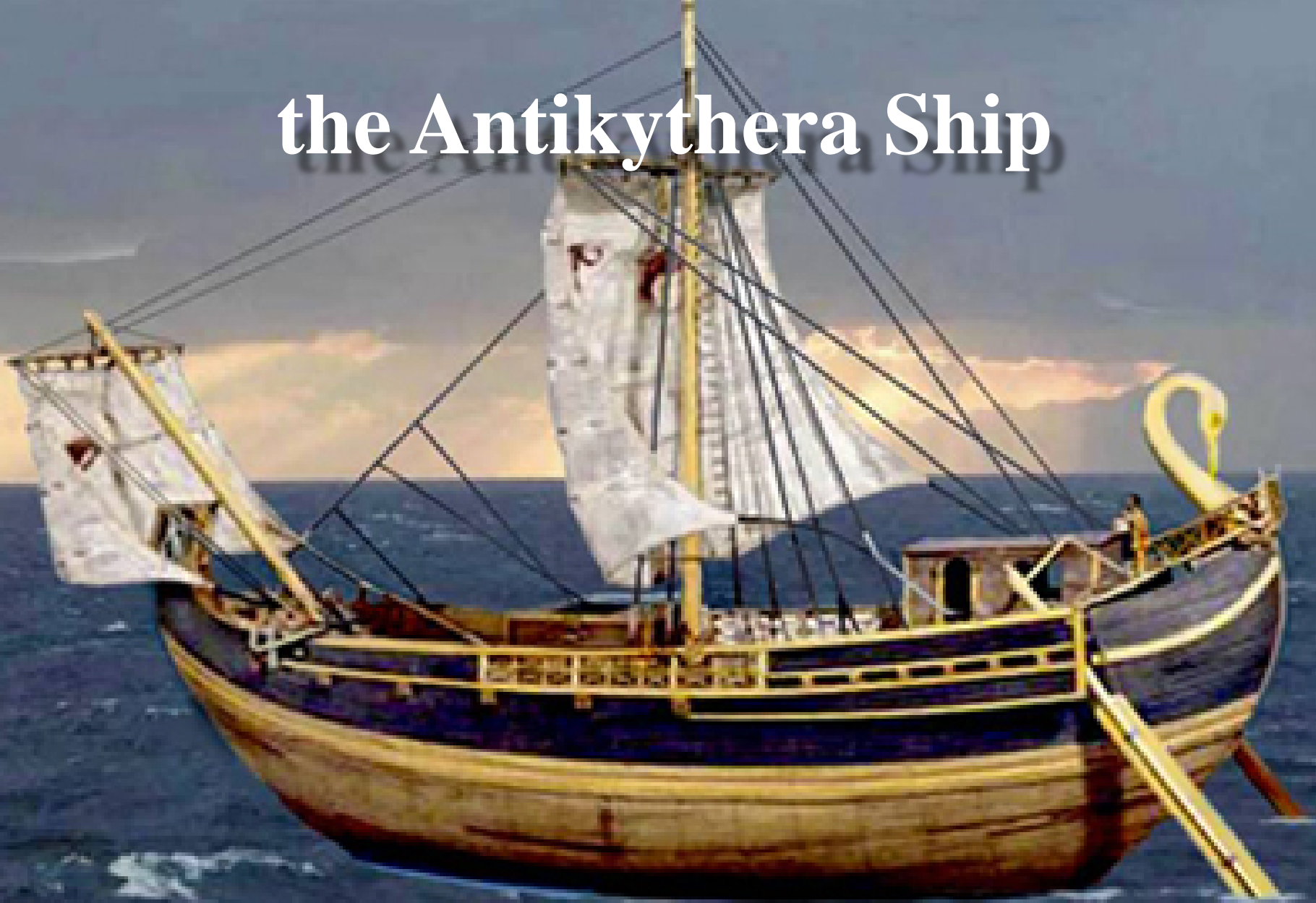
- bronze statues: 4th c. BC
- marble statues: 1st c. BC
copies earlier originals
- coins Pergamom ~ 60-80 BC
- carbon dating utensils: 65 BC
- ship of elm, wood often used by Roman for ships

Speculation:

- Loot by Sulla from Athens (86 BC), 1st Mithridatic War
- Destined for Rome (loved marvels Greek culture...)
- Lucian mentions loss one Sulla ship near Antikythera !



the Antikythera Ship



Roman cargo ship (Olkas, ~ 50 x 15m)

The Antikythera Shipwreck. The Ship, the Treasures, the Mechanism

National Archaeological Museum, Athens

A hull plank, bronze spikes and fragments of metal sheathing

1. Τμήμα πλάκας από το εξωτερικό κύβητος του κελύφους
220 π.Χ. (64)
Εργαστήριο Αρχαιολογίας ΜΕ 2011/11

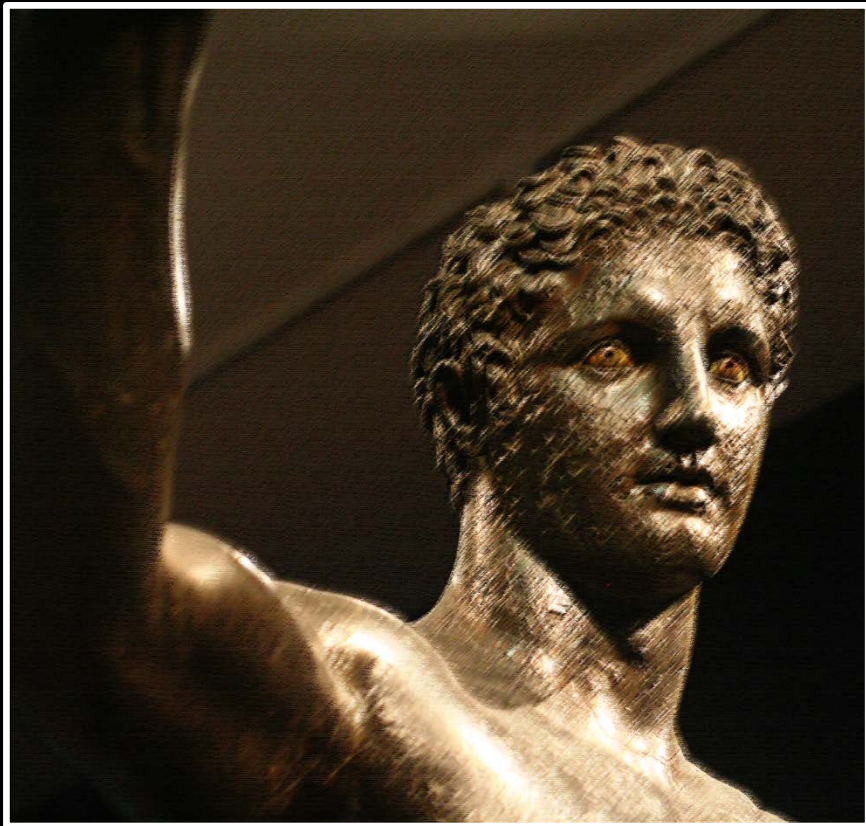
2. Τμήματα πολυβραχίονα φιδίλου
Αποτελούνται από εννέα βραχίονες, του κίτους
του κελύφους για το στερέωσή του κίτους από τον τοίχο γραφεί.

3. Τάβλα κίτου
Τμήματα από την περιοχή των σπυριών,
σε τμήματα, χωρίς ή με σπυρίδια.



the Marbles

Ephebe of Antikythera



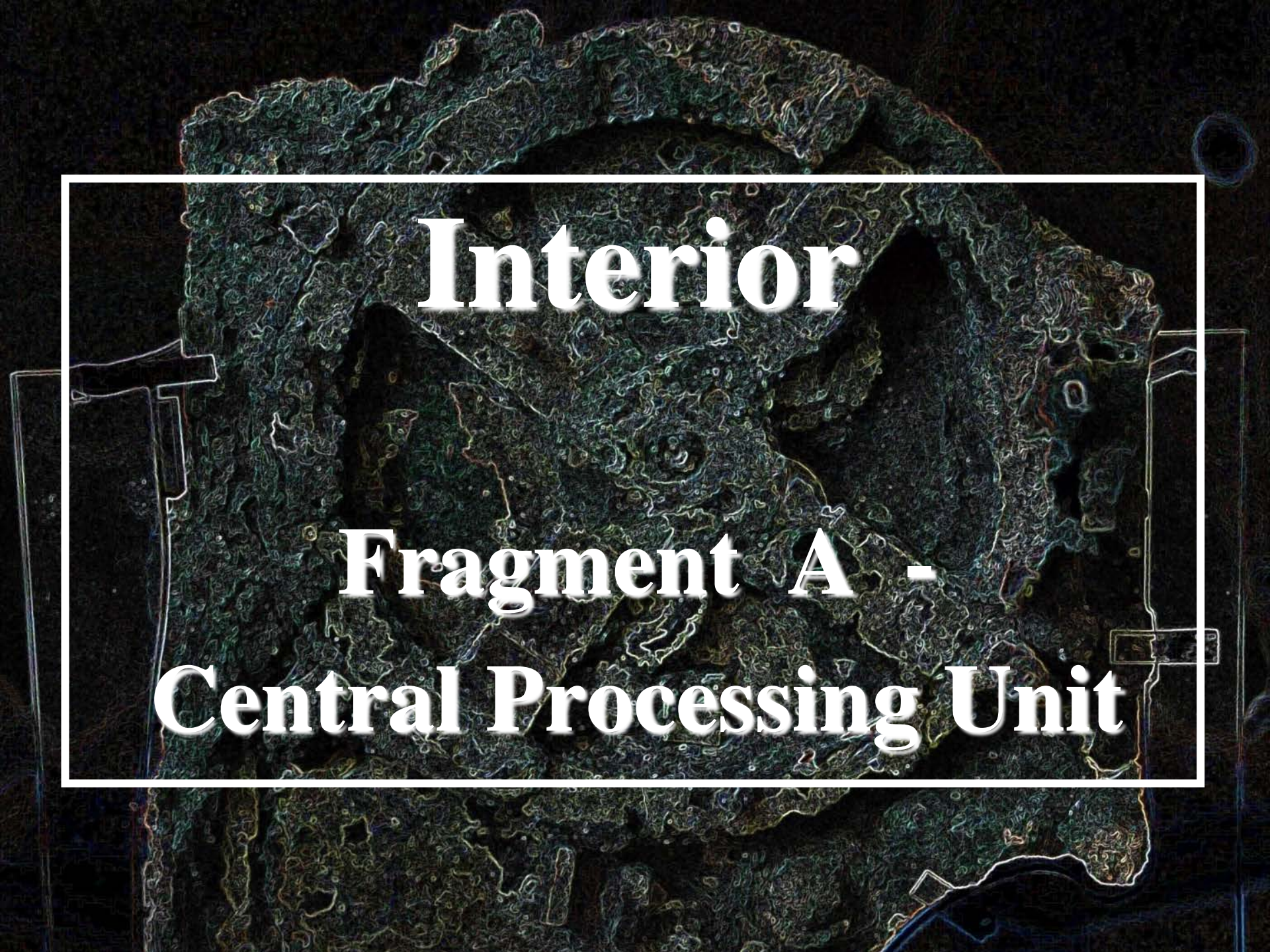
Coins & Jewelry



Pergamon. Silver cistophoric tetradrachme 85-76 BC.

precious and intricate golden jewelry





**Interior
Fragment A -
Central Processing Unit**

**Central
Mechanism**

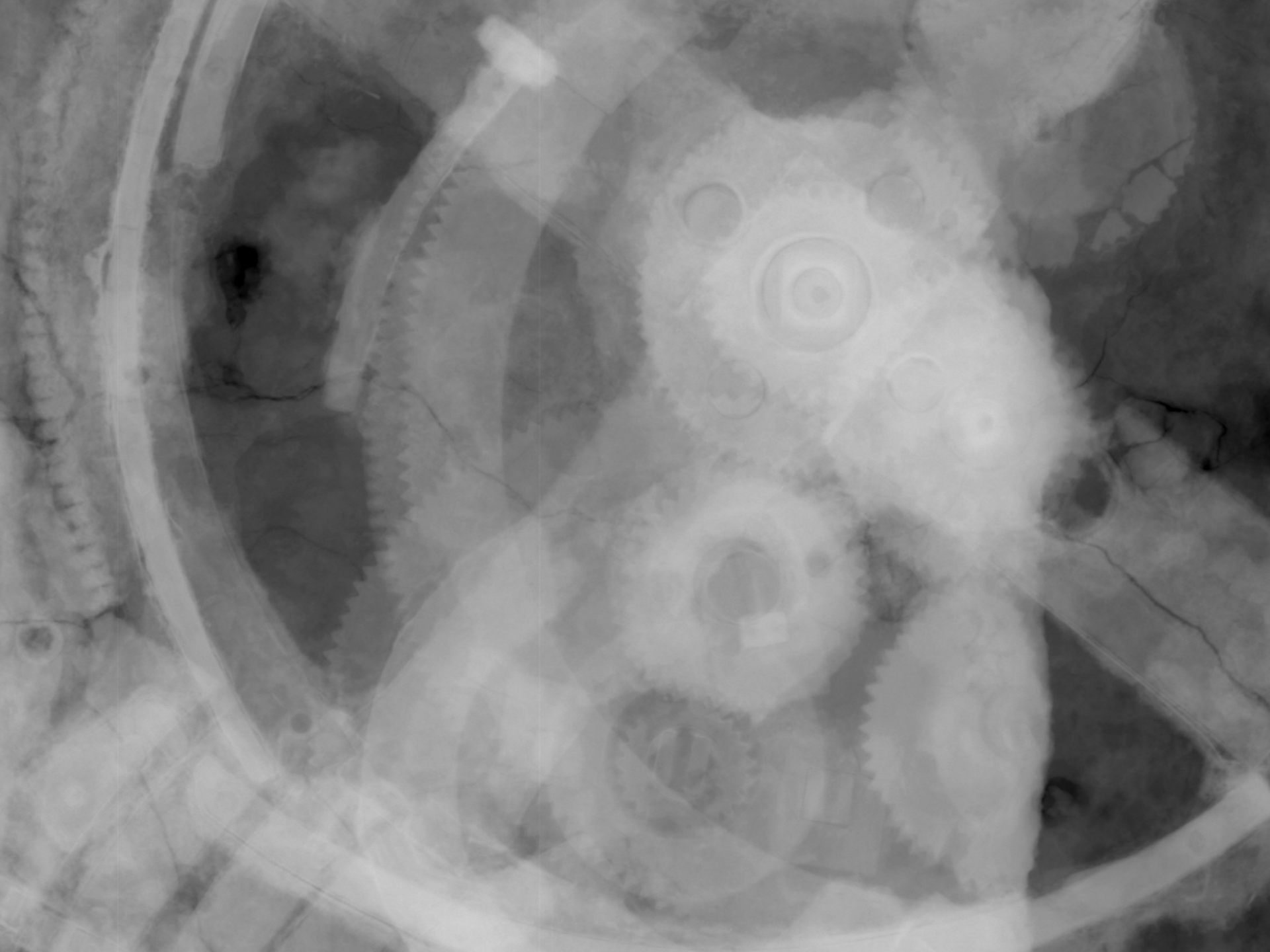


**Fragment A:
27 gearwheels !**

Interior

**AMRP
X-Tek X-ray
Tomography**

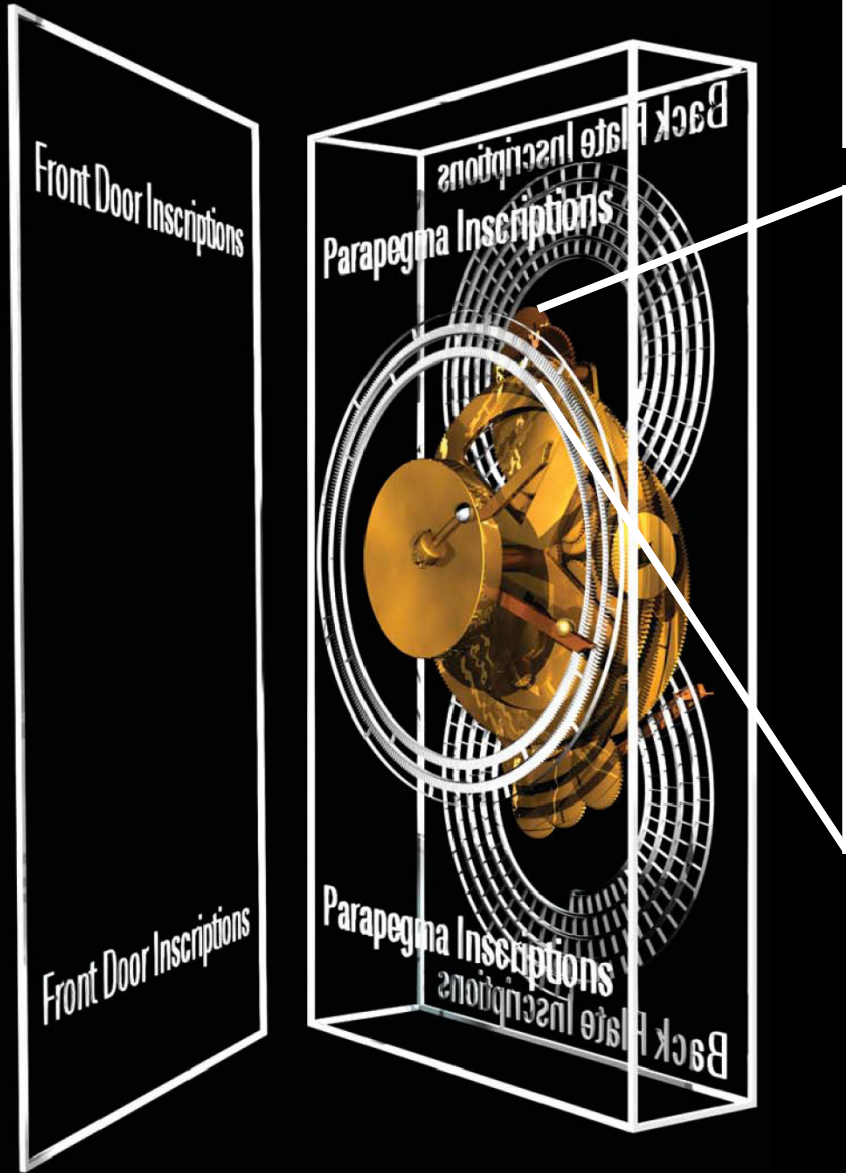






Fragment C:

Front Dial



Fragment C



Front Dial (part)

Front Dial

☐ Front dial, inner scale

position Sun and Moon in zodiac

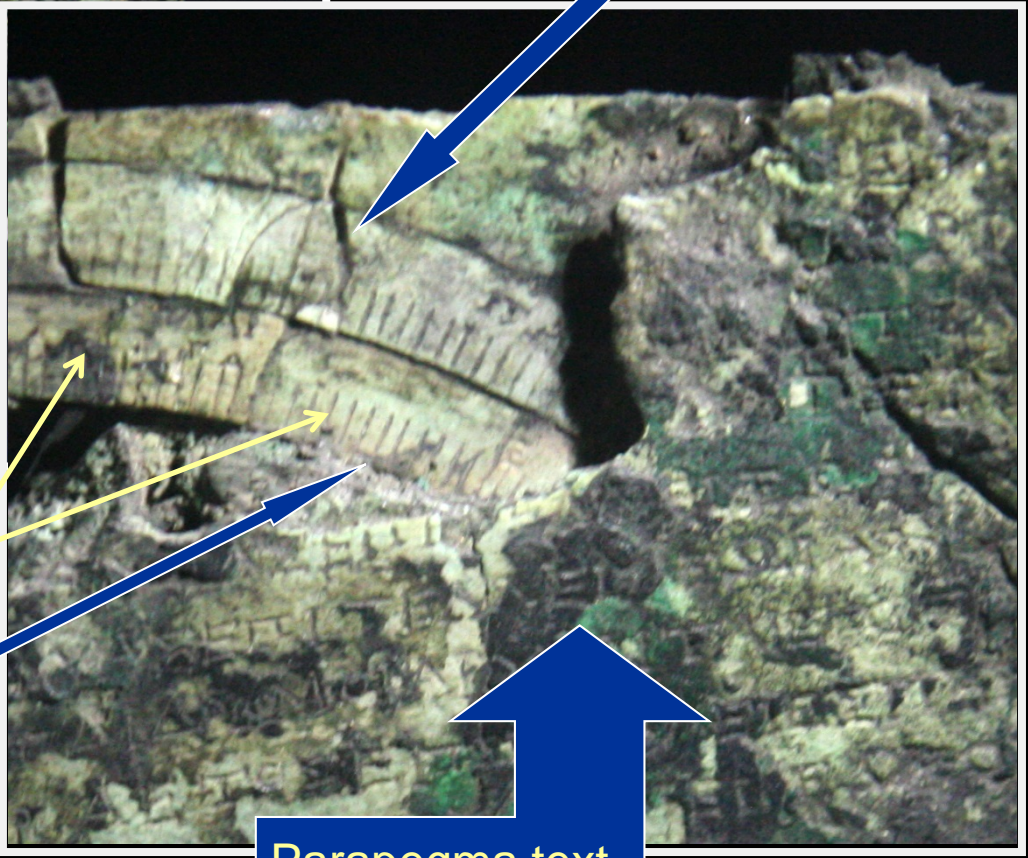
☐ Front dial, outer scale:

calendar Egyptian names months in Greek letters
(Egyptian calendar standard use Greek astronomy),
corresponding calendar of 365 days
adjustable for leap year



Front Dial (segment)

Egyptian Month
scale

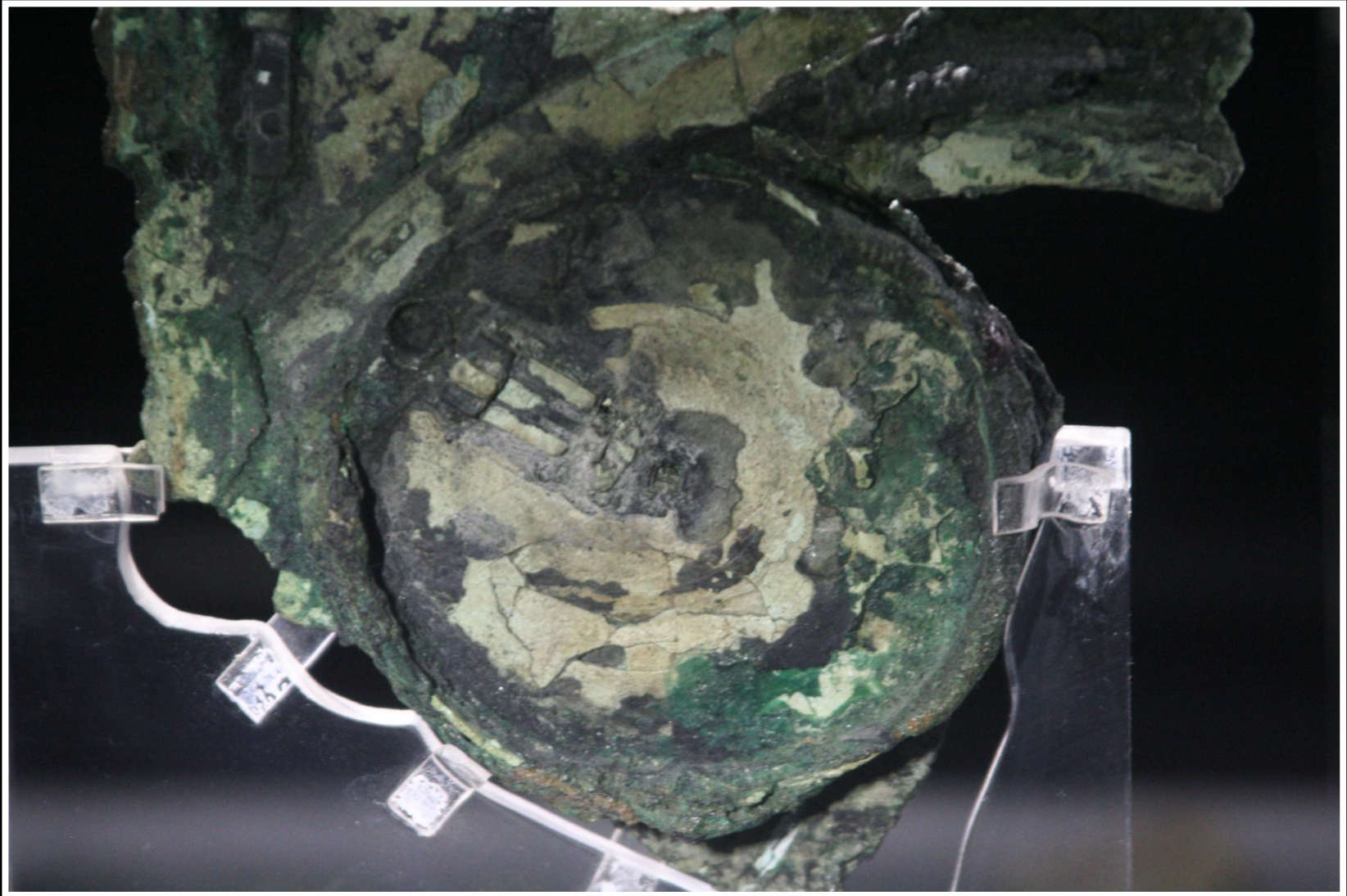


Parthenos (Virgo)
Xilai (Libra)

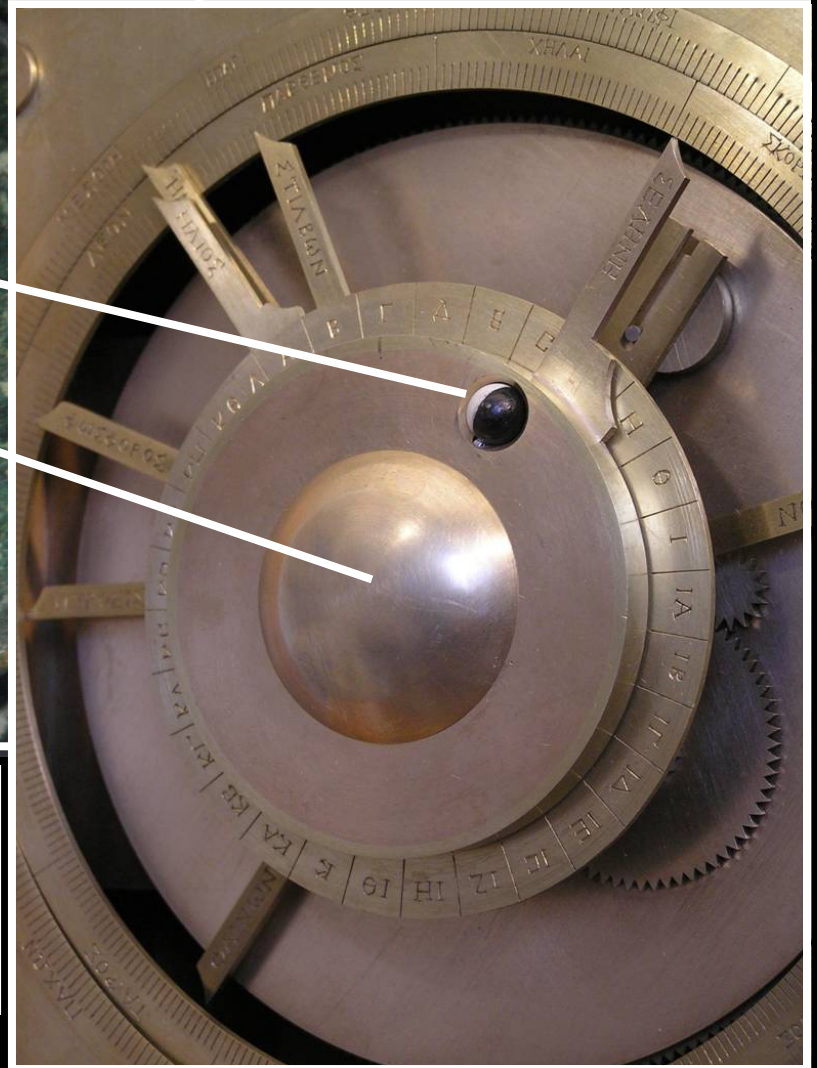
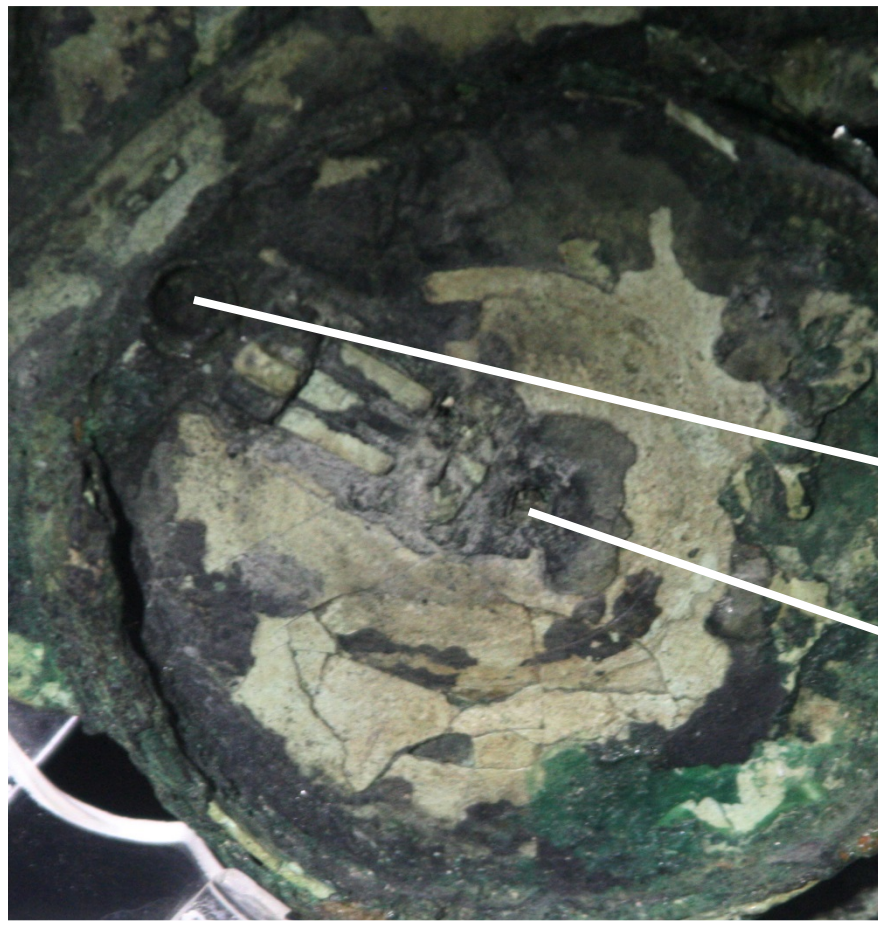
Zodiac scale

Parapegma text

Fragment C, back



Silver Moon Sphere



discovered by
M. Wright

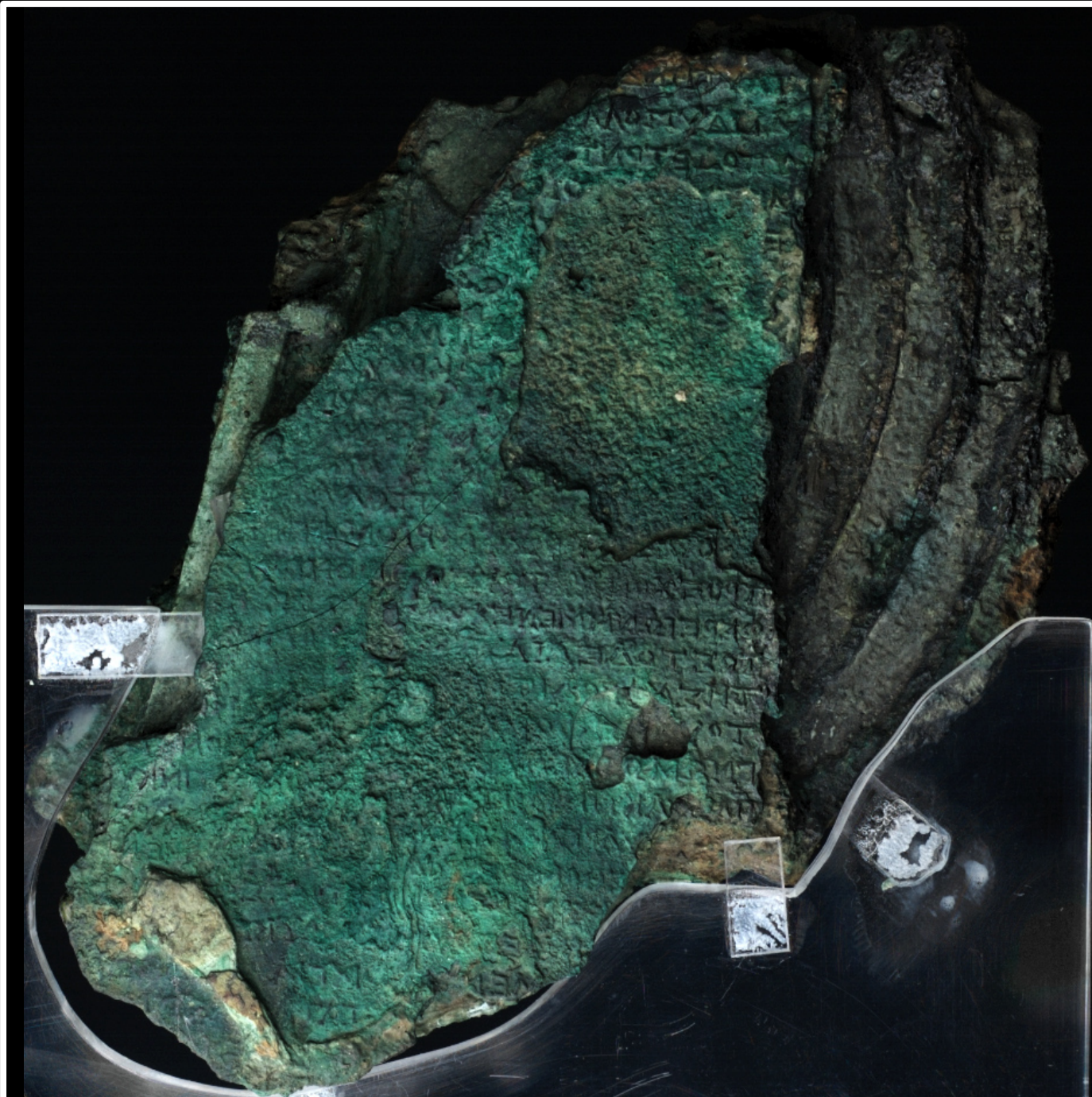
The background is a dark, almost black, surface with a complex, organic texture. It appears to be a close-up of a material like stone or metal, possibly showing signs of weathering or erosion. The texture is composed of many small, irregular shapes and lines, creating a sense of depth and complexity. A white rectangular box is superimposed over the center of the image, containing the word "Inscriptions" in a bold, white, serif font. The box has a thin white border and is centered both horizontally and vertically. The overall composition is simple and focused on the central text.

Inscriptions

Fragment B:

Backdoor

Inscriptions



Instrument covered with inscriptions:

- ☐ Doors
- ☐ Front- and Backside of instrument (outside dials)
- ☐ Dials
- ☐ Internally, on gear wheels

Inscriptions concern 3 different aspects:

- ☐ Technical, manual for the use of the instrument
- ☐ Astronomical
- ☐ Parapegma: “almanac”,
relating earthly matters (weather, harvest) to events on sky

- ☐ Inscriptions as dating tool, ie. they identify the time of manufacture:
 - type lettering: 2nd half second century BC
 - used old name planet Venus: changed around 100 BC

Line Number

15

ΤΟΣ ΤΩΔΕ ΔΙΔΥΜΟΝ
ΤΗΣ ΔΕ ΤΩΔΙΤΗΪ ΩΣ ΦΟΡΟΥ
ΦΟΡΟΥ

← ... Phosphorou ...
Venus !

20

ΥΑΡΕΛΣΙ ΥΡΣΕΝΤΟΣ
ΕΘΟΡ ΟΣ ΤΩΔΕ ΔΙΔΥΜΟΝ
ΙΝΟΣ ΟΣ ΚΥΚΛΑΙΕΤΑΣ

25

ΚΟΣΜΟΥ

← ... of the Cosmos ...

Technical Inscriptions:

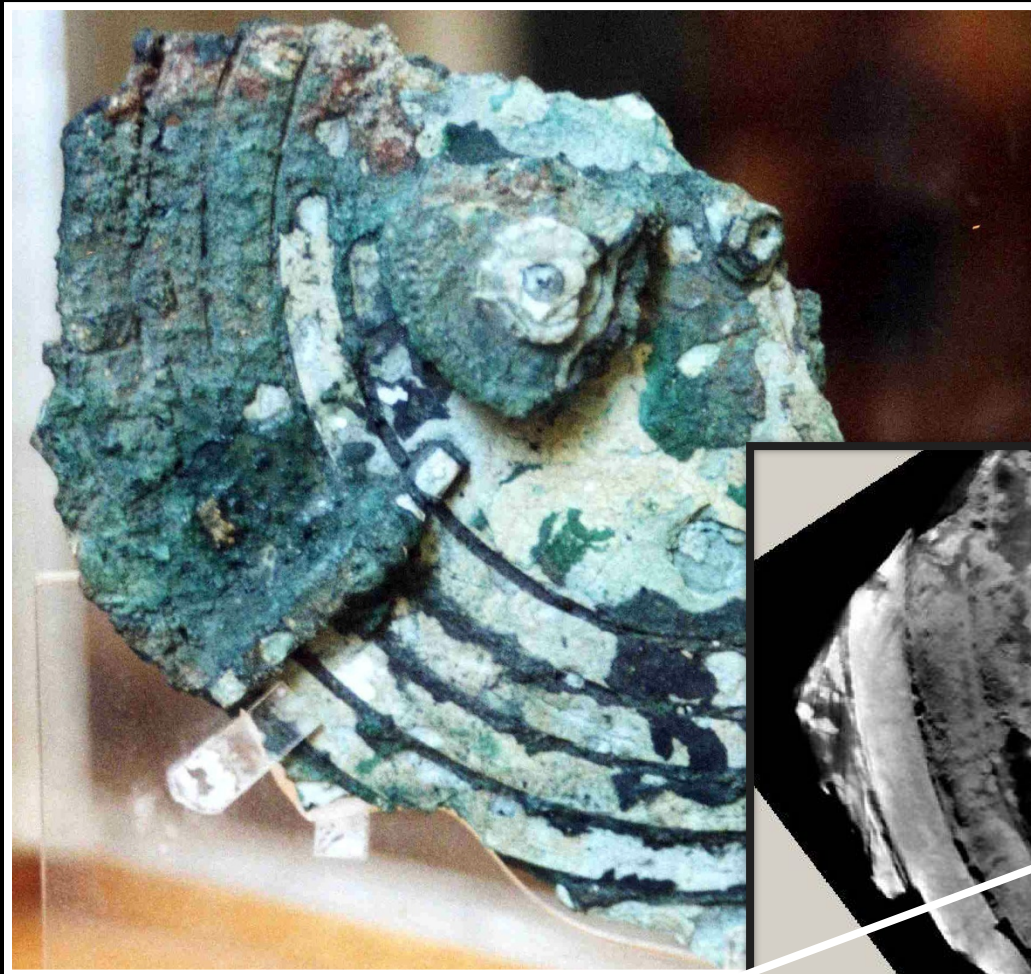
- ☐ “Tap”; “Gnomon”; “Perforations”; “Pointers”; “Gears”;
- ☐ “Spiral divided in 235 sections ...”
- ☐ “small golden ball”
- ☐ “small ball”

Astronomical Inscriptions:

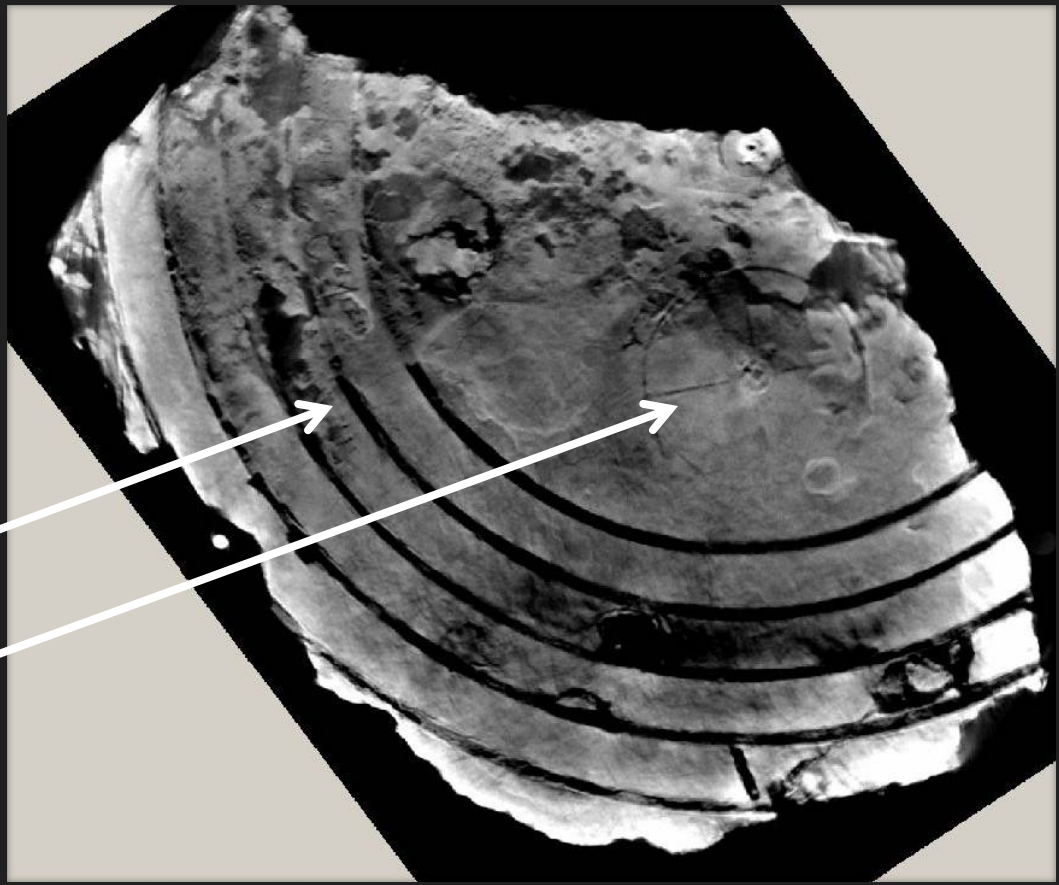
- ☐ “☐☐☐☐☐☐☐☐☐☐”: stationary point planets’ retrograde motion
- ☐ “Venus approaches the Sun”
- ☐ “The Hyades set in the evening”
- ☐ “Gemini begins to rise”, ...
- ☐ “ the 76 years, 19 years of the ...”

The image features a dark, almost black background with a complex, organic texture. This texture is composed of numerous fine, interconnected lines and shapes in shades of green, blue, and yellow, creating a dense, intricate pattern. A prominent white rectangular box is centered horizontally and vertically, containing the text "Back Dials" in a bold, white, serif font. The box has a thin white border. The overall appearance is that of a technical or scientific illustration, possibly related to a specific field of study like geology or materials science.

Back Dials



Fragment B
Back Dial (part)



Spiral Dial

Subsidiary Dial



Metonic
Cycle Dial:

5 spiral
235 glyphs

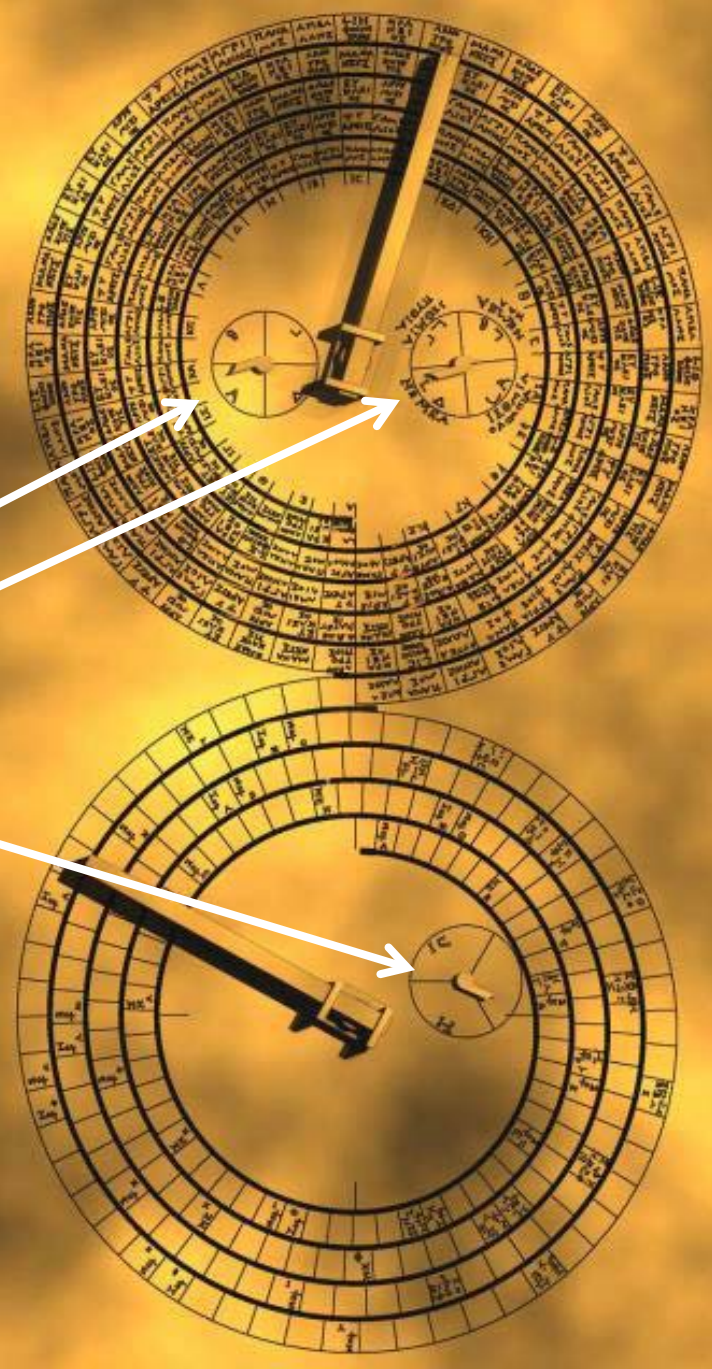
Callipic Cycle
Subdial

Olympic Cycle
Subdial

Exeligmos Cycle
Subdial

Saros
Cycle Dial:

4 spiral
223 glyphs



Astronomical Cycles

☐ Metonic Cycle

multiple of Tropical Year and Synodic Month

19 tropical years;
235 synodic months
254 sidereal months
6940 days

☐ Callippic Cycle

more accurate multiple
Tropical Year & Synodic Month

4 Metonic cycles - 1 days;
76 tropical years;
940 synodic months

☐ Saros Cycle

Eclipse cycle:
multiple of
Synodic, Draconic and Anomalistic month

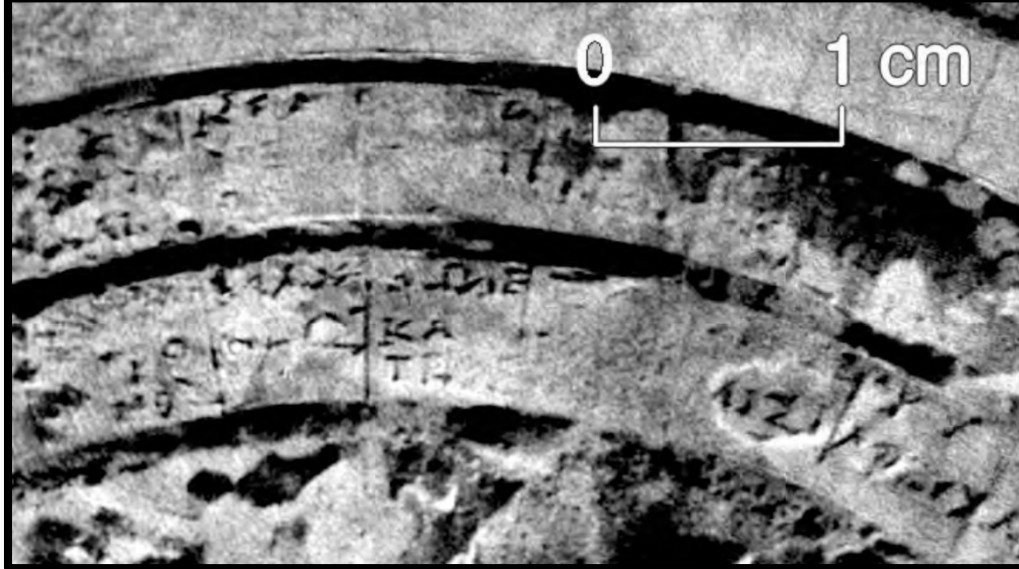
223 synodic;
242 draconic;
239 anomalistic:
18 yrs, 11 days, 8 hrs (6585 1/3 days)

☐ Exeligmos Cycle

3 Saros cycles:
following Exeligmos cycle, eclipse returns
at same location Earth

669 synodic;
726 draconic;
717 anomalistic:
54 yrs, 34 days (19756 days)

Metonic Cycle Dial



| | | | | |
|-----------|--|--|--|--|
| 125 A-3.2 | | | | Σ H ^M ω ^P H H ω ^P Γ Z |
| 131 F-3.1 | | | | Σ B H N ^Y ω ^P Θ H |
| 137 F-3.7 | | | | Σ H ^M ω ^P E H ω ^P IB Θ |
| 172 E-4.2 | | | | Σ H ω ^P Π |
| 178 A-4.2 | | | | Σ ω ^P Θ H ω ^P Θ P |
| 184 A-4.8 | | | | Σ H ^M ω ^P Δ H A Σ |
| 190 F-4.5 | | | | Σ H ^M ω ^P Θ T |

Saros Dial




Saros Dial



Saros Dial

glyph

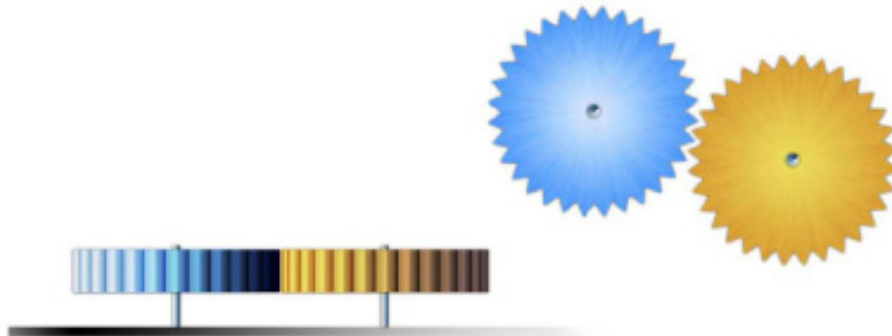


The image features a dark, textured background with a white border. The text is centered and reads "Antikythera Mechanism: Gear Train Reconstruction". The background has a complex, organic, and somewhat abstract appearance, possibly representing a reconstruction of the Antikythera mechanism's gear train. The text is in a bold, serif font, with the first line "Antikythera Mechanism:" and the second line "Gear Train Reconstruction".

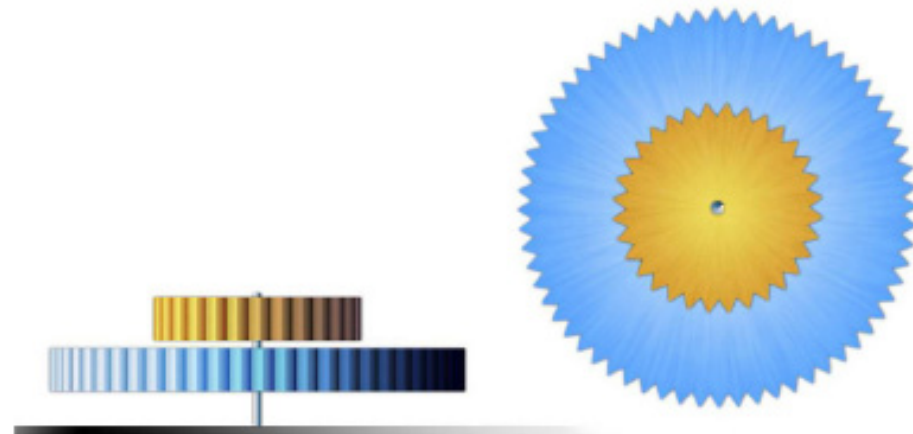
Antikythera Mechanism:
Gear Train Reconstruction



Gear Transmissions



Division



Multiplication

Example:

6 coupled gear wheels,
teeth: $r_1, r_2, s_1, s_2, t_1, t_2$:

$$f = \frac{r_1}{r_2} \times \frac{s_1}{s_2} \times \frac{t_1}{t_2}$$

Gear Train



courtesy: Niels Bos

| gear | #teeth |
|------|--------|
| b | 64 |
| l1 | 38 |
| l2 | 53 |
| m1 | 96 |
| m2 | 15 |
| n1 | 53 |

l

$$-\frac{64}{38} = -\frac{32}{19}$$

Gear Train



courtesy: Niels Bos

| gear | #teeth | |
|------|--------|---|
| b | 64 | |
| l1 | 38 | |
| l2 | 53 | $l \quad -\frac{64}{38} = -\frac{32}{19}$ |
| m1 | 96 | |
| m2 | 15 | $m \quad -\frac{32}{19} \times -\frac{53}{96} = \frac{53}{3 \times 19}$ |
| n1 | 53 | |

Gear Train



courtesy: Niels Bos

| gear | #teeth | |
|------|--------|---|
| b | 64 | |
| l1 | 38 | |
| l2 | 53 | $l \quad -\frac{64}{38} = -\frac{32}{19}$ |
| m1 | 96 | |
| m2 | 15 | $m \quad -\frac{32}{19} \times -\frac{53}{96} = \frac{53}{3 \times 19}$ |
| n1 | 53 | $n \quad \frac{53}{3 \times 19} \times -\frac{15}{53} = -\frac{5}{19}$ |

Gear Train



courtesy: Niels Bos

| gear | #teeth |
|------|--------|
| b | 64 |
| l1 | 38 |
| l2 | 53 |
| m1 | 96 |
| m2 | 15 |
| n1 | 53 |

 l

$$-\frac{64}{38} = -\frac{32}{19}$$

 m

$$-\frac{32}{19} \times -\frac{53}{96} = \frac{53}{3 \times 19}$$

 n

$$\frac{53}{3 \times 19} \times -\frac{15}{53} = -\frac{5}{19}$$

Exactly what we want for a
19-year 5-turn dial

AMRP Gear Train

Front dials

Zodiac • Egyptian calendar • Parapegma

Lunar phase

Hipparchos
sidereal month

Year

b0

q1

Input

Lost epicyclic gearing

Possibly
Hipparchos' solar mechanism
and planetary mechanisms

224 b1

a1

b2

38 64 38 c1

m1

96

53

l2

c2

48

24 d1

n1

53

15 m2

b3

32

32 e1

d2

127

p1

60

15 n2

e2

32

e3

223

e4

188

e5

53

o1

60

12 p2

27 m3

k1

50

50 e5

e4

f1

53

Pin and slot
Hipparchos' lunar mechanism

k2

50

50 e6

f2

30

g1

54

g2

20

h1

60

h2

15

i1

60

Callippic

Metonic × 5

Saros × 4

Exeligmos

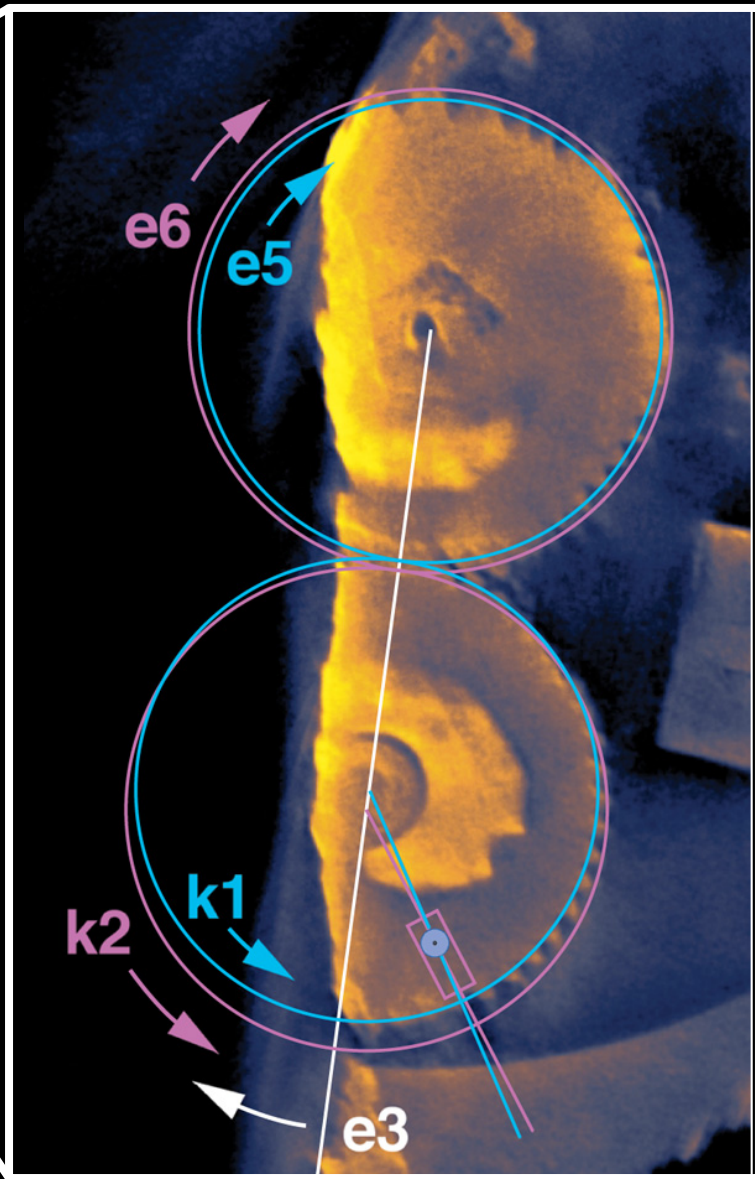
Luni-solar calendar

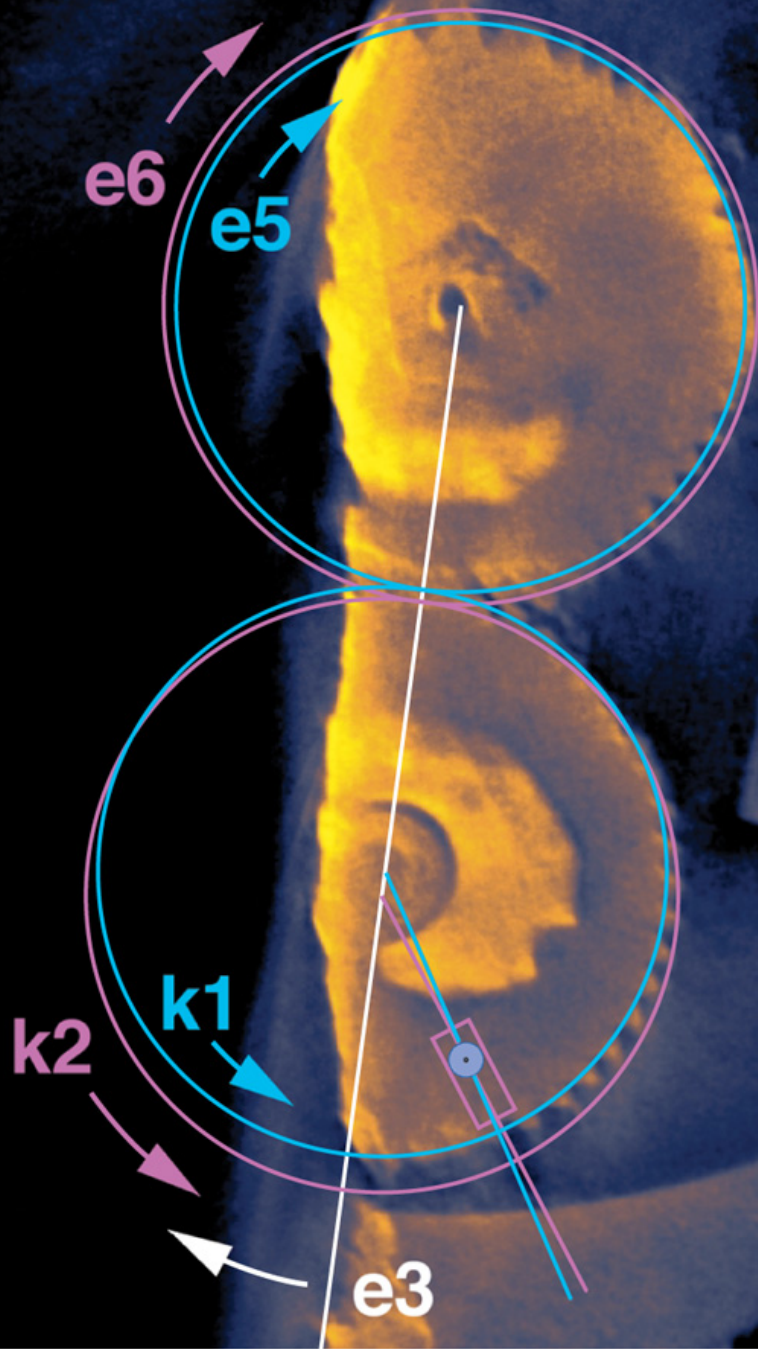
Eclipse prediction

Back dials

Hipparchus'

Spirograph ?





Pin-and-Slot Mechanism

Hipparchus'
Lunar Mechanism

Epicycle Theory

- ☐ describes
non-circular Moon orbit
- ☐ non-uniform motion
- ☐ differing apparent Moon size
- ☐ Noticeable: libration !

Apogee

Perigee



2004-12-26

405,363 km

29.94 arc-secs

Altitude @ 77.81°

2004-07-02

357,448 km

33.66 arc-secs

Altitude @ 21.72°

Moon Size

different distance along
orbit Moon

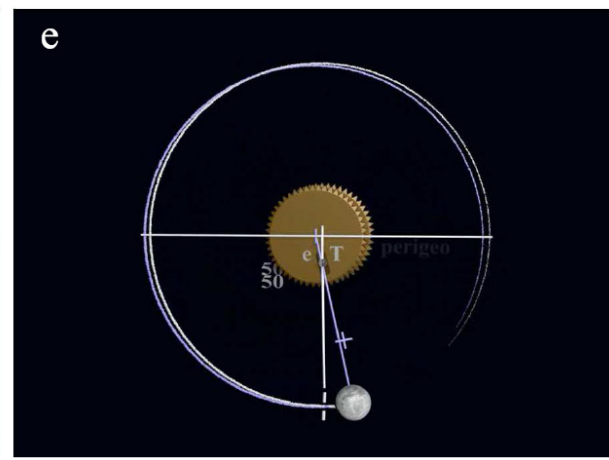
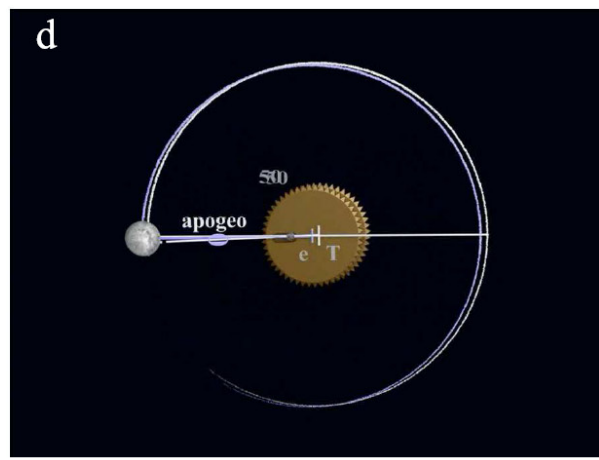
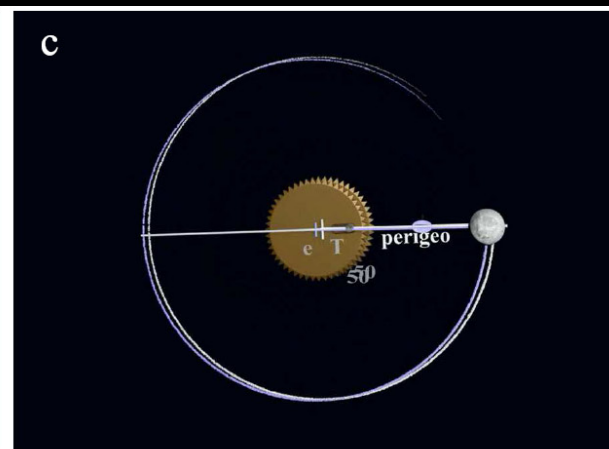
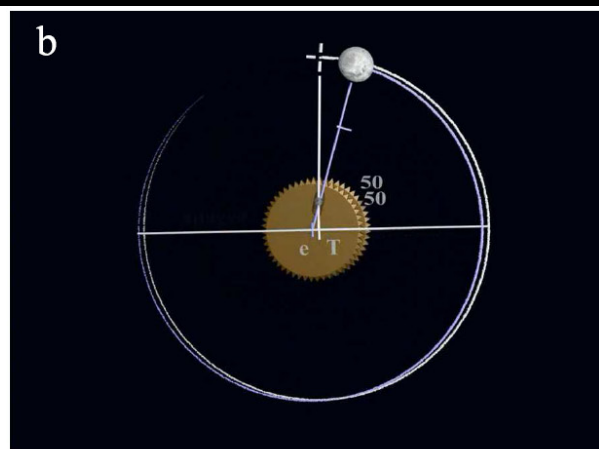
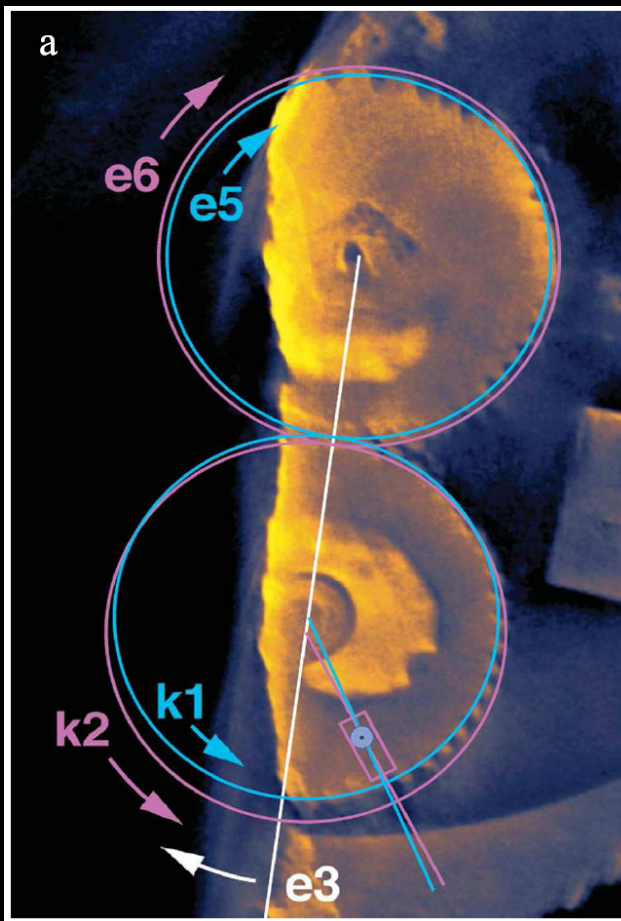
Moon Libration

We can see more than $\frac{1}{2}$ of
Moon surface, due to its
elliptical orbit

Date: 2005 Sep 1 02:23:28 UT



Hipparchus' Pin-Slot Gears



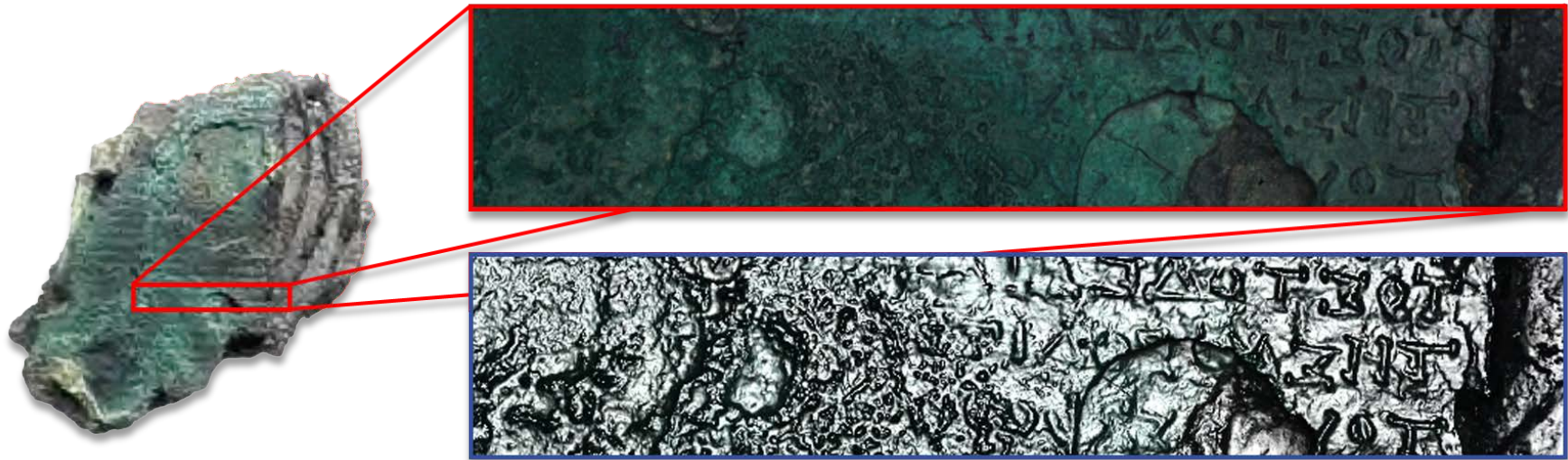


the Antikythera Planetarium ?

PhD project Niels Bos
Kapteyn Astron. Institute - Dept. Ancient History, RUG

the Antikythera Planetarium: indications

- Inscriptions on the surviving fragments
 - Early reading of “ΤΗΣΑΦΡΟΔΙΤΗ” (..of Venus..)
 - Freeth & Jones (2012)



the Antikythera Planetarium: indications

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 - Freeth & Jones (2012)
- Fragment A
 - Large size of the Sun wheel
 - Irregular features



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- Fragment A
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- Fragment D
 - Contains the only gear with no function





the Antikythera Planetarium: indications

- Inscriptions on the surviving fragments
 - Early reading of “ΤΗΣΑΦΡΟΔΙΤΗ” (..of Venus..)
 - Freeth & Jones (2012)
- Fragment A
 - Large size of the Sun wheel
 - Irregular features
- Fragment D
 - Contains the only gear with no function
- References found in ancient literature

Antikythera Mechanism

may be a planetarium following the Cosmos of Aristoteles

- Moon
- Mercury
- Venus
- Sun
- Mars
- Jupiter
- Saturn



A. Jones found all

5 planet names & Moon and Sun

- in inscriptions:
 - ordered like Cosmos
 - each with descriptive & theophoric name
 - e.g. Venus:
 - + Phosphoros
 - + star of Aphrodite

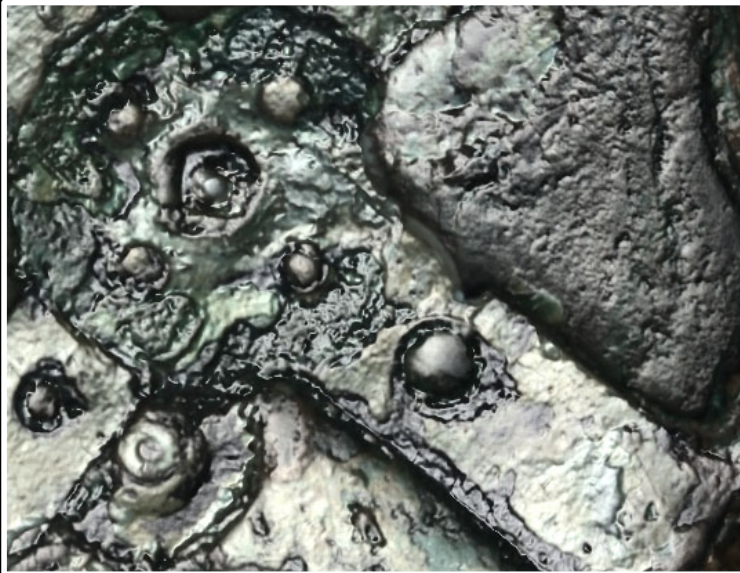
Freeth & Jones 2012 ISAW publ.

Mechanical Elements

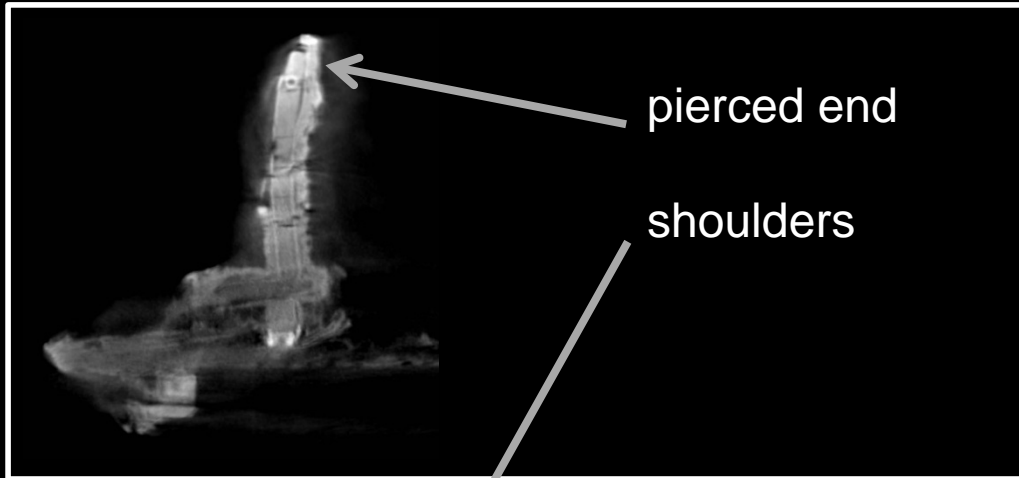
evidence of

pillars, bearing and
other fittings on the

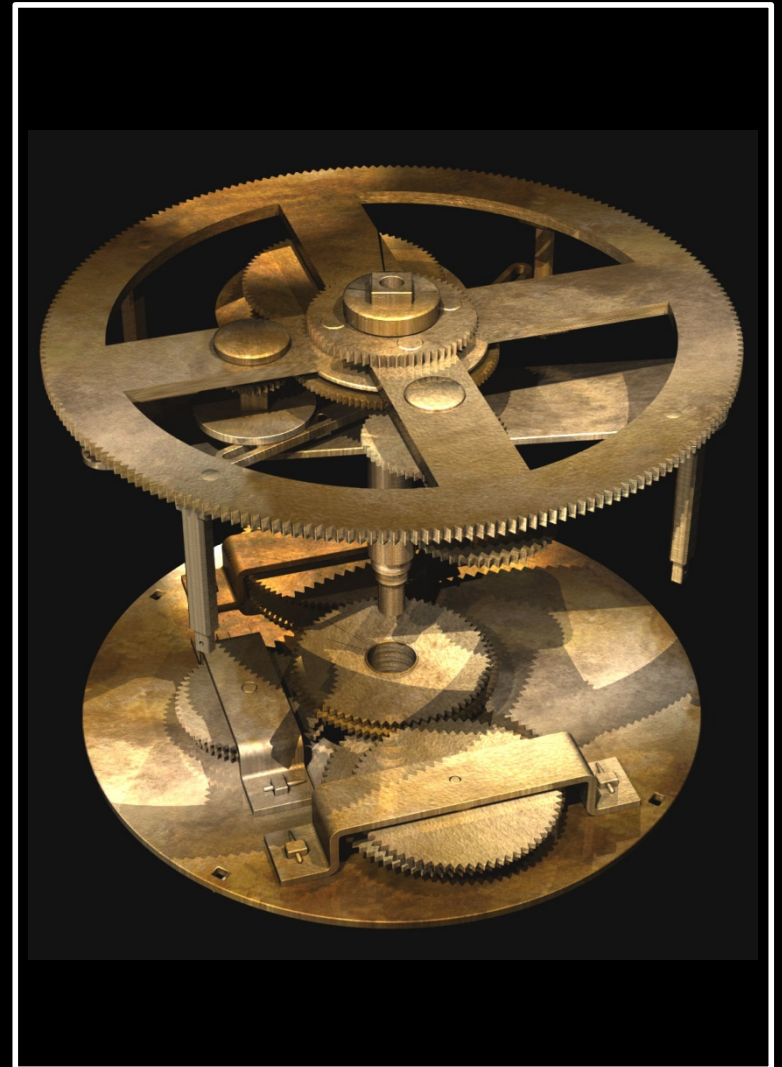
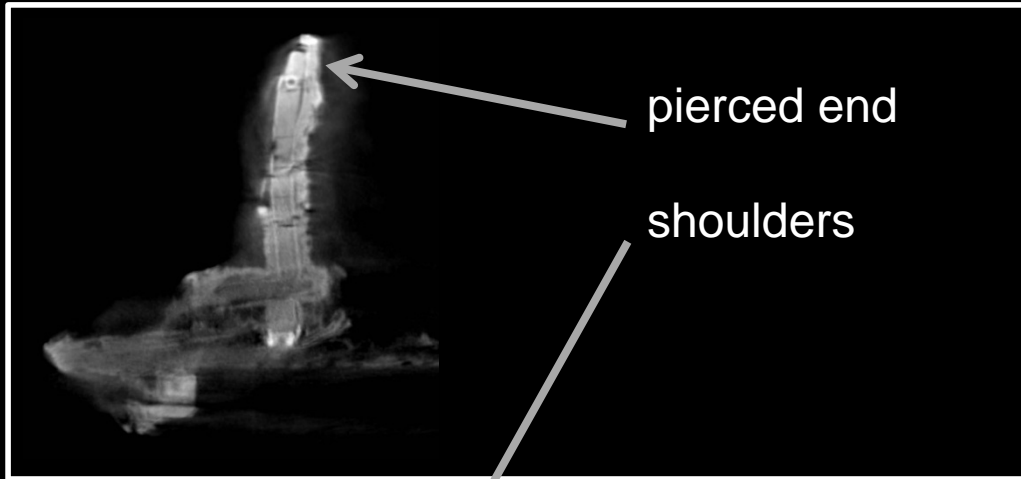
Main Drive Wheel



Mechanical Elements



Mechanical Elements

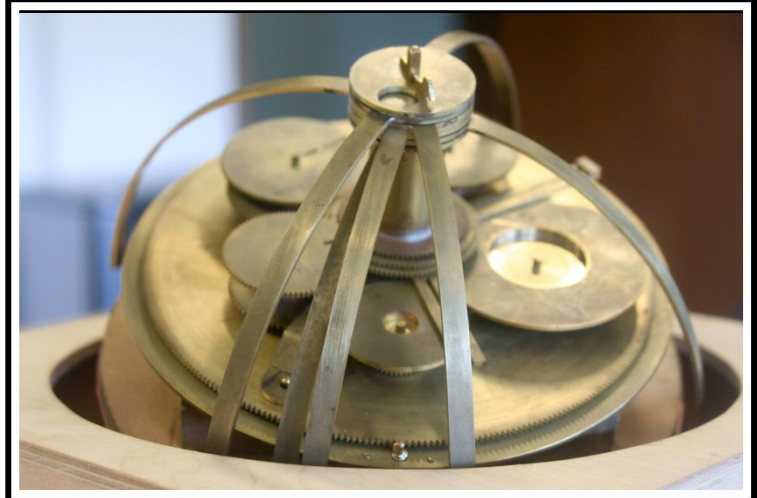




M. Wright

copper hardware models of

- Antikythera planetarium
- Archimedes Sphaera



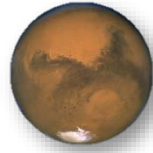
M. Wright

copper hardware model Antikythera planetarium
components

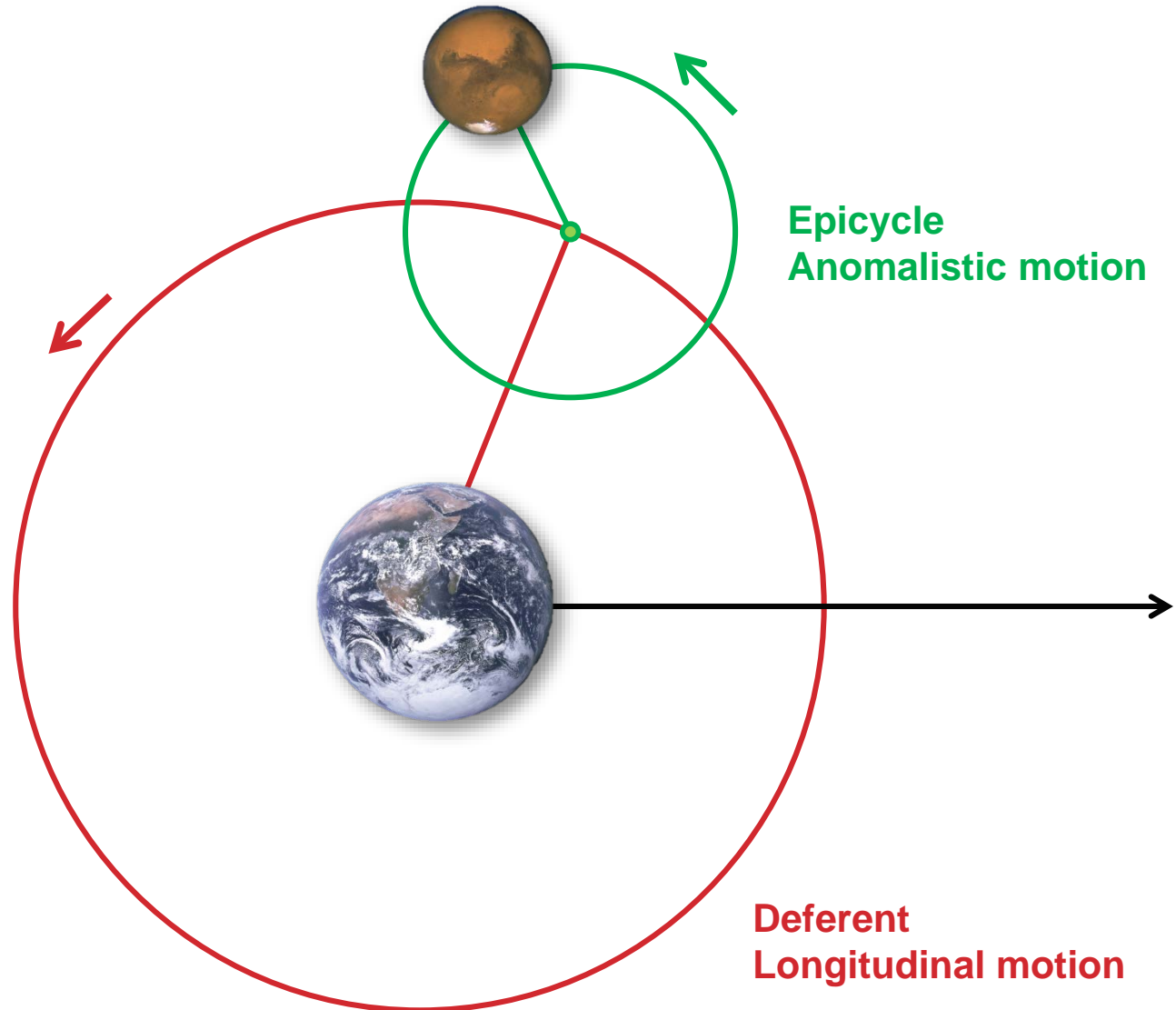




Geometric (Epicyle) planetary models

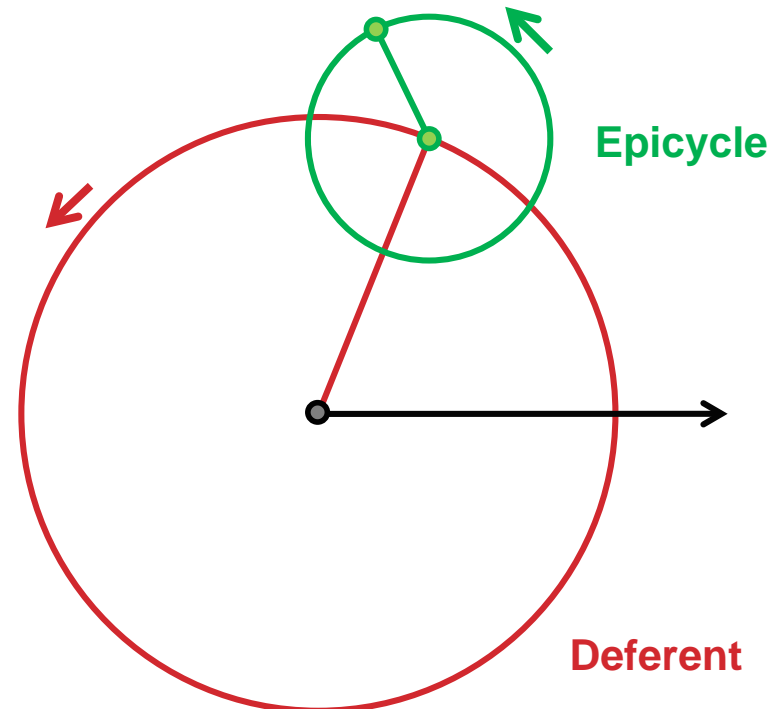


Geometric (Epicyle) planetary models



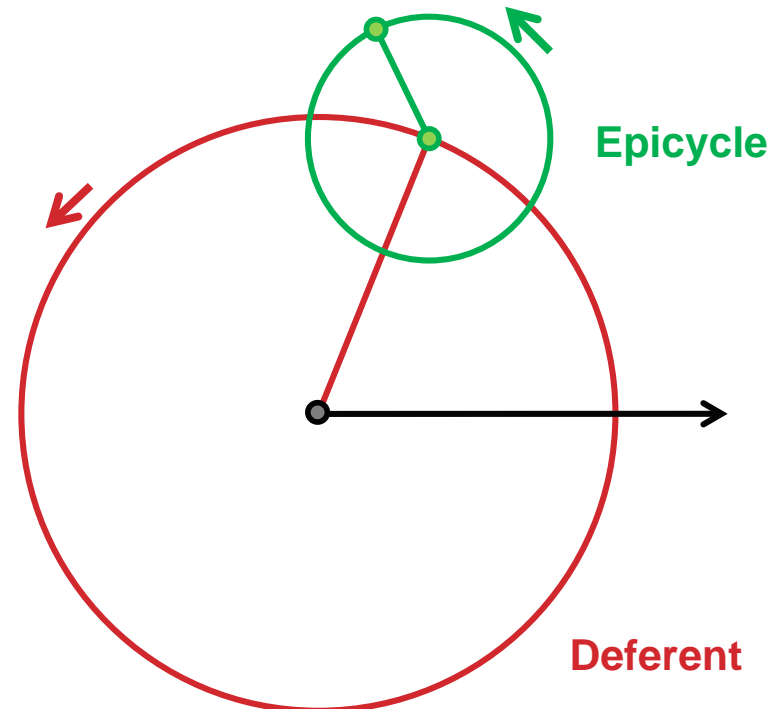
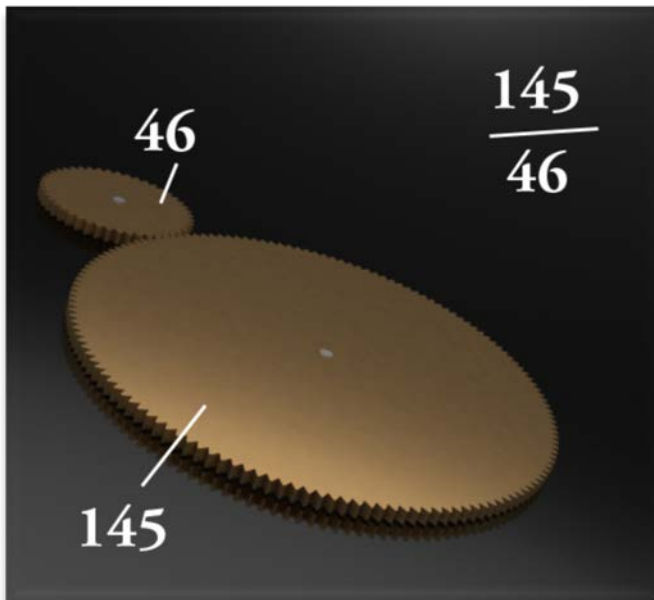
Periods to Gears

- Mercury's anomalistic motion: 145 cycles in 46 years.
- It makes $145/46$ cycles per year.
- Geared solution: $145/46$



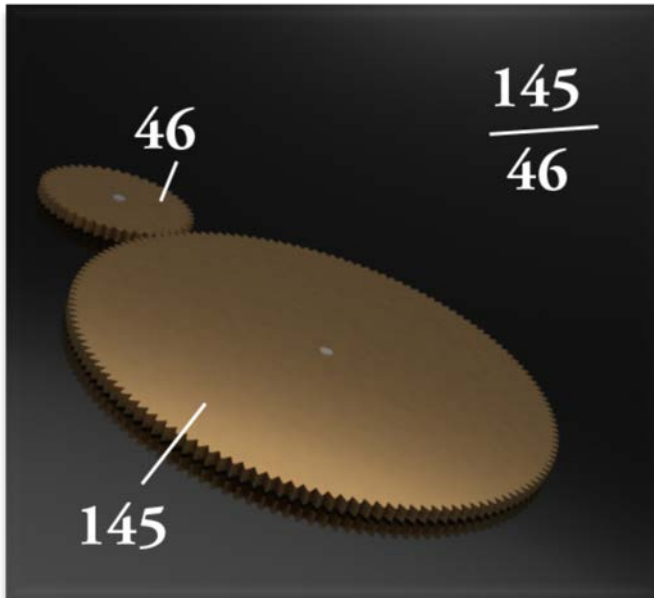
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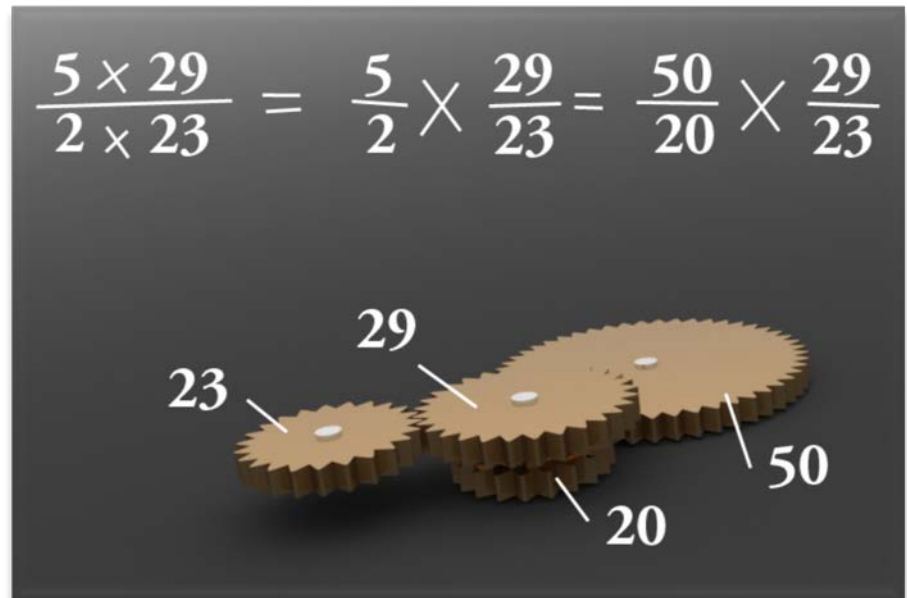
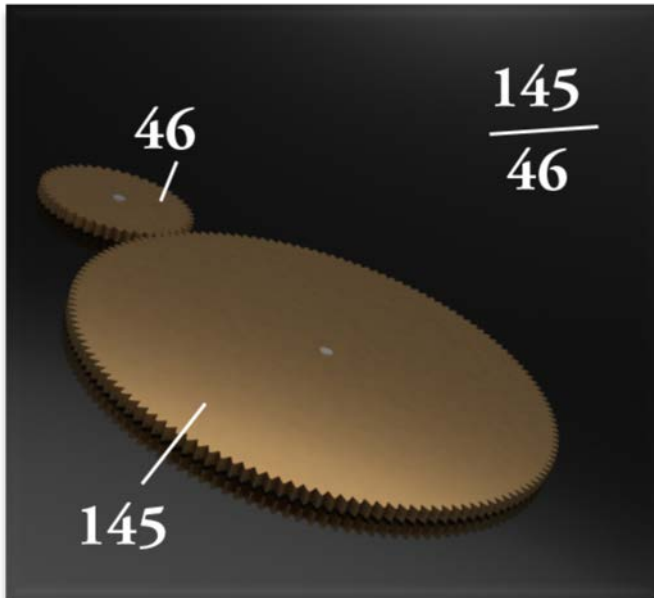
Periods to Gears

- Mercury's anomalistic motion: 145 cycles in 46 years.
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 - Factorizes into: 5×29 and 2×23



Periods to Gears

- Mercury's anomalistic motion: 145 cycles in 46 years.
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 - Factorizes into: 5×29 and 2×23



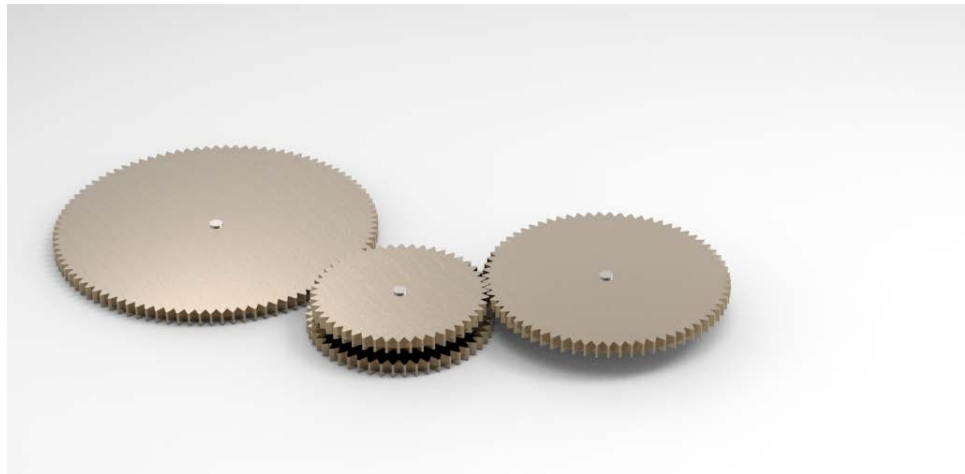
Venus: case study planetary geartrain

| Accuracy > 1e0: | | | | | | | |
|-----------------|---------------|-----|--------|--------|-------------------|---------------------------|-----|
| * 1 | 4500800.99764 | 97 | 3519.0 | 5626.0 | 2.22182673823e-07 | [3, 3, 17, 23][2, 29, 97] | [0] |
| * 2 | 3684799.00783 | 67 | 2881.0 | 4606.0 | 2.7138522332e-07 | [43, 67][2, 7, 7, 47] | [0] |
| * 3 | 2454400.99972 | 101 | 1919.0 | 3899.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 4 | 2149999.00033 | 43 | 3362.0 | 7995.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 5 | 2045120.99964 | 83 | 1281.0 | 5605.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 6 | 1638399.0002 | 61 | 5605.0 | 3519.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 7 | 1433759.00019 | 103 | 3519.0 | 5626.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| Accuracy > 1e5: | | | | | | | |
| * 1 | 785732.333369 | 97 | 3696.0 | 5626.0 | 2.22182673823e-07 | [3, 3, 17, 23][2, 29, 97] | [0] |
| * 2 | 700465.666715 | 79 | 6572.0 | 4606.0 | 2.7138522332e-07 | [43, 67][2, 7, 7, 47] | [0] |
| * 3 | 599850.999955 | 67 | 7504.0 | 3899.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 4 | 461719.000006 | 97 | 7220.0 | 7995.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 5 | 420303.761922 | 103 | 6901.0 | 5605.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 6 | 408000.999978 | 29 | 319.0 | 4977.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 7 | 374446.058831 | 109 | 4977.0 | 3696.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 8 | 326144.999994 | 17 | 6375.0 | 5626.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 9 | 322856.142858 | 113 | 1767.0 | 4606.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 10 | 314982.818173 | 71 | 2709.0 | 3899.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 11 | 312075.999993 | 73 | 7808.0 | 7995.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 12 | 298372.428569 | 71 | 6532.0 | 5605.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 13 | 294312.111101 | 109 | 2071.0 | 4977.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 14 | 289635.363643 | 59 | 4982.0 | 3696.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 15 | 287775.000001 | 23 | 5625.0 | 5626.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 16 | 271574.333394 | 67 | 3185.0 | 4606.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 17 | 232595.285708 | 67 | 6365.0 | 3899.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 18 | 231587.206895 | 89 | 5251.0 | 7995.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 19 | 221964.636362 | 109 | 1909.0 | 5605.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 20 | 205919.000001 | 23 | 805.0 | 4977.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 21 | 203360.999999 | 53 | 795.0 | 3696.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 22 | 190298.872337 | 43 | 6993.0 | 5626.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 23 | 186882.720932 | 103 | 6283.0 | 4606.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 24 | 185566.21739 | 97 | 3337.0 | 3899.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 25 | 182896.999999 | 71 | 7150.0 | 7995.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 26 | 182306.692308 | 109 | 3706.0 | 5605.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 27 | 165664.157893 | 107 | 4922.0 | 4977.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 28 | 162629.769233 | 31 | 6612.0 | 3696.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 29 | 157885.444444 | 107 | 5555.0 | 5626.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 30 | 154759.000003 | 73 | 2420.0 | 4606.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 31 | 151561.500001 | 97 | 7584.0 | 3899.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 32 | 146232.333335 | 107 | 2744.0 | 7995.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 33 | 141079.260867 | 59 | 2537.0 | 5605.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 34 | 140141.857144 | 109 | 3068.0 | 4977.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 35 | 135574.000002 | 53 | 3392.0 | 3696.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 36 | 131899.412697 | 89 | 6497.0 | 5626.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 37 | 125842.478261 | 73 | 2263.0 | 4606.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 38 | 123531.467534 | 67 | 7437.0 | 3899.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 39 | 122846.901639 | 31 | 5859.0 | 7995.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 40 | 117633.432432 | 83 | 6806.0 | 5605.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 41 | 116389.0 | 113 | 7280.0 | 4977.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 42 | 113251.83019 | 61 | 4693.0 | 3696.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 43 | 108527.829268 | 103 | 3479.0 | 5626.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 44 | 107862.333333 | 79 | 6325.0 | 4606.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 45 | 106426.263156 | 89 | 7905.0 | 3899.0 | 1.6218188551e-07 | [19, 19][2, 3, 16, 59] | [0] |
| * 46 | 104414.186812 | 107 | 7429.0 | 7995.0 | 1.6218188551e-07 | [19, 19, 23][3, 37, 107] | [0] |
| * 47 | 103599.000001 | 37 | 162.0 | 5626.0 | 1.6218188551e-07 | [2, 3, 3, 3, 3][7, 37] | [0] |

Best approximation

$$\text{Accuracy} = 2,22 \times 10^{-7}$$

$$\begin{aligned} \text{Period} &= 5626 / 3519 \\ &= (2 \times 29 \times 97) / (3 \times 3 \times 17 \times 23) \\ &= (97 / 51) \times (48 / 69) \end{aligned}$$



Venus: case study planetary geartrain

| Accuracy > 1e6: | | | | | | | |
|-----------------|---------------|-----|--------|---------|-------------------|----------------------------|-----|
| 1 | 4500800.99764 | 97 | 3519.0 | 5626.0 | 2.22182673823e-07 | [3, 3, 17, 23]/[2, 29, 97] | [0] |
| 2 | 3684799.00183 | 67 | 2881.0 | 4606.0 | 2.7138522332e-07 | [43, 67]/[2, 7, 7, 47] | [0] |
| 3 | 2454400.99972 | 101 | 1049.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 4 | 2149999.00033 | 43 | 3362.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 5 | 2049120.99984 | 83 | 7995.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 6 | 1638399.0002 | 61 | 1281.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 7 | 143933.00019 | 103 | 5605.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| Accuracy > 1e5: | | | | | | | |
| 1 | 785732.333369 | 97 | 3696.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 2 | 700465.666715 | 79 | 6572.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 3 | 599850.999955 | 67 | 7504.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 4 | 461719.000006 | 97 | 7220.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 5 | 420303.761922 | 103 | 6901.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 6 | 408000.999978 | 29 | 319.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 7 | 374446.058831 | 109 | 4977.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 8 | 326144.999994 | 17 | 6375.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 9 | 322856.142858 | 113 | 1767.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 10 | 314982.818173 | 71 | 2709.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 11 | 312075.999993 | 73 | 7808.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 12 | 298372.428569 | 71 | 6532.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 13 | 294312.111101 | 109 | 2071.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 14 | 280635.363643 | 59 | 4982.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 15 | 287775.00001 | 23 | 5625.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 16 | 271574.333324 | 67 | 3185.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 17 | 232595.265709 | 67 | 6365.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 18 | 231587.208995 | 89 | 5251.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 19 | 221964.636382 | 109 | 1909.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 20 | 205919.000001 | 23 | 805.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 21 | 203360.999999 | 53 | 795.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 22 | 190298.872337 | 43 | 6993.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 23 | 186882.720932 | 103 | 6283.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 24 | 185566.21739 | 97 | 3337.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 25 | 182896.999999 | 71 | 7150.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 26 | 182306.692308 | 109 | 3706.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 27 | 165664.157893 | 107 | 4922.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 28 | 162629.769233 | 31 | 6612.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 29 | 157885.444444 | 107 | 5555.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 30 | 154759.000003 | 73 | 2420.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 31 | 151561.500001 | 97 | 7584.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 32 | 146232.333335 | 107 | 2744.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 33 | 141079.260867 | 59 | 2537.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 34 | 140141.857144 | 109 | 3068.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 35 | 135574.000002 | 53 | 3392.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 36 | 131899.412697 | 89 | 6497.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 37 | 125842.478261 | 73 | 2263.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 38 | 123531.467534 | 67 | 7437.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 39 | 122846.901639 | 31 | 5859.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 40 | 117633.432432 | 83 | 6806.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 41 | 116389.0 | 113 | 7280.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 42 | 113251.83019 | 61 | 4693.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 43 | 108527.829268 | 103 | 3479.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 44 | 107862.333333 | 79 | 6325.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 45 | 106426.263156 | 89 | 7905.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 47, 53] | [0] |
| 46 | 104414.186812 | 107 | 7429.0 | 11877.0 | 9.57724261936e-06 | [17, 19, 23]/[3, 37, 107] | [0] |
| 47 | 103599.000001 | 37 | 162.0 | 259.0 | 9.65260282428e-06 | [2, 3, 3, 3, 3]/[7, 37] | [0] |

Sixth best approximation

$$\text{Accuracy} = 6,10 \times 10^{-7}$$

$$\begin{aligned} \text{Period} &= 2048 / 1281 \\ &= 2^{11} / (3 \times 7 \times 61) \\ &= (32 / 61) \times (64 / 21) \\ &= (96 / 61) \times (64 / 63) \end{aligned}$$

Venus: case study planetary geartrain

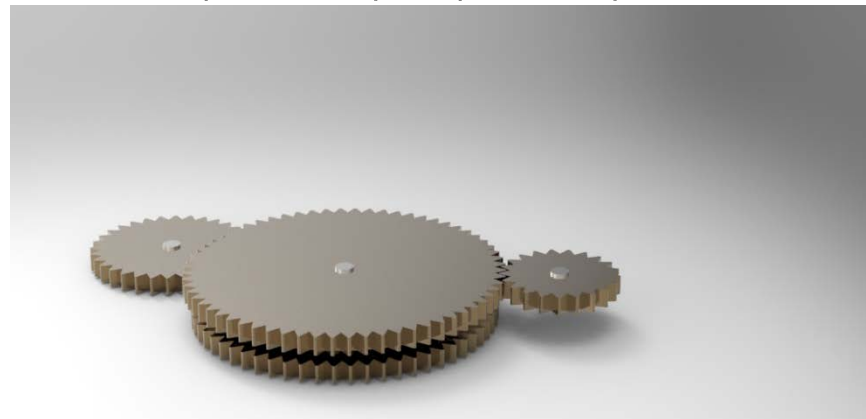
| Accuracy > 1e6: | | | | | | | |
|-----------------|---------------|-----|--------|--------|-------------------|---------------------------|-----|
| 1 | 4500800.99764 | 97 | 3519.0 | 5626.0 | 2.22182673823e-07 | [3, 3, 17, 23][2, 29, 97] | [0] |
| 2 | 3684799.00183 | 67 | 2881.0 | 4606.0 | 2.7138522332e-07 | [43, 67][2, 7, 7, 47] | [0] |
| 3 | 2454400.99972 | 101 | 1049.0 | 3892.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 4 | 2149999.00033 | 43 | 3362.0 | 3362.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 5 | 2049120.99984 | 83 | 7995.0 | 7995.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 6 | 1638399.0002 | 61 | 1281.0 | 1281.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 7 | 143933.00019 | 103 | 5605.0 | 5605.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |

| Accuracy > 1e5: | | | | | | | |
|-----------------|---------------|-----|--------|---------|-------------------|--------------------------|-----|
| 1 | 785732.333369 | 97 | 3696.0 | 3696.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 2 | 700465.666715 | 79 | 6572.0 | 6572.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 3 | 599850.999955 | 67 | 7504.0 | 7504.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 4 | 461719.000006 | 97 | 7220.0 | 7220.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 5 | 420303.761922 | 103 | 6901.0 | 6901.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 6 | 408000.999978 | 29 | 319.0 | 319.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 7 | 374446.058831 | 109 | 4977.0 | 4977.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 8 | 326144.999994 | 17 | 6375.0 | 6375.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 9 | 322856.142858 | 113 | 1767.0 | 1767.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 10 | 314982.818173 | 71 | 2709.0 | 2709.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 11 | 312075.999993 | 73 | 7808.0 | 7808.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 12 | 298372.428569 | 71 | 6532.0 | 6532.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 13 | 294312.111101 | 109 | 2071.0 | 2071.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 14 | 280635.363643 | 59 | 4982.0 | 4982.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 15 | 287775.00001 | 23 | 5625.0 | 5625.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 16 | 271574.333324 | 67 | 3185.0 | 3185.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 17 | 232595.265709 | 67 | 6365.0 | 6365.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 18 | 231587.208995 | 89 | 5251.0 | 5251.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 19 | 221964.636382 | 109 | 1909.0 | 1909.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 20 | 205919.000001 | 23 | 805.0 | 805.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 21 | 203360.999999 | 53 | 795.0 | 795.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 22 | 190298.872337 | 43 | 6993.0 | 6993.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 23 | 186882.720932 | 103 | 6283.0 | 6283.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 24 | 185566.21739 | 97 | 3337.0 | 3337.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 25 | 182896.999999 | 71 | 7150.0 | 7150.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 26 | 182306.692308 | 109 | 3706.0 | 3706.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 27 | 165664.157893 | 107 | 4922.0 | 4922.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 28 | 162629.769233 | 31 | 6612.0 | 6612.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 29 | 157885.444444 | 107 | 5555.0 | 5555.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 30 | 154759.000003 | 73 | 2420.0 | 2420.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 31 | 151561.500001 | 97 | 7584.0 | 7584.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 32 | 146232.333335 | 107 | 2744.0 | 2744.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 33 | 141079.260867 | 59 | 2537.0 | 2537.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 34 | 140141.857144 | 109 | 3068.0 | 3068.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 35 | 135574.000002 | 53 | 3392.0 | 3392.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 36 | 131899.412697 | 89 | 6497.0 | 6497.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 37 | 125842.478261 | 73 | 2263.0 | 2263.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 38 | 123531.467534 | 67 | 7437.0 | 7437.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 39 | 122846.901639 | 31 | 5859.0 | 5859.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 40 | 117633.432432 | 83 | 6806.0 | 6806.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 41 | 116389.0 | 113 | 7280.0 | 7280.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 42 | 113251.83019 | 61 | 4693.0 | 4693.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 43 | 108527.829268 | 103 | 3479.0 | 3479.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 44 | 107862.333333 | 79 | 6325.0 | 6325.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 45 | 106426.263156 | 89 | 7905.0 | 7905.0 | 1.6518128551e-07 | [16, 19][2, 3, 16, 53] | [0] |
| 46 | 104414.186812 | 107 | 7429.0 | 11877.0 | 9.57724261936e-06 | [17, 19, 23][3, 37, 107] | [0] |
| 47 | 103599.000001 | 37 | 162.0 | 259.0 | 9.65260282428e-06 | [2, 3, 3, 3, 3][7, 37] | [0] |

Sixth best approximation

$$\text{Accuracy} = 6,10 \times 10^{-7}$$

$$\begin{aligned} \text{Period} &= 2048 / 1281 \\ &= 2^{11} / (3 \times 7 \times 61) \\ &= (32 / 61) \times (64 / 21) \\ &= (96 / 61) \times (64 / 63) \end{aligned}$$



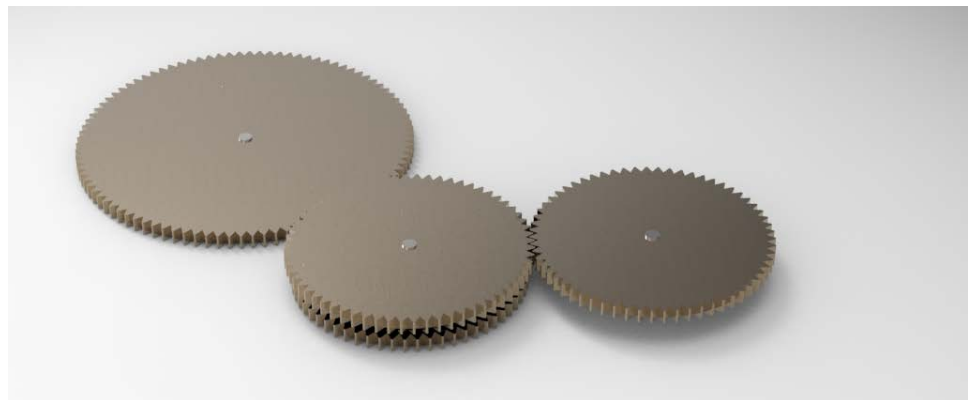
Venus: case study planetary geartrain

| Accuracy > 1e6: | | | | | | | |
|-----------------|---------------|-----|--------|--------|-------------------|----------------------------|-----|
| 1 | 4500800.99764 | 97 | 3519.0 | 5626.0 | 2.22182673823e-07 | [3, 3, 17, 23]/[2, 29, 97] | [0] |
| 2 | 3684799.00183 | 67 | 2881.0 | 4606.0 | 2.7138522332e-07 | [43, 67]/[2, 7, 7, 47] | [0] |
| 3 | 2454400.99972 | 101 | 1049.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 4 | 2149999.00033 | 43 | 3362.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 5 | 2049120.99984 | 83 | 7995.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 6 | 1638399.0002 | 61 | 1281.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 7 | 143933.00019 | 103 | 5605.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| Accuracy > 1e5: | | | | | | | |
| 1 | 785732.333369 | 97 | 3696.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 2 | 700465.666715 | 79 | 6572.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 3 | 599850.999955 | 67 | 7504.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 4 | 461719.000006 | 97 | 7220.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 5 | 420303.761922 | 103 | 6901.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 6 | 408000.999978 | 29 | 319.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 7 | 374446.058831 | 109 | 4977.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 8 | 326144.999994 | 17 | 6375.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 9 | 322856.142858 | 113 | 1767.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 10 | 314982.818173 | 71 | 2709.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 11 | 312075.999993 | 73 | 7808.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 12 | 298372.428569 | 71 | 6532.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 13 | 294312.111101 | 109 | 2071.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 14 | 280635.363643 | 59 | 4982.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 15 | 287775.00001 | 23 | 5625.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 16 | 271574.333324 | 67 | 3185.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 17 | 232595.265709 | 67 | 6365.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 18 | 231587.208995 | 89 | 5251.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 19 | 221964.636382 | 109 | 1909.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 20 | 205919.000001 | 23 | 805.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 21 | 203360.999999 | 53 | 795.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 22 | 190298.872337 | 43 | 6993.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 23 | 186882.720932 | 103 | 6283.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 24 | 185566.21739 | 97 | 3337.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 25 | 182896.999999 | 71 | 7150.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 26 | 182306.692308 | 109 | 3706.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 27 | 165664.157893 | 107 | 4922.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 28 | 162629.769233 | 31 | 6612.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 29 | 157885.444444 | 107 | 5555.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 30 | 154759.000003 | 73 | 2420.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 31 | 151561.500001 | 97 | 7584.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 32 | 146232.333335 | 107 | 2744.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 33 | 141079.260867 | 59 | 2537.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 34 | 140141.857144 | 109 | 3068.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 35 | 135574.000002 | 53 | 3392.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 36 | 131899.412697 | 89 | 6497.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 37 | 125842.478261 | 73 | 2263.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 38 | 123531.467534 | 67 | 7437.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 39 | 122846.901639 | 31 | 5859.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 40 | 117633.432432 | 83 | 6806.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 41 | 116389.0 | 113 | 7280.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 42 | 113251.83019 | 61 | 4693.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 43 | 108527.829268 | 103 | 3479.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 44 | 107862.333333 | 79 | 6325.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 45 | 106426.263156 | 89 | 7905.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 46 | 104414.186812 | 107 | 7429.0 | 3892.0 | 1.6518128551e-07 | [16, 19, 29, 43, 53] | [0] |
| 47 | 103599.000001 | 37 | 162.0 | 3892.0 | 1.6518128551e-07 | [2, 3, 3, 3]/[7, 37] | [0] |

Sixth best approximation

$$\text{Accuracy} = 6,10 \times 10^{-7}$$

$$\begin{aligned} \text{Period} &= 2048 / 1281 \\ &= 2^{11} / (3 \times 7 \times 61) \\ &= (32 / 61) \times (64 / 21) \\ &= (96 / 61) \times (64 / 63) \end{aligned}$$



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| Accuracy > 1e6: | | | | | | | |
|-----------------|---------------|-----|--------|--------|-------------------|----------------------------|-----|
| 1 | 4500800.99764 | 97 | 3519.0 | 5626.0 | 2.22182673823e-07 | [3, 3, 17, 23]/[2, 29, 97] | [0] |
| 2 | 3684799.00183 | 67 | 2881.0 | 4606.0 | 2.7138522332e-07 | [43, 67]/[2, 7, 7, 47] | [0] |
| 3 | 2454400.99972 | 101 | 1049.0 | 3888.0 | 1.6518128551e-07 | [16, 18, 19, 21, 23, 29] | [0] |
| 4 | 2149999.00033 | 43 | 3362.0 | 3362.0 | 1.6518128551e-07 | [16, 18, 19, 21, 23, 29] | [0] |
| 5 | 2049120.99984 | 83 | 7995.0 | 7995.0 | 1.6518128551e-07 | [16, 18, 19, 21, 23, 29] | [0] |
| 6 | 1638399.0002 | 61 | 1281.0 | 1281.0 | 1.6518128551e-07 | [16, 18, 19, 21, 23, 29] | [0] |
| 7 | 143933.00019 | 103 | 5605.0 | 5605.0 | 1.6518128551e-07 | [16, 18, 19, 21, 23, 29] | [0] |

| Accuracy > 1e5: | | | |
|-----------------|---------------|-----|--------|
| 1 | 785732.333369 | 97 | 3696.0 |
| 2 | 700465.666715 | 79 | 6572.0 |
| 3 | 599850.999955 | 67 | 7504.0 |
| 4 | 461719.000006 | 97 | 7220.0 |
| 5 | 420303.761922 | 103 | 6901.0 |
| 6 | 408000.999978 | 29 | 319.0 |
| 7 | 374446.058831 | 109 | 4977.0 |
| 8 | 326144.999994 | 17 | 6375.0 |
| 9 | 322856.142858 | 113 | 1767.0 |
| 10 | 291817.3 | 71 | 2709.0 |
| 11 | | 73 | 7808.0 |
| 12 | | 71 | 6532.0 |

Sixth best approximation

$$\text{Accuracy} = 6,10 \times 10^{-7}$$

$$\begin{aligned} \text{Period} &= 2048 / 1281 \\ &= 2^{11} / (3 \times 7 \times 61) \\ &= (32 / 61) \times (64 / 21) \\ &= (96 / 61) \times (64 / 63) \end{aligned}$$



| | | | | | | | |
|----|---------------|-----|--------|---------|-------------------|---------------------------|-----|
| 13 | | | | | | | |
| 1 | | | | | | | |
| 43 | | 103 | 3479.0 | | | | |
| 44 | | 79 | 6325.0 | | | | |
| | | 89 | 7905.0 | | | | |
| | 104414.186812 | 107 | 7429.0 | 11877.0 | 9.57724261936e-06 | [17, 19, 23]/[3, 37, 107] | [0] |
| .. | 103599.000001 | 37 | 162.0 | 259.0 | 9.65260282428e-06 | [2, 3, 3, 3, 3]/[7, 37] | [0] |

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| Accuracy > 1e6: | | | | | | | |
|-----------------|---------------|-----|--------|--------|-------------------|----------------------------|-----|
| 1 | 4500800.99764 | 97 | 3519.0 | 5626.0 | 2.22182673823e-07 | [3, 3, 17, 23]/[2, 29, 97] | [0] |
| 2 | 3684799.00183 | 67 | 2881.0 | 4606.0 | 2.7138522332e-07 | [43, 67]/[2, 7, 7, 47] | [0] |
| 3 | 2454400.99972 | 101 | 1049.0 | 3888.0 | 1.6518128551e-07 | [16, 18, 19, 21, 23, 29] | [0] |
| 4 | 2149999.00033 | 43 | 3362.0 | 7995.0 | 1.6518128551e-07 | [16, 18, 19, 21, 23, 29] | [0] |
| 5 | 2049120.99984 | 83 | 7995.0 | 1281.0 | 1.6518128551e-07 | [16, 18, 19, 21, 23, 29] | [0] |
| 6 | 1638399.0002 | 61 | 1281.0 | 5605.0 | 1.6518128551e-07 | [16, 18, 19, 21, 23, 29] | [0] |
| 7 | 143933.00019 | 103 | 5605.0 | | | | |

| Accuracy > 1e5: | | | |
|-----------------|---------------|-----|-------|
| 1 | 785732.333369 | 97 | 3696 |
| 2 | 700465.666715 | 79 | 6572 |
| 3 | 599850.999955 | 67 | 7504 |
| 4 | 461719.000006 | 97 | 7220 |
| 5 | 420303.761922 | 103 | 6901 |
| 6 | 408000.999978 | 29 | 319.0 |
| 7 | 374446.058831 | 109 | 4977 |
| 8 | 326144.999994 | 17 | 6375 |
| 9 | 322856.142858 | 113 | 1767 |
| 10 | 2918173 | 71 | 2709 |
| 11 | | 73 | 7808 |
| 12 | | 71 | 6532 |

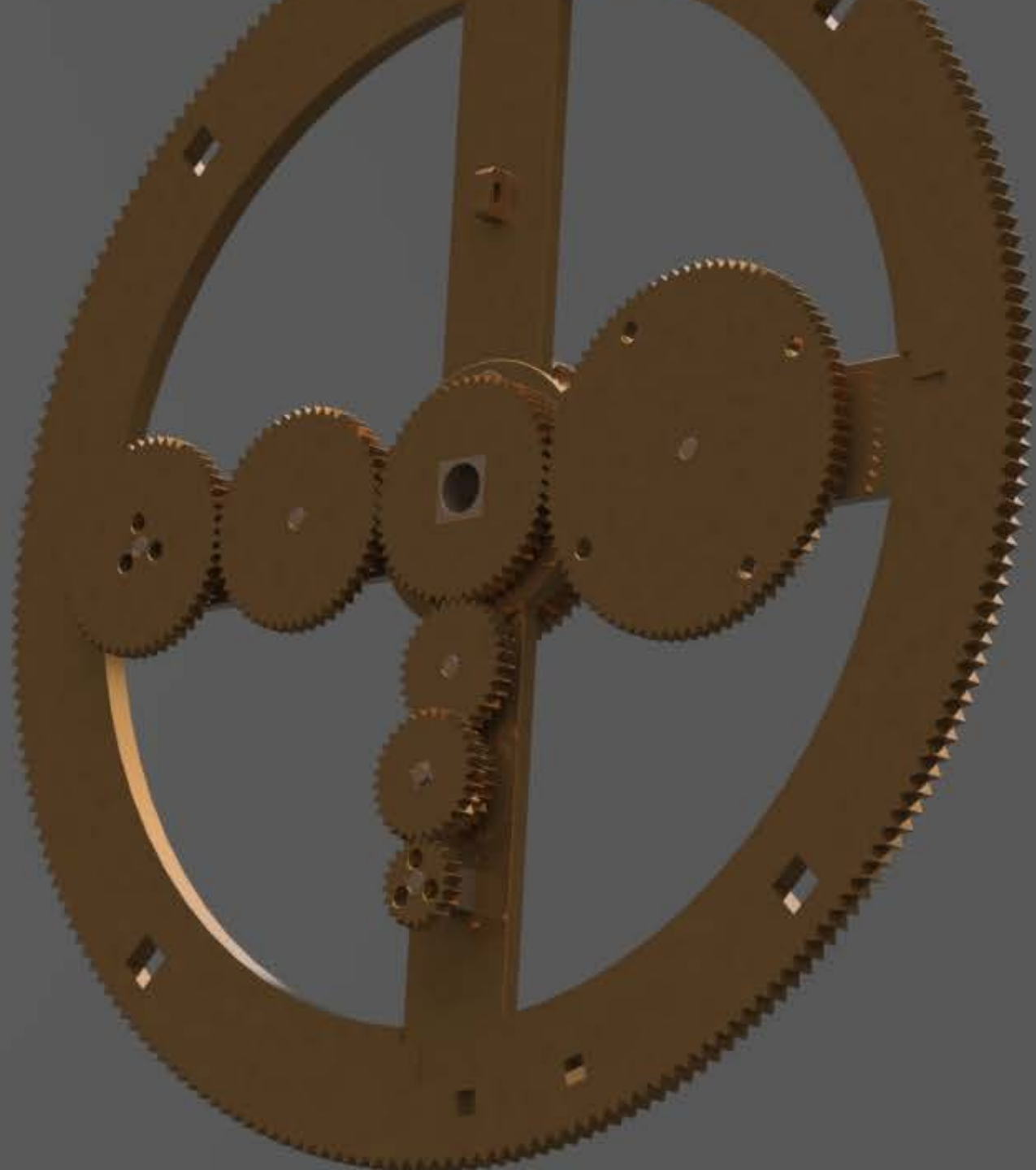
Sixth best approximation

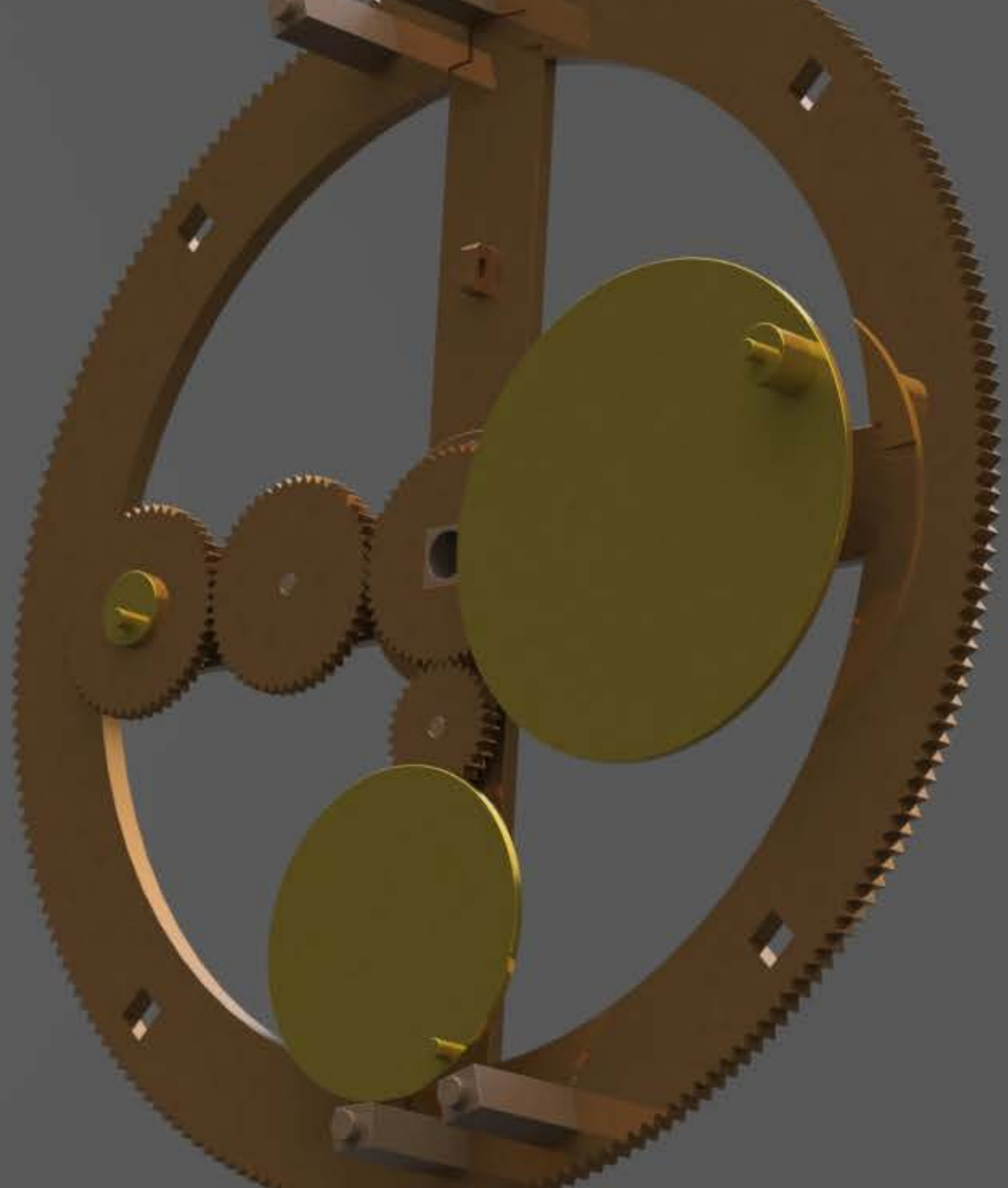
$$\text{Accuracy} = 6,10 \times 10^{-7}$$

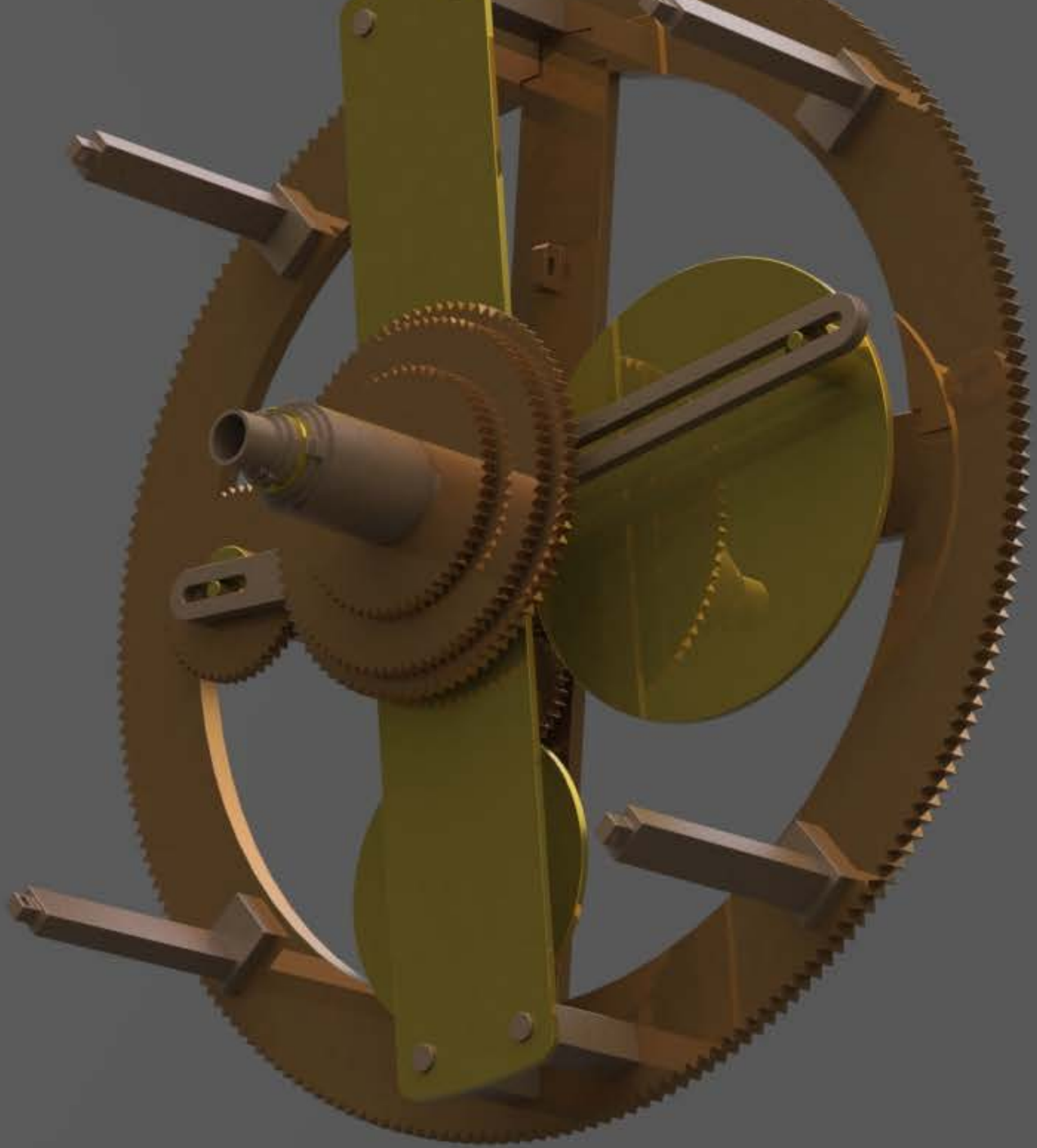
$$\begin{aligned} \text{Period} &= 2048 / 1281 \\ &= 2^{11} / (3 \times 7 \times 61) \\ &= (32 / 61) \times (64 / 21) \\ &= (96 / 61) \times (64 / 63) \end{aligned}$$

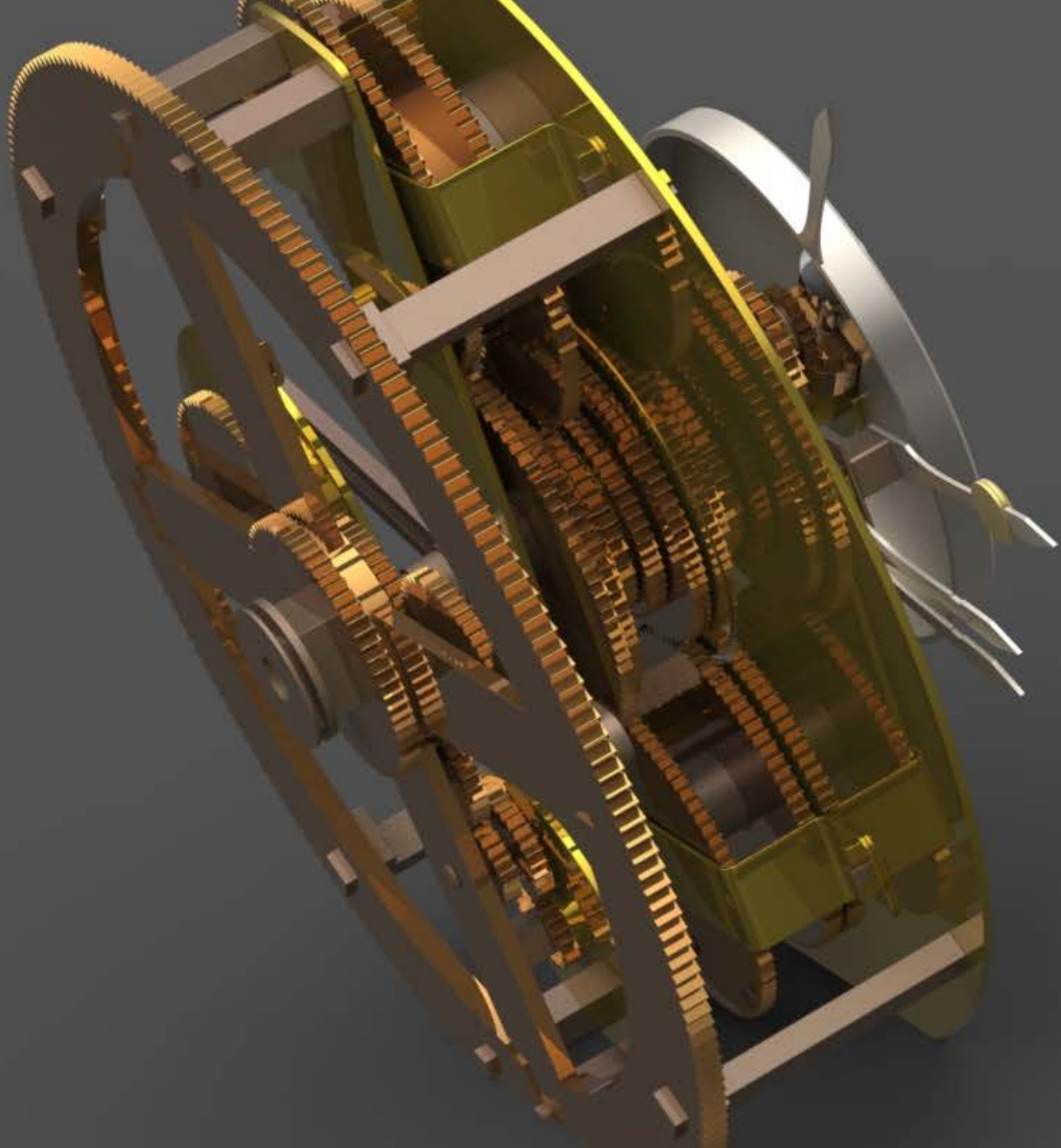


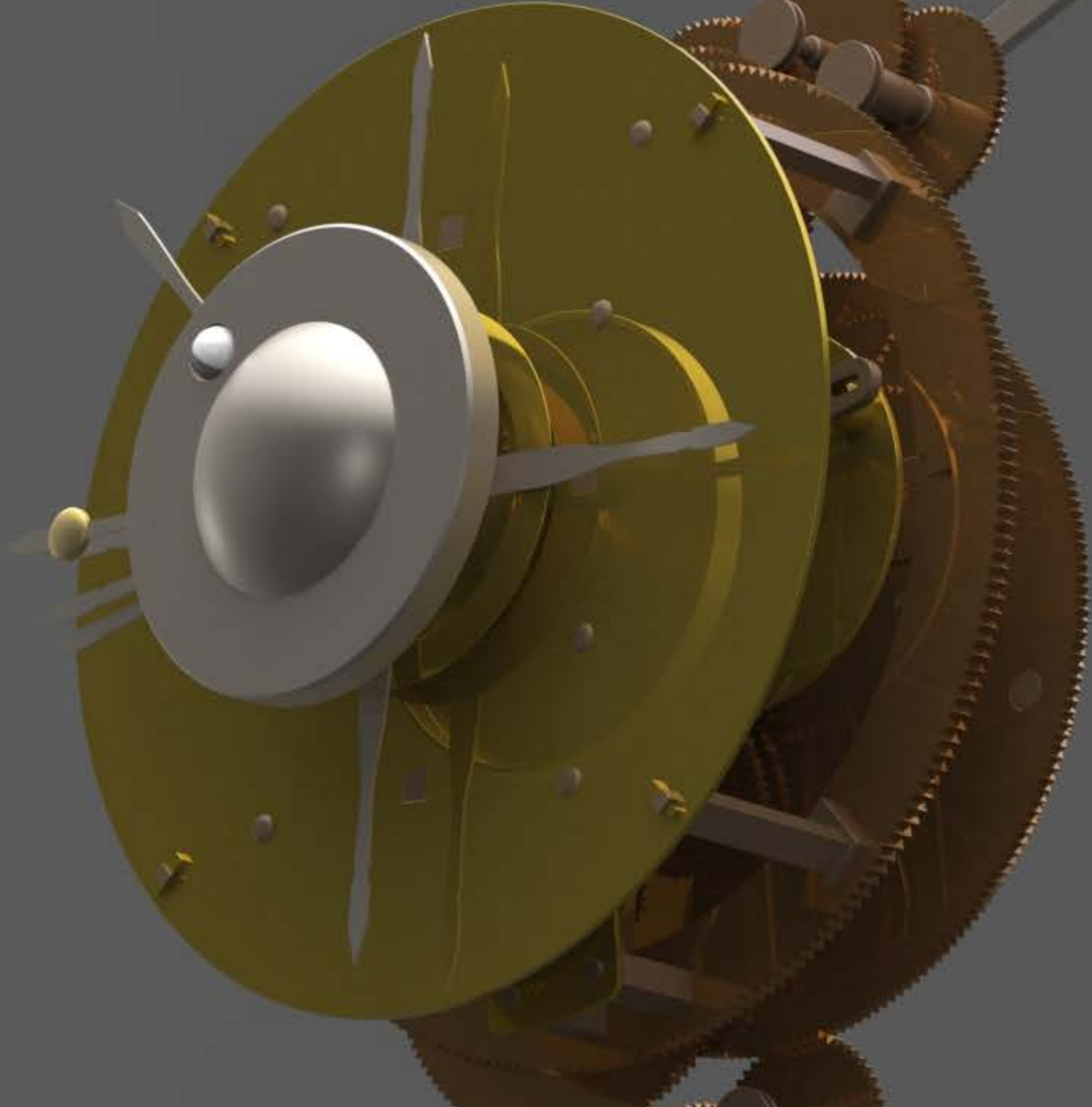
| | | | | | | | |
|----|---------------|-----|--------|---------|-------------------|---------------------------|-----|
| 13 | | | | | | | |
| 1 | | | | | | | |
| 43 | | 103 | 4693 | | | | |
| 44 | | 79 | 3479 | | | | |
| | | 89 | 6325 | | | | |
| | | 89 | 7905 | | | | |
| | 104414.186812 | 107 | 7429.0 | 11877.0 | 9.57724261936e-06 | [17, 19, 23]/[3, 37, 107] | [0] |
| .. | 103599.000001 | 37 | 162.0 | 259.0 | 9.65260282428e-06 | [2, 3, 3, 3, 3]/[7, 37] | [0] |











AMRP Reconstruction

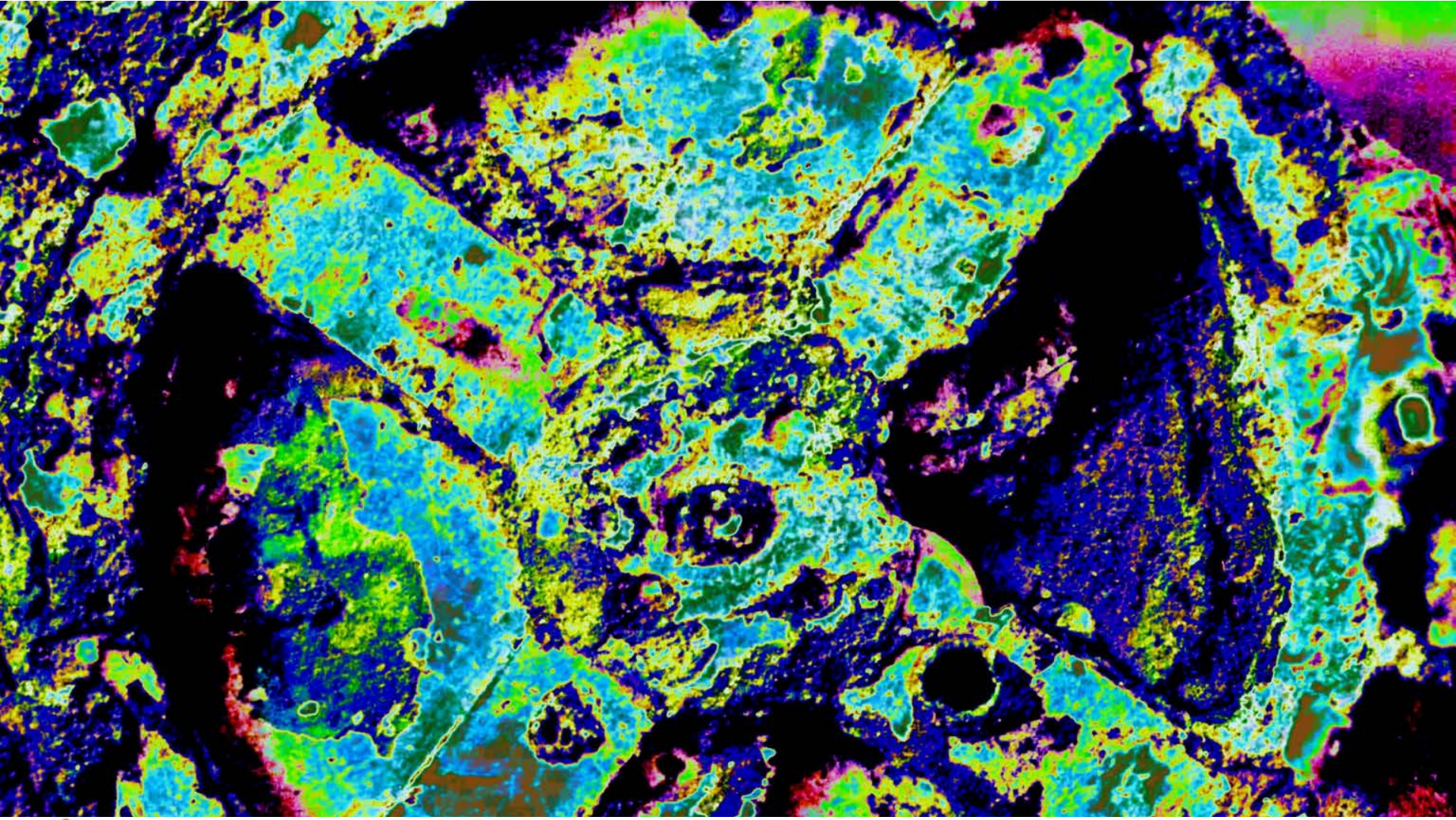
Gear Train Animation

A. Vicentini & M. Wright

Animation Vicentini & Wright



Rien van de Weijgaert



university of
groningen

faculty of mathematics
and natural sciences

kapteyn astronomical
institute

A black and white photograph of a person, likely a craftsman or instrument maker, working at a workbench in a workshop. The person is seen from the side, focused on their task. The workshop is filled with various tools, equipment, and materials, creating a sense of a busy, creative environment. The text "the Instrument Maker" is overlaid in the center of the image in a bold, white, serif font.

the Instrument Maker

Origin and Manufacture

? Where from ?

- Rhodos ...
- at the time one highly important and central port of the Mediterranean
- important centre of learning:
- ... Hipparchus 140-120 BC, Posidonius 92-51 BC, ...

? Who manufactured the machine ?

- the craftsman of the mechanism is (totally) unknown
- a highly skilled technician, the machine testimony of genius ...
- knowledge of astronomy (of the time) meticulous, state-of-the-art
- an advanced piece of equipment like the mechanism cannot be a stand-alone
- rather likely it is a representative of a gradual development ...

| | | | |
|----|----------------------|--|----------------------------|
| 1 | Archimedes | “De Sphaerae” | ca 260? -212 BC |
| 2 | Cicero | De Republica I,14 | 54-51 BC |
| 3A | Cicero | De natura deorum II, xxxiv | 45 BC |
| 3B | Cicero | De natura deorum II, xcvi | 45 BC |
| 4 | Cicero | Tusculan disputations I, 36 | ca 45 BC |
| 5 | Vitruvius | De Architectura 10.1.4 | ca 20 BC |
| 6 | Theon of Smyrna | Expositio rerum | ca 70 - ca 135 AD |
| 7 | Ptolomy | Almagest XIII, 2 | 120-150 AD |
| 8 | Ptolomy | Planetary Hypothesis | Mid 2 nd C AD |
| 9 | Galen | De Usu Partium | 169-176 AD |
| 10 | Sextus Empiricus | Adversus mathematicos, IX, 115 | 3 rd C AD |
| 11 | Pappus | Works VIII, 2 | 3 rd C AD |
| 12 | Agrestius Chromatius | quoted by St Sebastian and St Polycarp | 3 rd C AD |
| 13 | Lacantius | Institutiones divinae II, 5, 18 | 4 th C AD |
| 14 | Claudian | Carmina minora LI (LXVIII) | ca. 400 AD |
| 15 | Proclus | On Providence | ? 432 – ? 485 AD |
| 16 | Martianus Capella | De nuptiis | Early 5 th C AD |
| 17 | Nonus | Dionysiaca | 5 th C AD |
| 18 | John Philoponus | De Anima 106, 25 | 6 th C AD |
| | | | |
| 19 | Ovid | Fasti VI | 8 AD |
| 20 | Manilius | Astronomica Book 2, line 127 | 1 st C AD |
| 21 | Mesdomedes | (of Crete) Poem | Early 2 nd C AD |

Archimedes

Archimedes

(287-211/212 BC, Syracuse)

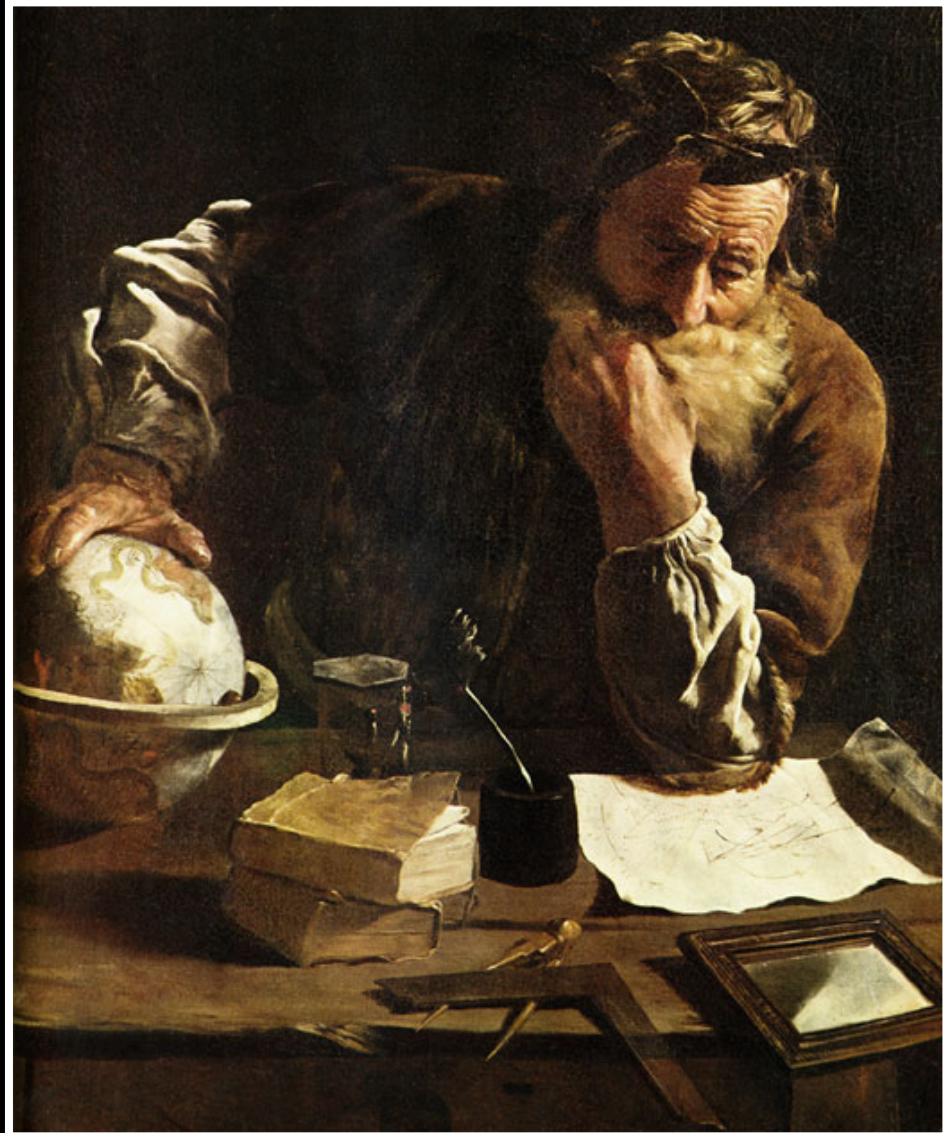
☐ Pappus of Alexandria:
Archimedes wrote book
“On Sphere-Making”

... is this the key ...

☐ Compare with

Archimedes Palimpsest:
... “On the Method” ...

Fundamentals Calculus,
Integral calculus ...



The discovery of Archimedes was all the more remarkable, because he had discovered how a single turn action could preserve these unequal orbits with their different speeds. When Galus moved this globe, the moon followed the sun by as many revolutions of the bronze globe as it does by days in the sky itself; the result was that the same eclipse of the sun occurred on the globe, and the moon fell into the space which was in the shadow of the earth. . .

learning began to explain the workings of this device, I decided that Archimedes had more genius than human nature seemed capable of possessing. Galus said that the invention of the other one, the solid globe, was old; it had been made by Thales of Miletus and then was marked out with the fixed celestial stars by Eudoxus of Cnidus, who he said was a pupil of Plato's. [...] But this new kind of globe included the motions of the sun and moon and the five stars that are known as "planets" or "wandering," something that not could be achieved in the solid globe. The discovery of Archimedes was all the more remarkable, because he had discovered how a single turn action could preserve these unequal orbits with their different speeds. When Galus moved this globe, the moon followed the sun by as many revolutions of the bronze globe as it does by days in the sky itself; the result was that the same eclipse of the sun occurred on the globe, and the moon the fell into the space which was in the shadow of the earth. . . (translation: Zetzel, 1999)

Cicero mentions two similar machines

For when Archimedes fastened on a globe the movements of moon, sun and five wandering stars, he, just like Plato's God who built the world in the "Timaeus", made one revolution of the sphere control several movements utterly unlike in slowness and speed. Now if in this world of ours phenomena cannot take place without the act of God, neither could Archimedes have reproduced the same movements upon a globe without divine genius''

Cicero,

Tusculan Disputations, Book I, Section XXV



**Plato Academy
Mosaic**

**Villa T. Siminius
Pompeii**



**Is this
Archimedes Sphere ?**

**Is this
an image of an
Antikythera Mechanism
related device ?**



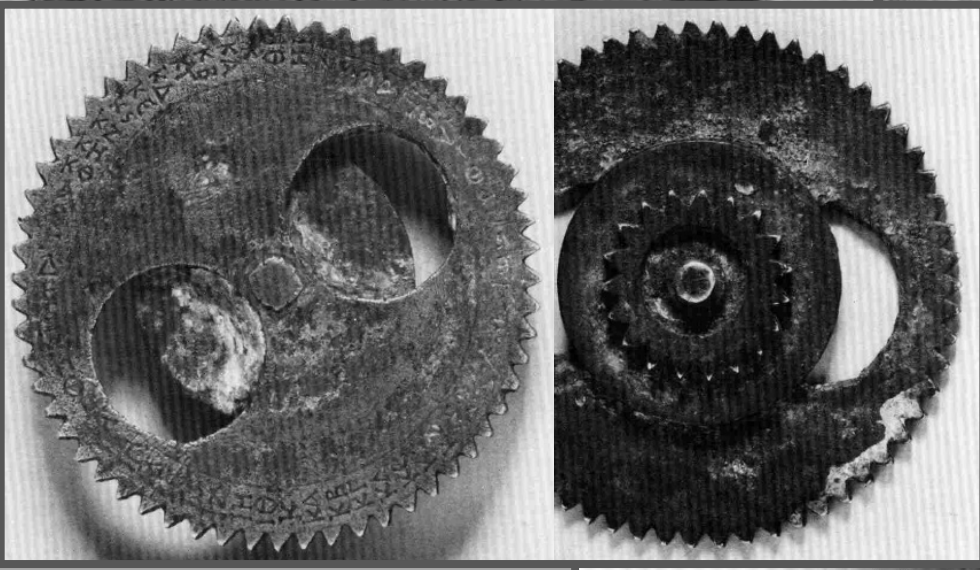
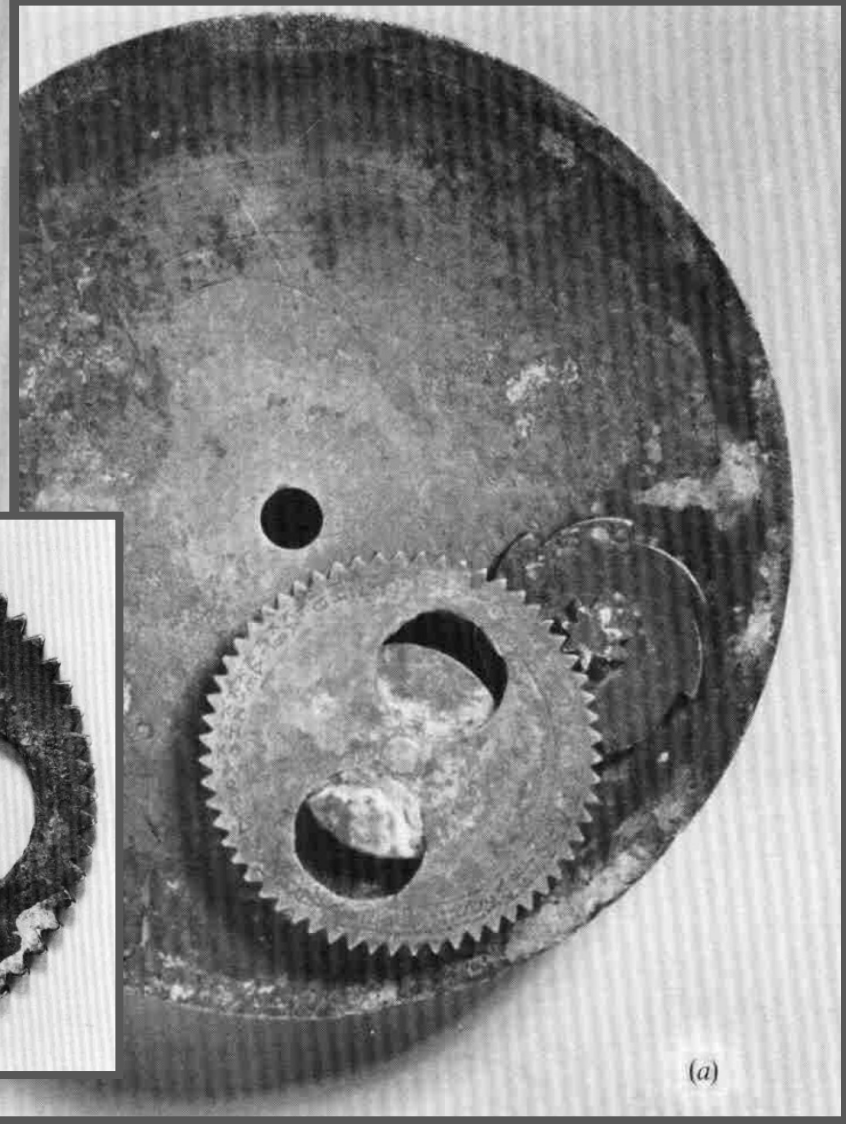
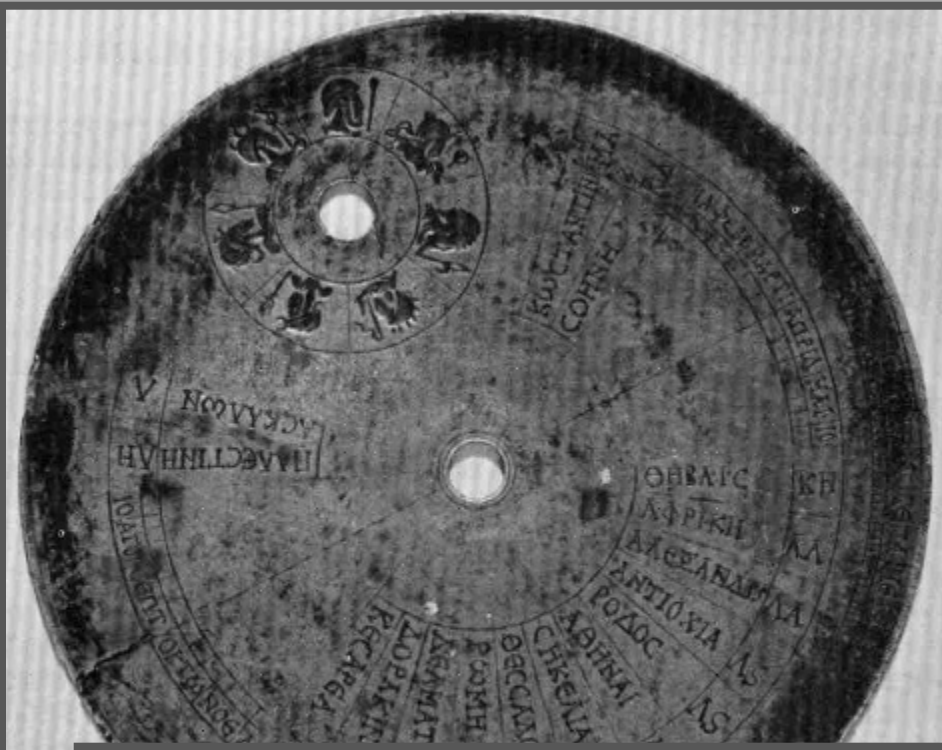
**ancient Technology,
Science & Innovation**

Roman Military Technology



Byzantine "Astrolabe"

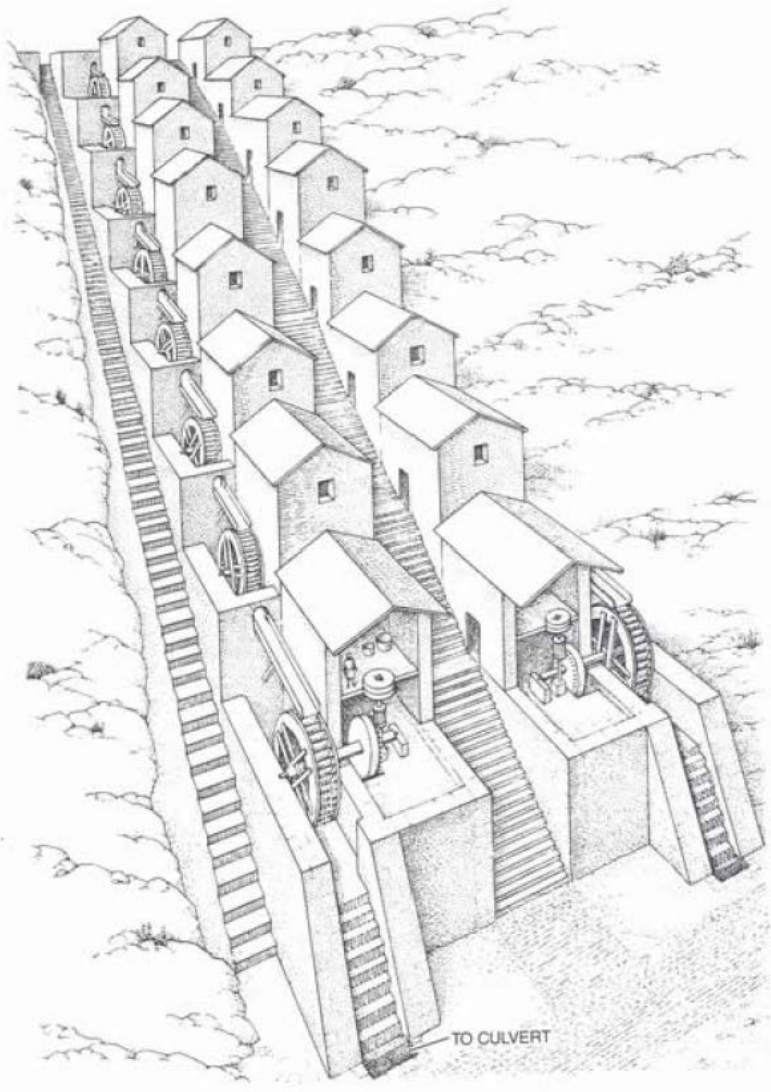
6th cent. AD !!!



(a)

Roman Industry

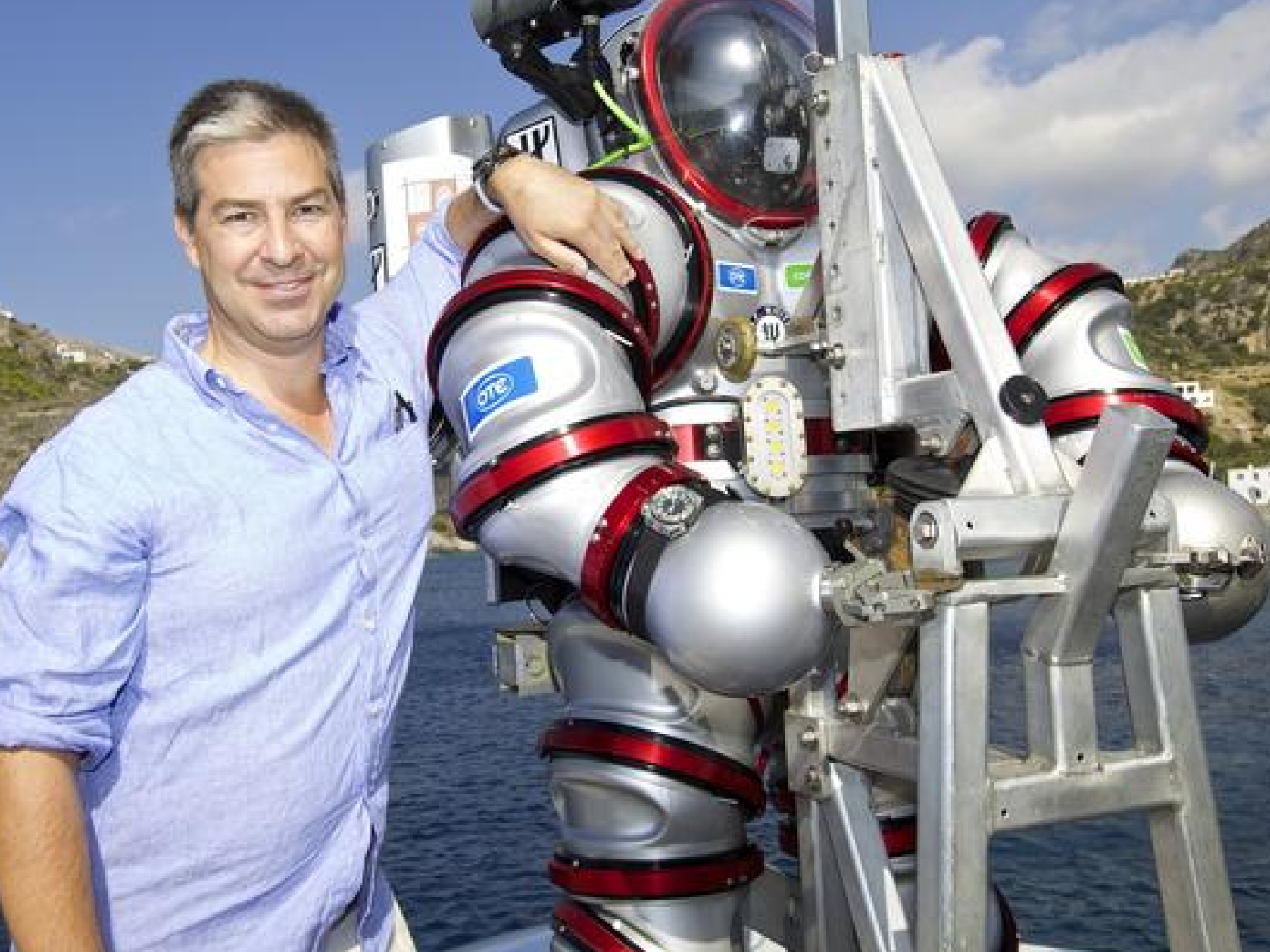
Did hightech find applications
In industrial activities ?



Barbegal:
industrial complex of watermills

A diver in full scuba gear is positioned in the center of the frame, viewed from above. The diver is holding a large, professional video camera with a prominent lens. The background is a deep blue underwater environment. The entire scene is framed by a white border.

Return to Antikythera



2013: Antikythera Ship Anchor



Images courtesy: Ephorate of Underwater Antiquities

2014: 200 kg 1.4 metre long anchor (by far largest known from antiquity)

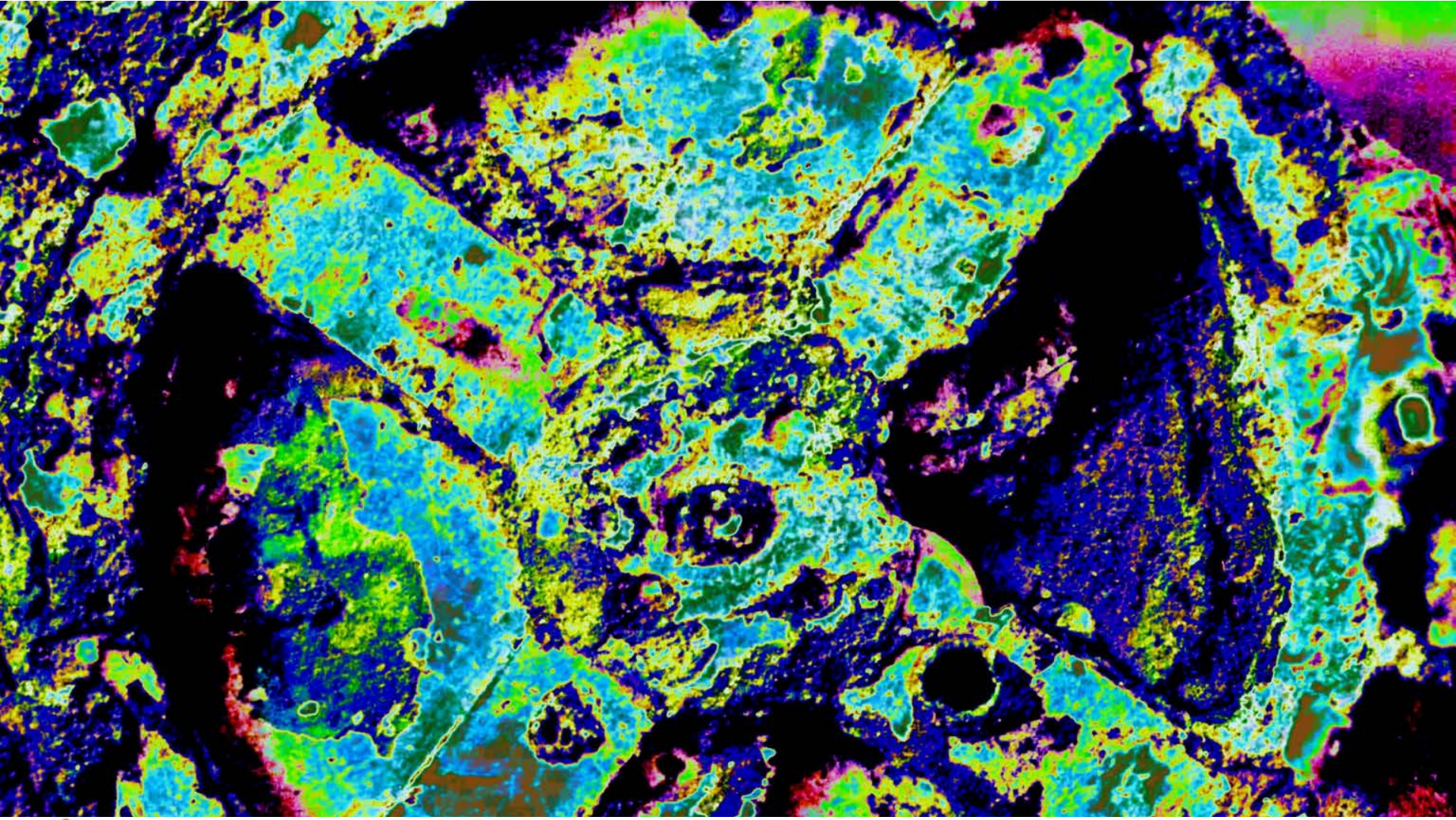
Titanic of the Ancient World ?



2014 diving expedition:

- Ceramics, amphorae, anchor 200 metres away from 1901 position.
- 2 Olkas cargo ships, or ...
- a giant 200 m giant grain ship ... (capacity, up to 1000 tons)

Rien van de Weijgaert



university of
groningen

faculty of mathematics
and natural sciences

kapteyn astronomical
institute